

# Sovereign Debt Without Default Penalties\*

PRELIMINARY DRAFT

Comments welcome

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## Abstract

The basic question regarding sovereign debt is why sovereign borrowers ever repay, provided that creditors have no power to foreclose on any of their assets. In this paper we suggest an answer: sovereign debt will be served as long as the median voter is a net loser from default. Default generates a reallocation of wealth from locals to foreigners, but also from local debtholders to local tax payers. Sovereign debt is stable as long as the median voter's interests are more aligned with the foreign lenders than with the local taxpayers. We further augment the model with elements of market microstructure theory to address the question how markets rationally use capital flows so as to infer the stability of debt structure. We show that foreign demand shocks can destabilise debt even though they are not fundamental. We also show that more volatile foreign demand reduces a country's debt capacity. Our work thus integrates elements of market microstructure theory into political-economy modeling.

## 1. Introduction

It is widely recognized that sovereign debt differs from corporate debt in that the debtor cannot credibly collateralize assets and grant the creditor an enforceable default-contingent

foreclosure right.<sup>1</sup> At the same time, most of the literature does assume that creditors can pre-commit (imperfectly) to inflict a default-contingent penalty – such as trade sanctions – on a sovereign debtor. Assuming, however, that default-contingent penalties are implementable, sovereign and corporate debt are actually not that dissimilar. In both cases the debtor repays under threat, the difference being more in the ability to adjust and refine the penalty; see Eaton and Gersovitz (1981) or Bulow and Rogoff (1989a, b) for classic references. The conclusion that both sovereign and corporate debt are supported by similar incentives has important practical implications: c.f. Krueger’s (2002) proposal for a Sovereign Debt Restructuring Mechanism, which is modeled after Chapter 11 of the US Bankruptcy Code.

In this paper we consider an alternative (extreme) case where no penalty for default is implementable, so that sovereign debt must be supported by an entirely different incentive scheme. The main idea is that sovereign debt is structured so that it is in the best interest of the median voter to serve it. Two elements in this structure are critical; first, the debt should be unregistered (allowing the creditor to remain anonymous) and transferable so that in case of sanctions foreigners can sell the bonds to locals who would obtain repayment. Second, the debt should remain at the level at which the median voter still has an incentive to repay. Note that when domestic and foreign creditors hold identical instruments, default benefits domestic tax-payers but harms domestic bond-holders, where the net effect depends on exact positions. The trick then is to find a level of debt where the interests of the median voter are more aligned with foreign bondholders rather than with local taxpayers. We demonstrate that our repayment mechanism has important implications regarding several open questions in this area such as debt restructuring and renegotiation or the relationship between liquidity shortage and sovereign-debt crisis. We give special attention to the case of market *opacity* where even the aggregate position of domestic and foreign bondholders is unobservable.

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<sup>1</sup>For an exceptional case where a creditor managed to threaten enforcement upon a sovereign lender see “Elliot Associates vs. the Republic of Peru”, discussed at length in IMF (2001). See also Zettelmeyer (2003), who estimates that, for a large pool of developing and emerging market countries, only 6.2% of outstanding debt is collateralised.

Our setting can be motivated by two observations. First, while sovereign default certainly disrupts the debtor’s operations, there is a widespread feeling that the implied penalty is insufficient to support the level of activity observed in sovereign-debt markets. For example, Eichengreen (1988) finds no evidence for a negative relationship between pre-WWII default and post-war lending.<sup>2</sup> Bulow and Rogoff (1989a) comment that, “admittedly, there are many uncertainties surrounding the actual damage which a lender can inflict on an LDC” following default. They therefore dismiss reputational models – where sovereigns repay just in order to preserve the capacity of further borrowing. Instead, Bulow and Rogoff (1989b) suggest that sovereign debt is enforced by penalties that creditors can enforce within their own jurisdictions, like trade sanctions. However, Tirole (2002) points out that such sanctions suffer from similar problems, particularly a severe free-rider problem amongst those countries who are supposed to enforce the sanctions, combined with a strong incentive to renegotiate ex-post-inefficient sanctions. Note that all these problems result from the sharp separation between domestic and foreign creditors, so that all domestic interests are unanimously aligned against serving the debt. We avoid the difficulty by assuming that *some* domestic agents hold ‘foreign’ debt, which breaks the unanimity and gives the median voter a genuine interest in serving the debt.

Second, it is widely recognized that a “large fraction of the [foreign-currency denominated] government debt was issued domestically and purchased by domestic banks”; see Roubini (2002).<sup>3</sup> Precise information on this “home bias” in sovereign debt is not available, because the market – in line with our theory – is highly opaque. Indeed, Gray (2003) reports that communication between the sovereign and its creditors is done via the clearing systems that deliver the message to the custodians, neither of whom would disclose the identity of the

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<sup>2</sup>Rose (2002) finds significant long-term decrease in bilateral trade flows following sovereign default. It is not clear, however, whether this should be interpreted as a penalty, or whether this is a result of the same crisis that caused the default. Also the results are limited to Paris-club defaults.

<sup>3</sup>See also Cline (2002) who stipulates that the primary subscribers to Argentina’s ‘megaswap’ of June 2001 were domestic pension funds.

bondholders.<sup>4</sup> Yet, there is much anecdotal evidence about domestic bondholders who use their political clout obtain repayment. For example, on May 7, 1999 the Russian government announced that it would pay \$333m interest on five out of seven tranches of MinFin bonds. The announcement was a great surprise as – according to one contemporary analyst – “none expected to get any money in May”.<sup>5</sup> A month later *The Economist* commented, cynically, that now that “a big chunk of ex-Soviet debt ... is held not by the original banks, but by hedge funds and other individuals” the repayment was actually “to the benefit of wealthy Russian individuals and institutions”.<sup>6</sup> Yet a year later, after reaching an agreement with the London Club to restructure \$32bn of Soviet-era debt (on February 11, 2000) *The Economist* reported that “some observers ... note sourly that it has delivered a hefty profit to Russian banks which bought up the least popular category [of the debt]... at time its price had fallen following the leak of draft scheduling terms”.<sup>7</sup>

It may be argued that in the examples above the pivotal player in the political game is not the median voter. It is thus worth emphasizing that our main point is not that all political systems follow a majority rule, but rather that the decision to repay sovereign debt is a political one; as noted in Tirole (2002), “domestic political constraints usually start biting before any technical capacity constraint (the physical ability to reimburse) is reached”. Moreover, various constituencies may have conflicting interests regarding repayment. These conflicting interests may be used in order to build a commitment mechanism that supports borrowing and repayment. Obviously, equilibrium depends on the exact parameters of the political system. In our formal modeling and most of the discussion here we use a majority rule, but it is actually very easy to adjust the model so that political power is not evenly distributed across the entire population.

Our approach can explain three important phenomena, which the conventional theory

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<sup>4</sup>Nevertheless, IMF (2003) reports that locals’ position increased sharply during the 1990s, reaching 40% by 2003; see Figure 4.15, based on data provided by PIMCO, a leading fixed-income firm.

<sup>5</sup>See Reuters report by Jukie Talkacheva on May 7, 14:51.

<sup>6</sup>See *The Economist*, June 3 1999.

