Exercises on Cournot, Tacit Collusion, and Entry Deterrence

MBA 299 — Spring 2003

1. Consider an industry in which market demand is \(2,000,000 - 50,000p\). There are two identical competitors, each of whom has a constant marginal cost of $10 per unit. The firms are Cournot competitors.

   (a) What is the Cournot equilibrium in terms of the quantities (capacities) of the two firms?

   (b) What is the profit of each firm in this equilibrium?

   (c) Suppose the two firms merged to form a monopoly. What would the monopoly’s profit be?

2. Consider an industry satisfying the assumptions of the Bertrand model. Suppose there are four firms in this industry. Assume market demand is \(D\) if \(p \leq v\) and 0 otherwise (i.e., there are \(D\) consumers, each of whom wishes to buy one unit if the price does not exceed \(v\)). Let \(c\) be the marginal cost. Show that if the common discount factor among the firms, \(\delta\), is .9, then the four firms can sustain tacit collusion at the monopoly price.

3. Same assumptions as the previous problem, except now the good in question has a very seasonal demand (e.g., it’s ice cream or gasoline). In particular, in the summer season (one of four seasons), demand is \(10D\) if \(p \leq v\). For the other three seasons it is \(D\) if \(p \leq v\). Assume that price is set simultaneously by the four firms at the beginning of each season and cannot be changed during the season. That is, a season is a period. As before, the discount factor across periods is .9. Some useful formulae are:

   • The discounting formula for every fourth period (i.e., summer) when the next fourth period is \(i\) periods away:
     \[
     \delta^i + \delta^{i+4} + \delta^{i+2\times4} + \cdots = \frac{\delta^i}{1 - \delta^4}.
     \]

   • The discounting formula when every fourth period is skipped starting at period \(i\) when the fourth period is \(j\) periods later:
     \[
     \delta^i + \delta^{i+1} + \text{skipped} + \delta^{i+3} + \cdots = \frac{\delta^i}{1 - \delta} - \frac{\delta^{i+j}}{1 - \delta^4}.
     \]

   (a) Show that if the current period is summer, then tacit collusion at \(p = v\) is not sustainable (i.e., firms do better to undercut than tacitly collude).
(b) Show that the firms can sustain tacit collusion if their prices are such that

\[ p_{\text{summer}} = \frac{v + 9c}{10} < v = p_{\text{not summer}}. \]

*Hint:* How would this make the situation like Problem 2?

(c) What does the last result suggest about a motive for seasonal sales?

(d) Suppose \( v = 100 \) and \( c = 0 \). If \( p_{\text{not summer}} = 100 \), what is the largest value of \( p_{\text{summer}} \) that is consistent with tacit collusion?

4. Consider an incumbent monopoly that is concerned about entry. Today, it chooses its price, \( p \). Provided \( p \leq v \), demand is \( D \). If \( p > v \), then demand is 0. It, of course, knows its marginal cost, \( c \). A potential entrant, however, knows only that \( c \) can take any value between 0 and \( 3v/4 \) with equal probability (i.e., \( c \) is distributed uniformly on the interval \( [0, 3v/4] \)). After observing the incumbent’s price in the first period, an entrant decides to enter or not. Because the entrant builds a modern factory, its marginal cost is 0. However, entry costs it \( K < 3Dv/8 \). If the entrant enters, then it learns the incumbent’s cost and prices just under that cost to steal the entire market (yes, this is a bit unrealistic, but making it more realistic won’t add much to the lesson to be learned). If the entrant stays out, then the incumbent is free to price as it likes in the second period. There are only two periods. Ignore discounting across periods (i.e., set \( \delta = 1 \)).

(a) Suppose the entrant stays out. What price should the incumbent charge in the *second* period? What are its second-period profits?

(b) If the entrant knew \( c \) prior to entry, for what values of \( c \) would it enter? For what values would it stay out?

(c) If the incumbent always charged \( p = v \) (the monopoly price) in the first period, regardless of \( c \), what would the entrant do if it were seeking to maximize its expected profit?

(d) In light of your answers to the last three parts, show that if \( c \leq K/D \) it pays for the monopolist to price at \( K/D \) if that will convince the entrant to stay out; but that it should price at \( v \) if \( c > K/D \) even if that will trigger entry.

(e) What does all this suggest about limit pricing as a means of deterring entry?