Sample Examination for MBA 201A (with Answers)

Instructions

- Please answer four of the following six questions.[1] Each question counts for 250 points. Do *not* answer more than four; if you do, your grade will be based on your four worst answers.
- Please make sure it is clear which questions you are answering and which answer goes with which question.
- Where you are asked to come up with either a numerical or algebraic answer, please put a box around your answer $\boxed{\text{like this.}}$
- Where you are asked for an essay (open-ended) question, please be as brief as possible in your answer. Remember a quality of good writing, especially in business, is brevity.
- If you use graphs in your answer, please make sure to label all components clearly *including the axes*.
- Please write as legibly as possible. If I can't read your answer, I will have no choice but to give you a poor mark for it.

[1] The real midterm may not offer you choice. For practice do each problem. See if you can do each problem in 30 minutes or less.

Question 1: Your firm, Dynamerica, Inc., believes that another firm, Munchausen Industries, has infringed on one of your firm's patents. You are contemplating suing Munchausen for patent infringement. The cost of a lawsuit (legal fees) is estimated to be \$10,000,000. If you win the suit, you will receive \$100,000,000 in compensation from Munchausen for the damages you have suffered. You will also be compensated (reimbursed) by Munchausen your legal fees. If you lose the suit, then you receive nothing from Munchausen. In addition, if you lose the suit or don't sue, then you will lose \$50 million in business to Munchausen. If you win the suit, you keep the \$50 million. Whether or not you will prevail in your suit is uncertain.

(a) Draw a decision tree for your problem. Be sure to label it clearly, including the payoffs.

Answer: See Figure 1.

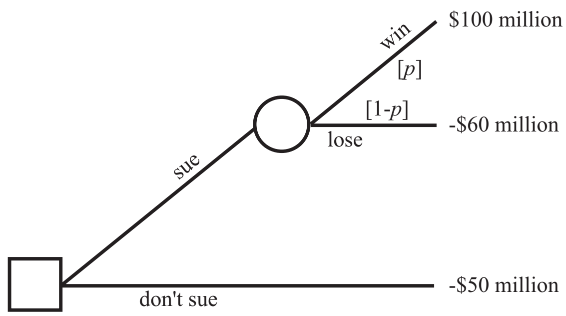

Figure 1: Tree for Problem 1(a).

(b) Let p denote the probability that you prevail in court (win the suit). For what range of values of p does it make sense to file the suit? (You are risk neutral.)

Answer: The expected value of suing (in millions of dollars) is

$$100p + (-60)(1-p) = 160p - 60\,.$$

Your payoff if you don't sue is -50 million dollars. Hence you should sue if

$$160p - 60 \geq -50\,;$$

hence, you should $\boxed{\text{sue if } p \geq 1/16}$.

Suppose you currently have a suit concerning the same patent ongoing against Adrubbya Manufacturing. The court in that case can render two rulings. One, it can rule against you (*i.e.,* rule in Adrubbya's favor). Such a ruling weakens your case against Munchausen, but such that you still might prevail. Your legal team estimates that the probability you would then win against Munchausen at 1/32. Two, it can rule in your favor. Such a ruling greatly strengthens your case against Munchausen. Your legal team estimates that the probability that you would then win against Munchausen at 22/32. Suppose that the Adrubbya-case ruling will come out any day now. Suppose too that the cost of delaying filing the Munchausen suit until the verdict in the Adrubbya case is learned is relatively small, on the order of a few hundred dollars. Assume that your legal team informs you that *right now* (*i.e.,* prior to the Adrubbya verdict), Dynamerica's chance of winning against Munchausen is 25% (*i.e.,* $p = 1/4$). Your team also estimates the probability of winning against Adrubbya to be 1/3.

(c) Draw a tree to reflect this new decision problem.

Answer: See Figure 2.

(d) Without doing any calculations (beyond those done in parts a–c), is the information gained by waiting to see the Adrubbya verdict valuable information? How do you know?