<sup>7</sup>The Economist, February 17, 2000.

could address with only some difficulty. First, that sovereign-bond lending have proven more stable than syndicated bank lending, resulting in the virtual “collapse” of the latter.<sup>8</sup> Indeed, throughout the 19th century, sovereign lending was dominated by bonds. Banks entered the market only during the second half of the 20th century. In the 1970s, default rates on sovereign-bank lending increased sharply, while sovereign default rates on bonds remained low. Moreover, Beers and Chambers (2003) report quite a few cases where sovereign debtors defaulted on their bank debt, but not on their bonds. Our account of this trend is straightforward: with weak penalties, maintaining the anonymity of the creditor and the transferability of the instrument are necessary conditions for repayment. Bolton and Jeanne (2005) suggest an alternative explanation, by which sovereigns serve their bonds because it is practically impossible to restructure when the creditors are so dispersed, just like in corporate-bond theory; see Gertner and Scharfstein (1991) or Bolton and Freixas (2000). Note, however, that this explanation still rests on the existence of a default penalty, which could be released to the benefit of any descending creditor in case of an attempted restructuring.

Second, as demonstrated in the previous paragraph, the conventional theory may explain either infrequent default on bonds or full service, but would struggle with partial default or restructuring. Indeed, Roubini (2002) points out the puzzling incidence of “recent cases where there were thousands of bondholders (Ukraine, Pakistan, Ecuador, Russia) [but nevertheless] unilateral exchange offers have had overwhelming success with 99% plus creditors accepting the offer”. Calculations by Sturzenegger and Zettelmeyer (2005) show a discounted write-down “clustered in the 25-35 percent range”. Again, our explanation is straightforward: debt is written down to the point that serves the interests of the median voter, and no further. Consistent with our explanation is the fact that in many cases the exchange offers were accompanied by threats to inflict even heavier write-downs on creditors that would reject the exchange offer; see Gray (2003). Indeed, the Russian restructuring was described

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<sup>8</sup>See IMF (2003).

by *The Economist* as a “confiscatory restructuring scheme”.<sup>9</sup> All that is evidence that the creditors were unable to inflict any penalty on the sovereign. All they could do to obtain payment was ‘hiding behind’ domestic holders of identical instruments who have the political clout to enforce repayment.

Third, our approach explains the increasingly closer link that is observed between liquidity shortages, erratic capital flows and sovereign debt crises along the lines of Kaminsky and Reinhart (1999).<sup>10</sup> It should be quite clear at this point that even under conditions of market transparency prices may respond sharply to even a slight change in the position of the median voter. However, when the market is opaque and the identity of creditors – whether domestic or foreign – cannot be directly observed, there is a need to infer positions according to the overall level of demand, like in a conventional micro-structure model; see Kyle (1985). That makes a smoother, S-shaped, price function but preserves the crisis-like non-linear relationship between overall demand and the price of the debt. In such a situation, a “sudden stop” phenomenon – as discussed in Calvo (1998) – may occur. When external capital flows drop, the market needs to infer whether this is a result of lower foreign demand (which is harmless) or whether the median voter was hit by a liquidity shortage and leaves the market (which is detrimental); see Detragiachi and Spilimbergo (2001) for some econometric evidence. Clearly, the probability of the latter is higher the lower is the overall level of demand. Hence, a sudden stop may create a sharp drop in the quality of sovereign debt. We develop this mechanism further and derive a result about the relationship between capital-account volatility on debt capacity. Suppose that the government wants to maintain a certain ‘quality’ (rating) of debt. Then it needs to adjust the amount of its borrowing to the position of the median voter. However, the more volatile capital flows are, the more likely it is that the government would make first/second type mistakes; e.g. borrow less when foreign capital flows are low. That would result in a lower expected debt capacity.

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<sup>9</sup> *The Economist*, June 3, 1999.

<sup>10</sup> To be precise, Kaminsky and Reinhart (1999) explore the relationship between a BOP crisis and a banking crisis; we add the sovereign debt into the equation.

Our work is related to some of the Political Economy literature that explores the role of the political process in addressing commitment problems arising from intergenerational redistribution (see Persson and Tabellini, 2002). Dixit and Londregan (2000) use this line of argument to show that repayment of sovereign debt becomes credible when constituencies with a lot of political clout hold large amounts of the debt. In contrast to Dixit and Londregan (2000) we focus specifically on a government’s ability to raise foreign debt. This allows us to derive new results regarding security design and debt renegotiation. Moreover, the inclusion of a market microstructure mechanism to price sovereign debt is novel to our model. This enables us to derive further results from the interaction between the voting and the pricing mechanisms.

In a recent paper, Sandleris (2005) proposes a model of sovereign debt payment without default penalties. In his paper the government has private information about the fundamentals of the economy and can use its repayment behavior as a signal of such information. The mechanism is thus very different from the one put forward in our paper.

The remainder of the paper proceeds as follows. Section 2 describes the basic set-up. Section 3 explores repayment and debt pricing in the full information benchmark. A bond market is introduced in Section 4 and the pricing implications of market opacity are investigated, while Section 5 extends the analysis to the issue of debt capacity. We offer a model extension that allows us to analyze partial default and debt restructuring in Section 6 and Section 7 concludes.

## **2. The model**

Our economy is active for two periods. At  $t = 0$  a certain amount of sovereign debt is placed in the markets, priced and allocated across domestic and foreign agents. Period  $t = 1$  has two stages. First, domestic agents vote on whether to serve the debt or to default. Later on we shall consider the possibility of partial default or “debt renegotiation”. However, in



its present specification, the model cannot be enriched by the consideration of the interim case. At the second stage, the voters' decision is implemented, the debt repaid (funded by a lump-sum tax,  $T$ ), and positions are wound up. We turn next to the details of this structure.

Local demand results from a measure-one of domestic agents who are indexed by  $i \in [0, 1]$ . Initial wealth endowments,  $w_i$ , are distributed as follows. Agents  $i \in [0, \mu_l]$ ,  $\mu_l < 1/2$ , have zero wealth, while agents  $i \in [\mu_h, 1]$ ,  $\mu_h > 1/2$ , are endowed with wealth  $W > 0$ . The remaining agents,  $i \in (\mu_l, \mu_h)$ , are exposed to a binary wealth shock, so that  $w_i = \tilde{\delta}W$ , where  $\tilde{\delta}$  equals  $\delta \in [0, 1)$  with probability  $\gamma$  and 1 with a probability  $1 - \gamma$ . Domestic agents care only about period-one consumption, so that

$$s_i = w_i, \tag{1}$$

where  $s_i$  denotes agent  $i$ 's savings.

At date 0 the government issues  $B$  one-period-maturity bonds with a one-dollar face value. These bearer notes are anonymous and freely transferable, particularly from domestic agents to foreigners and vice versa. To make things simple, we assume that these government bonds are the only available store of value, which absorb all of agents' savings.

In addition to domestic agents, there are foreigners who have a random (un-modelled) demand  $f$  for the government bond.  $f$  is denominated in dollars and has density  $h(f)$ .

