Prior User Rights

By CARL SHAPIRO*

Many inventions are discovered independently, but at roughly the same time, by two or more individuals or organizations. Famous examples include the light bulb, the telephone, and the integrated circuit. Independent invention is common for minor technological improvements. How should property rights to an invention be defined and awarded in such cases?

Patent law has struggled with this question for many years. The basic rule in the United States is that the patent is awarded to the first inventor, but this system can create some peculiar results.

Suppose Firm A invents something and files for a patent. Slightly later, before the invention is made public, Firm B independently invents the same thing. Firm A receives the patent and can prevent Firm B from developing the invention. In legal terms, a party accused of patent infringement cannot defend itself by showing it discovered the same invention independently. Would such an *independent invention defense* be desirable?

Alternatively, suppose that Firm A invents something, but decides *not* to file for a patent, perhaps because Firm A does not believe this invention is sufficiently novel and nonobvious to be patentable. Instead, Firm A uses the invention internally as a trade secret. Later, Firm B invents the same thing and files for a patent. Under U.S. law, Firm B is awarded the patent and usually can prevent Firm A from practicing the invention. In legal terms, Firm A, despite achieving and using the invention before Firm B obtained its patent, has no *prior user rights*. Would granting such rights be desirable?

Since 1999, U.S. law has provided for some prior user rights for patents on business methods, and Congress is considering legislation (H.R. 2795) that would greatly expand prior user rights.

* Haas School of Business, University of California, Berkeley, CA 94720-1900 (e-mail: shapiro@haas.berkeley.edu). I thank Joseph Farrell, Richard Gilbert, Michael Katz, Mark Lemley, and Josh Lerner for very helpful comments. All proofs are in a separate technical appendix available at www.e-aer.org/data/may06_app_P06066.pdf.

This paper explores the effects of awarding prior user rights. We abstract away from the details of which party discovered the invention before or after another, viewing slight differences in timing as random. With this abstraction, there is no difference between the independent invention defense and prior user rights.

Suppose two firms are conducting research and development (R&D) directed at a given invention. Prior user rights come into play only if *both* firms successfully discover the invention. In that event, without prior user rights, each firm has a 50-percent chance of getting the patent and obtaining a monopoly over the patented invention. Call monopoly profits π_M and welfare under monopoly W_M. With prior user rights, both firms have the right to use the invention, so duopoly results. Call each firm's duopoly profits π_D and duopoly welfare W_D. Assume combined duopoly profits are less than monopoly profits, $2\pi_D < \pi_M$, and welfare is less under monopoly than duopoly, $W_M < W_D$. So, the ex post effects of prior user rights are clear: if both firms discover the invention, prior user rights enhance competition, reduce joint profits, and increase welfare.

What about the ex ante effects of awarding prior user rights? Prior user rights reduce the return to achieving the invention. If the firms' R&D expenditures without prior user rights are socially excessive, awarding those rights has favorable ex ante and ex post effects. Stephen Maurer and Suzanne Scotchmer (2002) make this point in a static model with free entry in which each firm, by paying a fixed amount, can discover the invention with certainty, so all R&D expenditures by multiple firms are duplicative. We show the attractiveness of prior user rights extends well beyond situations in which equilibrium R&D expenditures are excessive.

I. R&D Expenditure Levels with Independent Projects

Suppose two firms are engaged in R&D competition, each choosing how much to spend on

R&D. Greater expenditures increase the chance of success, with diminishing returns. The cost of achieving a success probability p is $C(p)$ with $C(0) = 0$, $C'(p) > 0$, and $C''(p) > 0$. Success by one firm is independent of success by the other.

Two patent policy instruments are available: patent lifetime, T, and the strength of prior user rights. There is no discounting, the invention is useful during [0, 1], and the patent remains in force during $[0, T]$. After the patent expires, the market is competitive, the firms earn zero profits, and welfare is W_C. Stronger prior user rights are modeled by an increase in the probability, α, that prior user rights will be granted if both firms achieve the invention. Stronger prior user rights correspond to policy changes that lower the requirements for such rights to be granted, such as Congress is currently considering. If both firms are successful, each receives a flow payoff of $\pi_B = \alpha[\pi_D] + (1 - \alpha)[\pi_M/2]$ during $[0, T]$, for a payoff of $\pi_B T$. Flow welfare is $W_B = \alpha[W_D] + (1 - \alpha)[W_M]$ during $[0, T]$ and W_C during $[T, 1]$. If only one firm is successful, that firm's payoff is $\pi_M T$, and welfare is $W_M T + W_C(1 - T)$.

