A Note on Demand Estimation with Supply Information in Non-Linear Models

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Abstract

This paper compares demand estimation with and without full supply information in non-linear models. The main result is that, in a non-linear model, under general monotonicity conditions, there is a specification of the full supply model which is consistent with an arbitrarily specified auxiliary equation to account for endogeneity in demand estimation. In particular, there is a specification of the full supply model that is consistent with a typical linear auxiliary equation of the endogenous variable in a limited information approach of demand estimation. In general, in a non-linear model, a full supply model and the auxiliary equation specifications do not represent the same model, but the auxiliary equation model covers a "larger" range of the space of potential models than a fully specified supply model. The paper presents an application to multiple products competing on price with non-linear demand, and shows some simulation results of the relationship between the auxiliary equation and the full supply model in that application.

1 Introduction

Data from markets come from the interaction between potential customers and firms. In particular, the behavior of the firms depends on the behavior of the potential customers, which may be based on information that is not available to the researcher. That is, when modeling the potential customers behavior we need to account for the endogeneity of the behavior of the firms. The approaches that have been used to deal with this issue can be classified in two basic camps: a full information approach and a limited information approach (e.g., Hausman, 1984). Roughly speaking, the full information approach models all the economic agents' interaction in the system, explicitly assuming how the behavior of some agents depends on the behavior of other agents (constraints of parameters across equations). In the context of demand estimation this mean fully specifying the supply model. On the other hand, the limited information approach focuses on modeling the behavior of one type of agents and uses some "limited" information about the behavior of the other agents to identify the behavior of the target type of agents. In the context of demand estimation this typically means using exogenous variables (instruments) that are known (assumed) to be correlated with the firms' actions and independent of the demand error terms, and possibly use some form of auxiliary equation of the potential endogenous variables as a function of the instruments (a control function, see, for example, Petrin and Train, 2009).

It is well known that in linear models the full information estimation approach, although more efficient, puts more constraints on the model than a limited information approach. That is, if the assumptions made in the full information model are not true, then the full information approach may yield inconsistent estimates for the parameters, while the limited information approach may still yield consistent estimates if the "limited" information is true (e.g., Hausman, 1984). In other words, in a full information approach the mis-specification of one behavior equation affects the other behavior equations, while the limited information approach is not as prone to mis-specification (due to fewer assumptions).

However, the relationship between the full information and limited information approach in non-linear models is not as well understood.[1] This paper looks at this relationship when the limited information approach uses an auxiliary equation, in the context of demand estimation.[2]

The main result is that, under general monotonicity conditions, in a non-linear model there is a specification of the full supply model which is consistent with an arbitrarily specified auxiliary equation to account for endogeneity in demand estimation. That is, consider a demand system where the relationship between the firms' action and the behavior of the potential customers is monotonic. Then, if we specify an aux-

[1]For estimation approaches of non-linear systems of equations see, for example, Amemiya, 1985, Ch. 8.

[2]The results presented here can be seen as applying to general economic environments where there is interaction between economic agents and endogeneity problems.

iliary equation model of the endogenous variables, there is a specification of the full supply model where the monotonic assumptions hold and which is consistent with the auxiliary equation model. The proof is by construction of the specification of the full supply model that would make the auxiliary equation true. This construction involves the inverse function of the firms' behavior equations of the full supply model, which is guaranteed to exist given the monotonicity assumption. In particular, this result means that there is a specification of the full supply model that is consistent with a typical linear auxiliary model of the endogenous variable when a full supply model is not specified (e.g., Newey 1987, Rivers and Vuong 1988, Villas-Boas and Winer 1999, Blundell and Powell, 2004, Petrin and Train 2009).[3]

In general, in a non-linear model, a full information and a limited information specifications do not represent the same model (unlike in the linear-model case). However, we can recover the idea of the linear models that full information estimation (full supply and demand model) requires more assumptions than limited information estimation (just an auxiliary equation model). In a non-linear model this idea is represented by the result that the limited information model covers a "larger" range of the space of potential models than a full information model does, in a sense defined below. This result can be seen as just formalizing some intuition that one might have from the linear models, but it has important implications about when one needs to fully model structurally a market interaction, or when we can use auxiliary equations to consider the endogeneity of some variables in the demand system, when the only objective is demand estimation.

As discussed below, a well-known "alternative" limited information approach is to invert, if possible, the demand equation(s) to obtain the error term as a function of the observable variables, create a moment condition with the orthogonality of the error term and some instruments, and finally estimate the parameters with the general method of moments. However, in some cases either the inversion of the demand equation is not possible (for example, in the case of limited dependent variables), or is computationally difficult. One may also have concerns about the small sample statistical properties of the method of moments in the particular problem being studied. The researcher may therefore want to use in some cases a limited information approach with an auxiliary equation, and this paper discusses the relationship between this approach and a full information approach in the context of demand estimation.

