Hermalin Consulting
"The Nerd in the Know"™

To: Mr. William Jaeger
From: Benjamin E. Hermalin, Ph.D.
Re: Harvesting Decision

I have analyzed your harvesting decision problem. Figure 1 represents your problem in terms of a decision tree.

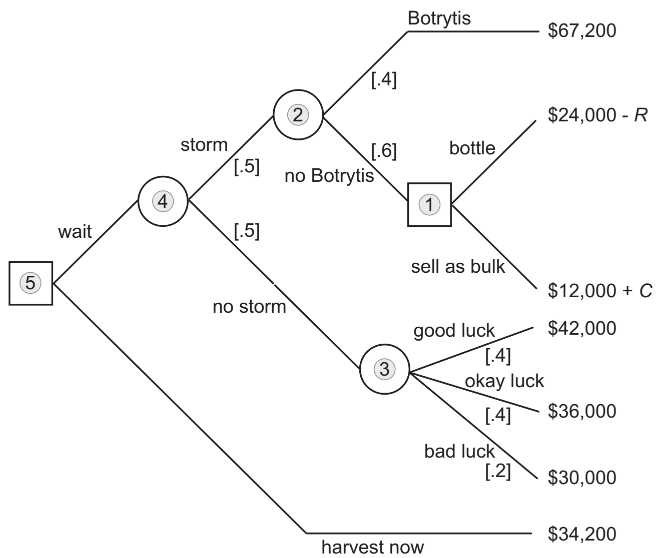

Figure 1: The harvesting decision represented as a decision tree.

In Figure 1, R denotes the monetary value of the loss of reputation you would suffer from bottling an inferior wine, while C denotes the cost saving from not bottling your own wine, should you choose to sell in bulk.

Assuming the information you gave me is correct (more on this later), R and C are irrelevant to the problem. Given the information you provided, it seems safe to assume that $C < \$15,000$.* Even assuming the worst, $R \geq \$12,000$ and $C = \$0$, the option of waiting has a greater expected value than the decision to harvest now: \$35,640 versus \$34,200.† If the worst is not true (*i.e.,* $R < \$12,000$ or $C > \$0$), the the option of waiting is even more valuable. Consequently, unless you strongly dislike risk, you should wait to harvest. Your *expected gain* from waiting is at least 4% (worst-case scenario). If the very best-case scenario in which the node (labeled as ①) is worth \$28,200 (see Technical Appendix for underlying calculations), your expected gain would be at least 18%.

Because your winery is a partnership rather than a stock firm, I assume that risk is an issue for you. On the other hand, Riesling represents only 4% of your

*Otherwise, you not find it profitable to produce a \$2.50 bottle of wine should the storm not occur and the acidity drops below 0.7%.

†I provide details in the attached technical appendix.

business, which suggests that this particular risk would not be very significant to you. Only a full evaluation of your attitudes toward risk—which I could conduct for an additional fee—would reveal whether this risk would be acceptable.

Finally, there is the accuracy of the numbers you gave me. I assume I can trust the prices you gave me. The analysis is largely independent of the storm probabilities. The analysis is, however, sensitive to the probability you gave me for the mold forming. If this probability were actually 34.8% or lower, then the expected-value maximizing decision would be to harvest now in the worse-case scenario.

Technical Appendix

Basic Analysis

The decision tree shown in Figure 1 is read left to right. That is, it starts at the decision node labeled ⑤. The tree is solved backwards.

Starting at the decision node labeled ①, you will (should) bottle provided $\$24,000 - R > \$12,000 + C$. The \$24,000 is calculated as number of bottles, $12,000 = 12 \times 1000$, times your estimate of a price of \$2.00/bottle. Not being a vintner myself, I was unsure of how to account for the 5 to 10% swelling of the grapes, which lessened their concentration. If that means you get 5 to 10% more wine, then the \$24,000 should be, at most, \$26,400. The \$12,000 figure comes from your estimate that grapes sold as bulk will yield only half as much revenue as grapes used in wine production. If the swelling effect also pertains to the grapes sold as bulk, then this number could be as large as \$13,200. The value of node ① is thus at least \$12,000 (if $R \geq \$12,000$, $C = \$0$, and the grape swelling yields no more wine). As noted, from the data provided, I feel I can assume the maximum possible cost savings from selling the wine as bulk is \$15,000. Hence, the maximum value of node ① is \$28,200.

The value if the Botrytis forms is 70% of the 12,000 bottles (allowing for the 30% reduction in juice) times the \$8/bottle price. That number is \$67,200.