A single firm whose rival's success rate is q chooses its own success rate p to maximize $\pi(p, q) = p(1 - q)T\pi_M + pqT\pi_B - C(p)$. The first-order condition for this firm is $\pi_p(p, q) = T[(1 - q)\pi_M + q\pi_B] - C'(p) = 0$. In the symmetric equilibrium, $[C'(p)/T\pi_M] = 1 - p[1 - (\pi_B/\pi_M)]$. The equilibrium success rate depends on the policy parameters, T and α. Welfare is

$$W(p, T, \alpha) = p^2[TW_B + (1 - T)W_C] + 2p(1 - p)[TW_M + (1 - T)W_C] - 2C(p).$$

THEOREM 1: *Suppose each firm chooses its R&D investment level, with greater investment increasing the chance of success, and with success at one firm independent of success at the other. Prior user rights are socially optimal if and only if the ratio of deadweight loss to profits is higher under monopoly than under duopoly.*

Prior user rights are an attractive feature of the patent system if duopoly delivers returns to innovators more efficiently, in terms of deadweight loss, than monopoly (see the deadweight loss to profit ratio test in Louis Kaplow, 1984.) Richard Gilbert and Shapiro (1990) show this condition holds if profits and welfare are concave in output, a very weak condition.

II. Diversification of Research Approaches

We now use the model from Partha Dasgupta and Eric Maskin (1987) to study how prior user rights affect firms' decisions to allocate fixed research budgets across R&D projects. Each of two firms can adopt an approach that is less correlated with its rival, but doing so reduces its probability of success. Dasgupta and Maskin established conditions under which the market is biased toward overly correlated project choices, but did not study prior user rights.

The first firm selects a project $x \in [0, ½]$ and the second firm selects a project $y \in [0, ½]$. Higher values correspond to projects that are less likely to succeed: the probability of success for project z is $p(z)$, with $p(0) > 0$, $p'(z) < 0$, and $p''(z) < 0$. Higher values of x and y correspond to research projects that are less correlated; the correlation between the two projects equals $1 - (x + y)$. The probability that both firms succeed is $B(x, y)$ and the probability that only the first firm succeeds is $A(x, y)$. Imposing symmetry, the probability that only the second firm succeeds is given by $A(y, x)$.

The first firm picks x to maximize $\Pi = A(x, y)\pi_M + B(x, y)\pi_B$, giving the first-order condition $A_x(x, y)\pi_M + B_x(x, y)\pi_B = 0$. Since $A(x, y) + B(x, y) = p(x)$, $A_x(x, y) + B_x(x, y) = p'(x) < 0$. Substituting into the first-order condition, $A_x(x, y)\pi_M + [p'(x) - A_x(x, y)]\pi_B = 0$, or $A_x(x, y)[\pi_M - \pi_B] + p'(x) = 0$. Since $\pi_M > \pi_B$, if x is chosen optimally, we must have $A_x(x, y) > 0$ and $B_x(x, y) < 0$. Since $\partial^2\Pi/\partial x\partial\pi_B = B_x(x, y) < 0$, prior user rights reduce π_B and cause the first firm to increase x. Prior user rights reduce the return if both firms are successful and, thus, cause each firm to select a less correlated research approach.

The symmetric equilibrium is characterized by $A_x(x, x)\pi_M + B_x(x, x)\pi_B = 0$. Welfare is given by $W(x, y, \alpha) = W_M(A(x, y) + A(y, x)) + W_B B(x, y)$. The direct effect of awarding stronger prior user rights is positive, so stronger prior user rights raise welfare if their indirect effects are also favorable for welfare, which will be true if the equilibrium is biased toward overly correlated projects.

THEOREM 2: *Suppose that each firm picks from a menu of R&D projects. Projects at one firm that are more likely to succeed are also more highly correlated with the other firm's projects. Strengthening prior user rights raises welfare if* $(\pi_B/\pi_M) > [(W_B - W_M)/W_M]$.

Strengthening prior user rights raises social welfare if each firm is biased toward joint versus sole discovery. The firm's trade-off is reflected in the ratio π_B/π_M. The social trade-off is reflected in the ratio $(W_B - W_M)/W_M$. If $\pi_B/\pi_M > (W_B - W_M)/W_M$, the equilibrium is biased toward joint discovery, and prior user rights help correct for this bias. With no prior user rights, $\alpha = 0$, $\pi_B/\pi_M = 1/2$, and $W_B = W_M$, so the inequality in theorem 2 is satisfied.

COROLLARY 2A: *At least some prior user rights are socially optimal.*

Since π_B decreases with α and W_B increases with α, the inequality in theorem 2 will be satisfied for all values of α if it is satisfied at $\alpha = 1$. Therefore, we also have:

COROLLARY 2B: *Full prior user rights are socially optimal if* $(\pi_D/\pi_M) > [(W_D - W_M)/W_M]$.

