# Choice Deferral and Search Fatigue*

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## Abstract

When gathering information to make decisions, individuals often have to delay making a decision because the process of gathering information is interrupted, and the individual is not yet ready to make a decision. The paper considers a model of choice deferral based on time-varying search costs, potentially based on search fatigue, in which individuals have to strategically decide whether to defer choice when information gathering is interrupted, taking into account the current available information, and when they will be able to resume gathering information. We find that individuals are more likely to defer choice when information gathering is interrupted less frequently, when individuals can resume gathering information sooner, and when they discount less the future. We also consider the case in which individuals incur costs of re-starting a process of information gathering, and cases in which the individual has greater or less information about the extent of search fatigue. The paper also considers optimal pricing and shows how pricing should respond to the length of consumer browsing sessions, and gaps between browsing sessions.

# 1. Introduction

When gathering information to make decisions, an individual often has to delay making a decision because the process of gathering information is interrupted, and the individual is not yet ready to make a decision. This interruption can be caused by time-varying search costs, potentially based on search fatigue. When information gathering is interrupted, individuals have to strategically decide whether to make a choice based on current available information or defer choice until they have a chance to gather further information. Choice deferrals often occur in the health, food, financial, entertainment, and general consumption domains.

For example, when a consumer searches online for information to decide whether to purchase a certain product, the consumer may at some point have to interrupt the search process without having yet sufficient diagnostic information, and at that point has to decide whether to purchase the product right away, or delay purchase until a future time when the consumer is again able to search for information on the product. The interruption could be based on search fatigue increasing the search costs, or because the consumer has to perform another task which temporarily increases the opportunity costs of searching for product information (Li, Capra, and Zhang 2020). For example, the consumer starts gathering information before dinner, and then the dinner time arrives and the consumer can choose to either purchase the product right away given the available information, or delay choice until the consumer has again a chance to look for information after dinner. Interruptions can also occur in offline shopping, when after shopping for a while the consumer may have to leave the store at some point, without having made a decision.

Another possibility could be a manager having to make a strategic decision, for example, on whether to launch a new product. The manager could be gathering information about whether to launch the product, and then have that process be interrupted with another managerial activity that may last some period of time. The manager then has to decide whether to launch the product right away, or wait until the manager has more time to analyze the potential success of the product launch.

In order to consider the possibility of choice deferral, we formulate a model in which an individual gathers information gradually to decide whether to adopt an alternative. The individual can be either in a state of low search costs or a state of high search costs, and move across states at some hazard rate. In the consumer setting, these hazard rates could be relatively high for online shopping as consumers could frequently start and stop browsing sessions, but be lower for offline shopping, especially if it is a store that is of difficult access.

To simplify the analysis, we consider that the individual has zero search costs in the low search costs state, and very high search costs in the high search costs state. Thus, the individual gathers information when she is in the low search costs state, and prefers not to gather information when she is in the high search costs state.

Suppose an individual has not gathered sufficient information to make a choice before the moment when the search costs increase, then at thatmoment, the individual has two options. The individual can either defer choice until she can resume gathering information at low search costs, or make an immediate choice using current information. In order to obtain strategic effects at the time when the search process is interrupted, we consider that the individual either discounts the future or has a fixed cost of re-starting the search process. Either of these possibilities leads the individual, at the time when the search costs increase, to potentially decide not to defer choice, even though the individual has not made a choice up to that point, a phenomenon which is termed in the paper as choice closure.[1] If there is neither discounting nor fixed costs of re-starting the search, individuals would just defer choice automatically when the search costs increase.

When gathering information (that is, in the low search costs state), the individual makes a choice if the individual obtains sufficiently diagnostic information. When the search costs increase, the consumer may decide to make a choice then (i.e., choice closure), because, even though the individual has not received sufficient positive information, the current evaluation is close enough such that it is better to make the choice now, than to wait for the search costs to come down again.

We find that individuals are more likely to defer choice when information gathering is interrupted less frequently, when individuals can resume gathering information sooner, and when they discount less the future. When individuals can resume gathering information sooner, or when individuals discount less the future, future information becomes in expectation more valuable when evaluated at the time when the deferral decision is made, and so individuals defer more choice. When information gathering is interrupted more frequently, individuals know that when they resume gathering information, they will be again interrupted quickly, therefore leading to a lower payoff from deferring choice. Seen the other way, individuals do more choice deferral when information gathering is interrupted less frequently. In terms of choice closure, we find that the extent of choice closure increases when information gathering is interrupted less frequently, when interruptions last longer, and when the individuals discount the future less.

[1] For a related concept, see, for example, Webster and Kruglanski (1994), Choi et al. (2008).

If the interruption to information gathering is caused by search fatigue, then as the individual does more search, the individual may be aware that she is getting more tired of search over time. To capture this, we consider an extension with three states. The individual moves from the full-rested search state to the fatigued search state, from the fatigued search state to the no-search state, then from the no-search state back to the full-rested search state. The individual expects information gathering to be interrupted sooner in the fatigued search state than in the fully-rested search state, reflecting her awareness of search fatigue. We show that the individual's choice deferral and choice closure behaviors are similar to those we find in the main model. We also find that the individual requires less positive information to make a choice in the fatigued search state than in the fully-rested search state, and the extent of this reduction is greater when the rate of fatigue is higher and when the individual discounts the future less.

We also consider the case in which individuals incur costs of re-starting a process of information gathering.[2] In that case, we do not need discounting for the choice deferral decision to be strategic. This case could be important empirically as the extent of time discounting between different opportunities to gather information may be relatively small (for example, days). The existence of costs of re-starting the information gathering phase can be seen as a possibility that leads to significant strategic effects at the time when the choice deferral decision is made.

We also derive a firm's optimal pricing strategy given the individuals' choice deferral behavior. If the initial expected value of adopting the alternative is low, the firm sets a price such that the consumer does not adopt it before gathering some information. In such a case, we find that the optimal price should be higher when the speed of information gathering is greater, when information gathering is interrupted less frequently, when the individuals can resume information gathering sooner, and when the individuals discount the future less. We also find that these comparatives statics are reversed if the initial expected value of adopting the alternative is sufficiently high. These results show how firms should use data on consumer browsing sessions to determine price, and provide managerial implications on how price should change following other interventions to reduce search fatigue or restart consumer search sooner, such as redesigning user interface, ad re-targeting, email marketing, and push notifications.

There is substantial work documenting the existence of choice deferrals by individuals, because of the inability to make a decision (see Anderson 2003, Chernev, Böckenholt, and

[2]See Byrne and de Roos (2022) on evidence on the existence of start-up search costs.

Goodman 2015, Scheibehenne, Greifeneder, and Todd 2010, for reviews). This work has characterized the causes for choice deferral, and its consequences. For example, this work has investigated the role of dominance relations, option desirability, attribute commonality, and attribute alignability on choice deferral (e.g., Chernev and Hamilton 2009, Dhar 1997, Gourville and Soman 2005, Tversky and Shafir 1982), and that the option of choice deferral may affect individual choices and affect behavioral effects (e.g., Dhar and Simonson 2003). Bhatia and Mullet (2016) consider a sequential learning model with the possibility of choice deferral which provides an explanation for several of the behavioral effects obtained. A significant explanation for not choosing has been choice overload, the existence of too many options may deter choice, which can also been seen as deferral of choice. Examples of work providing explanations for this effect of choice overload include Kamenica (2008), Kuksov and Villas-Boas (2010), Villas-Boas (2009). In this paper, the existence of multiple alternatives is not going to play any role, and the decision of choice deferral comes from the difficulty of the decision being made, and from time varying search costs (or, alternatively, time-varying information gained). There is also work showing evidence of search gaps and search fatigue when consumers search across multiple alternatives (e.g., Ursu, Zhang, and Honka 2022).

In relation to the existing literature, a significant innovation of this paper is to formally consider future choice opportunities once choice is deferred. That is, while in the existing literature choice deferral is considered as no choice, here we formally consider the possibility of future choices when the individuals have again a chance to search for information. This formulation allows us to study choice deferral and choice closure as strategic decisions, and as an application, our study shows how optimal pricing in e-commerce should depend on the lengths of consumer browsing sessions and the gaps between browsing sessions.

The remainder of the paper is organized as follows. The next section introduces a base model of choice deferral with discounting. Section 3 presents the analysis and results, and Section 4 considers the effect of awareness of search fatigue. Section 5 considers the case in which there are start-up search costs. Section 6 discusses optimal pricing. Section 7 concludes. The Appendix collects the proofs of the results.

## 2. The Model

Consider the following simple model of choice deferrals. A decision-maker (DM) is gradually collecting information about whether to adopt an alternative. Suppose time is continuous. The DM can be either in a "search" mode or in a "no-search" mode. In the search mode, the DM has zero search costs, while in the no-search mode, the DM's search costs are sufficiently high such that the DM does not search for information.

Whether the DM is in the "search" or in the "no-search" mode is exogenous. If the DM is in the search mode, the DM moves to the no-search mode with a constant hazard rate of $\lambda$. If the DM is in the no-search mode, the DM moves to the search mode with a constant hazard rate of $\beta$. When the DM is in the search mode, the DM updates the expected value of adopting the alternative, and can choose to adopt the alternative at any time. In the no-search mode, the DM does not receive any information. At the instance when the DM moves from the search mode to the no-search mode, if the DM's beliefs about the alternative are not sufficiently high, the DM may choose to defer choice until the DM is again in the search mode.

This set-up captures the idea that the DM sometimes has the ability to search for information, and other times cannot search for information. This can also be interpreted as search fatigue, as the DM suddenly has high search costs after some periods of information gathering, stops getting information on the alternative, and decides to delay making a choice until the DM has again a chance to learn more information about the alternative (the DM gets sufficiently rested such that the DM returns to the search mode). Another interpretation is that, instead of higher search costs, search fatigue makes additional search uninformative, so the DM has to rest for some periods before gathering information again.

At each moment in time, the DM has some expected value of the payoff of the alternative, which we denote by $x$. When the DM is in the search mode, $x$ evolves as a Brownian motion with a constant variance $\sigma^2$. This can be interpreted as the DM learning over time about equally important and independent attributes, and there being an infinite number of attributes (e.g., Branco, Sun, and Villas-Boas 2012).[3] When the DM is in the no-search mode, the expected value, $x$, stays fixed (as no information is gained). The payoff of not adopting the alternative is set at zero. The DM discounts the future at a continuous-time discount rate $r$ and does not incur any on-going search costs when learning information. The discount rate can also be seen as the rate at which the alternative disappears. For example, a consumer considering purchasing a product may find the product out of stock, or a manager considering launching a product may find that the opportunity has passed.

The optimal search behavior of the DM would be to adopt the alternative, when in the search mode, if the expected payoff of the alternative $x$ reaches a threshold $\overline{x}$. When

[3] Alternatively, this case could be seen as the limit case when there is a finite but large number of attributes.

in the no-search mode, the DM would adopt the alternative if the expected payoff of the alternative is above some threshold $\widetilde{x}$. Note that at the instant at which the DM moves from the search mode to the no-search mode, if $x \in [\widetilde{x}, \overline{x})$, the DM chooses to adopt the alternative immediately because of the costly delay of getting any additional information. This is the case of choice closure. If $x < \widetilde{x}$ at the instance when the DM moves from the search mode to the no-search mode, the DM decides not to adopt the alternative then, and waits until the DM switches again to the search mode and gain further information then. This is the case in which the DM defers choice. Note that this means that there is a positive mass probability of the DM adopting the alternative at an instant when the DM moves from the search to the no-search mode. The next section will study how to obtain the thresholds $\overline{x}$ and $\widetilde{x}$, and their properties. (Table 1 presents the notation used throughout the paper.)

*Assumptions and Extensions*

The existence of constant hazard rates of moving between the search and no-search mode allows the problem to be stationary so that the threshold of whether to adopt the alternative is constant over time. If the hazard rates of moving between the search mode and the no-search mode are not constant, then the thresholds of whether to adopt the alternative would also not be constant leading to significant complications in the analysis (it could still be characterized numerically, but analytical results would be difficult to obtain).

Note that this set-up can be interpreted as search fatigue leading to the DM stopping to search: the DM would be endowed with a search fatigue limit when starting a search process, but would not know when that search fatigue limit is. With a constant hazard rate the process is memoryless, and therefore, from the point of view of the DM, she gets search fatigue with the same likelihood, independently of how long the DM has been searching. If, however, the DM understands that she is getting more fatigued over time from search, we would expect the hazard rate of moving from the search mode to the no-search mode to be increasing in the length of time that the DM has been in the search mode. This would lead the threshold to adopt the alternative to vary over time (in fact, to decrease with the length of time in the search mode). This case is considered in Section 4 when search fatigue occurs over two stages.

Note that, similarly, we could expect the hazard rate of moving from the no-search mode to the search mode to be increasing in the length of time spent in the no-search mode, because a longer rest from search should lead to a greater likelihood of returning to search for information again. This possibility would not affect the results presented here, as the DM would prefer to continue waiting until the switch to the search mode, as that switch is expected to be sooner.

If learning is done with signals about the overall value of the product, or if attributes have unequal importance with the DM checking first the most important attributes, or if there is non-zero correlation between the attributes, then we would have $\sigma^2$ to be decreasing over time, leading again to a threshold to adopt the alternative that is varying (decreasing) over time, which is a more complicated case to consider. The case presented here can be seen as the extreme case if the amount of information learned over time is constant, in contrast to the other extreme case in which all information about the alternative is learned in one shot. The real world would be somewhere between these two extreme cases.

Note that discounting is crucial for the problem as presented. If there is no discounting, the DM would always defer choice when moving to the no-search mode, and choice deferral becomes non-strategic. One alternative to discounting is to have start-up search costs each time the search mode starts, and that case is considered in Section 5.

The base model assumes that that there are no ongoing search costs. If there are ongoing search costs when learning for information, then the DM would also have another threshold such that the DM permanently leaves the search process without adopting when the expected payoff of adopting the alternative drops below the threshold. We do not consider this case in the base model to simplify the analysis, as this case is not essential to obtain the strategic choice deferral effects. The ongoing search costs and the quitting threshold are considered in Section 5.

The assumption that the payoff of not adopting the alternative is zero is not without loss of generality. In fact, if the payoff of the outside option is positive, the DM has to consider the trade-off between losing the discounted payoff of the outside option and continuing to search for further information on the focal alternative. This would lead again to the existence of a lower threshold such that the DM leaves the search process by taking the outside option if the expected payoff of adopting the alternative drops below the threshold. We again do not consider this possibility to simplify the analysis, as this possibility is not essential to obtain the choice deferral effects.

## 3. Analysis

In order to consider the optimal decisions of the DM, we have to consider the expected present discounted value of the DM under the optimal decisions, depending on the state in which the DM is in. Let $V(x)$ be the expected discounted payoff for the DM if the DM is in the search mode, and $W(x)$ be the expected payoff for the DM if the DM is in the no-search mode, if the DM's current expected utility from adopting the alternative is $x$.

The Bellman equation for $V(x)$ for $x < \widetilde{x}$ can be written as

$$V(x) = (1 - \lambda\, dt)e^{-r\, dt} EV(x + dx) + \lambda\, dtW(x). \tag{1}$$

(Note that we could have $e^{-r\, dt} EW(x + dx)$ instead of $W(x)$ in (1) and the subsequent analysis would not change, as the second order terms in $(dt)^2$ disappear as $dt \to 0$.) The Bellman equation for $V(x)$ for $x \in (\widetilde{x}, \overline{x})$ can be written as

$$V(x) = (1 - \lambda\, dt)e^{-r\, dt} EV(x + dx) + \lambda\, dtx. \tag{2}$$

Applying Itô's Lemma to (2), we can obtain the following second-order differential equation in $V(x)$:

$$V(x) = \frac{\sigma^2}{2(r + \lambda)} V''(x) + \frac{\lambda}{r + \lambda} x \tag{3}$$

The Bellman equation for $W(x)$ can be written as

$$W(x) = \beta\, dtV(x) + (1 - \beta\, dt)e^{-r\, dt} W(x), \tag{4}$$

from which one can obtain $W(x) = \frac{\beta}{r+\beta} V(x)$. Substituting $W(x)$ into (1), and using Itô's Lemma, we can obtain the second-order differential equation in $V(x)$ for $x < \widetilde{x}$ as

$$r\frac{r + \beta + \lambda}{r + \beta} V(x) = \frac{\sigma^2}{2} V''(x). \tag{5}$$

Solving the above second-order differential equations for $V(x)$, and using value matching and smooth pasting of $V(x)$ at $\widetilde{x}$ and $\overline{x}$, $V(\widetilde{x}^-) = V(\widetilde{x}^+), V'(\widetilde{x}^-) = V'(\widetilde{x}^+)$, $V(\overline{x}) = \overline{x}$, $V'(\overline{x}) = 1$, and $W(\widetilde{x}) = \widetilde{x}$, we obtain a system of five equations (presented in the Appendix) to obtain $\widetilde{x}$ and $\overline{x}$.

Defining, $\delta = \overline{x} - \widetilde{x}$, $\mu = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$, and $\widetilde{\mu} = \sqrt{\frac{2(r+\lambda)}{\sigma^2}}$, we can obtain (see Appendix)

$$\beta(D-1)\left\{\mu(r+\lambda)\left[1+D-\frac{\delta\widetilde{\mu}}{D-1}(1+D^2)\right]+\widetilde{\mu}(r-\lambda)(D-1)-\delta\widetilde{\mu}^2 r(1+D)\right\}+ \\ (r+\lambda)[r(D^2-1)(\mu-\widetilde{\mu}^2\delta)+r\widetilde{\mu}(1+D^2)(1-\mu\delta)+2\widetilde{\mu}\lambda D]=0 \quad (6)$$

which determines $\delta$, where $D = e^{\widetilde{\mu}\delta}$. We can then also obtain $\widetilde{x}$ as a function of $\delta$ as

$$\widetilde{x} = \beta\frac{r + r\widetilde{\mu}\delta + \lambda D}{D[\widetilde{\mu}r(r+\beta+\lambda)+\mu(r+\beta)(r+\lambda)]-\widetilde{\mu}\beta r}. \quad (7)$$

Note that from (6) and (7) we can obtain that both $\delta/\sigma$ and $\widetilde{x}/\sigma$ are independent of $\sigma$. The reason is that the standard deviation of the DM's belief process in a unit of time is $\sigma$, and so all optimal thresholds are then proportional to $\sigma$.

In this model, $\widetilde{x}$ can be seen as a measure of the extent of choice deferral. When switching from the search mode to the no-search mode, the DM defers if and only if $x < \widetilde{x}$. On the other hand, $\delta = \overline{x} - \widetilde{x}$ can be seen as a measure of the extent of choice closure. When switching from the search mode to the no-search mode, the DM adopts the alternative immediately if $\widetilde{x} \leq x < \overline{x}$, even though the DM would not adopt the alternative if she is still in the search mode.

Figure 1 illustrates a sample path in which the individual makes the decision to take the alternative during the search mode after several choice deferrals. Figure 2 illustrates a sample path in which the individual makes the decision to take the alternative when switching from the search to the no-search mode (i.e., choice closure) after several choice deferrals.

