Browse or Experience*

Z. Eddie Ning
(Cheung Kong Graduate School of Business)

J. Miguel Villas-Boas
(University of California, Berkeley)

June, 2021

*Comments welcome. E-mail addresses: zhaoning@ckgsb.edu.cn and villas@haas.berkeley.edu.

Browse or Experience

Abstract

Consumers gain information about the evolving value of a product both prior to purchase and when owning a product. We consider a model where both these types of gaining information are possible. The information gained when owning the product may affect future product purchases. We characterize when the consumer chooses to purchase the product if the consumer does not own it, the expected interval of time between purchases, and the expected number of product purchases over time. We find that, keeping product duration fixed, the optimal fixed price is independent of the initial product valuation if that valuation is sufficiently low such that a consumer not owning the product does not purchase it immediately, and characterize how the price charged affects the consumer information gathering strategy. When the firm can also choose product duration and there are no costs of production, we find that the firm chooses an expected production duration that is infinitely small, and charges a flow price for the consumer to use the product. We also characterize how the extent of learning when owning and when not owning the product, the duration of the product, and the discount rate affect the optimal consumer and firm strategies.

1. Introduction

Consumers can gain information about the evolving value fit of a product, by both checking information about the product prior to owning it, or by experiencing the product when owning the product. In Nelson (1970)'s classification, for some products all information can be obtained prior to purchase, search goods, while for other products information is only obtained when using the product, experience goods. In fact, for most products, consumers can gain information on their value, both prior to and after purchase.

For example, when buying a car, a consumer can gain information on it (including test driving) prior to purchase, but will continue to gain information after purchase, when using it. Similarly, a consumer can gain information on a media subscription service when using its app, but can similarly gain information prior to subscribing by checking its catalogue, or reading reviews of the app. Such flow of information both before and after purchase causes a consumer's preferences for the product or service to evolve over time. There is also new information over time due to the changing environment or product features. For example, a consumer's desire to own an SUV changes as her commute condition or gas price changes. A consumer's evaluation of a media subscription service changes as her taste changes or as the content catalogue changes. Thus a consumer's expected utility from owning the product can continue to update even after multiple prior purchases.

We consider a model where consumers can gain information on a product both prior to and after purchase. The product is a durable good that lasts for some uncertain time.[1] If the expected value of the product is sufficiently low and the consumer does not own the product, the consumer may prefer to choose gaining further information about the product, until the information is sufficiently positive such that it is worth purchasing the product. But after purchasing the product the consumer will continue gaining information about its value, and the consumer will have to make a decision about whether to repurchase when the product breaks down. If the information received when owning the product is predominantly negative, the consumer will hold off on repurchasing the product for some time, and will only repurchase it if the consumer receives sufficiently positive information while not owning the product to repurchase the product again. If the information received when owning the product is predominantly positive, the consumer will repurchase the product immediately after the product breaks down.

[1] Note that it could be a service that the consumer does not receive all benefit of visiting immediately after consumption, and there is some uncertain time when the service may again be potentially needed.

We find that, keeping product duration fixed, the optimal constant price for the firm to charge if the expected initial valuation is sufficiently low is independent of the consumer's initial expected valuation. Choosing the price to charge determines the optimal information gathering by consumers, and the firm has to take into account the potential future revenue of the expected future repurchases by the consumer, therefore not lowering the price beyond a certain level, which makes the optimal price not varying with the expected initial valuation if that is sufficiently low.

When allowing for the firm to choose product duration, we find that the firm would prefer an infinitely short product duration under zero marginal costs. Thus with evolving preferences, the firm prefers renting over selling a durable product, even though there is no issue of price commitment (as in, e.g., Coase 1972 and Bulow 1982). By offering a shorter product duration, the firm can better extract the consumer's option value of delaying purchase to obtain information on the product.

In order to obtain these results, we fully characterize the optimal strategy for the consumer of when to purchase the product given its expected value going forward. The model characterizes rich dynamics where a customer may spend some time gaining information before purchasing the product, and after a product is purchased and then breaks down after some time, the consumer can wait a certain period of time to purchase it again, or repurchase it immediately. The model captures also the possibility that a consumer continues to derive value from using a product while owning it, even though, if the product were to break down, the consumer would not immediately repurchase the product. We characterize the certainty equivalent time until the next purchase, and the expected number of purchases by a consumer.

We find that a consumer delays purchase if the product's price is higher, the discount rate is higher, or the expected duration of the product is lower. If the price is greater, the discount rate is greater, or the expected duration of the product is lower, then the present value for the consumer of purchasing the product is lower, and the consumer delays the purchase until the expected valuation of the product is greater.

Potentially more interestingly, the consumer delays purchase when the information gained without owning the product is greater, and anticipates the purchase of the product when the information gained while owning/using the product is greater. That is, if more information can be gained by checking the product prior to purchase, the consumer delays purchase until the consumer finds sufficiently good news. If more information can be gained while owning the product, the consumer anticipates the purchase to gain further information on the product. We find that the effect of the greater information obtained prior to purchase dominates the effect of the greater information obtained after purchase, such that when the extent of information gained increases equally prior to and after purchase, the consumer chooses to delay purchase. The information gained prior to purchase allows the consumer to make better decisions immediately in the next purchase occasion. The information gained after purchase allows the consumer to make better decisions only in future purchase decisions, and therefore the former effect dominates.

We find that, after the consumer just made an initial purchase, the consumer makes more frequent repurchases when the ratio of the information gained while owning the product to the information gained without owning the product is smaller. That is, an increase in the information gained without owning the product delays the initial purchase, but leads to more repurchases after an initial purchase is made. On the other hand, an increase in the information gained while owning the product leads to a faster initial purchase, but less frequent repurchases afterward. The consumer is more cautious to buy a search good initially due to the option value of waiting but is quicker to buy it again, whereas the consumer is more eager to buy an experience good initially for its experimentation value but less likely to repurchase it afterward.

When we consider the optimal price charged by the firm, we can then obtain that, in equilibrium, the price and the extent to which the consumers delays purchase are decreasing in the discount rate, and increasing in the expected duration of the product and in the amount of information gained prior to and after purchase. Thus the effect of price change dominates the effects of changes in the discount rate or product duration. A greater expected duration of the product makes the firm increase so much the price, that consumers choose to delay purchase until they receive sufficient good news. That is, for products of greater duration, consumers wait to receive further information. Also interestingly, when the information gained prior to and after purchased increase, the firm chooses to increase its price, as now there is a possibility that the consumer gets more positive news about the product.

When the initial expected valuation by the consumer is high enough, the firm prices the product such that the consumer purchases the product immediately, and in this situation the optimal price is increasing in the initial expected valuation of the product.

When the firm is also allowed to decide on the product duration under optimal pricing, given the result on the optimal infinitely small product duration, we find that the optimal flow price is increasing in the amount of information gained (when the information gained is similar when owning or not owning the product) and decreasing in the discount rate.

The paper presents important managerial implications for firms in settings in which consumers learn information both prior to and after purchase. The paper presents implications both for pricing and product duration, as described above, while at the same time providing measures of the number of expected sales to a consumer over time.

This paper is related to the existing literature on gaining information prior to choosing an alternative (e.g., Roberts and Weitzman 1981, Moscarini and Smith 2001, Branco, Sun, and Villas-Boas 2012, Fudenberg, Strack, and Strzalecki 2018) with the difference that information continues to be gained after the choice is made, which may become useful in future choices.[2] In relation to that literature, the possibility of gaining information after purchase that can become relevant for future purchases makes the consumer anticipate the purchase, and the decision of when to purchase depends on what can be learned after purchase. The set-up presented here allows us also to consider the question of the expected number of purchases by a consumer, while the literature looking at information prior to purchase considered only the possibility of either one or zero purchases. Erdem and Keane (1996) considers empirically a setting in which consumers gain information both when owning and not owning a product, and consumers make a choice in every period. In relation to that paper, here we concentrate on the effects of owning or not owning a product, and when to make repurchases, consider optimal pricing and optimal product duration, and obtain general and sharp results on the consumers' and firm's strategic decisions.

There is also a literature focusing on information gained only while using the product (e.g., Bergemann and Välimäki 1996).[3] In relation to that literature this paper considers that the consumer can gain information prior to purchase, which delays purchase. There is also the possibility, not considered here, of learning about the quality of products by observing the actions of others, and potentially save on search costs (e.g., Tucker and Zhang 2011, Hendricks, Sorensen, and Wiseman 2012).

The remainder of the paper is organized as follows. The next Section presents the model, and Section 3 considers the optimal consumer behavior. Section 4 studies the expected length of time between purchases and the expected number of purchases. Section 5 presents the optimal pricing by the firm, the market equilibrium, and considers the case in which the firm can choose the product duration. Section 6 concludes.

[2]See also Ke, Shen, and Villas-Boas (2016), Che and Mierendorf (2019), and Ke and Villas-Boas (2019) for similar learning prior to choosing one alternative when there are more than two alternatives.

[3]See also Felli and Harris (1996) for a similar set-up in a labor market setting.

2. The Model

Consider the setting of a risk-neutral consumer who can purchase a product of varying value that lasts for some uncertain period, and then has to decide whether to repurchase it.

Let x be the current expected flow utility generated if the consumer owns the product. With additional information about the product, the current expected flow utility evolves over time. Let x_t be the current expected flow utility of owning the product at time t. Given the definition of additional information gained after time t, we know that x_t is a martingale, $E(x_{t+\Delta}|x_t) = x_t$, for any $\Delta > 0$. This also implies that the increments in x_t over time are uncorrelated.

We consider that the additional information arrives continuously over time to the consumers. This can also be seen as the limit when information arrives in discrete time periods and the length of the time period goes to zero. This allows us to obtain sharper results. In particular, we assume that x evolves continuously over time as a Brownian motion with zero drift, with potentially varying variance depending on whether the consumer currently owns the product. The flow utility of not owning the product is set at zero.[4]

The consumer could be an owner of the product in which case she can potentially gain some information over time about the product value. Also the consumer may not own the product, but may be learning the value of the product. We interpret the evolution of x as learning on a number of attributes for a certain period of time. Alternatively, we can also interpret the evolution of x as learning about changes in preferences if preferences evolve over time. We discuss possible specifics of what consumers learn in the Appendix and show cases where the information gained has constant variances when owning and not owning the product.

Let σ^2 be the variance of the Brownian motion when the consumer does not own the product, but is learning information about the product, and let s^2 be the variance of the Brownian motion when the consumer owns the product. We may expect that the consumer learns more attributes when owning the product than when not owning the product, which means that $s^2 \geq \sigma^2$. We could also think that some random attributes that were discovered in the past stop mattering as time goes by. In many cases the information gained during learning can have a decreasing variance over time, which can be seen as an intermediate situation between the case considered here and a situation where the consumer learns everything at

[4]For a similar framework, see, for example, Roberts and Weitzman (1981), Moscarini and Smith (2001), Branco, Sun, and Villas-Boas (2012), Fudenberg, Strack, and Strzalecki (2018).

the first encounter with the product.[5] In this way, the case considered here can be seen as the extreme case in which the importance of information gained is constant. Alternatively we could think of a situation of evolving preferences, where preferences may evolve at different speed when the consumer owns/uses the product than when the consumer does not own the product. Note also that the set-up presented allows both for learning prior to and after purchase, and does not depend on the attributes that can be learned through product experience after purchase to be the same as the attributes which can be learned prior to purchase. That is, the set-up is consistent with consumers being able to learn some attributes before purchase, and only being able to learn some other attributes after purchase. We consider the general case of $s^2 \neq \sigma^2$, but one interesting benchmark is the case of $s^2 = \sigma^2$, such the preferences evolve in the same way whether the consumer owns or does not own the product.

When the consumer owns the product we allow for the possibility of the consumer not using the product. That is, when the current utility x is negative, the consumer chooses not to use the product, as using it is detrimental with respect to not using it and getting an instant utility of zero. We assume that the consumer, while owning the product, learns at the same rate s^2 whether using or not using the product. One may consider that the extent of learning when owning but not using the product is smaller than the extent of learning when using the product. However, note that the extent of learning while owning but not using the product may be significantly greater than when not owning the product. That is, the extent of learning while owning but not using the product may be closer to the extent of learning while using the product than to the extent of learning when not owning the product. In order to simplify the analysis and not to add an additional parameter on the extent of learning while owning but not using the product, we set that extent of learning to be the same as when using the product, s^2. Considering the extent of learning while owning but not using the product as an additional parameter can be done in a relatively straightforward way in the analysis that follows.

Let λ be the hazard rate at which the product breaks down, and let P be the price of purchasing the product. When the product breaks down the consumer can decide to either not repurchase the product immediately and learn about the product with informativeness σ^2, or repurchase the product immediately and continue to learn about the product with informativeness s^2.

[5]The case in which the consumer learns everything at the first encounter with the product, is the typical search costs model, e.g., Weitzman (1979).

Consumers and the firm discount the future at the continuous discount rate r. Note that at state x the expected value of owning the product going forward is $\frac{x}{r+\lambda}$. In Section 4, we consider the possibility that r is composed by an actual discount rate $\widetilde{r}$ and a hazard rate β of the consumer exiting the market, with $r = \widetilde{r} + \beta$. This distinction between $\widetilde{r}$ and β is only relevant in Section 4 when we study the expected number of purchases in the lifetime of a consumer in this market. In the remainder of the paper, the only relevant construct is $\widetilde{r} + \beta$ which we define as r.

The optimal strategy of the consumer is going to be characterized by an $\overline{x}$ such that if the consumer does not own the product the consumer chooses to buy the product if $x \geq \overline{x}$.

We consider that the consumer does not have any costs of obtaining information, and that information comes to the consumer freely, even when not owning the product. This can be seen as the situation of the consumer getting information from friends or from media without acting to get that information. This assumption allows us to fully characterize the optimal strategy of the consumer by a unique threshold $\overline{x}$ for the expected current utility x at which the consumer decides to purchase the product if the consumer does not own the product yet, which simplifies the analysis. Were the consumer to also have costs (and the decision) of whether to keep learning information, the optimal strategy of the consumer would then need to be characterized with an additional threshold, below which the consumer would decide not to gather information on the product (see Branco, Sun, and Villas-Boas 2012).

We can see the discount rate r as playing the role of the costs of learning information (information-processing costs). The discount rate makes the consumer willing to purchase immediately if the product provides a sufficiently high value, as delaying purchase just delays the benefits of owning the product. Note, however, that a greater discount rate makes also the present value of owning the product lower, which may make the consumer more demanding on the value at which to purchase the product.

3. Optimal Consumer Behavior

Consider now the optimal consumer behavior of when to purchase or delay purchase. Let $W(x)$ be the expected present value of payoffs for the consumer if the consumer does not own the product and is getting information on the product, $x < \overline{x}$. Let $V(x)$ be the expected present value of payoffs for the consumer if the consumer owns the product and $x \geq \overline{x}$. Let $\widetilde{V}(x)$ be the expected value of payoffs for the consumer if the consumer owns the product and $x < \overline{x}$. We focus the initial presentation on the case in which at the optimum $\overline{x} > 0$, which will occur if $s^2 = \sigma^2$ or the product's price is high enough. We later consider also the situation in which the optimum has $\overline{x} < 0$.

When the consumer does not own the product and is searching for information we can obtain that the evolution of $W(x)$ is characterized by

$$W(x) = e^{-r\,dt} E[W(x + dx)]. \tag{1}$$

Using Itô's Lemma, we can get $rW(x) = W''(x)\frac{\sigma^2}{2}$. Note also that $\lim_{x\to-\infty} W(x) = 0$, as the present value of benefits has to approach zero when the current utility derived of potentially using the product goes to negative infinity. We can then obtain the solution for $W(x)$, presented in (x) in the Appendix.