Luís Cabral (1994) shows that this condition is satisfied in Cournot duopoly with linear demand and constant marginal costs. With homogeneous products and Bertrand competition, however, we have $\pi_D = 0$ and $W_D > W_M$, so this inequality is not satisfied. If competition is sufficiently severe, each firm will see little value in being one of two inventors, even though there is a social benefit of having a second inventor. Therefore, full prior user rights can cause the market to be biased toward projects that are less likely to succeed, but less correlated. In that case, the indirect effect of stronger prior user rights on welfare is adverse, but full prior user rights may still be optimal due to their favorable direct effect.

III. Allocation of R&D Budgets across Markets

We now ask how prior user rights affect firms' decisions to allocate their fixed R&D budgets across markets. Following Cabral (1994), each of two firms allocates its R&D budget between a smaller market, in which innovation is easier, and a larger market, in which innovation is harder. Success by one firm is independent of success by the other.

A firm that allocates a fraction x of its R&D budget to the smaller market will achieve the innovation in that market with probability $p(x)$, where $p'(x) > 0$ and $p''(x) < 0$. The larger market involves a lower probability of success, $p(1 - x)/\sigma$, where $\sigma > 1$, but a proportionately larger payoff, $\sigma\pi_M$ or $\sigma\pi_B$.

Suppose that the other firm is expected to allocate a fraction y of its budget to the smaller market. Therefore, the other firm is expected to succeed in the smaller market with probability $p(y)$ and in the larger market with probability $p(1 - y)/\sigma$. The payoff to the first firm is

$$p(x)p(y)\pi_B + p(x)(1 - p(y))\pi_M + \frac{p(1-x)}{\sigma}\frac{p(1-y)}{\sigma}\sigma\pi_B + \frac{p(1-x)}{\sigma}\frac{1-p(1-y)}{\sigma}\sigma\pi_M.$$

Total welfare in the symmetric Nash equilibrium is

$$W(x, \alpha) = p(x)^2 W_B + 2p(x)(1 - p(x))W_M + \left(\frac{p(1-x)}{\sigma}\right)^2 \sigma W_B + 2\frac{p(1-x)}{\sigma}\frac{1-p(1-x)}{\sigma}\sigma W_M.$$

Stronger prior user rights cause the firms to shift R&D resources into the larger market, where discovery by the rival is less likely, so prior user rights are less likely to come into play. Cabral (1994) proves the market is biased against R&D in the larger market if and only if $(\pi_B/\pi_M) > [(W_B - W_M)/W_M]$, so we have:

THEOREM 3: *Suppose each firm allocates its R&D budget between a smaller market and a larger market, in which innovation is more difficult. Stronger prior user rights cause the firms to shift their R&D budgets toward the larger market. Some prior user rights are always so-*

cially optimal. Full prior user rights are socially optimal if $(\pi_D/\pi_M) > [(W_D - W_M)/W_M]$.

Even if the inequality in Theorem 3 is not satisfied, full prior user rights may be optimal due to their favorable direct effect on welfare.

IV. Concluding Remarks

When nearly simultaneous, independent invention occurs, awarding one inventor a patent and the other the right to use the invention has very attractive properties. Competition is enhanced, innovation is rewarded with relatively little deadweight loss, and the private and social incentives to be the sole versus joint inventor are generally better aligned than in the absence of such rights.

The attractiveness of prior user rights is even stronger if we take into account the fact that a single patent lifetime is set for all industries and inventions, despite huge differences across inventions in their expected profit-to-cost ratios. Prior user rights *automatically* reduce the rewards precisely for those inventions with a high profit-to-cost ratio, since these are the inventions most likely to be discovered simultaneously. These are also the inventions that the patent system is most likely to overreward. From a Bayesian perspective, the fact that an invention was discovered independently by two or more parties is evidence that the profit-to-cost ratio for that invention was relatively high, so reducing the reward based on market power is attractive.

The appeal of prior user rights is especially great today, given mounting evidence that the patent system is out of balance, as argued by the Federal Trade Commission (2003), the National Academies of Science (2004), Adam Jaffe and Josh Lerner (2004), Shapiro (2004), and Mark Lemley and Shapiro (2005). Prior user rights can partially correct for problems caused when patents are issued for obvious or nearly obvious inventions, and for inventions that are not truly novel.

The main potential drawback associated with prior user rights is that they may encourage inventors to keep their inventions secret rather than disclosing them in patent applications. Vincenzo Denicolo and Luigi Franzoni (2004) develop a model in which a second party, who duplicates and patents an invention he knows had previously been discovered but kept secret, should be granted the right to exclude the inventor from using his invention. The effectiveness of patent disclosures is in doubt, however, especially in industries where scientists and engineers are instructed not to read patents for fear of triggering liability for willful infringement. Plus, the current patent system rewards applicants who are most aggressive in seeking patents over those who simply use their own inventions internally as trade secrets. The effects of encouraging inventors to adopt trade secret versus patent protection are not well understood. Further work is needed to compare the benefits of prior user rights, as described here, with any costs that result from inducing some inventors to seek trade secret rather than patent protection.

