

# Energy Efficiency and Commercial-Mortgage Valuation\*

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## Abstract

Energy efficiency is key to the future of the U.S. economy, and commercial buildings are among the largest users of energy. However, existing loan underwriting practices provide no incentive for building owners to make their buildings more energy efficient. In this paper, we develop a commercial-mortgage valuation, or underwriting, strategy that accounts for the energy risk of individual office buildings, this energy risk being a function of both the relative energy efficiency of the building and the characteristics of its location. Our method extends standard underwriting practices, which account for the expected dynamics of interest rates and office building prices over time, by including the expected dynamics of the electricity and gas prices as well as quantity dynamics appropriate to the location of the building. This allows lenders to explicitly take into account the effect of energy use and various alternative efficiency measures when underwriting commercial mortgages.

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# 1 Introduction

Commercial mortgage lending in the United States traditionally focuses on two key risk measures for underwriting mortgages: the loan-to-value ratio (LTVR), the ratio of the mortgage balance to the value of the building, and the debt-service-coverage ratio (DSCR), the ratio of the principal and interest payments on the mortgage debt to the net operating income of the building. These ratios are also monitored by bank regulators, such as the Federal Reserve banks, the Office of Thrift Supervision and the Comptroller of the Currency, because they are important indicators of the quality of commercial bank underwriting and the level of mortgage-related default risk exposure. These ratios are important because mortgage performance data have shown that mortgage borrowers are more likely to default as the LTVR approaches one (from below) and as the DSCRs approach one (from above).

The current practice of constructing the LTVR and the DSCR from the net operating income generated by commercial buildings presents an important potential impediment to the development of mortgage contracts that fully account for the risks inherent in either the level or the volatility of the energy use of commercial buildings. In practice, the actual and forecasted net operating income of a commercial building is constructed by: 1) aggregating the actual and forecasted contractual rental income from the tenants' leases; 2) subtracting the buildings' actual and forecasted operating expenses, including the costs of energy; and 3) adding back in the actual and forecasted energy-use expense reimbursements from the tenant to the landlord through a common lease contract known as the "triple net lease."

This aggregation practice thus "nets" out the energy risk exposure of buildings, other than those energy costs borne solely by the landlord due to vacancies, incorrect contracting on the appropriate level of energy reimbursements, or joint costs associated with common areas such as lobbies. For this reason, commercial loan underwriting decisions typically do not account for the risks associated with the level or volatility of a commercial building's energy costs. In addition, commercial mortgage underwriters currently have no actuarially validated comparative scoring systems for the level and volatility of energy costs that can be used to rate the quality of loan applications in a manner similar to that of the DSCR and the LTVR (the DSCR usually must be above 1.25 and LTVR typically must be 65% or less for a successful loan application).

There are two mechanisms through which the benefits of higher energy efficiency can become an instrumental component of the mortgage underwriting process.

1. Convince developers and lenders that there exist highly effective investments that will lower the energy costs of building operation. That is, the investments will earn a ROI (based on energy savings) that exceeds the mortgage interest rate (which becomes

the hurdle rate for the investment). The reason these investments have not already occurred is that the available information (including metrics and tools) has been too imprecise or inappropriately structured to convince the developers to make the investments and to convince the lenders to fund them. Transparent energy efficiency valuation metrics and tools tailored to the mortgage underwriting process will remove the frictions that have greatly inhibited energy-saving investments to date in the U.S. commercial-building sector.<sup>1</sup>

2. Even putting aside the direct operating-cost savings of energy-efficiency investments, lenders will likely recognize that the default risks created by high and volatile energy costs can be eliminated by requiring a high level of energy efficiency for buildings on which they will make loans. This is very much the same as lenders requiring that buildings be protected from earthquake and terrorism risks. This has not occurred to date because lenders have lacked metrics and tools to quantify the benefits. So here too, it is essential to develop transparent energy-efficiency valuation metrics and tools that can grade buildings on the basis of their exposure to the risks created by high and volatile energy costs.

The purpose of this paper is to develop a commercial-mortgage valuation, or underwriting, strategy that accounts for the energy risk of individual office buildings. The energy risk of an office buildings is a function of the relative energy efficiency of the building and the characteristics of its location. The regional component of energy risk arises because the United States does not have a single price setting market for either electricity or natural gas. As will be explained below, U.S. wholesale electricity markets (hubs) exhibit significant heterogeneity in both the level of prices and in their volatility. The importance of our new commercial mortgage underwriting methodology is that it will allow lenders to account for both the specific energy efficiency metrics of buildings, as well as for regional differences in energy price risk due to the location of buildings.

Our method extends standard underwriting practices that account for the expected dynamics of interest rates and office building prices over time to include the expected dynamics of the electricity price and quantity dynamics appropriate to the location of the building, and the dynamics of the wholesale forward prices for natural gas, all of which we benchmark to the Henry Hub for natural gas. The innovation in our methodology is, therefore, to explicitly model the income dynamics of the building that is the collateral for the loan by

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<sup>1</sup>Note that this holds true whether the building is owner-occupied or rented through triple net leases. While it obviously works for owner-occupied buildings, it will also work for triple net leases because once the tenants recognize the savings, they will be willing to pay higher net rents, which the developer can then take to the bank. This again requires transparent information and tools tailored to the mortgage underwriting process.

accounting for both the rent and operating cost, particularly the expected price and quantity of electricity and natural gas used to operate the building.

## 2 Traditional Commercial Mortgage Underwriting

Although there is considerable heterogeneity in commercial mortgage contracts, the key contracting features are: the mortgage principal, the mortgage contract rate (the interest rate paid by the borrower), the mortgage maturity (the date at which the mortgage principal balance is due in full), and the amortization schedule (the schedule for repayment of the mortgage principal). As part of the underwriting process, the contract terms are summarized in the loan-to-value ratio (LTVR) and debt-service-coverage ratio (DSCR) which are then used to verify that the loan meets the required underwriting standards, or to form the basis of required changes in the loan size. The LTVR and the DSCR are closely monitored by lenders, regulators, and investors as metrics of commercial mortgage loan quality. Once the lender knows the actual and forecasted net operating income for a commercial real estate building, the required DSCR (e.g., DSCR greater than or equal to 1.25) is used to determine the maximal periodic debt service that can be supported by the building's cash flows.

The discounted present value of a building's maximal debt service using the loan contract rate for discounting then provides an estimate of the maximal loan amount for the building. Since commercial buildings depreciate slowly, the price of a commercial building is also the discounted present value of the net operating income over a long horizon, which is often simply assumed to be infinity (without significant distortion in value). The benchmark for the LTVR (e.g., 65% or less) in combination with the building value thus determines another maximal estimate of the loan amount. These two maximal values may not be the same, so lenders typically use the lesser value. Since only the current net operating income is actually observed at the time the loan is made, the uncertainty in a building's future net operating income will be considered by lenders in setting the LTVR and DSCR standards for a loan. The valuation of commercial mortgage contracts is also affected by the existence of the default and prepayment options that are owned by the borrowers.<sup>2</sup>

In the traditional underwriting framework, the lender would determine the magnitude of the loan-to-value ratio, the maturity, and the coupon for a specific loan by pricing the loan as a function of the expected cash flows for the building, the expected dynamics of interest rates, and the likelihood that the borrower would exercise the embedded prepayment and default

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<sup>2</sup>The default option is the right of the mortgage borrower to default on the loan and thereby cancel the mortgage by returning the building to the bank. The prepayment option is the right of the mortgage borrower to buy back the mortgage from the bank by prepaying the principal balance of the mortgage

options. Interest rate dynamics are important in this process because market prices depend on the expected discounted present value of future mortgage contract payments. In addition, the dynamics of interest rates and building values both determine the expected values of the prepayment and default options that are held by the borrowers. Since commercial mortgages usually include important restrictions on prepayment, due to the inclusion of prepayment penalties (which compensate the lender if prepayment does occur) or the prohibition of prepayments (lock-outs), commercial mortgage valuation tools primarily focus on modeling the mortgage default options. Commercial mortgage default options, as with all options, are more valuable the longer the horizon of the contract and the greater the volatility of the underlying interest rate and price dynamics.

Since the influential paper of Schwartz and Torous (1989), mortgage valuation tools are based upon estimates of the conditional probabilities (hazard rates) of option exercise using mortgage performance data and proxy measures for the value of the default options (usually measured by the LTVR and ratio of the mortgage contract rate to current market interest rate) and then use Monte Carlo techniques to simulate out the interest rate and building price dynamics. The mortgage value is then computed as the discounted probability weighted (using the hazard rates) averages of forecasted cash flows. These modeling methods require detailed loan performance data sets (observations on default) as well as loan and building characteristics.

Traditionally, mortgage underwriting methods have focused exclusively on the effects of building prices and interest rates on the relative risk of the mortgage. Existing methods do not account for the effects of energy-efficiency-related shocks, due to the shocks on the consumption or pricing of energy factor inputs, on the level and volatility of net operating income or value, and thus on default. Because this information is not accounted for, currently lenders are unable to distinguish the relative risk of efficient versus inefficient commercial real estate buildings and consequently they do not currently risk-adjust the pricing of mortgages on buildings with different energy efficiency attributes. As a result, current building owners do not see differences in the cost of mortgage debt due to the relative energy efficiency of their buildings. Similarly, it is difficult to get energy retrofits financed due to the lack of existing underwriting methods that allow lenders to price the risk mitigation benefits of these retrofits. For lenders to accurately price these benefits, the traditional commercial mortgage valuation, or underwriting, strategies must be augmented to explicitly include energy-related sources of risk.

### 3 The Geography of Energy Risk in the U.S.

The electrical power system in the U.S. is organized into three major networks (see Clewlow and Strickland, 2000; Harris, 2006; Weron, 2006): The Eastern Interconnected System (approximately covering the Eastern Standard and Central time zones); the Western Interconnected System (the Mountain and Pacific time zones); and the Texas Interconnected System. Figure 1 presents the geographic location of all the wholesale power hubs in the U.S. The existence of these three network divisions and the physics of electricity transmission imply that there is not a national market for pricing electricity in the U.S. Instead, electricity trades in hub locations that correspond to the nodal structure of the U.S. natural gas pipeline and to the location of the major population centers.

As shown, in the Figure 1 there is considerable regional variation in the level of electricity forward prices. As will be discussed in more detail below, there is also considerable cross-sectional variability in the dynamics of electricity prices across regions and in their volatility. The electricity spot and forward markets are over-the-counter markets. Pricing information is assembled by a company called Platts that gathers information on the power forward market from active brokers and traders and through the non-commercial departments (back offices) of companies. Since October 2007 this information is complemented with the Intercontinental Exchange (ICE) quotes to form the Platts forward market power daily assessment. Since more liquid locations and shorter term packages trade more on ICE, while less liquid locations and longer term packages trade more over-the-counter (OTC), Platts is able to combine these sources to build a comprehensive picture of the forward market. Details of the methodology are described in the Platts Methodology and Specification Guide - Platts-ICE electricity Forward Curve (North America). Prices are reported in this market per million Watt hours (MWh).

In contrast to the wholesale electricity markets, the wholesale natural gas market we benchmark to the Henry Hub in Erath, Louisiana.<sup>3</sup> The Henry Hub is the pricing locus for natural gas futures contracts traded on the New York Mercantile Exchange (NYMEX). The Henry Hub interconnects with nine interstate and four intrastate pipelines: Acadian, Columbia Gulf Transmission, Gulf South Pipeline, Bridgeline, NGPL, Sea Robin, Southern Natural Pipeline, Texas Gas Transmission, Transcontinental Pipeline, Trunkline Pipeline, Jefferson Island, and Sabine Pipe Line LLC. The spot and future prices set at Henry Hub

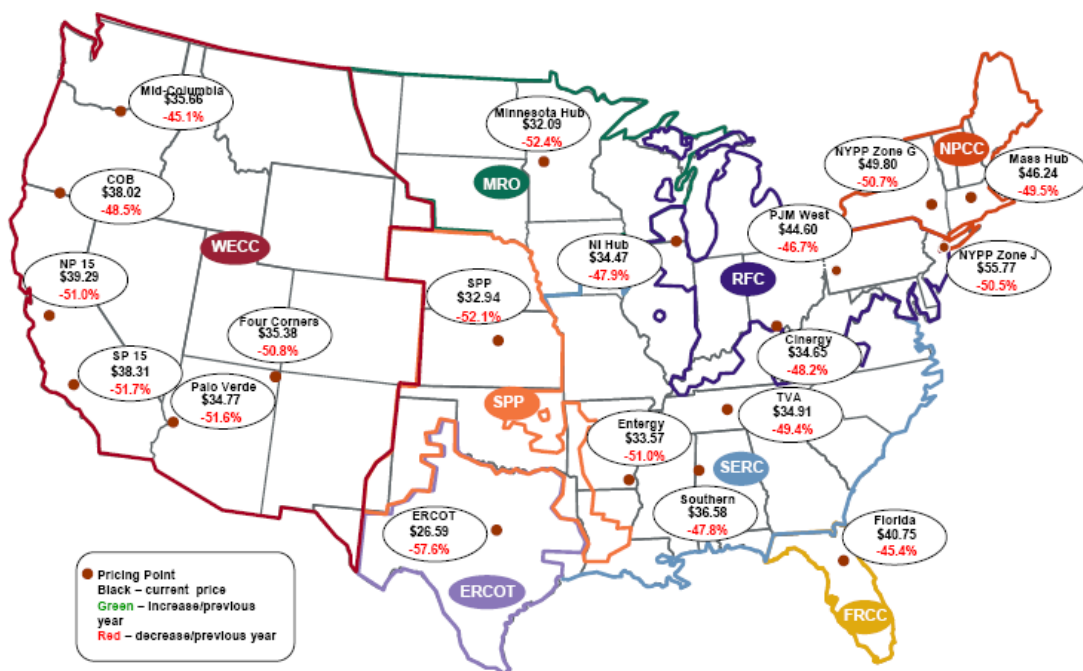
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<sup>3</sup>Although there is regional heterogeneity in price levels and these can be significant for some cities during the winter months, the term structure of volatility for Henry hub future prices is a very good first approximation for the term structure of volatilities of forward prices for other natural gas hubs. For this reason, we benchmark to the Henry Hub and the highly liquid NYMEX Henry Hub futures and options markets.



Figure 1: Federal Energy Regulatory Commission Geographic Location of the Power Hubs in the United States

This figure was obtained from the Federal Energy Regulatory Commission ([www.ferc.gov/oversight](http://www.ferc.gov/oversight)). It presents the geographic location of the hubs for electricity forward contract auctions in the U.S.. The average dollar value of the near contract over the year 2009 is presented for each hub and the percentage change in this average price from the average over the year 2008.



are denominated in dollars per millions of British thermal units (MMBtu) and are generally seen to be the primary price set for the North American natural gas market.

In Figure 2, we plot the nearest contract prices (\$ per MWh) for monthly delivery of on-peak electricity forwards for the ERCOT hub (the electricity hub for Texas) and contract prices (\$ per MMBtu) for monthly delivery of Henry Hub natural gas futures contracts from January 2002 to February 2010. Interestingly, although gas is an important fuel in the production of electricity and even though we are comparing forward contracts for geographically proximate markets, it is clear from Figure 2 that there have been periods of significant differences in the dynamics of monthly futures prices for natural gas and forward prices for electricity. Overall, the electricity forward price dynamics appear more volatile and they appear to exhibit a stronger seasonal component. Both series, are shown to mean revert, however, the speed of mean reversion of the electricity forward prices appears more rapid than that for natural gas futures. A feature in these time series dynamics, which is not apparent from Figure 2, is that they also exhibit important cross-sectional heterogeneity across the electricity hubs. As will be seen below, forward prices on the ERCOT hub and the relationship between forward prices and the maturity of the forward contracts frequently exhibit very different characteristics than those of the other electricity hubs in the U.S. There are also important differences across the three major networks.