Given the complexity of (6), it is difficult to analyze explicit expressions for $\overline{x}$ and $\widetilde{x}$ for the general case. We focus first on two limiting cases: (1) $\beta \to 0$, the case in which once the DM leaves the search mode the DM almost never comes back to search, and (2) $\beta \to \infty$, the case in which DM returns infinitely quickly to the search mode once the DM goes into the no-search mode. These two extreme cases can also be seen as benchmarks for the extent of choice deferral and the extent of choice closure. Without the opportunity to go back to the search mode, as $\beta \to 0$, the DM adopts the alternative myopically in the no-search mode – adopting as long as the payoff is positive ($\widetilde{x} \to 0$). It corresponds to the least extent of choice deferral. When the DM goes back from the no-search mode to the search mode immediately, as $\beta \to \infty$, she does not adopt the alternative prematurely without receiving enough positive signals when switching to the no-search mode. So, the adoption thresholds

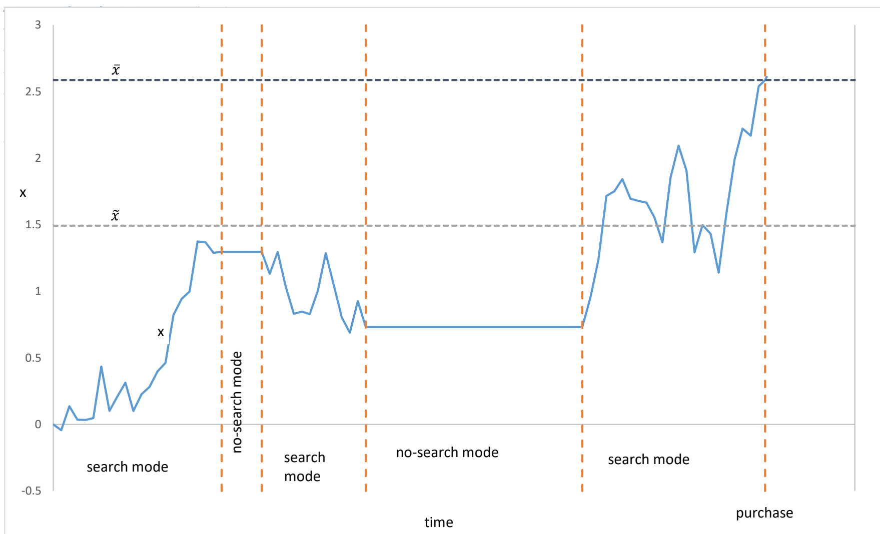



Figure 1: Example of sample path of individual expected payoff when making a decision during the search mode with $x_0 = 0, r = .05, \lambda = \beta = .5$, and $\sigma^2 = 1$. For these parameter values we have $\overline{x} \approx 2.59$ and $\widetilde{x} \approx 1.49$.

in both the search and no-search modes are identical ($\delta = \overline{x} - \widetilde{x} \to 0$). It corresponds to the least extent of choice closure. We then consider the general case and present some numerical illustrations.

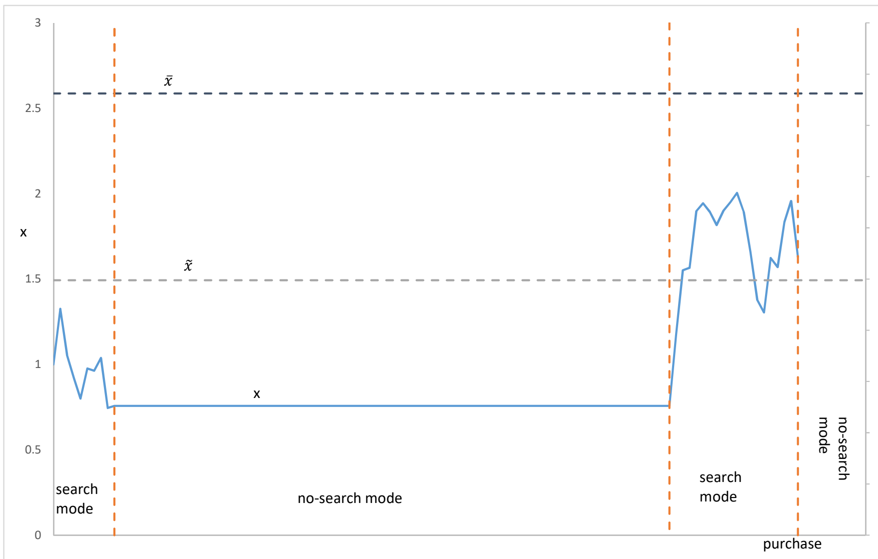



Figure 2: Example of sample path of individual expected payoff when making a decision when moving search to no-search mode (choice closure) with $x_0 = 1, r = .05, \lambda = \beta = .5$, and $\sigma^2 = 1$. For these parameter values we have $\overline{x} \approx 2.59$ and $\widetilde{x} \approx 1.49$.

*Case of* $\beta \to 0$ :

In the case of $\beta \to 0$ we have that $\widetilde{x} \to 0$, such that when the search mode ends the DM adopts the alternative as long as $x \geq 0$. We can also then obtain that $\overline{x}$ in the limit solves

$$e^{\mu\overline{x}}(1-\mu\overline{x})+\frac{\lambda}{r}=0. \tag{8}$$

From this we can obtain that $\overline{x} > 1/\mu$ and that at the limit $\overline{x}$ is increasing in $\sigma^2$ and decreasing in $\lambda$ and $r$.

For this case, we can obtain that $\widetilde{x}/\beta \to \frac{1}{r(\mu+\widetilde{\mu})}$, which shows that for $\beta$ small, $\widetilde{x}$ is increasing in both $\beta$ and $\sigma^2$, and decreasing in the discount rate $r$ and the hazard rate of moving to the no-search mode $\lambda$.

We collect these results in the following proposition.

**Proposition 1.** *Suppose that $\beta$ is sufficiently small. Then the purchase threshold in the search mode, $\overline{x}$ is increasing in $\sigma^2$ and decreasing in both $\lambda$ and $r$, and the purchase threshold in the no-search mode, $\widetilde{x}$, is increasing in both $\beta$ and $\sigma^2$, and decreasing in both $\lambda$ and $r$. The difference, $\delta = \overline{x} - \widetilde{x}$, is increasing in $\sigma^2$ and decreasing in both $\lambda$ and $r$.*

As the information gained in the search mode, $\sigma^2$, is greater the DM gains more from search, and chooses to search more, which results in both purchase thresholds to increase. When the discount rate increases, the present value of delaying purchase is reduced, and therefore the DM searches less, which means that both purchase thresholds fall. Similarly, when the likelihood of moving from the search mode to the no-search mode increases, the likelihood of being able to continue to search decreases, and therefore the DM prefers to make the adoption decision sooner, which means that both purchase thresholds fall.

For a fixed $\sigma^2$, the extent of choice deferral can be seen as increasing in $\widetilde{x}$ and, therefore, is decreasing in $\lambda$ and $r$. As the discount rate, $r$, or the rate at which the DM moves from the search to the no-search mode, $\lambda$, increases, the DM values future search sessions less. Thus, the expected benefit from deferring the choice and gaining more information decreases and the DM decreases the purchase threshold when in the no-search mode, $\widetilde{x}$. Therefore, the extent of choice deferral decreases.

The extent of choice deferral cannot be simply measured by the size of $\widetilde{x}$ when $\sigma^2$ changes. On one hand, the region where the DM defers choice increases in $\widetilde{x}$. On the other hand, however, the DM's belief changes more quickly as $\sigma^2$ increases. So, a larger $\widetilde{x}$ does not necessarily imply a greater extent of choice deferral. Since the standard deviation of the DM's belief processes in a unit of time is $\sigma$, $\widetilde{x}$ normalized by $1/\sigma$, $\widetilde{x}/\sigma$, is more appropriate to measure the extent of choice deferral for different $\sigma^2$. As we noted previously, since $\widetilde{x}/\sigma$ is independent of $\sigma$, we have that the extent of choice deferral does not depend on $\sigma^2$.

The extent of choice closure can be seen as increasing in $\delta$ for a given $\sigma^2$ and, therefore, is also decreasing in $\lambda$ and $r$. As the discount rate, $r$, or the rate at which the DM moves from the search to the no-search mode, $\lambda$, increases, the DM has a stronger incentive to make a faster decision in the search mode while she always adopts anything positive in the no-search mode. So, the extent of choice closure decreases in $\lambda$ and $r$.

Similar to the extent of choice deferral, the extent of choice closure cannot be simply measured by comparing the size of $\delta$ when $\sigma^2$ changes. Using normalized $\delta/\sigma$ to measure the extent of choice closure for different $\sigma^2$, which as we noted above does not vary with $\sigma$, we then have that the extent of choice closure does not depend on $\sigma^2$.

*Case of $\beta \to \infty$:*

In the case of $\beta \to \infty$ we have that $\delta \to 0$ and $\overline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$. This shows that, as one may expect, when the DM is more likely to come back to the search mode, the DM is more demanding on the expected payoff of the alternative to decide to adopt it (in comparison to the case of $\beta \to 0$).

In this case of $\beta \to \infty$ it is also interesting to see the rate at which $\delta$ converges to zero, and the rate at which $\overline{x}$ and $\widetilde{x}$ converge to $\sqrt{\frac{\sigma^2}{2r}}$.

To see this note that as $\beta \to \infty$ we can obtain from (6) that

$$\beta(D-1)^2 \to 2(r+\lambda) \tag{9}$$

from which we can obtain that[4]

$$\delta\sqrt{\beta} \to \sigma, \tag{10}$$

which shows that $\delta$ is increasing in the speed of learning during the search mode, $\sigma^2$, and decreasing in the rate at which the DM returns to the search mode from the no-search mode, $\beta$. Therefore, the extent of choice closure can be seen as decreasing in $\beta$. As the DM becomes more likely to return to the search mode, the expected waiting time in the no-search mode and loss from discounting are lower. So, the DM has a weaker incentive to make a premature decision in the no-search mode.

With this result on $\delta$ we can now turn to the thresholds $\overline{x}$ and $\widetilde{x}$. To see this we can

[4]Please see the derivation in the appendix.

obtain from (7), taking the limit when $\beta \to \infty$, that

$$\sqrt{\beta}\left(\widetilde{x}-\frac{1}{\mu}\right) \quad \to \quad -\sigma \tag{11}$$

$$\sqrt{\beta}\left(\overline{x}-\frac{1}{\mu}\right) \quad \to \quad 0, \tag{12}$$

from which we can obtain that $\widetilde{x}$ is lower than $1/\mu$ and that $\overline{x}$ approaches $1/\mu$ for $\beta$ large. We can also obtain that both $\widetilde{x}$ and $\overline{x}$, when $\beta$ is large, are increasing in $\sigma^2$ and decreasing in $r$ and $\lambda$. Also, $\widetilde{x}$ is increasing in $\beta$. So, the extent of choice deferral is increasing in $\beta$ and decreasing in $\lambda$ and $r$, same as when $\beta \to 0$. We summarize these results in the following proposition.

**Proposition 2.** *Suppose that $\beta$ is sufficiently large. Then the purchase threshold in both the search mode and in the no-search mode, $\overline{x}$ and $\widetilde{x}$, are increasing in $\sigma^2$ and decreasing in both $\lambda$ and $r$. The purchase threshold in the search mode, $\overline{x}$, is increasing in $\beta$, and the difference, $\delta = \overline{x} - \widetilde{x}$, is increasing in $\sigma^2$ and decreasing in $\beta$.*

*General Case*

The previous comparative statics results for $\overline{x}$ and $\widetilde{x}$ in the limiting cases, Proposition 1 and Proposition 2, extend to the case with general $\beta$.

**Proposition 3.** *The purchase threshold in the search mode, $\overline{x}$, and the purchase threshold in the no-search mode, $\widetilde{x}$, are increasing in both $\beta$ and $\sigma^2$, and decreasing in both $\lambda$ and $r$.*

We illustrate the results for the general case in Figures 3-6.

Figure 3 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ increase with the rate at which the individual switches from the no-search mode to the search mode, $\beta$, and that the difference $\overline{x}-\widetilde{x}$ decreases with $\beta$. Thus, the DM has a greater extent of choice deferral and a lesser extent of choice closure when the DM returns to the search mode sooner after interruptions to the search process.

Figure 4 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ decrease with the discount rate $r$, for a case of $\beta$ low ($\beta = .1$), and a case of $\beta$ high ($\beta = 5$). The figure also illustrates that the difference $\overline{x} - \widetilde{x}$ decreases in $r$, as shown in Proposition 1.[5] As discussed in the limiting

[5]Note that for $\beta = 5$, $\overline{x} - \widetilde{x}$ still decreases in $r$ even though the lines appear to be closer for $r$ small in Figure 4.

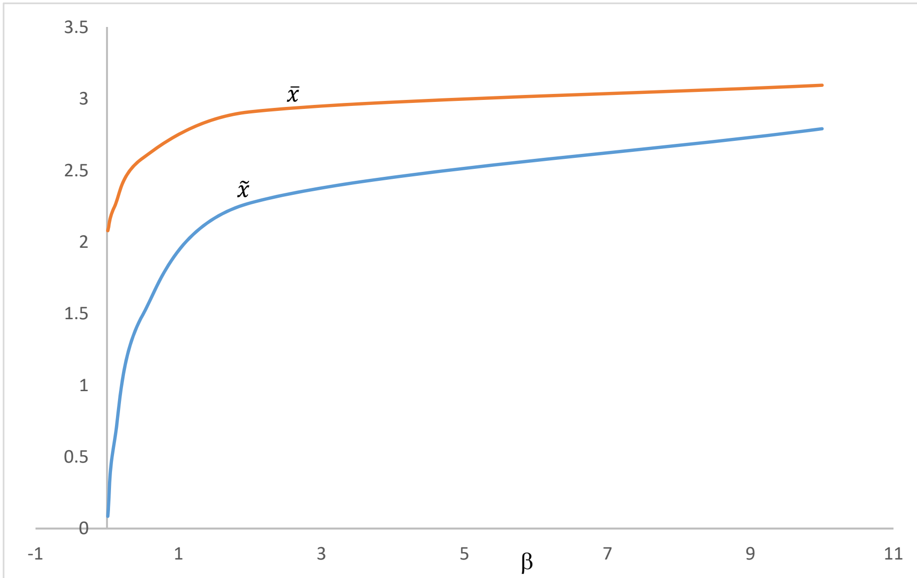



Figure 3: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of $\beta$ for $r = .05, \lambda = .5$, and $\sigma^2 = 1$.

case of $\beta \to 0$, a higher discount rate has a greater effect on the purchase threshold in the search mode, $\overline{x}$, which leads to a decrease in the difference $\overline{x} - \widetilde{x}$, which means a lower extent of choice closure. Note that both the extent of choice deferral and the extent of choice closure decrease with $r$, because a less patient DM has a stronger incentive to make a decision before search interruptions arrive by lowering the purchase threshold $\overline{x}$. It emphasizes that choice deferral and choice closure are not two completely opposite concepts. We also observe that the effect of $r$ on $\overline{x} - \widetilde{x}$ is smaller for a higher $\beta$, which corresponds to our finding that $\overline{x} - \widetilde{x}$ does not depend on $r$ at the limit of $\beta \to \infty$.

Figure 5 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ decrease with the rate at which the individual switches from the search mode to the no-search mode, $\lambda$, for a case of $\beta$ low ($\beta = .1$), and a case of $\beta$ high ($\beta = 5$). The figure also illustrates how the difference $\overline{x} - \widetilde{x}$ decreases in $\lambda$. Both the extent of choice deferral and the extent of choice closure decrease with $\lambda$. The rationale is similar to the one regarding the effect of the discount rate discussed above. When information gathering is interrupted more frequently, the DM has a stronger incentive to stop searching by lowering the purchase threshold in the search mode, $\overline{x}$. We

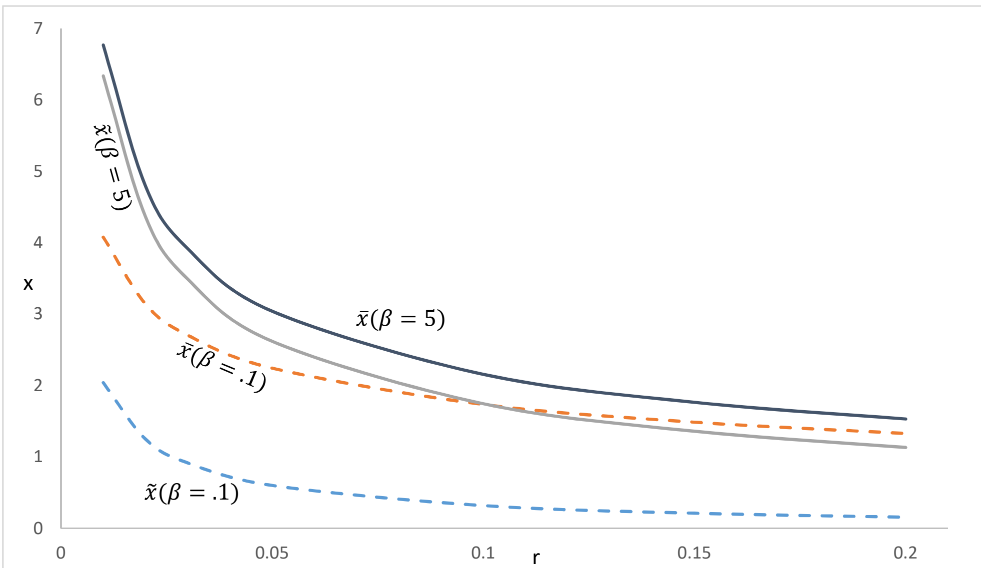



Figure 4: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of $r$ for $\lambda = .5$, $\sigma^2 = 1$, and $\beta = .1, 5$.

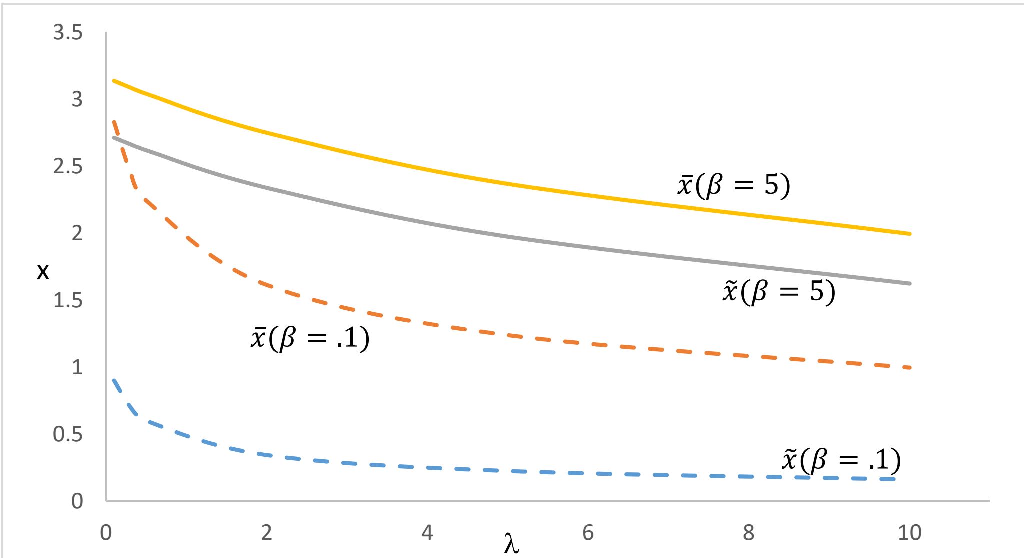



Figure 5: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of $\lambda$ for $r = .05$, $\sigma^2 = 1$, and $\beta = .1, 5$.

also observe that the effect of $\lambda$ on $\overline{x} - \widetilde{x}$ is smaller for a higher $\beta$, which corresponds to our finding that $\overline{x} - \widetilde{x}$ does not depend on $\lambda$ at the limit of $\beta \to \infty$.