REFERENCES

Cabral, Luís. "Bias in Market R&D Portfolios." *International Journal of Industrial Organization*, 1994, *12*(4), pp. 533–47.

Dasgupta, Partha and Maskin, Eric. "The Simple Economics of Research Portfolios." *Economic Journal*, 1987, *97*(387), pp. 581–95.

Denicolò, Vincenzo and Franzoni, Luigi A. "Patents, Secrets, and the First-Inventor Defense." *Journal of Economics and Management Strategy*, 2004, *13*(3), pp. 517–38.

Gilbert, Richard and Shapiro, Carl. "Optimal Patent Length and Breadth." *RAND Journal of Economics*, 1990, *21*(1), pp. 106–12.

Jaffe, Adam and Lerner, Josh. *Innovation and its discontents: How our broken patent system is endangering innovation and progress, and what to do about it*. Princeton: Princeton University Press, 2004.

Kaplow, Louis. "The Patent-Antitrust Intersection: A Reappraisal." *Harvard Law Review*, 1984, *97*(8), pp. 1813–92.

Lemley, Mark A. and Shapiro, Carl. "Probabilistic Patents." *Journal of Economic Perspectives*, 2005, *19*(2), pp. 75–98.

Maurer, Stephen M. and Scotchmer, Suzanne. "The Independent Invention Defense in Intellectual Property." *Economica*, 2003, *69*(276), pp. 535–47.

Merrill, Stephen A.; Levin, Richard C. and Myers, Mark B., eds., *A patent system for the*

21st century. Washington, DC: The National Academies Press, 2004.

Shapiro, Carl. "Patent System Reform: Economic Analysis and Critique." *Berkeley Technology Law Journal*, 2004, *19*(3), pp. 1017–47.

U.S. Federal Trade Commission. *To promote innovation: The proper balance between competition and patent law and policy*. Washington, DC: U.S. Federal Trade Commission, 2003. Available at http://www.ftc.gov/os/2003/10/innovationrpt.pdf.

Prior User Rights: Appendix*

Carl Shapiro†

May 2006

R&D Expenditure Levels with Independent Projects

Discounting could easily be incorporated into this model by redefining T to represent the ratio of the value of an annuity that lasts for the lifetime of the patent to the value of a perpetuity.

A. Proof of Theorem #1

If the patent lifetime *T*, is set optimally, given α, we must have $\frac{dW}{dT} = \frac{\partial W}{\partial p}\frac{\partial p}{\partial T} + \frac{\partial W}{\partial T} = 0$, so $\frac{\partial W}{\partial p} = \frac{\partial W}{\partial T} / \frac{\partial p}{\partial T}$. The welfare impact of strengthening prior user rights is given by $\frac{dW}{d\alpha} = \frac{\partial W}{\partial p}\frac{\partial p}{\partial \alpha} + \frac{\partial W}{\partial \alpha}$. Substituting for $\partial W / \partial p$, we get $\left.\frac{dW}{d\alpha}\right|_{T=T^*} = \frac{\partial p}{\partial \alpha}\frac{\partial W}{\partial T} / \frac{\partial p}{\partial T} + \frac{\partial W}{\partial \alpha}$, so $\left.\frac{dW}{d\alpha}\right|_{T=T^*} > 0$ if and only if $\frac{\partial p}{\partial \alpha}\frac{\partial W}{\partial T} / \frac{\partial p}{\partial T} + \frac{\partial W}{\partial \alpha} > 0$. Since $\frac{\partial W}{\partial T} < 0$, we have $\left.\frac{dW}{d\alpha}\right|_{T=T^*} > 0$ if and only if

$$\frac{\partial W}{\partial \alpha} / [-\frac{\partial W}{\partial T}] > [-\frac{\partial p}{\partial \alpha}] / \frac{\partial p}{\partial T}.$$

We now proceed to establish that this inequality is met.

---

* This is the Appendix to my paper, "Prior User Rights," (2006), *American Economic Review Papers and Proceedings*, May, vol. 96, no. 2. The paper and appendix together are available at my web site at http://faculty.haas.berkeley.edu/shapiro/prior.pdf. This Appendix alone is available at http://faculty.haas.berkeley.edu/shapiro/priorapp.pdf.

† Haas School of Business and Department of Economics, University of California at Berkeley.