Lastly, any slack between supply and demand is taken by a risk-neutral market maker, who also puts a fair price  $P$  on the bond, taking into consideration the risk of default. The fair-pricing assumption may be motivated by the interpretation of the market-maker as a competitive industry; see Kyle (1985). Section 4 provides more detail on the microstructure foundations of the bond market. The market maker may be local or foreign, provided his insignificant weight as a voter. It may thus be proper to interpret the market maker as the foreign financial sector.

At the first stage of period one, the decision about the default rate is taken by majority voting. As noted above, at this stage we consider either full repayment ( $\tilde{\alpha} = 1$ ) or complete

default ( $\tilde{\alpha} = 0$ ); no restriction is imposed on the current specification by ignoring the interim case. The voters' decision is taken by comparing the cost of serving the debt against the benefit. The cost is the lump-sum tax  $T$  that would be imposed in the next stage in order to repay the debt; the benefit is obtaining payment that equals the face value of his bonds,  $\frac{s_i}{P}$ . Hence, agent  $i$  votes in favor of repayment if

$$\frac{s_i}{P} \geq T. \quad (2)$$

and against repayment otherwise. In case of equality, we allow voters to play mixed strategies. In case of a non-zero measure of indifferent voters, we allow them to correlate their mixed strategies, so that default actually occurs with the intended probability. As in any analysis of this sort, the median voter, indexed  $m = 1/2$  plays a crucial role.

At the second stage of period 1 the voters' decision is implemented and the government finances the bond repayment by a lump sum tax  $T$  on all domestic agents (including those who had no wealth in period 0). The tax burden is thus

$$T = \tilde{\alpha}B. \quad (3)$$

### 3. Debt capacity in the full information benchmark

Consider the benchmark case in which  $\tilde{\delta}$  is publicly observable. An equilibrium is then defined as a mapping from local demand to price such that the price equals the probability of repayment given  $\tilde{\delta}$  and given that locals vote on repayment according to (2). Hence, the equilibrium price depends on the realized wealth-endowment of the median voter  $\tilde{\delta}W$ . Even though the economy's 'fundamentals' are thus realized, the repayment decision can still be random, because the pivotal voters may play (correlated) mixed-strategies. The fair-pricing condition can then be written as

$$P = E[\tilde{\alpha}|\tilde{\delta}]. \quad (4)$$

The equilibrium is characterized in the following Proposition.

**Proposition 1** *There exists a mixed-strategy equilibrium where  $P$  is both the fair price of the bond and the probability of serving the debt, such that*

$$P = \begin{cases} 1 & \text{if } B \leq w_m \\ \frac{w_m}{B} & \text{if } B > w_m \end{cases}. \quad (5)$$

*Proof.* Consider the case where  $B \leq w_m$ . Since  $\alpha$  is bounded (from above) by 1, its expected value  $P$  cannot exceed 1. Hence, condition (2) holds with strict inequality, repayment occurs with certainty, so that by (4)  $P = 1$ .

Now consider the case where  $B > w_m$ . The price of the bond cannot be 1, for then condition (2) is violated and default occurs with certainty, which is inconsistent with a fair price of 1. Similarly, the price of the bond cannot be zero, for then the median voter is allocated an infinite number of bonds and therefore vote for repayment. Hence, the price needs to be between zero and one, which implies that default occurs with a positive probability. For that to happen, the median voter must be indifferent between default and repayment, condition (2) holds with equality and  $P = \frac{w_m}{B}$ . ■

From Proposition 1 follows a result about the economy's debt capacity. In the first-best, the government can determine  $B$  contingent on the realization of  $\tilde{\delta}$ . Denote the government's dollar revenue from selling the bond by  $D = B \cdot P$ . Then,

**Corollary 1** *The maximum amount of funds  $\bar{D}$  that the government can raise is limited by the median voter's wealth  $\bar{D} = w_m$ .*

This follows directly from the price function given in (5). If  $B < w_m$  the government can increase  $B$  without affecting the repayment probability and thereby raise more funds. However, beyond the point that (2) holds with equality any increase in  $B$  is followed by a proportional fall in price without any change in revenue. The crucial point here is that the government's borrowing capacity is not affected by the economy's aggregate or per-capita wealth nor by the level of foreign demand, but by its median voter's political willingness to serve the debt. To see this, consider the wealth-shock case ( $\tilde{\delta} = \delta$ ) and then increase  $W$  and

decrease  $\delta$  proportionately so that the median voter's wealth remains unchanged. Wealth per capita increases unboundedly without this having any effect on the government's ability to borrow.

More interesting still is the mechanism through which external debt can be enforced. The usual puzzle about the incentive to serve external debt results from the fact that such service is a net transfer from the domestic economy to the rest of the world, and thus not in the best interest of the economy as a whole. However, our crucial point is that when domestic and foreign borrowing is done by the same instrument, defaulting on that instrument serves the interests of some local agents but undermines the interests of others. The critical question then is what effect the default has on the median voter.

**Remark 1** *In the absence of a default cost, the government can only raise external debt through a security that guarantees anonymity, i.e., the repayment decision cannot be made contingent on the identity of the security holder.*

The security design implication of the above discussion is against bank debt and in favour of tradeable bonds. When bonds are tradeable it is impossible to track properly foreign and domestic ownership of debt. Trade itself may allow some inference about ownership patterns, and this is essentially how debt will be priced in the subsequent analysis. When a government borrows directly from banks its default decision can be partial and targeted at those banks whose depositors (and shareholders) are primarily foreign. It is therefore unsurprising that attempts to finance large amounts of external debt through banks can largely be viewed as a failed experiment. After defaulting on their bank debt in the 1980s most countries have restructured their debt to (Brady) bonds (see Bolton and Jeanne (2005) and Standard and Poor's (2003) for a discussion of this issue).

To understand the repayment mechanism more clearly, we write the maximum external borrowing capacity  $\bar{E}$  as

$$\bar{E} = w_m - \int_{i=0}^1 \frac{w_i}{P} di. \quad (6)$$

From (6) it follows immediately that  $\bar{E}$  depends on the *distribution* of domestic wealth and the implied bond holdings. Now consider the no-wealth-shock case ( $\tilde{\delta} = 1$ ) and let  $\mu_h = 1$ . When a fraction  $\mu_l$  of the population that holds no bonds increases above 1/2, the median voter favors default, so that any attempt to raise any debt (internal or external) is doomed to fail. (For the sake of this discussion and the diagram below we relax the assumption that  $\mu_l < 1/2$  .) At the other extreme, when all domestic agents have identical positions in the bond ( $\mu_l = 0$ ), the government cannot raise any external debt either. For then, all domestic interests are aligned against serving the debt. External debt capacity is actually maximized when  $\mu_l$  approaches 1/2 from below (see Figure 1). Serving the debt implies a redistribution from non bondholders to both domestic and foreign bondholders. When fewer locals are bondholders, but the median voter is, then the local debt capacity is low, while the median-voter's interests are aligned with the foreign creditors. Hence the debt is served even though local debt capacity is low.