The interest in non-linear demand models has recently increased because of the interest in more careful modeling of the behavior of consumers in ways that are consistent with first principles (for example, assuming utility maximization as a basis for consumer choice), while continuing to have models that are easy to work with. This paper studies how the limited and full information approaches apply to such models.

The next section presents the model set-up, the question of the relationship between a full supply and

[3] See also Wooldridge (2002), pp. 472-477. For applications with demand and supply information see, for example, Berry et al. (1995), Besanko et al. (1998), Sudhir (2001), Villas-Boas and Zhao (2005).

an auxiliary equation model in demand estimation, and derives the main results. Section 3 presents an example of multiple products competing on price with non-linear demand, and shows simulation results of the relationship between the full supply and auxiliary equation model in that application. Finally, Section 4 presents concluding remarks.

2 Model and Results

In many circumstances one can write a non-linear model of demand that has the form

$$y = h(x, \varepsilon) \tag{1}$$

where y is demand (possibly a vectors of demands across products), x is a vector of observable variables that affect demand, ε are some unobservable variables that affect that demand, and where the function $h()$ is known/assumed. Example of this are where y is the choice of a consumer or the market share of a firm, x represents the prices charged by firms, ε represents some unobservable characteristics of the consumers, and $h()$ is obtained, for example, from some utility maximization by consumers (for example, a demand model based on Guadagni and Little, 1983).[4] In what follows we use the leading example where x is price, but the results apply to the variable x being any other action by the firms, or other observables.

The linear example of equation (1) is just

$$y = x\beta + \varepsilon.$$

This is an example that we will always come back to, and compare results with.

One crucial issue in the estimation of a model of this type is that the prices x may be set as a function of ε. That is, the price vector x is endogenous. In the example above, firms may set prices as a function of characteristics of the consumer preferences that are not observed by the researcher.

In general, one might also have some theory of how x is determined, the supply model. In the example above, one could, for instance, have the theory that firms fully observe ε and each other's costs, are choosing prices to maximize their own profits, and behave as in a Nash equilibrium.

From this theory, one might have

$$x = f(g(z, \eta), \varepsilon) \tag{2}$$

in which one knows the function $f()$ from the theory, and assumes a function $g()$ of some exogenous observ-

[4]In terms of the example above one might have to define one of the x as minus the price of that firm.

ables z and unobservables η, representing some construct that also affects x according to the theory. For example, $g()$ could be the marginal cost of production. Assume that both $f()$ and $g()$ are monotonic in its arguments. In particular, assume $\frac{\partial f}{\partial g}, \frac{\partial f}{\partial \varepsilon}, \frac{\partial g}{\partial z}, \frac{\partial g}{\partial \eta} > 0$. In terms of the example above, $g()$ could represent the marginal cost of production, z some input prices, and η some unobservables that affect the marginal cost of production. The price being set, according to the theory, is a function $f()$ of the marginal cost of production $g()$ and of the demand unobservables. The error term ε is assumed independent of z. The specification of the functions $f()$ and $g()$ constitute the full supply model.

To simplify the presentation we consider y, x, and z to be scalars. The extension to the multi-dimensional case is straightforward, but instead of a monotonicity condition we would then assume that $f()$ be invertible. One sufficient condition for $f()$ to be invertible is that g, having the same dimension as x, belongs to a convex set, and the Jacobian of $f()$ with respect to g is negative (or positive) semidefinite (Gale and Nikaido, 1965). If there is a dominant diagonal condition (for example, under price competition with products sufficiently differentiated, then the Nash equilibrium price of one firm may be more affected by its marginal cost than by the marginal costs of the other firms) then the condition above is satisfied. See also Vives (1999, pp.47-48).

Consider, for example, the case of two products, $x = [x_1, x_2]$, with marginal costs $g = [g_1, g_2]$, with g_i being the marginal cost of product i, which has price x_i. Then the system of equations (2) would be

$$\begin{aligned} x_1 &= f_1(g_1, g_2, \varepsilon) \\ x_2 &= f_2(g_1, g_2, \varepsilon). \end{aligned}$$

For the positive semidefinite condition we would just need $\frac{\partial f_i}{\partial g_i} > 0$ for all i, the price of product i, x_i, would be increasing in its marginal cost g_i, and $\frac{\partial f_1}{\partial g_1}\frac{\partial f_2}{\partial g_2} > \frac{\partial f_1}{\partial g_2}\frac{\partial f_2}{\partial g_1}$, which holds if the marginal costs of a product affect much more the price of that product than the price of the other product (the dominant diagonal condition).

In a linear example we would have

$$g(z, \eta) = z\gamma + \eta$$

and

$$x = z\lambda\gamma + \lambda\eta + \lambda'\varepsilon$$

where λ and λ' would be some function of β.