Regional differences in the wholesale price dynamics of electricity and the significant volatility of both natural gas and electricity forward price contracts are important for mortgage prices because energy costs are, on average, about 12% of base rents, and in many regions of the country as much as 30% of total costs.<sup>4</sup> Even though energy markets are regulated and most buildings do not pay the wholesale prices for power and natural gas, many real estate operating companies do now purchase their electricity from the wholesale market, as do some counties.<sup>5</sup> In addition, the wholesale markets reflect the true resource costs of energy consumption and these costs are incorporated, in time, into the rate schedules offered by regulated utility companies. Both because commercial real estate operators appear to be expanding their energy purchases through wholesale energy markets and because the dynamics of these markets affect the performance of commercial office buildings, the resource signals from these markets should be of concern to mortgage lenders who bear the residual default risk associated with the energy cost exposure of borrowers. Surprisingly, despite the fact that commercial buildings accounted for 18% of the total energy consumption in the U.S.,<sup>6</sup> traditional commercial mortgage underwriting processes do not account for the

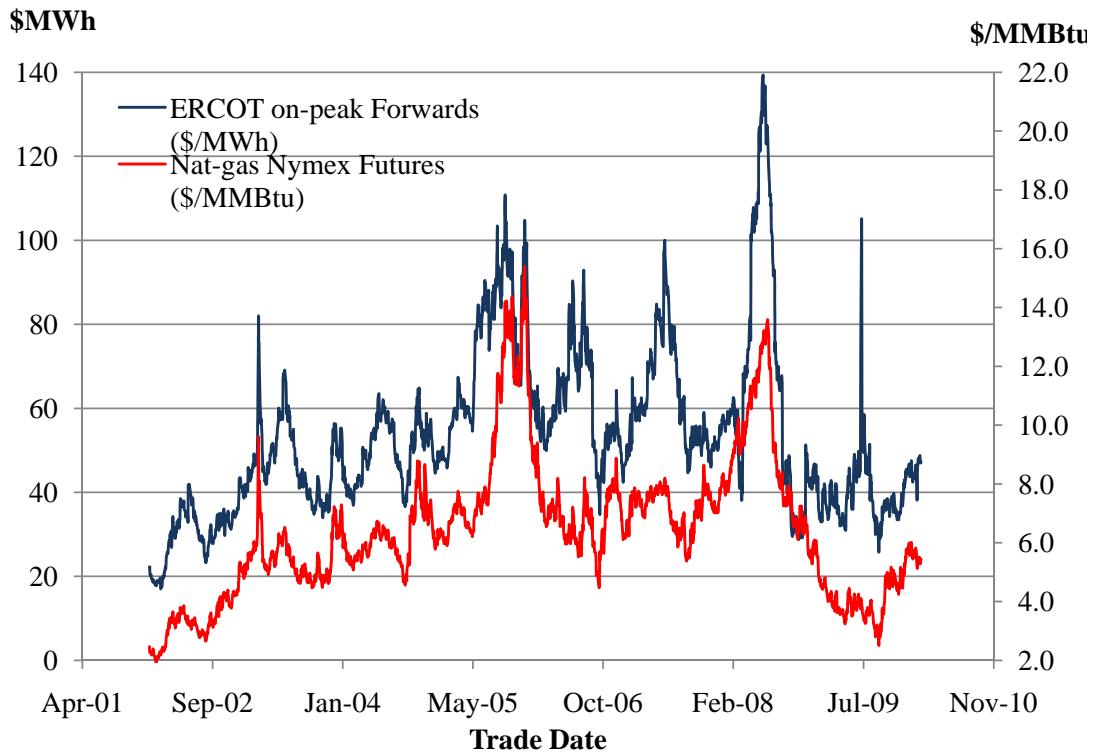
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<sup>4</sup>See BOMA Experience and Exchange Report for 2009, <http://www.boma.org/resources/benchmarking/Pages/default.aspx>, and authors' calculations based on building-owner interviews.

<sup>5</sup>See <http://www.sonomacountyenergy.org/lower.php?url=fnma-freddie-mac-letters>.

<sup>6</sup>See U.S. Department of Energy (2009).

Figure 2: Nearest Contract Price for the ERCOT Electricity Forward Contracts and Henry Hub Natural Gas Futures Contracts



performance risks associated with the energy efficiency of office buildings.

## 4 Underwriting Mortgage Energy Risk

Accurately underwriting of the energy efficiency of commercial office buildings requires the precise measurement of the energy efficiency of office buildings. Unfortunately, at present, there are no readily available methods for lenders to determine the expected level and volatility of a specific building's consumption of electricity and natural gas given the characteristics of the building's engineering systems, roof, window, lighting, and surface characteristics, along with its exposure to location factors such as wind, humidity, and temperature. At best, the lender will have a short history of the utility bills, however, there are no available metrics that allow lenders to readily measure the relationship between the existing building systems, the metering of the tenants, the relative occupancy levels, the commissioning for the equipment in the building, and the observed utility bills. Even if there is an Energy Star score for a building, lenders still would know nothing about the expected level and volatility of the future energy consumption of the rated building.<sup>7</sup>

Surprisingly, despite the current lack of available measures for the expected level and volatility of natural gas and electricity consumption of office buildings, most commercial mortgage lenders do require engineering reports on buildings as part of the underwriting process. These reports often, but not always, include detailed information on the engineering systems and the architectural features of the building and, at least in principle, they could be used to design measures of the relative energy efficiency of office buildings. At present, however, the engineering report is primarily used by lenders to determine the level of reserves that will be required to assure that borrowers can replace major building systems if these systems are found to be close to, or beyond, their usable lives. Other than reserve requirements, no other standardized information from these reports is used in the commercial loan funding decision. Since the property appraisal usually precedes the engineering report by several months in the underwriting process, the property appraisal is also not informed by the engineering analyzes of the major building systems and their implication for the absolute or relative energy efficiency of a specific building.

The engineering due diligence reports that are used in commercial mortgage underwrit-

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<sup>7</sup>The Energy Star rating program was designed by the EPA and the U.S. Department of Energy to promote energy efficiency in the U.S. commercial real estate sector is based on comparative national data obtained from the Commercial Building Energy Consumption Survey that set the annual benchmarks for energy usage levels across property types. The Energy Star measure of the building's energy efficiency is measured as the residual between the actual and predicted energy usage of the building using actual utility bills. However, to receive an Energy Star label, a building must score in the top quartile of the EPA's energy performance rating system and must meet designated indoor air quality standards.

ing processes are called Property Condition Assessments (PCAs). The Resolution Trust Corporation (RTC) was the first commercial real estate loan underwriter to formalize the use of PCAs in fulfilling its mandate to liquidate the commercial real estate mortgages of failed Savings and Loan Institutions in the early 1990s. Because the RTC was also instrumental in the development of the commercial mortgage backed securities market, PCAs became, and are now, a required component of standard underwriting processes for all securitized commercial mortgages in the U.S. In 1995, Standard & Poor's produced the first guide that defined the PCA process for commercial lending and the American Society for Testing and Materials (ASTM),<sup>8</sup> released the first vetted PCA standard called the ASTM E2018 - 08 in 1999.<sup>9</sup> The ASTM released a further standard, in April 2011, called the ASTM E2797 - 11 that is intended to augment E2018 PCA, on a voluntary basis, to include information on the energy use of the buildings.<sup>10</sup>

The PCA provides an analysis of ten major systems found in commercial real estate. These systems include: 1) building site (topography, drainage, retaining walls, paving, curbing); 2) lighting; 3) building envelope (windows and walls); 4) structural (foundation and framing); 5) interior elements (stairways, hallways, common areas); 6) roofing systems; 7) mechanical (heating, ventilation, and air conditioning); 8) plumbing; electrical; 9) vertical transportation (elevators and escalators); and 10) life safety, American Disability Act (ADA) requirements, building code compliance; air quality (fire codes, handicapped accessibility, water/mold). The PCA process generally consists of two phases: a site inspection and data analysis. These reports can cost building owners from \$15,000 to \$100,000 to complete, however, they are not required to be submitted in a standardized format.

The lack of standardized PCAs presents the lenders with a significant impediment to translating the information in the PCAs into usable measures of the expected level and volatility of a given building's energy consumption. Without standardization and methods to summarize the information in the PCAs, lenders cannot readily *connect the dots* between their loan underwriting and valuation protocols and quantifiable energy efficiency measures for specific buildings. As a result, it is difficult for lenders to truly "underwrite" the energy risk of commercial mortgages and, therefore, to provide cost-of-capital incentives for building owners to improve the energy efficiency of their buildings. In fact, in general in the United States, the embedded energy risk of commercial office mortgages is not priced

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<sup>8</sup>The ASTM is an international standards organization that develops and publishes voluntary consensus technical standards for a wide range of materials, products, systems, and services

<sup>9</sup>See ASTM E2018 - 08 Standard Guide for Property Condition Assessments: Baseline Property Condition Assessment process, <http://www.astm.org/Standards/E2018.htm>

<sup>10</sup>See ASTM E2797 - 11 Standard Practice for Building Energy Performance Assessment for a Building Involved in a Real Estate Transaction, <http://www.astm.org/Standards/E2797.htm>

by lenders. This means that energy efficient and energy inefficient buildings are offered the same mortgage terms, everything else equal, despite the potentially different default risk of these mortgages.

## **4.1 The Measurement of Expected Energy Consumption for Office Buildings**

Accurate modeling of the expected energy consumption of U.S. office buildings is a data and labor intensive process. If sufficient data are available, regression models can be used to relate the energy use of buildings to their characteristics. Fitted regression models can then be used to forecast the out-of-sample expected energy use for similar buildings. Of course, the heterogeneity of office buildings raises potentially important problems for the comparability of the out-of-sample fits and associated problems with omitted variable bias in the specification of the regression.

Simulation is another alternative that can be used to forecast the expected energy consumption of office buildings. Simulation models, however, require quite detailed data on building geometry, construction, equipment characteristics, occupancy characteristics and operation schedules.<sup>11</sup> Although there are several different available simulation tools, buildings with unusual construction or equipment may not correspond well to the underlying specifications of these models. Of course, the primary benefit of simulation tools is that they provide detailed quantitative results and annual energy use measures are standard outputs.

If neither regression nor simulation models are viable for a specific building, benchmark data may provide an option for estimating average energy use. This approach should be used with caution because benchmarking tools are used to compare a building's energy use to its peers, not to estimate the energy use of the building itself. However, to the extent that the peer buildings have similar characteristics, the average energy use of the peer buildings may be a reasonable proxy for a subject building. Several benchmark data sources are available, including the Commercial Building Energy Consumption Survey (CBECS) and the California Commercial End Use Survey (CEUS).

### **4.1.1 Empirically Benchmarked Office Building Energy Consumption**

Currently, the best option for measuring the expected energy consumption of office buildings is through benchmarking. Given the current state of available information for specific

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<sup>11</sup>For new construction these data are usually readily available in construction drawings and specifications. However, for existing buildings, if drawings and specifications are not available or are not current, data collection may be too very burdensome.

buildings, two data items, the location and the floor area, are generally known as a matter of course for all properties at any point in the standard mortgage underwriting process. The advantages of using empirical benchmarking for this purpose are:

1. It is based on measured energy use of existing buildings.
2. Given the lack of any information about building features, there would be no added value to using simulation because the primary benefit of simulation is the ability to model building features.

For the purposes of this study, the energy usage intensities (EUI) for electricity and natural gas were determined using EnergyIQ, which is a benchmarking tool that has been developed by the Lawrence Berkeley National Laboratory using the Commercial Buildings Energy Consumption Survey (CBECS) and California Commercial End-Use Survey (CEUS) (see Mathew, Mills, Bourassa, and Brook, 2008a; Mathew, Mills, Bourassa, Brook, and Piette, 2008b).

CBECS is a national sample survey that collects information on the stock of U.S. commercial buildings, their energy-related building characteristics, and their energy consumption and expenditures.<sup>12</sup> The 2003 CBECS contains 5,215 sample building records across the country which were statistically sampled and weighted to represent the entire stock of national wide commercial building. CEUS is a comprehensive study of commercial sector energy use in California, primarily designed to support the state’s energy demand forecasting activities (Itron (2006)). A stratified random sample of 2,790 commercial facilities was collected from the service areas of Pacific Gas and Electric, San Diego Gas & Electric, Southern California Edison, Southern California Gas Company, and the Sacramento Municipal Utility District. The sample was stratified by utility service area, climate region, building type, and energy consumption level. EnergyIQ allows a user to choose of these datasets and benchmark their building against the dataset is using various energy use intensity (EUI) metrics at the whole building level as well as for different end uses (lighting, heating, cooling etc.).

The CEUS data were used for buildings located in California, and CBECS for all other locations. Peer groups were defined based on building type (“office”), size and geographical region. The building sizes are classified as large (> 150,000 square feet), medium (25,000–150,000 square feet) and small buildings (< 25,000 square feet). The geographical regions in CBECS are based on nine census divisions, including East North Central, West North Central, New England, Middle Atlantic, South Atlantic, East South Central, West South Central, Mountain, and Pacific. In CEUS, there are seven geographical regions for California,

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<sup>12</sup>See Commercial Building Energy Consumption Survey 2003, Energy Information Administration (EIA), <http://ww.eia.gov/emeu/cbecs/contents.html>.

including South Coast, Central Coast, North Coast, Central Valley, South Inland, Mountain, Desert. Each peer group contained at least 20 buildings. If a peer group in CBECS contained less than 20, we used census region (typically consisting of two census divisions) as the geographical filter to increase the number of buildings for that peer group. The tool calculates the median EUIs for electricity and natural gas for each peer group, which were then applied to each of the building locations in the mortgage valuation dataset.

Our application of empirical benchmarking has two key limitations: These EUI estimates for each building in the mortgage valuation dataset do not account for their relative energy efficiency, because building asset and operations characteristics were not available for this analysis (i.e., all buildings in a given geographical region and of a given size have the same EUI). Secondly, the analysis does not account for differences in climate within a geographical region.

As shown in Table 1, there is considerable variability in the consumption levels of natural gas and electricity across regions and building types. In general, the western coastal regions appear to have lower consumption levels of both electricity and natural gas and the East Coast and Texas locations appear to have higher consumption levels. There are also important differences across buildings with different square footage. These reported median energy consumption variables, as will be discussed below, will have important implications for pricing the relative risk of mortgage across locations and building sizes.

## 5 Commercial Real Estate Mortgage Valuation with Energy Risk

As previously discussed, the traditional mortgage valuation process focuses on the dynamics of interest rates and building prices to model the market price of the mortgage cash flows. However, to account for energy risk, building prices must be decomposed into market rents minus total costs including the costs of energy expenditures. The canonical representation for the market price of a commercial real estate asset is the discounted present value of the asset's future net operating income. Since well-maintained office properties typically can be assumed to be long-lived assets, the market price per square foot of a commercial office building at the investor's purchase date ( $t = 0$ ) can be written as

$$P(0) = \sum_{t=1}^{\infty} \frac{E_0 [NOI(t)]}{(1 + i_t)^t}, \quad (1)$$



Table 1: CEUS Benchmarks for Median Energy Consumption by Building Size and Region

Market Name	CBECS (or CEUS) Region <sup>a</sup>	Large Building Consumption (Greater than 150,000 Square Feet)			Medium Building Consumption (25,000 to 150,000 Square Feet)			Small Building Consumption (Less than 25,000 Square Feet)		
		Electricity (kWh/psf-yr)	Gas (kBTU/psf-yr)	Electricity (kWh/psf-yr)	Gas (kBTU/psf-yr)	Electricity (kWh/psf-yr)	Gas (kBTU/psf-yr)	Electricity (kWh/psf-yr)	Gas (kBTU/psf-yr)	
Atlanta	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2			
Austin	West South Central	25.0	2.6	21.8	5.3	13.7	25.0			
Boston	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			
Charlotte	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2			
Chicago	East North Central	21.9	17.6	13.8	40.5	10.3	54.5			
Cincinnati	East North Central	21.9	17.6	13.8	40.5	10.3	54.5			
Cleveland	East North Central	21.9	17.6	13.8	40.5	10.3	54.5			
Dallas/Ft Worth	West South Central	25.0	2.6	21.8	5.3	13.7	25.0			
Detroit	East North Central	21.9	17.6	13.8	40.5	10.3	54.5			
East Bay/Oakland	Central Coast (CEUS)	13.8	20.5	12.0	13.4	9.9	12.2			
Hartford	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			
Houston	West South Central	25.0	2.6	21.8	5.3	13.7	25.0			
Indianapolis	East North Central	21.9	17.6	13.8	40.5	10.3	54.5			
Kansas City	Mid West	21.2	16.3	14.6	40.1	10.1	42.9			
Long Island (New York)	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			
Los Angeles	South Coast (CEUS)	14.2	6.5	13.8	7.4	12.5	9.7			
Marin/Sonoma	Central Coast (CEUS)	13.8	20.5	12.0	13.4	9.9	12.2			
Miami	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2			
Milwaukee/Madison	East North Central	21.9	17.6	13.8	40.5	10.3	54.5			
Minneapolis/St Paul	Mid West	21.2	16.3	14.6	40.1	10.1	42.9			
Nashville	East & West South Central	25.0	3.2	19.0	16.1	14.5	24.8			
New York City	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			
Northern New Jersey	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			
Orange (California)	South Coast (CEUS)	14.2	6.5	13.8	7.4	12.5	9.7			
Orlando	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2			
Philadelphia	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			
Pittsburgh	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			
Riverside (California)	South Inland (CEUS)	18.1	10.7	13.8	8.1	11.8	12.0			
Sacramento	Central Valley (CEUS)	13.3	15.2	13.1	12.6	10.1	17.6			
San Antonio	West South Central	25.0	2.6	21.8	5.3	13.7	25.0			
San Diego	South Coast (CEUS)	14.2	6.5	13.8	7.4	12.5	9.7			
San Francisco	Central Coast (CEUS)	13.8	20.5	12.0	13.4	9.9	12.2			
South Bay/San Jose	Central Coast (CEUS)	13.8	20.5	12.0	13.4	9.9	12.2			
St. Louis	Mid West	21.2	16.3	14.6	40.1	10.1	42.9			
Tampa/St Petersburg	South Atlantic	21.8	3.7	15.3	15.5	13.0	29.2			
Westchester/S Connecticut	NorthEast	18.4	31.3	7.9	34.7	10.0	44.1			