When the consumer owns the product and $x \geq \overline{x}$, we have that the expected present value of consumer payoffs has to satisfy

$$V(x) = x\,dt + e^{-r\,dt}\lambda\,dt\{E[V(x + dx)] - P\} + e^{-r\,dt}(1 - \lambda\,dt)E[V(x + dx)], \tag{2}$$

where the first term represents the expected flow utility of owning the product, the second term represents the possibility of the product breaking down, which occurs with probability $\lambda\,dt$, in which case the consumer buys the product again immediately with an expected net benefit of $E[V(x + dx)] - P$, and the third term represents the possibility of the product not breaking down, in which case the consumer gets the expected present value of consumer payoffs if owning the product after the evolution in x, $E[V(x + dx)]$. Using Itô's Lemma, (2) reduces to $rV(x) = x - \lambda P + V''(x)\frac{s^2}{2}$. Note that $\lim_{x\to\infty}[V(x) - (x - \lambda P)/r] = 0$, as when the current utility goes to infinity, the consumer is always buying the product when it breaks down, which generates an expected utility of $(x - \lambda P)/r$. Using this when solving the differential equation on $V(x)$, one obtains $V(x)$ as a function of one constant to be determined, presented in (xi) in the Appendix.

Consider now that the consumer owns the product and $x < \overline{x}$. We consider that the consumer can choose not to use the product if $x < 0$, so we will further divide this region into $(0, \overline{x})$ and $(-\infty, 0]$. Consider first the case of $x \in (0, \overline{x})$. In this region we have

$$\widetilde{V}(x) = x\,dt + e^{-r\,dt}\lambda\,dtW(x) + e^{-r\,dt}(1 - \lambda\,dt)E\widetilde{V}(x + dx), \tag{3}$$

where the first term represents the expected flow utility of owning the product, the second term represents the possibility of the product breaking down, which occurs with probability $\lambda\,dt$, in which case the consumer gets the expected present value of consumer payoffs if not owning the product, $W(x)$, and the third term represents the possibility of the product not breaking down, in which case the consumer gets the expected present value of consumer payoffs if owning the product after the evolution in x, $E[\widetilde{V}(x+dx)]$. Using Itô's Lemma, and solving the resulting differential equation, one obtains the solution for $\widetilde{V}(x)$, presented in (xii) in the Appendix.

For the case of $x \leq 0$ we can similarly obtain

$$\widetilde{V}(x) = e^{-r\,dt}\lambda\,dtW(x) + e^{-r\,dt}(1-\lambda\,dt)E\widetilde{V}(x+dx). \tag{4}$$

Note also that $\lim_{x\to-\infty}\widetilde{V}(x)=0$, as the expected utility when owning the product goes to zero when the current utility of using the product approaches negative infinity. Using this, when solving for (4), we can obtain $\widetilde{V}(x)$, presented in (xiii) in the Appendix.

Value matching and smooth pasting at both $\overline{x}$ and 0, $W(\overline{x}) = V(\overline{x}) - P, W'(\overline{x}) = V'(\overline{x}), V(\overline{x}) = \widetilde{V}(\overline{x}), V'(\overline{x}) = \widetilde{V}'(\overline{x}), \widetilde{V}(0^+) = \widetilde{V}(0^-)$, and $\widetilde{V}'(0^+) = \widetilde{V}'(0^-)$ allow us then to determine the constants of integration, and fully obtain $W(x), V(x)$, and $\widetilde{V}(x)$. Value matching and smooth pasting at $\overline{x}$ and 0 guarantee that $\overline{x}$ is the optimal threshold for the consumer to choose to purchase the product if the consumer does not own the product (e.g., Dixit 1993).

We can then obtain (derivation presented in the Appendix)

$$\widehat{\mu}\overline{x}+e^{-\widehat{\mu}\overline{x}}-1-\widehat{\mu}P(r+\lambda) = (\mu-\widehat{\mu})(r+\lambda)\frac{r(s^2/\sigma^2-1)}{\lambda-r(s^2/\sigma^2-1)}\left[\frac{\widetilde{\mu}}{\mu+\widetilde{\mu}}\frac{\overline{x}-\lambda P}{r}-\frac{\widetilde{\mu}}{\mu+\widetilde{\mu}}P+\frac{1}{r(\mu+\widetilde{\mu})}\right], \tag{5}$$

which determines $\overline{x}$, where $\mu=\sqrt{2r/\sigma^2}, \widetilde{\mu}=\sqrt{2r/s^2}$, and $\widehat{\mu}=\sqrt{2(r+\lambda)/s^2}$.

Note that if $s^2=\sigma^2$, (5) reduces to

$$\widehat{\mu}\overline{x}+e^{-\widehat{\mu}\overline{x}}-1-\widehat{\mu}P(r+\lambda)=0, \tag{6}$$

from which we can obtain that the threshold $\overline{x}$ increases in P, r, λ, and σ^2, under the constraint $s^2=\sigma^2$. Given the focus on learning information about the product both prior to and after purchase, this case can be seen as reasonable to consider. We also discuss below the case in which s^2 is much greater than σ^2, which in the limit is the case of experience goods, just learning information when owning the product.[6] Comparing (6) with (5), we can see that if $\sigma^2 < s^2$ but $(s^2 - \sigma^2)$ is small, we can have that $\overline{x}$ is decreasing in s^2. It is also interesting to observe that when $s^2 = \sigma^2$ the consumer does not purchase immediately when the present value of the current utility is equal to the price. That is, $\frac{\overline{x}}{\lambda+r} > P$. This is because the consumer wants to keep the option of not purchasing the product alive a little longer. The consumer wants to see if the expected current utility is sufficiently large before deciding to make the purchase.

Note that when $s^2 > \sigma^2$ and the price is relatively low, we may have $\overline{x} < 0$. The consumer would buy even though the consumer does not intend to use it immediately, because the consumer can get more information about the product by owning it. The optimal behavior in that case is derived in the Appendix. The consumer buys when $\overline{x}$ reaches

$$e^{\mu\overline{x}} = P\left[\widetilde{\mu}(\lambda + r) + r\widehat{\mu}\frac{\mu + \widetilde{\mu}}{\widehat{\mu} - \mu}\frac{r(1 - s^2/\sigma^2)}{r(1 - s^2/\sigma^2) + \lambda}\right] \tag{7}$$

Note that if s^2 and σ^2 are close, (7) can be approximated by

$$e^{\mu\overline{x}} = P\widetilde{\mu}(\lambda + r) \tag{8}$$

from which we can obtain that the threshold $\overline{x}$ increases in P, r, λ, and σ^2, and decreases in s^2, as in the case of $x > \overline{x}$. We state these results in the following proposition.

Proposition 1. *Suppose that the information gained prior to and after purchase are close to each other, s^2 close to σ^2. Then the threshold to purchase the product, $\overline{x}$, increases in the price charged, P, in the discount rate, r, in the hazard rate of the product breaking down, λ, in the amount of information gained without owning the product, σ^2, both when s^2 stays fixed and when $s^2 = \sigma^2$, and decreases in the amount of information gained while owning the product, s^2.*

When the present value of payoffs to the consumer of acquiring the product declines, which can occur when there is an increase in either the price or the discount rate, or a decrease in the expected duration of the product (greater λ), the consumer is more demanding on the expected current utility of using the product before deciding to purchase it (a greater $\overline{x}$). The effect of the discount rate on the purchase threshold is interesting. On the one hand a

[6] Also as discussed below, one can see the case of $s^2 = 0$ as the case of search goods as in that case after search, if the consumer purchases the product, the consumer will end up purchasing the product every time that the product breaks down (as x does not change when the consumer owns the product).

greater discount rate makes delaying having the benefits of owning the product more costly, which is a force toward decreasing the purchase threshold. On the other hand, as noted above, a greater discount rate lowers the present value of the benefits of owning the product, which is a force to increase the purchase threshold. The proposition shows that the latter effect dominates the former.

When the amount of information gained without owning the product, σ^2, increases, the consumer chooses to gain further information about the high current utility that the product can deliver, and $\overline{x}$ increases. When the amount information gained when owning the product, s^2, increases, the consumer chooses to anticipate the purchase of the product in order to gain further information for future repurchase, $\overline{x}$ decreases. The former effect dominates, as it relates to the immediate purchase, and when both σ^2 and s^2 increase in the same amount, we have that the consumer delays purchase, $\overline{x}$ increases. If we interpret the model as a consumer with evolving preferences, then the consumer delays purchase when her preferences are more volatile.

As an example of the value of $\overline{x}$ as it relates to the different parameters, for $P = 2, r = .05, \lambda = .2, \sigma^2 = .2$, and $s^2 = .25$ we can obtain $\overline{x} \approx .84$. Figure 1 illustrates a sample path with repeated purchases and product breakdowns given the evolution of preferences for these parameter values. Numerical analysis also suggests that the comparative statistics presented in Proposition 1 for the case when s^2 is close to σ^2 also hold for general s^2 and σ^2.

Extreme Case of Experience Goods

It is also interesting to consider the case in which the information gained when owning the product, s^2, is much greater than the information gained when not owning the product, σ^2. The extreme case is the one in which $\sigma^2 \to 0$, when we are in the experience goods case. In this case if $x < \overline{x}$ and the consumer does not own the product, the consumer never purchases the product. If $x \geq \overline{x}$, and the consumer does not own the product, or the product breaks down, the consumer buys the product immediately.

For the case in which the price P is not too low, we can obtain from (5) that the purchase threshold in the limit is determined by

$$(\widehat{\mu} + \widetilde{\mu}\frac{r+\lambda}{r})[\overline{x} - P(r+\lambda)] + e^{-\widehat{\mu}\overline{x}} + \frac{\lambda}{r} = 0. \tag{9}$$

One can then obtain that $\overline{x}$ increases in P, decreases in s^2, and increases in λ and r when

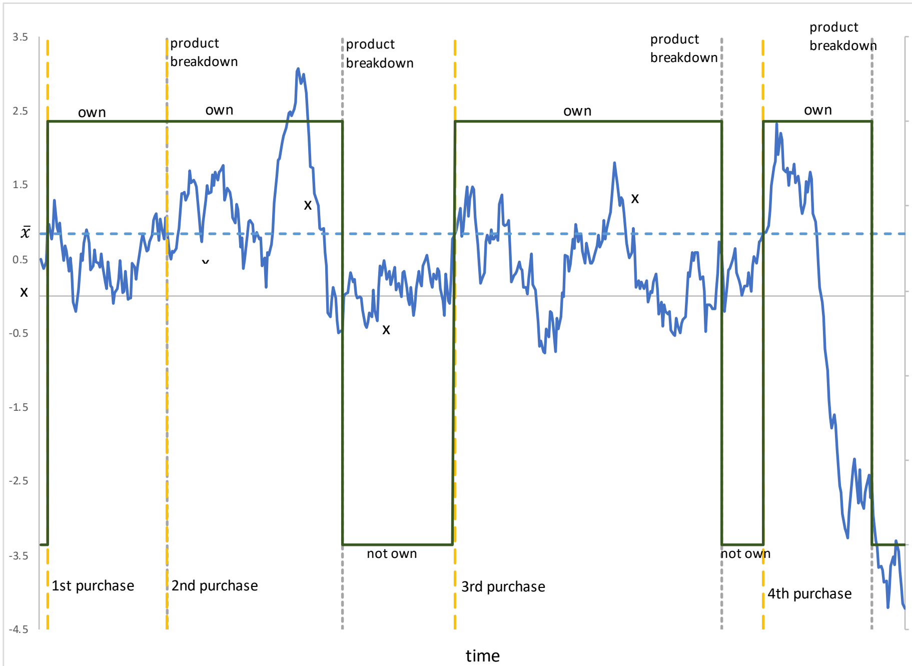

Figure 1: Example of sample path of consumer expected current utility with repeated purchases and product breakdowns for $P = 2$, $r = .05, \lambda = .02, \sigma^2 = .2, s^2 = .25$. For these parameter values we have $\overline{x} \approx .84$.

r is sufficiently large, which is as in Proposition 1. Numerical analysis suggests that these comparative statics also hold for r small.

For the case in which the price P is sufficiently low, we can obtain from (7) that $\overline{x} \to 0$, and, furthermore that

$$\lim_{\sigma^2 \to 0} \frac{\overline{x}}{\sigma} = \frac{1}{\sqrt{2r}} \ln\{P[\widetilde{\mu}(r+\lambda) - \widehat{\mu}r]\}. \tag{10}$$

Again one can confirm that for σ^2 small, $\overline{x}$ increases in P, λ, and r, and decreases in s^2, as in Proposition 1.

Case of Search Goods

Consider now the case in which the consumer gains information prior to purchase, and after owning the product the consumer will buy it again immediately once it breaks down. This case can be seen as the case of search goods, and can be obtained with $\sigma^2 > 0$ and

$s^2 = 0$. In such a case, when x reaches $\overline{x}$, the consumer purchases the product and the product value stops updating. When the product breaks down, the consumer buys again immediately, as $x = \overline{x}$. Thus when the consumer purchases the product, the consumer receives a total discounted utility of $[x-(r+\lambda)P]/r$, where x/r is the present value of utility from using the product, and $(r+\lambda)P/r$ is the present value of current and future prices that the consumer pays.

Value matching and smooth pasting at $\overline{x}$, $W(\overline{x}) = [x-(r+\lambda)P]/r$, and $W'(\overline{x}) = 1/r$, where $W(x)$ is given in (x) in the Appendix, allows us to determine $\overline{x}$. We get

$$\overline{x} = \sqrt{\frac{\sigma^2}{2r}} + (r+\lambda)P \tag{11}$$

from which it is straightforward to check that $\overline{x}$ increases in P, σ^2, r and λ, as in Proposition 1.

4. Expected Time to Next Purchase and Expected Number of Purchases

4.1. Expected Time to Next Purchase

In this Section we consider some properties in terms of timing of purchases and the expected number of purchases given the optimal consumer behavior. Consider the certainty equivalent time until the next purchase associated with the expected discount factor of length of time until the next purchase.[7] That is, if δ is the expected discount factor of the time until the next purchase, then $T = -\frac{1}{r}\ln\delta$ is the certainty equivalent time until the next purchase.

Consider first the case in which the consumer does now own the product. In that case if $x \geq \overline{x}$ the consumer purchases the product immediately and the discount factor of the length of time until the next purchase is one (the certainty equivalent time until the next purchase is zero). Consider now that $x < \overline{x}$. Let $\delta(x)$ be the expected discount factor of the time until the next purchase, and let $T(x)$ be the certainty equivalent time until the next purchase.

We have that

$$\delta(x) = e^{-r\,dt}E\delta(x+dx), \tag{12}$$

[7] Note that if $x < \overline{x}$ the expected length of time until the next purchase is infinity.

from which, using Itô's Lemma, value matching at $\overline{x}$, $\delta(\overline{x}) = 1$, and that the expected discount factor for $x \to -\infty$ approaches zero, $\lim_{x\to-\infty} \delta(x) = 0$, one can obtain

$$\delta(x) = e^{-\mu(\overline{x}-x)}, \tag{13}$$

where we recall $\mu = \sqrt{2r/\sigma^2}$, which yields $T(x) = \mu(\overline{x} - x)/r$.

Consider now the case when the consumer owns the product. Let $\widetilde{\delta}(x)$ be the expected discount factor of the time until the next purchase, and let $\widetilde{T}(x)$ be the certainty equivalent time until the next purchase.