This makes the value of the chance node labeled ② equal to somewhere between

$$\$34,080 = .6 \times \$12,000 + .4 \times \$67,200$$

and

$$\$43,800 = .6 \times \$28,200 + .4 \times \$67,200\,.$$

Node ③ is worth the expected value at that chance node. The payoffs at the end of each branch from that node are calculated as 12,000 bottles times estimated wholesale price. I considered three scenarios: good luck with sugar at 25%, which yields a wholesale price of \$3.50; okay luck with sugar at 20%, which yields a wholesale price of \$3; and bad luck when acidity fails to reach 0.7%, which yields a wholesale price of \$2.50. You estimated the likelihood of this last scenario as .2 and thought the other scenarios equally likely. Because probabilities sum to one, this means these other two scenarios each have a probability of .4 (= (1 − .2)/2). The expected value of node ③ is, therefore,

$$\$37,200 = .4 \times \$42,000 + .4 \times \$36,000 + .2 \times \$30,000\,.$$

You estimate the likelihood of the storm hitting as 50%. Hence, the expected value at node ④ is between

$$\$35,640 = .5 \times \underbrace{\$34,080}_{\text{min. EV ②}} + .5 \times \$37,200$$

and

$$\$40,500 = .5 \times \underbrace{\$43,800}_{\text{max. EV ②}} + .5 \times \$37,200\,.$$

Observe that any value in the range for the expected value of node ④ exceeds, \$34,200,[‡] the payoff from harvesting immediately. Hence, the reason for my concluding that you should wait.

Sensitivity Analysis

As the above analysis indicates, the conclusions are not sensitive to the values of C and R provided the other data are accurate.

I analyzed the sensitivity of this conclusion to the probability that a storm hits. Taking the other data as correct, the storm probabilities must be irrelevant if the values for nodes ② and ③ are *each* greater than \$34,200. The expected value calculated over two values that each exceed a third cannot be less than this third value. Node ③ is always greater than \$34,200. Hence, the only question is when is node ② less than \$34,200? Set the adjustment (if any) for the swelling of the grapes should they not be attacked by the botrytis mold to zero. (If it is 5%, then node ② will always be greater than \$34,200 regardless of the values of R and C.) Recall that the value of node ② is

$$EV_2 = .4 \times \$67,200 + .6 \times V_1\,,$$

where EV_2 is the expected value of node ② and V_1 is the value of node ①. Hence, if $EV_2 \leq \$34,200$, then

$$\$34,200 \geq .4 \times \$67,200 + .6 \times V_1\,.$$

[‡] 12,000 bottles times \$2.85/bottle.

This is equivalent to

$$\$7,320 \geq .6 \times V_1\,,$$

which, in turn, is equivalent to

$$\$12,200 \geq V_1\,.$$

This last expression can be satisfied only if $R \geq \$11,800$ and $C \leq 200$. This leads me to conclude that it is unlikely that the analysis is sensitive to the estimates of the storm probabilities.

In the worse case scenario, $R \geq \$12,000$ and $C = 0$, then $EV_2 = \$34,080$. If p is the probability of a storm, then wait would still be the best advice provided

$$\$34,080p + \$37,200(1-p) > \$34,200\,,$$

which holds provided $p < 25/26 \approx .96$. I think it unlikely that your estimate of the storm probabilities could be that far off.

While errors in the storm probabilities are highly unlikely to affect the conclusion that wait is the right course of action, the same cannot be said of the botrytis probabilities. Under the worse-case scenario ($R \geq \$12,000$ and $C = \$0$), the expected value of node ④ is given by

$$EV_4 = .5 \times \$37,200 + .5 \times \big(\$67,200b + \$12,000(1-b)\big)\,,$$

where b is the probability that the botrytis forms. The expectation EV_4 must exceed \$34,200 to make waiting the right strategy; that is,

$$.5 \times \$37,200 + .5 \times \big(\$67,200b + \$12,000(1-b)\big) > \$34,200\,.$$

This occurs only if $b > 8/23 \approx .348$. In the best-case scenario ($C = 15,000$ and swell factor is 10%), we need

$$.5 \times \$37,200 + .5 \times \big(\$67,200b + \$28,200(1-b)\big) > \$34,200\,,$$

or $b > 1/13 \approx .077$. To the extent the worse-case scenario is the more accurate, then this says that the analysis is very sensitive to your estimate that there is a 40% chance that the botrytis mold will form in the event of a storm.

A final point is that it could be the case that selling a botrytised Riesling adds to your brand value. If B is the dollar value of this brand enhancement, then the value of node ④ increases by $.02B$.

If you would like to conduct your own sensitivity analysis, I supply an Excel spreadsheet for you to use.