A further implication of the above discussion is that credible debt repayment requires a majority of local agents to have (substantial) bondholdings. Even though we have not modelled risk sharing it is clear that the substantial holdings required by the locals may conflict with efficient risk sharing. In particular, from a risk sharing perspective, locals of a small country should hold only a tiny amount of domestic bonds. This, however, would severely limit a country's ability to raise external debt. Reversing this argument, locals may be willing to hold larger amounts of debt than is optimal from a risk sharing perspective, because their country's bonds are worth more in the hands of the locals than in the hands of foreigners. A home bias in domestic bond holdings may thus emerge naturally from our

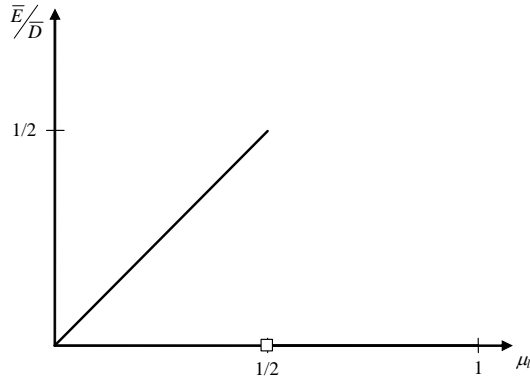


Figure 1: Shows the degree to which a country can leverage up its domestic debt capacity as a function of the fraction  $\mu_l$  of domestic agents who are bond holders. Leverage is maximised when default generates a strong redistribution of wealth from domestic bondholders to domestic tax payers. This aligns the median voter’s interest with that of the foreign creditors and thereby makes repayment of larger amounts of external debt credible.

framework. A proper analysis of this question is left for future research.<sup>11</sup>

#### 4. The bond market

In this Section we describe the microstructure of the bond market and thereby endogenize bond prices. In a setting where demand by domestic agents is publicly observable, such a treatment would be purely a theoretical exercise. Instead, we are interested in the situation in which markets are opaque in the sense that demand by locals cannot be distinguished from demand by foreigners. This is a realistic assumption to make. In particular, it would seem that opacity in our context is a necessary by-product of security design: In a first step we established that the government can only issue external debt, if the debt contract is

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<sup>11</sup>Kremer and Mehta (2000) identify a related reason for home bias: since repayment and the amount of tax are negatively correlated for locals, their risk exposure from holding the domestic bond is smaller than for foreigners, who are not ‘hedged’ through tax savings in the case of default. Hence, home bias may also result from risk sharing.

structured so as to force the borrower to make payments to the debtholders, indiscriminately of their identity. Once such a security is issued and is tradable on a secondary market, it is hard to see how one could trace ownership (and thereby distinguish domestic from foreign holdings): Any mechanism that allowed ownership to be traced, would facilitate discriminatory default on foreigners but not locals, and thereby undermine the government's ability to raise external debt.

Consider the following market mechanism to determine the price of a primary debt issue. The government issues  $B$  bonds. After  $\tilde{\delta}$  and  $f$  are realized, foreigners and locals submit their demands to a market maker (this could be similar to a book building financial intermediary). Note that the model set-up is so simple that we do not need to make any assumptions about who can observe the realizations of the random variables. However, the spirit of our model is that each agent can observe their own shock, but not the shocks of others (and therefore the aggregate shocks to foreign and local demand are not observed directly). The market maker observes total demand

$$d = f + l, \tag{7}$$

where  $l = \int_{i=0}^1 w_i di$  denotes local demand (denominated in dollars). Then the market maker sets the price of the bond contingent on  $B$  and  $d$  and meets order flow imbalances out of his own inventory. Following Kyle (1985) assume that the market maker faces Bertrand competition and therefore sets the price so as to break even in expectation on his trades, i.e., the price is equal to the expected bond value conditional on the information contained in order flow. This corresponds to our earlier assumption about informationally efficient prices, except that publicly available information is now restricted to aggregate demand  $d$ , which is a noisy signal of the local demand shock  $\tilde{\delta}$ . Since  $\mu_l \leq \frac{1}{2} \leq \mu_h$  there is genuine uncertainty regarding the median voter's position.

The market maker uses total order flow in order to update his belief about the median voter's position. Denote by  $g(d)$  the probability that the shock  $\tilde{\delta} = 1$  occurred conditional

on the demand observation  $d$ . If  $\tilde{\delta} = 1$ , then local demand is

$$l_1 = W(1 - \mu_l)$$

and if  $\tilde{\delta} = \delta$  then local demand is

$$l_\delta = W(1 - \mu_h + \delta(\mu_h - \mu_l)).$$

Using (7) the Bayesian update then yields

$$g(d) = \frac{(1 - \gamma)h(d - l_1)}{(1 - \gamma)h(d - l_1) + \gamma h(d - l_\delta)}. \quad (8)$$

This allows us a characterization of equilibrium prices when  $B = W$ .<sup>12</sup>

**Proposition 2** *If  $B = W$  then there exists a unique equilibrium with price*

$$P = \max\{\delta, g(d)\}. \quad (9)$$

*Proof.* Suppose  $g(d) > \delta$ . With probability  $g(d)$ ,  $\tilde{\delta} = 1$  and the median voter's position is  $\frac{W}{P} \geq B$ , so that repayment occurs. With complementary probability the median voter's position is  $\frac{\delta W}{P}$ . When  $P = g(d) > \delta$  then  $\frac{\delta W}{P} < B$  so default occurs. Hence, repayment occurs with probability  $g(d)$  and the corresponding fair price is then  $P = g(d)$ .

Suppose  $g(d) \leq \delta$ . Therefore, the probability that  $\tilde{\delta} = 1$  is smaller than  $\delta$ . If the median voter were to default for sure when  $\tilde{\delta} = \delta$ , then the price would drop below  $\delta$ . This, however, cannot be an equilibrium, because the median voter's position in that case would be larger than  $B$  and he would therefore vote in favour of repayment for sure. If the median voter always voted in favour of repayment, then the equilibrium price would have to be  $P = 1$ , which implies that the median voter has a strict preference for defaulting after  $\tilde{\delta} = \delta$ . The only equilibrium is therefore in mixed strategies over the default decision. The median voter is only indifferent between repayment and default after shock  $\tilde{\delta} = \delta$  if  $P = \delta$ . Denote by  $\theta$

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<sup>12</sup>It is straightforward to complete the characterisation of equilibrium for any level of  $B$ . For expositional clarity we focus on the case  $B = W$  which maximises the expected amount of funds raised ex ante.