If $x > \overline{x}$ we have that the evolution of $\widetilde{\delta}(x)$ over time has to satisfy

$$\widetilde{\delta}(x) = \lambda\, dt + (1 - \lambda\, dt)e^{-r\,dt} E\widetilde{\delta}(x + dx). \tag{14}$$

Note also that $\lim_{x\to\infty} \widetilde{\delta}(x) = \frac{\lambda}{\lambda+r}$, as when the current utility approaches infinity the product will be almost surely repurchased when it breaks down, and the expected discount factor of duration of the product given that it is functioning is $\frac{\lambda}{\lambda+r}$.

Consider now the case of $x < \overline{x}$. The evolution of $\widetilde{\delta}(x)$ over time has to satisfy

$$\widetilde{\delta}(x) = \lambda\, dt\, \delta(x) + (1 - \lambda\, dt)e^{-rdt} E\widetilde{\delta}(x + dx), \tag{15}$$

Note that as $x \to -\infty$, the expected discount factor until the next purchase approaches zero. Using the value of $\widetilde{\delta}(x)$ as x converges to plus and minus infinity, Itô's Lemma on (14) and (15), and value matching and smooth pasting at $\overline{x}$, $\widetilde{\delta}(\overline{x}^+) = \widetilde{\delta}(\overline{x}^-)$ and $\widetilde{\delta}'(\overline{x}^+) = \widetilde{\delta}'(\overline{x}^-)$, we can then obtain $\widetilde{\delta}(x)$, and therefore $\widetilde{T}(x)$ (see Appendix for the derivation).

When the consumer just purchased the product we can compute the certainty equivalent time until the next purchase as

$$\widetilde{T}(\overline{x}) = -\frac{1}{r} \ln\left[\frac{\lambda}{2(\lambda + r)} + \frac{\lambda(1 - \mu/\widehat{\mu})}{2[\lambda + r(1 - s^2/\sigma^2)]}\right]. \tag{16}$$

We can then obtain the following proposition characterizing the certainty equivalent time between two purchases.

Proposition 2. *After the consumer just made a purchase, the certainty equivalent time to the next purchase is increasing in the expected duration of the product,* $1/\lambda$, *and in the* *amount of information gained while owning the product, s^2, and is decreasing in the amount of information gained when not owning the product, σ^2, and in the discount rate, r.*

As the amount of learning when the consumer owns the product, s^2, is larger, the current utility can have greater negative shocks, leading the consumer not to repurchase the product immediately after the products breaks down. Similarly, as the amount of learning when the consumer does not own the product, σ^2, is larger, the current utility evolves faster while not owning the product, which could potentially lead to the consumer repurchasing the product sooner. Interestingly, note that the effect of s^2 on the certainty equivalent time to repurchase is different than its effect on the certainty equivalent time to the initial purchase. From Proposition 1, we see that a larger s^2 decreases $\overline{x}$, which shortens the certainty equivalent time if $x < \overline{x}$. Similarly, a larger σ^2 increases $\overline{x}$, which is a force toward increasing the time to purchase if the consumer does not own the product.

The effect of the expected duration of the product is also interesting. A greater expected duration of the product makes the certainty equivalent time to the next purchase increase, as the product lasts longer. However, the effect of product duration on the certainty equivalent time to repurchase is different than its effect on the certainty equivalent time to the initial purchase. From Proposition 1, a greater expected duration of the product makes the purchase threshold to be lower, which would be a force toward decreasing the certainty equivalent time to purchase if the consumer does not own the product. A greater discount rate decreases the certainty equivalent time to the next purchase as the discount factor for any time horizon decreases in the discount rate.

For the example considered above of $\lambda = .2, \sigma^2 = .2$, $s^2 = .25$, and $r = .05$, we have that the certainty equivalent time until the next purchase after one purchase is 8.1 units of time, compared to the expected duration of the product of 5 units of time. Figure 2 illustrates the evolution of the certainty equivalent time until the next purchase when not owning, $T(x)$, and when owning the product, $\widetilde{T}(x)$, as a function of the current utility of having the product, x.

As one would expect the certainty equivalent time until the next purchase is decreasing in the current expected utility. As the expected current utility of the product is lower, the consumer is more likely to delay the purchase of the product once it breaks down.

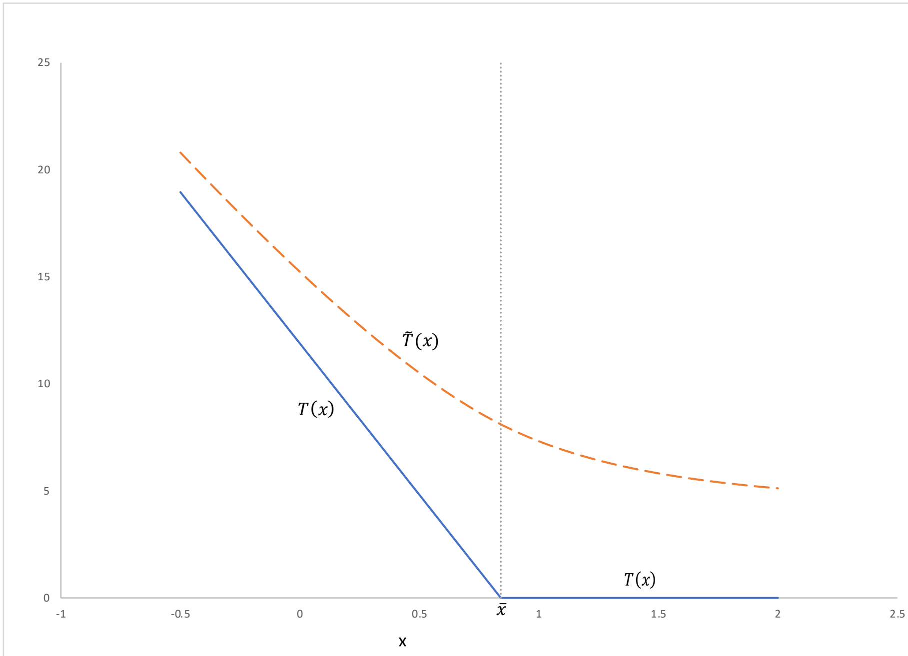

Figure 2: Evolution of the certainty equivalent time until the next purchase when not owning, $T(x)$, and when owning the product, $\widetilde{T}(x)$, as a function of the current utility of having the product, x, for $P = 2$, $r = .05, \sigma^2 = .2, s^2 = .25$ and $\lambda = .2$

4.2. Expected Number of Future Purchases

To construct the expected number of purchases given the optimal consumer behavior, we have to use the hazard rate β of the consumer dropping out of the market. Recall that the discount rate r considered above was composed of both the actual discount rate $\widetilde{r}$ and the hazard rate of the consumer dropping out of the market, $r = \widetilde{r} + \beta$. This hazard rate β of the consumer dropping out of the market captures the idea that consumers end up making a finite number of purchases in a category over their lifetime. In the remainder of the paper, this hazard rate can be folded into an overall discount rate r, but to study the actual number of purchases done by a consumer it has to be explicitly considered. We expect β to be much smaller that λ, the hazard rate of the product breaking down, such that a consumer makes multiple purchases over the consumer's lifetime.

Let $N(x)$ be the expected number of units purchased going forward given that the consumer starts at $x < \overline{x}$ and the consumer does not own the product. We have that $N(x)$

evolves over time as

$$N(x) = (1 - \beta\, dt) E N(x + dx). \tag{17}$$

Let $\widetilde{N}(x)$ be the expected number of future units purchased over time given that the consumer owns the product. As the consumer purchases the product immediately if the consumer does not own the product and $x = \overline{x}$, we have

$$N(\overline{x}) = 1 + \widetilde{N}(\overline{x}). \tag{18}$$

For $x \geq \overline{x}$ the evolution of $\widetilde{N}(x)$ over time has to satisfy

$$\widetilde{N}(x) = \lambda\, dt\, [1 + \widetilde{N}(x)] + (1 - \lambda\, dt - \beta\, dt) E \widetilde{N}(x + dx). \tag{19}$$

Consider now the evolution of $\widetilde{N}(x)$ for $x < \overline{x}$. This yields

$$\widetilde{N}(x) = \lambda\, dt\, N(x) + (1 - \lambda\, dt - \beta\, dt) E \widetilde{N}(x + dx). \tag{20}$$

Applying Itô's Lemma on (17), (19), and (20), solving the corresponding differential equations, and using value matching and smooth pasting at $\overline{x}$ for $\widetilde{N}(x)$, $\widetilde{N}(\overline{x}^+) = \widetilde{N}(\overline{x}^-)$ and $\widetilde{N}'(\overline{x}^+) = \widetilde{N}'(\overline{x}^-)$, we can then obtain the expected number of purchase going forward when $x < \overline{x}$ as (see Appendix for the derivation)

$$N(x) = e^{\eta(x-\overline{x})} \left[1 + \frac{\widehat{\eta} s^2/\sigma^2 - \eta(\lambda+\beta)/\beta}{\widehat{\eta}\frac{\beta(1-s^2/\sigma^2)}{\lambda} + \widetilde{\eta}(\frac{\lambda+\beta(1-s^2/\sigma^2)}{\lambda}) + \eta} + \frac{\lambda}{\beta} \right], \tag{21}$$

where $\eta = \sqrt{2\beta/\sigma^2}$, $\widetilde{\eta} = \sqrt{2\beta/s^2}$, and $\widehat{\eta} = \sqrt{2(\beta+\lambda)/s^2}$.

When $s^2 = \sigma^2$ this expression simplifies to

$$N(x) = \frac{1}{2} e^{\eta(x-\overline{x})} \left[\frac{\beta+\lambda}{\beta} + \sqrt{\frac{\beta+\lambda}{\beta}} \right]. \tag{22}$$

We can then obtain the following result:

Proposition 3. *Suppose that the amount of information gained without owning or while owning the product is not too different, s^2 close to σ^2. Then, the expected number of purchases going forward after the consumer just made a purchase is decreasing in the expected duration of the product, $1/\lambda$, in the hazard rate of the consumer dropping out of the market, β, and* *in the ratio s^2/σ^2. Starting from an initial current utility $x < \overline{x}$, the expected number of purchases going forward is decreasing in the price charged, P, in the actual discount rate, $\widetilde{r}$, and in the hazard rate of the consumer dropping out of the market, β, and is increasing (decreasing) in the amount of information gained under the constraint $s^2 = \sigma^2$ if the initial current utility is low (high) enough. The expected number of purchases at $x < \overline{x}$ is increasing in the expected duration of the product, $1/\lambda$, if the hazard rate of the consumer dropping out of the market is not too low.*

As one would regard as likely, the expected number of purchases going forward is decreasing in the hazard rate of the consumer dropping out of the market and in the price charged. More interestingly, an increase in the actual discount rate $\widetilde{r}$ leads to a lower expected number of purchases as the consumer discounts more the future benefits, and has therefore a lower present value of the benefits of buying the product. This then makes the consumer more demanding on the current expected utility of the product to decide to purchase the product, which results in a lower expected number of purchases going forward.

The effect of the amount of information gained before and after purchase on the expected number of purchases depends on the initial expected current utility, because of two conflicting market forces. On the one hand, a greater amount of information gained allows the current utility to move substantially over time, which is a force for the number of expected number of purchases to increase when the initial current utility is very low. On the other hand, a greater amount of information gained makes the consumer be more demanding in terms of current utility in order to decide to purchase the product (greater $\overline{x}$), and this is a force to reduce the expected number of purchases. The former market force dominates if the initial current utility is very low, while the latter dominates if the initial current utility is not too low.

The effect of the ratio s^2/σ^2 on the expected number of repurchases going forward after a purchase is negative. An increase in the ratio s^2/σ^2 means that the consumer after each purchase has a likelihood of receiving negative information through using the product, which may be difficult to recover from when the product breaks down, which yields a lower number of purchases going forward. This means that there are more repeated purchases when the product is more of a search good than when the product is more of an experience good, after the consumer makes an initial purchase.

The effect of the expected duration of the product has also opposing market forces on the expected number of purchases. On the one hand, after purchase, a longer expected duration of the product leads to a lower number of purchases going forward. On the other hand, initially, a longer duration of the product makes the consumer be more willing to purchase the product and be less demanding on the current utility (lower $\overline{x}$), which will lead to more purchases. Which market force dominates depends on the hazard rate of the consumer dropping out of the market, with an increase in the expected duration of the product leading to more purchases if the hazard rate of the consumer dropping out of the market is not too low.

Figure 3 illustrates how the expected number of purchases going forward when not owning, $N(x)$, and when owning the product, $\widetilde{N}(x)$, evolve as a function of the current utility x. Note that $N(x) = \widetilde{N}(x) + 1$ for $x > \overline{x}$, as in that case the consumer purchases the product immediately. Note that for $x < \overline{x}$ we have $N(x) < \widetilde{N}(x) + 1$, as in that region of x there is uncertainty that the consumer will make another purchase. Figures 4-9 illustrate the results in the proposition on how the expected number of purchases going forward evolve with the price charged, P, the actual discount rate, $\widetilde{r}$, the hazard rate of the consumer dropping out of the market, β, the amount of information gained without owning the product, σ^2, the amount of information gained when owning the product, s^2, and the hazard rate of the product breaking down, λ, respectively.

In particular, Figure 7 illustrates that depending on the value of the current utility x, the effect of the amount of information gained when not owning the product on the expected number of purchases going forward can be positive or negative. As discussed above, for a current utility that is not too low (the case of $x = 0$ in Figure 7), a greater amount of information gained makes the consumer be more demanding on the expected valuation needed to trigger a purchase, which decreases the expected number of purchases. This effect is reduced by the effect that immediately after purchase the expected number of purchases going forward decreases in the ratio s^2/σ^2. For a current utility that is relatively low (the case $x = -2$ in Figure 7) a greater amount of information gained makes the consumer more likely to reach the purchase threshold, and in that case the expected number of purchases going forward increases.

Figure 8 illustrates that an increase in the amount of information gained when owning the product increases the expected number of purchases going forward, but as argued above for the purchase threshold, this effect is smaller than the effect of the information gained when not owning the product, and therefore we obtain the result in the proposition that when the amounts of information while owning or not owning the product are the same, and there is an increase in the same amount on both types of information, the effect of σ^2

dominates, and we have the pattern of Figure 7.

Figure 9 illustrates how the effect of the expected duration of the product on the expected number of purchases going forward can be positive or negative, with it being positive if the hazard rate of the consumer dropping out of the market is not too low.

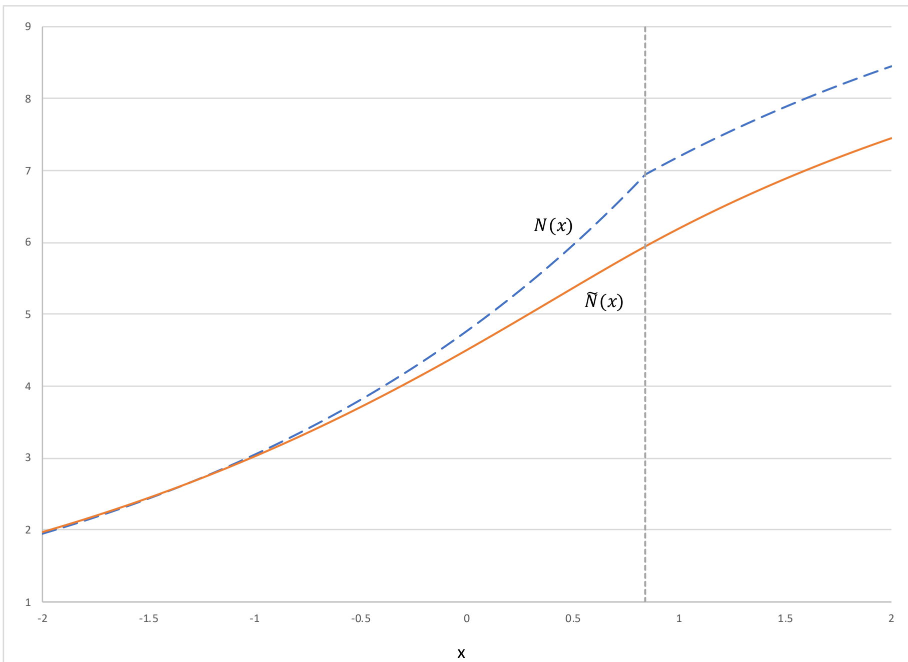

Figure 3: Evolution of the expected number of purchases going forward when not owning ($N(x)$), and when owning the product ($\widetilde{N}(x)$) as a function of the current utility of having the product, x, for $P = 2$, $r = .05, \beta = .02, \sigma^2 = .2, s^2 = .25$ and $\lambda = .2$.