Answer: Yes, it is valuable. You know this because a bad verdict pushes your probability of winning against Munchausen below 1/16, the cutoff value found in (b), so you won't sue Munchausen if you lose against Adrubbya. A good verdict gives you a probability of winning against Munchausen that exceeds 1/16. Hence, the information has the potential to affect your decision. It is, therefore, valuable by the fundamental rule of information.

(e) It suddenly occurs to you to wonder if the probabilities supplied by your legal team are internally consistent. Are they?

Answer: The prior probability of winning against Munchausen is 1/4. Hence, the probability of winning *before* you learn the Adrubbya verdict must also be 1/4. You can win against Munchausen along two paths: (i) lose against Adrubbya and win against Munchausen; and (ii) win against both. The sum of the probabilities of the two paths has to equal 1/4 for the probabilities to be internally consistent. The probability of path (i) is $(1 - 1/3) \times (1/32) = 2/96$. The probability of path (ii) is $(1/3) \times (22/32) = 22/96$. The sum is $24/96 = 1/4$. The probabilities are internally consistent.

Question 2: You are stationed overseas in a non-North American country that is currently having an election. There are two parties in this country, the KE party and the BC party. The company that you represent is seeking part of a valuable contract that the country will issue after the election. At the moment, preliminary discussions have indicated that your company will get 25% of the contract. The entire contract is worth \$100 million. It is legal under the laws of this country for your company to donate money to these political parties. Your company provides you \$75,000 to use for this purpose. As with politics