<sup>a</sup>These data were provided by the Lawrence Berkeley National Laboratory.

where  $P(0)$  is the market price per square foot at the investment date,  $t = 0$ ,  $E_0 [NOI(t)]$  is the expected net operating income per square foot at the  $t^{th}$  period, and the discount rate for cash flows at date  $t$ ,  $i_t$ , equals the riskless rate plus a risk premium. This can alternatively be written in terms of “risk-neutral” expectations (see Harrison and Kreps, 1979) as

$$P(0) = E_0^* \left[ \sum_{i=1}^{\infty} NOI(i \Delta t) e^{-\Delta t \sum_{j=0}^{i-1} \tilde{r}_j \Delta t} \right], \quad (2)$$

where  $\tilde{r}_t$  is the one-period riskless interest rate at date  $t$ . The net operating income per square foot of an office building is defined as

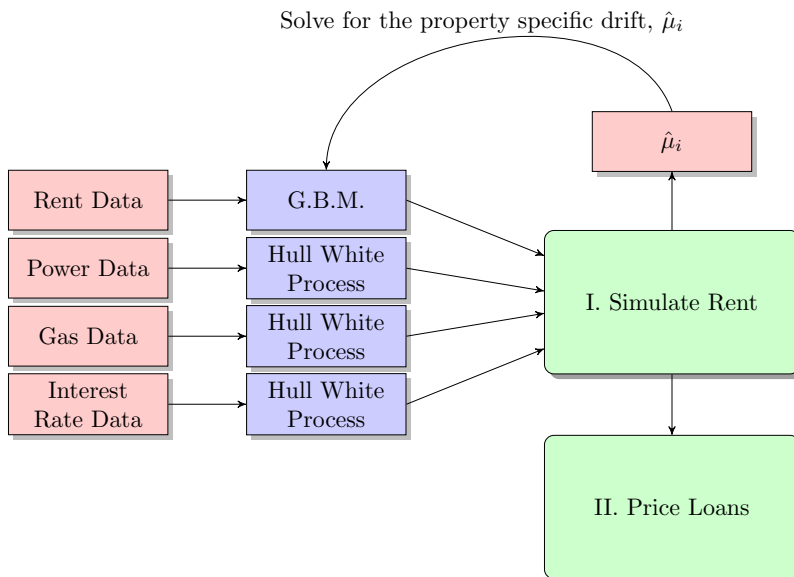
$$NOI(t) = c(t) - (p_{gas}(t) \times q_{gas}(t)) - (p_{elec}(t) \times q_{elec}(t)) - (p_{other}(t) \times q_{other}(t)), \quad (3)$$

where  $c(t)_{psf}$  is the rent per square foot,  $(p_{gas}(t) \times q_{gas}(t))$  is the total gas expense per square foot (the price  $p_{gas}(t)$  per square foot times the quantity of gas used per square foot  $q_{gas}(t)$ ),  $(p_{elec}(t) \times q_{elec}(t))$  is the total electricity expense per square foot (the price  $p_{elec}(t)$  per square foot times the quantity of electricity used per square foot  $q_{elec}(t)$ ), and other expenses per square foot,  $(p_{other}(t) \times q_{other}(t))$ .

The challenge of this decomposition is that the mortgage valuation problem must now account for the dynamics of four stochastic processes: 1) interest rates; 2) electricity forward prices at the appropriate geographic hub in which the property is located; 3) gas futures prices at the Henry Hub; and 4) office market rents for the building that is the collateral on the loan that is to be priced. A schematic for our proposed modeling strategy is presented in Figure 3. Moving from left to right in the Figure 3 schematic, our mortgage valuation protocol requires market specific data for interest rates; electricity prices, natural gas prices, and office market rents. As previously discussed, the electricity price data is specific to the electricity hub in which the building is located. We assume that the natural gas dynamics are determined by the NYMEX Henry Hub futures price dynamics that are common to all office buildings in the U.S. The interest rate process is also common across all buildings and the data for this process is U.S. Treasury data. The rental process must be calibrated for each building, as will be discussed in more detail below.

The data requirements for the augmented mortgage valuation protocol are also significant. Our valuation protocol requires the interest rate process, the electricity price process, and the natural gas price process to “match” (exactly fit) the observed term structure of interest rates or forward contract prices for every month for which we intend to price mortgage contracts. This requires that we collect monthly data series from 2002 through 2007 corresponding to the sample of mortgages that we will price. In addition, the natural gas and electricity

Figure 3: Flow Chart for the Mortgage Valuation Strategy



simulations also require information on the expected building specific consumption levels of natural gas and electricity per square foot. As previously discussed, we use the CEUS and CBECS benchmarking values by matching buildings to locations and their appropriate building size.<sup>13</sup>

As shown in Figure 3, the next component of the valuation protocol is to fit a Hull-White process for interest rates and exponential Hull-White processes for electricity and natural gas prices. As will be discussed below, these functional forms are commonly used in modeling these dynamics in both the practitioner and academic literatures. The price dynamics for each of the stochastic components of the model are fit exogenously using market data from each of the respective markets.

In *Stage I* of the modeling protocol, we solve for the implied risk-neutral drift,  $\hat{\mu}_i$ , of the building specific market rental process, assumed to be a Geometric Brownian motion (GBM), conditional on the estimated dynamics of the interest rates, electricity forward prices, and natural gas forward prices. The solution for this implied drift is the value that will exactly match the observed price of the building at the origination date of the mortgage given the market dynamics of the three other market fundamentals. Once the drift parameter of the building specific rent is optimally fit, the valuation component of the model, the *Stage II* component, applies the four stochastic factors: 1) interest rates; 2) electricity forward

<sup>13</sup>This strategy does not allow the demand for power or natural gas to fluctuate as a function of prices. However, there is considerable evidence that office buildings in the U.S. are sufficiently inefficient that they are unable to make such price related adjustments.

prices; 3) natural gas futures prices; and 4) the market rents for the building in a Monte Carlo simulation to compute the expected value of the contractual mortgage cash flows and the value of the embedded default option. To recap the stages of the modeling process:

1. Monthly data are assembled for U.S. interest rates; electricity forward prices by electricity hub; and natural gas forward prices for the Henry Hub;
2. The interest rate is fit to a Hull-White process and the gas and electricity price data are fit to exponential Hull-White processes. These processes are fit to exactly match the observed term structure of these series on a monthly frequency.
3. *Stage I*: Using the fitted dynamics of interest rates and energy forward prices, the long run mean, or drift, of the stochastic price process for a building's market rent dynamic is fit, assuming that the process follows a GBM, such that the estimated process exactly matches the observed building price at the origination of the mortgage.
4. *Stage II*: Using a four factor model (interest rates, natural gas forward prices; electricity hub forward prices, and the building specific rental price dynamic), Monte Carlo simulation is used to value the mortgage contract cash flows and the embedded default option.

## 5.1 Interest Rate Dynamics

In practice for mortgage valuation, interest rate models are fit to observed market data for the term structure of interest rates and the volatility of interest rates. The Hull and White (1990) model is a commonly assumed model for this application due to its flexibility in exactly matching observed term structures and volatilities. In the Hull and White (1990) model, the short-term riskless rate is assumed to follow the risk-neutral process

$$dr(t) = (\theta(t) - \alpha_r r(t)) dt + \sigma_r dW(t), \quad (4)$$

where  $dW(t)$  defines a standard Brownian motion under the risk-neutral measure, and  $\theta(t)$ ,  $\alpha_r$ ,  $\sigma_r$  and  $r_0$  (the starting rate at time zero) are the parameters that need to be estimated. The function  $\theta(t)$  is fit so that the model matches the yield curve for the U.S. LIBOR swap rate on September 30, 2004. Hull and White (1990) show that  $\theta(t)$  is given by

$$\theta(t) = F_t(0, t) + \alpha_r F(0, t) + \frac{\sigma_r^2}{2\alpha_r} (1 - e^{-2\alpha_r t}),$$

where  $F(0, t)$  is the continuously compounded forward rate at date 0 for an instantaneous loan at  $t$ . Parameters  $\alpha_r$  and  $\sigma_r$  are fit with maximum likelihood using U.S. Treasury curve data and implied caplet volatilities.

## 5.2 Rent Dynamics

The market rent of an office building, as discussed above is assumed to follow a geometric Brownian motion,

$$dC_t = \hat{\mu}C_t dt + \phi_C C_t dW_t, \quad (5)$$

where  $\hat{\mu}$  is the risk adjusted long run drift of the rental process and  $\phi_C$  is the volatility. The process defined by equation (5) is fit individually for each building that is the collateral for each mortgage. The results of this fitting process is will be discussed in detail below. The estimate for volatility was estimated in Stanton and Wallace (2011) to be  $\phi_C = 21.478$ , by solving for the implied volatility from a large sample of 9,778 office building loans originated between 2002 and 2007.

## 5.3 Electricity and Gas Dynamics

We calibrate the dynamics of electricity and natural gas prices following Schwartz (1997) and Clewlow and Strickland (1999), assuming the log of the spot price for electricity ( $e$ ) and natural gas ( $g$ ) follows a Hull and White (1990) process,

$$dS_{e,g}(t)/S_{e,g}(t) = [\mu_{e,g}(t) - \alpha_{e,g} \ln S_{e,g}] dt + \sigma_{e,g} dW(t). \quad (6)$$

To match the initial forward curve for electricity and futures curve for natural gas, we need to set

$$\mu_{e,g}(t) = \frac{\partial \ln F_{e,g}(0, t)}{\partial t} + \alpha_{e,g} \ln F_{e,g}(0, t) + \frac{\sigma_{e,g}^2}{4\alpha_{e,g}} (1 - e^{-2\alpha_{e,g}t}). \quad (7)$$

Clewlow and Strickland (1999) show that

$$F_{e,g}(t, T) = F_{e,g}(0, T) \left( \frac{S_{e,g}(t)}{F_{e,g}(0, t)} \right)^{\exp(-\alpha_{e,g}(T-t))} \exp \left[ -\frac{\sigma_{e,g}^2}{4\alpha_{e,g}} e^{-\alpha_{e,g}T} (e^{2\alpha_{e,g}t} - 1) (e^{-\alpha_{e,g}T} - e^{-\alpha_{e,g}t}) \right]. \quad (8)$$

In other words, the forward (futures) curve at any future time is simply a function of the spot price at that time, the initial forward (futures) curve, and the volatility function parameters for electricity and natural gas, respectively.

## 5.4 Data and Calibration

The data collection and processing procedures used to construct the needed monthly observations on electricity and natural gas forward prices by contract maturity are described in the Appendix.

### 5.4.1 Calibrating the Electricity Forward Curves

In Table 2, we report the estimation for the exponential Hull-White model parameters for the electricity forward curve. Our objective is to calibrate the parameters  $\alpha_e$  and  $\sigma_e$  in Equation (6), the stochastic differential equation describing the dynamics of the forward curve for electricity. As a first step, we pre-process the forward prices by tagging, at each trading date, the number of months out before delivery for each forward price. For example, the nearest contract (prompt contract) has *month out* equal to 1, the second to prompt contract is assigned with *month out* equal 2 and so forth.

In the pre-processed data, we also keep track of the source package related to the forward price entry. For example, in 12/28/2006 the on-peak PJM Western hub October-2008 contract has its quote derived from an annual package (*package length* equal 12). The next trading date, 12/29/2006, the source of the quote is now from a quarterly package (*package length* equal 3).

For a given forward price, we then calculate the volatility for each *month out*. For example, the on-peak PJM Western hub January-2009 contract has about 22 returns when it is 10 months out. Its *10-month-out* volatility is calculated as the standard deviation of the 22 daily returns and the result is then annualized. The same on-peak PJM Western hub January-2009 contract has about 22 return entries when the contract is 9 months out. We proceed in the same way to calculate its *9-month-out* instantaneous volatility. Finally, we compute the average volatility for each *month out* and *package length*. This procedure allows us to calibrate the term structure of instantaneous historical volatility as a function of maturity, while controlling for *package length*. The calibrated parameters  $\alpha_e$  and  $\sigma_e$  are estimated by regressing the logarithm of the average volatilities on *month out* measured in years and *package length*.

As shown in the Table 2, there is considerable heterogeneity across the electricity hubs in the fitted values of the speed of mean reversion,  $\alpha_e$ , of the exponential Hull-White process and in the volatility,  $\sigma_e$ . The results indicate that overall the higher volatility of forward prices is higher in the Western time zones than it is in the Eastern times, but it is the highest for the forward prices observed in the ERCOT hub. The speeds of adjustment to the long run drift,  $\alpha_e$ , are not as differentiated by regions as are the volatilities, however, again the ERCOT hub exhibits a higher speed of adjustment than any of the other over-the-counter markets. The effects of these differences will become more apparent in our discussion below.

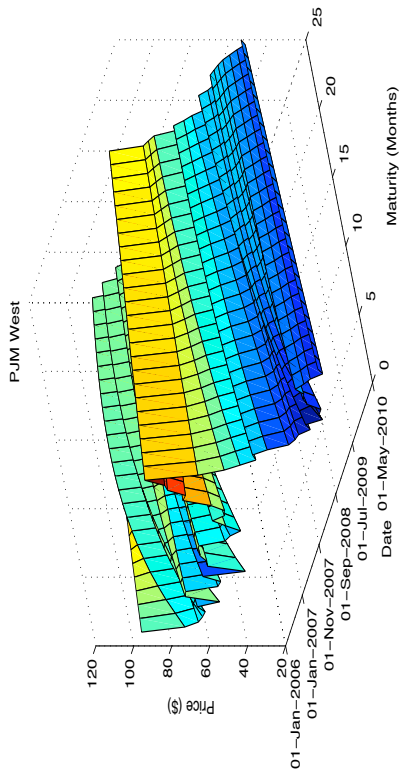
As shown in Figures 4 and 5, we graph a time series of our estimated forward price curves by the maturity of the contract out to twenty five months of maturity. In Figure 4, we present the ERCOT and Eastern time zone hubs and, in Figure 5, we present the Western network hubs. As is clear from these Figures, there is significant heterogeneity in the fitted forward

Table 2: Parameter Estimates,  $\alpha_e$  and  $\sigma_e$ , of the Exponential Hull-White Process for the Electricity Hubs (Average 2004–2010)

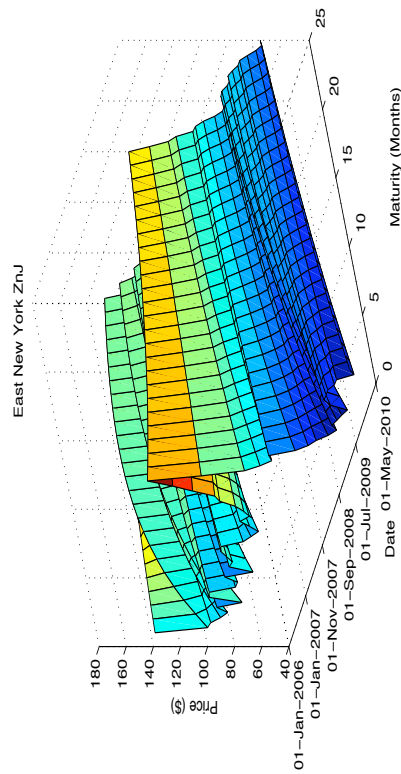
Region	$\alpha_e$	$\sigma_e$
East New York Zone J	0.352	0.313
ERCOT	0.417	0.525
Into Cinergy	0.231	0.384
Into Entergy	0.363	0.448
Into Southern	0.364	0.414
Into TVA	0.303	0.424
Mass Hub	0.279	0.353
Mid-Columbia	0.175	0.489
Northern Illinois Hub	0.190	0.437
North Path 15	0.236	0.457
Palo Verde	0.206	0.473
PJM West	0.272	0.347
South Path 15	0.212	0.446

price term structures both across hubs and between the Eastern, ERCOT, and Western networks, although there is more similarities within each power network. As shown, both the level and the slopes of the fitted forward price curves differ importantly over time. It is also interesting to note that these markets are often decoupled with some hubs exhibiting backwardated (downward sloping) forward curves while at the same time the forward curves for other hubs are in contango (upward sloping). The important differences in the time series dynamics and in the overall level of prices across the various maturities is also quite significant. Overall these curves suggest, that hub-specific heterogeneity in electricity pricing could potentially drive important differences in the relative default risk of mortgages collateralized by buildings located across these regions.