We call equations (1) and (2) the full information model. Estimating (1) and (2) together, one can, in general, obtain consistent estimates of the parameters in the demand function $h()$ and in the marginal cost function $g()$. This has been called the full information approach in the sense that one uses the information from the theory, which is the full supply model represented by equation (2), to estimate equation (1). However, one potential problem with this approach is that if the full supply model is incorrect one brings the mis-specification of equation (2) into the demand estimation of (1), which results in inconsistent estimates of the parameters of the demand equation (1).

In the linear case one well-known approach is to use z as instruments for x in equation (1) to obtain an estimate of β. This produces consistent estimates of β, which are not affected by any possible assumed relationship between λ or λ' and β.

Consider now the non-linear case. One approach, if the demand function $h(x, \varepsilon)$ is invertible, is to obtain $\varepsilon = h_{\varepsilon}^{-1}(x, y)$, and use some instruments z with the property that $E[\varepsilon|z] = 0$, to construct moment conditions, from which consistent estimates can be obtained with a general method of moments (GMM) estimator. For example, if $h(x, \varepsilon)$ is parametrically specified as $h(x, \varepsilon, \theta)$, then a consistent GMM estimator of θ could be obtained from the moment conditions $E[h_{\varepsilon}^{-1}(x, y, \theta)m(z)] = 0$ where $m(z)$ is a vector of nonlinear functions of the instruments (e.g., using only the demand moment in Berry et al. 1995, or Chintagunta 2001). This approach achieves consistent estimates of the parameters of the target behavior equation (1) which is robust to any mis-specification in (2), and is consistent with any specification of (2).

However, in some cases, the inversion of the demand function $h(x, \varepsilon)$ is not possible (for example, with limited dependent variables), or is computationally difficult, which may rule out the possibility of using that method of moments approach. In other cases the researcher may not be fully satisfied with the small sample statistical properties of the method of moments estimator in the particular problem being studied.

In those cases, another possibility may be to use an auxiliary equation of the endogenous variable(s) x without the constraints of the supply equation (2). The purpose of this paper is to identify conditions under which one can perform this limited information approach with an auxiliary equation in a non-linear model of demand estimation, that is consistent with the full supply model equation (2), and still obtain consistent estimates of the parameters of the demand equation (1) even if (2) is mis-specified. For estimation procedures with such auxiliary equations see, for example, Newey (1987), Rivers and Vuong (1988), or Blundell and Powell (2004).

In particular, suppose that one wants to substitute equation (2) with a linear equation

$$x = z\alpha + \widetilde{\eta} \tag{3}$$

where the parameters α are not constrained to be related to the parameters in the demand equation (1), and where we would like to estimate the parameters in equation (1) by estimating (1) and (3) together. We call equations (1) and (3) the limited information model. For the estimation of (1) and (3) together to be correct it must be that equation (3) is true given equation (2). In particular, given some assumed distribution of $\widetilde{\eta}$, are there a marginal cost function $g()$ and a distribution of η such that equation (3) is true? Moreover, is there an infinite number of triples of f, g and the distribution over η such that equation (3) is true, so that we can say that estimating (1) with the help of (3) produces consistent estimates for the parameters in (1) without being affected by some mis-specification in the pricing equation (2)?

Given a cumulative distribution of $(\widetilde{\eta}, \varepsilon)$ given z, and a pricing (supply) function f increasing in the first argument, we are looking for the existence of a marginal cost function $g()$ and a probability distribution of η such that

$$z\alpha + \widetilde{\eta} =^d f(g(z, \eta), \varepsilon), \tag{4}$$

where $=^d$ means that "it has the same probability distribution given z."[5]

Note that for the linear example the answer to this question is obvious, because, if $f()$ is linear, and we make $g()$ linear, having η distributed as a linear combination of $\widetilde{\eta}$ and ε makes the equality hold.[6]

For the case where $f()$ is non-linear, note that we can obtain the distribution of the marginal cost $g(z, \eta)$, given z, and given the probability distribution of $(\widetilde{\eta}, \varepsilon)$ and the pricing function f, by rewriting (4) as

$$g(z, \eta) =^d f^{-1}(z\alpha + \widetilde{\eta}, \varepsilon). \tag{5}$$

We can, similarly, obtain the joint distribution of $(g(z, \eta), \varepsilon)$. In general, $g(z, \eta)$ is not independent of ε. Denote the cumulative probability distribution of the marginal costs $g(z, \eta)$ given z as $\Phi(g(z, \eta); z)$. Note that, given the monotonicity of $f()$, this also means that the marginal cost $g(z, \eta)$ is stochastically increasing in z (that is, the probability distribution of $g(z_1, \eta)$ first order stochastically dominates the probability distribution of $g(z_2, \eta)$ if and only if $z_1 > z_2$).

Having obtained the distribution of the marginal costs $g(z, \eta)$ given z, we then have immediately the result that, given a distribution of $(\widetilde{\eta}, \varepsilon)$, there is a specification of the full information supply and demand model that represents the same model as the limited information model represented by (1) and (3).[7] We state this result in the following proposition.