5. Optimal Pricing and Market Equilibrium

Consider now the optimal pricing of a firm, assuming that the price chosen is fixed over time, and the optimal price is chosen to maximize the present value of expected profits over time given that the consumers start with an expected current utility x_0. To simplify the presentation we consider that when the firm makes the pricing decision all consumers have the same starting expected current utility x_0. Alternatively, the firm can have a prior distribution over x_0 and do optimal pricing given that prior distribution over x_0, with effects

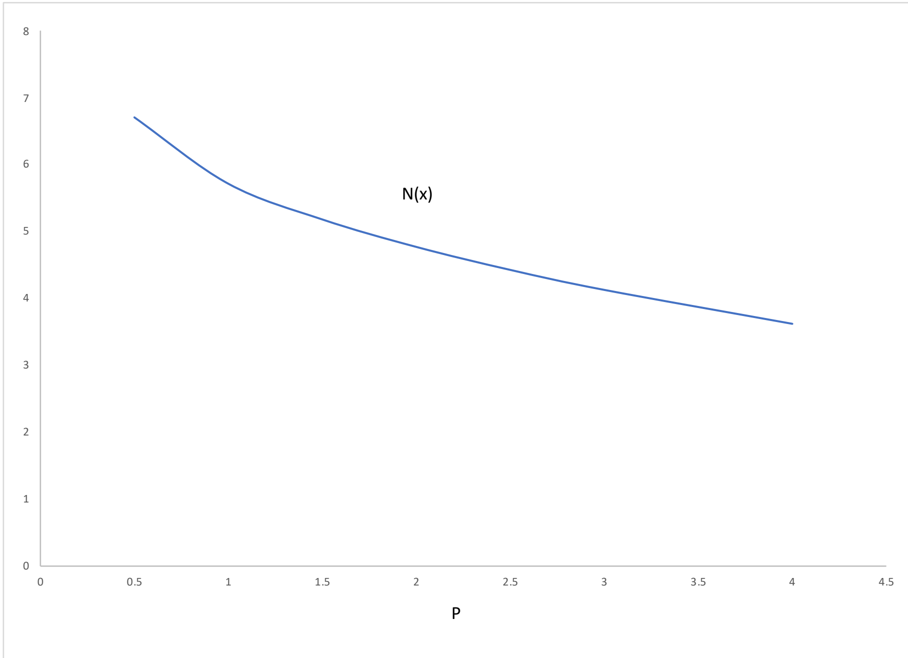

Figure 4: Evolution of the expected number of purchases going forward when not owning the product, $N(x)$, as a function of the price charged P for $x = 0$, $\widetilde{r} = .03, \beta = .02, \sigma^2 = .2, s^2 = .25$ and $\lambda = .2$.

that are similar to the ones considered here.[8]

To construct the present value of profits, let $G(x)$ be the discounted number of units purchased over time given that the consumer starts at $x < \overline{x}$ and the consumer does not own the product, and $\widetilde{G}(x)$ be the discounted number of units purchased over time given that the consumer owns the product. The construction of $G(x)$ and $\widetilde{G}(x)$ is similar to the construction of $N(x)$ and $\widetilde{N}(x)$ in subsection 4.2, and is presented in the Appendix. In particular, we can obtain

$$G(x) = e^{\mu(x-\overline{x})}\left[1 + \frac{\widehat{\mu}s^2/\sigma^2 - \mu(\lambda+r)/r}{\widehat{\mu}\frac{r(1-s^2/\sigma^2)}{\lambda} + \widetilde{\mu}(\frac{\lambda+r(1-s^2/\sigma^2)}{\lambda}) + \mu} + \frac{\lambda}{r}\right]. \tag{23}$$

Suppose that the consumer does not buy at $t = 0$. That is, $x_0 < \overline{x}$. As optimizing on the

[8]The case considered here can also be seen as the firm having information about the starting expected current utility of each consumer and doing personalized pricing. It would also be interesting to consider optimal pricing in this set-up under competition, but that is beyond the scope of this paper.

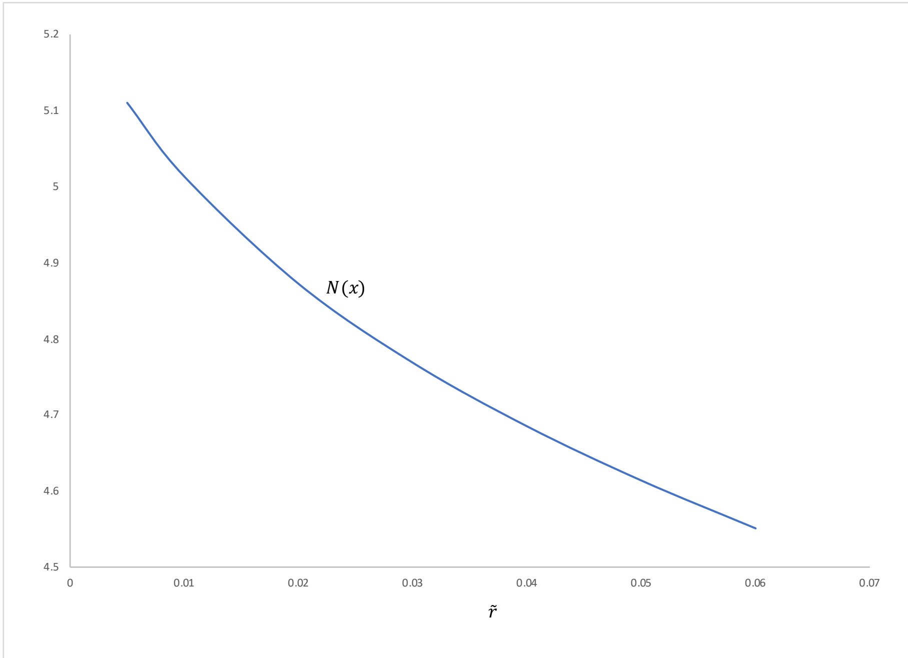

Figure 5: Evolution of the expected number of purchases going forward when not owning the product, $N(x)$, as a function of the actual discount rate $\widetilde{r}$ for $x = 0$, $P = 2$, $\beta = .02, \sigma^2 = .2, s^2 = .25$ and $\lambda = .2$.

price to charge, under zero production costs, is $\max_P PG(x_0)$, we can obtain the first order condition as

$$1 - \mu P \frac{\partial \overline{x}}{\partial P} = 0. \tag{24}$$

Concentrating on the case $s^2 = \sigma^2$, we can use (6) to obtain

$$\frac{\partial \overline{x}}{\partial P} = \frac{\widehat{X}(r + \lambda)}{\widehat{X} - 1}, \tag{25}$$

with $\widehat{X} = e^{\widehat{\mu \overline{x}}}$. Using this in the price first order condition, yields

$$P = \frac{\widehat{X} - 1}{\mu \widehat{X}(r + \lambda)}. \tag{26}$$

Note that the optimal price in this case of $x_0 < \overline{x}$ does not depend on the initial state x_0.

From (6) and (26) we can then determine the equilibrium P and $\overline{x}$. We can obtain the

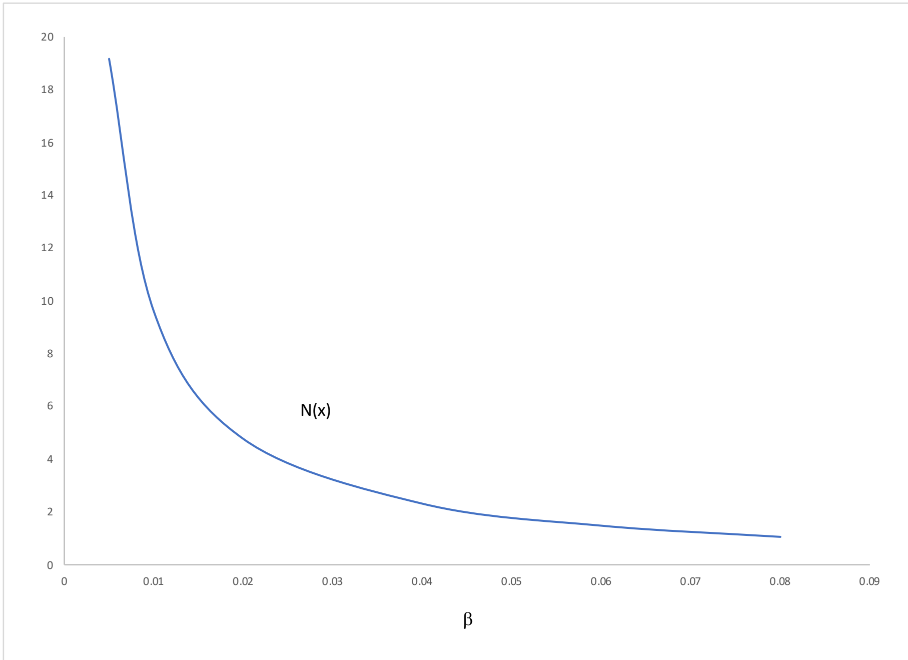

Figure 6: Evolution of the expected number of purchases going forward when not owning the product, $N(x)$, as a function of β for $x = 0$, $P = 2$, $\widetilde{r} = .03, \sigma^2 = .2, s^2 = .25$ and $\lambda = .2$.

equilibrium $\overline{x}$ by the implicit equation

$$\overline{x} = \frac{\widehat{X} - 1}{\widehat{X}} \left(\frac{1}{\widehat{\mu}} + \frac{1}{\mu} \right), \tag{27}$$

which yields that the equilibrium $\overline{x}$ and equilibrium price are decreasing in λ and r, and increasing in $s^2 = \sigma^2$.

Let x^* denote the solution to equation (27). That is, (26) gives the optimal price only if $x_0 < x^*$.

If $x_0 > \overline{x}$, then the customer would buy at $t = 0$. From the analysis in the Appendix, we have that for $x > \overline{x}$:

$$\widetilde{G}(x) = \frac{\widehat{\mu} s^2/\sigma^2 - \mu(\lambda + r)/r}{\widehat{\mu}\frac{r(1-s^2/\sigma^2)}{\lambda} + \widetilde{\mu}(\frac{\lambda + r(1-s^2/\sigma^2)}{\lambda}) + \mu} e^{\widetilde{\mu}(\overline{x} - x)} + \frac{\lambda}{r}. \tag{28}$$

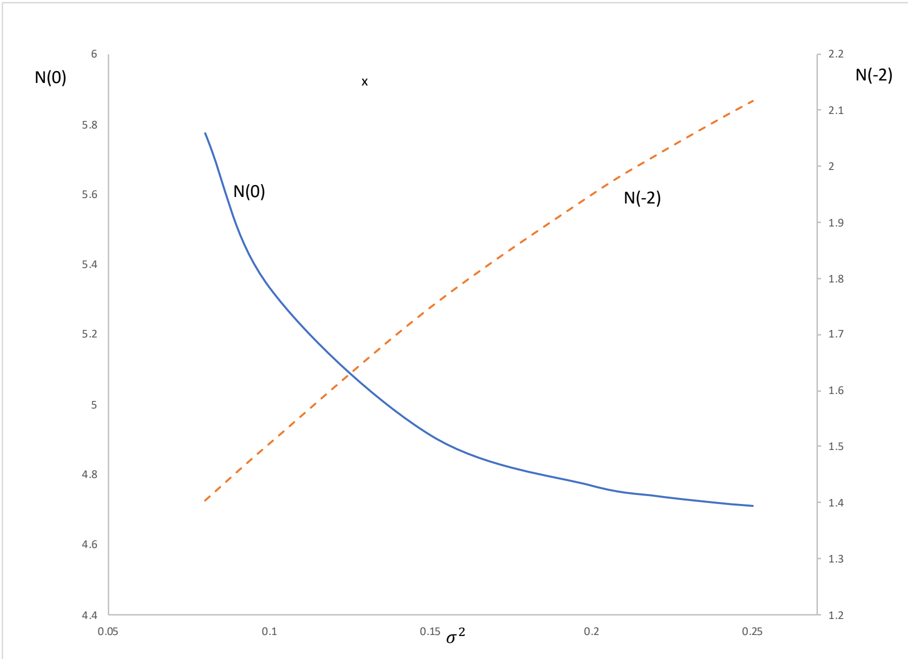

Figure 7: Evolution of the expected number of purchases going forward when not owning the product, $N(x)$, as a function of σ^2 for $x = 0$ and $x = -2$, with $P = 2$, $\widetilde{r} = .03, \beta = .02, s^2 = .25$ and $\lambda = .2$.

Focusing on the case of $s^2 = \sigma^2$, this simplifies to

$$\widetilde{G}(x) = \frac{1}{2}\left(\sqrt{\frac{\lambda + r}{r}} - \frac{\lambda + r}{r}\right) e^{\widetilde{\mu}(\overline{x} - x)} + \frac{\lambda}{r}. \tag{29}$$

The firm's objective is $\max_P P(1 + \widetilde{G}(x_0))$. We can obtain the first order condition as

$$P \frac{\partial \widetilde{G}(x_0)}{\partial \overline{x}} \frac{\partial \overline{x}}{\partial P} + 1 + \widetilde{G}(x_0) = 0. \tag{30}$$

Using (6), this simplifies to the following implicit equation which determines the optimal $\overline{x}$ as a function of x_0:

$$\frac{1}{2}\left[1 - \sqrt{\frac{\lambda + r}{r}}\right]\left[\widehat{\mu}\overline{x}\frac{\widehat{X}}{\widehat{X} - 1} - 1 + \sqrt{\frac{\lambda + r}{r}}\right] e^{\widetilde{\mu}(\overline{x} - x_0)} + \frac{\lambda + r}{r} = 0. \tag{31}$$

One can then use (6) to get the optimal price. Let $h(\overline{x}, x_0) = 0$ represent (31). Then note

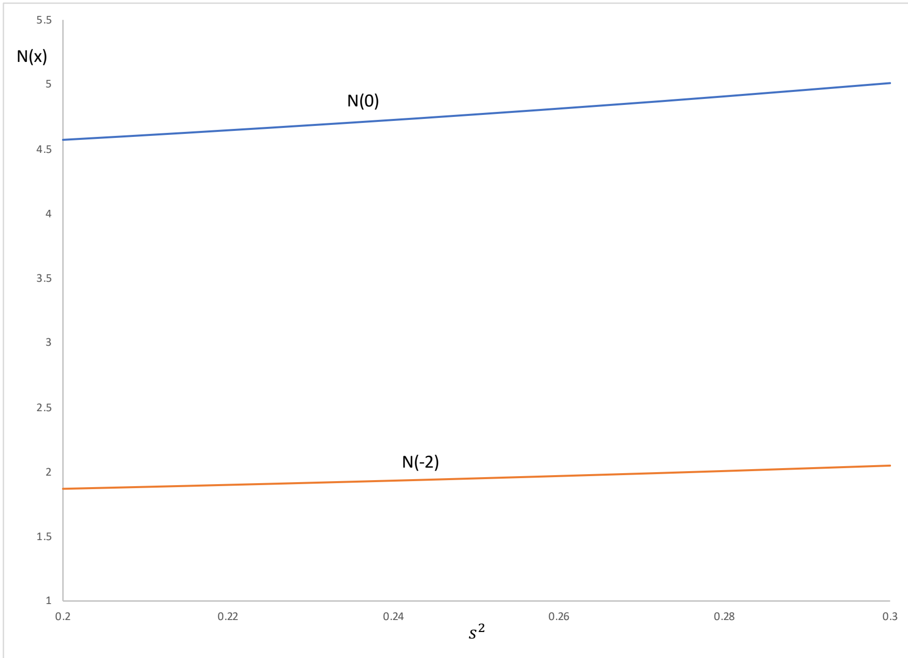

Figure 8: Evolution of the expected number of purchases going forward when not owning the product, $N(x)$, as a function of s^2 for $x = 0$ and $x = -2$, with $P = 2, \widetilde{r} = .03, \beta = .02, \sigma^2 = .2$ and $\lambda = .2$.

that $h(\overline{x}, x^*) = 0$ generates a $\overline{x} > x^*$. That is, for x_0 slightly above x^*, (31) does not specify the equilibrium $\overline{x}$. In fact, we can obtain x^{**} as the solution to $h(\overline{x}, \overline{x}) = 0$,

$$\widehat{\mu}\frac{\overline{x}\widehat{X}}{\widehat{X}-1} = \frac{\frac{\lambda}{r} + 2\sqrt{\frac{\lambda+r}{r}}}{\sqrt{\frac{\lambda+r}{r}} - 1}, \tag{32}$$

where $x^{**} > x^*$.