Figure 6 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ increase with the amount of information learned in the search mode, $\sigma^2$, for a case of $\beta$ low ($\beta = .1$), and a case of $\beta$ high ($\beta = 5$). The figure illustrates how the difference $\overline{x} - \widetilde{x}$ also increases in $\sigma^2$. But as discussed previously, when $\sigma^2$ changes, the extent of choice deferral and the extent of choice closure are measured by $\widetilde{x}/\sigma$ and $(\overline{x} - \widetilde{x})/\sigma$, respectively, and both values do not change with $\sigma^2$. Thus, the extent of choice deferral and the extent of choice closure do not depend on $\sigma^2$.

## 4. Two Search Modes

We now consider a set-up in which the DM can become aware of her increased fatigue over time. We consider this possibility with the existence of two search modes, the fully-

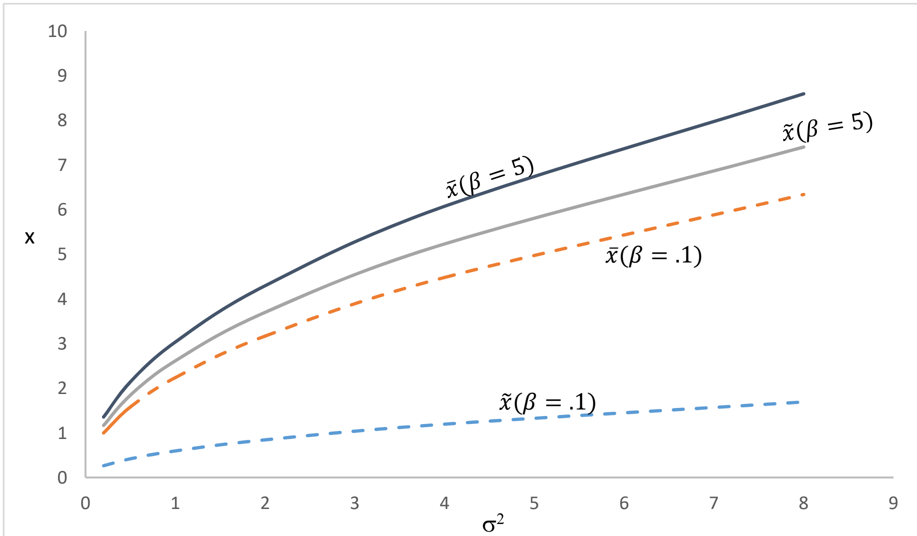



Figure 6: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of $\sigma^2$ for $r = .05$, $\lambda = .5$, and $\beta = .1, 5$.

rested search mode 1 and the fatigued search mode 2. The DM moves from search mode 1 to search mode 2 at a hazard rate of $\lambda_1$, and then from search mode 2 to the no-search mode at a hazard rate of $\lambda_2$. For simplicity, we assume $\lambda_1 = \lambda_2 = \lambda$ in our analysis and discuss the case of $\lambda_1 \neq \lambda_2$ at the end of the section. Once in the no-search mode, the DM moves to search mode 1 at a hazard rate of $\beta$. This captures the idea that the DM is aware of search fatigue because the DM realizes that the no-search mode will arrive sooner when she is in search mode 2 than when she is in search mode 1.

In the construction of optimal decision-making, we are looking for three thresholds, $\overline{x}, \underline{x}$, and $\widetilde{x}$, with $\widetilde{x} < \underline{x} < \overline{x}$, such that in search mode 1 the DM adopts the alternative if $x \geq \overline{x}$, in search mode 2 the DM adopts the alternative if $x \geq \underline{x}$, and in the no-search mode the DM adopts the alternative if $x \geq \widetilde{x}$.

Let $V_1(x)$ be the expected payoff in search mode 1, $V_2(x)$ be the expected payoff in search mode 2, and $W(x)$ be the expected payoff in the no-search mode. Consider the

Bellman equation in the no-search mode. We have

$$W(x) = \beta\, dt V_1(x) + (1 - \beta\, dt) e^{-r\, dt} W(x), \tag{13}$$

which leads to $W(x) = \frac{\beta}{r+\beta} V_1(x)$.

Consider now the Bellman equation in search mode 1. For $x \in (\underline{x}, \overline{x})$ we have

$$V_1(x) = (1 - \lambda\, dt) e^{-r\, dt} E V_1(x + dx) + \lambda\, dt x. \tag{14}$$

For $x < \underline{x}$ we have

$$V_1(x) = (1 - \lambda\, dt) e^{-r\, dt} E V_1(x + dx) + \lambda\, dt V_2(x). \tag{15}$$

Consider now the Bellman equation in search mode 2. For $x \in (\widetilde{x}, \underline{x})$ we have

$$V_2(x) = (1 - \lambda\, dt) e^{-r\, dt} E V_2(x + dx) + \lambda\, dt x. \tag{16}$$

Regarding the Bellman equation in search mode 2 for $x < \widetilde{x}$ we can obtain

$$V_2(x) = (1 - \lambda\, dt) e^{-r\, dt} E V_2(x + dx) + \lambda\, dt\, \frac{\beta}{r + \beta} V_1(x), \tag{17}$$

where we use that $W(x) = \frac{\beta}{r+\beta} V_1(x)$.

Applying Itô's Lemma to the Bellman equations, solving the corresponding differential equations, and using value matching and smooth pasting at the different thresholds, $V_1(\overline{x}) = \overline{x}, V_1'(\overline{x}) = 1, V_1(\underline{x}^+) = V_1(\underline{x}^-), V_1'(\underline{x}^+) = V_1'(\underline{x}^-), V_1(\widetilde{x}^+) = V_1(\widetilde{x}^-), V_1'(\widetilde{x}^+) = V_1'(\widetilde{x}^-), V_2(\widetilde{x}^+) = V_2(\widetilde{x}^-), V_2'(\widetilde{x}^+) = V_2'(\widetilde{x}^-), V_2(\underline{x}) = \underline{x}, V_2'(\underline{x}) = 1, \frac{\beta}{r+\beta} V_1(\widetilde{x}) = \widetilde{x}$, leads to a system of 11 equations (presented and analyzed in the Appendix) to obtain $\overline{x}_1$, $\overline{x}_2$, and $\widetilde{x}$.

We illustrate the results for the general case in Figures 7-10. We observe that $\widetilde{x}$ decreases in $\lambda$, increases in $\beta$, and decreases in $r$. Thus, the extent of choice deferral is greater when the search process is interrupted less frequently, when the DM returns to search mode sooner after an interruption, and when the DM discounts the future less. We observe that $\overline{x} - \widetilde{x}$ decreases in $\lambda$, $\beta$, and $r$. Thus, the extent of choice closure is greater when the search process is interrupted less frequently, when search interruptions last longer, and when the DM discounts the future less. These observations match those from the base model.

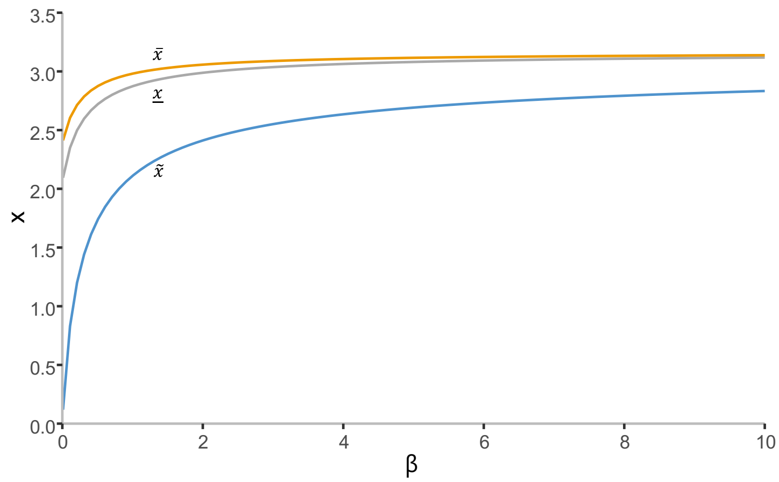



Figure 7: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of $\beta$ for $r = .05, \lambda = .5$, and $\sigma^2 = 1$.

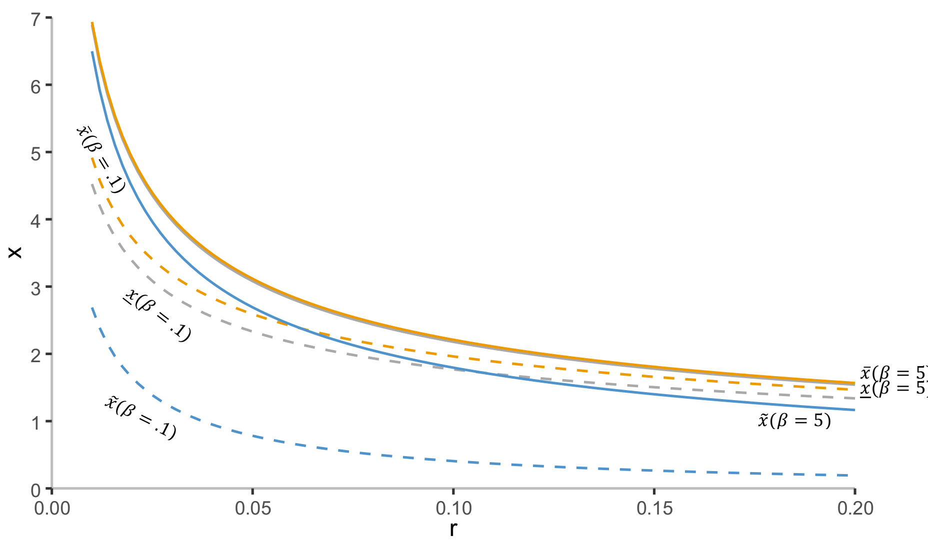



Figure 8: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of $r$ for $\lambda = .5$, $\sigma^2 = 1$, and $\beta = .1, 5$.

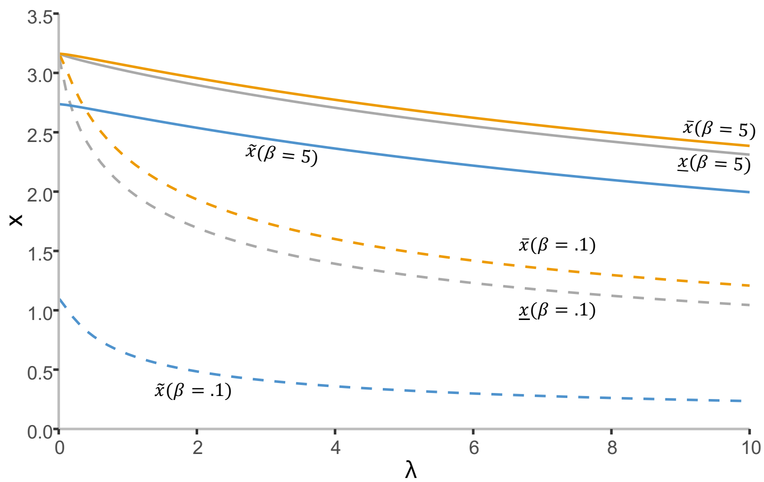



Figure 9: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of $\lambda$ for $r = .05$, $\sigma^2 = 1$, and $\beta = .1, 5$.

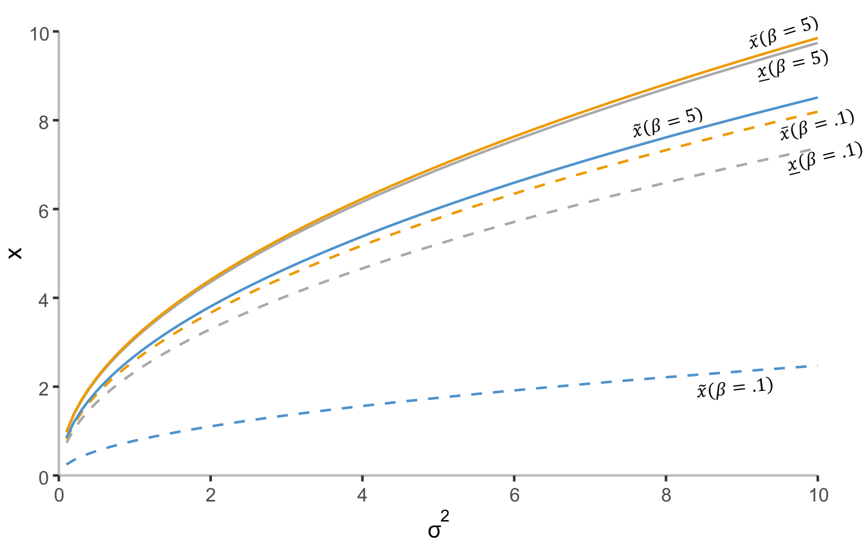



Figure 10: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of $\sigma^2$ for $r = .05$, $\lambda = .5$, and $\beta = .1, 5$.

Note that with two search modes, there are two types of choice closure. The first type of choice closure happens when the DM moves from search mode 1 to search mode 2. With $\underline{x} < \overline{x}$, the DM requires less positive information to adopt the alternative in the fatigued search mode 2 than in the fully-rested search mode 1, because the DM expects information gathering to be interrupted sooner. The extent of this choice closure is measured by $\overline{x} - \underline{x}$. The second type of choice closure happens when the DM moves from search mode 2 to the no-search mode. The extent of this choice closure is measured by $\underline{x} - \widetilde{x}$. From Figures 7-10, we observe that the extent of choice closure in search mode 2, $\underline{x} - \widetilde{x}$, is greater than the extent of choice closure in search mode 1, $\overline{x} - \underline{x}$, showing that greater fatigue leads to a greater extent of choice closure.

*Choice Closure Behaviors for $\beta$ and $\lambda$ Small*

To get sharper results on the DM's choice closure behaviors in different search modes, let us consider what happens when $\beta \to 0$, which makes $\widetilde{x} \to 0$. The Appendix presents the analysis for this case.

We can obtain the condition for the optimal $\underline{x}$ as

$$e^{\mu \underline{x}}(1 - \mu \underline{x}) + \frac{\lambda}{r} = 0, \tag{18}$$

as $\mu = \widetilde{\mu}$ for $\beta = 0$, which is intuitively the same condition as (8). Furthermore, we can obtain

$$\frac{\lambda}{4\widetilde{\mu}} + \frac{\lambda}{\widetilde{\mu}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\mu}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\mu}} - \frac{\lambda}{2}\underline{x}, \tag{19}$$

where $\overline{X} = e^{\widetilde{\mu}\overline{x}}$ and $\underline{X} = e^{\widetilde{\mu}\underline{x}}$, which determines $\overline{x}$ as a function of $\underline{x}$. For $\lambda \to 0$, we can then obtain $\underline{x}, \overline{x} \to \sqrt{\frac{\sigma^2}{2r}}$, and

$$\frac{\underline{x} - 1/\mu}{\lambda} \quad \to \quad \frac{1}{r\mu e} \tag{20}$$

$$\frac{\overline{x} - 1/\mu}{\lambda} \quad \to \quad \frac{1}{2r\mu}. \tag{21}$$

This then yields the following results.

**Proposition 4.** *Consider the two search modes case, and assume that $\lambda$ and $\beta$ are sufficiently small. Then, $\overline{x} - \underline{x}$ increases in $\lambda$ and $\underline{x} - \widetilde{x}$ decreases in $\lambda$. Both $\overline{x} - \underline{x}$ and $\underline{x} - \widetilde{x}$ decrease in the discount rate $r$ while $\underline{x}/\sigma, \overline{x}/\sigma$, and $\frac{\overline{x}-\underline{x}}{\sigma}$ do not depend on the amount of information gained during search $\sigma^2$.*

Note that the comparative statics from Proposition 4 match the numerical illustration shown in Figure 9. The extent of choice closure in search mode 2 can be seen as increasing in $\underline{x} - \widetilde{x} \approx \underline{x}$. As the discount rate, $r$, increases, the DM has a stronger incentive to make a faster decision in search mode 2, so the extent of choice closure in search mode 2 decreases in $r$. Similarly, when $\lambda$ increases, the DM experiences fatigue faster. Facing more interruptions to search, the DM has a stronger incentive to make a faster decision in search mode 2. Thus, the extent of choice closure decreases in $\lambda$. The effects of $\lambda$ and $r$ on the extent of choice closure in search mode 2 are similar to those in the base model.

The extent of choice closure in search mode 1, however, behaves differently. A greater rate of search fatigue $\lambda$ makes the DM more concerned about not being able to do further search. Thus, the DM has a stronger incentive to make a faster decision in both search modes, causing both $\overline{x}$ and $\underline{x}$ to decrease in $\lambda$. However, the effect is stronger in search mode 2 because a more fatigued DM expects search interruption to arrive sooner, causing $\overline{x} - \underline{x}$ to increase in $\lambda$.

As we discussed in the base model, $\overline{x} - \underline{x}$ and $\underline{x} - \widetilde{x}$ normalized by $1/\sigma$ can be seen as more appropriate measures of choice closure for different $\sigma^2$. According to the result above, the extent of either type of choice closure does not depend on $\sigma^2$.

Following the above discussion, we would then expect, in the case where the DM's rate of moving from search mode 1 to search mode 2, $\lambda_1$, is different from the DM's rate of moving from search mode 2 to the no-search mode, $\lambda_2$, the extent of choice closure in search mode 1 to increase in both $\lambda_1$ and $\lambda_2$, the extent of choice closure in search mode 2 to decrease in both $\lambda_1$ and $\lambda_2$, and the extent of choice deferral to decrease in both $\lambda_1$ and $\lambda_2$.

## 5. Start-Up Search Costs

The analysis above considered the strategic effects of choice deferral through discounting of future payoffs. We now consider the existence of start-up search costs in the beginning of the search mode and show that these start-up search costs yield strategic effects of choice deferral without discounting.

Consider the model of Section 2, but assume that the DM does not discount the future expected payoffs but has start-up search costs $F$ when moving to the search mode from the no-search mode.[6] Furthermore, let us consider that the DM has ongoing search costs $c$ per

[6]The start-up search costs $F$ can also be seen as capturing in some way the possible effects of hyperbolic discounting (Laibson 1997).

unit of time while in the search mode. The role of the search costs $c$ is to give the DM an incentive to stop search and adopt the alternative in the search mode. Without the ongoing search costs and discounting, the DM would keep on learning information without making a decision until there would be a switch from the search to the no-search mode.

The optimal decision-making will involve the existence of four thresholds, $\overline{x}, \widetilde{x}, \widehat{x}$, and $\underline{\widehat{x}}$, with $\overline{x} > \widetilde{x} \geq 0 \geq \widehat{x} > \underline{\widehat{x}}$ such that the DM adopts the alternative in the search mode if $x \geq \overline{x}$, adopts the alternative when switching from the search mode to the no-search mode if $x \geq \widetilde{x}$, defers choice when switching from the search to the no-search mode if $x \in (\widehat{x}, \widetilde{x})$, stops search without adopting the alternative when switching from the search to the no-search mode if $x \leq \widehat{x}$, and stops search in the search mode without adopting the alternative if $x < \underline{\widehat{x}}$.