Proposition 1. *Consider a non-linear demand and supply system composed of the behavior equations (1)*

[5]The presentation here is based on the complete probability distributions of the error terms. One should be able to obtain similar results when one only considers a set of moment conditions for the probability distributions of the error terms. Note that the same questions about equation (3) can also be raised if one takes a semiparametric approach with respect to the $\widetilde{\eta}$ error (Blundell and Powell, 2004).

[6]For example, if $(\widetilde{\eta}, \varepsilon)$ are distributed jointly normal, then η's probability distribution is a normal distribution.

[7]To simplify notation (and corresponding to what is typically presented in the literature), we present the discussion above with equation (3) linear in z and $\widetilde{\eta}$. Exactly the same result holds if equation (3) is non-linear and monotonic in z and $\widetilde{\eta}$.

and (2), the full information model (with the demand and the full supply model), where the pricing function f is given and monotonic. Then, for every limited information model, (1) and (3), there is a probability distribution over η and a marginal cost function $g(z, \eta)$, such that the specification of the full information model represents the same model as the limited information model.

The implication of this proposition is that, in general, the researcher can focus on a demand equation (e.g., equation (1)) with a limited information model, and not have to worry about the full supply model (the full information of the system). More specifically, the functional form of (3) is not inconsistent with the known properties of the full supply model, because there is a probability distribution over η and a marginal cost function $g(z, \eta)$ that can make it consistent.

We now turn to the idea that a full information model requires more assumptions than a limited information model. One first point to make is that in the typical specification with the full supply model one specifies both the probability distribution of η and the marginal cost functional form $g(z, \eta)$. From above, it is clear that for the full supply model to be completely specified we only need to specify the probability distribution of the marginal cost $g(z, \eta)$ given z.

More interestingly, note that from equation (5), for each pricing function $f()$ there is a probability distribution of the marginal cost $g(z, \eta)$ that satisfies the equality. That is, the limited information model represented by equation (3) can be true for an infinite number of supply models (and infinite-dimensional), represented by the pricing function $f()$. Because of the importance of the result we state it in the following proposition.

Proposition 2. *For every limited information model, equations (1) and (3), there is an infinite number of full supply models (a pricing function f and a probability distribution of marginal costs $g(z, \eta)$ given z in equations (1) and (2)) that represent the same model as the limited information model. For every specification of a full supply model, there is only one limited information model that represents the same model as that full information model.*

This means that in order to obtain consistent estimates of the parameters, the full information model requires more assumptions than the limited information model. A limited information model is more robust to mis-specification than a full information model. Note, however, that arbitrary specifications of a limited and full information model may not represent the same model. This is also immediate from equation (5). If the full supply model specifies a probability distribution for the marginal $g(z, \eta)$ that is different than the one implied by (5) then the limited and full information models represent different models. And it could be that the full information model is true, which means that the limited information model is mis-specified. Note then that this means that although one may interpret the limited information approach with the auxiliary equation as requiring less assumptions than the full information model, this limited information approach requires still more assumptions than the method of moments described above, as that method obtains consistent estimates of the parameters for any specification of the full information model (the function f and the probability distribution of $g(z, \eta)$ given z).

In order to gain further insight into the relationship between the full information model and the limited information model with an auxiliary equation, consider now the question of whether there is a function $g(z, \eta)$ and a probability distribution of η given z that is independent of z, such that equation (5) holds. Such probability distribution of η given z, independent of z, is often considered in full information specifications.

From equation (5), we know that the cumulative probability distribution of η given z is $\Phi(g(z, \eta); z)$. One can then obtain that for this to be independent of z we need

$$\frac{\partial g}{\partial z} = -\frac{\Phi_1}{\Phi_2} \tag{6}$$

where Φ_i represents the partial derivative of Φ with respect to its ith argument. That is, there is a function $g()$ and a probability distribution of η given z, independent of z, such that the limited information model is true. Note that from equation (6), we can see that, in general, the function $g(z, \eta)$ will not be linear in z and η, which is a typical specification in full information models. That is, if the true full information model is the one with the typical specification of a linear $g(z, \eta)$, then the typical specification of the limited information model (the linear equation (3)) is not true. Equation (6), in general, specifies a function $g(z, \eta)$ that is not linear in z and η as, in general, Φ_1 and Φ_2 will also be functions of η.