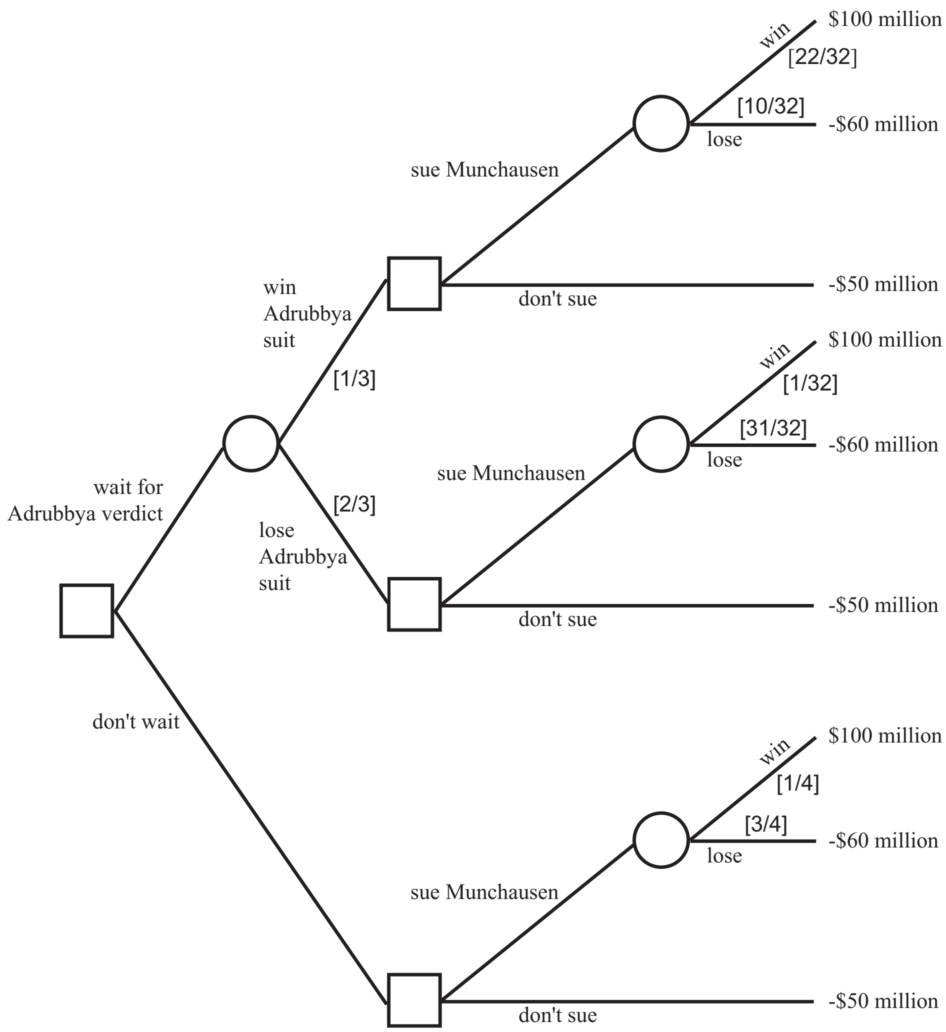

Figure 2: Tree for Problem 1(c).

everywhere, there is a *quid pro quo*. Specifically, for every thousand dollars you donate to the KE party, your percentage of the contract goes up by one percentage point from the preliminary 25% if KE wins (*e.g.,* if you donate \$3000, then you will have 28% of the contract if KE wins). Of course donations to KE displease the BC party, so it will reduce your percentage of the contract (from the 25%), should it win, by 1/2 percentage point for every thousand dollars donated to the KE party. Conversely, you can also donate to the BC party (you can, in fact, make donations to both). Every thousand dollars to the BC party means an increase of one percentage point should BC win (but BC will also penalize you if you've also made donations to KE, so the figure is the gross increase). The KE party is cooler than the BC party, so it doesn't penalize you for donations to the BC party if it wins.

The company for which you works informs you that it wants as big a percentage as possible of that contract. But, for reasons not made clear to you, you are to avoid any risk; that is, regardless of which party wins the election, your company's percentage should be the same.

At the time you must make your donations, the book makers are predicting that KE will win with probability .4 and, thus, that BC will win with probability .6.

How much do you donate to each party if you want to satisfy your company's requirements?

Answer: You are trying to come up with the optimal riskless portfolio (*i.e.,* this is like the ice-cream-company-umbrella-company example in the lecture notes). You need your payoff (in terms of percentage of the contract) to be as large as possible in the two states and equal. If KE wins, your payoff (in percentage terms) is $25 + k$, where k is your donation (in thousands) to the KE party. If BC wins, your payoff is $25 + b - k/2$, where b is your donation (in thousands) to the BC party. These two amounts must be equal, so

$$\begin{aligned} 25 + k &= 25 + b - k/2\,;\ \text{hence,} \\ b &= 3k/2\,. \end{aligned}$$

In addition to that equation, you have the requirement that $b + k = 75$. So we have $5k/2 = 75$ or $k = 30$. Hence, $b = 45$.

$\boxed{\text{Answer: donate \$30,000 to the KE party and \$45,000 to the BC party}}$.

Question 3: Spacely Sprockets, Inc., makes two products, A103 gears and B100 gears. The two kinds of gear each require the same amount of raw material per gear and the same amount of direct labor. In addition, regardless of type, every gear produced uses eight seconds of Spacely's one gear-stamping machine, which Spacely owns outright. The machine can stamp out only one gear at a time. The time and resources spent switching the stamping machine from one type of gear to the other are negligible. The wear and tear on the machine is negligible over the course of a year. Because, however, there is steady innovation in stamping machine technology, the resale value of the machine is assumed to drop 15% annually. The accounting system assigns a total cost per year of \$100,000 for using the machine. This is the only overhead charge and it is allocated between the two gears based on the percentage of direct labor-hours used for each type of gear. The plant currently produces 250,000 A103 gears and 750,000 B100 gears per year. The accounting system currently calculates the annual cost of the A103 line to be $275,000 and the annual cost of the B100 line to be $825,000.

(a) What is the best estimate of the marginal cost of an A103 gear? The marginal cost of a B100 gear? Why?