Let $V(x)$ be the value function for the DM when in the search mode and $x \in (\widetilde{x}, \overline{x})$, $\widetilde{V}(x)$ be the value function for the DM when in the search mode and $x \in (\widehat{x}, \widetilde{x})$, and $\widehat{V}$ be the value function for the DM when in the search mode and $x \in (\underline{\widehat{x}}, \widehat{x})$. Furthermore, recall that $W(x)$ is the value function of the DM when in the no-search mode.

The Bellman equation of value function when the DM is in the no-search mode (which is relevant for $x \in (\widehat{x}, \widetilde{x})$) can be written as

$$W(x) = \beta\, dt[\widetilde{V}(x) - F] + (1 - \beta\, dt)W(x), \tag{22}$$

from which we can obtain $W(x) = \widetilde{V}(x) - F$.

When the DM is in the search mode and $x \in (\widehat{x}, \widetilde{x})$ we can then write the Bellman equation of the value function as

$$\widetilde{V}(x) = -c\, dt + (1 - \lambda\, dt)E\widetilde{V}(x + dx) + \lambda\, dt[\widetilde{V}(x) - F]. \tag{23}$$

The Bellman equation for $x \in (\widetilde{x}, \overline{x})$ can be written as

$$V(x) = -c\, dt + (1 - \lambda\, dt)EV(x + dx) + \lambda\, dtx. \tag{24}$$

The Bellman equation for $x \in (\underline{\widehat{x}}, \widehat{x})$ can be written as

$$\widehat{V}(x) = -c\, dt + (1 - \lambda\, dt)EV(x + dx). \tag{25}$$

Applying Itô's Lemma on the Bellman equations, solving the corresponding differential equations, and using value matching and smooth pasting at each threshold, $V(\overline{x}) = \overline{x}, V'(\overline{x}) = 1, V(\widetilde{x}) = \widetilde{V}(\widetilde{x}), V'(\widetilde{x}) = \widetilde{V}'(\widetilde{x}), V(\widetilde{x}) - F \geq \widetilde{x}, \widetilde{V}(\widehat{x}) - F = 0, \widetilde{V}(\widehat{x}) = \widehat{V}(\widehat{x}), \widetilde{V}'(\widehat{x}) = \widehat{V}'(\widehat{x}), \widehat{V}(\underline{\widehat{x}}) = 0$, and $\widehat{V}'(\underline{\widehat{x}}) = 0$ leads to a system of 10 equations, from which we can obtain, $\overline{x}$, $\widetilde{x}$, $\widehat{x}$, and $\underline{\widehat{x}}$. [7]

We can obtain $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$, $\widetilde{x} = -\widehat{x}$, and

$$\widetilde{x} = \max\left\{\sqrt{\frac{\sigma^2}{2\lambda}}\frac{c}{2(\lambda F + c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}, 0\right\} \tag{26}$$

$$\overline{x} = \widetilde{x} + \frac{1}{\widehat{\mu}}\ln H, \tag{27}$$

where

$$H = 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}. \tag{28}$$

and $\widehat{\mu} = \sqrt{2\lambda/\sigma^2}$.

Noting that $(\widetilde{x} - \widehat{x})$ captures the extent of choice deferral and $(\overline{x} - \widetilde{x})$ captures the extent of choice closure, we can obtain the following results.

**Proposition 5.** *Consider that there are start-up and ongoing search costs. Then the extent of choice deferral decreases in the current search costs $c$, in the start-up search costs $F$, and in the rate at which the DM switches from the search to the no-search mode, $\lambda$. The extent of choice closure decreases in the ongoing search costs $c$ and in the rate at which the DM switches from the search to the no-search mode, $\lambda$, and increases in the start-up search costs $F$.*

The ongoing search costs $c$ play a similar role to the discount $r$ in the base model with discounting. An increase in ongoing costs lowers the present value of future payoffs and encourages the DM to make a choice faster in the search mode. Thus, both the extent of choice deferral and the extent of choice closure decrease in $c$. An increase in start-up search costs $F$ plays a similar role to a decrease in the rate of fatigue recovery $\beta$ in the base model by decreasing the benefits of deferring choice. Thus, a higher $F$ leads to a greater extent of choice closure and a lower extent of choice deferral. Note that because there is no discounting in this case, the rate at which the DM switches from the no-search mode to the search mode, $\beta$, does not affect the extent of choice deferral or closure in this model.

[7] The derivation of the solution is presented in the Appendix.

The comparative statics on the rate at which the DM switches from the search to the no-search mode, $\lambda$, are of the same directions as in the base model. The DM expects more search interruptions at a higher $\lambda$ which lowers the present value of future purchases and encourages the DM to make a choice faster in the search mode, causing both the extent of choice deferral and the extent of choice closure to decrease. This effect is consistent with the base model.

Given the above comparisons between the start-up costs model and the base model with discounting, if we consider adding start-up and ongoing search costs to the base model, we would expect the extent of choice deferral to decrease in the discount rate $r$, in the current search costs $c$, in the rate at which the DM switches from the search to the no-search mode $\lambda$, in the start-up search costs $F$, and increase in the rate at which the DM switches from the no-search mode to the search mode $\beta$. We would also expect the extent of choice closure to decrease in the discount rate $r$, in the current search costs $c$, in the rate at which the DM switches from the search to the no-search mode $\lambda$, in the rate at which the DM switches from the no-search mode to the search mode $\beta$, and increase in the start-up search costs $F$.

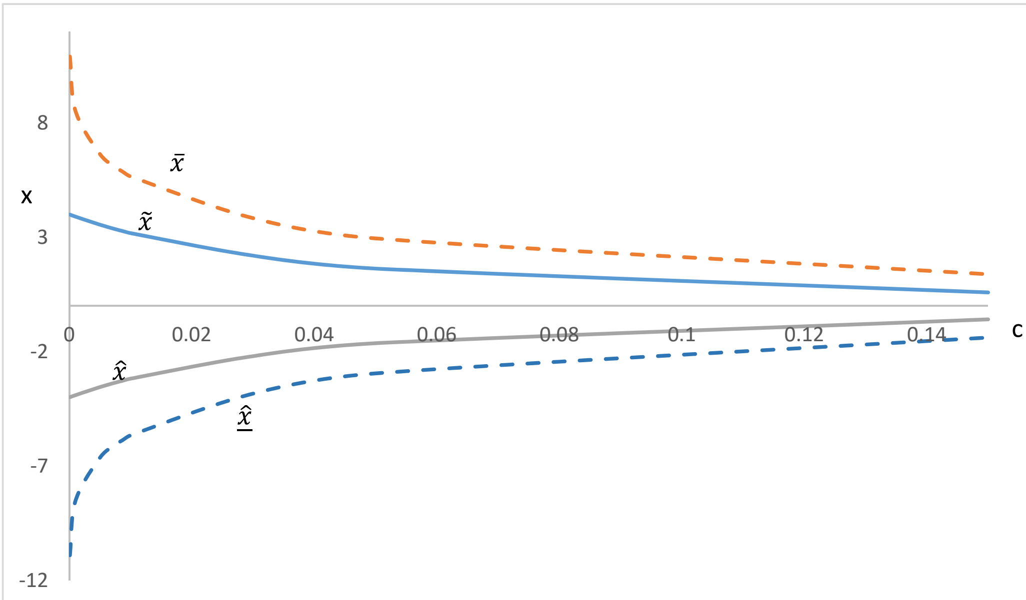



Figure 11: Evolution of the stop/search thresholds for the start-up search costs case as a function of $c$ for $\lambda = .5$, $\sigma^2 = 1$, and $F = .1$.

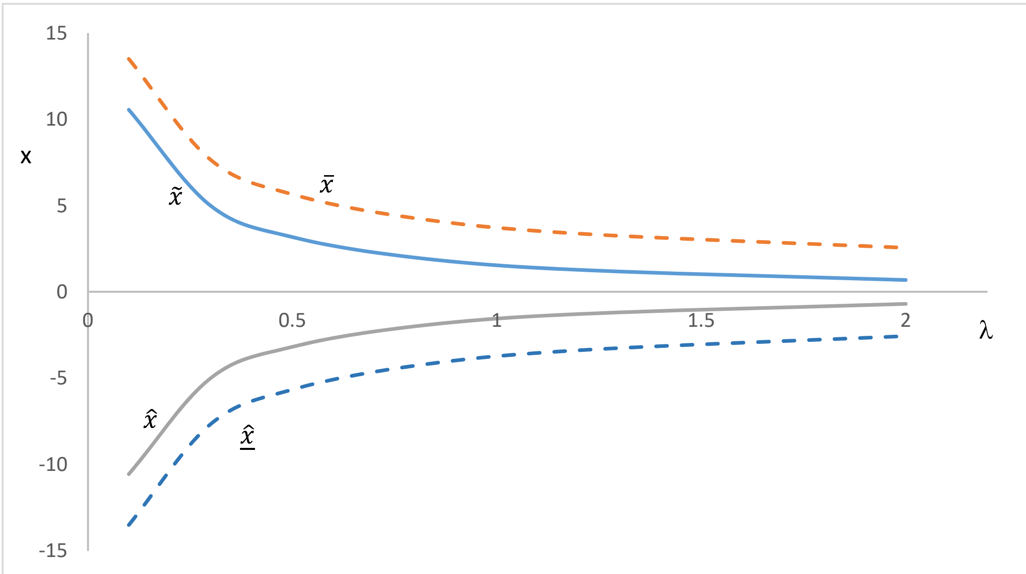



Figure 12: Evolution of the stop/search thresholds for the start-up search costs case as a function of $\lambda$ for $c = .01$, $\sigma^2 = 1$, and $F = .1$.

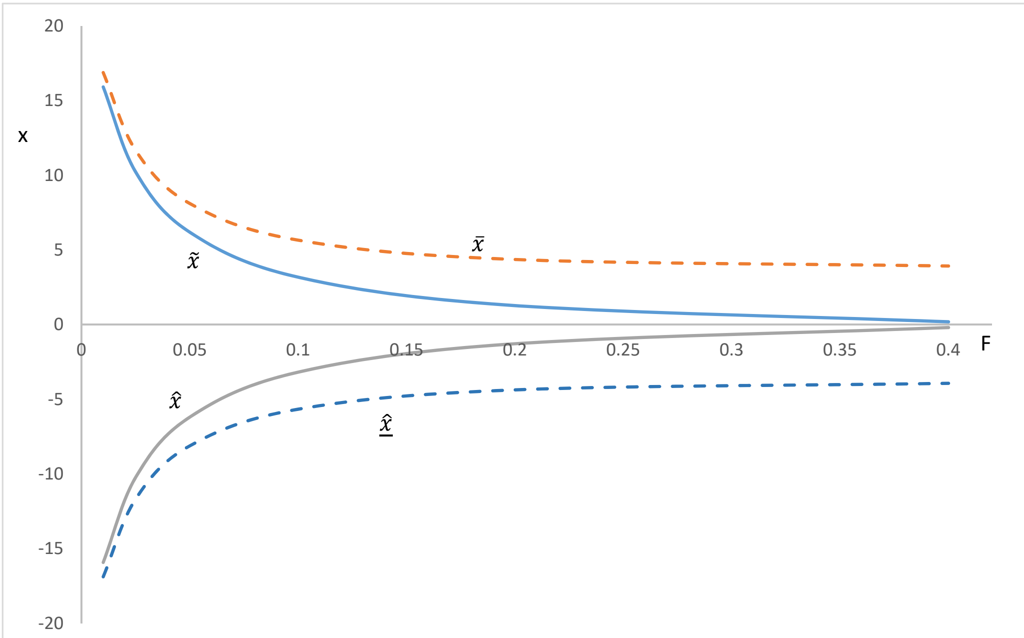



Figure 13: Evolution of the stop/search thresholds for the start-up search costs case as a function of $F$ for $\lambda = .5$, $\sigma^2 = 1$, and $c = .01$.

Figures 11, 12, and 13 illustrate how the thresholds $\overline{x}, \widetilde{x}, \widehat{x}$, and $\underline{\widehat{x}}$ evolve as a function of the ongoing search costs, $c$, the hazard rate of switching from the search to the no-search mode, $\lambda$, and the start-up search costs, $F$. We observe that both the extent of choice deferral and the extent of choice closure decrease in the frequency of search interruptions $\lambda$, similar to the base model. In particular, note that $\widetilde{x} = 0$ for $F$ sufficiently high. Intuitively, if the start-up search costs are high enough, the DM does not restart search, and chooses to adopt the alternative if $x > 0$, when switching from the search to the no-search mode.

## 6. Optimal Pricing

In this section, we derive the firm's optimal pricing strategy for the base model with discounting and one search mode. The analysis for the base model in Section 3 can be seen as describing the behavior of a DM facing a product with an exogenous price. Let $P$

denote the price, let $x$ denote the expected value of the payoff of the alternative as before, and let $y = x - P$ denote the expected payoff of the alternative minus the price. The DM then would adopt the alternative when $y$ reaches $\overline{x}$ in the search mode, and would adopt the alternative when $y$ reaches $\widetilde{x}$ in the no-search mode, where $\overline{x}$ and $\widetilde{x}$ are solutions to (6) and (7). Equivalently, the DM adopts when $x$ reaches $\overline{x} + P$ in the search mode or when $x$ reaches $\widetilde{x} + P$ in the no-search mode.

Let $V_f(x)$ be the expected discounted payoff for the firm if the DM is in the search mode, and $W_f(x)$ be the expected payoff for the firm if the DM is in the no-search mode.

The Bellman equation for $V_f(x)$ for $x < \widetilde{x} + P$ can be written as

$$V_f(x) = (1 - \lambda\, dt)e^{-r\,dt} EV(x + dx) + \lambda\, dt W_f(x). \tag{29}$$

The Bellman equation for $V_f(x)$ for $x \in (\widetilde{x} + P, \overline{x} + P)$ can be written as

$$V_f(x) = (1 - \lambda\, dt)e^{-r\,dt} EV_f(x + dx) + \lambda\, dt P. \tag{30}$$

Finally, the Bellman equation for $W_f(x)$ can be written as

$$W_f(x) = \beta\, dt V(x) + (1 - \beta\, dt)e^{-r\,dt} W_f(x), \tag{31}$$

from which one can obtain $W_f(x) = \frac{\beta}{r+\beta} V_f(x)$.

Given continuity of the value function at both $\overline{x} + P$ and $\widetilde{x} + P$, we have value matching of $V_f$ at both these points:

$$\begin{aligned} V_f(\overline{x} + P) &= P && (32) \\ V_f(\widetilde{x} + P^+) &= V_f(\widetilde{x} + P^-). && (33) \end{aligned}$$

Furthermore, given infinite variation of $x$ around $\widetilde{x} + P$, we also have smooth pasting at that point,

$$V_f'(\widetilde{x} + P^+) = V_f'(\widetilde{x} + P^-). \tag{34}$$

Applying Itô's Lemma to the Bellman equations, solving the corresponding differential equations, and using equations (32)-(34), we can solve for the firm's value function $V_f(x)$. The analysis is presented in the Appendix.

The optimal price, $P^*$, depends on the initial position, $x_0$. Suppose $P^* \leq x_0 - \overline{x}$ (that is, $x_0 \geq \overline{x} + P^*$), then the DM adopts the alternative at $x_0$ in both the search mode and the no-search mode. In this case, because the DM purchases immediately at $P^*$, the firm's profit strictly increases in $P^*$. Thus any price strictly below $x_0 - \overline{x}$ cannot be optimal. So we must have $P^* \geq x_0 - \overline{x}$. And there is a $x_0^{**}$, defined below, such that $P^* = x_0 - \overline{x}$ for $x_0 > x_0^{**}$.

Now consider the case where $P^* > x_0 - \widetilde{x}$ (that is, $x_0 < \widetilde{x} + P^*$). In this case, the DM does not adopt the alternative at $x_0$ in both the search mode and the no-search mode. And there is a $x_0^*$, defined below, when we will be in this case for $x_0 < x_0^*$.

We can obtain the value function of the firm in this region of $x_0$ as

$$V_f(x_0) = \frac{2r + \lambda(e^{\widetilde{\mu}\delta} + e^{-\widetilde{\mu}\delta})}{(\widetilde{\mu} + \mu)e^{\mu(\widetilde{x}+P)+\widetilde{\mu}\delta} + (\widetilde{\mu} - \mu)e^{\mu(\widetilde{x}+P)-\widetilde{\mu}\delta}} \frac{\widetilde{\mu}P}{r + \lambda} e^{\mu x_0}$$

Taking the derivative of $V_f(x_0)$ with respect to $P$, we have

$$\text{sign}\left\{\frac{\partial V_f(x_0)}{\partial P}\right\} = \text{sign}\{1 - \mu P\}. \tag{35}$$

We then have that for $x_0 < x_0^*$, the optimal price is $P^* = 1/\mu$. We can also then obtain that $x_0^* = \widetilde{x} + 1/\mu$.

Finally, consider the case where $P^* \in [x_0 - \overline{x}, x_0 - \widetilde{x}]$ (that is, $x_0 \in [\widetilde{x} + P^*, \overline{x} + P^*]$). In this case, the DM adopts at $x_0$ in the no-search mode but does not adopt at $x_0$ in the search mode. This is the case in which $x_0 \in [x_0^*, x_0^{**}]$. Let us denote $V_f(x, P)$ as the value function for $x \in (\widetilde{x} + P, \overline{x} + P)$, where we emphasize that that value function also depends on the price $P$.

In this case, the optimal interior price is obtained by differentiating $V_f(x, P)$ evaluated at $x = x_0$, with respect to price and making that derivative equal to zero. That equality determines the optimal price $P^*(x_0)$ implicitly by some function $h(P^*(x_0), x_0) = 0$, defined in the Appendix. We can then define $x_0^{**}$ by making $x_0^{**} - \overline{x} = V_f(x_0^{**}, P^*(x_0^{**}))$ and $P^*(x_0^{**}) \in \arg\max_P V_f(x_0^{**}, P)$. As discussed in the Appendix we may, or may not, have continuity of the price function at $x_0^{**}$. If we have continuity of the price function at $x_0^{**}$, which it can be obtained to occur, for example, if $\lambda/r$ and $\beta$ are small enough, we also have that $x_0^{**}$ satisfies $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$. As also discussed in the Appendix, we have that the optimal price is declining in $x_0$ at any existing discontinuity, and will be declining in $x_0$ for some region of $x_0 \in [x_0^*, x_0^{**}]$ if the price function is continuous for $\beta$ small. For $\beta$ large we can obtain that the price function is continuous. Furthermore, the price function is monotonic for $\beta \to \infty$.[8]

We summarize the optimal pricing strategy and comparative statics in the following Proposition.

**Proposition 6.** *The optimal price depends on $x_0$ in the following way:*

1. *For $x_0$ sufficiently low ($x_0 < x_0^* = \widetilde{x} + 1/\mu$), the optimal price is $P^* = 1/\mu$, which does not depend on $x_0$. The DM does not adopt the alternative at $x_0$ in either the search mode or the no-search mode. In that case, the optimal price increases in $\sigma^2$ and $\beta$, and decreases in $r$ and $\lambda$. If $\lambda$ and $\beta$ change simultaneously with a fixed ratio of $\lambda/\beta$, then the optimal price decreases in $\lambda$ and $\beta$.*

2. *For intermediate $x_0$ ($x_0 \in [x_0^*, x_0^{**}]$), the optimal price is in the range of $[1/\mu, x_0]$, and the DM does not adopt the alternative at $x_0$ in the search mode but adopts the alternative at $x_0$ in the no-search mode.*

3. *For $x_0$ sufficiently high ($x_0 > x_0^{**}$), the optimal price is $x_0 - \overline{x}$, and the DM adopts the alternative without searching. In that case, the optimal price decreases in $\sigma^2$ and $\beta$, and increases in $r$ and $\lambda$.*

Interestingly, the firm uses price to induce different choice deferral behaviors at $x = x_0$. When $x_0$ is low, the firm sets a price such that the DM defers choice if search is interrupted at $x = x_0$. For intermediate $x_0$, the firm sets a price to induce choice closure at $x = x_0$, i.e., the DM does not adopt the alternative in the search mode but adopts the alternative if search is interrupted at $x = x_0$. For higher $x_0$, the firm sets a price such that the DM adopts the alternative immediately even in the search mode.