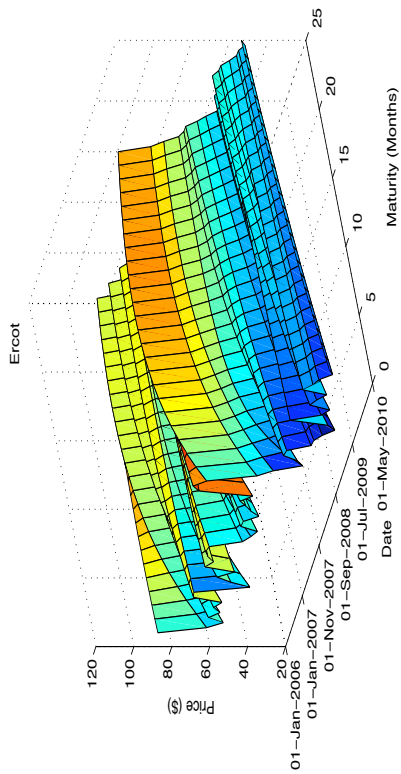
**Cross-sectional Differences** Figure 6 presents a snapshot at four dates: 1) January 1, 2006; 2) April 1, 2008; May 9, 2009; and March 30, 2010; for a cross-section of the fitted electricity forward curves for all the electricity hubs in the sample. As is clear from these cross-sections there are some dates, *e.g.* for May 9, 2009 and March 30, 2010, when the forward curves have very similar shapes although the level of prices to differ importantly. Whereas on other dates, *e.g.* for January 1, 2006, the Western hubs appear to move together and for other dates, *e.g.* for April 1, 2008, the ERCOT hub has a significantly different shape (it is in contango) while the term structure of forward rates for the other hubs are downward sloping. Again, Figure 6 strongly suggests that the cross-sectional differences in the risk of electricity exposure should be important in mortgage pricing across regions.



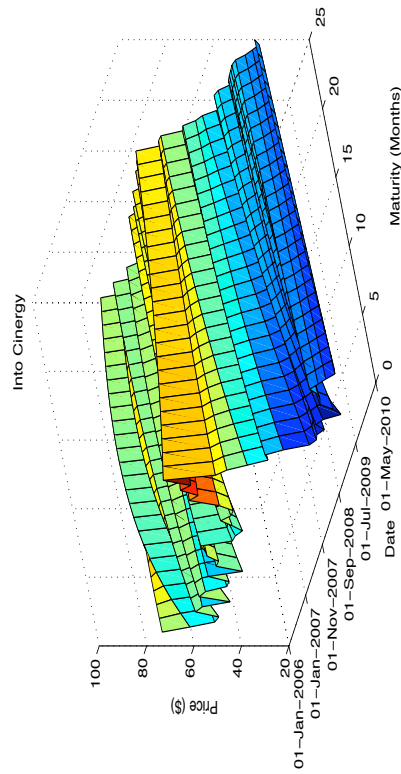
(a) ERCOT Hub



(b) East New York Zone J Hub



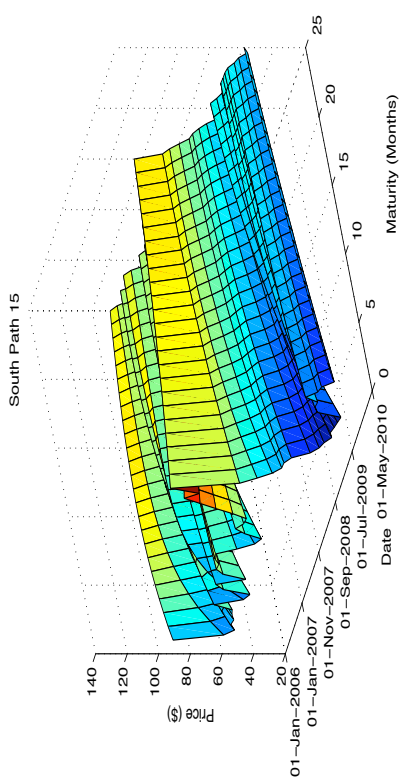
(c) PJM West Hub



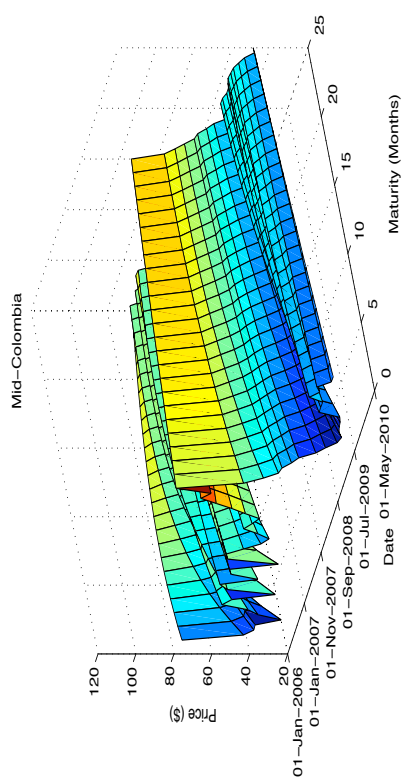
(d) Into Cinergy Hub

Figure 4: **Fitted Electricity Forward Curves, 2006–2010.** This figure plots our calibrated electricity forward curves for a selection of the ERCOT and Eastern electricity hubs. We plot the forward curve per month by the maturity of the contract.

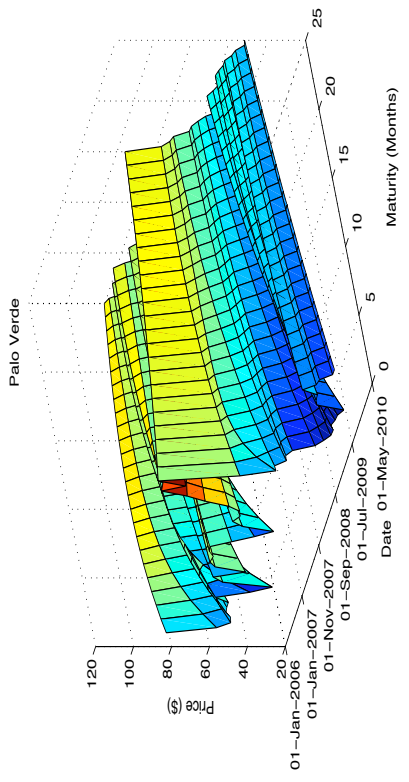




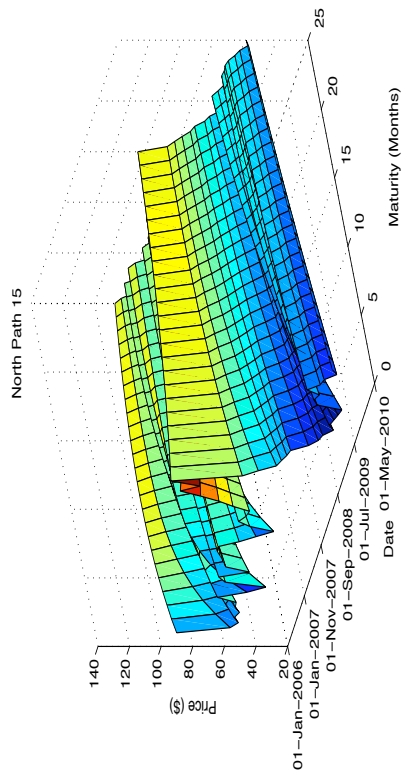
(a) Palo Verde Hub



(b) South Path 15 Hub

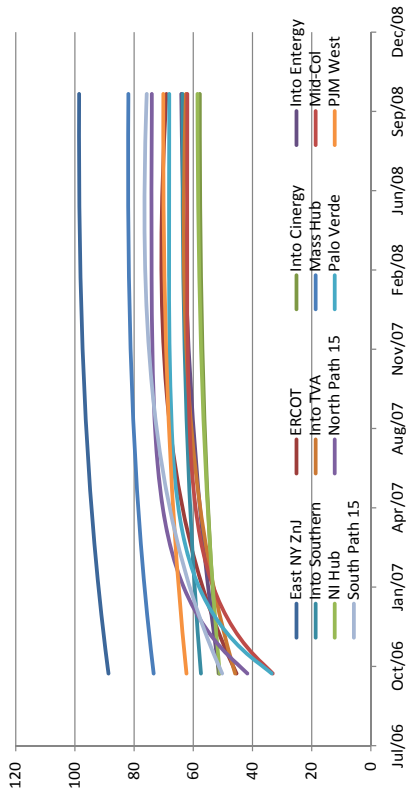


(c) North Path 15 Hub

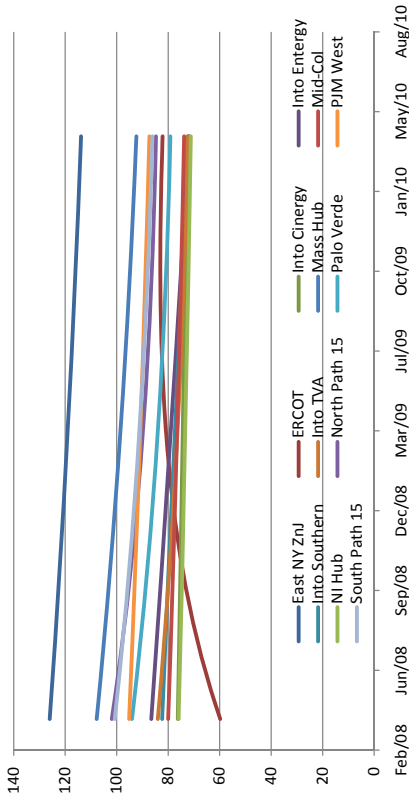


(d) Mid Columbia Hub

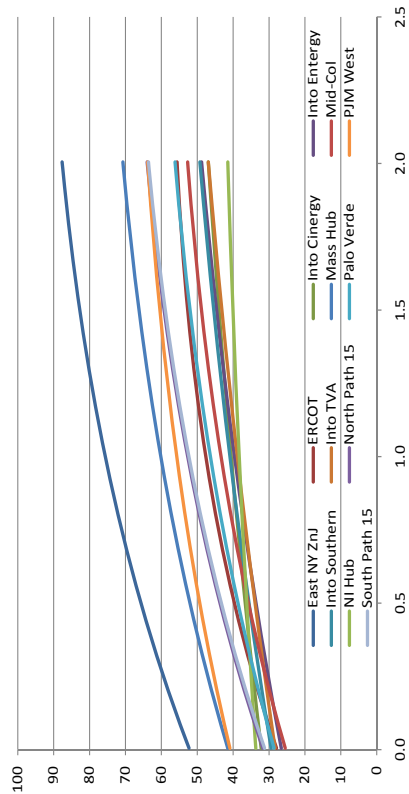
Figure 5: **Fitted Electricity Forward Curves, 2006–2010.** This figure plots our calibrated electricity forward curves for a selection of the Western electricity hubs. We plot the forward curve per month by the maturity of the contract.



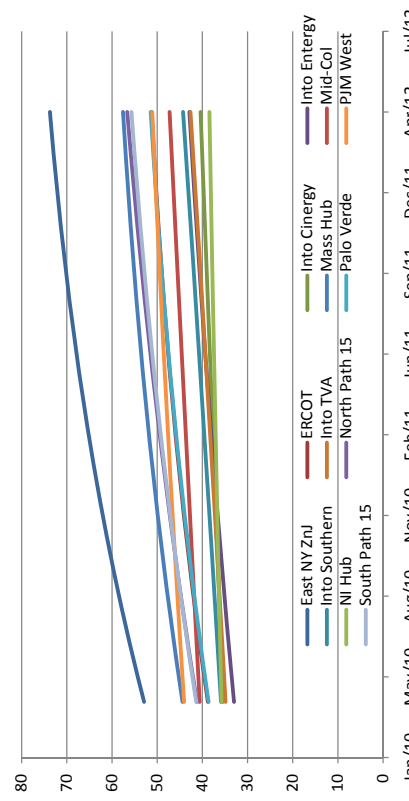
(a) January 1 2006



(b) April 1, 2008



(c) May 9, 2009



(d) March 30, 2010

Figure 6: Cross-Section of Fitted Electricity Forward Curves across the Electricity Hubs on Specific dates. This figure presents a cross section of forward curves for each hub on a single date.

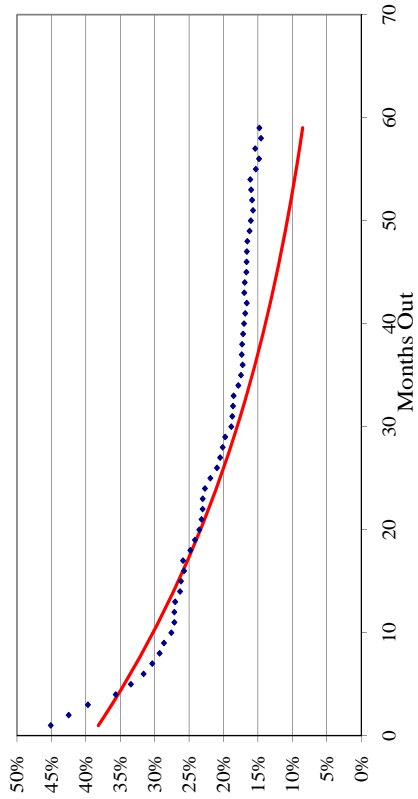
**Characteristics of Volatility** As was discussed previously, there is considerable volatility in electricity price dynamics. In Figures 7 and 8, we graph a cross-sectional comparison for the historical volatilities by maturity for the forward contracts up through November 2007. The Figures plot both the observed level of volatility by maturity and a fitted term structure of volatility. As volatility is an important determinant of the value of embedded mortgage default options, it is interesting to again note the important cross-sectional differences in the level and the slopes of these term structures. Interestingly, as shown in Figure 7 the level of volatility at the short maturity contracts is considerably higher than that for the Eastern time zone hubs. The ERCOT hub exhibits the highest volatility in the short maturity contracts across all of the hubs. Here again, these results suggest important potential differential in the expected value of embedded mortgage default options for mortgage written on building located across these hubs. As shown, these volatility levels exceed the volatility of rents,  $\phi_C = 21.478\%$ , and that of interest rates is about  $\sigma_r = 2.21\%$  (see Veronesi, 2010).

#### 5.4.2 Calibrating the Natural Gas Futures Curves

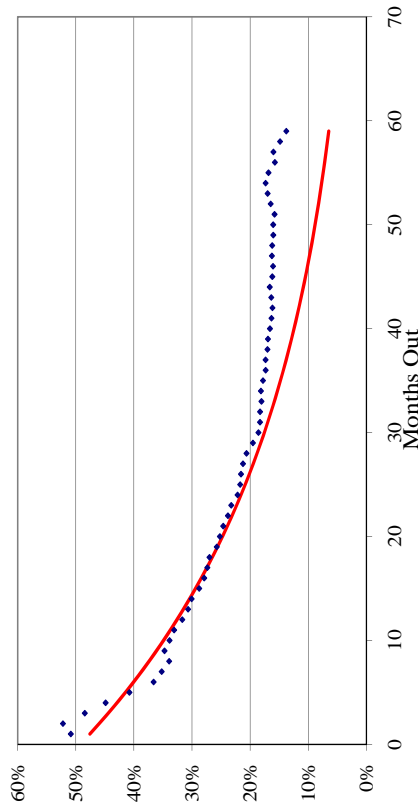
As previously discussed, there is only one major pricing hub for natural gas, the Henry Hub. As for the electricity hubs, we estimate the parameters for the exponential Hull-White process using data from Henry Hub NYMEX futures and options on NYMEX futures contracts. The time series average from 2004 through 2010 for  $\alpha_g$  is 79.1% (standard deviation, 3.7%) and is 58.7% (standard deviation, 1.4%) for  $\sigma_g$ . These values are again importantly larger than the volatility values for either the interest rate or building rent process.

**Time Series Dynamics** In Figure 9, we graph the NYMEX Henry Hub futures contract curves over time from 2006 through 2010. As shown there is significant times series variation in the shape and level of the natural gas futures price curve as a function of the maturity of the contracts. Again, the curves are backwarddated in some periods and in contango in others.