The left-hand side of this inequality is easy to calculate. As noted above, $dW_B / d\alpha = W_D - W_M$,

so $\frac{\partial W}{\partial \alpha} = p^2 T(W_D - W_M)$. From the definition of $W(p,T,\alpha)$ we also get

$-\frac{\partial W}{\partial T} = p^2(W_C - W_B) + 2p(1-p)(W_C - W_M)$. Therefore, we have

$$\frac{\partial W}{\partial \alpha} / [-\frac{\partial W}{\partial T}] = \frac{pT(W_D - W_M)}{p[W_C - W_B] + 2(1-p)[W_C - W_M]}.$$

We now look more closely at the $p(T,\alpha)$ function to obtain an expression for the right-hand side of above inequality .

Using the condition that defines the symmetric equilibrium level of p, we get

$\frac{\partial p}{\partial T} = \frac{(1-p)\pi_M + p\pi_B}{C''(p) + T(\pi_M - \pi_B)}$ and $-\frac{\partial p}{\partial \alpha} = \frac{pT(\pi_M / 2 - \pi_B)}{C''(p) + T(\pi_M - \pi_B)}$ so we have

$$[-\frac{\partial p}{\partial \alpha}] / \frac{\partial p}{\partial T} = \frac{pT(\frac{\pi_M}{2} - \pi_D)}{(1-p)\pi_M + p\pi_B}.$$

So, we have $\left.\frac{dW}{d\alpha}\right|_{T=T^*} > 0$ if and only if

$$\frac{(W_D - W_M)}{p[W_C - W_B] + 2(1-p)[W_C - W_M]} > \frac{\frac{\pi_M}{2} - \pi_D}{(1-p)\pi_M + p\pi_B}.$$

Substituting using $W_B = (1-\alpha)W_M + \alpha W_D$ and $\pi_B = (1-\alpha)\pi_M / 2 + \alpha\pi_D$, this becomes

$$\frac{(W_D - W_M)}{p[W_C - (1-\alpha)W_M - \alpha W_D] + 2(1-p)[W_C - W_M]} > \frac{\frac{\pi_M}{2} - \pi_D}{(1-p)\pi_M + p[(1-\alpha)\pi_M / 2 + \alpha\pi_D]}.$$

Collecting terms, this becomes

$$\frac{(W_D - W_M)}{(2-p)[W_C - W_M] - \alpha p[W_D - W_M]} > \frac{\pi_M - 2\pi_D}{(2-p)\pi_M - \alpha p[\pi_M - 2\pi_D]}.$$

Inverting both sides and simplifying gives

$$\frac{W_C - W_M}{W_D - W_M} < \frac{\pi_M}{\pi_M - 2\pi_D}.$$

Inverting again and simplifying gives $\frac{2\pi_D}{\pi_M} > \frac{W_C - W_D}{W_C - W_M}$. Defining the monopoly deadweight loss as $DWL_M = W_C - W_M$ and the duopoly deadweight loss as $DWL_D = W_C - W_D$, granting stronger prior user rights raises welfare if and only if $\frac{DWL_M}{\pi_M} > \frac{DWL_D}{2\pi_D}$, as asserted in the text.

B. Ratio of Profits to Deadweight Loss

Gilbert and Shapiro (1990) show that the ratio of deadweight loss to profits rises with price is profits and welfare are both concave in output. Here we establish an alternative sufficient condition. The material in this section was developed jointly with Joseph Farrell.

Call the demand function $X(p)$. Assume that output can be produced at constant marginal cost *c*. Denote by $L(p)$ the deadweight loss if the price is *p*. [For this subsection alone, *p* denotes price, not the probability of discovery.] Denote by $\Pi(p) = (p-c)X(p)$ the total profits if price is *p*. Under what circumstances is the ratio $L(p)/\Pi(p)$ increasing in price *p* in the range $c \le p \le p^M$, where p^M is the monopoly price?

The ratio $L(p)/\Pi(p)$ is increasing in *p* if and only if $L'(p)/\Pi'(p) > L(p)/\Pi(p)$. We look at each of these ratios in turn.

By definition, $L(p) = \int_c^p [X(t) - X(p)]dt$, so $L'(p) = (p-c)[-X'(p)]$.
$\Pi'(p) = (p-c)X'(p) + X(p) = X(p) - L'(p)$. Therefore, we get

$$\frac{\Pi'(p)}{L'(p)} = \frac{X(p) - L'(p)}{L'(p)} = -1 + \frac{X(p)}{-(p-c)X'(p)} = -1 + \frac{p}{p-c}\left[\frac{X(p)}{-pX'(p)}\right] = -1 + \frac{1}{mE(p)}$$, where

$m \equiv \frac{p-c}{p}$ is the Lerner Index and $E(p) \equiv -\frac{pX'(p)}{X(p)}$ is the absolute value of the elasticity of demand. Inverting this equation, we get $\frac{L'(p)}{\Pi'(p)} = \frac{mE(p)}{1-mE(p)}$. Assuming that $\Pi'(p) > 0$ for $p < p^M$, we know that $mE(p) < 1$ in this range; only at $p = p^M$ do we get $mE(p) = 1$.