The optimal price is constant if the DM's prior belief is sufficiently low, $x_0 < \widetilde{x}+1/\mu$. The firm would need to charge too low a price (potentially even negative) to convince the DM to adopt the alternative without learning any information, which is not profitable. It is better to charge a higher price, hoping that the DM will receive enough positive signals and adopt the alternative at a high price. Therefore, the firm sets a constant price, $1/\mu$, such that the DM does not adopt the alternative at $x_0$ in either the search mode or the no-search mode. In this region of $x_0 < \widetilde{x} + 1/\mu$, one can see that the optimal price increases in $\sigma^2$ and $\beta$, and decreases in $r$ and $\lambda$. Intuitively, when $\lambda$ increases or when $\beta$ decreases, the DM is expected to spend a larger fraction of time in the no-search mode, exhibiting stronger search fatigue.

[8] Numerical analysis also suggests that the price function is monotonic in $x_0$ for $\beta$ large.

When the DM faces more frequent and longer disruptions of information gathering, the firm should charge a lower price to prevent the DM from deferring choice. Online stores can often track consumers over different browsing sessions. Our result suggests that firms should factor in the lengths of browsing sessions and gaps between browsing sessions in setting their prices.

Another implication of our findings is that the firm should change its price following its efforts to intervene with the DM's search/no-search pattern. For example, in online retail, firms may re-design interfaces to reduce consumer fatigue, so that consumers stay longer in a browsing session. Firms may also use instruments such as ad retargeting, push notifications, or email marketing to bring back previous visitors more quickly. The price should increase if these efforts are successful.

In this case, we also find that the optimal price decreases when the DM switches between the search mode and the no-search mode more frequently, even if the long-term fraction of time in each mode remains constant. That is, assuming $\lambda/\beta = \alpha$ for some fixed $\alpha$, we find that $P^*$ decreases in $\lambda$ (or $\beta$) for low $x_0$. This is relevant when there is a change in the shopping environment such that consumers enter and exit search more or less frequently. For example, consumers shopping on mobile devices may have their browsing sessions disrupted and resumed more frequently than consumers shopping on computers. In that case, even if the overall time spent on shopping does not change for consumers on mobile devices, the firm should consider setting a lower price on mobile devices compared to the price on computers.

When the DM's prior belief about the alternative is higher, the optimal price depends on $x_0$ and the comparative statics may be reversed. This is because the firm can already obtain a high profit even if the DM does not receive additional information. The firm prefers to increase the adoption likelihood by setting a price such that the DM adopts the alternative at $x_0$ in the no-search mode and may even adopt it at $x_0$ in the search mode.

If $x_0 > x_0^{**}$ the firm charges a price equal to $x_0 - \overline{x}$ to induce the DM to adopt the alternative without searching. In this case, the optimal price increases in $x_0$ linearly. If $x_0 \in [x_0^*, x_0^{**}]$ the firm charges a higher price so that the DM would adopt the alternative at $x_0$ in the no-search mode but not in the search mode. In this case, the optimal price is in the interval $[x_0 - \overline{x}, x_0 - \widetilde{x}]$. There are two opposing effects of the prior belief $x_0$ on the price. On one hand, both the upper bound and the lower bound of the optimal price increase in $x_0$. This effect drives the price higher. On the other hand, the firm's loss from non-adoption is also higher. So, the firm has an incentive to induce the DM to adopt the alternative sooner. This effect drives the price lower. As a result, the optimal price can be non-monotonic in $x_0$

when $x_0$ is intermediate. The intuition for the opposing effects and non-monotonicity of the optimal price can also be seen in a two-period model, which we analyze in the Appendix.

Figures 14-18 illustrate how the optimal price depends on the initial belief $x_0$ and on the different parameters. Regions I corresponds to Proposition 6.1, when $x_0$ is low and the firm's optimal price induces choice deferral at $x = x_0$. Regions II corresponds to Proposition 6.2, when $x_0$ is intermediate and the firm's optimal price induces choice closure at $x = x_0$. Regions III corresponds to Proposition 6.3, when $x_0$ is high and the firm's optimal price induces the DM to adopt the alternative immediately without searching.

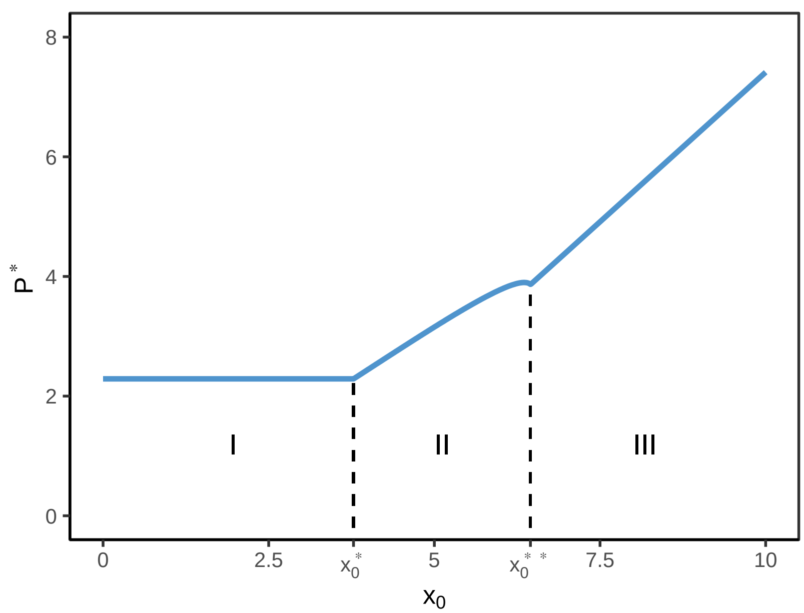



Figure 14: Example of the optimal price $P^*$ as a function of $x_0$ for $r = .05, \lambda = .5$, $\beta = .5$, and $\sigma^2 = 1$. For these parameter values we have $x_0^* \approx 3.78$ and $x_0^{**} \approx 6.45$. Region I: $x_0 < x_0^*$ (the DM adopts in neither the search nor the no-search mode); Region II: $x_0 \in [x_0^*, x_0^{**}]$ (the DM adopts in the no-search mode only); Region III: $x_0 > x_0^{**}$ (the DM adopts in both the search and no-search mode).

Figure 14 illustrates how the optimal price varies with the initial position, $x_0$. When $x_0$ is low, the DM does not take the alternative initially in both the search mode or the no-search mode, and the optimal price does not depend on $x_0$. When $x_0$ is high, the DM adopts the alternative without searching, and the optimal price increases in $x_0$ linearly. When $x_0$ is intermediate, $x_0 \in [x_0^*, x_0^{**}]$, the DM adopts the alternative at $x_0$ in the no-search mode but not in the search mode. The two opposing effects of $x_0$ on the price discussed above lead to

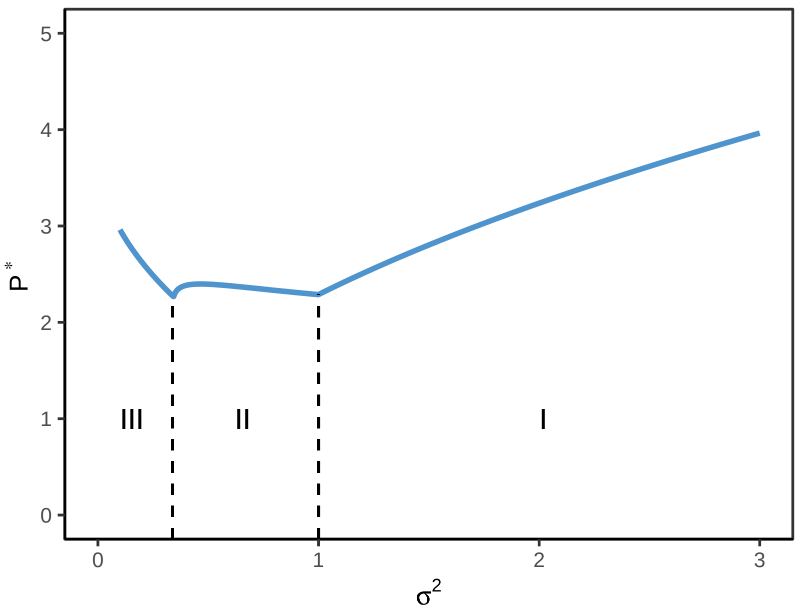



Figure 15: Example of the optimal price $P^*$ as a function of $\sigma^2$ for $r = .05, \beta = .5$, $\lambda = .5$, and $x_0 = 3.78$. Region I: $x_0 < x_0^*$ (the DM adopts in neither the search nor the no-search mode); Region II: $x_0 \in [x_0^*, x_0^{**}]$ (the DM adopts in the no-search mode only); Region III: $x_0 > x_0^{**}$ (the DM adopts in both the search and no-search mode).

the non-monotonicity of the optimal price.

Figure 15 illustrate the comparative statics of $P^*$ with regard to the speed of learning $\sigma^2$, noting the DM's different behaviors across different regions of the optimal prices. Since $\widetilde{x}$ and $1/\mu$ increase in $\sigma^2$, the condition $x_0 < x^* = \widetilde{x} + 1/\mu$ (Region I) is more likely to be satisfied for larger $\sigma^2$. So, fixing $x_0$, the optimal price is $1/\mu$ and the DM adopts the alternative at $x_0$ in neither the search or no-search mode when $\sigma^2$ is large. Since $1/\mu$ increases in $\sigma^2$, the optimal price increases in $\sigma^2$ in that region (Region I). Intuitively, the firm has a higher incentive to encourage search when the signal is more informative. By charging a higher price, the firm can encourage the consumer to search more. In contrast, the optimal price is $x_0 - \overline{x}$ and the DM adopts the alternative at $x_0$ without searching for small $\sigma^2$. Since $\overline{x}$ increases in $\sigma^2$, the optimal price decreases in $\sigma^2$ in that region (Region III). Intuitively, the firm wants to avoid search when the signal is not very informative and the consumer also wants to reduce search in that case. The less informative the search process (lower $\sigma^2$), the easier it is for the firm to convince the consumer to adopt the alternative without searching. So, the firm can charge a higher price as $\sigma^2$ decreases, for small $\sigma^2$.

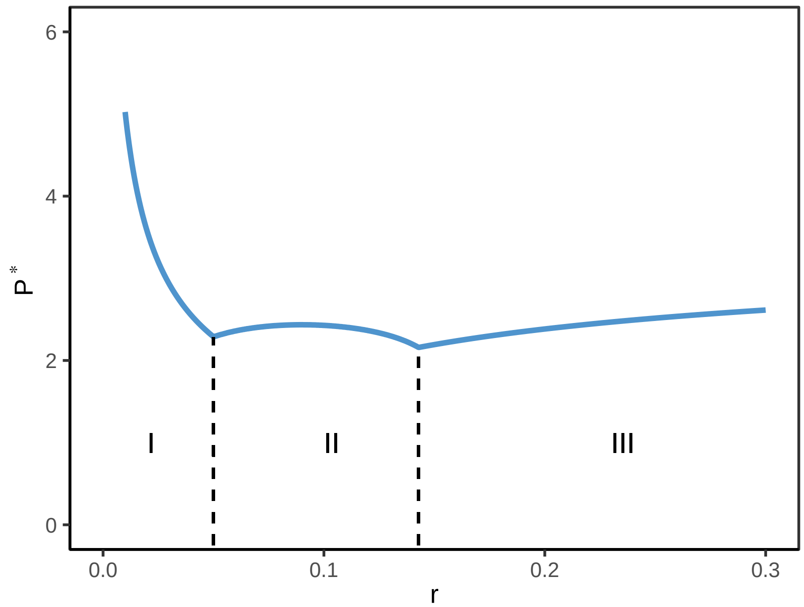
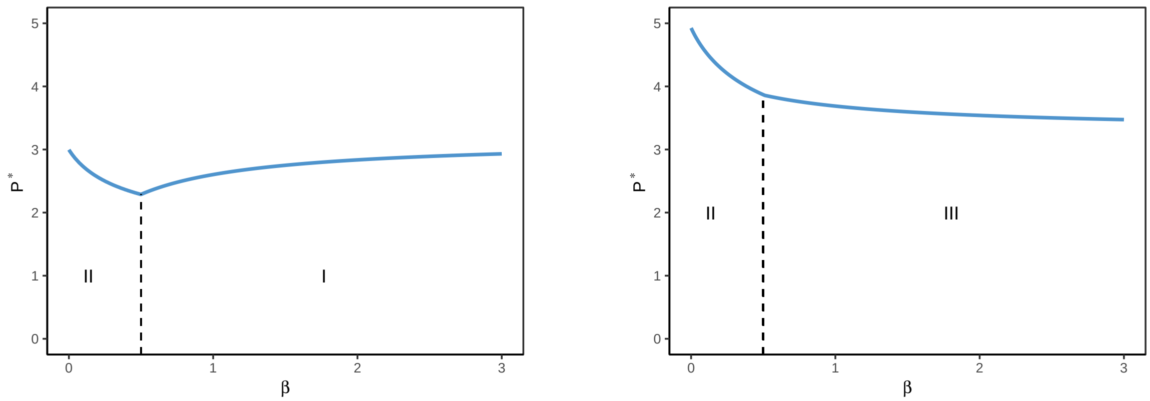



Figure 16: Example of the optimal price $P^*$ as a function of $r$ for $\lambda = .5, \beta = .5$, $\sigma^2 = 1$, and $x_0 = 3.78$. Region I: $x_0 < x_0^*$ (the DM adopts in neither the search nor the no-search mode); Region II: $x_0 \in [x_0^*, x_0^{**}]$ (the DM adopts in the no-search mode only); Region III: $x_0 > x_0^{**}$ (the DM adopts in both the search and no-search mode).

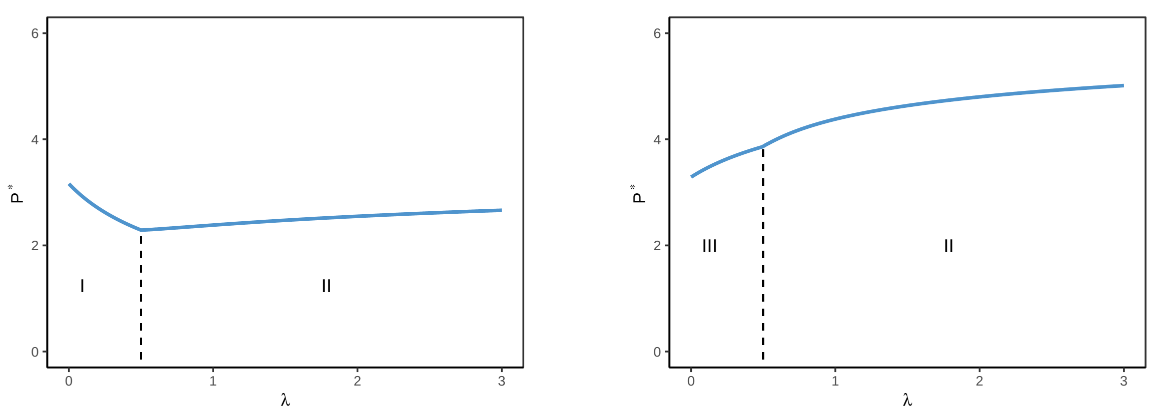
Figure 16 illustrates how the optimal price varies with the discount rate $r$. The optimal price is $1/\mu$ when $r$ is small. Therefore, the optimal price decreases in $r$ in that region. The optimal price is $x_0 - \overline{x}$ for large $r$. Since $\overline{x}$ decreases in $r$, the optimal price increases in $r$ in that region. The comparative statics with respect to the discount rate $r$ may be non-monotonic for intermediate values of the parameters (Region II) because the optimal price is in the range of $[x_0 - \overline{x}, x_0 - \widetilde{x}]$ and there are two opposing effects in that region, as discussed above.

The effects of $\beta$ and $\lambda$ are illustrated in Figures 17 and 18. When $x_0$ is low, as illustrated in Figure 17a, the firm wants to encourage search, so it charges a price such that the DM does not adopt the alternative immediately in the search mode (Region I and Region II). The firm has a higher incentive to encourage search when the disruption of search is shorter (larger $\beta$). So, for $\beta$ large, the firm charges a price such that the DM defers choice at $x_0$ (Region I). The shorter the disruption of search is, the easier it is for the firm to induce the DM to search. So, the optimal price increases in $\beta$ in this region. When $x_0$ is high, as illustrated in Figure 17b, the firm's loss from delayed adoption is higher, and the firm prefers




(a) $x_0 = 3.78$ (b) $x_0 = 6.45$

Figure 17: Example of the optimal price $P^*$ as a function of $\beta$ for $r = .05, \lambda = .5$, and $\sigma^2 = 1$. Region I: $x_0 < x_0^*$ (the DM adopts in neither the search nor the no-search mode); Region II: $x_0 \in [x_0^*, x_0^{**}]$ (the DM adopts in the no-search mode only); Region III: $x_0 > x_0^{**}$ (the DM adopts in both the search and no-search mode).




(a) $x_0 = 3.78$ (b) $x_0 = 6.45$

Figure 18: Example of the optimal price $P^*$ as a function of $\lambda$ for $r = .05, \beta = .5$, and $\sigma^2 = 1$. Region I: $x_0 < x_0^*$ (the DM adopts in neither the search nor the no-search mode); Region II: $x_0 \in [x_0^*, x_0^{**}]$ (the DM adopts in the no-search mode only); Region III: $x_0 > x_0^{**}$ (the DM adopts in both the search and no-search mode).

to induce choice closure at $x_0$ (Region II and Region III). It is harder to convince the DM to adopt the alternative in the search mode as the disruption of search becomes shorter (a higher $\beta$ implies a higher $\overline{x}$). So, the firm finds it less attractive to encourage search in the search mode as $\beta$ increases. For large enough $\beta$, the firm charges $x_0 - \overline{x}$, which decreases in $\beta$.

For similar reasons, the firm encourages the DM to search in both the search and no-search mode by charging $1/\mu$ for small $\lambda$ when $x_0$ is low (Region I), as illustrated by Figure 18a. The optimal price decreases in $\lambda$ because it is harder to encourage search as the search gets interrupted more frequently. In contrast, the firm charges $x_0 - \overline{x}$ for small $\lambda$ when $x_0$ is high (Region III), and the optimal price increases in $\lambda$.

## 7. Concluding Remarks

When searching for information to make a decision, an individual often faces interruptions to the information-gathering process due to time-varying search costs, potentially based on search fatigue. At the time when search is interrupted, decision-makers may decide to defer choices until they can gather information again because they do not have sufficient diagnostic information. Alternatively, when facing interruptions, decision-makers may strategically decide to make a choice immediately, even if they do not have sufficient diagnostic information, a behavior which the paper refers to as choice closure.