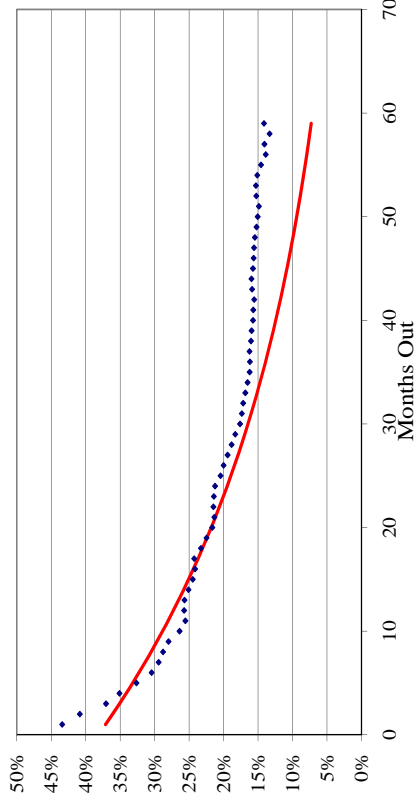
**Implied Volatility** In contrast with the electricity markets, we were able to gain access to a third-party dataset describing the term structure of at-the-money (ATM) implied volatilities of NYMEX Henry Hub futures contracts on a daily basis over the analysis period. This third-party dataset was built by backing out implied volatilities from market quotes of put and call option premia with strike prices near to the closing prices of the underlying futures contracts. Implied volatilities for near to ATM strikes were derived from straight application of the Black (1976) option pricing formula for futures contracts. The ATM implied volatilities were constructed by interpolation across nearby strikes.



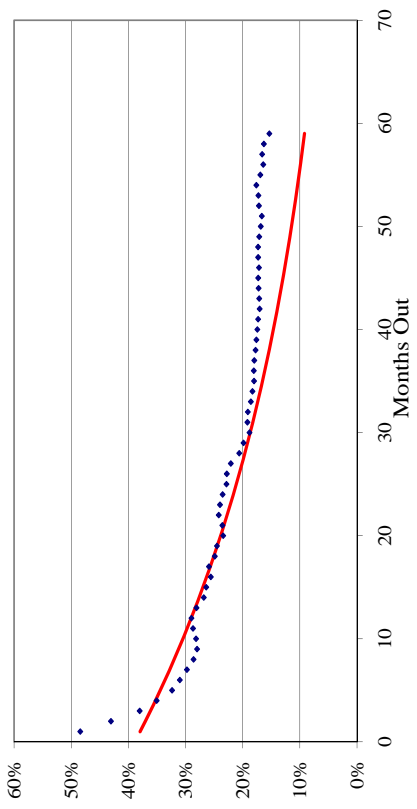
(a) Palo Verde Hub



(b) South Path 15 Hub

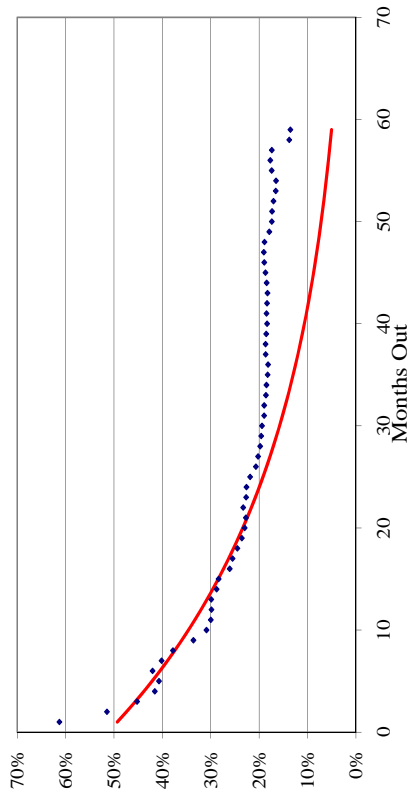


(c) North Path 15 Hub

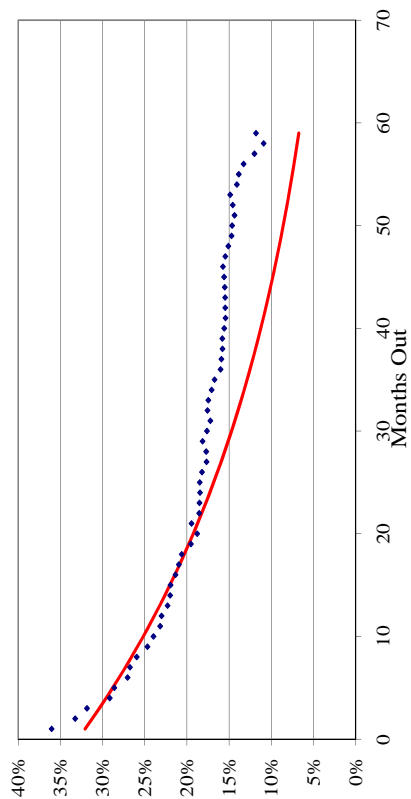


(d) Mid Columbia Hub

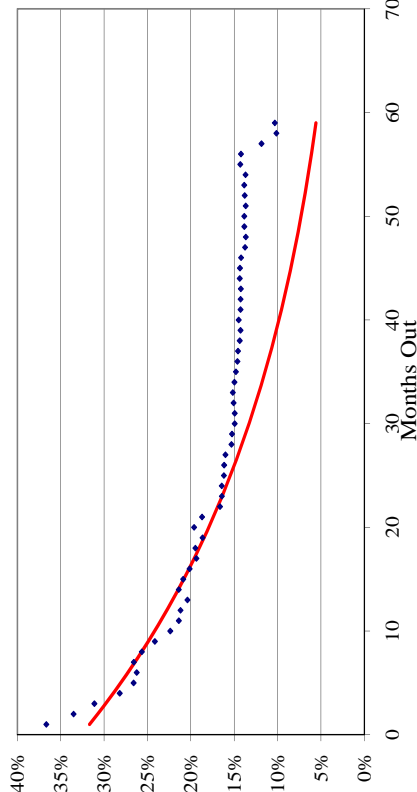
**Figure 7: Cross-Section of Historical Volatilities by Maturity of the Forward Contract on November 11, 2007.** This figure plots the historical volatilities for a selection of the Western Hubs. The dotted curve is the computed historical volatility for the forward contracts of given maturities. The solid line plots the fitted volatility curve.



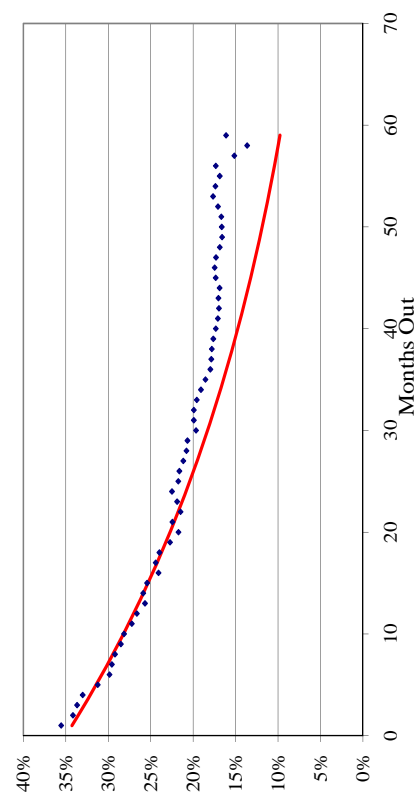
(a) ERCOT



(b) PJM East Hub



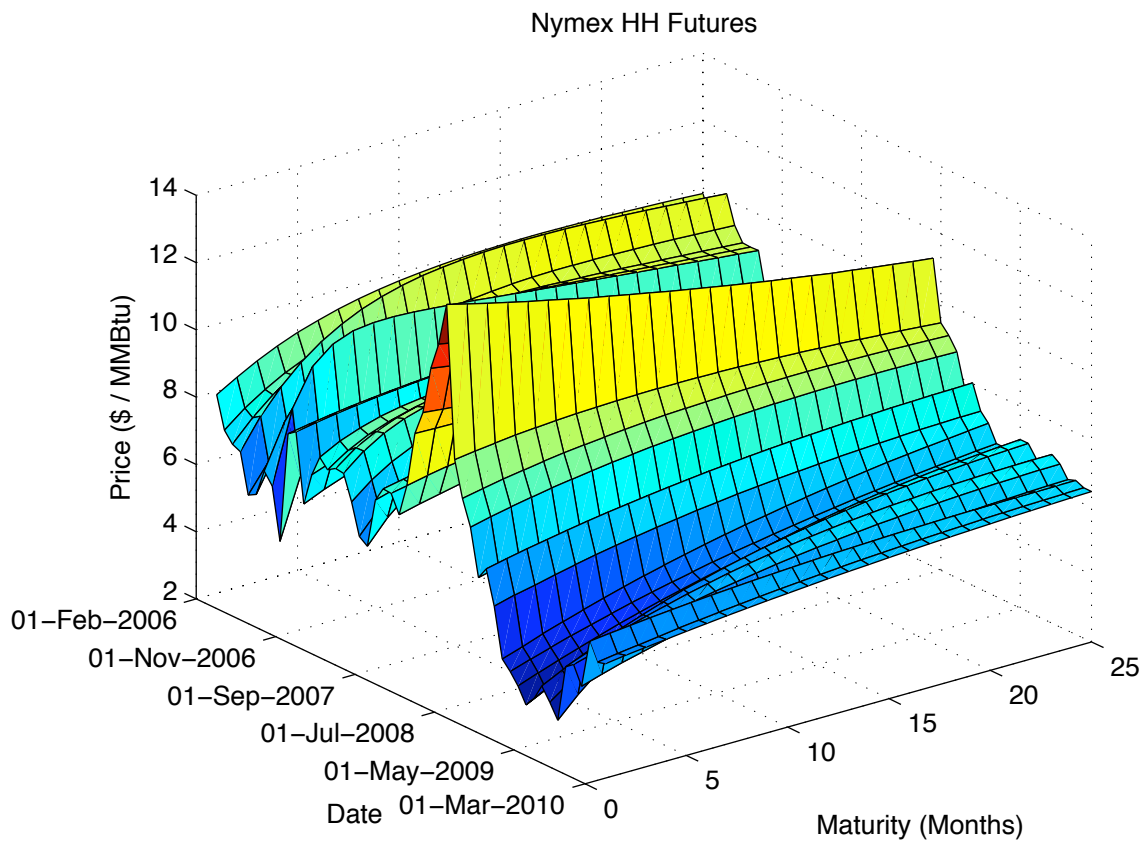
(c) East New York Zone J Hub



(d) Mid Columbia Hub

**Figure 8: Cross Section of Historical Volatilities by Maturity of the Forward Contract on November 11, 2007.** This figure plots the historical volatilities for a selection of the ERCOT and Eastern Hubs. The dotted curve is the computed historical volatility for the forward contracts of given maturities. The solid line plots the fitted volatility curve.

Figure 9: Estimated NYMEX Henry Hub Futures Contract Curves



From Black (1976), the implied volatility defines the variance of terminal future prices through the relationship

$$Var_t [\ln F_g(t, T)] = \overline{\sigma}_g^2 (T - t), \quad (9)$$

with  $\overline{\sigma}_g$  expressing the implied volatility.

Also, from the stochastic equation for futures contracts, we express the terminal variance of the logarithm of the futures price at maturity as

$$Var_t [\ln F_g(t, T)] = \int_t^T \sigma_g^2 e^{-2\alpha_g(T-u)} du = \frac{\sigma_g^2}{2\alpha_g} [1 - e^{-2\alpha_g(T-t)}] \quad (10)$$

By combining equations (9) and (10), the implied volatility is then expressed as

$$\overline{\sigma}_g = \sqrt{\frac{\sigma_g^2}{2\alpha_g(T-t)} [1 - e^{-2\alpha_g(T-t)}]}. \quad (11)$$

Although we were not able to characterize seasonal patterns in the term structure of instantaneous volatility for the electricity markets, we were able to do so for the natural gas markets. We identified the seasonal pattern by simply averaging the implied volatility by contract month (1 = January, 12 = December) over the whole time period.

For each trading date, the parameters  $\alpha_g$  and  $\sigma_g$  are calibrated by minimizing the sum of squares of the residuals of the term structure of ATM implied volatilities. We fit a curve of the form

$$\overline{\sigma}_g(t, T; m) = \sqrt{\frac{(\sigma_g \times sf_m)^2}{2\alpha_g(T-t)} [1 - e^{-2\alpha_g(T-t)}]}, \quad (12)$$

where  $m$  is the month of expiration of the futures contract and  $sf_m$  is the corresponding seasonal scaling factor tabulated above.

Figure 10 and Figure 11 plot the term structure of implied volatilities from options on NYMEX futures contracts on November 1, 2007 and for November 2, 2009. We plot the observed non-seasonally adjusted implied volatilities (the red plot); the non-seasonally adjusted fitted term structure of volatility (the blue plot); and the de-seasonalized fitted term structure of volatilities (the black plot) for the two dates. As shown in Figure 10 and Figure 11, there is considerable heterogeneity in the implied volatilities across option maturities at short maturities, but the curves fall to about the same level of implied volatility for the longer maturity options. Nevertheless, these volatilities are comparable to those of the electricity hubs and importantly exceed the levels of either interest rate volatility or the volatility of office rents.

Figure 10: NYMEX Natural Gas Implied Volatilities, November 1, 2007

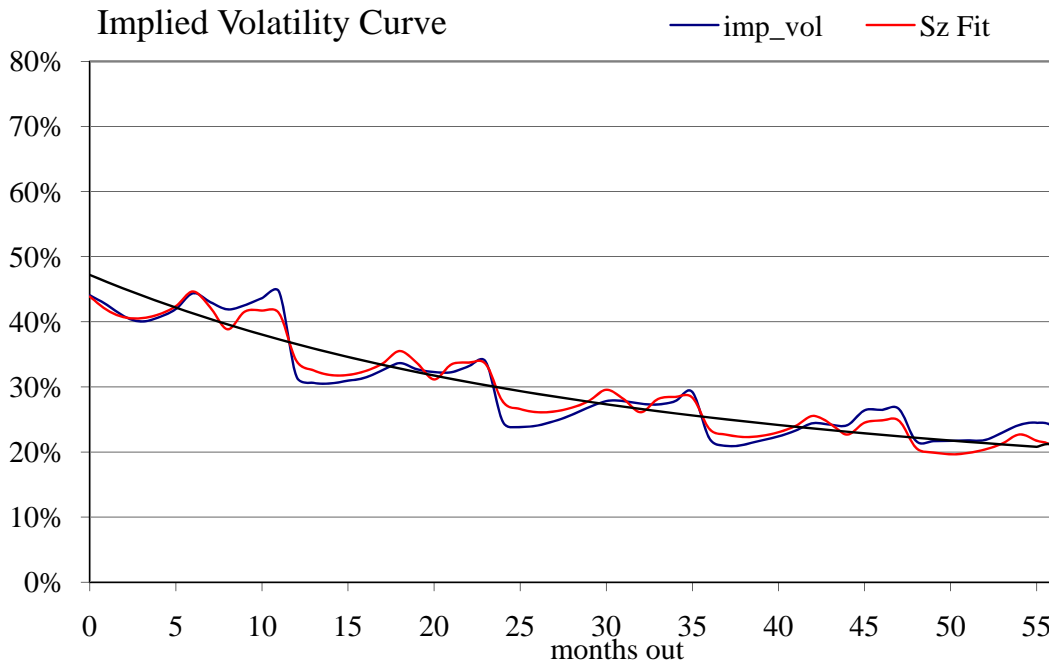
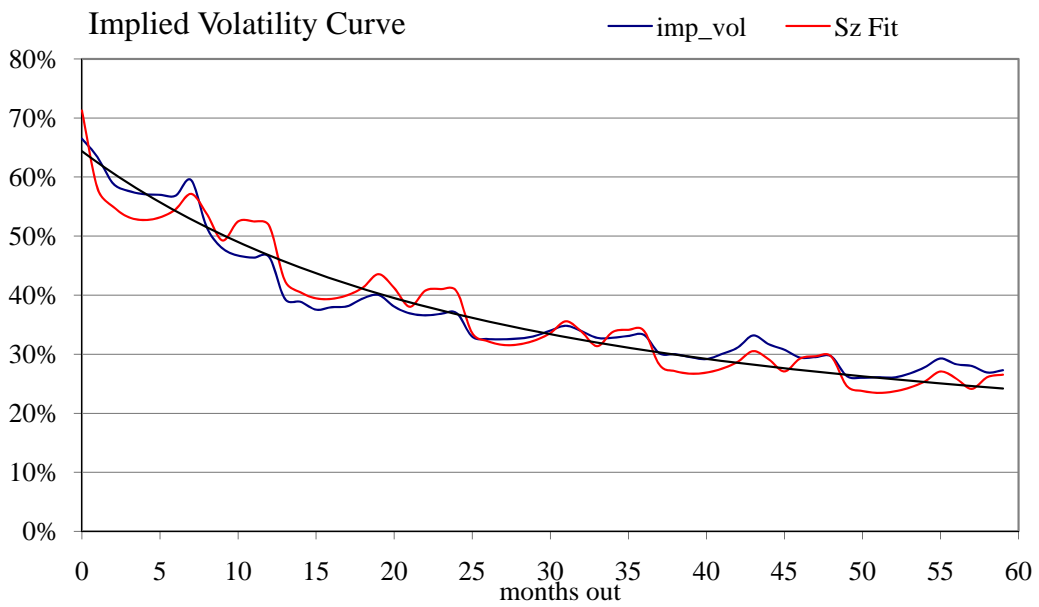


Figure 11: NYMEX Natural Gas Implied Volatilities, November 1, 2009





## 6 Two-Part Valuation Strategy

Overall, the fitted factor dynamics for interest rates, energy forward prices, and rents suggest that the energy prices could induce important volatility into cash flows and, therefore into building prices over time. Since commercial mortgage are long contracts, these results indicate that the volatility of energy costs to the building owner could swamp other costs such as janitorial services and the cost of building management staff.