We now look at the first-order approximations to $L'(p)/\Pi'(p)$ and $L(p)/\Pi(p)$ for values of *p* near *c*. We express these in terms of *m*, which is zero at $p = c$. Using the above calculation, we have $\frac{L'(p)}{\Pi'(p)} \approx mE(c)$ for values of *p* near *c*. . From the definition of $L(p)$, for values of *p* near *c* we get the approximation $L(p) \approx \frac{1}{2}[p-c][X(c) - X(p)] \approx \frac{1}{2}[p-c][-(p-c)X'(c)]$. Some simple algebra shows that this expression is approximately equal to $\frac{1}{2} mE(c)\Pi(p)$. Therefore, for values of *p* near *c*, we have $\frac{L(p)}{\Pi(p)} \approx \frac{1}{2} mE(c)$. We have thus shown that in the neighborhood of $p = c$, the ratio $L'(p)/\Pi'(p)$ rises with *p* twice as rapidly as does the ratio $L(p)/\Pi(p)$. Both of these ratios approach zero as $p \to c$. This reflects the fact that the deadweight loss is second-order small in $p - c$ when price is near marginal cost.

Using $\frac{L'(p)}{\Pi'(p)} = \frac{mE(p)}{1-mE(p)}$, we know that $L'(p)/\Pi'(p)$ rises with *p* if $mE(p)$ rises with *p*, i.e. if $\left(\frac{p-c}{p}\right)E(p)$ rises with *p*. Suppose that this condition is satisfied.

Now suppose that $d[L(p)/\Pi(p)]/dp = 0$ for some value of *p*, as it must if $L(p)/\Pi(p)$ is ever to decline with *p*, since $L(p)/\Pi(p)$ is increasing with *p* near $p = c$ (and we are assuming all functions are smooth). Call p_0 the lowest value of *p* at which $d[L(p)/\Pi(p)]/dp = 0$. So, for $p < p_0$, $L(p)/\Pi(p)$ is increasing, which we know requires that $L'(p)/\Pi'(p) > L(p)/\Pi(p)$.

We must have $L(p)/\Pi(p) = L'(p)/\Pi'(p)$ at $p = p_0$. Since $L(p)/\Pi(p)$ is locally constant with respect to *p* at $p = p_0$, and since $L'(p)/\Pi'(p)$ is increasing in *p* (by assumption), this could only happen if $L'(p)/\Pi'(p)$ were *less* than $L(p)/\Pi(p)$ for values of *p* just below p_0. But this contradicts the fact that $L'(p)/\Pi'(p) > L(p)/\Pi(p)$ for $p < p_0$. We have therefore proven:

If $(\frac{p-c}{p})E(p)$ rises with *p*, then the ratio of deadweight loss to monopoly profits also rises with *p* for prices between marginal cost and the monopoly price.

C. Uniqueness and Stability of the Symmetric Equilibrium

For ease of notation, we write $k = 1 - \frac{\pi_B}{\pi_M}$, so the first-order condition is $\frac{C'(p)}{T\pi_M} = 1 - kq$. Note that $1/2 \le k \le 1$; when $\alpha = 0$, $\pi_B = \pi_M / 2$ and $k = 1/2$, and when $\alpha = 1$, $\pi_B = \pi_D$, and $k = 1 - \pi_D / \pi_M$.

The first-order condition for the choice of *p* is given by $C'(p)/T\pi_M = 1 - kq$. The slope of the first firm's best response function is therefore given by $dp/dq = -kT\pi_M / C''(p)$. The symmetric equilibrium is stable if and only if the first firm's best-response schedule is steeper than the second firm's at that point. Since the payoffs are symmetric, this is true if and only if the absolute value of the slope of the *p* best-response curve is greater than unity at the symmetric equilibrium. So, we get stability of the symmetric equilibrium if and only if $kT\pi_M > C''(p)$ at the point where $C'(p)/T\pi_M = 1 - kp$. The necessary and sufficient condition for stability, $kT\pi_M > C''(p)$, can be written as $kpT\pi_M > pC''(p)$. From the first-order condition, we have $kpT\pi_M = T\pi_M - C'(p)$, so the stability condition can be written as $T\pi_M - C'(p) > pC''(p)$ or $T\pi_M > C'(p) + pC''(p) = C'(p)[1+E]$ where $E \equiv pC''(p)/C'(p)$ is the elasticity of the cost function with respect to the success probability. Dividing this inequality by $T\pi_M$ gives $[C'(p)/T\pi_M][1+E] < 1$. Finally, substituting using the first-order condition we get the necessary and sufficient condition for stability as $(1-kp)(1+E) < 1$.