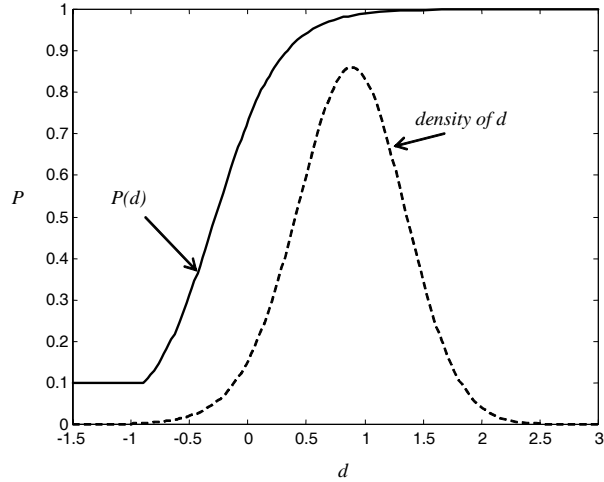


Figure 2: Gives the bond price as a function of underlying demand (solid line) and the density of demand (dashed line). Parameter values are  $W = 1$ ,  $\sigma^2 = 0.2$ ,  $\gamma = 0.05$ ,  $\delta = 0.1$ ,  $\mu_l = 0.1$ , and  $\mu_h = 0.9$ . A small probability of a shock to local demand can give rise to a strong drop in price for a small reduction in overall demand.

the probability with which the median voter after shock  $\tilde{\delta} = \delta$  votes in favour of repayment when he is indifferent. Then  $\theta$  is determined such that the price is informationally efficient:

$$\delta = g(d) + \theta(1 - g(d)). \quad (10)$$

Since  $\delta \geq g(d)$  it is clear that a  $\theta \in [0, 1]$  exists such that (10) can be satisfied for any  $\delta$ . ■

#### 4.1 Foreign demand shocks and debt crisis

In order to understand better how the price of debt varies with underlying demand, let us consider the example where foreign demand  $f$  is normally distributed with mean zero and variance  $\sigma^2$ . In this case the Bayesian update (8) can be calculated explicitly by substituting  $h(f)$  with the normal density. Figure 2 gives the resulting price function.

From Figure 2 we can see that the bond price is a non-linear function of total demand  $d$ . At high realizations of  $d$  it is relatively certain that local demand is sufficiently high to

guarantee repayment. Therefore, the price is relatively insensitive to changes in  $d$  - *the bond market is stable*. For lower realizations of  $d$  price can become extremely sensitive to changes in  $d$ . This occurs in the region where the market maker's inference over local demand is very noisy. The market maker therefore responds strongly to variations in underlying demand. The price of debt drops sharply in response to a decline in demand - *the bond market is unstable*. This can be understood as a crisis phenomenon: a small shock is enough to push the country over the brink.

Two points are noteworthy about Figure 2. Firstly, the price of the bond falls when foreigners sell it, even though their demand has no impact on the bond's intrinsic value. When foreigners sell, the realization of total demand is low. Due to market opacity, the demand shock cannot be identified as non-fundamental and drives down the bond price. As the picture shows, a non fundamental outflow of capital can thus have a substantial effect on the market price of the bond.

Secondly, our model provides an interesting variation on existing market microstructure models. In that type of setting price effects are commonly generated by the presence of traders who are privately informed about an asset's fundamental value. The market maker can then infer some of that information from order flow and adjusts prices in response. In our setting there are no privately informed traders. Instead, demand itself determines the bond's fundamental value via the political economy considerations that underlie the governments default decision. The information contained in total demand about its *composition* then generates a link between price and demand.

Note that more standard market microstructure models tend to generate a linear relationship between equilibrium price and demand (for an overview see O'Hara (1995)). An exception is Germain and Dridri (2001) who present a model where privately informed traders receive a signal with binary distribution ('buy' or 'sell'), and normally distributed noise. The resulting price function corresponds to our function  $g(d)$  and thus also features regions of relative price stability and a region where the price changes sharply in response to demand.

An important difference between that model and our analysis is that in Germain and Dridri (2001) the price function is driven by the signal distribution which is essentially arbitrary. By contrast the price function in our model is driven by the binary payoff distribution, which is endogenous and a robust feature of the political economy set-up that determines default.

## 5. Debt capacity

This Section addresses the issue how the country's ability to raise funds is affected by the underlying volatility of foreign capital flows. Foreign capital flows are non-fundamental but have a price impact because of market opacity. Market opacity is the cost the government incurs in return for the ability to issue external debt. The framework developed above allows us to quantify this cost of foreign demand volatility in terms of the government's ability to issue external debt.

In principle we can think about two slightly different mechanisms for selling debt depending on whether the face value of debt can or cannot be adjusted in response to observing aggregate demand. In the first scenario, the government may have to choose a quantity  $B$  of bonds before demand shocks are realized. In that case it sells  $B$  to a financial intermediary who subsequently acts as the market maker in the secondary market for the bonds. The intermediary would then be willing to pay the expected future secondary market price  $E(P)$  for the bonds. In the second setting, a financial intermediary carries out something akin to a bookbuilding exercise, before  $B$  is chosen. That is, the intermediary (and the government) can observe aggregate demand for the bonds before they determine the face value of the bonds that will be issued.

Since the second mechanism allows  $B$  to be conditioned on more information, it is not surprising that debt capacity is higher in that case. More interestingly, we show that an increase in the volatility of foreign demand has a negative impact on debt capacity in either case.

5.1 *The face value of debt  $B$  is chosen before demand shocks are realized*

Once we know the price function (9) we can calculate the expected amount of money  $K$  that the government can raise when it issues  $B$  bonds. Denote by  $H(f)$  the distribution function of  $f$ , i.e.,  $H(f) = \int_{-\infty}^f h(s)ds$ . Moreover, define  $d^*$  by  $\delta = g(d^*)$ . We can then calculate the expected price at which the government can issue the bond.

**Lemma 1** *If the government issues  $B = W$  bonds, their expected price will be*

$$E(P) = (1 - \gamma)(1 - (1 - \delta)H(d^* - l_1)) + \gamma\delta H(d^* - l_\delta). \quad (11)$$

Proof see Appendix.

Using (11) it is straightforward to calculate the country's debt capacity  $K = B \cdot E(P)$  as a function of the underlying distribution of foreign demand.<sup>13</sup> Going back to our earlier example of normally distributed foreign demand, the expected price can be calculated for different levels of foreign demand volatility  $\sigma$ . The result is provided in the next Proposition and illustrated in Figure 3.

**Proposition 3** *An increase in the volatility of foreign demand reduces debt capacity.*

Proof see Appendix.

The expected price of debt and therefore debt capacity, falls when foreign demand is more volatile. Given that all agents in the economy are risk neutral and that debt is fairly priced this result is non trivial. In order to understand it, we need to consider how the market mechanism allocates bonds across the population of locals. In the extreme case where foreign demand is certain, the equilibrium price fully reflects the underlying realization of local demand. If local demand is high ( $\tilde{\delta} = 1$ ) then the equilibrium price is  $P = 1$  and debt capacity is fully utilized at  $W = B$ . If local demand is low ( $\tilde{\delta} = \delta$ ), then the price drops to

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<sup>13</sup>Again it is straightforward to show that debt capacity is maximised if the government issues  $B = W$ .

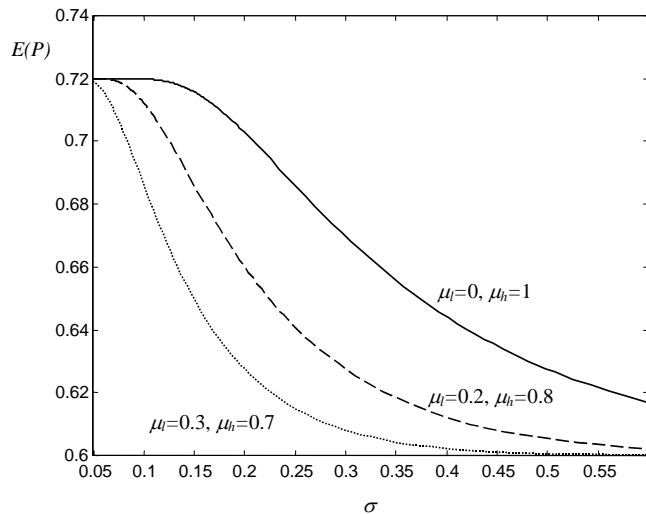


Figure 3: Shows the expected bond price (and debt capacity) as a function of the volatility  $\sigma$  of foreign demand. Parameter values are  $\gamma = 0.4$ ,  $\delta = 0.3$  and  $W = 1$ . The solid, dashed and dotted line have the following values for  $(\mu_l, \mu_h)$ , respectively:  $(0,1)$ ,  $(0.2,0.8)$ , and  $(0.3,0.7)$ .

