Contingent Convertible Bonds and Capital Structure Decisions

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Abstract

This paper provides a formal model of contingent convertible bonds (CCBs), debt instruments that automatically convert to equity if and when the issuing firm or bank reaches a specified level of financial distress. We develop closed-form solutions for the value of CCBs with market-based conversion triggers and show that under certain conditions on CCB parameters, the equilibrium is unique. We also show that CCBs can increase firm value and reduce the chance of costly bankruptcy or bailout if properly implemented. Nonetheless, shareholders of overleveraged or too-big-to-fail firms may resist straight-debt-for-CCB swaps due to the debt overhang effect or the loss of the government subsidy. CCBs can also create incentives to manipulate the stock market when the conversion value of the CCB is too low or too high.

1 Introduction

This paper provides a formal model of contingent convertible bonds (CCBs), a new instrument being considered for the reform of prudential bank regulation following the recent financial crisis, that also offers potential value as a component of corporate capital structure for all types of firms. CCBs are debt instruments that automatically convert to equity if and when the issuing firm or bank reaches a specified level of financial distress. While qualitative discussions of CCBs are available in the literature, this is the first paper to develop a complete and formal model of their properties.

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§Raviv (2004) analyzes CCBs using a contingent claims approach, while Pennacchi (2011) primarily focuses on CCB pricing.
CCBs have been proposed by academics (Flannery (2005, 2009, 2010), Duffie (2009), Squam Lake Working Group on Financial Regulation (2009), and McDonald (2013)) and endorsed for further study by bank regulators (Bernanke (2009), Dudley (2009), and Flaherty (2010)). Furthermore, both the House and Senate 2010 financial reform bills require studies of CCBs for regulatory applications and provide regulatory approval for their use. The Financial Stability Board of the G20 and the Bank for International Settlement are also studying CCBs. A number of banks, including Lloyds Banking Group, Credit Suisse and UBS, have issued CCBs totalling $70 billion since 2009.

The key contribution of the current paper is to provide a formal financial model in which the effects of alternative CCB contract provisions can be analytically evaluated. We develop closed-form solutions for CCB value by adapting the Leland (1994) model. The paper provides analytic propositions concerning CCB attributes and develops implications for structuring CCBs to maximize their general benefits. Our results apply equally well to the addition of CCBs to the capital structure of corporations generally, as well as for their specific application as a tool for prudential bank regulation. Our main conclusion is that CCBs can increase firm value and reduce the chance of costly bankruptcy or bailout, but only if properly implemented.

We model a CCB as a bond that pays a coupon continuously in time until conversion. The bond automatically converts into equity when the firm’s stock price drops for the first time to a predetermined level, in which case CCB holders receive a specified number of shares of common stock in exchange for their bonds. The CCB can be issued in addition to straight debt and equity.

We start our analysis with two important conditions that have to be satisfied for the correct implementation of the CCB. Condition 1 states that the firm does not default before or at conversion. In other words, the CCB conversion trigger has to be set high enough so that the firm has sufficient asset value to deliver the promised payoff to the CCB holders at conversion. If this condition is not satisfied, the CCB becomes equivalent to the straight debt with no additional benefits for the firm.

Condition 2 requires that the firm’s equity value strictly increases with the asset value. This condition is satisfied whenever the firm’s stock price is an increasing function of the asset value.

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2There have also been proposals for contingent capital instruments that are not bonds. Kashyap, Rajan, and Stein (2008) propose an insurance contract that provides banks with capital when certain triggering events occur and Hart and Zingales (2011) focus on the use of credit swaps. Wall (2009) provides a survey of this evolving literature.

3The House bill is 111th Congress, first session, H.R. 4173. The Senate bill is 111th Congress, second session, S.3217.

4Avdijev, Kartasheva and Bogdanova (2013) examine recent developments and trends in the market for CCBs.
satisfied when the conversion value of the CCB is sufficiently high, i.e., the CCB converts into a relatively high value of equity. We show that Conditions 1 and 2 lead to the existence of a unique equilibrium in equity and bond prices when a stock price is used as a conversion trigger. This equilibrium is equivalent to an equilibrium with a corresponding asset-based conversion trigger.

Our analysis shows that the effect of CCBs on the firm’s equity and debt values critically depends on the regulatory environment. If a sufficiently small amount of CCBs is included into a de novo capital structure, it would crowd out equity without reducing the amount of straight debt. Thus, if not regulated, CCBs can result in higher total leverage, a higher tax shield, and the same level of default risk. However, if there is a regulatory constraint that fixes the total amount of debt the firm can issue, replacing some amount of straight debt with CCBs in a de novo capital structure would increase the firm and equity values and lower bankruptcy costs.

If an overleveraged firm replaces a portion of existing straight debt with a newly issued CCB, it will increase the total value of the firm and lower bankruptcy costs. However, the equity value will decrease due to the debt overhang effect. All the gains in firm value plus a portion of equity value are passed on to the original debt holders. Thus, equity holders are not likely to initiate the straight-debt-for-CCB swap on their own and are likely to resist regulators trying to implement it.

If a too-big-to-fail firm (TBTF) replaces a portion of straight debt with a CCB, it will reduce the chance of a bailout. However, such a swap will result in losses for equity holders due to the reduction in the value of the government subsidy. Hence, equity holders are likely to resist the implementation of capital requirements involving straight-debt-for-CCB swaps.

A CCB with a stock-based conversion trigger can create incentives to manipulate the stock market for both equity and CCB holders. We show that if the conversion value of the CCB is too low, the equity holders may benefit from triggering early conversion via manipulation of the stock price. On the other hand, if the conversion value of the CCB is too high, CCB holders may profit from the stock manipulation.

We build on Leland (1994), which is a natural setting to study CCB properties, since it provides a fully dynamic platform with endogenous default and allows for closed-form solutions. The Leland model

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*A According to Avdjiev, Kertasheva and Bogdanova (2013), CCBs account for a small fraction of total bank debt.*
has been successful applied in recent studies of other fixed-income debt security innovations, although none analyze the case of a bond conversion triggered by financial distress. Bhanot and Mello (2006) study corporate debt that includes a rating trigger such that a rating downgrade requires the equity holders to compensate the bondholders with early debt redemption or other benefits. Manso, Strulovici, and Tchistyi (2010) study performance-sensitive debt (PSD), a class of debt obligations whose interest payments depend on some measure of the borrower’s performance.

The rest of this paper is organized as follows. In Section 2, we present our formal model. We derive closed-form solutions for a CCB, straight debt and equity, and discuss Conditions 1 and 2, which lead to the existence of the unique equilibrium. Section 3 analyzes how a CCB affects the optimal capital structure. Section 4 studies the debt overhang effect when a CCB is used to replace existing straight debt. Section 5 considers a setting with government bailouts of too-big-to-fail firms. Section 6 focuses on equity market manipulations. Section 7 discusses the policy implications of our findings.

2 Model

We use the capital structure model of Leland (1994) to analyze CCBs. In this model, a firm has productive assets that generate after-tax cash flows with the following dynamics

\[
\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dB_t^Q,
\]

where \(\mu\) and \(\sigma\) are constant, and \(B_t^Q\) defines a standard Brownian motion under the risk-neutral measure. By assumption, the risk-free rate, \(r\), is such that \(\mu < r\), and the tax rate \(\theta \in (0, 1)\). At any time \(t\), the market value of assets, \(A_t\), is defined as the expected present value of all future cash flows,

\[
A_t = \mathbb{E}^Q \left[ \int_t^\infty e^{-r(s-t)} \delta_s ds \right] = \frac{\delta_t}{r - \mu}.
\]

The dynamics for \(A_t\) are: \(dA_t = \mu A_t dt + \sigma A_t dB_t^Q\).

Unlike CCBs, PSD obligations charge a higher interest rate as the borrower’s performance deteriorates, which leads to earlier default compared to fixed-rate debt of the same market value.
The capital structure of the firm includes equity and a straight bond. The bond pays a tax-deductible coupon $c_b$, continually in time, until default. At default, fraction $\alpha \in [0, 1]$ of assets are lost.

Equity holders operate the firm. At current time $t$, the liquidation policy of the firm maximizes equity value:

$$ W(A_t; c_b) \equiv \sup_{\tau \in \mathcal{T}} \mathbb{E}^Q \left[ \int_{t}^{\tau} e^{-r(s-t)} (\delta_s - (1-\theta)c_b) \, ds \right] $$

where $\mathcal{T}$ is the set of stopping times and $(\delta_s - (1-\theta)c_b)$ is the after-tax dividend at time $s$, $t \leq s \leq \tau$. The optimal liquidation time is the first time $\tau(A_B) = \inf\{s : A_s \leq A_B\}$ that the asset level falls to some sufficiently low boundary $A_B > 0$. Leland (1994) shows that, at any time $t < \tau(A_B)$, the optimal default boundary

$$ A_B = \beta(1-\theta)c_b, \quad (2) $$

where $\beta = \frac{r}{\gamma(1+\gamma)}$ and $\gamma = \frac{1}{\sigma^2} \left[ (\mu - \frac{\sigma^2}{2}) + \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2r\sigma^2} \right]$. $A_B$ does not depend on $A_t$ and increases in $c_b$. Equity holders liquidate the firm the first time equity value drops to zero.

We allow the firm to add a CCB to a capital structure that includes equity and a straight bond. The CCB pays a tax-deductible coupon $c_c$ continually in time until conversion. The bond fully converts into equity when asset value falls to a pre-determined level $A_C$. The time of conversion is denoted by $\tau(A_C) = \inf\{s : A_s \leq A_C\}$. At $\tau(A_C)$, CCB holders receive equity, valued at its market price, in the amount of $\left(\lambda \frac{c_c}{\gamma} \right)$. The coefficient $\lambda$ is the CCB contract term that determines the ratio of equity value to the face value of the bond, $\frac{c_c}{\gamma}$. When $\lambda = 1$, the equity value equals the face value of the bond. When $\lambda < 1 (\lambda > 1)$, the equity value is at a discount (premium) to the face value.

We require that the following condition always holds.

**Condition 1 (No Default Before Conversion):** $c_b, c_c, \lambda$ and $A_C$ are such that the firm does not default before conversion at any $A_t \geq A_C$.

Before conversion, equity holders’ value and their decision to liquidate depend on the characteristics of
both bonds, and so does Condition 1. One implication of this condition is that the CCB issue is sufficiently small. The CCBs claim on the firm’s assets does not affect equity value to the extent that equity holders would want to default before conversion – equity value remains positive. Economically, a small CCB issue can be justified by either potential limits imposed by regulators or the market capacity to absorb this kind of debt.

When Condition 1 is violated and the firm is liquidated before conversion, at each time \( s \geq t \) before default, equity holders receive a dividend in the amount of \((\delta_s - (c_b + c_c)(1 - \theta))\). The firm optimally defaults when equity value drops to zero at \( A_B = \beta(1 - \theta)(c_b + c_c) \), based on equation \(^2\). The default boundary is the same whether the capital structure of the firm includes a straight bond paying coupon \( c_b \) and a CCB paying coupon \( c_c \), or whether it includes only a straight bond paying \((c_b + c_c)\). If Condition 1 is violated, issuing a CCB becomes redundant from the point of view of default risk.

**Lemma 1.** If the capital structure of the firm includes equity, a straight bond and a CCB, the optimal default boundary, \( A_B \), is determined only by the size of the straight debt coupon, \( c_b \), and is given by equation \(^2\).

**Proof.** Based on Condition 1, there is no default prior to or at conversion. After conversion, the maximum-valuation problem of equity holders is the same as in Leland (1994), when the capital structure includes only equity and a straight bond paying coupon \( c_b \). Hence, the same \( A_B \).

We derive closed-form solutions for the values of various claims on the firm’s assets prior to conversion.