We now provide a sufficient condition for the symmetric equilibrium to be the only equilibrium. The equation defining the symmetric equilibrium is $\frac{C'(p)}{T\pi_M} = 1 - kp$.

Suppose there were an asymmetric equilibrium with $p > q$. Then we must have $C'(p)/T\pi_M = 1 - kq$ and $C'(q)/T\pi_M = 1 - kp$. Taking ratios of these two first-order conditions, we would have $C'(p)(1-kp) = C'(q)(1-kq)$. There can be no such asymmetric equilibrium if the function $C'(p)(1-kp)$ is monotonic in *p*. This expression is decreasing in *p* if and only if $pC''(p)/C'(p) < kp/(1-kp)$, which we can write as $E(1-kp) < kp$. This is the same as the stability condition, $(1+E)(1-kp) < 1$.

To illustrate using an example, suppose that $C(p) = [\gamma p + \beta p^2/2]T\pi_M$, so $C'(p) = [\gamma + \beta p]T\pi_M$ and $C''(p) = \beta T\pi_M$. Then the symmetric equilibrium level of *p* is given by $p^* = \frac{1-\gamma}{k+\beta}$. An interior equilibrium requires that $p^* > 0$, so $\gamma < 1$, and that $p^* < 1$, so $\beta + \gamma > 1 - k$. The condition for stability is that $\beta < k$. So long as these three conditions are satisfied, we have a stable interior equilibrium.

Diversification of Research Approaches

A. Proof of Theorem #2

We are interested in exploring the welfare effects of granting stronger prior user rights. Differentiating with respect to α, we get

$$\frac{dW(x,\alpha)}{d\alpha} = \frac{\partial W(x,\alpha)}{\partial x}\frac{dx}{d\alpha} + \frac{\partial W(x,\alpha)}{\partial \alpha}.$$

As usual, the direct effect of awarding stronger prior user rights is positive, since $\partial W/\partial\alpha = B(x,y)\partial W_B/\partial\alpha = B(x,y)(W_D - W_M) > 0$. The text establishes that $dx/d\alpha > 0$, so a sufficient condition for stronger prior user rights to raise welfare is that $\partial W/\partial x > 0$ at the equilibrium.

Using the definition of *W*, we have $W(x,y,\alpha) = W_M(A(x,y) + A(y,x)) + W_B B(x,y)$. Differentiating with respect to *x*, we have $W_x(x,y,\alpha) = W_M(A_x(x,y) + A_x(y,x)) + W_B B_x(x,y)$. By symmetry, $A_x(y,x) = A_y(x,y)$. So $W_x(x,y,\alpha) = W_M(A_x(x,y) + A_y(x,y) + B_x(x,y)) + (W_B - W_M)B_x(x,y)$. Evaluating this at a symmetric point where $x = y$ gives

$$W_x(x,x,\alpha) = W_M(A_x(x,x) + A_y(x,x) + B_x(x,x)) + (W_B - W_M)B_x(x,x).$$

Since $A(x,y) + B(x,y) = p(x)$, we know that $A_y(x,y) + B_y(x,y) = 0$. By symmetry, $B(x,y) = B(y,x)$, so $B_x(x,x) = B_y(x,x)$. Therefore we must have $A_y(x,x) + B_y(x,x) = A_y(x,x) + B_x(x,x)$. Since the left-hand side of this expression is zero, the right-hand side must also equal zero, so we get

$$W_x(x,x,\alpha) = W_M A_x(x,x) + (W_B - W_M)B_x(x,x).$$

From the condition characterizing the symmetric equilibrium, $A_x(x,x)\pi_M + B_x(x,x)\pi_B = 0$. Solving this for $B_x(x,x)$, substituting, and simplifying gives

$$W_x(x,x,\alpha) = W_M A_x(x,x)[1 - \frac{W_B - W_M}{W_M}\frac{\pi_M}{\pi_B}]$$

at the symmetric equilibrium. Therefore, $W_x(x,x,\alpha) > 0$ at the symmetric equilibrium if and only if $\frac{\pi_B}{\pi_M} > \frac{W_B - W_M}{W_M}$.