We can then obtain that if $x_0 < x^*$, we have $\overline{x}$ defined by (27), if $x_0 > x^{**}$, we have $\overline{x}$ defined by (31), and for $x_0 \in [x^*, x^{**}]$, we have $\overline{x} = x_0$. For an example of $s^2 = \sigma^2 = .25, \lambda = .2$, and $r = .05$, we can obtain $x^* \approx 2.18$, and $x^{**} \approx 4.84$.

We collect some of these results in the following proposition:

Proposition 4. *Consider optimal pricing and the case in which the amount of information gained while owning or not owning the product is relatively close, s^2 close to σ^2. Then the purchase threshold is independent of x_0 for $x_0 < x^*$, and increasing in x_0 for $x_0 > x^*$, with*

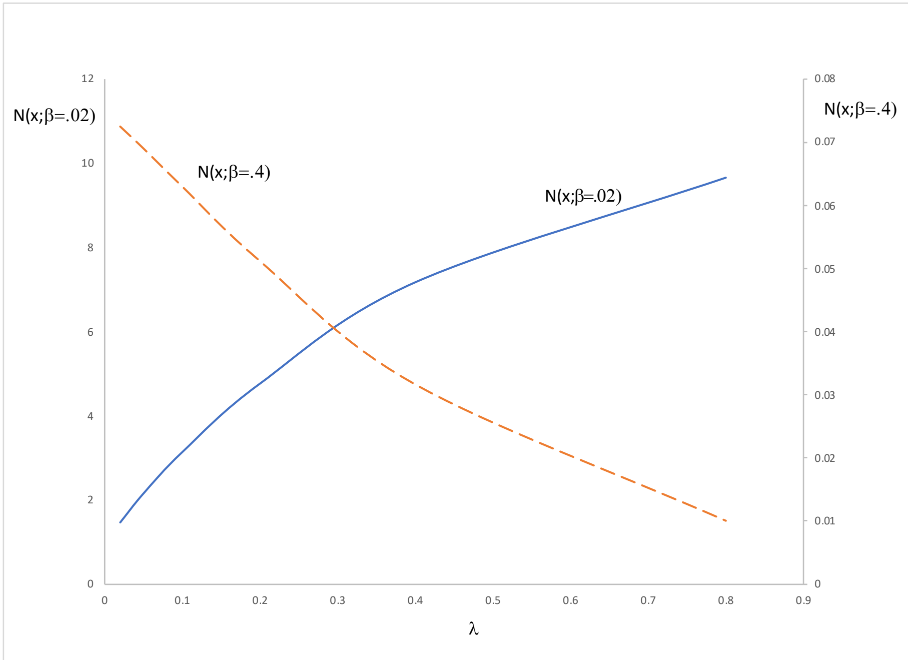

Figure 9: Evolution of the expected number of purchases going forward when not owning the product, $N(x)$, as a function of λ for $\beta = .02$ and $\beta = .4$, with $x = 0$, $P = 2$, $\widetilde{r} = .03$, $s^2 = .25$, and $\sigma^2 = .2$.

$\overline{x} = x_0$ for $x_0 \in [x^*, x^{**}]$, and $\overline{x} < x_0$ for $x_0 > x^{**}$. The optimal price is decreasing in λ and r for all x_0. The purchase threshold under optimal pricing is non-increasing in λ for all x_0, and is decreasing in r for $x_0 < x^*$, and can be either increasing or decreasing in r for $x_0 > x^{**}$. An increase in σ^2 under the constraint of $s^2 = \sigma^2$ leads to an increase in the purchase threshold and the optimal price for $x_0 < x^*$, a decrease in the optimal price for $x_0 \in [x^*, x^{**}]$, and an increase or decrease of the price and purchase threshold for $x_0 > x^{**}$.

When x_0 is low, the optimal price and the purchase threshold do not depend on x_0. The firm has to take into account the potential future revenue of the expected future repurchases by the consumer, therefore does not lower the price beyond a certain level, which makes the optimal price not varying with the expected initial valuation if that valuation is sufficiently low. In that region, an increase in the duration of the product or decrease in the discount rate make the product more valuable and the firm optimally increases its price, with a resulting increase in the purchase threshold. Similarly, when the amount of information gained increases, we have that both the consumer is more likely to get information that makes the consumer willing to purchase the product, and the consumer has a higher potential of having greater benefits after purchase. Both of these effects lead then the firm to raise its price, and the purchase threshold to increase.

For higher x_0, the effect of the duration of the product on the purchase threshold and the optimal price are in the same direction as when x_0 is low. Similarly, if $x_0 \in [x^*, x^{**}]$ the effect of the discount rate on the optimal price is also in the same direction as in the x_0 low condition. The effect of the discount rate on the purchase threshold for $x_0 > x^{**}$ can be either positive or negative. When $x_0 > x^{**}$ the consumer starts in a region when the consumer is likely to make a repurchase when the product breaks down, which increases the effect of the discount rate on the value of the product, which may make then the consumer be more demanding on the purchase threshold.

The effect of the information gained on the optimal price when x_0 is not too low is interesting. In the region where $x_0 \in [x^*, x^{**}]$ the purchase threshold is set at x_0. As the amount of information gained is a force for the consumer to be more demanding on the purchase threshold while keeping the price fixed, the firm has then to decrease its price for the purchase threshold to remain fixed at x_0. The same effect also holds for $x_0 > x^{**}$, but in that region the firm has some flexibility in increasing $\overline{x}$, and therefore, in that region, the purchase threshold can increase or decrease with the amount of information gained.

To illustrate the equilibrium, Figure 10 presents the evolution of the optimal price P and the resulting purchase threshold $\overline{x}$ as a function of the initial current utility x_0. Figures 11-13 illustrates how the optimal price and the resulting purchase threshold vary with λ, r, and $s^2 = \sigma^2$ for x_0 in the different regions, as presented in Proposition 4. Figure 12 illustrates that the purchase threshold can either increase or decrease with r for $x_0 > x^{**}$, and presents an effect of the price declining with r for that region of x_0. Figure 13 illustrates a case where the purchase threshold increases in σ^2 with $s^2 = \sigma^2$ for $x_0 > x^{**}$, which was a possibility in the proposition, and presents that the optimal price can either increase or decrease.

Effect of the Product Duration on Profit

We now investigate the effect of the product duration on the firm's profit under optimal pricing. Assume that $s^2 = \sigma^2$ and x_0 is not too high so that the consumer does not buy at time 0. The optimal price is determined by (26), and the discounted number of purchases is

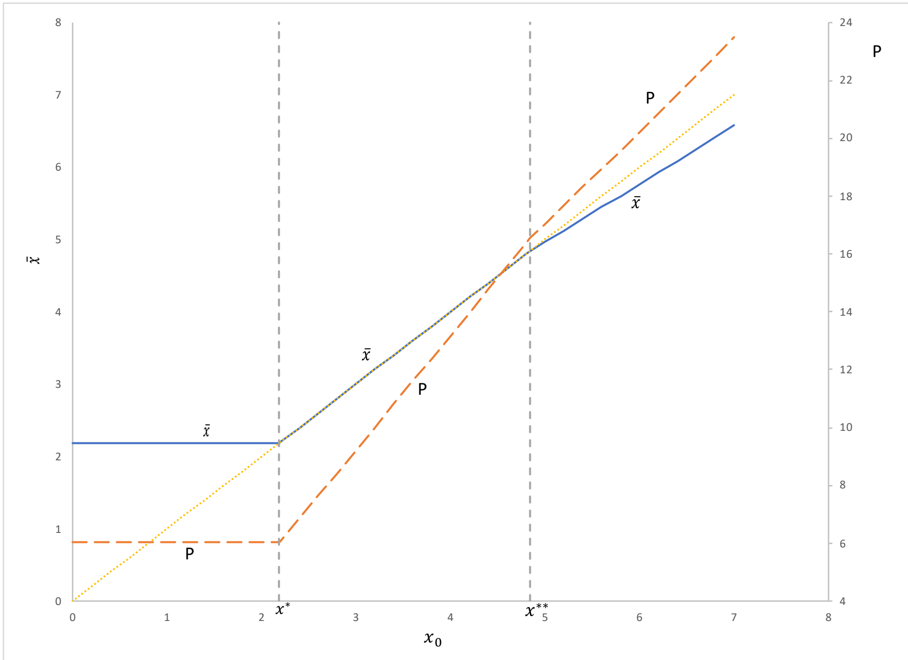

Figure 10: Evolution of the optimal price P and of the resulting purchase threshold $\overline{x}$ as a function of the initial current utility x_0 for $s^2 = \sigma^2 = .25$, $\lambda = .2$, and $r = .05$.

given by (23). Then, the firm's expected profit $\Pi(x_0) = PG(x_0)$ is

$$\Pi(x_0) = \frac{1}{2}\frac{\widehat{X}-1}{\mu\widehat{X}(r+\lambda)}e^{\mu(x_0-\overline{x})}\left[\frac{r+\lambda}{r}+\sqrt{\frac{r+\lambda}{r}}\right] \tag{33}$$

Using (27), this becomes

$$\Pi(x_0) = \frac{\overline{x}}{2r}e^{\mu(x_0-\overline{x})} \tag{34}$$

Taking the derivative of $\Pi(x_0)$ with respect to $\overline{x}$, we get

$$\frac{d\Pi(x_0)}{d\overline{x}} = \frac{1-\mu\overline{x}}{2r}e^{\mu(x_0-\overline{x})} \tag{35}$$

From (27) we get that $\overline{x} \to \frac{1}{\mu}$ as $\lambda \to \infty$. Given that $\overline{x}$ decreases in λ, this implies that $\overline{x} > \frac{1}{\mu}$ for all finite λ. Thus we have

$$\frac{d\Pi(x)}{d\lambda} = \frac{d\Pi(x)}{d\overline{x}}\frac{d\overline{x}}{d\lambda} > 0 \tag{36}$$

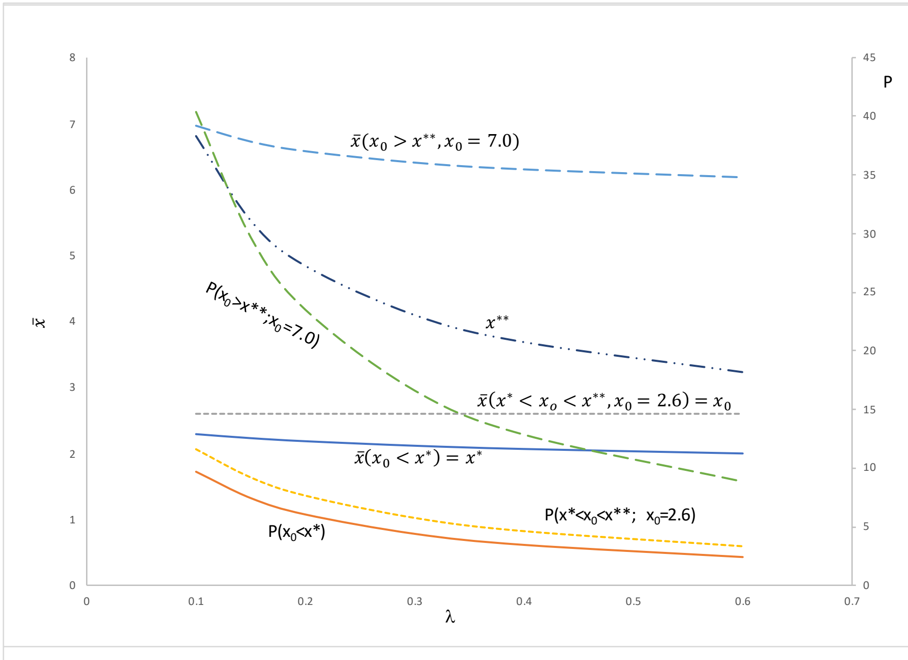

Figure 11: Evolution of the optimal price P and of the purchase threshold $\overline{x}$ as a function λ for $s^2 = \sigma^2 = .25$, and $r = .05$.

which means that the firm's expected profit monotonically increases in λ.

What happens at the limit of $\lambda \to \infty$? At the limit, $\overline{x} = \frac{1}{\mu}$, so the expected profit becomes

$$\Pi(x_0) = \frac{e^{\mu x_0 - 1}}{2r\mu} \tag{37}$$

Let Pdt denote the flow price in the limit. The consumer pays for the product whenever $x - P > 0$. Thus we have $Pdt = \overline{x}dt = \frac{1}{\mu}dt$, which means that the optimal flow price is increasing in the amount of information gained, $\sigma^2 = s^2$, and decreasing in the discount rate r. We summarize these results in the following proposition.