### 6.1 Part I: Solving for Building-Specific Rental Drift

In order to obtain reliable mortgage values, it is important first to ensure that the valuation model we are using is consistent with the current price of the underlying building. As previously discussed, therefore, in *Stage I* of the valuation strategy on a given date, we fit the interest rate process, the electricity forward process, and the natural gas futures process, then solve for the implied building-specific, risk-adjusted drift for market rents,  $\mu_i$ , assuming a volatility of 21.478% (see Stanton and Wallace, 2011). The implied drift is the value that makes the valuation model exactly match the observed price of the building at the origination date of the mortgage, given the market dynamics of the three other market fundamentals.

Valuation of the building is performed using Monte Carlo simulation with antithetic variates to estimate the price as the (risk-neutral) expectation of future cash flows,<sup>14</sup>

$$P_t = E_t^* \left[ \sum_{k=1}^{\infty} CF_{t+k \Delta t} e^{-\Delta t \sum_{j=0}^{k-1} \tilde{r}_{t+j \Delta t}} \right]. \quad (13)$$

Estimating the expectation in Equation (13) involves three steps:

1. Simulate 10,000 paths for rent, interest rates, gas prices, and electricity prices using the risk-neutral processes described above.
2. Calculate the monthly building cash flow (NOI) along each path from Equation (3).
3. Discount each path's cash flows back to the present, and average across all paths.

We repeat this process for various different values of  $\hat{\mu}$  in Equation (5), searching numerically until we find the value that makes the building price produced by the Monte Carlo valuation equal to the known price of the building at the mortgage origination date.<sup>15</sup>

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<sup>14</sup>For details see, for example, Boyle (1977); Boyle, Broadie, and Glasserman (1997); Glasserman (2004).

<sup>15</sup>In performing this search, it is important to use the same set of random numbers for each valuation.

## 6.2 Part II: Solving for Mortgage Value

Valuing mortgages using Monte Carlo is very similar to the process described above for calibrating the risk-neutral drift. Specifically, we start by writing the mortgage value as the risk-neutral expected present value of its future cash flows using Equation (13) again. Then we use Monte Carlo simulation to estimate the expectation the same way as above.

1. Simulate 10,000 paths for rent, interest rates, gas prices, and electricity prices using the risk-neutral processes described above.
2. Calculate the monthly cash flows for the mortgage along each path.
3. Discount each path's cash flows back to the present, and average across all paths.

While structurally similar, there are two significant differences between the two valuations, both related to step 2, the calculation of the mortgage cash flows along each path:

1. Commercial mortgages include embedded default options, and when borrowers exercise these options, this affects both the amount and the timing of the mortgage cash flows. To model the borrowers' default behavior, we therefore introduce an empirical hazard model, a model for the estimated conditional probability that a mortgage will default given its survival times, into *Stage II*.
2. Because the likelihood of default at any instant depends on the loan-to-value ratio (LTV), we need a way to estimate the building's value not just at the mortgage origination date, but rather at every date along every path.

We now discuss each of these differences in detail.

### 6.2.1 Empirical Default Hazard Model

Following standard mortgage-valuation practice (see Schwartz and Torous, 1989), the default hazard for the loans is estimated using a time-varying-covariate hazard model with a log-logistic baseline hazard.<sup>16</sup> Our model also includes controls for loan characteristics including the amortization structure, the loan coupon, amortizing maturity of the loan, the principal due date on the loan, the time varying loan-to-value ratios of the building, and a measure of the difference between the coupon on the loan and the time varying 10 year Treasury rate which is the measure for current interest rates.

We estimate the proportional-hazard model using a sample of 8,497 loans on commercial office buildings that were originated between 2002 and 2007. These data were obtained from Trepp LLC loan-level performance data and include all the origination information on

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<sup>16</sup>For details on hazard models see, for example, Cox and Oakes (1984).

Table 3: Office Loan-level Estimates for the Default Hazard

	Coeff. Est.	Std. Err.
$\gamma$	0.0019***	0.00026
$p$	1.94387***	0.0898
Current Coupon minus Treasury(t)	0.1613**	0.04561
Loan-to-Value Ratio(t)	0.5771**	0.02225
Number of Observations		8,497

$t$  statistics in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

the mortgages along with monthly performance records. The estimated hazard rate is the conditional probability that a mortgage will terminate in the next instant, given that it has survived up until then. Hazard models comprise two components: 1) a baseline hazard that determines the termination rates simply as a function of time and 2) shift parameters for the baseline defined by the time-varying evolution of exogenous determinants of prepayment and default. We define default as a 90-day delinquency on the loan, and model its occurrence via the hazard function

$$\pi(t) = \pi_0(t)e^{\beta\nu}, \quad \text{where} \quad (14)$$

$$\pi_0(t) = \frac{\gamma p (\gamma t)^{p-1}}{1 + (\gamma t)^p}. \quad (15)$$

The first term on the right-hand side of Equation (14) is the log-logistic baseline hazard, which increases from the origination date ( $t = 0$ ) to a maximum at  $t = \frac{(p-1)^{1/p}}{\gamma}$ . This is shifted by the factor  $e^{\beta\nu}$ , where  $\beta$  is a vector of parameters and  $\nu$  a vector of covariates including the end-of-month difference between the current coupon on the mortgage and U.S. Treasury rates and the current loan-to-value ratio of the mortgage.

The results of our hazard models are reported in Table 3. As expected, there is a statistically significant, positive coefficient on the difference between the coupon rate on the mortgage and the observed 10 year Treasury rate and a statistically significant and large positive coefficient on the current loan-to-value ratio of the loan. Thus, our empirical hazard suggests that loans will default when the difference between the coupon on the loan and the current 10 year U.S. Treasury rate is large and, reasonably, when the value of the loan relative to the value of the building is high.

## 6.2.2 Empirical Building Value Estimator

In order to estimate default rates along each path, we need an estimate of the building value at every time along each path. In principle, we could just perform a new simulation at every time step along every path, but this would be computationally infeasible. Instead, therefore, we construct an empirical model for building value as a function of current NOI, interest rates and other variables. This is similar to Boudoukh, Richardson, Stanton, and Whitelaw (1997), who used nonparametric regression to estimate the value of mortgage-backed securities as a function of interest rates.<sup>17</sup> Assuming a constant expected growth rate for NOI per square foot and a flat term structure of risk-adjusted discount rates the value,  $P(t)$ , of an office building per square foot would be given by the Gordon growth model (in logs),

$$\ln P(t) = \ln NOI(t) - \ln(i - g), \quad (16)$$

where  $g$  is the market growth rate for net operating income,  $i$  is the risk-adjusted discount rate, and we assume  $i - g > 0$ . This is the basis of our empirical valuation estimator, which we adjust via the inclusion of other explanatory variables.

Based on Equation (16), we fit the following estimator for building values:

$$\ln P(t) = \beta_0 + \beta_1 \ln NOI(t) + \beta_2 \ln i(t) + \beta_3 \ln (p_{gas}(t) \times q_{gas}) + \beta_4 \ln (p_{elec}(t) \times q_{elec}), \quad (17)$$

where  $\ln P(t)$  is the natural log of the price per square foot of the building on the transaction month  $t$ ,  $\ln NOI(t)$  is the natural log of the annual net operating income per square foot on the transaction month  $t$ ,  $\ln i(t)$  is the natural log of the ten-year Treasury rate for the transaction month  $t$ ,  $p_{gas}(t)$  is the average spot price of gas per kBTU for the transaction month  $t$ , and  $q_{gas}$  is the annual benchmark level of natural gas consumption (kBTU) per square foot for buildings of a corresponding size and location to those reported in Table 1,  $p_{elec}(t)$  is average spot price per kWh of electricity for the transaction month  $t$ , and  $q_{elec}$  is the annual benchmark level of electricity consumption (kWh) per square foot for buildings of a corresponding size and location to those reported in Table 1.

To estimate our building value estimator, we construct a data set that combines two separate transaction data sets: 1) the CoStar Group data; 2) the Trepp LLC data. The Costar data is a comprehensive data set that is maintained by leasing and sales brokers in commercial real estate industry. The data offers comprehensive coverage of transactions

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<sup>17</sup>We also use this model in the drift calibration. We simulate out to year 10, then use the empirical valuation model to estimate the building's "terminal value" in year 10. This is similar to the use of short-cut methods such as valuation multiples in estimating the terminal value when valuing a business (see Berk and DeMarzo, 2007).

across the U.S., although its best geographic coverage is for Western States. We use only the CoStar transactions that are arms-length and confirmed market transactions.<sup>18</sup> The data also include information on the overall building characteristics (building and lot square footage, typical floor area square footage, numbers of floors, etc), how many tenants, the location, and quality characteristics of the building, information on the first and second lien amounts, and the lien periodic payment amounts. For a subset of these data, there is also information on the annual net operating income at sale, the gross rent at sale, and the operating expenses at sales. We then further restricted our sample to the transactions for which we have complete information on transaction characteristics as well as complete information on the annual net operating income at sale, the gross income at sale, and the total annual expenses at sale. This further restriction generated a sample of 1,540 observations from the CoStar transaction data.

Our second data set is obtained from Trepp LLC. Trepp is a data vendor widely regarded as the most accurate source of data on the securitized commercial loan market in the U.S. We restricted the Trepp commercial loan data to those loans that were for transactions and for which we had information on the annual net operating income at sale, the gross rent at sale, and the operating expenses at sales. This restriction leaves us 3,551 transactions. One limitation of the Trepp data is that we only have the underwritten appraised value of the building at the loan origination, rather than the true sales prices. We therefore assume that the appraised value is the market price of the office building. As shown in Table 4 the two samples are not that different. Trepp has slightly more expensive buildings, however, the sample distributions for the revenues and operating expenses levels are comparable for the two data sources. Given this comparability we merge the two data sets together for a total transactions data set of 5,092 observations.

As shown in Table 4 the two data set are quite comparable in revenues and expenses per square foot. The sample of Trepp transactions appear to have sold for a slightly higher price than those of CoStar. We consider this differences, however, to be minor and we proceed to fit our building value estimator on the joint sample of 5,091 office transactions. The summary statistics for the merged transaction data are presented in Table 5. As shown in Table 5 overall these are fairly large office buildings with an average transaction price of about \$195 per square foot. Annual rents were averaged about \$22 per square foot and annual operating expenses averaged about \$7.7 per square foot. The electricity and

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<sup>18</sup>We eliminate all transactions for which there was a “non-arms-length” condition of sale due to such factors as a 1031 Exchange, a foreclosure, a sale between related entities, a title transfer, among other conditions. All of these sale conditions would affect prices due to the trading of tax basis in the case of 1031 exchanges or the auction structure in the case of foreclosure. Instead, we focus only on market transactions between unrelated persons.

Table 4: Sale Transactions Summary Statistics

	N	Mean	Standard Deviation	Minimum	Maximum
CoStar Sample					
Annual Price (\$ per Square Foot)	1540	174.20	105.00	6.04	737.25
Annual Revenue (\$ per Square Foot)	1540	21.85	9.21	10.00	134.56
Annual Expenses (\$ per Square Foot)	1540	7.56	4.17	1	73.11
Trepp Sample					
Annual Price (\$ per Square Foot)	3551	205.19	102.81	10.03	872.30
Annual Revenue (\$ per Square Foot)	3551	22.11	9.20	10.03	169.55
Annual Expenses (\$ per Square Foot)	3551	7.80	3.83	1.05	76.89

Table 5: Sale Transactions Summary Statistics

Variable	N	Mean	Std. Deviation	Minimum	Maximum
Annual Price (\$ per Square Foot)	5091	195.81	104.44	6.04	872.30
Annual Revenue (\$ per Square Foot)	5091	22.03	9.20	10.00	169.55
Annual Expenses (\$ per Square Foot)	5091	7.72	3.94	1.00	76.89
Ten Year Treasury Rate (%)	5091	4.50	0.00	2.90	5.28
Gas Spot Price (\$ per kBTU)	5091	0.01	0.00	0.00	0.01
Electricity Spot Price (\$ per kWh)	5091	0.07	0.02	0.03	0.16
Building Size (Square Feet)	5091	100350.34	127589.63	15575.00	998770.00
Annual Electricity Consumption (kWh per Square Foot)	5091	1.19	0.35	0.42	3.20
Annual Gas Consumption (kBTU per Square Foot)	5091	6.53	3.89	0.01	75.81

gas consumption information for each building was obtained from the CEUS and CBECS benchmark information provided in Table 1 and discussed above.

The results of estimating our building value estimator are reported in Table 6. As shown, the estimator explains about 68% of the observed variance in building prices in the same. As expected the log of net operating income has a statistically significant and positive effect on log price per square foot and the log of the 10 year Treasury rate has a statistically significant and negative effect on log price per square foot. We include the additional covariates to capture the additional effects of energy costs on building transactions prices per square foot. As shown in Table 6, we find that the log of natural gas and electricity prices have a positive effect on log price and the benchmarked consumption levels of gas and electricity for buildings have a negative effect on log prices.

## 7 Valuation Application

Following Figure 3, the valuation of a specific loan requires data for the market term structure of interest rates and volatility, the market energy and natural gas forward curves and their volatility, and the calibrated market rent process for the building. These data need to be

Table 6: Estimation Results for the Office Building Valuation using the Trepp and CoStar Merged Data

Variable	Coefficient Estimate	Standard Error
Intercept	2.760***	0.068
Natural Log of Net Operating Income per square foot	0.898***	0.010
Natural Log of the 10 Year Treasury Rate	-7.100***	1.168
Natural log (Gas Spot Price $\times$ Gas Consumption)	-0.071***	0.009
Natural log (Electricity Spot Price $\times$ Electricity Consumption)	-0.201***	0.017
$R^2$	0.690	

Includes city fixed effects

$t$  statistics in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

collected for the same date, since the model is pricing the mortgage relative to current market data. Usually, the date that would be selected would be the mortgage origination date, however, the modeling strategy could be applied to value the mortgage contract at any point prior to its maturity. Typically, at origination, the lender is seeking to select the contractual features of the commercial loan such that its market price, given observed market fundamentals, is equal to the amount of principal borrowed. Thus, the lender typically structures the contract so that its market price at origination is equal to par.<sup>19</sup>

## 7.1 Loan Valuation Results

To implement our valuation strategy, we first find all of the loans in the Trepp LLC data set that were originated in September 2004. There were thirteen such loans, and we have CEUS/CBECS natural gas and electricity consumption data for the cities in which these loans were originated. We then fit the Hull-White model for the term structure of interest rates on that date and obtain the estimated values of  $\alpha_r = .128$  and  $\sigma_r = 0.12$ . Similarly, we use the location of the loan to identify the appropriate electricity forward curve for September 2004. These are presented in Table 7. As shown in Table 7, even in this small sample the differences between the hubs is quite apparent. We also fit the forward curve for natural gas NYMEX futures and options at the Henry Hub in September 2004, obtaining a value of 0.512 for  $\sigma_g$  and 0.837 for  $\alpha_g$ . Finally, conditional on the fitted dynamics of interest rates and the electricity and natural gas forward prices, we fit the drift of the GBM for the building specific rent processes for each loan in *Stage I* of the valuation strategy. These are

<sup>19</sup>That is the ratio of the market price of the discounted cash flows mortgage net the embedded option values to the principal on the loan is equal to one hundred percent.