Answer: The first step in calculating marginal cost is to separate variable costs from overhead costs. Because the value of the machine does not vary with production *per se* (wear and tear is negligible), there is no variable component to $100,000 charge for the machine. Because the accounting system allocates the only overhead, the machine cost, based on production levels, we can easily extract it from the $275,000 and $825,000 currently allocated to the A103 and B100, respectively. Current production levels have 75% of production in B100 gears, therefore 75% of $100,000, or $75,000, is allocated to the B100 line. Similar calculations reveal that $25,000 is allocated to the A103 line. Subtracting these figures from the annual cost numbers, we find total variable costs for both lines. These are: $750,000 for the B100 and $250,000 for the A103.

Because we are told that each gear takes the same amount of resources to produce, we can assume that marginal costs are constant. Since we are producing 750,000 B100 gears at a total variable cost of $750,000, Spacely's $\boxed{\text{marginal cost of a B100 gear is \$1/gear}}$. We find the same answer for the A103 gear.

(b) Suppose that the firm can earn a return of 5% annually on capital invested in the market. If the accountants have calculated the $100,000 figure correctly (*i.e.,* consistent with what economics dictates), what is the current market value of the gear-stamping machine?

Answer: Recall that

$$\text{capital cost} = rV_0 + (V_0 - V_1)\,,$$

where r is the market rate of return on capital, V_0 is the current market value, and V_1 is the future market value. We know that $V_1 = .85V_0$ (because the market value decreases by 15%). The capital cost is $100,000 and $r = .05$, hence, we can rewrite the above equation as

$$100,000 = .05V_0 + V_0 - .85V_0 = .2V_0\,.$$

It follows, then, that $\boxed{V_0 = \$500,000}$.

Suppose that Spacely can make only 1 million gears per year total. Suppose, initially, that Spacely has a contract with Amalgamated Automatons to supply it with 250,000 A103s and a contract with Boston Bionics to supply it with

750,000 B100s. Amalgamated pays Spacely \$2 per gear and Boston pays Spacely \$3 per gear. Just before the year is to start, Boston Bionic asks if it can increase its order by 100,000 B100 gears, for a total order of 850,000. The price Boston is willing to pay per gear remains \$3 per gear.

(c) Spacely asks you what the cost will be of producing these extra 100,000 B100s assuming that Amalgamated is willing to accommodate by reducing its order by 100,000. What do you tell Spacely?

Answer: Observe that under both alternatives, the original deal or an increase in B100s, one million gears will be produced. Because B100s cost as much to produce as A103s, the same production expenditures will be incurred under either alternative; hence, these expenditures are sunk. What does Spacely forgo if it sells the additional 100,000 B100s? It forgoes the \$2/gear it would get from Amalgamated times 100,000 gears, or $\boxed{\$200{,}000}$.

(d) After preliminary discussions with Amalgamated, it is clear that if Spacely won't sell Amalgamated 250,000 A103s, then Amalgamated will cancel the entire contract (*i.e.,* buy zero A103 gears). Now what is the cost of producing these extra 100,000 B100s for Boston Bionics?

Answer: The alternatives are the same, except now if you produce the extra 100,000 B100s for Boston Bionics, you forgo the \$2/gear on the 100,000 gears that you would have received from Amalgamated *plus* the lost profits on the remaining 150,000 A103s that, now, Amalgamated doesn't buy. Because the machine cost is overhead, the contribution to profit of these 150,000 A103s is

$$(\$2 - \$1) \times 150,000 = \$150,000\,.$$

Add that to the \$200,000 calculated in part (c), to arrive at a cost of $\boxed{\$350{,}000}$.

Question 4: Your company's product, an electronic device, requires 1 cubic centimeter (cc) of compound X per device produced. It requires \$2 worth of other raw materials and \$1 worth of labor per device produced. Overhead cost per period is \$100,000. You can purchase compound X on the spot market at \$5/cc. Alternatively, you can enter into a contract with Ofda-Bridge, a chemical company. Ofda-Bridge will supply you with as much compound X as you want per period at a price of \$2/cc, but you must pay it a stocking fee of \$18,000 per period. Compound X is notoriously unstable, so it cannot be stored for more than a very short amount of time.