**Proposition 1.** If the capital structure of the firm includes equity, a straight bond and a CCB, then for any \( t \leq \tau(A_C) \),

(i) firm value

\[
G(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r} \left(1 - \frac{A_t}{A_B}\right)^\gamma + \frac{\theta c_c}{r} \left(1 - \frac{A_t}{A_C}\right)^\gamma - \alpha A_B \left(\frac{A_t}{A_B}\right)^\gamma
\]
(ii) equity value

\[
W(A_t; c_b, c_c) = \frac{A_t}{r} - \frac{c_b (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^{-\gamma} \right) - \frac{c_c (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right) - A_B \left( \frac{A_t}{A_B} \right)^{-\gamma} - \left( \frac{c_c}{r} \right) \left( \frac{A_t}{A_C} \right)^{-\gamma}
\]

(iii) straight bond value

\[
U^B(A_t; c_b, c_c) = \frac{c_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^{-\gamma} \right) + \left( \frac{A_t}{A_B} \right)^{-\gamma} (1 - \alpha A_B).\]

(iv) CCB value

\[
U^C(A_t; c_b, c_c) = \frac{c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right) + \left( \frac{A_t}{A_C} \right)^{-\gamma} \left( \frac{c_c}{r} \right)
\]

(v) tax savings

\[
TB(A_t; c_b, c_c) = \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right)
\]

(vi) bankruptcy costs

\[
BC(A_t; c_b, c_c) = \alpha A_B \left( \frac{A_t}{A_B} \right)^{-\gamma}
\]

It follows from Proposition 1 that the values of straight debt and bankruptcy costs are not affected by the presence of the CCB. Tax savings include

1. savings associated with the straight bond

\[
TB^B(A_t; c_b, c_c) = \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^{-\gamma} \right)
\]
2. and savings associated with the CCB

\[ TB^C(A_t; c_b, c_c) = \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right). \tag{3} \]

Figure 1 plots equity value as a function of asset value. Before conversion, equity value is computed based on the closed-form solution from Proposition 1. After conversion, it is based on the solution that corresponds to the case when the capital structure of the firm includes only equity and straight debt. It represents the value of new equity holders (the former CCB holders) and the value of old equity holders (who ran the firm solely before conversion). In the top subfigure of Figure 1, equity value monotonically declines as \( A_t \) approaches the conversion boundary from the right-hand side and increases by the amount of \( \lambda \hat{\tau} = 9 \) at \( A_C \). The value of equity is always positive before conversion. Condition 1 holds.

In the bottom subfigure of Figure 1, equity value is also positive before conversion, and Condition 1 holds. The key difference from the previous subfigure is that before conversion, equity value as a function of asset value is non-monotonic. As \( A_t \) approaches \( A_C \) from the right-hand side, equity value first decreases and then increases. The intuition is as follows. First, when \( A_t \) is high and relatively far from \( A_C \), changes in asset value have a dominant effect on equity value. As asset value declines, so does equity value. Second, as \( A_t \) continues to decline and gets closer to \( A_C \), equity holders face a higher probability of eliminating the liability to pay coupon \( c_c \). Since \( \lambda = 0.1 \) is small, the value that equity holders are giving up at conversion, \( \lambda \hat{\tau} = 3.5 \), is relatively low compared to the face value of the bond, \( \hat{\tau} = 70 \). The positive effect from potentially not having to make coupon payments to CCB holders dominates the negative effect due to lower asset value realizations and a higher probability of having to give up value at conversion. As asset value approaches \( A_C \), equity value increases.

The issue of non-monotonicity of equity value in asset value becomes important when addressing CCB implementation. Working with asset values is convenient when performing analytical calculations. However, since equity prices are observable while asset values are not, in practice a CCB conversion rule based on equity value is more relevant. When equity price, \( W(A_t; c_b, c_c) \), is strictly increasing in \( A_t \) before conversion, there is a one-to-one correspondence between equity and assets values. \( A_C \) and \( W_C = W(A_C; c_b, c_c) \)

\[ \text{See equation (13) in Leland (1994).} \]
Figure 1: **Equity value as a function of asset value, when Condition 1 holds.**
The lines plot equity value, $W$, for a range of asset value realizations, $A_t$, before (*solid line*) and after (*dashed line*) CCB conversion. By assumption, $r = 5\%$, $\mu = 1\%$, $\sigma = 15\%$, $\theta = 35\%$, $\alpha = 30\%$ and $c_b = $5. Based on (2), $A_B = $43.7.

(a) **Monotonicity before conversion.** $c_t = $0.5, $\lambda = 0.9$, $A_C = $70.

(b) **Non-monotonicity before conversion.** $c_t = $3.5, $\lambda = 0.05$, $A_C = $70.
become interchangeable, and the conversion rule can be rephrased – the CCB converts into equity when equity price drops to $W_C$. In the future, when necessary, we will explicitly avoid non-monotonicity of equity value in $A_t$ by imposing the following condition.

**Condition 2 (Monotonicity of Equity Value):** $c_b, c_c, A_C$ and $\lambda$ are such that equity value, $W(A_t; c_b, c_c)$, is strictly increasing in asset value, $A_t$, before conversion at any $A_t \geq A_C$.

Although Condition 2 further limits the parameter space defined by Condition 1, it is not too restrictive for practical purposes. As in the bottom example of Figure 1, it is violated when the amount of equity CCB holders receive at conversion is substantially smaller than the face value of the bond. As we argue in Section 5, CCBs with sufficiently low conversion ratios create an incentive for equity holders to expedite conversion by artificially driving the stock price down, and therefore should be restricted by regulators.

The following Lemma gives an easy-to-verify condition on the parameters of the model, for which Condition 2 holds.

**Lemma 2.** Condition 2 holds, i.e., equity value $W(A_t; c_b, c_c)$ is strictly increasing in the asset value $A_t$ for any $A_t > A_C$ if and only if

$$\lambda > 1 - \theta - \left(1 - \left(\frac{A_C}{A_B}\right)^{-(1+\gamma)}\right)\frac{A_C}{\gamma r}.$$  

### 2.1 Existence and Uniqueness of the Equilibrium with a Stock Price Conversion Trigger

Proposition 1 describes the unique equilibrium when the asset level is used as the conversion trigger. An important property of this equilibrium is that the equity value immediately after the conversion is equal to the equity value just before the conversion plus the conversion value of the CCB:

$$W(A_C, c_b, 0) = W(A_C, c_b, c_c) + \lambda \frac{c_c}{r}.$$  

(4)

When Conditions 1 and 2 are satisfied, this equilibrium can be implemented using a stock price conversion trigger. Without loss of generality, we normalize the initial number of shares to one. Thus, the stock price
$S_t$ is equal to the equity value before the conversion. In the equilibrium, the conversion occurs when the stock price drops to $S_C = W(A_C, c_b, c_c)$ for the first time, in which case the CCB holders receive $n = \frac{S}{S_C}$ new shares of stock.

To show that this equilibrium is unique, we consider a situation in which the stock trigger $S_C$ leads to conversion at some asset level $A' \neq A_C$. We note that the stock value after the conversion is not affected by the CCB and is uniquely determined by $\frac{W(A', c_b, 0)}{1+n}$, where $W(A', c_b, 0)$ is the monotone function of the asset level $A'$, as in Leland (1994). If $A' > A_C$, then the stock price immediately after the conversion is going to be greater than $S_C$. Indeed, taking into account (4), we can write that

$$\frac{W(A', c_b, 0)}{1+n} > \frac{W(A_C, c_b, 0)}{1+n} = \frac{W(A_C, c_b, c_c) + \lambda c_c}{1+n} = \frac{S_C + nS_C}{1+n} = S_C.$$  

This cannot be an equilibrium, because it allows an arbitrage opportunity. One can buy shares at the price of $S_C$ just before the conversion, and sell them at the higher price after the conversion. Similarly, if $A' < A_C$, then the stock price immediately after the conversion is going to be lower than $S_C$, which is also inconsistent with the no-arbitrage condition. Thus, the equilibrium with a stock price conversion trigger is unique.

The following theorem summarizes our finding.

**Theorem 1.** If Conditions 1 and 2 are satisfied for $c_b, c_c, \lambda$ and $A_C$, then there exists a unique equilibrium with the stock trigger $S_C = W(A_C, c_b, c_c)$ that results in the same stock and bond values as those with the asset-based trigger $A_C$.

Due to the fact that asset-based triggers result in more convenient analytical expressions, our subsequent analysis uses $A_C$ as a conversion trigger. However, Theorem 1 assures that the resulting equilibrium in stock and bond prices can be implemented using a corresponding stock trigger.

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8Unlike in Sundaresan and Wang (2013), there is a unique equilibrium in our continuous time setting.
3 Optimal Capital Structure Decisions

The main question we address in this section is whether the firm would add a CCB to a de novo capital structure. We also highlight important policy implications associated with regulating CCBs.

3.1 De Novo Capital Structure

Consider that, at time $t$, the firm has no debt but can leverage up by choosing from two options. First, equity holders can issue an optimal amount of straight debt as in Leland (1994) and no CCBs. Second, they can fix the size of a CCB ex-ante and solve for a new optimal amount of straight debt by maximizing firm value from Proposition 1. In the second case, the assumption is that the CCB parameters and the resulting (new) optimal straight debt coupon satisfy Condition 1. Which of the two capital structures would equity holders prefer?

Proposition 2. If an unlevered firm chooses to leverage up by issuing a fixed-size CCB and an optimal amount of straight debt, so that Condition 1 does not bind, then compared to the optimal capital structure that does not include CCBs ($c_c = 0$).

(i) optimal straight debt coupon is the same

$$c_b^* = \frac{A_t}{\beta(1 - \theta)} \left( \frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[ (\gamma + 1)\left( \frac{\theta}{r} + \alpha \beta (1 - \theta) \right) \right]^{\frac{1}{\gamma}}$$

(ii) firm value is higher by the amount of tax savings associated with $c_c$

$$G(A_t; c_b^*, c_c) = G(A_t; c_b^*, 0) + TB_C^C(A_t; c_b^*, c_c)$$

(iii) adjusted for tax benefits, equity is crowded out by the CCB one-to-one

$$W(A_t; c_b^*, c_c) = W(A_t; c_b^*, 0) - UC(A_t; c_b^*, c_c) + TB_C^C(A_t; c_b^*, c_c)$$
(iv) tax savings are higher by the amount of savings associated with the CCB

\[
TB(A_i; c_b^*, c_c) = TB(A_i; c_b^*, 0) + TB^C(A_i; c_b^*, c_c)
\]

(v) straight debt value and bankruptcy costs are the same

\[
U^B(A_i; c_b^*, c_c) = U^B(A_i; c_b^*, 0),
\]
\[
BC(A_i; c_b^*, c_c) = BC(A_i; c_b^*, 0).
\]

The intuition behind the above results is as follows. The firm does not default before conversion due to Condition 1. After conversion, equity holders’ value-maximization problem is the same as if the CCB was not part of the capital structure. This leads to the same optimal default boundary. Since the CCB does not change the timing of default, equity holders issue the same optimal amount of straight debt as in the case when CCBs are not considered (item (v)). The straight debt coupon (item (i)) and bankruptcy costs (item (v)) are also the same. Tax benefits (item (iv)) and firm value (item (ii)) increase by the amount of tax savings associated with coupon \(c_c\). Finally, since operational cash flows and the straight debt coupon are the same, making payments to CCB holders reduces equity value. Adjusted for CCB tax savings, equity value decreases by a dollar for each dollar of contingent capital debt (item (iii)).

Equity holders have an incentive to issue contingent capital debt – they increase firm value by taking advantage of additional tax benefits. The new type of debt crowds out equity without reducing the amount of straight debt. Proposition 2 suggests that, if not regulated, CCBs will result in higher total leverage, higher tax subsidies, and the same level of default risk.