Proposition 5. *Consider that the firm could choose the duration of the product, that $\sigma^2 = s^2$, and that the expected current utility is low enough, $x_0 < 1/\mu$. Then the firm chooses an infinitely short duration for the product, and the optimal price is $\sqrt{\frac{\sigma^2}{2r}}$ per unit of time.*

Proposition 5 suggests that the firm prefers renting over selling a product with durability. The mechanism at work here is different than that of the previous literature on the durable

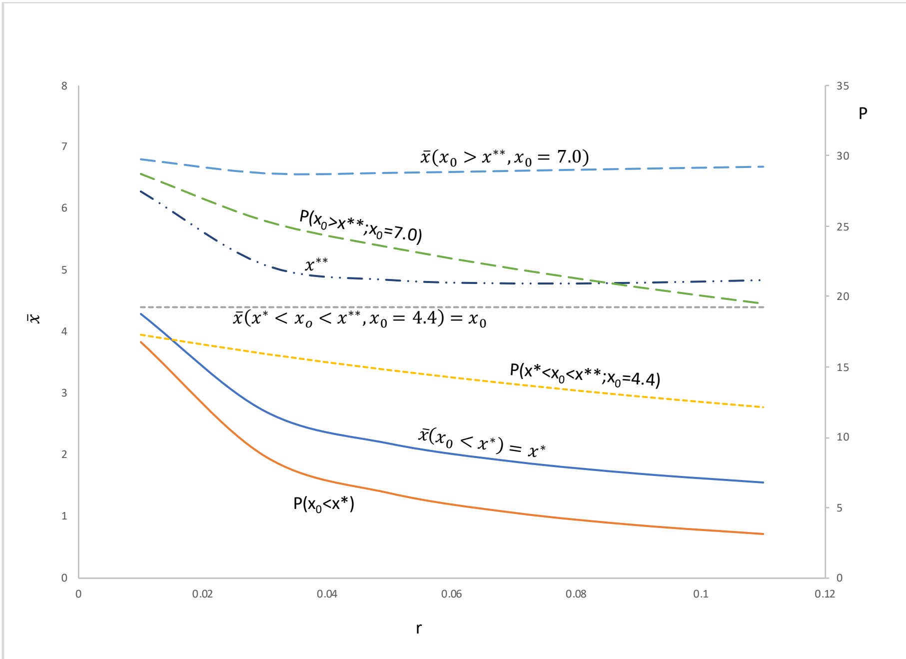

Figure 12: Evolution of the optimal price P and of the purchase threshold $\overline{x}$ as a function r for $s^2 = \sigma^2 = .25$, and $\lambda = .2$.

goods monopolist. The traditional argument for renting over selling durable goods (or decreasing durability) is that rational consumers expect that a monopolist without the ability to commit to future prices to lower price and increase supply over time to capture residual demand (e.g., Coase 1972, Bulow 1982). In Stokey (1979, 1981), a monopolist with commitment power is indifferent between renting and selling. In the current model, the firm prefers lower durability even though it has commitment power on price, which is constant over time. If a consumer has to make an irreversible purchase decision, then there is an option value in delaying purchase. The consumer can choose to acquire more information and preserve the option to make a purchase at a later time. Such delaying is not in the interest of the firm. With a high durability, a firm has to charge a higher price. This further exacerbates the problem because the option value of delaying purchase is higher when the consumer has to make a more costly decision with a longer-lasting impact. Thus a higher durability results in an increase in price and in the purchase threshold and a decrease in the expected discounted life-time value of the consumer. However, the firm can effectively eliminate this option value of delaying by switching to continuously renting the product/service. A consumer's decision

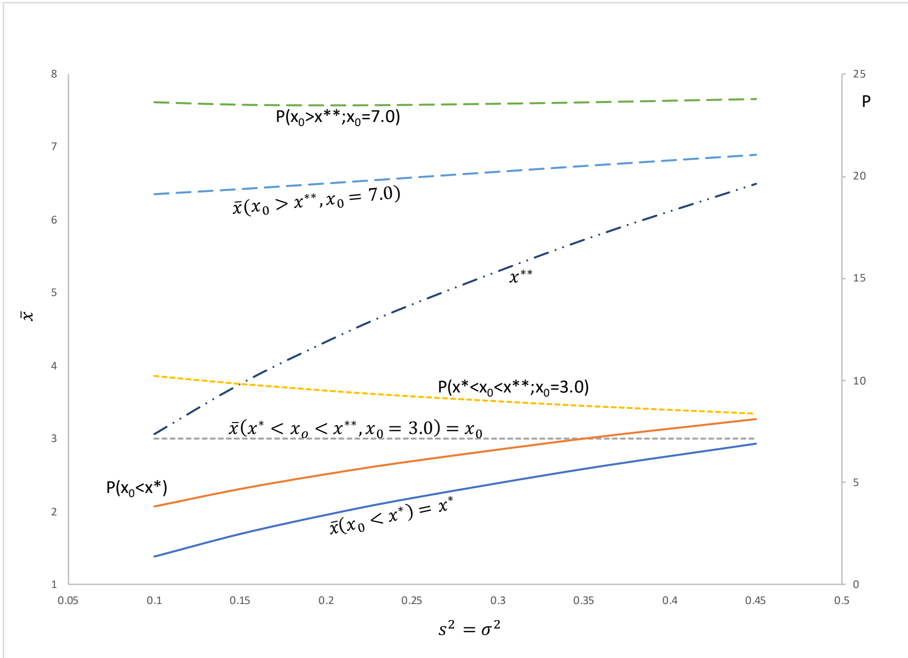

Figure 13: Evolution of the optimal price P and of the purchase threshold $\overline{x}$ as a function $s^2 = \sigma^2$ for $r = .05$, and $\lambda = .2$.

to rent does not limit her future choices, but only affects her current flow utility. So there is no option value of delaying as long as the current flow utility from renting is positive.

Note that this result on the optimality of the infinitely short duration of the product continues to hold if $s^2 > \sigma^2$ and s^2 not too far from σ^2. If s^2 is much greater than σ^2, then the optimal $\overline{x}$ may end up being negative, and then the shortest duration possible will no longer be optimal. Note also that we assume the marginal costs of production to be zero. If the marginal cost is linear in the discounted duration the product, that is, $C(\lambda) = K/(r+\lambda)$ for some cost parameter K, then the cost can be normalized into x and P and all results will continue to hold. But if, for example, there is a fixed cost per transaction that is independent of λ, which can be a production cost or a transaction cost paid by the consumer, then the infinitely short duration will no longer be optimal.

6. Conclusion

This paper studies the possibility of a consumer deciding when to purchase and repurchase a product as preferences evolve over time. The paper generates rich dynamics of when a consumer owns a product, and decides when to repurchase it when the product breaks down. We characterize the optimal strategy of the consumer, and then compute the market equilibrium.

The model can be seen as considering a mixture of search and experience goods, where the consumer decides when to learn information about the product prior to purchase, or learn information about the product while using it. In particular, when the consumer gains more information when using the product, the consumer becomes less demanding on the expected value of the product to decide to make a purchase, but on the other hand makes less frequent repurchases after the initial purchase.

We can construct the optimal price to charge given an initial expected current utility of the product. We find that if the initial expected current utility is low enough such that firm does not want to get the consumer to make an immediate purchase, the optimal price is independent of the initial valuation. On the other hand when the initial expected current utility is sufficiently high the firm may want to price such that consumer purchase the product immediately, and in that the case the optimal price is increasing in the initial expected valuation.

We can obtain that while a greater information gained increases prices for low initial valuations, it can decrease prices for higher initial valuations. We also find that if the firm could choose the product duration it would choose the smallest one possible, which can be interpreted as a rental pricing mechanism.

It would be interesting to explore in future research the possibility of having a subscription model where consumers commit to subscribe for some period of time. It would also be interesting to study the possibility of product returns in this environment of evolving preferences, and allowing for the possibility of prices evolving over time.

APPENDIX

Specifics of Consumer Learning:

1. Learning about Attributes:

In the interpretation of consumer learning while owning or not owning the product, we can consider that consumers are leaning about attributes of equal importance in each instant in time, with the overall utility being the sum of the deviation to the mean of each attribute's contribution. See Branco, Sun, and Villas-Boas (2012) as an example. If the consumer learns more attributes per unit of time when owning than when not owning the product, we have $s^2 > \sigma^2$.

Consider T as the mass of attributes. The main text presents the case of $T \to \infty$.

Alternatively, we could have T distributed exponentially with parameter ψ, with the consumers not knowing T. In that case, if the consumer does not own the product and information on all the number of attributes has been obtained, the consumer gets a present value of utilities $W_T(x) = \max[0, \frac{x-\lambda P}{r}]$. If the consumer owns the product all information on all attributes have been obtained, the consumer gets a present value of utilities of

$$V_T(x) = \frac{\max[0, x] + \lambda W_T(x)}{r + \lambda}. \tag{i}$$

When the consumer owns the product and $x > \overline{x}$ and the consumer has not yet checked all attributes, we have that (2) is replaced with

$$\begin{aligned} V(x) &= x\,dt + e^{-r\,dt}(1 - \psi\,dt)\left[\lambda\,dt\{E[V(x+dx)] - P\} + (1 - \lambda\,dt)E[V(x+dx)]\right] \\ &\quad + e^{-r\,dt}\psi\,dt\left[\lambda\,dtW_T(x) + (1 - \lambda\,dt)V_T(x)\right]. \end{aligned} \tag{ii}$$

When the consumer owns the product and $x \in (0, \overline{x})$, we have that (3) and (4) are replaced with

$$\begin{aligned} \widetilde{V}(x) &= \max[0, x\,dt] + e^{-r\,dt}(1 - \psi\,dt)[\lambda\,dtW(x) + (1 - \lambda\,dt)E\widetilde{V}(x+dx)] \\ &\quad + e^{-r\,dt}\psi\,dt[\lambda\,dtW_T(x) + (1 - \lambda\,dt)V_T(x)]. \end{aligned} \tag{iii}$$

We can then conduct the analysis as in the main text and obtain the (stationary) threshold $\overline{x}$, which is now a function of ψ as well. The case presented in the main text is the case in which $\psi \to 0$.

2. Signals of Product Value:

Consider an alternative model, where the true value of the product, $\widehat{x}$, evolves over time, as

$$d\widehat{x}_t = \widetilde{\sigma} dW_t \tag{iv}$$

where W_t is a Wiener process. The decision maker observes a noisy signal S_t^x at time t, which follows

$$S_t^x = \widehat{x}_t + \widetilde{s} V_t \tag{v}$$

or

$$dS_t^x = d\widehat{x}_t + \widetilde{s} dV_t \tag{vi}$$

where V_t is a Wiener process. That is, the signal is only on the change of $\widehat{x}$.

For simplicity, assume that $\widehat{x}_0$ is known. Then we have $\widehat{x}_t \sim \mathcal{N}(\widehat{x}_0, \widetilde{\sigma}^2 t)$, and noise $S_t^x - \widehat{x}_t \sim \mathcal{N}(0, \widetilde{s}^2 t)$. The posterior mean x_t is

$$x_t = \frac{\frac{\widehat{x}_0}{\widetilde{\sigma}^2 t} + \frac{N_t}{\widetilde{s}^2 t}}{\frac{1}{\widetilde{\sigma}^2 t} + \frac{1}{\widetilde{s}^2 t}} \tag{vii}$$

$$= \frac{\widetilde{s}^2}{\widetilde{\sigma}^2 + \widetilde{s}^2} \widehat{x}_0 + \frac{\widetilde{\sigma}^2}{\widetilde{s}^2 + \widetilde{\sigma}^2} N_t \tag{viii}$$

Because $S_t^x \sim \mathcal{N}(\widehat{x}_t, \widetilde{s}^2 t)$ and $\widehat{x}_t \sim \mathcal{N}(\widehat{x}_0, \widetilde{\sigma}^2 t)$, we have $X_t^x \sim \mathcal{N}(\widehat{x}_0, (\widetilde{\sigma}^2 + \widetilde{s}^2)t)$. Thus

$$\mathbb{E}[(x_t - \widehat{x}_0)^2] = \frac{\widetilde{\sigma}^4}{\widetilde{\sigma}^2 + \widetilde{s}^2} t \tag{ix}$$

so x_t evolves with a constant variance that is decreasing in $\widetilde{s}^2$. With infinite noise, x_t does not update, and with no noise, x_t updates with variance $\widetilde{\sigma}^2$, which is intuitive.

If $\widetilde{s}_n^2$ is variance of the noise when not owning the product, and $\widetilde{s}_o$ is the variance of the noise when owning the product, we can then obtain the representation as in the main text with $\sigma^2 = \frac{\widetilde{\sigma}^4}{\widetilde{\sigma}^2 + \widetilde{s}_n^2}$ and $s^2 = \frac{\widetilde{\sigma}^4}{\widetilde{\sigma}^2 + \widetilde{s}_o^2}$. Note that a more informative signal leads to a higher variance on x_t. If the consumer gets more information when owning the product than not owning the product, or $\widetilde{s}_o \leq \widetilde{s}_n$, then we have $s^2 \geq \sigma^2$.

Still another variation could be the case in which the true value $\widehat{x}$ can take only two values, $\{-1, 1\}$, and both search and experience leads to a continuous evolution of the consumer belief about whether the consumer is facing the good or bad product. Although tractable, such a model leads to a more complex analysis than the one presented in the main text.

Derivation of $\overline{x}$ in the Case of $\overline{x} > 0$:

From the differential equation on W(x), and using $\lim_{x\to-\infty} W(x) = 0$, one obtains

$$W(x) = A_1 e^{\mu x} \tag{x}$$

where $\mu = \sqrt{2r/\sigma^2}$, and A_1 is a constant to be determined.[9]

Using Itô's Lemma in (2) one obtains $rV(x) = x - \lambda P + V''(x)\frac{s^2}{2}$. Note that $\lim_{x\to\infty}[V(x) - (x - \lambda P)/r] = 0$, as when the current utility goes to infinity, the consumer is always buying the product when it breaks down, which generates an expected utility of $(x - \lambda P)/r$. Using this when solving the differential equation on $V(x)$, one obtains

$$V(x) = A_2 e^{-\widetilde{\mu} x} + \frac{x - \lambda P}{r}, \tag{xi}$$

where $\widetilde{\mu} = \sqrt{2r/s^2}$, and A_2 is a constant to be determined.

Using Itô's Lemma on (3), and solving the resulting differential equation, this yields

$$\widetilde{V}(x) = A_3 e^{\widehat{\mu} x} + A_4 e^{-\widehat{\mu} x} + \frac{x}{r + \lambda} + \frac{\lambda A_1}{r(1 - s^2/\sigma^2) + \lambda} e^{\mu x}, \tag{xii}$$

where $\widehat{\mu} = \sqrt{2(r + \lambda)/s^2}$, and A_3 and A_4 are constants to be determined.

Similarly, from (4), and using the fact that the expected utility when owning the product goes to zero when the current utility of using the product approaches negative infinity, one obtains

$$\widetilde{V}(x) = A_5 e^{\widehat{\mu} x} + \frac{\lambda A_1}{r(1 - s^2/\sigma^2) + \lambda} e^{\mu x}, \tag{xiii}$$

where A_5 is a constant to be determined.

Value matching and smooth pasting at both $\overline{x}$ and 0, $W(\overline{x}) = V(\overline{x}) - P, W'(\overline{x}) = V'(\overline{x}), V(\overline{x}) = \widetilde{V}(\overline{x}), V'(\overline{x}) = \widetilde{V}'(\overline{x}), \widetilde{V}(0^+) = \widetilde{V}(0^-)$, and $\widetilde{V}'(0^+) = \widetilde{V}'(0^-)$ yields

[9]Note that the general solution of the differential equation $rW(x) = W''(x)\frac{\sigma^2}{2}$ is $W(x) = A_1 e^{\mu x} + \widetilde{A}_1 e^{-\mu x}$, where A_1 and $\widetilde{A}_1$ are constants. The condition $\lim_{x\to-\infty} W(x) = 0$ then yields $\widetilde{A}_1 = 0$. Similar derivations are also used in the remainder of the paper when appropriate.

$$
\begin{aligned}
A_1\overline{X} &= \frac{A_2}{\widetilde{X}} + \frac{\overline{x}-\lambda P}{r} - P && \text{(xiv)}\\
\mu A_1\overline{X} &= -\frac{\widetilde{\mu}A_2}{\widetilde{X}} + \frac{1}{r} && \text{(xv)}\\
A_5 &= A_3 + A_4 && \text{(xvi)}\\
A_5 - \frac{1}{\widehat{\mu}(r+\lambda)} &= A_3 - A_4 && \text{(xvii)}\\
\frac{A_2}{\widetilde{X}} + \frac{\overline{x}-\lambda P}{r} &= A_3\widehat{X} + \frac{A_4}{\widehat{X}} + \frac{\overline{x}}{r+\lambda} + \frac{\lambda A_1\overline{X}}{r(1-s^2/\sigma^2)+\lambda} && \text{(xviii)}\\
-\frac{\widetilde{\mu}A_2}{\widetilde{X}} + \frac{1}{r} &= \widehat{\mu}A_3\widehat{X} - \frac{\widehat{\mu}A_4}{\widehat{X}} + \frac{1}{r+\lambda} + \frac{\mu\lambda A_1\overline{X}}{r(1-s^2/\sigma^2)+\lambda} && \text{(xix)}
\end{aligned}
$$

where $\overline{X} = e^{\mu\overline{x}}$, $\widetilde{X} = e^{\widetilde{\mu}\overline{x}}$, and $\widehat{X} = e^{\widehat{\mu}\overline{x}}$. We can then solve (xiv)-(xix) for A_1, A_2, A_3, A_4, A_5, and $\overline{x}$.