$P = \delta$ . The price reduction is important, because it allows more bonds to be allocated to the median voter compared to if the price had stayed high at  $P = 1$ . This is crucial, because it stabilizes the underlying bond issue: even when local (and therefore the median voter's) demand is low, prices adjust so as to allow the median voter to hold enough bonds to repay with positive probability.

The allocation of bonds changes when foreign demand is volatile. The price now reflects underlying local demand with noise and therefore distorts the bond allocation. Distortions are possible in two directions. The price can be too high or too low, compared to the benchmark price that would obtain if local demand were publicly observable. If foreign demand is high and local demand is low, the price will end up too high. This reduces the number of bonds that will be allocated to locals: foreign demand crowds out local demand. The bond will then not be repaid and anticipating this possibility the bond price will be lower than 1 even when overall demand is high. This has a negative effect on the amount

of funds that can be raised. Conversely, if local demand is high and foreign demand is low the bond price may be too low. In this case, the median voter gets a large allocation of bonds. However, this increased allocation does not improve the bond price since local demand was high and repayment therefore occurs anyway. The pricing error introduced by the combination of foreign demand volatility and market opacity therefore has an asymmetric effect on debt capacity: overpricing crowds out the already low local demand and reduces the repayment probability, while underpricing has no effect on the repayment probability. On balance this reduces debt capacity.

The impact of volatility on debt capacity depends on the variability of local demand. Since a small variability in local demand can have a significant effect on the stability of sovereign debt, a correspondingly small amount of foreign noise can therefore have a large impact on debt capacity. This can also be seen from Figure 3, which provides the relationship between  $E(P)$  and  $\sigma$  for different values of  $\mu_l$  and  $\mu_h$ . The variability of local demand can be calculated as

$$l_1 - l_\delta = W(1 - \delta)(\mu_h - \mu_l).$$

When  $\mu_h - \mu_l$  is large local demand variability is large and therefore aggregate demand is relatively informative, even when foreign demand is somewhat volatile. As  $\mu_h - \mu_l$  becomes smaller the inferences about local demand that can be drawn from total demand become more noisy. An increase in foreign demand volatility therefore has a stronger negative impact on debt capacity. In the example a 20% volatility in foreign demand (measured in fraction of the outstanding face value of debt) reduces the country's overall debt capacity by 13%.

## 5.2 *The face value of debt $B$ can be conditioned on aggregate demand*

Consider now the case where the government can condition the face value of bonds  $B$  on the realization of aggregate demand  $d$ . This would correspond to a situation where the financial intermediary charged with the bond issue can carry out bookbuilding before selling

the bonds. The question then is what issuance policy would maximize the expected dollar amount of debt raised? The following Lemma answers this question.

**Lemma 2** *Debt capacity is maximized at  $W$  when the face value of bonds issued is*

$$B = \frac{W}{g(d)}.$$

*Proof.* If the government issues  $B = \frac{W}{g(d)}$  bonds, then repayment occurs when local demand is high  $\tilde{\delta} = 1$ , but not otherwise. This follows directly from the repayment condition on the median voter. The fair price of the bond is then  $P = g(d)$  and the revenue raised is exactly  $W$ . If  $B > \frac{W}{g(d)}$  then the price has to fall until the median voter is willing to repay when  $\tilde{\delta} = 1$  (otherwise repayment would never occur, which cannot be an equilibrium). Willingness to repay requires  $\frac{W}{P} \geq B$  and from this it follows directly that  $B \cdot P \leq W$ . Hence,  $W$  is indeed the maximum debt capacity. Moreover, reducing  $B$  to a level that would be compatible with repayment when  $\tilde{\delta} = \delta$  reduces debt capacity, because it would require  $\frac{\delta W}{P} \geq B$  and therefore  $B \cdot P \leq \delta W$ . ■

It is clear from the above that debt capacity is bigger when the government can condition on the observation of aggregate demand before setting  $B$ . This is not surprising, since aggregate demand is informative about the likelihood that the median voter has high demand, and therefore prices and  $B$  can be adjusted to ensure that a constant amount  $W$  can be raised. Alternatively, one could think about this mechanism in terms of a slightly different contract between the government and the issuing intermediary. The government makes a request to the intermediary to raise  $W$  worth of debt and determine the required face value of debt after having observed aggregate demand. Note that in this case foreign demand volatility has no impact on debt capacity.

This is not to say, however, that foreign demand volatility has no impact on the quality of the debt that is issued. According to the issuance policy the face value and the price of debt do depend on the realization of aggregate demand. In particular, if demand  $d$  falls, the government issues more and lower quality debt. If the government is concerned about the

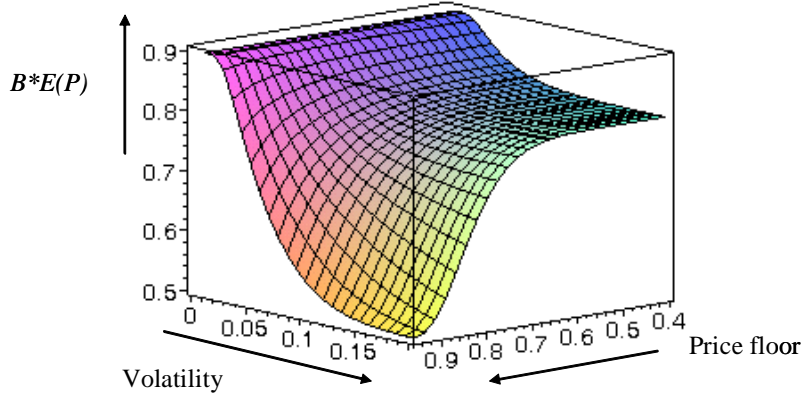


Figure 4: Illustrates the impact of foreign demand volatility on debt capacity, when the government imposes a lower bound on debt quality (and therefore price). Parameter values are:  $W = 1$ ,  $\mu_h = 0.6$ ,  $\mu_l = 0.4$ ,  $\delta = 0.5$  and  $\gamma = 0.2$ .

quality of its debt *per se* and not just revenues raised, then demand volatility does have an impact on debt capacity.

To see this consider a government that imposes a lower bound on the quality of its debt. Since the equilibrium price of debt exactly reflects the quality of debt, the government can simply impose a lower bound on the price of debt that it is willing to issue. Suppose the government has thus chosen a lower bound, or a price floor  $P_{floor}$  for its debt. In that case it has to modify its issuance rule as follows. If  $g(d) \geq P_{floor}$  then the government issues  $B = \frac{W}{g(d)}$  like before. When  $g(d) < P_{floor}$  the government reduces  $B$  to  $\frac{\delta W}{P_{floor}}$  so as to maintain a constant quality of debt. The government thus loses revenue whenever demand drops below the point where  $g(d) = P_{floor}$ .