3.2 De Novo Capital Structure with Limits on Total Amount of Debt

In this section we assume that regulators impose a limit on the total amount of debt that the firm can issue. As before, at time \(t\), the firm is unlevered but can leverage up by choosing from two options. First, equity holders can issue an optimal amount of straight debt, \(U^B(A_i; c_b^*, 0)\), and no CCBs. Second, they can issue
a straight bond, $U^B(A_t; \tilde{c}_b, c_c)$, and a CCB, $U^C(A_t; \tilde{c}_b, c_c)$, such that Condition 1 holds. The difference from Section 3.1 is that the total amount of debt in the second case cannot exceed the amount of straight debt in the first case. This leads to the following formal constraint:

$$U^B(A_t; \tilde{c}_b, c_c) + U^C(A_t; \tilde{c}_b, c_c) = U^B(A_t; c^*_b, 0).$$

(6)

Equation (6) implies that, given parameters $\tilde{c}_b, A_C$ and $\lambda$, CCB coupon $c_c$ is

$$c_c = \frac{U^B(A_t; c^*_b, 0) - U^B(A_t; \tilde{c}_b, c_c)}{\frac{1}{2} \left( 1 - (1 - \lambda) \left( \frac{A_C}{\lambda} \right)^\gamma \right)}$$

Will the firm still choose to issue contingent capital debt?

**Proposition 3.** A capital structure that includes an optimal amount of straight debt and no CCBs compares to a capital structure that includes a CCB and a straight bond subject to regulatory constraint (6) in the following way:

(i) the difference in firm value equals the difference in equity value,

$$G(A_t; \tilde{c}_b, c_c) - G(A_t; c^*_b, 0) = W(A_t; \tilde{c}_b, c_c) - W(A_t; c^*_b, 0)$$

(ii) bankruptcy costs are lower in the presence of the CCB, $BC(A_t; \tilde{c}_b, c_c) < BC(A_t; c^*_b, 0)$

(iii) if coupon $c_c$ is sufficiently small, firm value is higher in the presence of the CCB, i.e., there exists $c_1$, such that $G(A_t; \tilde{c}_b, c_c) > G(A_t; c^*_b, 0)$ for any $c_c \in (0, c_1)$

Regulatory constraint (6) fixes the total amount of debt. Therefore, changes in firm value as a result of replacing straight debt with the CCB can only be due to changes in equity value.

Given Condition 1, the firm does not default before conversion. After conversion, equity holders’ default policy is not affected by the CCB. Based on equation (2), the optimal default boundary $\bar{A}_B =$
In the case of the optimal amount of straight debt, default boundary \( A^*_B = \beta (1 - \theta) c^*_b \). Since \( \bar{c}_b < c^*_b \), it follows that \( \bar{A}_B < A^*_B \) which results in lower expected bankruptcy costs.

The key result is that equity holders gain from replacing some amount of straight debt with a CCB. Firm and equity values increase because reduced amounts of straight debt result in lower bankruptcy costs.

### 4 Partially Replacing Existing Straight Debt with a CCB

We continue with the case when a CCB replaces a portion of already existing straight debt, but not necessarily in the optimal amount. Assume that at time \( t \) the capital structure of the firm consists of equity and straight debt paying coupon \( \hat{c}_b \). The firm wants to issue a CCB and swap it for a portion of straight debt in order to reduce \( \hat{c}_b \) to \( \bar{c}_b \), where \( \bar{c}_b < \hat{c}_b \). Once the announcement is made, the market value of straight debt that is still paying \( \hat{c}_b \), will rise from \( U^B(A_t, \hat{c}_b, 0; \hat{A}_B) \) to \( U^B(A_t, \bar{c}_b, 0; \bar{A}_B) \) to reflect a lower default boundary due to a lesser amount of straight debt after the swap. Here, \( U^B(A_t, \hat{c}_b, 0; \hat{A}_B) \) denotes the value of straight debt paying coupon \( \hat{c}_b \) and defaulting at \( \hat{A}_B = \beta (1 - \theta) \hat{c}_b \), while \( U^B(A_t, \bar{c}_b, 0; \bar{A}_B) \) is the value of straight debt paying coupon \( \bar{c}_b \) and defaulting at \( \bar{A}_B = \beta (1 - \theta) \bar{c}_b \). For the straight debt holders to be indifferent between exchanging their holdings for the CCB and continuing to hold straight debt, the following should be true

\[
U^B(A_t, \hat{c}_b, 0; \hat{A}_B) = U^B(A_t, \bar{c}_b, c_c; \bar{A}_B) + U^C(A_t, \bar{c}_b, c_c; \bar{A}_B),
\]

i.e., the ex post announcement value of the existing straight debt should equal \( U^B(A_t, \bar{c}_b, c_c; \bar{A}_B) \), the value of straight debt that remains after the swap, plus \( U^C(A_t, \bar{c}_b, c_c; \bar{A}_B) \), the value of the newly issued CCB, which is used to buy the existing debt.

The same amount of contingent capital debt can be issued with different coupons and conversion-triggering asset levels. The firm, for example, could pick \( \bar{c}_b \) and \( A_C \), and solve (7) for

\[
c_c = \frac{\left( \bar{c}_b - \hat{c}_b \right) \left( 1 - \left( \frac{A_C}{A_B} \right)^\gamma \right)}{1 - (1 - A) \left( \frac{A_C}{A_B} \right)^\gamma},
\]
We analyze if equity holders would be willing to replace some of the existing straight debt with a CCB and what effect this replacement would have on the total value of the firm.

**Proposition 4.** If a leveraged firm replaces a portion of straight debt with a CCB then

(i) the value of equity decreases, \( W(A_t, \hat{c}_b, c_c) - W(A_t, \bar{c}_b, 0) < 0 \)

(ii) however, if \( \hat{c}_b \geq c_b^* \) and \( \lambda \geq 2 - \left( \frac{A_t}{A_C} \right)^\gamma \), then there exists \( c_2 \) such that firm value increases:

\[
G(A_t, \bar{c}_b, c_c) > G(A_t, \hat{c}_b, 0), \text{ for } c_c \in (0, c_2)
\]

(iii) the cost of bankruptcy decreases, \( BC(A_t, \bar{c}_b, c_c) < BC(A_t, \hat{c}_b, c_c) \).

Proposition 4 says that if the firm is over-levered, \( \hat{c}_b \geq c_b^* \), it will benefit from replacing some amount of straight debt with a CCB. Firm value increases, because the CCB-for-straight-debt swap reduces bankruptcy costs. However, equity holders will not replace any amount of existing straight debt with a CCB voluntarily as their value decreases due to debt overhang effect. All the gains in firm value plus a portion of the equity value are passed on to original debt holders.

The key economic result of Section 4 is that, if an over-levered firm decides to partially replace existing straight debt with a CCB, firm value will increase while bankruptcy costs and the amount of risky straight debt will decrease. Equity holders, however, will never initiate this kind of debt replacement on their own and are likely to resist regulators trying to implement it due to the debt overhang effect.

## 5 Too-Big-To-Fail Firms

In this section we look at firms that are 'too big' for the government to let them fail, as they pose systemic risk. We assume that the government bails out a too-big-to-fail firm (TBTF) by taking over its assets at the point of bankruptcy and committing to making payments to debt holders. We study a partial replacement of straight debt with a CCB and its effect on the government subsidy.

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9Note, that condition \( \lambda \geq 2 - \left( \frac{A_t}{A_C} \right)^\gamma \) would normally be satisfied, because \( 2 - \left( \frac{A_t}{A_C} \right)^\gamma \leq 1 \) for \( A_t \geq A_C \).
We consider a firm with a capital structure that includes equity and straight debt, paying coupon $c_b$. The firm reaches bankruptcy when the asset value hits the default boundary, $A_B$, for the first time. At that point, if the government decides to step in to prevent bankruptcy, it will obtain assets worth $A_B$ and an obligation to pay $c_b$ forever with the risk-free value of $\frac{c_b}{r}$. Therefore, the value of the government subsidy at the time of bankruptcy is $\frac{c_b}{r} - A_B$.

The optimal time to default $\tau(A_B)$ solves the equity value-maximization problem. A government subsidy kicks in at time $\tau(A_B)$ and covers only straight debt obligations. Therefore, it affects neither the timing of default nor the value of $A_B = \beta(1 - \theta)c_b$. This implies that equity value remains the same, provided that the capital structure does not change. The government guarantee benefits only the debt holders and does not subsidize equity. Thus, at any time $t$ before bankruptcy, the value of the subsidy is

$$S(A_t, c_b, 0) = \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma}.$$  \hfill (8)

By definition, a government subsidy prevents the firm from going into default. Therefore, it eliminates bankruptcy costs, $BC(A_t, c_b) = 0$, and makes straight debt default-free, $U^B(A_t, c_b, 0) = \frac{c_b}{r}$.

Given Condition 1, the time of default and the value of assets at default do not depend on whether a CCB is present. Therefore, the value of the subsidy is not affected by the CCB,

$$S(A_t; c_b, c_c) = S(A_t; c_b, 0).$$ \hfill (9)

**Proposition 5.** If the government issues a guarantee for straight debt, then

(i) the larger the amount of debt, the larger the subsidy, $\frac{dS(A_t; c_b, 0)}{dc_b} > 0$

(ii) firm value increases to

$$G(A_t; c_b, 0) = A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma}.$$ \hfill (10)
The intuition is as follows. First, government losses at bankruptcy, $c_{br} - A_B$, increase in $c_{br}$\(^{10}\). Second, larger amounts of straight debt increase the probability of default. Both result in higher expected present value of the subsidy. Adding the subsidy and eliminating bankruptcy costs result in higher firm value.

Since firm value strictly increases with the amount of straight debt due to the subsidy, equity holders would try to issue as much straight debt as possible, collect the proceeds as dividends and default immediately after the issuance. Although the market would know that the firm is going to default, due to the presence of the guarantee, the firm would still be able to sell the debt as if it was risk-free. To avoid this from happening, the government should set limits on how much debt a TBTF firm is allowed to issue\(^{11}\).

We assume that the government limits the amount of straight debt of a TBFT firm by $U_B(A_t, c_{gb}, 0)$, where $c_{gb}$ is the highest allowed straight debt coupon. Alternatively, the firm can issue straight debt with coupon $\bar{c}_b$ and a CCB with coupon $c_c$, such that the total amount of debt is the same,

\[
U_B(A_t, \bar{c}_b, 0) = U_B(A_t, c_{gb}, 0) + U_C(A_t, \bar{c}_b, c_c) \quad (11)
\]

Given the risk-free value of straight debt, $\frac{c_{br}}{r}$, and the closed-form solution for $U_C(A_t; \bar{c}_b, c_c)$ from Proposition\(^{11}\) we can solve equation (11) for

\[
\bar{c}_b = c_{gb} - c_c \left(1 - (1 - A \left(\frac{A_t}{A_C}\right)^\gamma)\right). \quad (12)
\]

**Proposition 6.** If a TBTF firm replaces a portion of straight debt with a CCB, so that the total amount of debt remains the same, then

(i) subsidy value decreases, $S(A_t, \bar{c}_b, c_c) < S(A_t, c_{gb}, 0)$

(ii) firm value decreases, $G(A_t, \bar{c}_b, c_c) < G(A_t, c_{gb}, 0)$

(iii) equity value decreases, $W(A_t, \bar{c}_b, c_c) < W(A_t, c_{gb}, 0)$.

\(^{10}\)Given equation 2, $\frac{c_{br}}{r} - A_B = \frac{1 + \theta \phi c_{br}}{1 + \phi}.$

\(^{11}\)The government sets capital requirements for major financial institutions.
An important difference between contingent capital and straight debt is that CCB value is not affected by the government guarantees. Indeed, while straight debt holders receive payments from the government after the bailout, CCB holders become equity holders at conversion and are not getting any money from the government. Since CCB holders do not benefit from the guarantee, the value of the government subsidy decreases.

The reduction in firm value is efficient from the government’s point of view. It is caused by the reduction in bailout subsidy and tax benefits, but not by an increase in bankruptcy costs.

The reduction in firm value translates into losses for equity holders. Hence, they are likely to resist the implementation of capital requirements involving straight-debt-for-CCB swaps. This is similar to the result in Section 4. However, here equity value declines not due to the debt overhang effect but due to a reduction in government subsidy.