Table 7: Historical Averages of  $\alpha_e$  and  $\sigma_e$ , for the Exponential Hull-White Electricity Process for the Thirteen Loan Sample

Electricity Hub	Network	$\alpha_e$	$\sigma_e$
ERCOT	Texas Interconnected System	0.42	0.53
East NY Zone J	Eastern Interconnected System	0.35	0.31
Into Cinergy	Eastern Interconnected System	0.23	0.38
Into TVA	Eastern Interconnected System	0.30	0.42
North Path 15	Eastern Interconnected System	0.24	0.46
South Path 15	Eastern Interconnected System	0.21	0.45

reported in Table 9.

To implement our model, we need information on each of the key contract elements for every loan that will be modeled. As shown in Table 8, across the thirteen loans there is considerable variety in the building square footage, the value of the building, the size of the loan, the coupon rates, and the loan-to-value ratios. Several of the buildings are in a common electricity hub, so that even in this small sample we can compare across hubs and across buildings and loans with differing characteristics. As reported in Table 8, the loans are quite similar in their amortization maturities and in their balance-due maturities. Not surprisingly, because these loans were all originated in the same month, they have quite similar coupons although the loans with higher loan-to-value ratios appear to have higher coupons, as expected, and loans on large buildings have higher coupons. Overall, the loans are collateralized by large buildings. The average building size is about eighty-nine thousand square feet with a standard deviation of about fifty three thousand square feet. The average building price of the thirteen-loan sample is about \$13 million, with a standard deviation of about \$7.4 million. The minimum building price in the sample is \$4.4 million. As is clear from Table 8, the expected energy consumption by region is highly variable.

We carry out three simulation exercises for each loan:

1. We value the loan conditional on the dynamics of interest rates, rents, and energy prices but without default risk. Thus, this pricing exercise values the contractual features of the loan and compares their present value with the observed balance on the loan. The idea behind this analysis is that the discounted present value of the loan at origination should equal the balance on the loan at origination if the loan was fairly priced. This valuation exercise, of course, ignores the default risk of the mortgages so we expect our model should price at a premium compared to the balance of the loan since the actual loan should have been structured to account for the risk of default.
2. In the second set of simulations we allow the mortgages to default. However, we do not enable the energy channels. Instead, we follow the traditional modeling procedure



of accounting for only the rent and interest rate dynamics in valuing a commercial mortgage contract with an embedded default option.

3. In the third set of simulations for each loan, we enable the energy channel. Here again we allow for mortgage default. In this set of simulations, our mortgage valuation strategy accounts for the dynamics of power and natural gas in valuing the commercial mortgage contract with an embedded default option.

We report the results of our mortgage valuations in Table 9. In the second, third and fourth columns of the Table 9, we report the geographic location of the loan and its electricity hub. In the fifth through seventh columns of Table 9, table we report the initial loan-to-value ratio, the building square footage, and the building value at the origination of the loan. In column eight, we report the *Stage I* estimate of the risk adjusted drift of rents for the building.

The simulation results are reported in the last three columns of Table 9. As shown, on average, the estimated value of the loans when default is not enabled is quite close to the observed balance on the loans at origination in September 2004. This result appears, at face, surprising because it suggests that lenders were, generally, underpricing the default options on these loans at the time these loans were originated.<sup>20</sup>

In column 10 of Table 9, we report the results of explicitly pricing the default option in the thirteen loans originated in September 2004 using the traditional mortgage valuation methodology, which controls for the effects of rent and interest rate dynamics to value the embedded default options. As shown, the default options embedded in these mortgage are valuable and all of the loans appear to be worth significantly less than the balance of the loans at origination. An important caveat is that our hazard function (see Table 3 for the coefficient estimates) is estimated using loan-level performance data from 2002 through 2010. These data include a higher incidence of 90-day delinquencies, our proxy for default, than would have been forecasted using available loan-level mortgage data prior to the origination date of these loans. Thus, our model is forecasting more default than lenders could have forecasted using the same model prior to September 2004, simply because the incidence of mortgage delinquencies in earlier vintages of commercial mortgages were nearly nonexistent. Nevertheless, for the purposes of the simulations reported here, the elevated levels of forecasted default is useful because it allows us to highlight the differences in mortgage valuations that are obtainable from using a two-factor versus a four-factor valuation model. Our results should, however, not be interpreted to mean that on average lenders were *ex ante*

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<sup>20</sup>At the time these loans were originated the historical default levels (from 1996 through 2004) for securitized commercial mortgages were less than 4%. Thus, the empirical hazard models that were implemented during this period predicted *de minimis* levels of expected default and the forecasted value of commercial mortgages was essentially the discounted present value of the default-free contract cash flows.

Table 8: Summary Statistics for the Characteristics of the Thirteen Mortgages Used in the Simulations, September 2004

Loan Number	Origination Date	Annual			Annual			Annual			Loan to		
		Electricity Consumption (kWh/psf-yr)	Gas Consumption (kBtu/psf-yr)	Gross Rents (\$ psf)	Non-Energy Expenses (\$ psf)	Maturity (Months)	Balance Due (Months)	Value Ratio (%)	Building Price (\$ 000)	Loan Coupon (%)	Building Size (000 Sqft)		
Loan1	Sept. 2004	13.8	40.5	13.7	3.78	360	120	74.0	7,610.0	5.7	70.7		
Loan2	Sept. 2004	13.8	40.5	10.4	3.48	360	120	76.2	8,000.0	5.6	124.9		
Loan3	Sept. 2004	18.4	31.3	19.2	7.87	360	120	77.1	22,000.0	5.8	176.2		
Loan4	Sept. 2004	18.4	31.3	17.1	5.93	120	120	59.5	21,500.0	5.3	206.4		
Loan5	Sept. 2004	13.8	7.4	21.6	6.7	360	120	67.9	9,560.0	5.9	41.6		
Loan6	Sept. 2004	13.8	7.4	24.4	8.43	360	122	74.8	16,200.0	5.5	77.0		
Loan7	Sept. 2004	15.3	15.5	10.2	2.76	360	120	74.8	12,000.0	5.6	123.4		
Loan8	Sept. 2004	13.8	8.1	16.0	5.32	360	120	65.2	11,100.0	6.2	82.4		
Loan9	Sept. 2004	13.1	12.6	23.3	5.17	360	120	77.9	14,350.0	5.8	71.5		
Loan10	Sept. 2004	13.1	12.6	16.5	2.96	360	120	78.7	6,350.0	5.7	41.1		
Loan11	Sept. 2004	10.1	17.6	21.3	5.99	360	120	70.9	4,360.0	5.9	24.1		
Loan12	Sept. 2004	21.8	5.3	18.6	5.83	300	120	61.4	6,000.0	6.1	43.3		
Loan13	Sept. 2004	12.0	13.4	25.3	6.54	240	240	69.3	27,000.0	5.5	123.8		
Average		14.7	18.7	18.3	5.4	327.7	129.4	71.4	12,771	5.7	92.8		
Standard Deviation		3.0	12.2	4.7	1.7	69.1	31.9	6.1	6,788.0	0.3	53.0		

mispricing the default options in these thirteen loans. We now know *ex post* that this was true, however, the point of our exercise is to show that *ex ante* the traditional mortgage valuation methodology would have mispriced the loans because it ignores the effect of energy price dynamics on the value of commercial mortgage default.

The results of valuing the thirteen mortgages, again as of their origination date in September 2004, using the full four-factor mortgage valuation model, are reported in column 11 of Table 9. As shown in Table 9, the inclusion of the energy channels generates mortgage values that are now on average about 8.89% below the value of the mortgages using the traditional modeling approach (which ignores the energy channel in valuing the embedded default options). For the most part these valuation reductions are larger for mortgages that are collateralized by larger buildings (which exhibit higher energy consumption on average) and mortgages with higher loan-to-value ratios. For some of the smaller buildings the effects are significantly smaller. Ignoring the dynamics of the energy factor inputs would lead lenders to significantly misprice the default risk of mortgages. On average, in the Trepp loan origination data, gas and electricity benchmarked expenditures represent about 19% of observed overall expenditures per square foot. Our results suggest that by not explicitly accounting for the important volatility of the energy related expenditures would lead to important overall levels of expected mortgage mispricing.

Based on the results in Table 10, we report the number of points that lenders would have had to charge the borrowers on origination day for the default options. On average, the lenders would have had to charge about 18.8 points to assure that the market price of the loan with the embedded default options was valued at the same amount that was dispersed to the borrower in principal. Of course, this conclusion assumes that the loan contract terms would remain the same. A more likely outcome would be that lenders, once they account for the dynamics of interest rates, rents, and energy using a full valuation framework, would instead alter the original loan terms, particularly the loan-to-value ratio, such that the loan would price to par.

Recent research has shown that it is not unusual to see 10–20% savings in energy consumption in some buildings with very simple energy recommissioning retrofits because the existing operations of many commercial buildings is very inefficient (see Mills, 2009). In the spirit of this type of retrofit, we reduce the benchmarked electricity and natural gas consumption of each building by 20% and then re-estimate the value of the mortgage on the more efficient, but otherwise, equivalent building. As shown in the last column of Table 11 the value of the mortgages on these buildings is now, on average, about 1.3% higher than the value reported in column 11 in Table 9. The size of these elasticities varies importantly across buildings, mortgage contract structures, and regions. Overall, the reductions in energy con-

Table 9: Mortgage Valuation Results, September 2004

Loan	City	State	Electricity Hub	Building Value (000)	Loan to Value (%)	Building Size (Sqft) (000)	Initial Balance (000)	$\hat{\mu}$	Non Defaultable		No Energy Effects		Energy Effects		Difference (%)
									Mortgage Value (000)	Mortgage Value (000)	Mortgage Value (000)	Defaultable Mortgage Value (000)	Mortgage Value (000)	Defaultable Mortgage Value (000)	
Loan 1	Cincinnati	OH	Into Cinergy	7,610	74	71	5,631	-0.018	5,783	5,243	4,881	6.90			
Loan 2	Cincinnati	OH	Into Cinergy	8,000	76	125	6,096	-0.0202	6,200	5,225	4,838	7.41			
Loan 3	Hartford	CT	East New York Z J	22,000	77	176	16,962	-0.0124	17,166	14,225	12,498	12.14			
Loan 4	Hartford	CT	East New York Z J	21,500	60	206	12,793	-0.0188	12,632	11,847	10,326	12.84			
Loan 5	Los Angeles	CA	South Path 15	9,560	68	42	6,488	-0.021	6,745	6,187	5,778	6.61			
Loan 6	Los Angeles	CA	South Path 15	16,200	75	77	12,118	-0.0098	12,140	10,607	9,573	9.75			
Loan 7	Orlando	FL	Into TVA	12,000	75	123	8,972	-0.0056	9,049	8,101	7,403	8.62			
Loan 8	Riverside	CA	South Path 15	11,100	65	82	7,237	-0.0099	7,301	6,901	6,339	8.14			
Loan 9	Sacramento	CA	North Path 15	14,350	78	71	11,179	-0.0223	11,156	10,316	9,084	11.94			
Loan 10	Sacramento	CA	North Path 15	6,350	79	41	4,997	-0.0196	5,111	4,682	4,365	6.77			
Loan 11	Sacramento	CA	North Path 15	4,360	71	24	3,091	-0.0184	3,189	2,956	2,712	8.25			
Loan 12	San Antonio	TX	ERCOT	6,000	61	43	3,684	-0.0137	3,845	3,557	3,369	5.29			
Loan 13	San Francisco	CA	North Path 15	27,000	69	124	18,711	-0.0192	18,835	16,470	14,663	10.97			
Average				12,772	71	93	9,074	-0.016	9,165.54	8,178.23	7,371.46	8.89			
Standard Deviation				7,065	6	55	4,957	0.005	4,936.12	4,213.43	3,639.51	2.42			

Table 10: Points That Would Be Required to Price the Mortgages at Par for Mortgage Origination, September 2004

Loan	City	Electricity Hub	Energy Channels		Points Required to Price at Par (Percent)
			Initial Balance (000)	Defaultable Mortgage Value (000)	
Loan 1	Cincinnati	Into Cinergy	5,631	4,881	13.3
Loan 2	Cincinnati	Into Cinergy	6,096	4,838	20.6
Loan 3	Hartford	East New York Zone J	16,962	12,498	26.3
Loan 4	Hartford	East New York Zone J	12,793	10,326	19.3
Loan 5	Los Angeles	South Path 15	6,488	5,778	10.9
Loan 6	Los Angeles	South Path 15	12,118	9,573	21.0
Loan 7	Orlando	Into TVA	8,972	7,403	17.5
Loan 8	Riverside	South Path 15	7,237	6,339	12.4
Loan 9	Sacramento	North Path 15	11,179	9,084	18.7
Loan 10	Sacramento	North Path 15	4,997	4,365	12.6
Loan 11	Sacramento	North Path 15	3,091	2,712	12.3
Loan 12	San Antonio	ERCOT	3,684	3,369	8.6
Loan 13	San Francisco	North Path 15	18,711	14,663	21.6
Average			9,074	7,371	18.8
Standard Deviation			4,957	3,640	26.6

sumption appears to benefit the higher loan-to-value ratio mortgages and larger buildings. This result, admittedly based on a very small sample, suggests that energy related retrofits should affect the mortgage cost of capital.

## 8 Conclusion

In this paper, we develop a commercial-mortgage valuation, or underwriting, strategy that accounts for the energy risk of individual office buildings in terms of the relative energy efficiency of the buildings and the characteristics of their locations. Our method extends standard underwriting practices, which account for the expected dynamics of interest rates and office building prices over time, by including the expected dynamics of the electricity and gas prices as well as quantity dynamics appropriate to the location of the building. Our proposed method allows lenders to explicitly take into account the effect of energy use and various alternative efficiency measures when underwriting commercial mortgages. We find that, relative to the traditional modeling strategy, our proposed strategy leads to an 8.5% reduction in the mispricing of the default risk of commercial mortgages. The valuation framework can also be applied to price the benefits of energy retrofits, in terms of their effect on the relative risk of commercial mortgage.

Table 11: Percentage Change in the Value of the Defaultable Loan for a 20% Immediate Reduction in the Building's Required Electricity and Natural Gas Consumption, September 2004

Loan	City	Electricity Hub	Mortgage Value with 20% Energy Decrease (000)	Percent Change in Mortgage Value (%)
Loan 1	Cincinnati	Into Cinergy	4,927	0.94
Loan 2	Cincinnati	Into Cinergy	4,892	1.12
Loan 3	Hartford	East New York Zone J	12,625	1.02
Loan 4	Hartford	East New York Zone J	10,453	1.23
Loan 5	Los Angeles	South Path 15	5,802	0.42
Loan 6	Los Angeles	South Path 15	9,774	2.10
Loan 7	Orlando	Into TVA	7,586	2.47
Loan 8	Riverside	South Path 15	6,373	0.54
Loan 9	Sacramento	North Path 15	9,249	1.82
Loan 10	Sacramento	North Path 15	4,408	0.99
Loan 11	Sacramento	North Path 15	2,778	2.43
Loan 12	San Antonio	ERCOT	3,389	0.59
Loan 13	San Francisco	North Path 15	14,848	1.26
Average				1.30
Standard Deviation				0.69

Overall, the valuation method has been shown to be tractable for actual market applications to price real mortgage products. In lieu of having standardized methods to use the engineering reports that are already part of the commercial mortgage underwriting process, we have shown that benchmarks for the energy consumption of office buildings determined by the square footage of the building and its location are adequate to differentiate the relative energy risk of commercial mortgages. Further development of relative efficiency measurement tools in conjunction with further refinements in our mortgage valuation framework will assure that the energy risk of commercial office buildings can be assessed as a matter of course in the mortgage underwriting process.

# A Appendix: Energy Data Construction

We extract the energy forward curve pricing from the forward contract auctions for electricity and from the futures contracts auctions for natural gas. We follow Benth, Koekebakker, and Ollmar (2007), Benth, Cartea, and Kiesel (2008), Geman and Roncoroni (2006) and Riedhauser (2000), in the construction of these curves.

## A.1 Forward Market for Electricity

The forward market for electricity is organized around the trading of different standard packages covering the on-peak and off-peak periods. Trading occurs for delivery hubs located at the Eastern-Central regions and delivery hubs located in the Western region of the continental United States. The Eastern-Central standard forward package covers the following markets: New England, New York (several hubs), Ontario, PJM, MISO, ERCOT South, Into Entergy, Into Southern and Into TVA. The Western packages cover NP15 and SP15 among others. Packages for the Eastern-Central hubs differ from those traded for the Western hub on two dimensions: the way on-peak and off-peak are defined and the delivery months of the forward packages.