(a) What is your company's marginal-cost schedule (*i.e.,* $MC(\cdot)$) per period?

Answer: The key is to determine the level of output, $\bar{q}$, such that if you know you are going to produce more than $\bar{q}$ it pays to enter into a contract with Ofda-Bridge, but to buy from the spot market if you know you will produce less than $\bar{q}$. If you produce exactly $\bar{q}$, then your are indifferent; hence,

$$2\bar{q} + 18,000 = 5\bar{q}\,.$$

Therefore, $\bar{q} = 6000$. Recall that

$$MC(q) \times 1 \text{ unit} = C(q) - C(q-1)\,,$$

because units are discrete. The cost schedule, assuming you buy from the spot market if you produce 5999 or fewer devices and from Ofda-Bridge if you produce 6000 or more.[2] So we have the following:

$$C(q) = \begin{cases} 0\,, \text{ if } q = 0 \\ 8q + 100,000\,, \text{ if } 1 \leq q \leq 5999 \\ 5q + 118,000\,, \text{ if } 6000 \leq q \end{cases}.$$

This yields

$$MC(q) = \begin{cases} 100,008\,, \text{ if } q = 1 \\ 8\,, \text{ if } 2 \leq q \leq 5999 \\ (30,000 + 118,000) - (47,992 + 100,000) = 8\,, \text{ if } q = 6000 \\ 5\,, \text{ if } 6001 \leq q \end{cases}.$$

(b) What is your company's average-cost schedule (*i.e.,* $AC(\cdot)$) per period?

Answer: From your answer to (a), we have

$$AC(q) = \begin{cases} 8 + \frac{100,000}{q}\,, \text{ if } 1 \leq q \leq 5999 \\ \\ 5 + \frac{118,000}{q}\,, \text{ if } 6000 \leq q \end{cases}.$$

(c) Does your company enjoy increasing, decreasing, constant, or U-shaped returns to scale? How do you know?

Answer: From your answer to (b), we can see that average cost is falling everywhere, except possibly at 6000. However,

$$\begin{aligned} AC(5999) = 8 + \frac{100,000}{5999} > 8 + \frac{100,000}{6000} &= 8 + \frac{50}{3} \\ &= 24\frac{2}{3}\,, \end{aligned}$$

while

$$\begin{aligned} AC(6000) = 5 + \frac{118,000}{6000} &= 5 + \frac{59}{3} \\ &= 24\frac{2}{3}\,. \end{aligned}$$

So AC is falling everywhere. If AC is falling everywhere, then the firm enjoys increasing returns to scale.

[2] Because you're indifferent at 6000, the calculations will yield the same answer if you buy from the spot market if you produce 6000 or fewer devices.

Question 5: I.M. Nervis is considering the following decision. He has just inherited 100 shares of stock in Post, Black, and Branch (PBB), an oil-drilling company that specializes in building and repairing oil pipelines in the Middle East (plus serving army personnel over-priced meals). If President Bush is reelected, then PBB stock will increase to \$144 per share. If John Kerry wins, then PBB stock will drop to \$64 per share. The stock is currently trading at \$112 per share.

(a) Assuming the stock market is rational, what do these data tell you about the market's assessment of the probability that Kerry will win?

Answer: Let p be the probability that Kerry wins. If the market is rational, then

$$112 = 64p + 144(1-p)\,.$$

Hence, $80p = 32$ or $p = 32/80 = 4/10 \boxed{= \ 2/5}$.

Mr. Nervis is risk averse. Tests reveal that his utility from y dollars is $\sqrt{y}$.

(b) What is Mr. Nervis's certainty equivalent value for the PBB shares that he holds?

