ANSWERS TO SELECTED EXERCISES FROM
"INTRODUCTION TO GAME THEORY & THE BERTRAND TRAP"

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		Co-Worker: relaxed	Co-Worker: hyper
You	relaxed	$\frac{P}{2}$, $\frac{P}{2}$	0, $P-H$
	hyper	$P-H$, 0	$\frac{P}{2}-H$, $\frac{P}{2}-H$

Figure 1: The Game of Question 2 in Normal Form

2. (a) See Figure 1.
   (b) If $P/2 < H$, then $P - H < P/2$, hence if one worker will play *relaxed*, the best response for the other worker is *relaxed*. If one worker will play *hyper*, the best response for the other worker is *relaxed*. Therefore, *relaxed* is a dominant strategy.
   (c) The analysis of the last part is reversed if $P/2 > H$; now *hyper* is a best response to both possible strategies of the other worker; hence, *hyper* is now the dominant strategy.
   (d) If $P/2 > H$, the equilibrium is that both workers are *hyper*. But collectively they would be better off if they were both *relaxed*. They are, thus, in a Prisoners' Dilemma (*i.e.,* they are rat-racing against each other).
   (e) Your payoff is
   $$\left(\frac{1}{2} + \frac{w_y - w_r}{2}\right) P - H w_y \,.$$
   Your marginal benefit of more work is, thus, $P/2$, and your marginal cost is H. If $P/2 > H$, you choose the maximum w_y regardless of your co-worker's strategy. If $P/2 < H$, you choose the minimum w_y regardless of your co-worker's strategy. The game is symmetric, so the Nash equilibrium is maximum effort from both parties if $P/2 > H$ and the Nash equilibrium is minimum effort from both parties if $P/2 < H$.

3. Let p be the probability that Column plays *Left*. Expected payoff from *Up* is, therefore, $6(1-p)$. Expected payoff from *Middle* is $3p + 7(1-p)$.

Row	Column: Left	Column: Right
Up	8, 5	2, 4
Middle	5, 4	6, 3

Figure 2: The Game of Figure 7 with *Down* deleted.

Row	Column: Left
Up	8, 5
Middle	5, 4

Figure 3: Game of Figure 2 with *Right* "pruned" (deleted).

Clearly, $3p + 7(1 - p) > 6(1 - p)$ for all p. Hence, *Middle* dominates *Up*, which means *Up* can never be a best response.

4. Let p be the probability that Column plays *Left*. Expected payoff from *Down* is, therefore, 4. Expected payoff from *Middle* is $5p + 6(1 - p)$. Clearly the latter value is greater than 4 for all p. That is, *Middle* strictly dominates *Down*.

5. (a) *Right* is a best response to *Down*, while *Left* is a best response to either *Up* or *Middle*. Thus Column has no dominant strategy and, therefore, the game cannot have a solution in dominant strategies.

   (b) As noted in Exercise 4, *Down* is a dominat*ed* strategy for Row. So deleting it, we're left with the game in Figure 2. In the "pruned" game — that is, the game in Figure 2 — *Left* is a dominant strategy for Column, which means *Right* is a dominated strategy and can, thus, be deleted. Doing so, leaves the game of Figure 3. In this game, *Up* dominates *Middle*; or, conversely, *Middle* is dominated and should, thus, be deleted. This leaves just the top cell of Figure 3. Hence the solution in IDSDS is Row plays *Up* and Column plays *Left*.

   (c) *Up* is a best response to *Left* (8 beats 5 or 4). *Left* is a best response to *Up* (5 beats 4). So *Up* and *Left* are mutual best responses; that is, they are a Nash equilibrium.

6. (a) Regardless of rival's price, there is at least a demand of 50 for a given firm if it prices at \$5 or below. Given that the given firm can sell

no more than 50, this means that regardless of its rival's price, it sells 50 units if its price is \$5 or less and 0 units if its price exceeds \$5. Clearly, the best course of action is to set price at \$5. Similar reasoning for the rival shows it, too, would do best to charge \$5. Therefore both charging a price of \$5 is a Nash equilibrium.

(b) Let $p_r \geq 1$ be the price a firm anticipates it rival will charge. Its profit as a function of its own price, p_o, is

$$\pi_o = \begin{cases} 0\,, & \text{if } p_o > p_r \\ 50(p_o - 1)\,, & \text{if } p_o = p_r \\ 100(p_o - 1)\,, & \text{if } p_o < p_r \end{cases} .$$

Clearly, the best response is, thus, $p_o = p_r - \varepsilon$, ε an arbitrarily small positive number, if $p_r > 1$. A best response is $p_o = p_r$ if $p_r = 1$. Given this, the only mutual best responses are $p_o = p_r = 1$.

(c) Call the two firms B and s for big and small, respectively. Capacity of B is 100 and capacity of s is 50. As a function of p_B, the best response for s is

$$p_s = \begin{cases} 5\,, & \text{if } p_B > 5 \\ p_B - \varepsilon\,, & \text{if } 5 \geq p_B > 1 \\ \geq 1\,, & \text{if } p_B = 1 \end{cases} .$$

As a function of p_s, the best response for B is

$$p_B = \begin{cases} 5\,, & \text{if } p_s \leq 3 \text{ or } p_s > 5 \\ p_s - \varepsilon\,, & \text{if } 3 < p_s \leq 5 \end{cases} .$$

It should be clear that there are *no* mutual best responses in pure strategies given these best-response functions.

(d) Limiting capacity is a way to avoid the Bertrand trap.

7. See Figure 4.

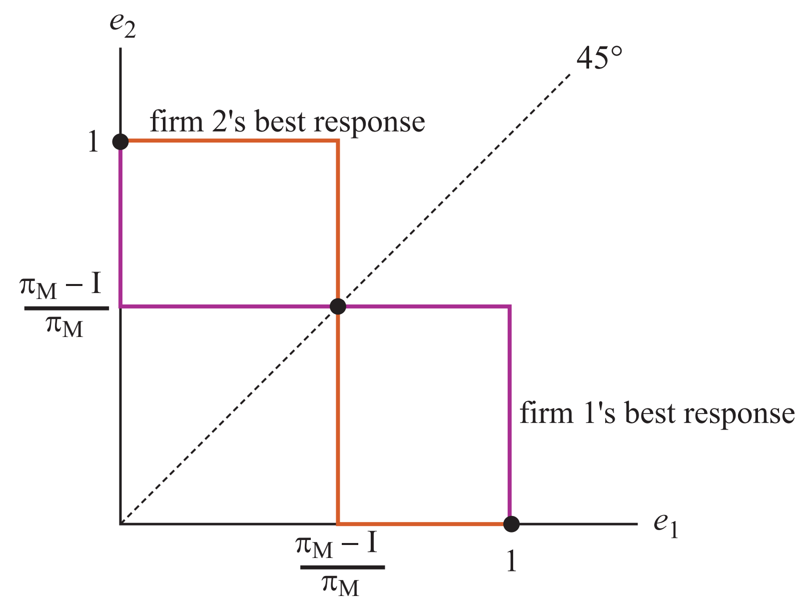

Figure 4: The three equilibria of the entry game from Page 16 of "Introduction to Game Theory & the Bertrand Trap." The equilibria are denoted by large dots. The best-response correspondence for firm 2 is the orange curve. The best-response correspondence for firm 1 is the purple curve.