From (xiv) and (xv) we can obtain

$$
A_1\overline{X} = \frac{\widetilde{\mu}}{\mu+\widetilde{\mu}}\frac{\overline{x}-\lambda P}{r} - \frac{\widetilde{\mu}}{\mu+\widetilde{\mu}}P + \frac{1}{r(\mu+\widetilde{\mu})}. \tag{xx}
$$

From (xvi) and (xvii) we can obtain

$$
A_4 = \frac{1}{2\widehat{\mu}(r+\lambda)}. \tag{xxi}
$$

Using (xiv) and (xviii) we can obtain

$$
A_3\widehat{X} = A_1\overline{X}\frac{r(1-s^2/\sigma^2)}{r(1-s^2/\sigma^2)+\lambda} + P - \frac{A_4}{\widehat{X}} - \frac{\overline{x}}{r+\lambda}. \tag{xxii}
$$

Using then (xv), (xx), (xxi), and (xxii) in (xix) we can then obtain (5) which determines $\overline{x}$.

Derivation of $\overline{x}$ in the Case of $\overline{x} < 0$:

Let $W(x)$ be the expected present value of payoffs for the consumer if the consumer does not own the product and is getting information on the product, $x < \overline{x} < 0$. Let $V(x)$ be the expected present value of payoffs for the consumer if the consumer owns the product and $x \geq \overline{x}$. Let $\widetilde{V}(x)$ be the expected value of payoffs for the consumer if the consumer owns the product and $x < \overline{x}$.

When the consumer does not own the product and is searching for information we can obtain that the evolution of $W(x)$ is characterized by

$$W(x) = e^{-r\,dt} E[W(x + dx)]. \tag{xxiii}$$

Using Itô's Lemma, we can get $rW(x) = W''(x)\frac{\sigma^2}{2}$, from which we can obtain

$$W(x) = D_1 e^{\mu x} + \widetilde{D}_1 e^{-\mu x} \tag{xxiv}$$

where $\mu = \sqrt{2r/\sigma^2}$ and D_1 and $\widetilde{D}_1$ are constants to be determined. Note that $\lim_{x\to-\infty} W(x) = 0$, so we obtain $\widetilde{D}_1 = 0$.

When the consumer owns the product and $x \geq 0$, we have that the expected present value of consumer payoffs has to satisfy

$$V(x) = x\,dt + e^{-r\,dt}\lambda\,dt\{E[V(x + dx)] - P\} + e^{-r\,dt}(1 - \lambda\,dt)E[V(x + dx)] \tag{xxv}$$

Using Itô's Lemma, this reduces to $rV(x) = x - \lambda P + V''(x)\frac{s^2}{2}$. Solving this differential equation one obtains

$$V(x) = \widetilde{D}_2 e^{\widetilde{\mu}x} + D_2 e^{-\widetilde{\mu}x} + \frac{x - \lambda P}{r}, \tag{xxvi}$$

where $\widetilde{\mu} = \sqrt{2r/s^2}$, and D_2 and $\widetilde{D}_2$ are constants to be determined. Note that $\lim_{x\to\infty}[V(x) - (x - \lambda P)/r] = 0$. We then have that $\widetilde{D}_2 = 0$.

Consider now that the consumer owns the product and $\overline{x} \leq x < 0$. In this region, the consumer would not use the product, but would repurchase if the product breaks down. The expected present value of consumer payoffs has to satisfy

$$V(x) = e^{-r\,dt}\lambda\,dt\{E[V(x + dx)] - P\} + e^{-r\,dt}(1 - \lambda\,dt)E[V(x + dx)]. \tag{xxvii}$$

Using Itô's Lemma, and solving the resulting differential equation, this yields

$$V(x) = D_3 e^{\widetilde{\mu}x} + D_4 e^{-\widetilde{\mu}x} - \frac{\lambda}{r}P, \tag{xxviii}$$

where $\widetilde{\mu} = \sqrt{2r/s^2}$, and D_3 and D_4 are constants to be determined.

Finally, for the case of $x < \overline{x}$, the expected present value of consumer payoffs has to satisfy

$$\widetilde{V}(x) = e^{-r\,dt}\lambda\,dtW(x) + e^{-r\,dt}(1 - \lambda\,dt)E\widetilde{V}(x + dx) \tag{xxix}$$

which yields

$$\widetilde{V}(x) = D_5 e^{\widehat{\mu}x} + \widetilde{D}_5 e^{-\widehat{\mu}x} + \frac{\lambda D_1}{r(1 - s^2/\sigma^2) + \lambda} e^{\mu x}, \tag{xxx}$$

where D_5 and $\widetilde{D}_5$ are constants to be determined. Noting that $\lim_{x \to -\infty} \widetilde{V}(x) = 0$, we obtain $\widetilde{D}_5 = 0$.

Value matching and smooth pasting at both $\overline{x}$ and 0, $W(\overline{x}) = V(\overline{x}) - P, W'(\overline{x}) = V'(\overline{x}), V(\overline{x}) = \widetilde{V}(\overline{x}), V'(\overline{x}) = \widetilde{V}'(\overline{x}), V(0^+) = V(0^-)$, and $V'(0^+) = V'(0^-)$ yields

$$\begin{aligned}
D_1 \overline{X} &= D_3 \widetilde{X} + \frac{D_4}{\widetilde{X}} - \frac{\lambda P}{r} - P && \text{(xxxi)} \\
\mu D_1 \overline{X} &= \widetilde{\mu} D_3 \widetilde{X} - \widetilde{\mu} \frac{D_4}{\widetilde{X}} && \text{(xxxii)} \\
D_3 \widetilde{X} + \frac{D_4}{\widetilde{X}} - \frac{\lambda P}{r} &= D_5 \widehat{X} + \frac{\lambda D_1 \overline{X}}{r(1 - s^2/\sigma^2) + \lambda} && \text{(xxxiii)} \\
D_3 \widetilde{X} - \frac{D_4}{\widetilde{X}} &= \frac{\widehat{\mu}}{\widetilde{\mu}} D_5 \widehat{X} + \frac{\mu}{\widetilde{\mu}} \frac{\lambda D_1 \overline{X}}{r(1 - s^2/\sigma^2) + \lambda} && \text{(xxxiv)} \\
D_3 + D_4 &= D_2 && \text{(xxxv)} \\
D_3 - D_4 &= -D_2 + \frac{1}{r\widetilde{\mu}} && \text{(xxxvi)}
\end{aligned}$$

where $\overline{X} = e^{\mu \overline{x}}, \widetilde{X} = e^{\widetilde{\mu}\overline{x}}$, and $\widehat{X} = e^{\widehat{\mu}\overline{x}}$. We can then solve (xxxi)-(xxxvi) for D_1, D_2, D_3, D_4, D_5, and $\overline{x}$.

From (xxxv) and (xxxvi) we can obtain

$$D_3 = \frac{1}{2r\widetilde{\mu}} \tag{xxxvii}$$

From (xxxi) and (xxxiii) we can obtain

$$\frac{r(1 - s^2/\sigma^2)}{r(1 - s^2/\sigma^2) + \lambda} D_1 \overline{X} = D_5 \widehat{X} + P \tag{xxxviii}$$

Using (xxxii) and (xxxiv) we can obtain

$$\frac{r(1 - s^2/\sigma^2)}{r(1 - s^2/\sigma^2) + \lambda} D_1 \overline{X} = \frac{\widehat{\mu}}{\mu} D_5 \widehat{X} \tag{xxxix}$$

Combining (xxxviii) and (xxxix), we get

$$D_1\overline{X} = P\frac{\widehat{\mu}}{\widehat{\mu}-\mu}\frac{r(1-s^2/\sigma^2)}{r(1-s^2/\sigma^2)+\lambda} \tag{xl}$$

From (xxxi) and (xxxii) we can obtain

$$D_1\overline{X} = \frac{1}{r(\mu+\widetilde{\mu}})\overline{X} - \frac{\widetilde{\mu}}{\mu+\widetilde{\mu}}\frac{\lambda+r}{r}P \tag{xli}$$

Combining (xl) and (xli) we get the closed-form solution for $\overline{x}$ as in (7):

$$e^{\mu\overline{x}} = P\left[\widetilde{\mu}(\lambda+r) + r\widehat{\mu}\frac{\mu+\widetilde{\mu}}{\widehat{\mu}-\mu}\frac{r(1-s^2/\sigma^2)}{r(1-s^2/\sigma^2)+\lambda}\right] \tag{xlii}$$

Derivation of the Expected Discount Factor until the Next Purchase:

Using $\lim_{x\to\infty}\widetilde{d}(x) = \frac{\lambda}{\lambda+r}$, and Itô's Lemma in (14), and solving the resulting differential equation, yields

$$\widetilde{\delta}(x) = B_1 e^{-\widehat{\mu}x} + \frac{\lambda}{\lambda+r} \tag{xliii}$$

where we recall that $\widehat{\mu} = \sqrt{2(r+\lambda)/s^2}$, and B_1 is a constant to be determined.

From (15) we can obtain the differential equation

$$(r+\lambda)\widetilde{\delta}(x) = \lambda e^{-\mu(\overline{x}-x)} + \frac{s^2}{2}\widetilde{\delta}''(x). \tag{xliv}$$

Note that as $x \to -\infty$, the expected discount factor until the next purchase approaches zero. Using this when solving the differential equation (xliv) yields

$$\widetilde{\delta}(x) = B_2 e^{\widehat{\mu}x} + \frac{\lambda}{\lambda + r(1-s^2/\sigma^2)}e^{-\mu(\overline{x}-x)}, \tag{xlv}$$

where B_2 is a constant to be determined.

Value matching and smooth pasting at $\overline{x}$, $\widetilde{\delta}(\overline{x}^+) = \widetilde{\delta}(\overline{x}^-)$ and $\widetilde{\delta}'(\overline{x}^+) = \widetilde{\delta}'(\overline{x}^-)$, yields B_1 and B_2 below, which fully determines $\widetilde{\delta}(x)$, and therefore $\widetilde{T}(x)$.

$$B_1 = \frac{\lambda e^{\widehat{\mu}\overline{x}}}{2}\left[\frac{1-\mu/\widehat{\mu}}{\lambda+r(1-s^2/\sigma^2)} - \frac{1}{\lambda+r}\right] \quad \text{(xlvi)}$$

$$B_2 = \frac{\lambda e^{-\widehat{\mu}\overline{x}}}{2}\left[\frac{1}{\lambda+r} - \frac{1+\mu/\widehat{\mu}}{\lambda+r(1-s^2/\sigma^2)}\right] \quad \text{(xlvii)}$$

Derivation of the Expected Number of Purchases:

Let $N(x)$ be the expected number of units purchased going forward given that the consumer starts at $x < \overline{x}$ and the consumer does not own the product. We have that $N(x)$ evolves over time as

$$N(x) = (1-\beta\, dt)EN(x+dx). \quad \text{(xlviii)}$$

Note that $\lim_{x\to-\infty} N(x) = 0$, as the number of expected purchases going forward approaches zero, when the current utility of owning/using the product goes to negative infinity. Using this when solving for (xlviii) yields

$$N(x) = C_1 e^{\eta x} \quad \text{(xlix)}$$

where $\eta = \sqrt{2\beta/\sigma^2}$, and C_1 ia a constant to be determined.

Let $\widetilde{N}(x)$ be the expected number of future units purchased over time given that the consumer owns the product. As the consumer purchases the product immediately if the consumer does not own the product and $x = \overline{x}$, we have

$$N(\overline{x}) = 1 + \widetilde{N}(\overline{x}). \quad \text{(l)}$$

For $x \geq \overline{x}$ the evolution of $\widetilde{N}(x)$ over time has to satisfy

$$\widetilde{N}(x) = \lambda\, dt\,[1+\widetilde{N}(x)] + (1-\lambda\, dt - \beta\, dt)E\widetilde{N}(x+dx). \quad \text{(li)}$$

Note also that $\lim_{x\to\infty} \widetilde{N}(x) = \lambda/\beta$. To see this, we can obtain that the expected duration of the consumer in the market is $1/\beta$, and the expected duration of the product is $1/\lambda$. Then, if the consumer always repurchased the product when it broke down, the consumer would make on "average" λ/β purchases going forward. As the current utiliy of owning the product approaches infinity, the consumer behaves as if always repurchasing the product when it breaks down, and therefore the expected number of purchases going forward, $\widetilde{N}(x)$

approaches λ/β. Using this, when solving for (li), yields

$$\widetilde{N}(x) = C_2 e^{-\widetilde{\eta}x} + \frac{\lambda}{\beta}, \tag{lii}$$

where $\widetilde{\eta} = \sqrt{2\beta/s^2}$. and C_2 is a constant to be determined.

Consider now the evolution of $\widetilde{N}(x)$ for $x < \overline{x}$. This yields

$$\widetilde{N}(x) = \lambda\, dt\, N(x) + (1 - \lambda\, dt - \beta\, dt) E\widetilde{N}(x + dx). \tag{liii}$$

By Itô's Lemma, this can be written as

$$(\beta + \lambda)\widetilde{N}(x) = \lambda C_1 e^{\eta x} + \frac{s^2}{2}\widetilde{N}''(x). \tag{liv}$$

Note that $\lim_{x\to-\infty} \widetilde{N}(x) = 0$ as when the current utility x of owning the product approaches negative infinity, the consumer is expected not to make any more purchases going forward. Using this when solving for (liv) yields

$$\widetilde{N}(x) = C_3 e^{\widehat{\eta}x} + C_1 \frac{\lambda}{\lambda + \beta(1 - s^2/\sigma^2)} e^{\eta x}, \tag{lv}$$

where $\widehat{\eta} = \sqrt{2(\beta + \lambda)/s^2}$, and C_3 is a constant to be determined.

The value matching and smooth pasting conditions at $\overline{x}$ presented in the text yield

$$\frac{C_2}{\widetilde{Y}} + \frac{\lambda}{\beta} \quad = \quad C_3\widehat{Y} + C_1\overline{Y}\frac{\lambda}{\lambda + \beta(1 - s^2/\sigma^2)} \tag{lvi}$$

$$-\widetilde{\eta}\frac{C_2}{\widetilde{Y}} \quad = \quad \widehat{\eta}C_3\widehat{Y} + \eta C_1\overline{Y}\frac{\lambda}{\lambda + \beta(1 - s^2/\sigma^2)}. \tag{lvii}$$

where $\overline{Y} = e^{\eta\overline{x}}, \widetilde{Y} = e^{\widetilde{\eta}\overline{x}}$, and $\widehat{Y} = e^{\widehat{\eta}\overline{x}}$. Using (l), (lvi), and (lvii) we can obtain C_1, C_2, and C_3 as a function of $\overline{x}$.[10]

Using (l) in both (lvi) and (lvii) we can solve for $C_2/\widetilde{Y}$ to obtain

$$\frac{C_2}{\widetilde{Y}} = \frac{\widehat{\eta}s^2/\sigma^2 - \eta(\lambda + \beta)/\beta}{\widehat{\eta}\frac{\beta(1-s^2/\sigma^2)}{\lambda} + \widetilde{\eta}(\frac{\lambda+\beta(1-s^2/\sigma^2)}{\lambda}) + \eta}. \tag{lviii}$$

[10] Note that we do not have smooth pasting at $\overline{x}$ between $N(x)$ and $\widetilde{N}(x)$ as there is no optimality decision on the derivation of these functions. Note also that the smooth pasting condition for $\widetilde{N}(x)$ at $\overline{x}$ is not an optimality condition, but it is rather due to the infinite variation of the Brownian motion.