6 Equity Market Manipulations

In this section we demonstrate that both CCB and equity holders may have incentives to manipulate the stock price when an equity based trigger is used for conversion. We assume Conditions 1 and 2 hold and the CCB converts into equity when the equity value drops to $W_C = W(A_C; c_b, c_c)$, where $A_C$ is the asset level corresponding to the pre-conversion equity value $W_C$.

6.1 Manipulation by CCB Holders

We start with the case when CCB holders attempt manipulation motivated by potential profits. They might buy a CCB when the stock price of the firm is above the conversion-triggering level $W_C$, drive the price down by spreading negative news, short selling equity, etc. in order to trigger conversion, and then sell the equity obtained as the result of conversion when the price corrects upwards.

As shown Figure we consider three time instances $t, t_+ \text{ and } t_{++}$, where $t < t_+ < t_{++}$. For simplicity, we assume that the time interval $t_{++} - t$ is short so that we can ignore changes in the asset value and

---

12 We emphasize that this statement is true only when Condition 1 holds.
discounting. At time $t$ the market value of assets, $A_t$, is uncertain. $A_t$ is equal to $A_H$ with probability $p$ or $A_L$ with probability $(1 - p)$. The market value of equity, $W_t$, is observed at time $t$ before the uncertainty about $A_t$ is resolved. If $W_t \leq W_C$, then at time $t_+$ CCB converts into equity. Otherwise, there is no conversion at $t_+$. The true market value of assets, $A_t$, is observed at $t_++$. $A_H$, $A_L$ and $A_C$ are such that $A_L < A_C < A_H$. Thus, if the asset value is $A_L$ and there was no conversion at $t_+$ CCB converts into equity at time $t_++$. Otherwise, there is no conversion at $t_++$. 

Observe: $W_t$

Do not observe: $A_t$

- If $W_t > W_C \rightarrow$ No conversion
- If $W_t \leq W_C \rightarrow$ Conversion

$A_t = A_H \rightarrow$ No conversion

$A_t = A_L \rightarrow$ No conversion

$A_t = A_C \rightarrow$ Conversion

Figure 2: Equity price, asset value and CCB conversion decisions.

We first consider the case with no conversion at time $t_+$, i.e., $W_t > W_C$. If $A_t = A_H$, then there is no conversion and the value of old equity at time $t_+$ is $W(A_H; c_b, c_c)$. On the other hand, if $A_t = A_L$, then the CCB is converted into equity at $t_+$, and the value of old equity is equal to the value of total post-conversion equity minus the value of new equity issued to replace the CCB holders: $W(A_L; c_b, 0) - \lambda c_r$. 

CCB holders can drive down the equity price by convincing the market that the probability of $A_t = A_H$ is $p' < p$. We assume that if the market believes that the probability of $A_t = A_H$ is $p$, the value of equity is above $W_C$ and, therefore there is no conversion at time $t_+$. On the other hand, if the market believes that the probability is $p'$, the CCB does convert into equity at time $t_+$. The expected true value of equity after manipulation is given by

$$\tilde{W}_{t_+} = pW(A_H; c_b, 0) + (1 - p)W(A_L; c_b, 0).$$

However, because of manipulation, the CCB is converted as if the equity price is

$$\tilde{W}_{t_+} = p'W(A_H; c_b, 0) + (1 - p')W(A_L; c_b, 0).$$
At conversion, CCB holders receive equity with the market value of $\lambda c_c$. Since the equity is undervalued due to manipulation, the expected payoff to the CCB holders is equal to

$$\Pi' = \lambda \frac{c_c}{r} \frac{pW(A_H; c_b, 0) - (1 - p)W(A_L; c_b, 0)}{p'W(A_H; c_b, 0) - (1 - p')W(A_L; c_b, 0)}.$$ 

If there is no manipulation that triggers conversion, the expected payoff to the CCB holders is

$$\Pi_t = pUC(A_H; c_b, c_c) + (1 - p)\frac{c_c}{r}.$$ 

Consider the difference between these two values

$$\Pi'_t - \Pi_t = \lambda \frac{c_c}{r} \left( \frac{p - p'}{p'W(A_H; c_b, 0) - W(A_L; c_b, 0)} - p \left( \frac{U^C(A_H; c_b, c_c) - \lambda \frac{c_c}{r}}{r - \left( \frac{A_H}{A_C} \right)^\gamma} \right) \right).$$ 

By using the closed-form solution for $UC(A_H; c_b, c_c)$ from Proposition [1] and rearranging terms, we get

$$\Pi'_t - \Pi_t = \lambda \frac{c_c}{r} \left( \frac{p - p'}{p'W(A_H; c_b, 0) - W(A_L; c_b, 0)} - p \left( 1 - \lambda \frac{c_c}{r} \right) \right).$$ 

(13)

It’s easy to see that if $\lambda = 0$, based on equation [13], $\Pi'_t < \Pi_t$ and, therefore, CCB holders do not have an incentive to manipulate the market. Also from [13], $\Pi'_t - \Pi_t$ is strictly increasing in $\lambda$ and the value of $\lambda$ for which the difference in the two payoffs is zero is

$$\lambda^* = \frac{p \left( 1 - \left( \frac{A_H}{A_C} \right)^\gamma \right)^\gamma}{p' \frac{p - p'}{W(A_H; c_b, 0) - W(A_L; c_b, 0)} + p \left( 1 - \left( \frac{A_H}{A_C} \right)^\gamma \right)^\gamma}.$$ 

Clearly, $\lambda^* > 0$. Also, since $\frac{p - p'}{p'W(A_H; c_b, 0) - W(A_L; c_b, 0)} > 0$, $\lambda^* < 1$.

The above argument is summarized in Theorem [2].

**Theorem 2.** CCB holders do not manipulate the equity market if and only if $\lambda \leq \lambda^*$, where $\lambda^* \in (0, 1)$ is
The intuition behind this result is simple. At conversion, CCB holders give up a stream of future coupon payments for the equity valued at \( \lambda \). A small \( \lambda \) would mean that even after we account for the appreciation of equity post manipulation, the value CCB holders receive is too small compared to the value of future coupon payments they need to give up. Therefore, CCB holders will not try to force conversion. On the other hand, a high \( \lambda \) increases payoff to the CCB holders at conversion and their profit from manipulation.

Based on equation (14), there are two major drivers behind the above value of \( \lambda^* \). The first is the distance between the probabilities \( p \) and \( p' \). The bigger the difference \( (p - p') \), the lower \( \lambda^* \). The greater the magnitude of possible manipulation, the lower the conversion ratio should be in order to avoid manipulation. The second driver is the difference between equity values for asset realizations \( A_H \) and \( A_L \). Here again, the bigger the difference \( (W(A_H; c_b, 0) - W(A_L; c_b, 0)) \), the lower \( \lambda^* \). The greater the initial uncertainty, the lower should the conversion ratio be.

One important policy implication of Theorem 2 is that having the conversion value equal to the par value \( (\lambda = 1) \) is not the best way to implement CCBs in practice. A CCB with \( \lambda = 1 \) has a number of favorable properties, e.g., it is always priced at par in the absence of manipulation, i.e., \( U_C(A_t; c_b, c_c) = c_c \), which may make it attractive to investors looking for safe assets. However, it also gives strong incentives to the CCB holders to manipulate the stock market, which would make CCB pricing a much more complicated problem. A more practical solution would be to make the conversion value strictly lower than the par value, i.e., \( \lambda < \lambda^* \). In this case, the CCB would be priced as a risky debt, but it would eliminate the manipulation incentives.

### 6.2 Manipulation by Equity Holders

We turn to the case when equity holders might attempt to manipulate the market. If the conversion value of the CCB is low compared to the value of the future coupon payments, the equity holders may increase the
value of their holdings at the expense of the CCB holders by triggering early conversion via manipulation of the equity price.

Assuming that \( A_t > A_C \), the payoff to the original equity holders is equal to \( W(A_t; c_b, c_c) \) if they do not manipulate the market. If the equity holders manipulate the market, the CCB holders will receive \( n = \frac{\lambda_c}{W(A_C; c_b, c_c)} \) new shares. We assume that manipulation is short-lived and does not lead to the destruction of the firm’s assets. The total value of the equity after the manipulation will rebound to \( W(A_t; c_b, 0) \). Hence, the actual payoff to the CCB holders will be

\[
\frac{n}{1 + n} W(A_t; c_b, 0) = \frac{\lambda_c}{W(A_C; c_b, c_c) + \lambda_c} W(A_t; c_b, 0)
\]

\[
= \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}.
\]

Here, we used the fact that \( W(A_C; c_b, c_c) + \lambda_c = W(A_C; c_b, 0) \).

The payoff to the original equity holders after the manipulation will be \( W(A_t; c_b, 0) - \frac{\lambda_c}{r} \frac{W(A_C; c_b, 0)}{W(A_C; c_b, 0)} \).

They will not manipulate the market if

\[
W(A_t; c_b, c_c) \geq W(A_t; c_b, 0) - \frac{\lambda_c}{r} \frac{W(A_C; c_b, 0)}{W(A_C; c_b, 0)}.
\]

**Theorem 3.** The equity holders will not manipulate the equity market at time \( t \) if and only if \( \lambda \geq \lambda^* \), where

\[
\lambda^* = (1 - \theta) \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right) \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)} - \left( \frac{A_t}{A_C} \right)^\gamma < (1 - \theta).
\]

Intuitively, the equity holders will manipulate the market only when the conversion value of the CCB is sufficiently low, i.e., \( \lambda < \lambda^* \). In this case, it is cheaper for the equity holders to convert the CCB than keep paying the coupon \( c_c \). The bigger the difference \( A_t - A_C \), the lower \( \lambda^* \). This is because the CCB holders are given shares whose real value is higher than \( W_C \), when the stock is manipulated.

Finally, we note that there may be additional costs associated with manipulations by both equity and CCB holders that we did not take into account. These could include implementation costs, potential penal-
ties, legal fees, etc. One should also remember that stock manipulations are against the securities laws. The additional costs would make manipulations less profitable and result in higher $\lambda^*$ and lower $\lambda^{**}$. We believe that for reasonable parameters there will be non-empty manipulation-proof interval $[\lambda^*, \lambda^{**}]$.

7 Summary and Policy Conclusions

This paper has provided a formal model of CCBs with market based conversion triggers. In terms of prudential bank regulation, CCBs provide a new instrument that allows banks or firms to recapitalize in an automatic and dependable fashion whenever their capital reaches a distressed level. We have shown that CCBs can reduce the chance of costly bankruptcy or bailout and increase the value of the issuing firm. However, whether this will be the case in practice depends on the parameters of the CCBs and actions of regulators. In particular, an asset-based conversion trigger should be set sufficiently high that the firm does not default before or at conversion, as required by Condition 1. Condition 2, requiring that the stock price is monotone in the asset level, is needed to successfully implement CCBs with stock-based triggers.

Regulators should require banks to substitute CCBs for straight debt, and not for equity, in their capital structure. Otherwise, CCBs would crowd out equity without reducing the amount of straight debt. This finding is in line with Flannery (2010) who has suggested that banks be presented with the choice of raising their capital ratio by a given amount or raising their capital ratio by a smaller amount as long as it is combined with a specified amount of CCBs.

Replacing straight debt with a CCB in the capital structure of a financially distressed firm will alleviate the financial distress and increase the total value of the firm. However, equity holders are likely to resist such a swap due to the debt overhang effect. Regulators have to either enforce the straight-debt-for-CCB swap, or come up with a mechanism that limits losses for the equity holders.[13] Equity holders of too-big-to-fail firms will be also unenthusiastic about issuing CCBs due to the loss of the government subsidy associated with the straight debt.