Answer: Recall that $U(CE) = \mathbb{E}U$. Hence,

$$\begin{aligned}\sqrt{CE} &= \frac{2}{5}\sqrt{64 \times 100} + \frac{3}{5}\sqrt{144 \times 100} \\ &= \frac{2}{5} \times 8 \times 10 + \frac{3}{5} \times 12 \times 10 \\ &= 32 + 72 = 104\,.\end{aligned}$$

Hence $CE = 104^2 = 10,816$ dollars.

(c) Schools Я Us is a company that supplies public schools. If President Bush is reelected, each share of Schools Я Us will be worth \$30. If Senator Kerry wins, each share will be worth \$80. Mr. Nervis approaches you and asks you to design a risk-free portfolio for him. For tax reasons, it is essentially that he does not receive any money from such trades. He is also strapped for cash at the moment, so he can't put up more money other than the negligible amount necessary to cover brokerage fees. Is it feasible to design such a portfolio for Mr. Nervis? (Note, fractional shares are okay.) If it is, what is Mr. Nervis's new portfolio?

Answer: From above, we know Mr. Nervis has an expected payoff of \$11,200 from his 100 shares of PBB. The question is designing a portfolio with the same payoff in both states of the world (Bush or Kerry wins). Let α be the number of shares of PBB Mr. Nervis has in his new portfolio. Let β be the number of shares of Schools Я Us (SЯU) in his new portfolio.

We require that

$$\begin{aligned} 11,200 &= \underbrace{30\beta + 144\alpha}_{\text{payoff Bush wins}} \\ &= \underbrace{80\beta + 64\alpha}_{\text{payoff Kerry wins}} \,. \end{aligned}$$

Observe, we thus have,

$$50\beta = 80\alpha \text{ or } \frac{5}{8}\beta = \alpha \,.$$

Plugging into the top line, we have

$$\begin{aligned} 11,200 &= 30\beta + 144 \times \frac{5}{8}\beta \\ &= 120\beta \,; \end{aligned}$$

hence,

$$\frac{280}{3} = 93\frac{1}{3} = \beta \,.$$

It follows, then, that $\alpha = 175/3$. So the new portfolio is

$$\boxed{280/3 = 93\tfrac{1}{3} \text{ shares of SЯU and } 175/3 = 58\tfrac{1}{3} \text{ shares of PBB}} \,.$$

We know the new portfolio is feasible because you can always slide up and down the fair-odds line without cost. Specifically, each share of SЯU is worth $30 \times 3/5 + 80 \times 2/5 = 50$. So we sell $100 - 175/3 = 41\frac{2}{3}$ shares of PBB, which yields \$4666.67. We need to buy 280/3 shares of SЯU at \$50/share, which requires \$4666.67; as we knew had to be the case, Mr. Nervis can adjust his portfolio to obtain a risk-free portfolio for free.

Question 6: Your company is considering introducing a new product. If this product is popular on a nationwide basis, your company will earn \$20 million per year. If it is unpopular, your company will earn just \$2 million per year. It costs \$60 million and takes one year to build a plant capable of supplying the nation. Historically, new products in this product category have proved popular one out of five times. Your company's return on capital is 10%. For convenience, assume that the plant, if built, will last essentially forever.

(a) Given the above information, should your company (which is risk neutral) introduce this new product or not?

Answer: If the product is popular, your firm earns \$20 million a year starting one year from today. That is a perpetuity, so its net present value (NPV) is \$200 million. Likewise, if the product is unpopular, it has an NPV of \$20 million. The expected NPV is

$$\frac{1}{5} \times 200 + \frac{4}{5} \times 20 = 56$$

million dollars. This is less than \$60 million, so you shouldn't introduce the new product.