However, the function $g(z, \eta)$ can be approximated by a Taylor expansion of equation (5) to a linear function as

$$g(z, \eta) =^d \frac{\partial f^{-1}}{\partial z\alpha + \widetilde{\eta}}(z_0\alpha, 0) \cdot ((z - z_0)\alpha + \widetilde{\eta}) + \frac{\partial f^{-1}}{\partial \varepsilon}(z_0\alpha, 0) \cdot \varepsilon. \tag{7}$$

3 An Example

Consider as a simple example the case of demand for J products, each sold by a separate firm, and in which each firm sets the price according to the Nash equilibrium of the market. In order to make the presentation clearer, denote the endogenous variable x as P for price (vector of prices). Suppose that the demand for the product j takes the form

$$y_j = h(P, \varepsilon) = \int_{\beta_i} \frac{e^{\beta_{ij0} + \beta_{i1} P_j + \varepsilon_j}}{1 + \sum_{j'=1}^{J} e^{\beta_{ij'0} + \beta_{i1} P_{j'} + \varepsilon_{j'}}} dF(\beta_i) \tag{8}$$

where β_i is a vector with elements, $\beta_{i10}, \beta_{i20,}, ..., \beta_{iJ0}, \beta_{i1}$, which can be seen as specific for each individual. The researcher observes the vector of prices P and demands y in each period, but does not observe the vector of demand shocks per period ε. Denote the marginal costs of production for firm j as $g_j(z,\eta)$, where z represents some exogenous observable input prices, and η are some shocks that are not observed by the researcher.

In the full information model, each firm is maximizing her profit with respect to price after observing the shocks ε and η. The first order condition of such maximization for product j results in

$$y_j(P,\epsilon,\beta) + [P_j - g(z_j,\eta_j)]\frac{\partial y_j}{\partial P_j}(P,\epsilon,\beta) = 0 \tag{9}$$

which is the implicit version of equation (2), and defines the price P as a function of vectors of z, ε, and η.

Consider now the limited information equation corresponding to equation (3) as

$$P = \alpha_0 + \alpha_1 z + \widetilde{\eta}. \tag{10}$$

The question in the previous section was whether, given a joint probability distribution for $(\varepsilon, \widetilde{\eta})$, is there a function $g(z,\eta)$, such that there is a probability distribution over $g(z,\eta)$ given z, such that equation (10) is true given equation (9). In order to see this, we can use equations (9) and (10) to obtain equation (5) in this example as

$$g(z,\eta) =^d \alpha_0 + \alpha_1 z + \widetilde{\eta} + \Delta(P,\epsilon,\beta)^{-1}\cdot y(P,\epsilon,\beta) \tag{11}$$

where $\Delta(P,\epsilon,\beta)$ is a J and J diagonoal matrix whose jth element is defined by $\frac{\partial y_j}{\partial P_j}(P,\epsilon,\beta)$. It is then immediate to obtain the distribution of $g(z,\eta)$ given z, given the joint distribution over $(\varepsilon, \widetilde{\eta})$.

In order to get some sense of such distributions, we run the following simulation with 5 firms (i.e., $J = 5$). Suppose $\alpha_0 = (3.8, 3.9, 4.0, 4.1, 4.2)'$ and $\alpha_1 = (0.8, 0.9, 1.0, 1.1, 1.2)'$. We drew 1,000 pairs of $\beta = (\beta_{10}, \beta_{20}, ..., \beta_{50}, \beta_1)$ from normal distributions where β_0 is each distributed normal with a zero mean and variance of 0.1, whereas β_1 is distributed normal with a mean of -0.3 and variance of 0.01. We also generated 1,000 pairs of $(\varepsilon, \widetilde{\eta})$ from the following joint normal distribution for the values of $z = 5, 5.2, 5.4, 5.6, 5.8$.

$$
\begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_5 \\ \tilde{\eta}_1 \\ \vdots \\ \tilde{\eta}_5 \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \begin{pmatrix} 0.18 & & 0 \\ & \ddots & \\ 0 & & 0.22 \end{pmatrix} & \begin{pmatrix} 0.1 & & 0.05 \\ & \ddots & \\ 0.05 & & 0.1 \end{pmatrix} \\ \begin{pmatrix} 0.1 & & 0.05 \\ & \ddots & \\ 0.05 & & 0.1 \end{pmatrix} & \begin{pmatrix} 0.18 & & 0.07 \\ & \ddots & \\ 0.07 & & 0.22 \end{pmatrix} \end{pmatrix} \right)
$$

The distribution of $g(z, \eta)$ is presented in Figures 1-5 for each z. Other parameter values were tried and led to similar distributions.

The linear regression of the price P on z yielded an $R^2 = .95$. The mean demand across all observations was .06 with a standard deviation of .02, a minimum of .01, and a maximum of .20. In order to check the approximation (7) for this example, we also run the linear regression of $g(z, \eta)$ on z, which yielded a $R^2 = .84$.[8] In order to have a sense whether the additive separable $g(z, \eta)$ under the linear model yielded a distribution for η that was "approximately" independent of z, we run Kolmogorov-Smirnov tests comparing the distributions of the residuals of the regression of $g(z, \eta)$ on z for each z. The results of these tests are presented in Table 1. It can be seen that the tests generally do not reject the hypothesis that the distributions of the residuals in the linear model across z are the same. However, this is just a small simulation for some parameter values, and these tests of the additive separable approximation (7) could lead to different results for other parameter values or other functional forms.