We have also shown that when the conversion value of a CCB with a stock-based conversion trigger

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[13] Issuing CCBs to replace maturing straight debt would eliminate the debt overhang costs. However, waiting for the straight debt to mature may be risky when the firm is already experiencing some financial distress.
is too high or too low, it can create incentives to manipulate the stock market for CCB holders or equity holders correspondingly. In particular, when the conversion value is equal to the par value, the CCB would be always priced at par in the absence of manipulation. However, CCB holders would increase their payoff if they trigger early conversion.

Finally, our analysis focuses on the financial health of an individual firm. It shows that CCBs can reduce financial distress, whether caused by idiosyncratic or aggregate shocks. The overall benefits of CCBs for the entire financial system are likely to be greater than our analysis suggests due to the negative externalities of individual bank failures. Moreover, these benefits are likely to be highest during a systemic crisis.

We conclude with comments on important topics for future research. One useful extension would fully determine the firm’s optimal capital structure in the presence of CCBs. In particular, our analysis has been static in the sense that we assume the firm’s entire CCB issue is converted into equity at a single point when the trigger is activated. We suspect, however, that the CCB benefits would expand further if the bonds could be converted in a sequence of triggers and/or that banks can issue new CCBs after the existing bonds were converted. A second factor is that our analysis has assumed that both the CCBs and straight debt have an unlimited maturity in the fashion of a consol. We expect that an analysis with finite maturity bonds would find lower debt overhang costs of swapping CCBs for straight debt. While we do not expect this to change our basic results, it should be confirmed.
Appendix: Proofs

Proof of Proposition 1 Based on Duffie (2001), for a given constant $K \in (0, A_t)$, the market value of a security that pays one dollar at the hitting time $\tau(K) = \inf\{s : A_s \leq K\}$ is, at any $t < \tau(K)$,

$$E^Q_t \left[ e^{-r(\tau(K)-t)} \right] = \left( \frac{A_t}{K} \right)^\gamma. \tag{A.1}$$

We use this result to derive closed-form solutions for the values of straight debt, a CCB, tax benefits and bankruptcy costs as the present values of the corresponding cash flows. Equation (A.1) is applied repeatedly with different values for $K$.

$$U^B(A_t; c_b, c_c) = E^Q_t \left[ \int_t^{\tau(A_b)} e^{-r(s-t)} c_b ds + e^{-r(\tau(A_b)-t)} (1 - \alpha) A_B \right]$$

$$= E^Q_t \left[ \frac{c_b}{r} (1 - \left( \frac{A_t}{A_B} \right)^\gamma) + \left( \frac{A_t}{A_B} \right)^\gamma (1 - \alpha) A_B. \right]$$

$$U^C(A_t; c_b, c_c) = E^Q_t \left[ \int_t^{\tau(A_c)} e^{-r(s-t)} c_c ds + e^{-r(\tau(A_c)-t)} \left( \frac{c_c}{r} \right) \right]$$

$$= E^Q_t \left[ \frac{c_c}{r} (1 - e^{-r(\tau(A_c)-t)}) + e^{-r(\tau(A_c)-t)} \left( \frac{c_c}{r} \right) \right]$$

$$= \frac{c_c}{r} \left[ 1 - \left( \frac{A_t}{A_c} \right)^\gamma \right] + \left( \frac{A_t}{A_c} \right)^\gamma \left( \frac{c_c}{r} \right).$$

$$TB(A_t; c_b, c_c) = E^Q_t \left[ \int_t^{\tau(A_b)} e^{-r(s-t)} \theta c_b ds + \int_t^{\tau(A_c)} e^{-r(u-t)} \theta c_c du \right]$$

$$= \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^\gamma \right) + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right).$$

$$BC(A_t; c_b, c_c) = E^Q_t \left[ e^{-r(\tau(A_b)-t)} \alpha A_B \right] = \alpha A_B \left( \frac{A_t}{A_B} \right)^\gamma.$$

At any time $t$ before conversion, the following budget equation holds:

$$A_t + TB(A_t; c_b, c_c) = W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + U^C(A_t; c_b, c_c) + BC(A_t; c_b, c_c).$$
Therefore,

\[
W(A_t; c_b, c_c) = A_t + TB(A_t; c_b, c_c) - U^B(A_t; c_b, c_c) - U^C(A_t; c_b, c_c) - BC(A_t; c_b, c_c)
\]

\[
= A_t + \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^\gamma \right) + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right) - \frac{c_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^\gamma \right) - \frac{c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right) (1 - \lambda) A_B
\]

\[
= A_t + \frac{c_b (\theta - 1)}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^\gamma \right) + \frac{c_c (\theta - 1)}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right) - \alpha A_B \left( \frac{A_t}{A_B} \right)^\gamma - \frac{1}{r} \left( \frac{c_b}{A_B} \right) \left( \frac{A_t}{A_B} \right)^\gamma - \frac{1}{r} \left( \frac{c_c}{A_C} \right) \left( \frac{A_t}{A_C} \right)^\gamma.
\]

Finally,

\[
G(A_t; c_b, c_c) = W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + U^C(A_t; c_b, c_c)
\]

\[
= A_t + \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^\gamma \right) + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right) - \alpha A_B \left( \frac{A_t}{A_B} \right)^\gamma.
\]

\[\Box\]

**Proof of Lemma**

Differentiating \( W(A_t, c_b, c_c) \) with respect to \( A_t \) gives

\[
\frac{\partial W(A_t, c_b, c_c)}{\partial A_t} = 1 - \left( \frac{A_t}{A_B} \right)^{-\gamma} - \gamma c_c (1 - \theta - \lambda) \left( \frac{A_t}{A_C} \right)^{-\gamma+1}.
\]

If \( \lambda \geq 1 - \theta \), \( \frac{\partial W(A_t, c_b, c_c)}{\partial A_t} > 0 \) for all \( A_t > A_B \). Since \( \gamma > 0 \), \( \frac{\partial W(A_t, c_b, c_c)}{\partial A_t} \) is increasing in \( A_t \), when \( \lambda < 1 - \theta \).

Thus, \( W(A_t, c_b, c_c) \) is increasing in \( A_t \) for all \( A_t > A_C \), if and only if

\[
\left. \frac{\partial W(A_t, c_b, c_c)}{\partial A_t} \right|_{A_t = A_C} = 1 - \left( \frac{A_C}{A_B} \right)^{-\gamma} - \gamma c_c (1 - \theta - \lambda) \frac{A_C}{A_B} > 0,
\]

which is equivalent to

\[
\lambda > 1 - \theta - (1 - \gamma) \frac{A_C}{A_B} c_c.
\]
Proof of Proposition 2. Equity holders chose \( c_b \) to maximize total firm value, \( G(A_t; c_b, c_c) \),

\[
\max_{c_b \geq 0} G(A_t; c_b, c_c) \equiv \max_{c_b \geq 0} [A_t + TB(A_t; c_b, c_c) - BC(A_t; c_b, c_c)].
\]

Proposition 1 and equation (2) lead to:

\[
\max_{c_b \geq 0} \left[ \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{\beta(1 - \theta) c_b} \right)^\gamma \right) + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^\gamma \right) - \alpha \beta (1 - \theta) c_b \left( \frac{A_t}{\beta(1 - \theta) c_b} \right)^\gamma \right].
\]

FOCs:

\[
\frac{\theta}{r} - \frac{\theta(y + 1)}{r} \left( \frac{\beta(1 - \theta)}{A_t} \right)^\gamma c_b^\gamma - \alpha(y + 1) \frac{\beta(1 - \theta)^{(y+1)}}{A_t^\gamma} c_b^\gamma = 0.
\]

The solution to the above equation does not depend on \( c_c \):

\[
c_b^* = \frac{A_t}{\beta(1 - \theta)} \left( \frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[ (y + 1) \left( \frac{\theta}{r} + \alpha \beta (1 - \theta) \right) \right]^{\frac{1}{\gamma}}.
\]

This proves item (i). Item (v) follows from the fact that straight debt value and bankruptcy costs depend on \( c_b \) but not on \( c_c \). As for item (iv), based on Proposition 1 and (3),

\[
TB(A_0; c_b^*, c_c) = \frac{\theta c_b^*}{r} \left( 1 - \left( \frac{A_0}{A_B} \right)^\gamma \right) + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_0}{A_C} \right)^\gamma \right) = TB(A_0; c_b^*, 0) + TB^C(A_0; c_b^*, c_c).
\]

By re-grouping the terms in the formula for the value of equity from Proposition 1, we get

\[
W(A_0; c_b^*, c_c) = A_0 - \frac{c_b^*(1 - \theta)}{r} \left( 1 - \left( \frac{A_0}{A_B} \right)^\gamma \right) - A_B \left( \frac{A_0}{A_B} \right)^\gamma - \frac{c_c(1 - \theta)}{r} \left( 1 - \left( \frac{A_0}{A_C} \right)^\gamma \right) + \frac{A_0}{A_C} \left( \frac{A_0}{A_C} \right)^\gamma + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_0}{A_C} \right)^\gamma \right)
\]

\[
= W(A_0; c_b^*, 0) - U^C(A_0; c_c) + TB^C(A_0; c_b^*, c_c)
\]
as in item (iii). Finally,

\[ G(A_0; c'_b, c_c) = A_0 + TB(A_0; c'_b, c_c) - BC(A_t, c_b) \]
\[ = \left[ A_0 + \frac{\theta \bar{c}_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right) ^{-\gamma} \right) - \alpha A_B \left( \frac{A_t}{A_B} \right) ^{-\gamma} \right] + \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right) ^{-\gamma} \right). \]
\[ = G(A_0; c'_b, 0) + TB^C(A_0; c'_b, c_c). \]

This proves item (ii). □

**Proof of Proposition 3** Since firm value is the sum of equity and debt values, and given regulatory constraint (6),

\[ G(A_t; \bar{c}_b, c_c) - G(A_t; c'_b, 0) = W(A_t; \bar{c}, c_c) + UC(A_t; \bar{c}_b, c_c) + UB(A_t; \bar{c}_b, c_c) - W(A_t; c'_b, 0) - UB(A_t; c'_b, 0) \]
\[ = W(A_t; \bar{c}_b, c_c) - W(A_t; c'_b, 0). \]

The difference in firm values equals the difference in equity values.

We denote \( G(A_t; \bar{c}_b, c_c) - G(A_t; c'_b, 0) \) by \( \Delta G \). Based on equity values given by Proposition 1,

\[ \Delta G = - \frac{\bar{c}_b (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_B} \right) ^{-\gamma} \right) - \frac{c_c (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_C} \right) ^{-\gamma} \right) - \bar{A}_B \left( \frac{A_t}{A_B} \right) ^{-\gamma} - \frac{c_c}{r} \left( \frac{A_t}{A_C} \right) ^{-\gamma} + \frac{c'_b (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_B} \right) ^{-\gamma} \right) + A_B \left( \frac{A_t}{A_B} \right) ^{-\gamma}. \]

(A.2)

Based on debt values given by Proposition 1 and constraint (6),

\[ \frac{\bar{c}_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right) ^{-\gamma} \right) + \frac{c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right) ^{-\gamma} \right) - \frac{c'_b}{r} \left( 1 - \left( \frac{A_t}{A_B} \right) ^{-\gamma} \right) = (1 - a) A_B \left( \frac{A_t}{A_B} \right) ^{-\gamma} - (1 - a) A_B \left( \frac{A_t}{A_B} \right) ^{-\gamma} - \left( \frac{a}{r} \right) \left( \frac{A_t}{A_C} \right) ^{-\gamma}. \]

(A.3)

By multiplying (A.3) by \( (1 - \theta) \) and using it in (A.2) we get

\[ \Delta G = (\theta + a - \theta a) A_B \left( \frac{A_t}{A_B} \right) ^{-\gamma} - \bar{A}_B \left( \frac{A_t}{A_B} \right) ^{-\gamma} - \theta \left( \frac{a}{r} \right) \left( \frac{A_t}{A_C} \right) ^{-\gamma}. \]
Since $\beta = \frac{\gamma}{\gamma + 1}$ and, based on equation (2), $\bar{A}_B = \beta(1 - \theta)\bar{c}_b$.