We compute the standard on-peak forward packages in Eastern and Central markets are 5x16 packages (5 days per week and 16 hours per weekday from 7:00 Am to 22:59 PM), which include power delivered during on-peak hours on weekdays and exclude weekends and holidays.<sup>21</sup> Similarly, on-peak forward packages in Western markets are 6x16 packages, which include power delivered during the 16 on-peak hours each day Monday through Saturday and exclude Sundays and holidays. The off-peak standard packages, the forward market trade 5x8 (5 days per week and 8 hours per day) plus a 2x24 package, this includes power for delivery during the eight off-peak hours each weekday, plus all 24 hours (around the clock) on weekends. The standard off-peak forward package for the Western markets is a 6x8 delivery block plus a 1x24 delivery block, this includes power for delivery during the eight off-peak hours Monday through Saturday plus all 24 hours (around the clock) on Sunday.

For the Eastern-Central markets, on-peak and off-peak contracts are formulated for the prompt month (nearest contract), second month, third month, and balance-of-the-year in seasonal or single month packages, two full years in seasonal or single-month packages and two subsequent calendar year packages. Separate seasonal and single-month packages include the January-February winter package, the March-April spring package, May, June, the July-August summer package, September and the fourth quarter (from October to December).

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<sup>21</sup>Power market holidays are defined by the North American Electric Reliability Corp. (NERC).



The following example illustrates, for a given trading date, the typical term-structure of contracts for packages traded in the Eastern-Central hubs. Suppose today's date is 5/15/2009. At this time, the market is trading the following forward packages:

- 2009 - June-2009 (prompt month), July-2009 (second month), August-2009 (third month), September-2009 (single month package), fourth-quarter 2009,
- 2010 - January-February-2010 (winter package), March-April-2010 (spring package), May-2010 (single month package), June-2010 (single month package), July-August-2010 (summer package), September-2010 (single month package), fourth-quarter 2010,
- 2011 - January-February-2011 (winter package), March-April-2011 (spring package), May-2011 (single month package), June-2011 (single month package), July-August-2011 (summer package), September-2011 (single month package), fourth-quarter 2011,
- 2012 - year-2012 (calendar year package), 2013 - year-2013 (calendar year package).

For the Western markets, on-peak and off-peak packages are formulated for the prompt month, second month, balance of the year in quarters, two full years in quarters, and two subsequent calendar year packages.

As before, suppose today's date is 5/15/2009. At this time, the market is trading the following forward packages for a Western hub:

- 2009 - June-2009 (prompt month), July-2009 (second month), August-2009 (third month), third-quarter 2009 (July-September package), fourth-quarter 2009 (October-December package),
- 2010 - first-quarter 2010 (January-March package), second-quarter 2010 (April-June package), third-quarter 2010 (July-September package), fourth-quarter 2010 (October-December package),
- 2011 - first-quarter 2011 (January-March package), second-quarter 2011 (April-June package), third-quarter 2011 (July-September package), fourth-quarter 2011 (October-December package),
- 2012 - year-2012 (calendar year package),
- 2013 - year-2013 (calendar year package),

A significant portion of transactions are realized over-the-counter (OTC). Transactions also occur on specialized exchanges such as the Intercontinental Exchange (ICE). Often trading parties take their existing OTC transactions for clearing into ICE. This mechanism mitigates counterparty risk since parties are now at arms-length and are subject to margin calls as prices for the forward packages fluctuate.

The energy volume for a typical package is 50 MWh (Megawatt-hour) times the number of on-peak or off-peak hours depending on the type of package. If for example, the parties are

trading an on-peak September-2011 contract delivering into the PJM Western hub. Assuming that there are 22 NERC weekdays for this month. The total volume for such a package would be then calculated as  $(50 \times 22 \times 16)$  MWh.

Parties engage in financial settlement a few days after the ending date of the package. As in the example above, the settlement for the September-2011 package occurs in the beginning of October-2011. The long party receives (pays) the difference between the floating price (calculated, in this case, as the arithmetic average of the arithmetic averages of the hourly on-peak real-time prices posted by PJM Interconnection, LLC, on their official website) and the agreed forward price at the time of contracting.<sup>22</sup>

## A.2 Dataset: Platts-Ice Forward Curve

In this section we describe the source of data for constructing our power forward price level and slope database. We contracted the raw data for Platts (the data vendor) and worked out a daily forward curve for power on-peak and off-peak comprising selected trading hubs serving large cities in The United States.

Raw data from Platts is formatted with single entries for each forward package (see Appendix A.1). For a given trading date, a power hub, and a type of contract - on peak and off-peak - there are single entries for the mark-to-market price for each forward package. This scheme characterizes a whole term-structure of power prices for a given trading date. Table 12 describes for each hub and contract type the time series of the related forward curves.

After inspecting the raw data, we noted the following:

1. We did not find any significant gaps on trade dates for all time series.
2. The time series for on-peak New York Zone-J has gaps on the term-structure from the beginning of the time series 1/2/2002 to 1/11/2005. Consequently, we discarded these raw entries when calibrating instantaneous volatilities.
3. The ERCOT (all zones) time series has data up to 11/25/2008. Consequently, we extended the time series for ERCOT by appending the ERCOT-South time series starting in 11/26/2008.
4. We discarded raw entries with trade dates later than the beginning of the delivery period. For example, we found on 3/6/2008 a quote for the on-peak East NY Zone J Mar/Apr 2008 package. Note that on this trade is already into the delivery period of the contract which starts on 1/3/2008. Though these types of trades are valid,

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<sup>22</sup>We refer the reader to the Intercontinental Exchange website for more details on how the contracts are traded and settled.

Table 12: The Earliest and Latest Trading Dates for the Power Hubs

This table presents the periods for which we have data on the forward and spot prices for each electricity hub.

Contract Type	Region Name	Minimum Trade Date	Maximum Trade Date
on-peak	East New York Zone J	1/2/2002	4/23/2010
on-peak	ERCOT	1/2/2002	4/27/2010
on-peak	Mass Hub	8/30/2002	4/23/2010
on-peak	NI Hub	1/2/2002	4/23/2010
on-peak	North Path 15	12/31/2001	4/23/2010
on-peak	PJM West	3/5/2001	4/23/2010
on-peak	South Path 15	12/31/2001	4/23/2010
on-peak	Into Cinergy	3/1/2001	10/23/2010
on-peak	Into Entergy	3/1/2001	10/23/2010
on-peak	Into Southern	12/1/2005	10/23/2010
on-peak	Into TVA	1/1/2002	10/23/2010
on-peak	Mid Columbia	3/1/2001	10/23/2010
on-peak	Palo Verde	3/1/2001	10/23/2010
off-peak	East New York Zone J	1/31/2007	4/27/2010
off-peak	Mass Hub	2/7/2007	4/23/2010
off-peak	NI Hub	1/31/2007	4/23/2010
off-peak	North Path 15	5/31/2006	4/23/2010
off-peak	PJM West	1/31/2007	4/23/2010
off-peak	South Path 15	5/31/2006	4/23/2010
off-peak	Into Cinergy	3/1/2001	10/23/2010
off-peak	Into Entergy	3/1/2001	10/23/2010
off-peak	Into Southern	12/1/2005	10/23/2010
off-peak	Into TVA	1/1/2002	10/23/2010
off-peak	Mid Columbia	3/1/2001	10/23/2010
off-peak	Palo Verde	3/1/2001	10/23/2010

Table 13: The Maximum Number of Months out by Trading Year for the On-peak PJM Western and NP15 Hubs

This table shows that the length of the forward curve has increased for more recent years.

Region Name	Year of Trade Date	maximum Month Out
North Path 15	2001	35
North Path 15	2002	41
North Path 15	2003	41
North Path 15	2004	41
North Path 15	2005	59
North Path 15	2006	59
North Path 15	2007	59
North Path 15	2008	59
North Path 15	2009	59
North Path 15	2010	59
PJM West	2001	41
PJM West	2002	41
PJM West	2003	41
PJM West	2004	41
PJM West	2005	41
PJM West	2006	47
PJM West	2007	59
PJM West	2008	59
PJM West	2009	59
PJM West	2010	59

the quote corresponds to parties trading on information related to the balance of the delivery package. Consequently, since the structure of the contract is now different from the original package, we discard these entries.

5. The length of the forward curve increases for more recent years. We illustrate this by showing below the maximum number of months out by trading year for the on-peak PJM Western hub and NP15 hub.

The raw data from Platts contains a field called “symbol” indicating the type of contract, the package, the power hub, and the year related to the package. The “symbol” field is coded with 7 characters with the following structure and the last two characters is a two digit code for the hub name. The other codes are described in Table 14. Table 15 describes Platts coding for the sub-fields seasonal package and Contract Length.

Table 14: The Symbol Codes from Platts

This table presents the symbol code keys from Platts that are used to identify the class of forward and spot contracts and the location of the trading hub.

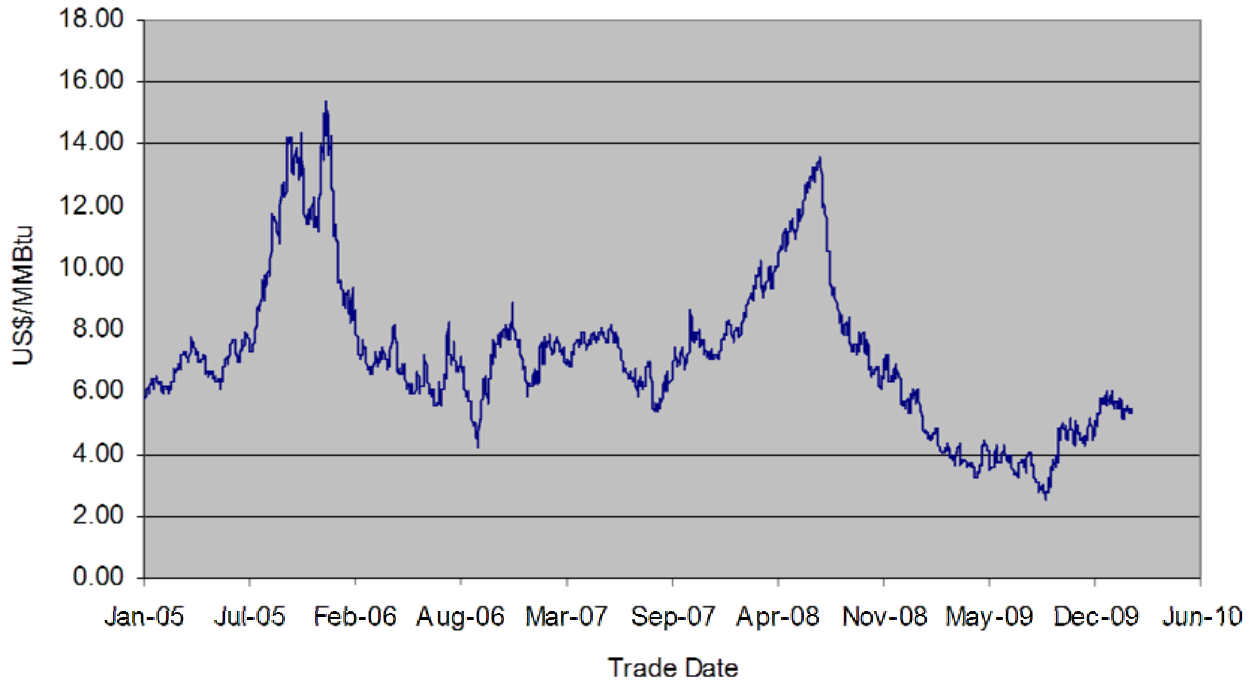
Position	sub-field size	Sub-field indicator	comments
1-Jan	1	on-peak / off-peak	“F” = on-peak, “O” = off-peak
3-Feb	2	seasonal package	see table below
5-Apr	2	power hub name	see table below
7-Jun	2	package year	—

Table 15: Codes for the Sub-Field Seasonal Packages and their Contract Length

This table presents the codes used by Platts to identify seasonal packages, their start and end dates, and the forward contract length.

Contract Code	Contract Description	Start Month	End Month	contract Length
	(seasonal package)			
AA	January	1	1	1
AB	February	2	2	1
AC	March	3	3	1
AD	April	4	4	1
AE	May	5	5	1
AF	June	6	6	1
AG	July	7	7	1
AH	August	8	8	1
AI	September	9	9	1
AJ	October	10	10	1
AK	November	11	11	1
AL	December	12	12	1
AN	January-February	1	2	2
AP	March-April	3	4	2
AT	July-August	7	8	2
AY	Year	1	12	12
Q1	First Quarter	1	3	3
Q2	Second Quarter	4	6	3
Q3	Third Quarter	7	9	3
Q4	Fourth Quarter	10	12	3

Figure 12: **NYMEX Natural Gas Prompt Month (nearest contract) Daily Quotes**  
This figure was computed from daily quotes for NYMEX natural gas prompt month (nearest contract futures contracts.



### A.3 Futures Market for Natural Gas

There is a very active market for natural gas in The United States. Following deregulation of the wholesale market for natural gas in mid 1990s, the New York Mercantile Exchange (NYMEX) launched the trading of monthly futures contracts with similar characteristics to those of crude oil. The standard NYMEX natural gas futures contracts specify physical delivery of 10,000 MMBtu (millions of British thermal unit) ratably delivered into Henry Hub - Louisiana. Until early 2000, NYMEX provided monthly contracts covering maturities about 36 months out. More recently, the range of maturities has been extended and it now covers more than six years (72 months) on a monthly basis. The NYMEX website provides more details on how the contracts are traded and the rules for settlement. Figure 12 shows NYMEX's prompt month (nearest contract) daily quotes starting in 2005.

There is an extensive network of natural gas pipelines connecting the production basins to large consumption areas (mainly large populated urban centers). Wholesale physical natural gas trading occurs in different hubs distributed in the continental United States. These hubs are key points in the pipeline grid characterized by either being interconnections between major pipelines and/or access points to public utility gas companies. Of all those hubs,

Table 16: PJM Western hub Packages

An example for the quotes on-peak PJM Western hub 2009 packages.

Forward Package	Description	MTM
September-2009	Prompt month	\$37.75
October-2009	Second month	\$37.25
November-2009	Third month	\$40.75
Fourth quarter-2009	fourth quarter	\$41.60

Henry Hub is the benchmark for price quotation. Henry Hub’s importance stems from both as being an interconnecting point for multiple pipelines and as being the most liquid point for trading spot and futures contracts. Prices for other hubs (spot and OTC forwards) are typically quoted as a basis to Henry Hub. These basis quotes are a very small fraction of the full benchmark quote.

From the modeling standpoint, our goal is to determine for each hub and type of contract the term structure of instantaneous volatilities. This is a key component for characterizing the stochastic behavior of both forward prices and the spot price,<sup>23</sup> which in turn allows us to price contingent claims. With that in mind, we need to define the term-structure of instantaneous volatilities for different maturities on a monthly basis. Consequently, our first challenge is to expand the initial raw forwards package dataset to reflect entries a monthly basis. However, this task should be done in such a way that non-arbitrage conditions hold. As an example, suppose the trade date is 8/15/2009 and we have the following quotes for on-peak PJM Western hub 2009 packages:

We want to assign an individual quote for September-2009, October-2009, November-2009, and December-2009 with the highest granularity. We calculate the first 3 months by simply assigning the same monthly quotes as in the packages. As for December-2009, we would violate a non-arbitrage condition if we assigned this entry with the fourth quarter-2009 quote of 41.60. To preserve the possibility of arbitrage the December-2009 quote should be assigned the value of  $(3 \times 41.60) - (40.75 + 37.25) = \$46.50$ .

There are some other situations, for instance, that for a given trade date we find, for the second year out, only quotes for the January-February seasonal package and the yearly package. In this case we assign the January-February package quote for the individual January and February entries in the new decomposed table. We also indicate that the

<sup>23</sup>The spot price is viewed as a special case of a forward contract with zero time to maturity.

source of the quote comes from a seasonal package of size 2. Finally, we create 10 individual entries for the months March through December with their quotes calculated as  $[(12 \times \text{yearly package quote}) - (2 \times \text{January-February quote})]/10$ . We indicate that the source of these 10 entries is the yearly package and that each quote corresponds to a new synthetic package of 10 months. On a later trade date more packages (price information) are added to the second year out. This increases the granularity of the quotes for the second year out. Say on a further trade date that the market begins trading the July-August seasonal package in addition to the already existing January-February and yearly packages. In the new table, individual quotes for the months March through June and September through December are now calculated as  $[(12 \times \text{yearly package quote}) - (2 \times \text{January-February quote}) - (2 \times \text{July-August quote})]/8$ , and each quote corresponds to a new synthetic package of 8 months.



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