4 Conclusion

This paper presents some results on the relationship of demand estimation with and without full supply information in non-linear models. Given general monotonicity conditions, any limited information supply model with an auxiliary equation can be shown to be true with a specification of the full information supply and demand model for non-linear models. Furthermore, it can be argued that a limited information model makes less assumptions for estimation than a full information model, as for any limited information model there is an infinite number (infinite-dimensional) of full information models for which the limited information model is true. However, the typical applications of the limited information model (additive separable error term) and full information model (additive separable error, in which the distribution is independent of the exogenous variables) cannot both be true in general in a non-linear model. An example of static competition

[8] The parameters on z were estimated to be between 0.841 and 1.241 with standard errors of approximately.02. The constant terms were estimated to be between -0.66 and -0.08 with standard errors of .19.

of multiple products was presented, where the complete specification of the full information model was recovered given a limited information specification.

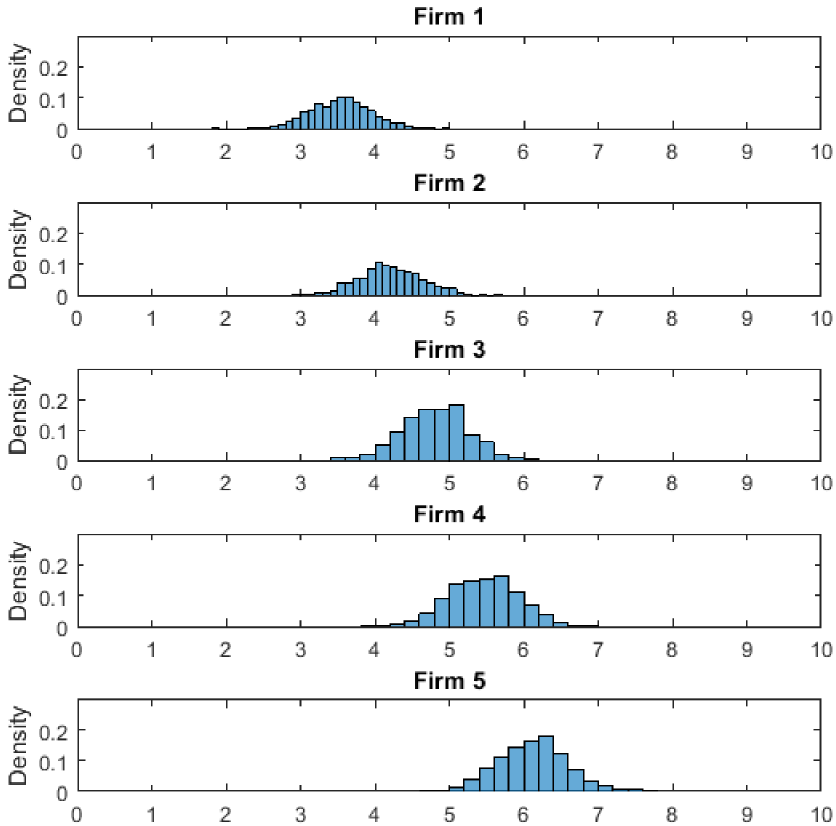

Figure 1: Density of g(z, η) for z = 5 in simulation

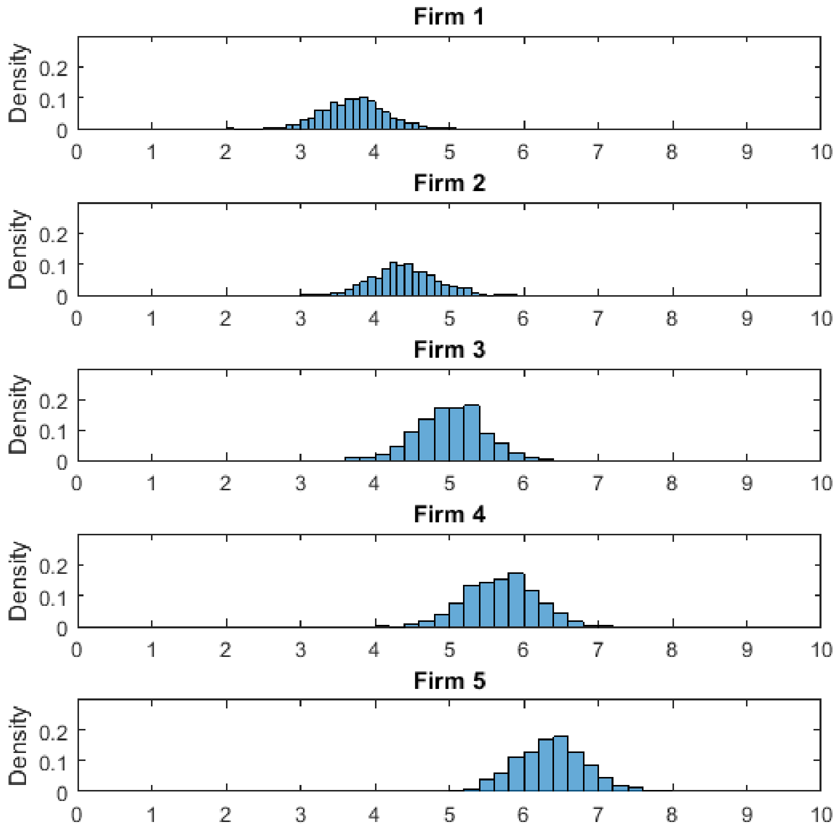

Figure 2: Density of g(z, η) for z = 5.2 in simulation

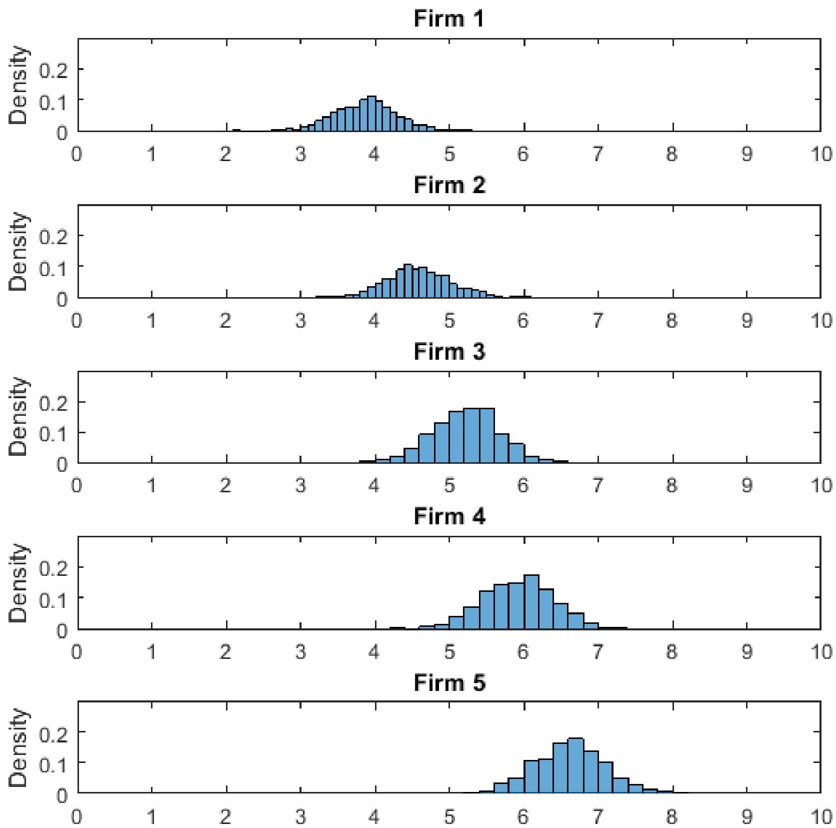

Figure 3: Density of g(z, η) for z = 5.4 in simulation

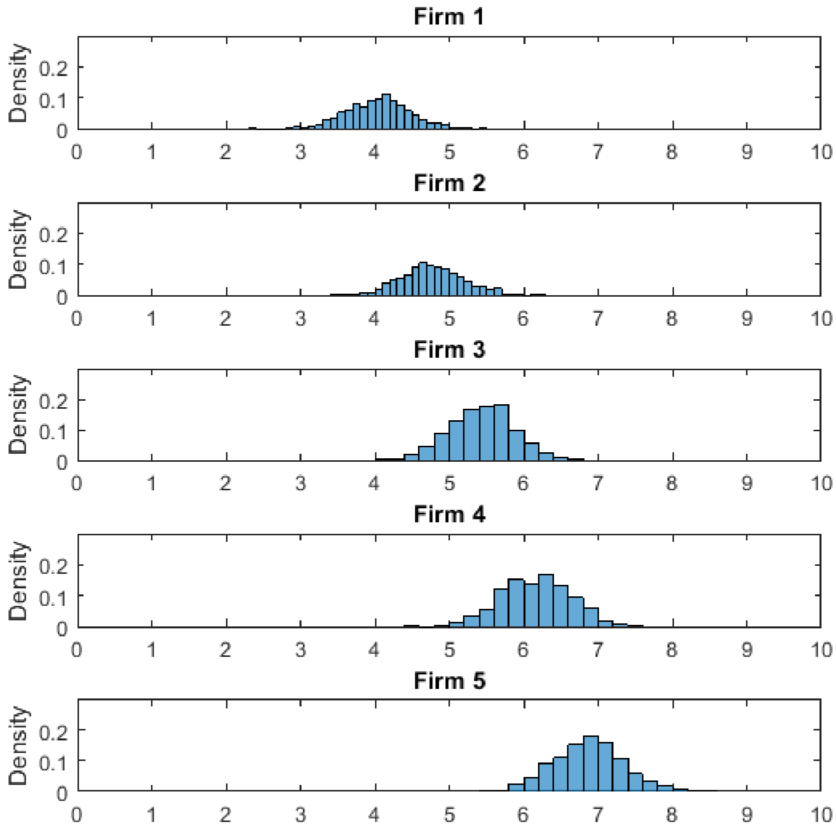

Figure 4: Density of g(z, η) for z = 5.6 in simulation

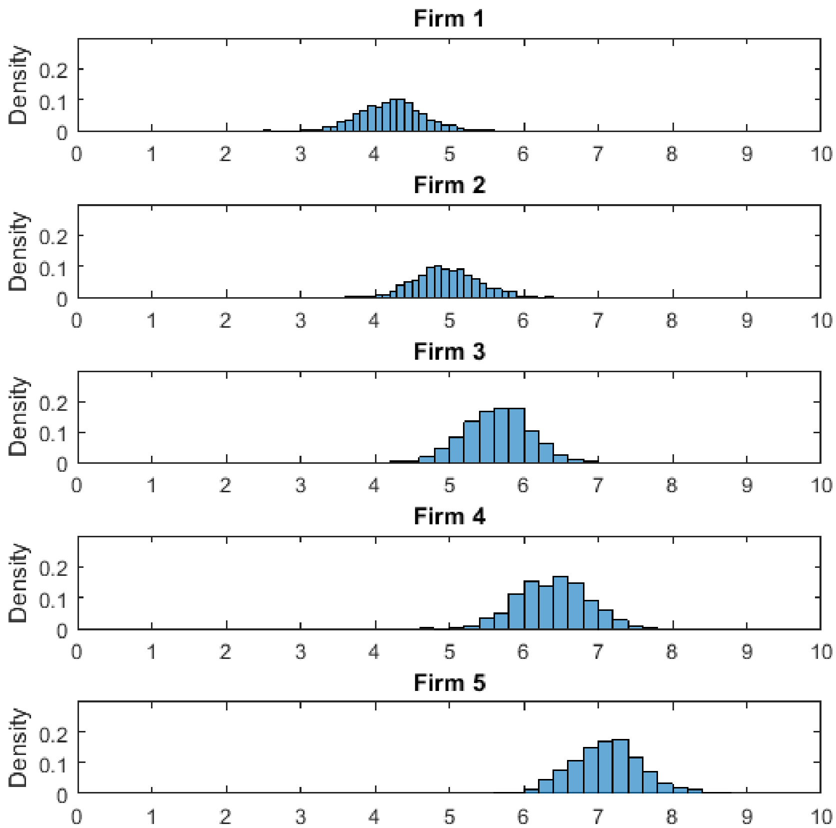

Figure 5: Density of g(z, η) for z = 5.8 in simulation

Table 1: Kolmogorov-Smirnov Tests of Residuals of Linear Model across z

Firm 1

z	5	5.2	5.4	5.6	5.8
5		0.080 (0.527)	0.055 (0.915)	0.070 (0.696)	0.090 (0.377)
5.2			0.070 (0.696)	0.060 (0.854)	0.050 (0.960)
5.4				0.055 (0.915)	0.060 (0.854)
5.6					0.070 (0.696)
5.8					