\[
\frac{\partial \Delta G}{\partial \bar{c}_b} = -\frac{(\theta + \alpha - \theta \alpha)\gamma(1 - \theta)}{r} \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \frac{\theta \lambda}{r} \left(\frac{A_t}{A_C}\right)^{-\gamma} \frac{\partial c_c}{\partial \bar{c}_b}.
\] (A.4)

Based on equation (A.3) and the Implicit Function Theorem,

\[
\frac{\partial c_c}{\partial \bar{c}_b} = -\frac{1}{r} \frac{(\gamma + 1)\gamma}{\left(\frac{A_t}{A_C}\right)^{-\gamma}} + \frac{1}{r} \frac{\lambda}{\left(\frac{A_t}{A_C}\right)^{-\gamma} - 1 + \lambda}.
\]

It follows, that

\[
\frac{\partial \Delta G}{\partial \bar{c}_b} = -\frac{(\theta - \alpha)\gamma(1 - \theta)}{r} \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} + \frac{\lambda}{\left(\frac{A_t}{A_C}\right)^{-\gamma} - 1 + \lambda} \times \\
\left(\frac{\theta}{r} - \frac{\theta(\gamma + 1)}{r} \left(\frac{A_t}{A_B}\right)^{-\gamma} + \frac{\theta(1 - \alpha)(1 - \theta)}{r} \left(\frac{A_t}{A_B}\right)^{-\gamma}\right).
\]

The implicit assumption is that when the firm issues the CCB at time $t$, the observed asset value is above the conversion boundary, $A_C < A_t$. Therefore, $\frac{\lambda}{\left(\frac{A_t}{A_C}\right)^{-\gamma} - 1 + \lambda} < 1$. By using this inequality and regrouping terms in the above equation, we get that

\[
\frac{\partial \Delta G}{\partial \bar{c}_b} < -\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \left[\frac{\alpha \gamma(1 - \theta)}{r} + \theta(\gamma + 1)\right] + \frac{\theta}{r}.
\]

At the point when the capital structure includes only an optimal amount of straight debt and no CCBs, $\bar{c}_b = c_b^*$,

\[
\frac{\partial \Delta G}{\partial \bar{c}_b} \bigg|_{\bar{c}_b = c_b^*} < -\left(\frac{A_t}{A_B^*}\right)^{-\gamma} \left[\frac{\alpha \gamma(1 - \theta)}{r} + \theta(\gamma + 1)\right] + \frac{\theta}{r}.
\] (A.5)
Given that \( A^*_B = \beta(1 - \theta)c^*_b \) and based on (5),

\[
\frac{\beta(1 - \theta)c^*_b}{A_t} = \left( \frac{\theta}{r} \right)^\gamma \left( \frac{1}{r} + \frac{1 + \alpha}{\beta} \right)^{-\frac{\gamma}{\gamma}}
\]

and

\[
\left( \frac{A_t}{A^*_B} \right)^{-\gamma} = \left( \frac{\theta}{r} \right) \left( \frac{\alpha \gamma(1 - \theta) + \beta \gamma(\gamma + 1)}{r} \right)^{-1}
\]

By using this in (A.5), we get that

\[
\frac{\partial \Delta G}{\partial \bar{c}b} \bigg|_{\bar{c}b = c^*_b} < \frac{-\theta}{r} + \frac{\theta}{r} = 0.
\]

This means that there exists \( \bar{c}_1 \) such that, for any \( c_c \in (0, \bar{c}_1) \), \( \Delta G \leq 0 \). Given that \( G(A_0, A^*_B; c^*_b, 0) \) is fixed, for any \( c_c \in (0, \bar{c}_1) \), \( G(A_0, A^*_B; c^*_b, c_c) \geq G(A_0, A^*_B; c^*_b, 0) \). This proves the second part of the proposition.

As for the bankruptcy costs, based on equation (A.2), \( A^*_B = \beta(1 - \theta)c^*_b \) and \( \bar{A}_B = \beta(1 - \theta)\bar{c}_b \). Given the closed-form solutions from Proposition 1

\[
BC(A_0, \bar{A}_B; \bar{c}_b) - BC(A_0, A^*_B; c^*_b) = \alpha \bar{A}_B \left( \frac{A_0}{\bar{A}_B} \right)^{-\gamma} - \alpha A^*_B \left( \frac{A_0}{A^*_B} \right)^{-\gamma}
\]

\[
= \alpha \bar{c}_b \beta(1 - \theta) \left( \frac{\bar{c}_b \beta(1 - \theta)}{A_0} \right)^{-\gamma} - \alpha c^*_b \beta(1 - \theta) \left( \frac{c^*_b \beta(1 - \theta)}{A_0} \right)^{-\gamma}
\]

\[
= (\bar{c}_b^\gamma - c^*_b^\gamma) \alpha \left( \frac{\beta(1 - \theta)^\gamma + 1}{A_0} \right)
\]

Since \( \bar{c}_b < c^*_b \), the last term is strictly negative. Therefore, \( BC(A_0, \bar{A}_B; \bar{c}_b) < BC(A_0, A^*_B; c^*_b) \).

**Proof of Proposition 4**

Denote \( W(A_t, \bar{A}_B; \bar{c}_b, c_c) - W(A_t, \bar{A}_B; \bar{c}_b, 0) \) by \( \Delta W \). When the capital structure of the firm includes only equity and straight debt, the closed-form solution for the value of equity is

\[
W(A_t, \bar{A}_B; \bar{c}_b, 0) = A_t - \frac{\bar{c}_b(1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \bar{A}_B \left( \frac{A_t}{\bar{A}_B} \right)^{-\gamma}.
\]
Based on \((A.6)\) and the closed-form solution for \(W(A_t, \tilde{A}_B; \tilde{c}_b, c_t)\) from Proposition 1

\[
\Delta W = A_t - \frac{\tilde{c}_b (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^{-\gamma} \right) - \frac{c_t (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right) - \tilde{A}_B \left( \frac{A_t}{A_B} \right)^{-\gamma} - \left( \frac{\tilde{c}_b}{r} \right) \left( \frac{A_t}{A_C} \right)^{-\gamma} - \frac{c_t (1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_B} \right)^{-\gamma} \right) + \tilde{A}_B \left( \frac{A_t}{A_B} \right)^{-\gamma}
\]

Multiply both sides of (7) by \((1 - \theta)\) and use the result to reduce the first three terms after the equal sign above to get

\[
\Delta \hat{W} = -\theta \left( \frac{\tilde{c}_b}{r} \right) \left( \frac{A_t}{A_C} \right)^{-\gamma} + \frac{\tilde{c}_b (1 - \theta)}{r} \left( \left( \frac{A_t}{A_B} \right)^{-\gamma} - \left( \frac{A_t}{A_B} \right)^{-\gamma} \right) + \tilde{A}_B \left( \frac{A_t}{A_B} \right)^{-\gamma}.
\]

(A.7)

Continue by showing that \(\Delta \hat{W} < 0\). From (A.7)

\[
\Delta \hat{W} = \frac{\tilde{c}_b (1 - \theta)}{r} \left( \frac{A_t}{A_B} \right)^{-\gamma} - \tilde{A}_B \left( \frac{A_t}{A_B} \right)^{-\gamma} - \left( \frac{\tilde{c}_b (1 - \theta)}{r} \right) \left( \frac{A_t}{A_B} \right)^{-\gamma} - \theta \left( \frac{\tilde{c}_b}{r} \right) \left( \frac{A_t}{A_C} \right)^{-\gamma}
\]

\[= -(H(\tilde{A}_B) - H(\tilde{A}_B)) - \theta \left( \frac{\tilde{c}_b}{r} \right) \left( \frac{A_t}{A_C} \right)^{-\gamma}, \quad (A.8)\]

where \(H(X) = \frac{\tilde{c}_b (1 - \theta)}{r} \left( \frac{A_t}{X} \right)^{-\gamma} - \frac{1}{X} \left( \frac{A_t}{X} \right)^{-\gamma} - (1 + \gamma) \left( \frac{A_t}{X} \right)^{-\gamma}\). \(H(X)\) is such that

\[
H'(X) = \gamma \frac{\tilde{c}_b (1 - \theta)}{r} \left( \frac{A_t}{X} \right)^{-\gamma} - (1 + \gamma) \left( \frac{A_t}{X} \right)^{-\gamma} = \left( \frac{A_t}{X} \right)^{-\gamma} \frac{\tilde{c}_b}{r} (1 - \theta) - (1 + \gamma)X.
\]
Since $\hat{A}_B = \frac{y(1-\theta c_b)}{r(1+y)}$

\[
H'(\hat{A}_B) = \left( \frac{A_t}{\hat{A}_B} \right)^{-\gamma} \frac{1}{\hat{A}_B} \left( \frac{\hat{c}_b}{r} (1-\theta) - (1+y)\frac{y(1-\theta)c_b}{r(1+y)} \right)
\]
\[
= \left( \frac{A_t}{\hat{A}_B} \right)^{-\gamma} \frac{1}{\hat{A}_B} \left( \frac{\hat{c}_b}{r} (1-\theta) - (1-\theta)\frac{\hat{c}_b}{r} \right)
\]
\[
= 0.
\]

It is also clear from the above that if $0 < X < \hat{A}$, then $H'(X) > 0$. $H(X)$ is an increasing function of $X$ on $(0, \hat{A}_B)$. Since $0 < \hat{A}_B < \hat{A}_B$, $H(\hat{A}_B) > H(\hat{A}_B)$ and, based on (A.8), $\Delta \hat{W} < 0$. The value of equity always decreases. This proves item (i).

Denote $G(A_t, \hat{A}_B; \hat{c}_b, c_c) - G(A_t, \hat{A}_B; \hat{c}_b, 0)$ by $\Delta \hat{G}$. When the capital structure of the firm includes only equity and straight debt, the closed-form solution for the total value of the firm is

\[
G(A_t, \hat{A}_B; \hat{c}_b, 0) = A_t + \frac{\hat{c}_b \theta}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) - \alpha \hat{A}_B \left( \frac{A_t}{\hat{A}_B} \right)^{-\gamma}.
\]  \hspace{1cm} (A.9)

Given (A.9) and the closed-form solution for $G(A_t, \hat{A}_B; \hat{c}_b, c_c)$ from Proposition [1]
Multiply both sides of (7) by \(\theta\) and replace \(\frac{c_b}{r} \left(1 - \left(\frac{A_i}{A_C}\right)^{-\gamma}\right)\) above to get

\[
\Delta \hat{G} = \frac{\theta c_b}{r} \left(1 - \left(\frac{A_i}{A_B}\right)^{-\gamma}\right) + \frac{\theta (\hat{c}_b - \hat{c}_b)}{r} \left(1 - \left(\frac{A_i}{A_B}\right)^{-\gamma}\right) - \theta \left(\frac{A_i}{A_C}\right)^{-\gamma} \left(\frac{\hat{c}_b}{r}\right) - \\
\frac{\hat{c}_b \theta}{r} \left(1 - \left(\frac{A_i}{A_B}\right)^{-\gamma}\right) + \alpha \left(\hat{A}_B \left(\frac{A_i}{A_B}\right)^{-\gamma} - \hat{A}_B \left(\frac{A_i}{A_B}\right)^{-\gamma}\right) - \\
\frac{\hat{c}_b \theta}{r} \left(\frac{A_i}{A_B}\right)^{-\gamma} - \left(1 - \alpha\right) \hat{A}_B \left(\frac{A_i}{A_B}\right)^{-\gamma} - \\
\left(\frac{\hat{c}_b}{r} \left(\frac{A_i}{A_B}\right)^{-\gamma} - \left(1 - \alpha\right) \hat{A}_B \left(\frac{A_i}{A_B}\right)^{-\gamma}\right).
\]