An increase in foreign demand volatility now will have an impact on the expected revenue raised. This is because with higher demand volatility, the likelihood that demand shocks will force the government to reduce the size of the bond issue increases. In expectation debt capacity therefore falls when foreign demand volatility increases. Figure 4 illustrates this



effect. As foreign demand volatility increases the expected amount of debt raised falls. This effect is more pronounced when the government is more concerned with debt quality. The higher the lower bound on quality (price floor), the stronger is the impact that volatility has on debt capacity.

## 6. Partial default

In this Section we extend the basic model to allow for partial default. So far the simple structure of the model implied that the median voter always had a strict preference over repayment or default, so that the preferred fraction of repayment was always a corner solution  $\alpha \in \{0, 1\}$  (see condition (2)). An important policy issue that this simple structure cannot address is that of debt restructuring. Of particular interest is the question how the ease of debt renegotiation affects debt capacity (see Bolton and Jeanne (2005)). Understanding this issue is crucial in throwing light on the role of collective action clauses in sovereign lending or the usefulness of a statutory approach to sovereign debt restructuring.

Consider therefore the following modification to our model. As before assume that the population can be divided into three cohorts  $l, m$  and  $h$  by increasing initial wealth, so that for  $i \in l = [0, \mu_l]$  agents have wealth  $w^l = 0$ , for  $i \in m = (\mu_l, \mu_h)$  they have wealth  $w^m \geq 0$  and for  $i \in h = [\mu_h, 1]$  they have wealth  $w^h > 0$ , and  $w^h \geq w^m$ . Suppose also that agents now receive additional income at date 1, denoted by  $y^i$ . In the spirit of a life-cycle model of income, we assume that agents' date 0 wealth is inversely related to their date 1 income, so that  $y^l \geq y^m \geq y^h = 0$ . Hence, one could interpret cohort  $l$  as the young generation who earn no income currently ( $w^l = 0$ ), but will have highest earnings in the next period, cohort  $m$  is then a medium generation that earns now and later, and cohort  $h$  corresponds to an old generation who earn a lot now and nothing later. Denote domestic per capita wealth at

dates 0 and 1 by

$$\begin{aligned}\bar{w} &= (\mu_h - \mu_l) w^m + (1 - \mu_h) w^h, \\ \bar{y} &= \mu_l y^l + (\mu_h - \mu_l) y^m.\end{aligned}$$

As before assume that all agents care only about date 1 consumption and the only store of wealth is the government bond. It therefore follows that (1) holds.

Suppose that the government finances the bond repayment through a proportional tax on date 1 income (labour income plus income from liquidating financial assets). Alternatively, one could interpret the tax as a consumption tax, which boils down to the same in this simple model. If the government chooses to repay a fraction  $\alpha$  of its liabilities, it needs to raise tax  $T = \alpha B$  and set the tax rate  $\tau$  such that

$$\tau \left( \bar{y} + \alpha \frac{\bar{w}}{P} \right) = \alpha B.$$

We can then determine the tax rate as a fraction of  $\alpha$  :

$$\tau(\alpha) = \frac{\alpha B}{\bar{y} + \alpha \frac{\bar{w}}{P}}. \quad (12)$$

If the government proposes to repay a fraction  $\alpha$  of its debt, agent  $i$  will end up with net wealth in date 1 given by

$$W_i(\alpha) = \left( y_i + \alpha \frac{w_i}{P} \right) \left( 1 - \frac{\alpha B}{\bar{y} + \alpha \frac{\bar{w}}{P}} \right). \quad (13)$$

Different cohorts now have differing preferences over  $\alpha$ . This can be seen by taking the first derivative of  $W_i(\alpha)$  with respect to  $\alpha$  for agents belonging to each cohort. Taking the first derivative of (13) yields

$$W_i'(\alpha) = \frac{w_i}{P} - B \left( \lambda(\alpha) \frac{y_i}{\bar{y}} + (1 - \lambda(\alpha)) \frac{w_i}{\bar{w}} \right), \quad (14)$$

where  $\lambda(\alpha) = \left( \frac{\bar{y}}{\bar{y} + \alpha \frac{\bar{w}}{P}} \right)^2$ .

The  $l$ -types clearly benefit from a lower  $\alpha$ : with  $w^l = 0$ , they hold no bonds but pay taxes and therefore always benefit from default. The  $h$ -types on the other hand have a

large position  $\frac{w^h}{P}$  in the bond but share the tax burden with all other agents. They therefore lose most from default. The  $m$ -types' preferred  $\alpha$ , however, may not necessarily be a corner solution. This is captured in the following result. Denote by  $\alpha_m^* = \arg \max W_m(\alpha)$ .

**Lemma 3** *The  $m$ -types' wealth  $W_m(\alpha)$  is maximized at an interior solution  $0 < \alpha_m^* < 1$  if*

$$\frac{w^m}{\bar{w}} > \frac{y^m}{\bar{y}}, \quad (15)$$

and

$$\frac{y^m}{\bar{y}} < \frac{w^m}{B \cdot P} < \lambda(1) \frac{y^m}{\bar{y}} + (1 - \lambda(1)) \frac{w^m}{\bar{w}}. \quad (16)$$

*Proof.* We can take the second derivative of (13) with respect to  $\alpha$  and verify that it is negative if and only if (15) holds. It follows that the first-order condition is satisfied at a maximum if (15) holds. For an interior solution we require that  $W'_m(\alpha = 0) > 0$ . Straightforward calculations show that this is the case if and only if  $\frac{y^m}{\bar{y}} < \frac{w^m}{B \cdot P}$ . Moreover, we require that  $W'_m(\alpha = 1) < 0$ , which holds if and only if  $\frac{w^m}{B \cdot P} < \lambda(1) \frac{y^m}{\bar{y}} + (1 - \lambda(1)) \frac{w^m}{\bar{w}}$ . ■

For low values of  $\alpha$   $m$ -types benefit from an increase in  $\alpha$ . Since they hold some bonds, they benefit directly from more repayment. Moreover, since their wealth depends itself on the default rate, they have little taxable income when  $\alpha$  is small, and the increase in the tax burden therefore falls most heavily on the  $l$  cohort. When  $\alpha$  increases, the share of total taxable income that the  $m$  types contribute also increases and at some point this effect more than outweighs the gains from higher repayment on the bond. Depending on the parameter values, there may therefore be an interior  $0 < \alpha^* < 1$  at which  $W^m(\alpha)$  is maximized. Suppose for the remainder of this Section that conditions (15) and (16) are satisfied.

Consider then the following (standard) set-up of the electoral process (see Persson and Tabellini, 2002). Two parties  $a$  and  $b$  compete in the election by simultaneously proposing their electoral platform  $\alpha_a$  and  $\alpha_b$ , respectively. Each party chooses their platform so as to maximize their probability of winning the election given the other party's platform. The election outcome is given in the following proposition.