(b) Suppose it is possible to build a small plant, which will be producing in six months. The output from that plant is sufficient for one test market. Your test market is Rochester, New York. From past experience, a product that is popular in Rochester will be popular nationwide nine out of ten times. Because test products that prove unpopular in Rochester are usually not released nationwide, your inference about what lack of popularity in Rochester implies is limited to the following: 78.75% of all new products are unpopular both nationwide and in Rochester. Suppose that a test plant will cost \$5 million to build. Suppose, because of various promotional pricings and advertising, that, regardless of its popularity, the net earnings from Rochester are essentially zero. Should you build the test plant? (Note $.7875 = 63/80$.)

Answer: Figure 3 shows the relevant tree. From part (a), we know that without a test, we won't build a plant, so our payoff is zero. Observe that a number of probabilities are unknown. We cannot solve the tree until we determine these probabilities. Although the probabilities are unknown, a number of relations among them are known. First the total probability the product is unpopular nationwide is the sum of the probabilities of reaching terminal nodes ② and ⑤, which are $\alpha/10$ and $(1-\alpha)(1-\beta)$, respectively. Moreover, this total probability is 4/5; hence,

$$\frac{1}{10}\alpha + (1-\alpha)(1-\beta) = \frac{4}{5}\,.$$

In addition the probability of a product being unpopular nationwide and unpopular in Rochester—that is, the probability of node ⑤—is 63/80. In other words,

$$(1-\alpha)(1-\beta) = \frac{63}{80}\,.$$

But, then, $\alpha/10 = 4/5 - 63/80 = 1/80$. Hence, $\alpha = 1/8$. This, in turn, implies that $7/8 \times (1-\beta) = 63/80$ or $1-\beta = 63/70$, so $\beta = 1/10$. What about the payoffs? At nodes ③ and ⑥, the firm is out the \$5 million spent on building the test plant. At nodes ① and ④, the expected net present value is the \$150 million calculated previously, but discounted one more year because of the delay imposed by building the test factory; so the expected NPV is $\$140/1.1 = \$127.27$ million minus the \$5 million cost of the test plant. Similar calculations yield the payoffs at nodes ② and ⑤. Calculating all the expected payoffs, we see that expected NPV of building the test plant is \$8.864 million. We should build it.

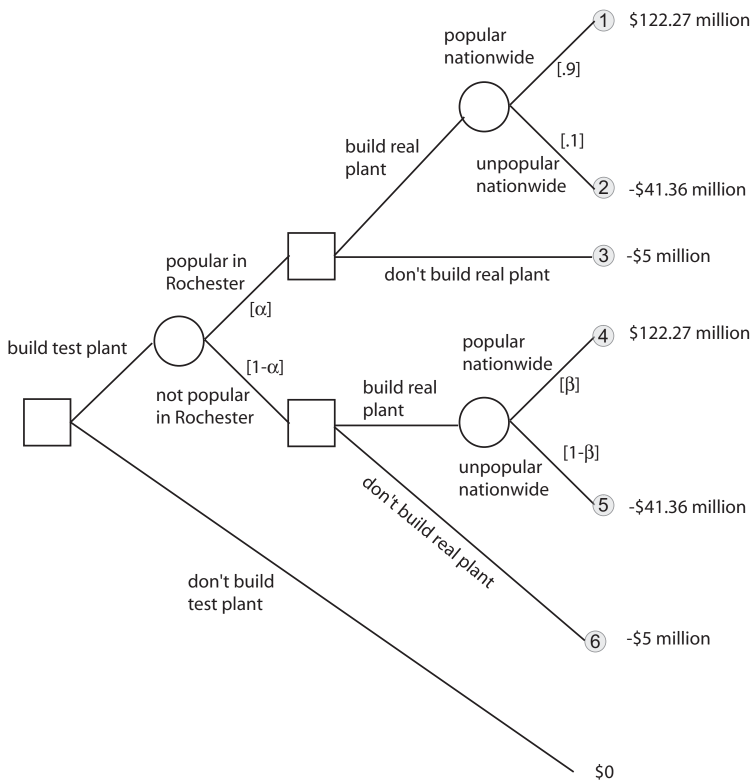

Figure 3: Tree for Problem 6(b).