Note: Proposition 3 in Dasgupta and Maskin (1987) provides conditions under which the market research portfolio consists of projects that are too highly correlated, so that $dx/d\alpha > 0$ in my notation. However, they assume that welfare is the same whether one or both firms are successful: $W_B = W_M$ in my notation. This condition holds at $\alpha = 0$, so Proposition 3 in Dasgupta and Maskin (1987), combined with the definition of prior user rights adopted in this paper, implies Corollary #2A, i.e., that some prior user rights are optimal. However, their analysis must be extended, as shown here, to study the effects of stronger prior user rights away from $\alpha = 0$.

B. Second-Order Condition and Best-Response Functions

As calculated by Dasgupta and Maskin, using my notation,

$B(x, y) = (x + y)p(x)p(y) + [1-(x + y)](p(x) + p(y))/2$ and

$A(x, y) = [1+(x + y)]p(x)/2 - [1-(x + y)]p(y)/2 - (x + y)p(x)p(y)$.

The second-order condition for the first firm is $A_{xx}\pi_M + B_{xx}\pi_B < 0$. A sufficient condition for this to hold (which is necessary if π_B is sufficiently small) is that $A_{xx} < 0$. Direct calculations show that $A_{xx}(x, y) = p'(x)[1- p(x) - p(y)] + p''(x)[1+(x + y)(1- p(y))]/2$. This expression is negative if $p(x)$ and $p(y)$ are each no larger than one-half, which they must be if $p(0) \leq 1/2$. However, we could have if $p(x) + p(y) > 1$ and if $p''(x)/p'(x)$ is small. In that case, the second-order condition is not satisfied, and the first firm should increase *x* to a higher level at which the first-order condition again holds to find the optimal level of *x*, avoiding a local minimum at a lower value of *x*.

The first-order condition for the first firm is $A_x(x, y)\pi_M + B_x(x, y)\pi_B = 0$. This firm's best-response function is downward sloping if $A_{xy}(x, y)\pi_M + B_{xy}(x, y)\pi_B < 0$, which we write as $\pi_M[A_{xy}(x, y) + B_{xy}(x, y)] - B_{xy}(x, y)[\pi_M - \pi_B] < 0$. Since $A(x, y) + B(x, y) = p(x)$, $A_y(x, y) + B_y(x, y) = 0$, and $A_{xy}(x, y) + B_{xy}(x, y) = 0$ as well, so this inequality is satisfied if and only if $B_{xy}(x, y) > 0$. Since $B_{xy}(x, y) = p'(x)[p(y) - 1/2] + p'(y)[p(x) - 1/2] + (x + y)p'(x)p'(y)$, this inequality is satisfied so long as $p(x)$ and $p(y)$ are each no larger than one-half, which they must be if $p(0) \leq 1/2$.

Allocation of R&D Budgets Across Markets: Proof of Theorem #3

The welfare effect of strengthening prior user rights is given by

$$\frac{dW}{d\alpha} = \frac{\partial W}{\partial x}\frac{dx}{d\alpha} + \frac{\partial W}{\partial \alpha}.$$

As usual, we know that the $\partial W / \partial \alpha > 0$, because $\partial W_B / \partial \alpha = W_D - W_M > 0$.

We show here that each firm will shift away from the smaller market and towards the larger market as prior user rights are strengthened. Formally, we show that $\partial x / \partial \alpha < 0$. The first firm picks x to maximize $\pi(x, y, \alpha)$. Since $d\pi_B / d\alpha = \pi_D - \pi_M / 2 < 0$, $\partial x / \partial \alpha < 0$ if and only if $\pi_x(x, y, \alpha)$ rises with π_B.

Differentiating $\pi(x, y, \alpha)$ with respect to π_B gives $p(x)p(y) + \sigma[p(1-x)/\sigma][p(1-y)/\sigma]$. Differentiating this with respect to x gives $p'(x)p(y) - p'(1-x)p(1-y)/\sigma$. This is positive if and only if $[p'(x)/p'(1-x)] > [p(1-y)/p(y)]/\sigma$. We now show that this expression is positive at the symmetric equilibrium, i.e., $\frac{p'(x)}{p'(1-x)} > \frac{p(1-x)}{p(x)}\frac{1}{\sigma}$ at the symmetric equilibrium.

In a symmetric equilibrium, Cabral shows (Equation A.4) that we must have

$\frac{p'(x)}{p'(1-x)} = \frac{\pi_M - (\pi_M - \pi_B)p(1-x)/\sigma}{\pi_M - (\pi_M - \pi_B)p(x)}$. So, we are attempting to show that $\frac{\pi_M - (\pi_M - \pi_B)p(1-x)/\sigma}{\pi_M - (\pi_M - \pi_B)p(x)} > \frac{p(1-x)/\sigma}{p(x)}$. Cross-multiplying and simplifying, this is equivalent to $p(x) > p(1-x)/\sigma$, i.e., that the equilibrium probability of success is greater in the smaller market, a condition that Cabral establishes.