**Proposition 4** *The winning proposal is  $\alpha_m^*$ .*

*Proof.* The three cohorts of the electorate can be divided according to their preferences over  $\alpha$ . It is clear from the previous discussion that all agents in cohort  $l$  will vote for the smallest  $\alpha$  available. Moreover, under our maintained assumptions, the  $h$ -types' preferred  $\alpha$  is larger than that of the  $m$ -types:  $\alpha_h^* \geq \alpha_m^*$ . Since preferences over  $\alpha$  are single peaked within each cohort, we can apply the median voter theorem, i.e., the unique sub-game perfect equilibrium of electoral competition yields  $\alpha_a = \alpha_b = \alpha_m^*$ . ■

Once we can allow for partial default, we can address the issue of how the ease of renegotiating sovereign debt contracts affects repayment behavior and debt capacity. When the sovereign can renegotiate freely, one can expect the outcome of debt restructuring to correspond to  $\alpha_m^*$ . When renegotiation is inhibited by collective action problems, the sovereign is effectively constrained to either repay in full, or not to repay at all (any partial repayment would not be accepted by some bondholders holding out for better terms, which undermines the sovereign's ability to settle to repay some but not all of its debt). A full analysis of the outcome of renegotiation with and without constraints on  $\alpha$  is left for future research.

## 7. Conclusions

The preceding analysis shows how political economy considerations can help understand sovereign borrowers apparent ability to access international debt markets. It provides a framework that combines explicitly elements from political economy modeling with aspects of price formation in a market microstructure setting. A number of results have been derived regarding liquidity and financial crises, debt capacity and debt restructuring. Beyond the interest of the results themselves, the paper hopes to illustrate the power of the underlying approach to a wider range of problems.

One example is the 'home bias' puzzle, which is a direct corollary of our central hypothesis. Applied to equity markets, one could think of a situation in which companies may benefit

from local political support, for example by receiving public contracts, subsidies etc. The degree to which policy makers yield to a company's lobbying pressure may well be influenced by the fraction of the electorate amongst its shareholders. As a result companies with a local shareholder base may perform better than companies that do not.

Another application concerns the currency denomination of corporate foreign borrowing. As Calvo and Guidotti (1990) pointed out, a government may have an incentive to devalue its exchange rate if corporations have all their foreign debt denominated in local currency. The approach espoused in this paper may help throw light on exchange rate volatility and currency crises in emerging markets.

## 8. Appendix

*Proof of Lemma 1.* Denote by  $k(d)$  the density of  $d$ . We can write

$$\begin{aligned} k(d) = & (1 - \gamma) h(d - W(1 - \mu_l)) + \\ & \gamma h(d^* - W(1 - \mu_h + \delta(\mu_h - \mu_l))). \end{aligned} \quad (17)$$

On the interval  $d \leq d^*$ , the price is  $\delta$  and for  $d > d^*$  it is  $g(d)$ . The probability that  $d \leq d^*$  is given by

$$\begin{aligned} \text{prob}(d \leq d^*) = & (1 - \gamma) H(d^* - W(1 - \mu_l)) \\ & + \gamma H(d^* - W(1 - \mu_h + \delta(\mu_h - \mu_l))) \end{aligned}$$

For  $d > d^*$  we can calculate the expected price from

$$\int_{d^*}^{\infty} k(s)g(s)ds.$$

Using (8) and (17) this simplifies to

$$\int_{d^*}^{\infty} (1 - \gamma) h(s - W(1 - \mu_l))ds.$$

This expression can be rearranged to yield

$$(1 - \gamma)(1 - H(d^* - W(1 - \mu_l))).$$

Adding up  $\delta \text{prob}(d \leq d^*)$  and  $(1 - \gamma)(1 - H(d^* - W(1 - \mu_l)))$  yields  $E(P)$ . ■

*Proof of Proposition 3.*

We can calculate debt capacity  $K$  as a function of the variance  $\sigma^2$  by using our result from (11).

$$K(\sigma^2) = \int_{-\infty}^{d^*} \gamma \delta h(s - l_\delta) - (1 - \gamma)(1 - \delta)h(s - l_1) ds$$

We can then take the first derivative with respect to  $\sigma^2$  which is

$$\begin{aligned} \frac{dK(\sigma^2)}{d\sigma^2} &= (\gamma \delta h(d^* - l_\delta) - (1 - \gamma)(1 - \delta)h(d^* - l_1)) \frac{\partial d^*}{\partial \sigma^2} \\ &+ \int_{-\infty}^{d^*} \gamma \delta \frac{\partial h(s - l_\delta)}{\partial \sigma^2} - (1 - \gamma)(1 - \delta) \frac{\partial h(s - l_1)}{\partial \sigma^2} ds. \end{aligned}$$

Using the definition of  $d^*$  it follows that  $\gamma \delta h(d^* - l_\delta) - (1 - \gamma)(1 - \delta)h(d^* - l_1) = 0$ . Taking the normal density for  $h$ , we can calculate explicitly the derivative of  $h$  with respect to  $\sigma^2$  and therefore re-write the integral as

$$\frac{1}{\sqrt{2\pi\sigma^2}2\sigma^2} \int_{-\infty}^{d^*} \gamma \delta e^{-\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}} \left( \frac{(s-l_\delta)^2}{\sigma^2} - 1 \right) - (1-\gamma)(1-\delta) e^{-\frac{1}{2}\frac{(s-l_1)^2}{\sigma^2}} \left( \frac{(s-l_1)^2}{\sigma^2} - 1 \right) ds. \quad (18)$$

In the next step we can calculate the integral  $\int_{-\infty}^{d^*} e^{-\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}} \frac{(s-l_\delta)^2}{\sigma^2} ds$  by making the following

substitution: Let  $u'(s) = e^{-\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}} \frac{(s-l_\delta)}{\sigma^2}$  and  $v(s) = s - l_\delta$ . Using this substitution we know that  $u(s) = -e^{\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}}$  and  $v'(s) = 1$ . Integration by parts then yields

$$\begin{aligned} &\int_{-\infty}^{d^*} e^{-\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}} \frac{(s-l_\delta)^2}{\sigma^2} ds \\ &= \left[ -e^{-\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}} (s-l_\delta) \right]_{-\infty}^{d^*} + \int_{-\infty}^{d^*} e^{-\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}} ds. \end{aligned}$$

If we substitute this expression into (18) we can write the condition that  $\frac{dK}{d\sigma^2} < 0$  as

$$\gamma \delta \left[ -e^{-\frac{1}{2}\frac{(s-l_\delta)^2}{\sigma^2}} (s-l_\delta) \right]_{-\infty}^{d^*} - (1-\gamma)(1-\delta) \left[ -e^{-\frac{1}{2}\frac{(s-l_1)^2}{\sigma^2}} (s-l_1) \right]_{-\infty}^{d^*} < 0.$$

This is the same as

$$\gamma\delta e^{-\frac{1}{2}\frac{(d^*-l_\delta)^2}{\sigma^2}}(d^* - l_\delta) - (1 - \gamma)(1 - \delta)e^{-\frac{1}{2}\frac{(d^*-l_1)^2}{\sigma^2}}(d^* - l_1) > 0.$$

From the definition of  $d^*$  and from  $l_\delta < l_1$  it follows that the above inequality holds. ■

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