We continue by showing that \(\Delta \hat{G} > \Delta \hat{W}\). Based on (A.7) and the above result

\[
\Delta \hat{G} - \Delta \hat{W} = \frac{\hat{c}_b}{r} \left(\frac{A_i}{A_B}\right)^{-\gamma} - \left(1 - \alpha\right) \hat{A}_B \left(\frac{A_i}{A_B}\right)^{-\gamma} - \\
\left(\frac{\hat{c}_b}{r} \left(\frac{A_i}{A_B}\right)^{-\gamma} - \left(1 - \alpha\right) \hat{A}_B \left(\frac{A_i}{A_B}\right)^{-\gamma}\right) = F(\hat{A}_B) - F(\hat{A}_B), \quad (A.10)
\]

where \(F(X) \equiv \frac{\hat{c}_b}{r} \left(\frac{A_i}{X}\right)^{-\gamma} - \left(1 - \alpha\right) X \left(\frac{A_i}{X}\right)^{-\gamma}\). \(F(X)\) is such that

\[
F'(X) = \gamma \frac{\hat{c}_b}{r} \left(\frac{A_i}{X}\right)^{-\gamma} \frac{1}{X} - \left(1 - \alpha\right) (1 + \gamma) \left(\frac{A_i}{X}\right)^{-\gamma} = \left(\frac{A_i}{X}\right)^{-\gamma} \frac{1}{X} \left(\gamma \frac{\hat{c}_b}{r} - \left(1 - \alpha\right) (1 + \gamma) X\right).
\]

Note that \(\hat{A}_B = \frac{\lambda(1 - \theta) c_b}{\gamma (1 + \gamma)}\) and therefore

\[
F'(\hat{A}_B) = \left(\frac{A_i}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \left(\gamma \frac{\hat{c}_b}{r} - \left(1 - \alpha\right) (1 + \gamma) \frac{\gamma (1 - \theta) c_b}{r (1 + \gamma)}\right) = \left(\frac{A_i}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \left(\gamma \frac{\hat{c}_b}{r} - \left(1 - \alpha\right) (1 - \theta) \frac{\hat{c}_b}{r}\right) = \left(\frac{A_i}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \gamma \frac{\hat{c}_b}{r} \left(\gamma - \left(1 - \alpha\right) (1 - \theta)\right).
\]
By assumption, \( \alpha \in [0, 1] \) and \( \theta \in (0, 1) \), so \((1 - (1 - \alpha)(1 - \theta)) > 0\). It follows that \( F'(\hat{A}_B) > 0 \) for all \( 0 < X \leq \hat{A}_B \), and, since \( 0 < \bar{A}_B < \hat{A}_B, F(\hat{A}_B) > F(\bar{A}_B) \). Finally, based on (A.10), \( \Delta \hat{G} > \Delta \hat{W} \).

We continue by proving the last statement of item (ii).

\[
\frac{\partial \Delta \hat{G}}{\partial \bar{c}_b} = -\gamma \left( \frac{A_r}{A_B} \right)^{\gamma} \frac{1}{A_B} \frac{\bar{c}_b}{r} - \alpha(1 + \gamma) \left( \frac{A_r}{A_B} \right)^{\gamma} \frac{\partial \hat{A}_B}{\partial \bar{c}_b} - \theta \left( \frac{A_r}{A_C} \right)^{\gamma} 1 \frac{1 - \left( \frac{A_r}{A_B} \right)^{\gamma}}{1 - (1 - \lambda) \left( \frac{A_r}{A_C} \right)^{\gamma}}.
\]

where \( \frac{\partial \hat{A}_B}{\partial \bar{c}_b} = \beta(1 - \theta) \). Based on the above

\[
\frac{\partial \Delta \hat{G}}{\partial \bar{c}_b} \bigg|_{\bar{c}_b = \hat{c}_b} = -\left( \frac{A_r}{A_B} \right)^{\gamma} \frac{\gamma \bar{c}_b \theta}{r} + \alpha(1 + \gamma) \beta(1 - \theta) + \theta \left( \frac{A_r}{A_B} \right)^{\gamma} \frac{1 - \left( \frac{A_r}{A_B} \right)^{\gamma}}{1 - (1 - \lambda) \left( \frac{A_r}{A_C} \right)^{\gamma}}
\]

For \( \left( \frac{A_r}{A_C} \right)^{\gamma} - (1 - \lambda) \geq 1 \) or \( \lambda \geq 2 - \left( \frac{A_r}{A_C} \right)^{\gamma} \)

\[
\frac{\partial \Delta \hat{G}}{\partial \bar{c}_b} \bigg|_{\bar{c}_b = \hat{c}_b} \leq -\left( \frac{A_r}{A_B} \right)^{\gamma} \left( \frac{\gamma \theta}{r} + \alpha(1 + \gamma) \beta(1 - \theta) \right) + \theta \left( \frac{A_r}{A_B} \right)^{\gamma} \frac{1 - \left( \frac{A_r}{A_B} \right)^{\gamma}}{1 - (1 - \lambda) \left( \frac{A_r}{A_C} \right)^{\gamma}}.
\]
Next, assume that \( \hat{c}_b \geq c_b^* \). Then, based on (5)

\[
\hat{c}_b \geq \frac{A_t}{\beta(1-\theta)} \left( \frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left( (y+1)^{\frac{\theta}{r}} + \alpha \beta (1-\theta) \right)^{\gamma}
\]

\[
\beta(1-\theta) \hat{c}_b \geq \frac{A_t}{\beta(1-\theta)} \left( \frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left( (y+1)^{\frac{\theta}{r}} + \alpha \beta (1-\theta) \right)^{\gamma}.
\]

Based on (2)

\[
\frac{\hat{A}_B}{A_t} \geq \left( \frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left( (y+1)^{\frac{\theta}{r}} + \alpha \beta (1-\theta) \right)^{\gamma},
\]

\[
\left( \frac{A_t}{\hat{A}_B} \right)^{\gamma} \geq \frac{\theta}{r} \left( (y+1)^{\frac{\theta}{r}} + \alpha \beta (1-\theta) \right)^{-\gamma},
\]

\[
-\left( \frac{A_t}{\hat{A}_B} \right)^{\gamma} \leq -\frac{\theta}{r} \left( (y+1)^{\frac{\theta}{r}} + \alpha \beta (1-\theta) \right)^{-\gamma}.
\]

(A.12)

By using (A.12) in (A.11), we get

\[
\frac{\partial \Delta \hat{G}}{\partial \hat{c}_b} \bigg|_{\hat{c}_b = \hat{c}_b} \leq -\frac{\theta}{r} + \frac{\theta}{r} = 0.
\]

This means that there exists \( c_2 \) such that, for any \( c_c \in (0, c_2) \), \( \Delta \hat{G} \leq 0 \). Given that \( G(A_0, A_B^*; c_b^*, 0) \) is fixed, for any \( c_c \in (0, c_2) \), \( G(A_0, \hat{A}_B; \hat{c}_b, c_c) \geq G(A_0, A_B^*; c_b^*, 0) \).

Finally, we prove item (iii) of the proposition. Since \( \hat{c}_b > \check{c}_b \), based on (A.2), the optimal default-triggering boundary drops from \( \hat{A}_B = \beta(1-\theta)\hat{c}_b \) to \( \check{A}_B = \beta(1-\theta)\check{c}_b \). Given this and the closed-form solution for the cost of bankruptcy from Proposition 1

\[
BC(A_t, \hat{A}_B; \hat{c}_b) - BC(A_t, \check{A}_B; \check{c}_b) = \alpha \hat{A}_B \left( \frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \alpha \check{A}_B \left( \frac{A_t}{\check{A}_B} \right)^{-\gamma}
\]

\[
= \alpha \check{c}_b \beta(1-\theta) \left( \frac{\check{c}_b(1-\theta)}{A_t} \right)^{-\gamma} - \alpha \hat{c}_b \beta(1-\theta) \left( \frac{\hat{c}_b(1-\theta)}{A_t} \right)^{-\gamma}
\]

\[
= (\check{c}_b^{\gamma+1} - \hat{c}_b^{\gamma+1}) \alpha \frac{(\beta(1-\theta)^{\gamma+1})}{A_t^{\gamma}}.
\]

Since \( \check{c}_b < \hat{c}_b \), the last term above is strictly negative. Therefore, \( BC(A_t, \hat{A}_B; \hat{c}_b) < BC(A_t, \check{A}_B; \check{c}_b) \). \( \square \)
Proof of Proposition 5 We can use (2) to rewrite (8) as

\[ S(A_t; c_b, c_c) = \left( \frac{c_b}{r} - c_b(1 - \theta)\beta \right) \left( \frac{A_t}{c_b(1 - \theta)\beta} \right)^\gamma = c_b \left( \frac{1}{r} - (1 - \theta)\beta \right) \left( \frac{c_b(1 - \theta)\beta}{A_t} \right)^\gamma. \]

Given that \( \beta = \frac{\gamma r}{(1+\gamma)}, \)

\[ \frac{dS(A_t; c_b, 0)}{dc_b} = \left( \frac{1 + \gamma}{r} - (1 + \gamma)(1 - \theta)\beta \right) \left( \frac{A_t}{A_B} \right)^\gamma = \frac{1}{r} (1 + \gamma) \theta \left( \frac{A_t}{A_B} \right)^\gamma > 0. \]

Since \( BC(A_t; \hat{c}_b, 0) = 0, \) the total value of the firm in the presence of government subsidy when it does not issue a CCB is

\[ G(A_t; c_b, 0) = A_t + TB(A_t; c_b, 0) + S(A_t; c_b, 0). \]

Based on the closed-form solution for \( TB(A_t; c_b, 0), \) and equation (8)

\[ G(A_t; c_b, 0) = A_t + \frac{\theta c_b}{r} (1 - \left( \frac{A_t}{A_B} \right)^\gamma) + \left( \frac{c_b}{r} - A_B \right) \left( \frac{A_t}{A_B} \right)^\gamma. \]

Given the budget equation, it is easy to see that \( G(A_t; c_b, 0) \) from above is higher than the total value of the firm before the guarantee was issued by

\[ S(A_t; c_b, 0) + BC(A_t; c_b) = \left( \frac{c_b}{r} - A_B \right) \left( \frac{A_t}{A_B} \right)^\gamma + \alpha A_B \left( \frac{A_t}{A_B} \right)^\gamma > 0. \]

\[ \Box \]

Proof of Proposition 6 Consider a TBTF firm with a capital structure that includes equity and straight debt paying coupon \( c_b. \) We try to analyze the effect of replacing a portion of straight debt with a CCB on the values of government subsidy and equity.
In the presence of government subsidy at any time $t$ the following budget equation holds

$$A_t + TB(A_t; \hat{c}_b, 0) + S(A_t; \hat{c}_b, 0) = W(A_t; \hat{c}_b, 0) + UB(A_t; \hat{c}_b, 0).$$ (A.13)

There are no bankruptcy costs, $BC(A_t; \hat{c}_b, 0) = 0$, and debt is risk-free, $UB(A_t; \hat{c}_b, 0) = \frac{\hat{c}_b}{r}$.

The firm is to replace a portion of its current straight debt with a CCB paying $c_c$. The remaining straight debt will be paying coupon $\bar{c}_b$, such that $\bar{c}_b < \hat{c}_b$. The government guarantee remains in place, so straight debt will still be risk-free, $UB(A_t; \bar{c}_b, c_c) = \frac{\bar{c}_b}{r}$. As before, straight debt holders should be indifferent between exchanging their holdings for a CCB and continuing to hold straight debt

$$UB(A_t; \bar{c}_b, 0) = UC(A_t; \bar{c}_b, c_c) + UB(A_t; \bar{c}_b, c_c).$$ (A.14)

Equation (A.14) is equivalent to equation (7) in Section 4. The key difference, though, is that after a TBTF firm announces its decision to replace straight debt with a CCB the value of existing straight debt does not change.