This paper investigates how the extent of choice deferral and the extent of choice closure responds to different environmental factors. We find that there is a greater extent of choice deferral when information gathering is interrupted less frequently, when individuals can resume gathering information sooner, and when individuals discount the future less. We also find that there is a greater extent of choice closure when information gathering is interrupted less frequently, when search interruptions last longer before individuals can resume gathering information, and when individuals discount the future less.

We investigate the effects of search fatigue by considering what happens when there are different stages in the search process with subsequently higher fatigue levels, showing that greater fatigue leads to a greater extent of choice closure. We also investigate the effects of start-up and ongoing costs, in which case we can obtain strategic choice deferral and choice closure behaviors without time discounting. We find that the extent of choice deferral decreases in the ongoing search costs and in the start-up search costs, and the extent of choice closure decreases in the ongoing search costs but increases in the start-up search costs.

In terms of pricing, we find that the optimal price may be non-monotonic in consumers' initial beliefs about the product. For a low enough initial belief, we find that the optimal price increases when the speed of learning during information gathering is higher, when information gathering is interrupted less frequently, when consumers can resume gathering information sooner after interruptions, and when the firm and consumers discount the future less. These results suggest that firms should use data on consumer browsing sessions when determining price, and price should change following interventions to reduce search fatigue or restart consumer search sooner, such as redesigning user interface, ad re-targeting, email marketing, and push notifications.

Table 1: Notation

| Variable | Description |
|---|---|
| $x$ | expected value of adopting the alternative |
| $r$ | continuous discount rate |
| $\lambda$ | hazard rate of the DM moving from the search mode to the no-search mode |
| $\beta$ | hazard rate of the DM moving from the no-search mode to the search mode |
| $\overline{x}$ | adoption threshold in the search mode |
| | adoption threshold in search mode 1 (two search modes model) |
| $\widetilde{x}$ | adoption threshold in the no-search mode |
| $V(x)$ | expected payoff for the DM in the search mode |
| | expected payoff for the DM in the search mode when $x > \widetilde{x}$ (search costs model) |
| $W(x)$ | expected payoff for the DM in the no-search mode |
| $\delta$ | $\overline{x} - \widetilde{x}$ |
| $\mu$ | $\sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ |
| $\widetilde{\mu}$ | $\sqrt{\frac{2(r+\lambda)}{\sigma^2}}$ |
| $\lambda_1$ | hazard rate from search mode 1 to search mode 2 (two search modes model) |
| $\lambda_2$ | hazard rate from search mode 2 to the no-search mode (two search modes model) |
| $\underline{x}$ | adoption threshold in search mode 2 (two search modes model) |
| $V_1(x)$ | expected payoff for the DM in search mode 1 (two search modes model) |
| $V_2(x)$ | expected payoff for the DM in search mode 2 (two search modes model) |
| $F$ | start-up search costs (search costs model) |
| $c$ | ongoing search costs per unit of time (search costs model) |
| $\widehat{x}$ | threshold to stop search in the no-search mode (search costs model) |
| $\underline{\widehat{x}}$ | threshold to stop search in the search mode (search costs model) |
| $\widetilde{V}(x)$ | expected payoff for the DM in the search mode for $x \in (\widehat{x}, \widetilde{x})$ (search costs model) |
| $\widehat{V}(x)$ | expected payoff for the DM in the search mode for $x \in (\underline{\widehat{x}}, \widehat{x})$ (search costs model) |
| $\widehat{\mu}$ | $\sqrt{\frac{2\lambda}{\sigma^2}}$ |
| $P$ | price |
| $P^*(x_0)$ | the firm's optimal price if $x = x_0$ at time 0 |
| $y$ | $x - P$ |
| $V_f(x)$ | expected payoff for the firm if the DM is in the search mode |
| $W_f(x)$ | expected payoff for the firm if the DM is in the no-search mode |
| $V_f(x, P)$ | $V_f(x)$ for $x \in (\widetilde{x} + P, \overline{x} + P)$ |
| $x_0^*$ | $\widetilde{x} + 1/\mu$ |
| $h(P, x)$ | equation defined in (lxxxviii) |
| $x_0^{**}$ | the solution to the implicit equation $x_0^{**} - \overline{x} = V_f(x_0^{**}, P^*(x_0^{**}))$ |

# APPENDIX

Derivation of Solution to Base Case with Discounting:

Given that $\lim_{x\to-\infty} V(x) = 0$, as the expected payoff of the DM has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (5) satisfies

$$V(x) = A_1 e^{\mu x} \tag{i}$$

where $A_1$ is a constant to be determined.

Similarly, applying Itô's Lemma to (2), we can obtain the solution to the second order differential equation in $V(x)$ for $x \in (\widetilde{x}, \overline{x})$ as

$$V(x) = A_2 e^{\widetilde{\mu}x} + A_3 e^{-\widetilde{\mu}x} + \frac{\lambda}{r+\lambda}x, \tag{ii}$$

where $A_2$ and $A_3$ are constants to be determined.

Using value matching and smooth pasting of $V(x)$ at $\widetilde{x}$ and $\overline{x}$, $V(\widetilde{x}^-) = V(\widetilde{x}^+), V'(\widetilde{x}^-) = V'(\widetilde{x}^+)$, $V(\overline{x}) = \overline{x}$, and $V'(\overline{x}) = 1$, and $W(\widetilde{x}) = \widetilde{x}$, we obtain the following system of five equations to obtain $\widetilde{x}, \overline{x}, A_1, A_2$, and $A_3$.

$$\begin{aligned}
A_2 e^{\widetilde{\mu}\overline{x}} + A_3 e^{-\widetilde{\mu}\overline{x}} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} && \text{(iii)}\\
\widetilde{\mu} A_2 e^{\widetilde{\mu}\overline{x}} - \widetilde{\mu} A_3 e^{-\widetilde{\mu}\overline{x}} + \frac{\lambda}{r+\lambda} &= 1 && \text{(iv)}\\
A_2 e^{\widetilde{\mu}\widetilde{x}} + A_3 e^{-\widetilde{\mu}\widetilde{x}} + \frac{\lambda}{r+\lambda}\widetilde{x} &= A_1 e^{\mu\widetilde{x}} && \text{(v)}\\
\widetilde{\mu} A_2 e^{\widetilde{\mu}\widetilde{x}} - \widetilde{\mu} A_3 e^{-\widetilde{\mu}\widetilde{x}} + \frac{\lambda}{r+\lambda} &= \mu A_1 e^{\mu\widetilde{x}} && \text{(vi)}\\
\frac{\beta}{r+\beta} A_1 e^{\mu\widetilde{x}} &= \widetilde{x}. && \text{(vii)}
\end{aligned}$$

Using (iii)-(vii), we can obtain a system of two equations to obtain $\widetilde{x}$ and $\overline{x}$ as

$$\begin{aligned}
e^{\widetilde{\mu}(\overline{x}-\widetilde{x})} &= \frac{\frac{r}{r+\lambda}\overline{x} + \frac{r}{\widetilde{\mu}(r+\lambda)}}{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) + \frac{1}{\widetilde{\mu}}\left(\mu\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)} && \text{(viii)}\\
e^{\widetilde{\mu}(\overline{x}-\widetilde{x})} &= \frac{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) - \frac{1}{\widetilde{\mu}}\left(\mu\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)}{\frac{r}{r+\lambda}\overline{x} - \frac{r}{\widetilde{\mu}(r+\lambda)}}. && \text{(ix)}
\end{aligned}$$

Using $\delta = \overline{x} - \widetilde{x}$ we can rewrite (viii) and (ix), as a system of equations for $\delta$ and $\widetilde{x}$ as

$$\begin{aligned}
\widetilde{x} &= \beta \frac{r + r\widetilde{\mu}\delta + \lambda D}{D[\widetilde{\mu}r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] - \widetilde{\mu}\beta r} && \text{(x)} \\
\widetilde{x} &= \beta \frac{\lambda + rD - \widetilde{\mu}r\delta D}{\widetilde{\mu}r\beta D + \mu(r+\beta)(r+\lambda) - \widetilde{\mu}r(r+\beta+\lambda)} && \text{(xi)}
\end{aligned}$$

where $D = e^{\widetilde{\mu}\delta}$. Using (x) and (xi) we can obtain (6) in the main text, from which we can obtain $\delta$. We can then use (x) or (xi) to obtain $\widetilde{x}$.

Derivation of equation (10) in the Base Case: Since $\delta \to 0$ as $\beta \to +\infty$ and $D = 1 + \widetilde{\mu}\delta + o(\delta)$, we have

$$\begin{aligned}
& \beta(D-1)^2 \to 2(r+\lambda) \\
\Rightarrow\ & \beta[\widetilde{\mu}\delta + o(\delta)]^2 = 2(r+\lambda) + o(1) \\
\Rightarrow\ & \beta\widetilde{\mu}^2\delta^2 = 2(r+\lambda) + o(1) \\
\Rightarrow\ & \beta\delta^2 = \sigma^2 + o(1) \\
\Rightarrow\ & \sqrt{\beta}\delta \to \sigma, \text{ as } \beta \to +\infty
\end{aligned}$$

Derivation of equation (11) and (12) in the Base Case:

$$\begin{aligned}
& \frac{\widetilde{x} - 1/\mu}{\delta} \\
=& \frac{\beta \frac{r + r\widetilde{\mu}\delta + \lambda D}{D[\widetilde{\mu}r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] - \widetilde{\mu}\beta r} - 1/\mu}{\delta} \\
=& \frac{\beta\mu(r + r\widetilde{\mu}\delta + \lambda D) - D[\widetilde{\mu}r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] + \widetilde{\mu}\beta r}{\delta\mu\{D[\widetilde{\mu}r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] - \widetilde{\mu}\beta r\}} \\
=& \frac{\beta\mu[r + r\widetilde{\mu}\delta + \lambda(1 + \widetilde{\mu}\delta + o(\delta))] - (1 + \widetilde{\mu}\delta + o(\delta))[\widetilde{\mu}r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] + \widetilde{\mu}\beta r}{\delta\mu\{(1 + \widetilde{\mu}\delta + o(\delta))[\widetilde{\mu}r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] - \widetilde{\mu}\beta r\}} \\
=& \frac{-\beta r\widetilde{\mu}^2 + o(\beta)}{\mu[\beta\mu(r+\lambda) + o(\beta)]}
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \lim_{\beta\to+\infty} \frac{\widetilde{x} - 1/\mu}{\delta} = \lim_{\beta\to+\infty} \frac{-r\widetilde{\mu}^2 + o(1)}{\mu^2(r+\lambda) + o(1)]} = -1 \\
\Rightarrow\ & \widetilde{x} = 1/\mu - \delta + o(\delta) \\
& \bar{x} = \widetilde{x} + \delta = 1/\mu + o(\delta)
\end{aligned}$$

When $\beta \to \infty$, we have shown that $\delta\sqrt{\beta} \to \sigma$. Therefore,

$$\begin{aligned}
\sqrt{\beta}\left(\widetilde{x} - \frac{1}{\mu}\right) &\to -\sigma \\
\sqrt{\beta}\left(\overline{x} - \frac{1}{\mu}\right) &\to 0
\end{aligned}$$

Proof of Proposition 3:

We provide proof for the comparative statics w.r.t. $\sigma^2$. The proofs for the comparative statics w.r.t. $\beta$, $r$, and $\lambda$ are similar.

Given $\sigma_s^2 < \sigma_\ell^2$ and the corresponding cutoff beliefs $(\overline{x}_s, \widetilde{x}_s)$, $(\overline{x}_\ell, \widetilde{x}_\ell)$, we want to show that $\overline{x}_\ell \geq \overline{x}_s$ and $\widetilde{x}_\ell \geq \widetilde{x}_s$.

Suppose $\overline{x}_\ell < \overline{x}_s$. Then $V_s(\overline{x}_\ell) > \overline{x}_\ell$, because the DM keeps searching for information when the belief is $\overline{x}_\ell$ and $\sigma^2 = \sigma_s^2$. Also, $V_\ell(\overline{x}_\ell) = \overline{x}_\ell$, because the DM takes the alternative when the belief is $\overline{x}_\ell$ and $\sigma^2 = \sigma_\ell^2$. Therefore, $V_s(\overline{x}_\ell) > V_l(\bar{x}_l)$.

However, one can see that the DM can achieve a payoff of at least $V_s(\bar{x}_\ell)$ when $\sigma^2 = \sigma_\ell^2$ and the belief is $\overline{x}_\ell$ by using the optimal strategy when $\sigma^2 = \sigma_s^2$ (which may be sub-optimal when $\sigma^2 = \sigma_\ell^2$). Therefore, $V_s(\overline{x}_\ell) \leq V_\ell(\overline{x}_\ell)$, a contradiction. So, $\overline{x}_\ell \geq \overline{x}_s$.

Now suppose that $\widetilde{x}_\ell < \widetilde{x}_s$. Then $W_s(\widetilde{x}_\ell) > \widetilde{x}_\ell$, because the DM defers the choice when the belief is $\widetilde{x}_\ell$ and $\sigma^2 = \sigma_s^2$. Also, $W_\ell(\widetilde{x}_\ell) = \widetilde{x}_\ell$ because the DM takes the alternative in the no-search mode when the belief is $\widetilde{x}_\ell$ and $\sigma^2 = \sigma_\ell^2$. Therefore, $V_s(\widetilde{x}_\ell) = \frac{r+\beta}{\beta} W_s(\widetilde{x}_\ell) > \frac{r+\beta}{\beta} W_\ell(\widetilde{x}_\ell) = V_\ell(\widetilde{x}_\ell)$.

However, one can see that the DM can achieve a payoff of at least $V_s(\widetilde{x}_\ell)$ when $\sigma^2 = \sigma_\ell^2$ and the belief is $\widetilde{x}_\ell$ by using the optimal strategy when $\sigma^2 = \sigma_s^2$ (which may be sub-optimal when $\sigma^2 = \sigma_\ell^2$). Therefore, $V_s(\widetilde{x}_\ell) \leq V_\ell(\widetilde{x}_\ell)$, a contradiction. So, $\widetilde{x}_\ell \geq \widetilde{x}_s$.

Derivation of the Optimal Decision-Making in the Two Search Modes Case: Applying Itô's Lemma to (14) and solve the differential equation, we can obtain

$$V_1(x) = B_3 e^{\widetilde{\mu}x} + B_4 e^{-\widetilde{\mu}x} + \frac{\lambda}{r+\lambda}x, \tag{xii}$$

where $B_3$ and $B_4$ are constants to be determined.

Applying Itô's Lemma to (16) and solve the differential equation, we can obtain

$$V_2(x) = B_1 e^{\widetilde{\mu}x} + B_2 e^{-\widetilde{\mu}x} + \frac{\lambda}{r+\lambda}x, \tag{xiii}$$

where $B_1$ and $B_2$ are constants to be determined. We can then use (xiii) in (15) to obtain that for $x \in (\widetilde{x}, \underline{x})$, solving the corresponding differential equation,

$$V_1(x) = B_5 e^{\widetilde{\mu}x} + B_6 e^{-\widetilde{\mu}x} + \frac{\lambda^2}{(r+\lambda)^2} x + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)} x[B_2 e^{-\widetilde{\mu}x} - B_1 e^{\widetilde{\mu}x}], \tag{xiv}$$

where $B_5$ and $B_6$ are constant to be determined.

Putting together (15) and (17) for $x < \widetilde{x}$, we obtain a system of differential equations

$$(r+\lambda)V_2(x) = \frac{\sigma^2}{2} V_2''(x) + \lambda \frac{\beta}{r+\beta} V_1(x) \tag{xv}$$

$$(r+\lambda)V_1(x) = \frac{\sigma^2}{2} V_1''(x) + \lambda V_2(x) \tag{xvi}$$

which has the solution

$$V_2(x) = \widetilde{B}_1 e^{z_1 x} + \widetilde{B}_2 e^{z_2 x} \tag{xvii}$$

$$V_1(x) = \sqrt{\frac{r+\beta}{\beta}} \left[\widetilde{B}_2 e^{z_2 x} - \widetilde{B}_1 e^{z_1 x}\right] \tag{xviii}$$

where $z_1 = \sqrt{\widetilde{\mu}^2 + \frac{2\lambda}{\sigma^2}\sqrt{\frac{\beta}{r+\beta}}}$, and $z_2 = \sqrt{\widetilde{\mu}^2 - \frac{2\lambda}{\sigma^2}\sqrt{\frac{\beta}{r+\beta}}}$, and $\widetilde{B}_1$ and $\widetilde{B}_2$ are constants to be determined, where we use that $\lim_{x\to-\infty} V_1(x) = \lim_{x\to-\infty} V_2(x) = 0$.

Value matching and smooth pasting at the different thresholds, $V_1(\overline{x}) = \overline{x}, V_1'(\overline{x}) = 1, V_1(\underline{x}^+) = V_1(\underline{x}^-), V_1'(\underline{x}^+) = V_1'(\underline{x}^-), V_1(\widetilde{x}^+) = V_1(\widetilde{x}^-), V_1'(\widetilde{x}^+) = V_1'(\widetilde{x}^-), V_2(\widetilde{x}^+) = V_2(\widetilde{x}^-), V_2'(\widetilde{x}^+) = V_2'(\widetilde{x}^-), V_2(\underline{x}) = \underline{x}, V_2'(\underline{x}) = 1, \frac{\beta}{r+\beta} V_1(\widetilde{x}) = \widetilde{x}$, lead to the following system of 11 equations to obtain the 11 unknowns, $\overline{x}_1, \overline{x}_2, \widetilde{x}, B_1, B_2, B_3, B_4, B_5, B_6, \widetilde{B}_1$, and $\widetilde{B}_2$.

$$
\begin{aligned}
B_3\underline{X} + B_4/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} &= B_5\underline{X} + B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2}\underline{x} - \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\underline{xX} \\
&\quad + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2\underline{x}/\underline{X} && \text{(xix)} \\
\widetilde{\mu}B_3\underline{X} - \widetilde{\mu}B_4/\underline{X} + \frac{\lambda}{r+\lambda} &= \widetilde{\mu}B_5\underline{X} - \widetilde{\mu}B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2} - \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\underline{X} - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_1\underline{xX} \\
&\quad + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2/\underline{X} - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_2\underline{x}/\underline{X} && \text{(xx)} \\
B_5\widetilde{X} + B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\widetilde{x} &- \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \\
&\quad \sqrt{\frac{r+\beta}{\beta}}\left[\widetilde{B}_2e^{z_2\widetilde{x}} - \widetilde{B}_1e^{z_1\widetilde{x}}\right] && \text{(xxi)} \\
\widetilde{\mu}B_5\widetilde{X} - \widetilde{\mu}B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2} &- \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\widetilde{X} - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2/\widetilde{X} \\
&\quad - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \sqrt{\frac{r+\beta}{\beta}}\left[z_2\widetilde{B}_2e^{z_2\widetilde{x}} - z_1\widetilde{B}_1e^{z_1\widetilde{x}}\right] && \text{(xxii)}
\end{aligned}
$$

$$
\begin{aligned}
B_1\widetilde{X} + B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\widetilde{x} &= \widetilde{B}_1e^{z_1\widetilde{x}} + \widetilde{B}_2e^{z_2\widetilde{x}} && \text{(xxiii)} \\
\widetilde{\mu}B_1\widetilde{X} - \widetilde{\mu}B_2/\widetilde{X} + \frac{\lambda}{r+\lambda} &= z_1\widetilde{B}_1e^{z_1\widetilde{x}} + z_2\widetilde{B}_2e^{z_2\widetilde{x}} && \text{(xxiv)} \\
B_3\overline{X} + B_4/\overline{X} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} && \text{(xxv)} \\
\widetilde{\mu}B_3\overline{X} - \widetilde{\mu}B_4/\overline{X} + \frac{\lambda}{r+\lambda} &= 1 && \text{(xxvi)} \\
B_1\underline{X} + B_2/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} &= \underline{x} && \text{(xxvii)} \\
\widetilde{\mu}B_1\underline{X} - \widetilde{\mu}B_2/\underline{X} + \frac{\lambda}{r+\lambda} &= 1 && \text{(xxviii)} \\
\sqrt{\frac{\beta}{r+\beta}}\left(-\widetilde{B}_1e^{z_1\widetilde{x}} + \widetilde{B}_2e^{z_2\widetilde{x}}\right) &= \widetilde{x}, && \text{(xxix)}
\end{aligned}
$$

where $\overline{X} = e^{\widetilde{\mu}\overline{x}}$, $\underline{X} = e^{\widetilde{\mu}\underline{x}}$, and $\widetilde{X} = e^{\widetilde{\mu}\widetilde{x}}$.