Using (l) we can then obtain (21).

Proof of Proposition 3:

The comparative statics for the expected number of purchases going forward immediately after a purchase can be directly obtained from evaluating (22) at $\overline{x}, N(\overline{x})$. The comparative statics for λ and β are straightforward. To get the comparative statics with respect to the ratio s^2/σ^2, let $\varepsilon = \sqrt{s^2/\sigma^2}$ and $w = \sqrt{1+\lambda/\beta}$. Then, we can obtain

$$\widetilde{N}(\overline{x}) = w^2 - 1 + \frac{w(w-1)(\varepsilon^2 - w\varepsilon)}{1-\varepsilon^2+(w-1)(1+\varepsilon)}, \tag{lix}$$

where the derivative to ε is negative.

The comparative statics with respect to $\beta, \widetilde{r}$, and P, of the expected number of purchases going forward after an initial current utility of $x < \overline{x}$ can be directly obtained by differentiating (22). The comparative statics with respect to $s^2 = \sigma^2$ and λ require a little more analysis.

Consider first the comparative statics with respect to $s^2 = \sigma^2$. We can obtain:

$$\frac{\partial N(x)}{\partial \sigma^2}_{\sigma^2=s^2} = \left[\frac{\partial \eta}{\partial \sigma^2}(x-\overline{x}) - \eta \frac{\partial \overline{x}}{\partial \sigma^2}_{\sigma^2=s^2}\right] N(x). \tag{lx}$$

As both $\frac{\partial \eta}{\partial \sigma^2}(x-\overline{x})$ and $\eta\frac{\partial \overline{x}}{\partial \sigma^2}_{\sigma^2=s^2}$ are positive, we can see that the size of $|x-\overline{x}|$ determines the sign of $\frac{\partial N(x)}{\partial \sigma^2}_{\sigma^2=s^2}$, which is positive (negative) is X is low (high) enough.

Consider now the comparative statics with respect to λ. We can obtain

$$\frac{\partial N(x)}{\partial \lambda} = \frac{1}{2}e^{\eta(x-\overline{x})}\left[\frac{1}{\beta}\left(1+\frac{1}{2}\sqrt{\frac{\beta}{\lambda+\beta}}\right) - \eta\frac{\partial \overline{x}}{\partial \lambda}\right], \tag{lxi}$$

which is negative if β is not too low.

Derivation of $G(x)$ and $\widetilde{G}(x)$:

We have that $G(x)$ evolves over time as

$$G(x) = e^{-r\,dt} EG(x+dx), \tag{lxii}$$

which yields

$$G(x) = C_1 e^{\mu x} + C_2 e^{-\mu x} \tag{lxiii}$$

where C_1 and C_2 are constants to be determined. As $\lim_{x\to-\infty} G(x) = 0$, we have $C_2 = 0$.

Let $\widetilde{G}(x)$ be the discounted number of future units purchased over time given that the consumer owns the product. As the consumer purchases the product immediately if the consumer does not own the product and $x = \overline{x}$, we have

$$G(\overline{x}) = 1 + \widetilde{G}(\overline{x}). \tag{lxiv}$$

For $x \geq \overline{x}$ the evolution of $\widetilde{G}(x)$ over time has to satisfy

$$\widetilde{G}(x) = \lambda\, dt\, [1 + \widetilde{G}(x)] + (1 - \lambda\, dt)e^{-r\, dt} E\widetilde{G}(x + dx), \tag{lxv}$$

which yields

$$\widetilde{G}(x) = C_3 e^{\widetilde{\mu}x} + C_4 e^{-\widetilde{\mu}x} + \frac{\lambda}{r}, \tag{lxvi}$$

where C_3 and C_4 are constants to be determined. As $\lim_{x\to\infty} \widetilde{G}(x) = \lambda/r$, we have $C_3 = 0$.

Consider now the evolution of $\widetilde{G}(x)$ for $x < \overline{x}$. This yields

$$\widetilde{G}(x) = \lambda\, dt\, G(x) + (1 - \lambda\, dt)e^{-r\, dt} E\widetilde{G}(x + dx). \tag{lxvii}$$

By Itô's Lemma, this can be written as

$$(r + \lambda)\widetilde{G}(x) = \lambda C_1 e^{\mu x} + \frac{s^2}{2}\widetilde{G}''(x), \tag{lxviii}$$

which yields

$$\widetilde{G}(x) = C_5 e^{\widehat{\mu}x} + C_6 e^{-\widehat{\mu}x} + C_1 \frac{\lambda}{\lambda + r(1 - s^2/\sigma^2)} e^{\mu x}, \tag{lxix}$$

where C_5 and C_6 are constants to be determined. As $\lim_{x\to-\infty} \widetilde{G}(x) = 0$, we have $C_6 = 0$.

Using value matching and smooth pasting at $\overline{x}$ for $\widetilde{G}(x)$, $\widetilde{G}(\overline{x}^+) = \widetilde{G}(\overline{x}^-)$ and $\widetilde{G}'(\overline{x}^+) = \widetilde{G}'(\overline{x}^-)$, yields

$$\frac{C_4}{\widetilde{X}} + \frac{\lambda}{r} = C_5 \widehat{X} + C_1 \overline{X} \frac{\lambda}{\lambda + r(1 - s^2/\sigma^2)} \tag{lxx}$$

$$-\widetilde{\mu}\frac{C_4}{\widetilde{X}} = \widehat{\mu} C_5 \widehat{X} + \mu C_1 \overline{X} \frac{\lambda}{\lambda + r(1 - s^2/\sigma^2)}. \tag{lxxi}$$

Using (lxiv), (lxx), and (lxxi) we can obtain C_1, C_4, and C_5 as a function of $\overline{x}$.[11]

[11] Note that we do not have smooth pasting at $\overline{x}$ between $G(x)$ and $\widetilde{G}(x)$ as there is no optimality decision

Using (lxiv) in both (lxx) and (lxxi) we can solve for $C_4/\widetilde{X}$ to obtain

$$\frac{C_4}{\widetilde{X}} = \frac{\widehat{\mu}s^2/\sigma^2 - \mu(\lambda + r)/r}{\widehat{\mu}\frac{r(1-s^2/\sigma^2)}{\lambda} + \widetilde{\mu}(\frac{\lambda+r(1-s^2/\sigma^2)}{\lambda}) + \mu}. \tag{lxxii}$$

Using (lxiv) we can then obtain (23).

PROOF OF PROPOSITION 4:

Given the presentation in the text, we have the characterization of the equilibrium as a function of x_0. To see that $\overline{x}$ is increasing in x_0 for $x_0 > x^{**}$ we can see that the right hand side of (31) is decreasing in $\overline{x}$ and increasing in x_0. To see that $\overline{x} < x_0$ for $x_0 > x^{**}$ we can just obtain that the total differentiation of (31) with respect to x_0 and $\overline{x}$ yields $\frac{\partial \overline{x}}{\partial x_0} < 1$.

For the comparative statics with respect to λ, r, and σ^2 (under the constraint $s^2 = \sigma^2$) let us consider each region of x_0 separately.

For $x_0 < x^*$, note that we can write (27) as

$$\overline{x} - \frac{\widehat{X} - 1}{\widehat{X}}\left(\frac{1}{\widehat{\mu}} + \frac{1}{\mu}\right) = 0. \tag{lxxiii}$$

The derivative of the left hand side with respect to $\overline{x}$ can be obtained to be $1 - a/(e^a - 1)$, after using (27), where $a = \widehat{\mu}\overline{x}$. We can then obtain that that derivative is positive for $a > 0$. Taking the derivative of (lxxiii) with respect to $\widehat{\mu}$ we can obtain that it has the same sign of $(e^a - 1)^2 - a^2 e^a$, which is positive for $a > 0$. Then, as $\widehat{\mu}$ is increasing in λ we can obtain $\frac{\partial \widehat{x}}{\partial \lambda} < 0$.

The derivative of the left hand side of (lxxiii) with respect to r is equal to the derivative with respect to λ, which we saw was positive, plus $\frac{\widehat{X}-1}{\widehat{X}}\frac{1}{\mu^2}\frac{\partial \mu}{\partial r} > 0$, which then yields $\frac{\partial \widehat{x}}{\partial r} < 0$.

The derivative of the left hand side of (lxxiii) with respect to σ^2 under the constraint σ^2 is composed with two terms, one though $\widehat{\mu}$ and the other through μ, where both are negative (the first one has the opposite sign of the derivative with respect to λ and the second is $\frac{\widehat{X}-1}{\widehat{X}}\frac{1}{\mu^2}\frac{\partial \mu}{\partial \sigma^2} < 0$). This then yields $\frac{\partial \widehat{x}}{\partial \sigma^2}_{|s^2=\sigma^2} > 0$.

on the derivation of these functions. Similarly to the case of $\widetilde{N}(x)$ above, note also that the smooth pasting condition for $\widetilde{G}(x)$ at $\overline{x}$ is not an optimality condition, but it is rather due to the infinite variation of the Brownian motion.

To check the effect on P we can use (27) in (6) to obtain

$$P = \overline{x}\left(\frac{1}{\lambda + r} + \frac{1}{\sqrt{r(\lambda + r)}}\right). \tag{lxxiv}$$

Given the results on the comparative statics on $\overline{x}$ we can then immediately obtain $\frac{\partial P}{\partial \lambda}, \frac{\partial P}{\partial r} < 0$, and $\frac{\partial P}{\partial \sigma^2}_{|s^2=\sigma^2} > 0$.

For $x_0 \in [x^*, x^{**}]$, we can just use the derivation in Proposition 1 as the purchase threshold is fixed at x_0, and we can just compute the effect of λ, r, and σ^2, under the constraint of $s^2 = \sigma^2$, by just total differentiating (6) with respect to P and each of the variables under interest. We can then obtain that the optimal price is decreasing in λ, r, and σ^2, under the constraint of $s^2 = \sigma^2$.

Consider now the case of $x_0 > x^{**}$. Consider first the effect of λ. We can obtain that the derivative of the right hand side of (31), using also that expression, has the sign equal to the sign of $1 - C_1/2 - C_2/2$ where

$$C_1 = \sqrt{1 + \frac{\lambda}{r}} \Big/ \left(\sqrt{1 + \frac{\lambda}{r}} - 1\right), \tag{lxxv}$$

$$C_2 = \frac{\frac{ae^a}{e^a - 1}\left(1 - \frac{a}{e^a - 1}\right) + \sqrt{1 + \frac{\lambda}{r}}}{\frac{ae^a}{e^a - 1} - 1 + \sqrt{1 + \frac{\lambda}{r}}}. \tag{lxxvi}$$

We can then obtain that $C_1, C_2 > 1$, so that this derivative is negative, which implies that $\overline{x}$ is decreasing in λ given that the right hand side of (31) is decreasing in $\overline{x}$. From (6) we can then obtain that P is also decreasing in λ given Proposition 1.

Consider now the effect of r. The derivative of the right hand side of (31) with respect to r can be obtained to be

$$-\frac{\lambda}{r^2} + \frac{\lambda}{2r^2}\frac{\sqrt{1 + \frac{\lambda}{r}}}{\sqrt{1 + \frac{\lambda}{r}} - 1} - \frac{1}{2r}\frac{\frac{ae^a}{e^a - 1}\left(1 - \frac{a}{e^a - 1}\right) - \frac{\lambda}{r}\sqrt{1 + \frac{\lambda}{r}}}{\frac{ae^a}{e^a - 1} - 1 + \sqrt{1 + \frac{\lambda}{r}}} + \left(1 + \frac{\lambda}{r}\right)(x_0 - \overline{x})\frac{1}{\sqrt{2rs^2}} \tag{lxxvii}$$

which can be either negative or positive. For example, it can be negative for $x_0 = x^{**}$ and λ large; it can be positive for λ small and x_0 large. Then, $\overline{x}$ can either increase or decrease with an increase in r.

Finally, consider the effect of σ^2, under the constraint $s^2 = \sigma^2$. The sign of the derivative of the right hand side of (31) can be obtained to be the same as the sign of

$$-\frac{ae^a}{e^a-1}\left(1-\frac{a}{e^a-1}\right)e^{\widetilde{\mu}(\overline{x}-x_0)}+2\widetilde{x}(x_0-\overline{x})\frac{1+\frac{\lambda}{r}}{\sqrt{1+\frac{\lambda}{r}}-1}, \qquad \text{(lxxviii)}$$

which can be either positive or negative. For example, it can be positive if x_0 is sufficiently large; and it can be negative if $x_0 = x^{**}$.

REFERENCES

Bergemann, D., and J. Välimäki (1996), "Learning and Strategic Pricing," *Econometrica,* **64(5)**, 1125-1149.

Branco, F., M. Sun, and J.M. Villas-Boas (2012), "Optimal Search for Product Information," *Management Science,* **58(11)**, 2037-2056.

Bulow, J. (1982), "Durable Goods Monopolists," *Journal of Political Economy,* **90(2)**, 314-332.

Che, Y.-K., and K. Mierendorff (2019), "Optimal Dynamic Allocation of Attention," *American Economic Review,* **109(8)**, 2993-3029.

Coase, R.H. (1972), "Durability and Monopoly," *Journal of Law and Economics,* **15(1)**, 143-149.

Dixit, A. (1993), *The Art of Smooth Pasting,* Harwood Academic Publishers: Chur, Switzerland.

Erdem, T., and M. Keane (1996), "Decision-Making under Uncertainty: Capturing Dynamic Brand Choice Processes in Turbulent Consumer Goods Markets," *Marketing Science,* **15(1)**, 1-20.

Felli, L., and C. Harris (1996), "Learning, Wage Dynamics, and Firm-Specific Human Capital," *Journal of Political Economy,* **104(4)**, 838-868.

Fudenberg, D., P. Strack, and T. Strzalecki (2018), "Speed, Accuracy, and the Optimal Timing of Choices," *American Economic Review,* **108(12)**, 3651-84.

Hendricks, K., A. Sorensen, and T. Wiseman (2012), "Observational Learning and Demand for Search Goods," *American Economic Journal: Microeconomics,* **4(1)**, 1-31.

Ke, T.T., Z.-J.M. Shen, and J.M. Villas-Boas (2016), "Search for Information on Multiple Products," *Management Science,* **62(12)**, 3576-3603.

Ke, T.T., and J.M. Villas-Boas (2019), "Optimal Learning before Choice," *Journal of Economic Theory,* **180**, 383-437.

Moscarini, G., and L. Smith (2001), "The Optimal Level of Experimentation," *Econometrica,* **69(6)**, 1629-1644.

Nelson, P. (1970), "Information and Consumer Behavior," *Journal of Political Economy,* **78(2)**, 311-329.

Roberts, K., and M.L. Weitzman (1981), "Funding Criteria for Research, Development, and Exploration Projects," *Econometrica,* **49(5)**, 1261-1288.

Stokey, N. (1979), "Intertemporal Price Discrimination," *Quarterly Journal of Economics,* **93(3)**, 355-371.

Stokey, N. (1981), "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics,* **12(1)**, 112-128.

Tucker, C., and J. Zhang (2011), "How Does Popularity Information Affect Sales? A Field Experiment," *Management Science,* **57(5)**, 828-842.

Weitzman, M.L. (1979), "Optimal Search for the Best Alternative," *Econometrica,* **47(3)**, 641-654.