After the firm replaces a portion of its straight debt with a CCB, the following budget equation will hold at any time $t$

$$A_t + TB(A_t; \bar{c}_b, c_c) + S(A_t; \bar{c}_b, c_c) = W(A_t; \bar{c}_b, c_c) + UB(A_t; \bar{c}_b, c_c) + UC(A_t; \bar{c}_b, c_c).$$ (A.15)

Given (A.13), (A.14) and (A.15),

$$W(A_t; \bar{c}_b, 0) - W(A_t; \bar{c}_b, c_c) = TB(A_t; \bar{c}_b, 0) - TB(A_t; \bar{c}_b, c_c) + S(A_t; \bar{c}_b, 0) - S(A_t; \bar{c}_b, c_c).$$ (A.16)

Since $\bar{c}_b > \hat{c}_b$, based on Proposition 5 (part i), $S(A_t; \bar{c}_b, 0) - S(A_t; \bar{c}_b, c_c) > 0$. This proves the first part of the proposition.

Denote $W(A_t; \bar{c}_b, 0) - W(A_t; \bar{c}_b, c_c)$ by $\Delta \bar{W}$. When the capital structure of the firm includes only equity
and straight debt, the closed-form solution for the value of tax benefits is

\[ TB(A_t; \hat{c}_b, 0) = \frac{\theta \hat{c}_b}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right). \quad (A.17) \]

Given (A.16), (A.17), the closed-form solution for the value of tax benefits from Proposition 1, and equation (8) for the value of government subsidy is

\[ \Delta \tilde{W} = \frac{\theta \hat{c}_b}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right) - \frac{\theta \hat{c}_b}{r} \left( 1 - \left( \frac{A_t}{\bar{A}_B} \right)^\gamma \right) - \frac{\theta \tilde{c}_b}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right) + \left( \frac{\hat{c}_b}{r} - \hat{A}_B \right) \left( \frac{A_t}{\hat{A}_B} \right)^\gamma - \left( \frac{\tilde{c}_b}{r} - \bar{A}_B \right) \left( \frac{A_t}{\bar{A}_B} \right)^\gamma. \quad (A.18) \]

By multiplying both sides of equation (A.14) by \( \theta \) and using the closed-form solutions for the values of straight debt and CCBs, we get

\[ \frac{\hat{c}_b \theta}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right) + \left( \frac{A_t}{\hat{A}_B} \right)^\gamma (1 - \alpha) \hat{A}_B \theta = \frac{\hat{c}_b \theta}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right) + \left( \frac{A_t}{\hat{A}_B} \right)^\gamma (1 - \alpha) \hat{A}_B \theta + \frac{\tilde{c}_b \theta}{r} \left( 1 - \left( \frac{A_t}{\bar{A}_B} \right)^\gamma \right) + \theta \left( \frac{\tilde{c}_c}{r} \right) \left( \frac{A_t}{\bar{A}_B} \right)^\gamma. \]

By rearranging terms

\[ \frac{\hat{c}_b \theta}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right) - \frac{\tilde{c}_b \theta}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right) - \frac{c_c \theta}{r} \left( 1 - \left( \frac{A_t}{\hat{A}_B} \right)^\gamma \right) = \left( \frac{A_t}{\hat{A}_B} \right)^\gamma (1 - \alpha) \hat{A}_B \theta + \theta \left( \frac{\tilde{c}_c}{r} \right) \left( \frac{A_t}{\bar{A}_B} \right)^\gamma \left( 1 - \alpha \right) \bar{A}_B \theta. \quad (A.19) \]
Now we can use (A.19) in (A.18) to get

\[
\Delta \tilde{W} = \left( \frac{A_t}{A_B} \right)^{\gamma - \gamma} (1 - \alpha) \tilde{A}_B \theta + \theta \left( \frac{A_t}{A_C} \right)^{\gamma} - \left( \frac{A_t}{A_B} \right)^{\gamma} (1 - \alpha) \tilde{A}_B \theta + \\
\left( \frac{\tilde{c}_b}{r} - \tilde{A}_B \right) \left( \frac{A_t}{A_C} \right)^{\gamma} - \left( \frac{\tilde{c}_b}{r} - \tilde{A}_B \right) \left( \frac{A_t}{A_B} \right)^{\gamma}
\]

\[
= \theta \left( \frac{A_t}{A_C} \right)^{\gamma} + \left( \frac{\tilde{c}_b}{r} - \left( \frac{A_t}{A_C} \right)^{\gamma} - \left( \frac{\tilde{c}_b}{r} - \left( \frac{A_t}{A_B} \right)^{\gamma} \right)ight)
\]

\[
= \theta \left( \frac{A_t}{A_C} \right)^{\gamma} + \\
\left( 1 - ((1 - \alpha) \theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} \left( \frac{\tilde{c}_b}{r} - \left( \frac{A_t}{A_C} \right)^{\gamma} - \left( \frac{\tilde{c}_b}{r} - \left( \frac{A_t}{A_B} \right)^{\gamma} \right) \right)ight).
\]

(A.20)

In (A.20) the first term on the right-hand side is positive, \( \theta \left( \frac{A_t}{A_C} \right)^{\gamma} > 0 \), and, for \( \tilde{c}_b > \frac{\alpha}{t} \), \( \left( \frac{\tilde{c}_b}{r} - \left( \frac{A_t}{A_C} \right)^{\gamma} - \left( \frac{\tilde{c}_b}{r} - \left( \frac{A_t}{A_B} \right)^{\gamma} \right) \right)^{\gamma} > 0 \). Finally,

\[
1 - ((1 - \alpha) \theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} > 1 - (\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} = 1 - (1 - \theta^2) \frac{\gamma}{(1 + \gamma)} > 0.
\]

All the terms on the right-hand side of equation (A.20) are positive. Therefore, \( \Delta \tilde{W} > 0 \). This proves the third part of the proposition.

The only statement that remains to be proved is that \( G(A_t; \tilde{c}_b, 0) - G(A_t; \tilde{c}_b, c_c) > 0 \).

Denote \( G(A_t; \tilde{c}_b, 0) - G(A_t; \tilde{c}_b, c_c) \) by \( \Delta \tilde{G} \). When the capital structure of the firm includes only equity

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and straight debt the closed-form solution for the value of tax benefits is

\[
TB(A_t; c_B^e, 0) = \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B^e} \right)^{-\gamma} \right).
\]  

(A.21)

Given (A.21), the closed-form solution for the value of tax benefits from Proposition 1, and equation (8) for the value of government subsidy is

\[
\Delta \tilde{G} = \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B^e} \right)^{-\gamma} \right) - \frac{\theta c_b}{r} \left( 1 - \left( \frac{A_t}{A_B^g} \right)^{-\gamma} \right) - \frac{\theta c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right) + \left( \frac{c_b}{r} - A_B^g \right) \left( \frac{A_t}{A_B^e} \right)^{-\gamma} - \left( \frac{c_b}{r} - A_B^g \right) \left( \frac{A_t}{A_B^g} \right)^{-\gamma}.
\]  

(A.22)

By multiplying both sides of equation (11) by \(\theta\) and using the closed-form solutions for the values of straight debt and CCBs, we get

\[
\frac{c_b}{r} \left( 1 - \left( \frac{A_t}{A_B^e} \right)^{-\gamma} \right) + \left( \frac{A_t}{A_B^e} \right)^{-\gamma} (1 - \alpha)A_B^e \theta = \frac{\tilde{c}_b}{r} \left( 1 - \left( \frac{A_t}{A_B^g} \right)^{-\gamma} \right) + \left( \frac{A_t}{A_B^g} \right)^{-\gamma} (1 - \alpha)\tilde{A}_B \theta + \frac{c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right) + \theta \left( \frac{\lambda_c}{r} \right) \left( \frac{A_t}{A_C} \right)^{-\gamma}.
\]

By rearranging terms

\[
\frac{c_b}{r} \left( 1 - \left( \frac{A_t}{A_B^e} \right)^{-\gamma} \right) - \frac{\tilde{c}_b}{r} \left( 1 - \left( \frac{A_t}{A_B^g} \right)^{-\gamma} \right) - \frac{c_c}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right) = \left( \frac{A_t}{A_B^e} \right)^{-\gamma} (1 - \alpha)A_B^e \theta + \theta \left( \frac{\lambda_c}{r} \right) \left( \frac{A_t}{A_C} \right)^{-\gamma} - \left( \frac{A_t}{A_B^g} \right)^{-\gamma} (1 - \alpha)A_B^g \theta.
\]  

(A.23)
Given (2) and \( \beta \), now we can use (A.23) in (A.22) to get

\[
\Delta \tilde{G} = \left( \frac{A_r}{A_B} \right)^{-\gamma} (1 - \alpha) \tilde{A}_B \theta + \theta \left( \frac{c_c}{r} \right) \left( \frac{A_r}{A_C} \right)^{-\gamma} - \left( \frac{A_r}{A_B} \right)^{-\gamma} (1 - \alpha) A^*_B \theta + \\
\left( \frac{c_b}{r} - A^*_B \right) \left( \frac{A_r}{A_B} \right)^{-\gamma} - \left( \frac{c_b}{r} - \tilde{A}_B \right) \left( \frac{A_r}{A_B} \right)^{-\gamma}
\]

\[
= \theta \left( \frac{c_c}{r} \right) \left( \frac{A_r}{A_C} \right)^{-\gamma} + \left( \frac{c_b}{r} \left( \frac{A_r}{A^*_B} \right)^{-\gamma} - \frac{c_b}{r} \left( \frac{A_r}{A_B} \right)^{-\gamma} \right) - \\
((1 - \alpha) \theta + 1) \frac{\gamma}{(1 + \gamma)} \left( \frac{c_b}{r} \left( \frac{A_r}{A^*_B} \right)^{-\gamma} - \frac{c_b}{r} \left( \frac{A_r}{A_B} \right)^{-\gamma} \right)
\]

Finally, for \( c^*_b > c_b, \left( \frac{c_b}{r} \left( \frac{A_r}{A^*_B} \right)^{-\gamma} - \frac{c_b}{r} \left( \frac{A_r}{A_B} \right)^{-\gamma} \right) > 0 \).

All the terms on the right-hand side of equation (A.24) are positive. Therefore, \( \Delta \tilde{G} > 0 \).

\[\Box\]

**Proof of Theorem 3** Equity holders will not manipulate the market if

\[
W(A^*; c_b, c_c) - W(A^*; c_b, 0) - \frac{A^*_c}{r} W(A^*; c_b, 0) \geq 0.
\]
Based on Proposition 1

\[ W(A_t; c_b, c_c) - \left[ W(A_t; c_b, 0) - \lambda \frac{c_c}{r} W(A_t; c_b, 0) \right] = -\frac{c_c(1 - \theta)}{r} \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right) - \frac{\lambda c_c}{r} \left( \frac{A_t}{A_C} \right)^{-\gamma} + \frac{\lambda c_c}{r} W(A_t; c_b, 0) W(A_C; c_b, 0). \] (A.25)

The right-hand side of equation (A.25) is strictly increasing in \( \lambda \) and the value of \( \lambda \) for which the difference in the equity values is zero is

\[ \lambda^{**} = \frac{(1 - \theta) \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right)}{W(A_t; c_b, 0) W(A_C; c_b, 0) - \left( \frac{A_t}{A_C} \right)^{-\gamma}}. \]

For values of \( \lambda \) higher or equal to \( \lambda^{**} \) the right-hand side of equation (A.25) is going to be non-negative and equity holders will not have an incentive to manipulate the market. On the other hand, for values of \( \lambda \) lower than \( \lambda^{**} \) they will manipulate the market. Note that \( W \) is an increasing function of the asset value \( A_t \).

Therefore,

\[ \lambda^{**} < \frac{(1 - \theta) \left( 1 - \left( \frac{A_t}{A_C} \right)^{-\gamma} \right)}{W(A_C; c_b, 0) W(A_C; c_b, 0) - \left( \frac{A_t}{A_C} \right)^{-\gamma}} = (1 - \theta). \]

\( \square \)
References