Putting together (xxv) and (xxvi) one obtains

$$
\begin{aligned}
2B_3\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x}+\frac{1}{\widetilde{\widetilde{\mu}}}\right) && \text{(xxx)}\\
2B_4/\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x}-\frac{1}{\widetilde{\widetilde{\mu}}}\right). && \text{(xxxi)}
\end{aligned}
$$

Putting together (xxvii) and (xxviii) one obtains

$$
\begin{aligned}
2B_1\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\widetilde{\mu}}}\right) && \text{(xxxii)}\\
2B_2/\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x}-\frac{1}{\widetilde{\widetilde{\mu}}}\right). && \text{(xxxiii)}
\end{aligned}
$$

Putting together (xix) and (xx) one obtains

$$
\begin{aligned}
2B_3\underline{X}+\frac{\lambda}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\widetilde{\mu}}}\right)=2B_5\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x}+\frac{1}{\widetilde{\widetilde{\mu}}}\right)-\frac{\lambda}{2(r+\lambda)}B_1\underline{X}(1+2\widetilde{\mu}\underline{x})\\
&+\frac{\lambda}{2(r+\lambda)}B_2/\underline{X} && \text{(xxxiv)}\\
2B_4/\underline{X}+\frac{\lambda}{r+\lambda}\left(\underline{x}-\frac{1}{\widetilde{\widetilde{\mu}}}\right)=2B_6/\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x}-\frac{1}{\widetilde{\widetilde{\mu}}}\right)-\frac{\lambda}{2(r+\lambda)}B_2/\underline{X}(1-2\widetilde{\mu}\underline{x})\\
&+\frac{\lambda}{2(r+\lambda)}B_1\underline{X}. && \text{(xxxv)}
\end{aligned}
$$

Putting together (xxi) and (xxii) one obtains

$$
\begin{aligned}
2B_5\widetilde{X}+\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}+\frac{1}{\widetilde{\widetilde{\mu}}}\right) &- \frac{\lambda}{2(r+\lambda)}B_1\widetilde{X}(1+2\widetilde{\mu}\widetilde{x})+\frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X}=\\
&\sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{\widetilde{B}}_1\left(1+\frac{z_1}{\widetilde{\widetilde{\mu}}}\right)+\widetilde{\widetilde{B}}_2\left(1+\frac{z_2}{\widetilde{\widetilde{\mu}}}\right)\right) && \text{(xxxvi)}\\
2B_6/\widetilde{X}+\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}-\frac{1}{\widetilde{\widetilde{\mu}}}\right) &- \frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X}(1-2\widetilde{\mu}\widetilde{x})+\frac{\lambda}{2(r+\lambda)}B_1\widetilde{X}=\\
&\sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{\widetilde{B}}_1\left(1-\frac{z_1}{\widetilde{\widetilde{\mu}}}\right)+\widetilde{\widetilde{B}}_2\left(1-\frac{z_2}{\widetilde{\widetilde{\mu}}}\right)\right) && \text{(xxxvii)}
\end{aligned}
$$

where $\widetilde{\widetilde{B}}_1 = \widetilde{B}_1 e^{z_1\widetilde{x}}$ and $\widetilde{\widetilde{B}}_2 = \widetilde{B}_2 e^{z_2\widetilde{x}}$. Putting together (xxiii) and (xxiv) one obtains

$$2B_1\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x} + \frac{1}{\widetilde{\mu}}\right) = \widetilde{\widetilde{B}}_1\left(1 + \frac{z_1}{\widetilde{\mu}}\right) + \widetilde{\widetilde{B}}_2\left(1 + \frac{z_2}{\widetilde{\mu}}\right) \tag{xxxviii}$$

$$2B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x} - \frac{1}{\widetilde{\mu}}\right) = \widetilde{\widetilde{B}}_1\left(1 - \frac{z_1}{\widetilde{\mu}}\right) + \widetilde{\widetilde{B}}_2\left(1 - \frac{z_2}{\widetilde{\mu}}\right). \tag{xxxix}$$

Using (xxix) we obtain $\widetilde{\widetilde{B}}_2 = \widetilde{\widetilde{B}}_1 + \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}$, which we can then substitute in (xxxvi)-(xxxix). Using the resulting equations (xxxviii) and (xxxix) we can obtain

$$\frac{2\widetilde{\mu} - z_1 - z_2}{2\widetilde{\mu} + z_1 + z_2}\left[2B_1\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x} + \frac{1}{\widetilde{\mu}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1 + \frac{z_2}{\widetilde{\mu}}\right)\right] = 2B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x} - \frac{1}{\widetilde{\mu}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1 - \frac{z_2}{\widetilde{\mu}}\right). \tag{xl}$$

Using (xxxii) and (xxxiii) in (xl) one can then obtain

$$\frac{2\widetilde{\mu} - z_1 - z_2}{2\widetilde{\mu} + z_1 + z_2}\left[\frac{\widetilde{X}}{\underline{X}}\frac{r}{r+\lambda}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right) + \frac{\lambda}{r+\lambda}\left(\widetilde{x} + \frac{1}{\widetilde{\mu}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1 + \frac{z_2}{\widetilde{\mu}}\right)\right] = \frac{\underline{X}}{\widetilde{X}}\frac{r}{r+\lambda}\left(\underline{x} - \frac{1}{\widetilde{\mu}}\right) + \frac{\lambda}{r+\lambda}\left(\widetilde{x} - \frac{1}{\widetilde{\mu}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1 - \frac{z_2}{\widetilde{\mu}}\right), \tag{xli}$$

which is an equation on only $\underline{x}$ and $\widetilde{x}$. Note that when $\beta \to \infty$ we have $\underline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$ and (xli) is satisfied.

Let $\delta_1 = \underline{x} - \widetilde{x}, \delta_2 = \overline{x} - \underline{x}, D_1 = e^{\widetilde{\mu}\delta_1}$, and $D_2 = e^{\widetilde{\mu}\delta_2}$. Using (xxxiv) and (xxxvi) to take out $B_5$, and using $B_1$ from (xxxii), $B_2$ from (xxxiii), $B_3$ from (xxx), and $\widetilde{C}_1$ from (xxxviii), we can obtain

$$\frac{1}{D_2}\left(\overline{x} + \frac{1}{\widetilde{\mu}}\right) = \frac{\lambda + r}{r}G_1(\underline{x}, \widetilde{x}), \tag{xlii}$$

where

$$G_1(\underline{x}, \widetilde{x}) = D_1\Bigg[-\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x} + \frac{1}{\widetilde{\mu}}\right) + \frac{r+\beta}{\beta}\widetilde{x}\left(1 + \frac{z_2}{\widetilde{\mu}}\right) + \sqrt{\frac{r+\beta}{\beta}}\frac{z_2 - z_1}{2\widetilde{\mu} + z_1 + z_2}\Bigg[\frac{1}{D_1}\frac{r}{r+\lambda}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right) + \frac{\lambda}{r+\lambda}\left(\widetilde{x} + \frac{1}{\widetilde{\mu}}\right) - \widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\left(1 + \frac{z_2}{\widetilde{\mu}}\right)\Bigg]\Bigg] - \frac{\lambda r}{4(r+\lambda)^2}\Bigg(\underline{x}(3 + 2\widetilde{\mu}\delta_1 + D_1^2) + \frac{1}{\widetilde{\mu}}(5 + 2\widetilde{\mu}\delta_1 - D_1^2)\Bigg). \tag{xliii}$$

Similarly, using (xxxv) and (xxxvii) to take out $B_6$, and using $B_1$ from (xxxii), $B_2$ from (xxxiii), $B_4$ from (xxxi), and $\widetilde{C}_1$ from (xxxviii), we can obtain

$$D_2\left(\overline{x} - \frac{1}{\widetilde{\mu}}\right) = \frac{\lambda + r}{r} G_2(\underline{x}, \widetilde{x}), \tag{xliv}$$

where

$$G_2(\underline{x}, \widetilde{x}) = \frac{1}{D_1}\Bigg[ -\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x} - \frac{1}{\widetilde{\mu}}\right) + \frac{r+\beta}{\beta}\widetilde{x}\left(1 - \frac{z_2}{\widetilde{\mu}}\right) + \sqrt{\frac{r+\beta}{\beta}} \frac{z_1 - z_2}{2\widetilde{\mu} + z_1 + z_2}\Bigg[\frac{1}{D_1}\frac{r}{r+\lambda}\Bigg(\underline{x} + \frac{1}{\widetilde{\mu}}\Bigg) + \frac{\lambda}{r+\lambda}\left(\widetilde{x} + \frac{1}{\widetilde{\mu}}\right) - \widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\left(1 + \frac{z_2}{\widetilde{\mu}}\right)\Bigg]\Bigg] - \frac{\lambda r}{4(r+\lambda)^2}\Bigg(\underline{x}(3 - 2\widetilde{\mu}\delta_1 + \frac{1}{D_1^2}) - \frac{1}{\widetilde{\mu}}(5 - 2\widetilde{\mu}\delta_1 - \frac{1}{D_1^2})\Bigg). \tag{xlv}$$

Note then that (xli), (xlii), and (xliv) is a system of equations for $\overline{x}, \underline{x}$, and $\widetilde{x}$. Note also that putting (xlii) and (xliv) together one obtains

$$\overline{x}^2 = \frac{(\lambda + r)^2}{r^2} G_1 G_2 + \frac{1}{\widetilde{\mu}^2}, \tag{xlvi}$$

which determines $\overline{x}$ as a function of $\underline{x}$ and $\widetilde{x}$. Plugging it in (xlii), we can then use (xli) and (xlii) to solve for $\underline{x}$ and $\widetilde{x}$.

Derivation of Optimal Decision-Making for $\beta = 0$ in the Two Search Modes Case:

In the case of $\beta \to 0$ and $\widetilde{x} \to 0$, we obtain $z_1, z_2 \to \widetilde{\mu}$, and for $x < \widetilde{x} = 0$ we obtain

$$V_2(x) = \widehat{B}_1 e^{\widetilde{\mu} x} \tag{xlvii}$$

$$V_1(x) = \widehat{B}_2 e^{\widetilde{\mu} x} - \frac{\lambda \widehat{B}_1}{\sigma^2 \widetilde{\mu}} e^{\widetilde{\mu} x}, \tag{xlviii}$$

where $\widehat{B}_1$ and $\widehat{B}_2$ are constants to be determined.

We then have that the condition $\frac{\beta}{r+\beta} V_1(\widetilde{x}) = \widetilde{x}$, (xxix), is no longer required, and that conditions (xxi)-(xxiv), are replaced by the conditions

$$
\begin{aligned}
B_5 + B_6 &= \widehat{B}_2 && \text{(xlix)}\\
\widetilde{\mu}B_5 - \widetilde{\mu}B_6 + \frac{\lambda^2}{(r+\lambda)^2} &- \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1 + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2 &&\\
&= \widetilde{\mu}\widehat{B}_2 - \frac{\lambda\widehat{B}_1}{\sigma^2\widetilde{\mu}} && \text{(l)}\\
B_1 + B_2 &= \widehat{B}_1 && \text{(li)}\\
\widetilde{\mu}B_1 - \widetilde{\mu}B_2 + \frac{\lambda}{r+\lambda} &= \widetilde{\mu}\widehat{B}_1, && \text{(lii)}
\end{aligned}
$$

respectively.

Using (li) and (lii) we can obtain $B_2 = \frac{\lambda}{2\widetilde{\mu}(r+\lambda)}$. Using this in (xxxiii), we can obtain the condition for the optimal $\underline{x}$ as

$$e^{\mu\underline{x}}(1-\mu) + \frac{\lambda}{r} = 0, \qquad \text{(liii)}$$

as $\mu = \widetilde{\mu}$ for $\beta = 0$, which is intuitively the same condition as (8). Using (xxxii) and (xxiii) we can then also obtain $\widehat{B}_1 = \frac{r}{2(r+\lambda)}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right)\frac{1}{\underline{X}} + \frac{\lambda}{2\widetilde{\mu}(r+\lambda)}$.

Note also that in this case (xxxvii) is replaced by

$$2B_6 - \frac{\lambda^2}{\widetilde{\mu}(r+\lambda^2)} - \frac{\lambda}{2(r+\lambda)}B_2 + \frac{\lambda}{2(r+\lambda)}B_1 = \frac{\lambda\widehat{B}_1}{2(r+\lambda)}. \qquad \text{(liv)}$$

Using (liv) and (xxxv) to substitute away $B_6$, we can then use $B_1, B_2$, and $B_4$ obtained above to yield

$$\frac{\lambda}{4\widetilde{\mu}} + \frac{\lambda}{\widetilde{\mu}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\mu}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\mu}} - \frac{\lambda}{2}\underline{x}, \qquad \text{(lv)}$$

which determines $\overline{x}$ as a function of $\underline{x}$.

Proof of Proposition 4:

For $\beta = 0$ and $\lambda$ sufficiently small, the extent of choice closure in search mode 1 is approximated by

$$\overline{x} - \underline{x} \approx \frac{\lambda}{r\mu}\left(\frac{1}{2} - \frac{1}{e}\right) = \sqrt{\frac{\lambda}{r+\lambda}\frac{\sigma^2}{2r}}\frac{1}{\sqrt{r}}\left(\frac{1}{2} - \frac{1}{e}\right) \qquad \text{(lvi)}$$

The extent of choice closure in search mode 2 is approximated by

$$\underline{x} - \widetilde{x} \approx \frac{1}{\mu}\left(\frac{\lambda}{re} + 1\right) = \frac{\sigma^2}{2r}\left(\sqrt{\frac{\lambda}{r+\lambda}}\frac{1}{\sqrt{r}}\frac{1}{e} + \sqrt{\frac{r+\lambda}{r}}\right) \tag{lvii}$$

The comparative statics follow immediately.

Derivation of Solution for Start-Up Search Costs Case:

Using Itô's Lemma on equation (23) and solving the corresponding differential equation, we can obtain

$$\widetilde{V}(x) = \frac{\lambda F + c}{\sigma^2}x^2 + a_1 x + a_0, \tag{lviii}$$

where $a_1$ and $a_0$ are constants to be determined.

Using Itô's Lemma on equation (24) and solving the corresponding differential equation, one obtains

$$V(x) = C_1 e^{\widehat{\mu}x} + C_2 e^{-\widehat{\mu}x} + x - c/\lambda, \tag{lix}$$

Using Itô's Lemma on equation (25) and solving the corresponding differential equation, one obtains

$$\widehat{V}(x) = C_3 e^{\widehat{\mu}x} + C_4 e^{-\widehat{\mu}x} - c/\lambda. \tag{lx}$$

If $\widetilde{x} > 0$, then value matching and smooth pasting at $\overline{x}, \widetilde{x}$, $\widehat{x}$, and $\underline{\widehat{x}}$ leads to $V(\overline{x}) = \overline{x}, V'(\overline{x}) = 1, V(\widetilde{x}) = \widetilde{V}(\widetilde{x}), V'(\widetilde{x}) = \widetilde{V}'(\widetilde{x}), V(\widetilde{x}) - F = \widetilde{x}, \widetilde{V}(\widehat{x}) - F = 0, \widetilde{V}(\widehat{x}) =$

$\widehat{V}(\widehat{x}), \widetilde{V}'(\widehat{x}) = \widehat{V}'(\widehat{x}), \widehat{V}(\underline{\widehat{x}}) = 0$, and $\widehat{V}'(\underline{\widehat{x}}) = 0$, which are the conditions

$$\begin{aligned}
C_1 e^{\widehat{\mu}\overline{x}} + C_2 e^{-\widehat{\mu}\overline{x}} &= c/\lambda && \text{(lxi)}\\
C_1 e^{\widehat{\mu}\overline{x}} - C_2 e^{-\widehat{\mu}\overline{x}} &= 0 && \text{(lxii)}\\
C_1 e^{\widehat{\mu}\widetilde{x}} + C_2 e^{-\widehat{\mu}\widetilde{x}} &= F + c/\lambda && \text{(lxiii)}\\
\widehat{\mu}[C_1 e^{\widehat{\mu}\widetilde{x}} - C_2 e^{-\widehat{\mu}\widetilde{x}}] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(lxiv)}\\
C_3 e^{\widehat{\mu}\widehat{x}} + C_4 e^{-\widehat{\mu}\widehat{x}} &= F + c/\lambda && \text{(lxv)}\\
\widehat{\mu}[C_3 e^{\widehat{\mu}\widehat{x}} - C_4 e^{-\widehat{\mu}\widehat{x}}] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(lxvi)}\\
C_3 e^{\widehat{\mu}\underline{\widehat{x}}} + C_4 e^{-\widehat{\mu}\underline{\widehat{x}}} &= c/\lambda && \text{(lxvii)}\\
C_3 e^{\widehat{\mu}\underline{\widehat{x}}} - C_4 e^{-\widehat{\mu}\underline{\widehat{x}}} &= 0 && \text{(lxviii)}\\
a_0 + a_1\widetilde{x} + \frac{\lambda F + c}{\sigma^2}\widetilde{x}^2 &= \widetilde{x} + F && \text{(lxix)}\\
a_0 + a_1\widehat{x} + \frac{\lambda F + c}{\sigma^2}\widehat{x}^2 &= F. && \text{(lxx)}
\end{aligned}$$

From (lxi) and (lxii) we can obtain $C_1 = \frac{c}{2\lambda}e^{-\widehat{\mu}\overline{x}}$ and $C_2 = \frac{c}{2\lambda}e^{\widehat{\mu}\overline{x}}$. Similarly, (lxvii) and (lxviii) we can obtain $C_3 = \frac{c}{2\lambda}e^{-\widehat{\mu}\underline{\widehat{x}}}$ and $C_4 = \frac{c}{2\lambda}e^{\widehat{\mu}\underline{\widehat{x}}}$. Using this in the other equations we can then obtain

$$\begin{aligned}
\frac{c}{2\lambda}e^{\widehat{\mu}(\widetilde{x}-\overline{x})} + \frac{c}{2\lambda}e^{-\widehat{\mu}(\widetilde{x}-\overline{x})} &= F + \frac{c}{\lambda} && \text{(lxxi)}\\
\widehat{\mu}\left[\frac{c}{2\lambda}e^{\widehat{\mu}(\widetilde{x}-\overline{x})} - \frac{c}{2\lambda}e^{-\widehat{\mu}(\widetilde{x}-\overline{x})}\right] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(lxxii)}\\
\frac{c}{2\lambda}e^{\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})} + \frac{c}{2\lambda}e^{-\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})} &= F + \frac{c}{\lambda} && \text{(lxxiii)}\\
\widehat{\mu}\left[\frac{c}{2\lambda}e^{\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})} - \frac{c}{2\lambda}e^{-\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})}\right] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(lxxiv)}\\
\frac{\lambda F + c}{\sigma^2}(\widetilde{x}^2 - \widehat{x}^2) + a_1(\widetilde{x} - \widehat{x}) &= \widetilde{x}. && \text{(lxxv)}
\end{aligned}$$

From (lxxi) and (lxxiii) we can obtain $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$. Using (lxxi) we can also obtain $e^{\widehat{\mu}(\widetilde{x}-\overline{x})} = 1/H$, where

$$H = 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}. \qquad \text{(lxxvi)}$$

Using $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$ in (lxxii) and (lxxiv) we can obtain

$$a_1 = \frac{1}{2} - \frac{\lambda F + c}{\sigma^2}(\widetilde{x} + \widehat{x}). \qquad \text{(lxxvii)}$$

Substituting in (lxxv) one obtains $\widetilde{x} = -\widehat{x}$ and $a_1 = 1/2$. Using this in (lxxii) one obtains

$$\widetilde{x} = \sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F + c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}}. \tag{lxxviii}$$

If $\widetilde{x} = 0$, the DM may strictly prefer stopping search without adopting the alternative to deferring choice. So, the value matching condition $V(\widetilde{x}) - F = \widetilde{x}$ needs to be replaced by $V(\widetilde{x}) - F \geq \widetilde{x}$. In that case, $\widetilde{x}$ will be 0 rather than $\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F+c)}}$. Therefore, in general,

$$\widetilde{x} = \max\left\{\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F + c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}}, 0\right\} \tag{lxxix}$$

Proof of Proposition 5:

The derivations for the comparative statics with regard to $F$ and the comparative statics of the extent of choice closure with regard to $c$ are straightforward.