Firm 2

z	5	5.2	5.4	5.6	5.8
5		0.060 (0.854)	0.110 (0.167)	0.060 (0.854)	0.095 (0.312)
5.2			0.075 (0.611)	0.095 (0.312)	0.065 (0.779)
5.4				0.110 (0.167)	0.090 (0.377)
5.6					0.120 (0.104)
5.8					

Firm 3

z	5	5.2	5.4	5.6	5.8
5		0.095 (0.312)	0.095 (0.312)	0.095 (0.312)	0.080 (0.527)
5.2			0.045 (0.985)	0.055 (0.915)	0.070 (0.696)
5.4				0.055 (0.915)	0.060 (0.854)
5.6					0.060 (0.854)
5.8					

Firm 4

z	5	5.2	5.4	5.6	5.8
5		0.060 (0.854)	0.085 (0.449)	0.085 (0.449)	0.100 (0.256)
5.2			0.095 (0.312)	0.070 (0.696)	0.105 (0.208)
5.4				0.135* (0.047)	0.085 (0.449)
5.6					0.140* (0.036)
5.8					

Firm 5

z	5	5.2	5.4	5.6	5.8
5		0.100 (0.256)	0.080 (0.527)	0.065 (0.779)	0.075 (0.611)
5.2			0.060 (0.854)	0.065 (0.779)	0.090 (0.377)
5.4				0.050 (0.960)	0.040 (0.997)
5.6					0.065 (0.779)
5.8					

The Table presents the value of the Kolmogorov-Smirnov statistic D for each pair of distributions of residuals determined by a pair of z's. The p-value is presented in parenthesis.

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