According to (26) and $\widetilde{x} = -\widehat{x}$, if $\widetilde{x} > 0$, then

$$\text{sign}\left\{\frac{\partial(\widetilde{x} - \widehat{x})}{\partial c}\right\} = \text{sign}\left\{\frac{8Fc^2}{2c + \lambda F} - \sigma^2\right\} \tag{lxxx}$$

First consider $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$. Since $\widetilde{x} > 0$ is equivalent to $\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F+c)}} > 0 \Leftrightarrow 8F(2c + \lambda F) - \sigma^2 < 0 \Leftrightarrow c < \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$, we have $\frac{\partial(\widetilde{x}-\widehat{x})}{\partial c} < 0$ if $c < \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$ and $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$, according to (lxxx). $\widetilde{x} = 0$ and thus $\frac{\partial(\widetilde{x}-\widehat{x})}{\partial c} = 0$ if $c \geq \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$ and $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$.

Now consider $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 \geq 0$. We have shown in the previous case that $\widetilde{x} = 0$ is equivalent to $8F(2c + \lambda F) - \sigma^2 \geq 0$. Since $8F(2c + \lambda F) - \sigma^2 > \frac{8Fc^2}{2c+\lambda F} - \sigma^2 \geq 0$, $\widetilde{x}$ is always 0 when $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 > 0$.

In sum, the extent of choice closure always (weakly) decreases in $c$.

Now let's look at the comparative statics with regard to $\lambda$. One can see that $H$ is increasing in $\lambda$. Therefore, $\frac{1-H^2}{H} = \frac{1}{H} - H$ is decreasing in $\lambda$ and $\widetilde{x} - \widehat{x} = 2\widetilde{x} = \sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{2(\lambda F+c)}}$ is decreasing in $\lambda$. So, the extent of choice deferral decreases in $\lambda$.

$$(27) \Rightarrow \overline{x} - \widetilde{x} = \frac{1}{\widehat{\mu}} \ln H = \sqrt{\frac{\sigma^2}{2\lambda}} \ln \left[ 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1} \right]$$

$$\Rightarrow \text{sign}\left\{ \frac{\partial(\overline{x} - \widetilde{x})}{\partial \lambda} \right\} = \text{sign}\left\{ -\ln H + \frac{2\lambda F}{c\sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}} \right\} = \text{sign}[G(\lambda)] \qquad \text{(lxxxi)}$$

, where $G(\lambda) := -\ln H + \frac{2\lambda F}{c\sqrt{(\frac{\lambda F}{c}+1)^2-1}}$. One can see that $G(0) = 0$ and $G'(\lambda) \propto -(\frac{\lambda F}{c})^2 < 0$. Therefore, $G(\lambda) < 0, \forall \lambda > 0$. (lxxxi) then implies that $\frac{\partial(\overline{x}-\widetilde{x})}{\partial \lambda} < 0$. So, the extent of choice closure decreases in $\lambda$.

Some Analysis of Optimal Pricing:

Substituting $W_f(x) = \frac{\beta}{r+\beta} V_f(x)$ into (29), and using Itô's Lemma, we can obtain the second order differential equation in $V_f(x)$ for $x < \widetilde{x} + P$ as

$$r \frac{r + \beta + \lambda}{r + \beta} V_f(x) = \frac{\sigma^2}{2} V_f''(x). \qquad \text{(lxxxii)}$$

Given that $\lim_{x \to -\infty} V_f(x) = 0$, as the expected payoff of the firm has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (lxxxii) satisfies

$$V_f(x) = \widetilde{A}_1 e^{\mu x} \qquad \text{(lxxxiii)}$$

where $\widetilde{A}_1$ is a constant to be determined.[9]

Similarly, applying Itô's Lemma to (30), we can solve the resulting second order differential equation in $V_f(x)$ for $x \in (\widetilde{x} + P, \overline{x} + P)$ as

$$V_f(x) = \widetilde{A}_2 e^{\widetilde{\mu} x} + \widetilde{A}_3 e^{-\widetilde{\mu} x} + \frac{\lambda}{r + \lambda} P \qquad \text{(lxxxiv)}$$

where $\widetilde{A}_2$ and $\widetilde{A}_3$ are constants to be determined.

[9]Recall that $\mu = \sqrt{\frac{2r}{\sigma^2} \frac{r+\beta+\lambda}{r+\beta}}$ and $\widetilde{\mu} = \sqrt{\frac{2(r+\lambda)}{\sigma^2}}$.

Conditions (32)-(34) can be written as:

$$P = \widetilde{A}_2 e^{\widetilde{\mu}(\overline{x}+P)} + \widetilde{A}_3 e^{-\widetilde{\mu}(\overline{x}+P)} + \frac{\lambda}{r+\lambda}P \tag{lxxxv}$$

$$\widetilde{A}_1 e^{\mu(\widetilde{x}+P)} = \widetilde{A}_2 e^{\widetilde{\mu}(\widetilde{x}+P)} + \widetilde{A}_3 e^{-\widetilde{\mu}(\widetilde{x}+P)} + \frac{\lambda}{r+\lambda}P \tag{lxxxvi}$$

$$\widetilde{A}_1 \mu e^{\mu(\widetilde{x}+P)} = \widetilde{A}_2 \widetilde{\mu} e^{\widetilde{\mu}(\widetilde{x}+P)} - \widetilde{A}_3 \widetilde{\mu} e^{-\widetilde{\mu}(\widetilde{x}+P)}. \tag{lxxxvii}$$

Taking the derivative of (lxxxiv) with respect to price and making it equal to zero, yields the optimal price for $x_0 \in [x_0^*, x_0^{**}]$. This yields an equation $h(P, x_0) = 0$ which is represented by

$$h(P, x_0) = \frac{\lambda}{r+\lambda}P^* + \widetilde{A}_2(1-\widetilde{\mu}P^*)e^{\widetilde{\mu}x_0} + \widetilde{A}_3(1+\widetilde{\mu}P^*)e^{-\widetilde{\mu}x_0} = 0, \tag{lxxxviii}$$

where $\widetilde{A}_2$ and $\widetilde{A}_3$ are both functions of price.

In order to obtain some more specific results we consider two particular cases.

*The Case of $\beta \to 0$ for $x_0 \in [x_0^*, x_0^{**}]$ :*

When $\beta \to 0$, we have $\mu \to \widetilde{\mu}, \widetilde{x} \to 0$, and $e^{\mu\overline{x}}(1 - \mu\overline{x}) + \frac{\lambda}{r} = 0$ (from which we recall that $\overline{x} > 1/\mu$).

From (lxxxiv)-(lxxxvii) we can obtain that in the limit

$$V_f(x_0) = \frac{P}{r+\lambda}\left[\frac{r(\mu\overline{x}+1)}{2}e^{\mu(x_0-\overline{x}-P)} - \frac{\lambda}{2}e^{\mu(P-x_0)} + \lambda\right]$$

$$\text{sign}\left\{\frac{\partial V_f(x)}{\partial P}\right\} = \text{sign}\left\{\lambda + (1-\mu P)\frac{r(\mu\overline{x}+1)}{2}e^{\mu(x_0-\overline{x}-P)} - (1+\mu P)\frac{\lambda}{2}e^{\mu(P-x_0)}\right\}.$$

Note that in this case we have $x_0^* \to 1/\mu$. So, for $x_0 > x_0^*$, we can obtain that $\frac{\partial V_f(x_0)}{\partial P} > 0$ for $P = 1/\mu$. Furthermore, we can obtain that $\frac{\partial V_f(x_0)}{\partial P} < 0$ for $P = x_0$. So, we have that for $x_0 \in [x_0^*, x_0^{**}]$ we have that $P^* \in [1/\mu, x_0]$. If the price function is continuous at $x_0^{**}$, then from the definition of $x_0^{**}$ we can also obtain for this case of $\beta \to 0$ that $x_0^{**} \to \overline{x} + \frac{r+\lambda}{r\mu^2\overline{x}}$.

To check whether the price function is continuous at $x_0^{**}$, we can check whether for $x_0^{**}$ obtained by $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$ we have that $V_f(x_0^{**}, P)$ is concave in the price $P$ when $P = x_0^{**} - \overline{x}$. This condition yields, using (8), $\lambda/r < 2a^2 - 1$ where $a > 1$ satisfies $e^a(a-1) - 2a^2 + 1 = 0$.[10] For $\lambda/r > 2a^2 - 1$ we then have then that the price function

[10]This yields $a \approx 1.94$ and $2a^2 - 1 \approx 6.51$.

cannot be continuous at $x_0^{**}$ and we then have $x_0^{**}$ obtained by $x_0^{**} - \overline{x} = V_f(x_0^{**}, P^*(x_0^{**}))$ and $P^*(x_0^{**}) \in \arg\max_P V_f(x_0^{**}, P)$, and that the optimal price falls at the discontinuity, $\lim_{x_0 \nearrow x_0 **} P^*(x_0) > x_0^{**} - \overline{x}$.

For the case in which the price function is continuous at $x_0^{**}$ we can also obtain that the price function is not monotonic in $x_0$ for $x_0 \in [x_0^*, x_0^{**}]$. Note that $\frac{\partial^2 V_f(x_0)}{\partial P \partial x_0}_{|x_0 = x_0^*} > 0$ so that the optimal price is increasing in $x_0$ for $x_0$ close to $x_0^*$. Note also that $\frac{\partial^2 V_f(x_0)}{\partial P \partial x_0}_{|x_0 = x_0^{**}}$ when the price function is continuous at $x_0^{**}$ can be negative if $\mu \overline{x} < \sqrt{1 + \lambda/r}$. Using (8), we can then obtain that this condition always holds when the price function is continuous. In this case of $\beta \to 0$ and continuous price function, we then obtain that the optimal price is decreasing in $x_0$ for $x_0$ close to $x_0^{**}$.

Since $P^* < x_0 - \widetilde{x}$ for $x_0 > x_0^*$, the DM adopts the alternative at $x_0$ in the no-search mode. Since $P = x_0 - \overline{x}$ is optimal for $x_0 > x_0^{**}$, the DM adopts the alternative at $x_0$ in the search mode in that case. We can show that the firm's value function decreases in the price $P$ when $x_0$ is high enough. When the DM's prior belief about the alternative is high enough, the firm can already obtain a high payoff by inducing the DM to adopt the alternative immediately without searching.

*The Case of $\beta \to \infty$ for $x_0 \in [x_0^*, x_0^{**}]$ :*

When $\beta \to \infty$, we have $\overline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$. Therefore, the interval $[x_0 - \overline{x}, x_0 - \widetilde{x}]$ disappears and the possible optimal prices are $P \geq x_0 - \widetilde{x}$. From the previous analysis, one can see that the optimal price is

$$P^* = \begin{cases} 1/\mu, \ if \ 1/\mu > x_0 - \widetilde{x} \\ x_0 - \widetilde{x}, \ otherwise. \end{cases}$$

To get that the price function is continuous for $\beta$ large, we can obtain that $\frac{\partial^2 V_f(x,P)}{\partial P^2}_{|x = x_0^{**}, P = x_0^{**} - \overline{x}}$ is strictly negative for $\beta \to \infty$ and $x_0^{**}$ satisfying $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$. The result that the price function is monotonic for $\beta \to \infty$ is straightforward to obtain since we have $x_0^{**} - x_0^* \to 0$ for $\beta \to \infty$.

## Optimal Pricing in a Two-period Model:

Consider a similar setup in discrete time. There are two periods, $t = 1, 2$. The DM can search for information about the alternative at most twice. Given the belief at the beginning of each period, $x_t$, the DM's belief will become $x_t + \Delta$ or $x_t - \Delta$ with equal probability if she is in the search mode and decides to search. The DM can adopt the alternative without searching, after searching once, or after searching twice. She may switch from the search to the no-search mode with probability $\lambda$ at the end of the first period. The discount factor per period of both the firm and the DM is $\widehat{\delta}$. Let us first consider the optimal search strategy of the DM, where $y_t = x_t - P$.

**Proposition 7.** *Suppose* $4\widehat{\delta} + (1-\lambda)\widehat{\delta}^2 > 4$.[11] *If* $y_0 \geq \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta$ *the DM adopts the alternative without searching. If* $y_0 \in [\Delta, \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta)$ *the DM adopts the alternative after receiving a positive signal, receiving a negative signal and then a positive signal, or switching to the no-search mode at the end of period one. If* $y_0 \in [-\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, \Delta)$ *The DM adopts the alternative after receiving a positive signal or receiving a negative signal and then a positive signal. If* $y_0 \in [-\Delta, -\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta)$ *the DM adopts the alternative after receiving two positive signals, or receiving a positive signal and then switching to the no-search mode. If* $y_0 \in [-2\Delta, -\Delta]$ *the DM adopts the alternative after receiving two positive signals*

From this proposition, one can see that the DM's adoption likelihood is piecewise constant and non-decreasing in $x_0$. Therefore, the firm will only choose from prices such that $y_0 = x_0 - P \in \{\frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, \Delta, -\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, -\Delta, -2\Delta\}$. Denote those price schemes by $P_1(x_0) = x_0 - \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, P_2(x_0) = x_0 - \Delta, P_3(x_0) = x_0 + \frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, P_4(x_0) = x_0 + \Delta, P_5(x_0) = x_0 + 2\Delta$. Note that the price increases from $P_1(x_0)$ to $P_5(x_0)$ for a given $x_0$. The corresponding profits are: $\Pi_1(x_0) = x_0 - \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, \Pi_2(x_0) = (\frac{\widehat{\delta}}{2} + \frac{\lambda\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 - \Delta), \Pi_3(x_0) = (\frac{\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 + \frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta), \Pi_4(x_0) = (\frac{\lambda\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 + \Delta), \Pi_5(x_0) = \frac{1-\lambda}{4}\widehat{\delta}^2(x_0 + 2\Delta)$. By plotting the firm's profits from all the candidate price schemes in Figure A.1, we can illustrate the optimal pricing strategy. The firm's expected payoff from charging each pricing scheme is linear in $x_0$. A lower pricing scheme leads to a higher adoption likelihood, and thus corresponds to a profit function with a higher slope and lower intercept. When the prior belief $x_0$ is low, the firm charges the highest candidate price $P_5(x_0)$, which increases in $x_0$ linearly. The intuition is that the DM only cares about $y_0 = x_0 - P$. So, the firm can charge a higher price to induce the same adoption likelihood when the prior belief increases. As $x_0$ increases to a certain level, however, the firm switches from charging a price given by $P_5(x_0)$ to charging a lower price given by $P_4(x_0)$. Intuitively, as $x_0$ and the price increase, the firm's loss from non-adoption is larger. Therefore, the firm has a higher incentive to induce the DM to search less and adopt more while the cost of doing so, $P_5(x_0) - P_4(x_0)$, does not depend on $x_0$. When this incentive becomes strong enough, the optimal price has

[11] If this condition is not satisfied, the threshold of adopting the alternative without searching is different. But the intuition of the entire analysis is the same. We omit the presentation of that case for simplicity.

a discrete downward jump, as illustrated in Figure A.2. Then, the optimal price remains as given by $P_4(x_0)$ and increases in $x_0$ linearly until it switches from the pricing function $P_4(x_0)$ to $P_3(x_0)$ and decreases discontinuously. The optimal price then remains given $P_3(x_0)$ and increases in $x_0$ linearly until it switches from $P_3(x_0)$ to $P_1(x_0)$. For $x_0$ high enough, the optimal price is always given by $P_1(x_0)$, low enough such that the DM adopts the alternative without searching. In sum, each time the firm switches from one pricing scheme to another with a higher slope, the optimal price decreases discontinuously. In all other places, the optimal price increases in $x_0$ linearly.

The optimal price as a function of $x_0$ is smoother in continuous time. But non-monotonicity and discontinuity may still arise due to the effects we identify in discrete time.

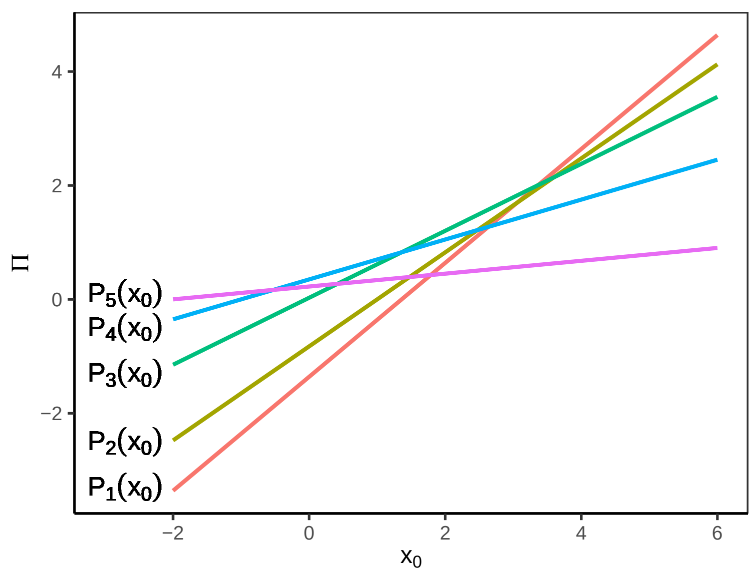



Figure A.1: Example of the firm's profit $\Pi$ as a function of $x_0$ for $r = .95, \lambda = .5$, and $\Delta = 1$.

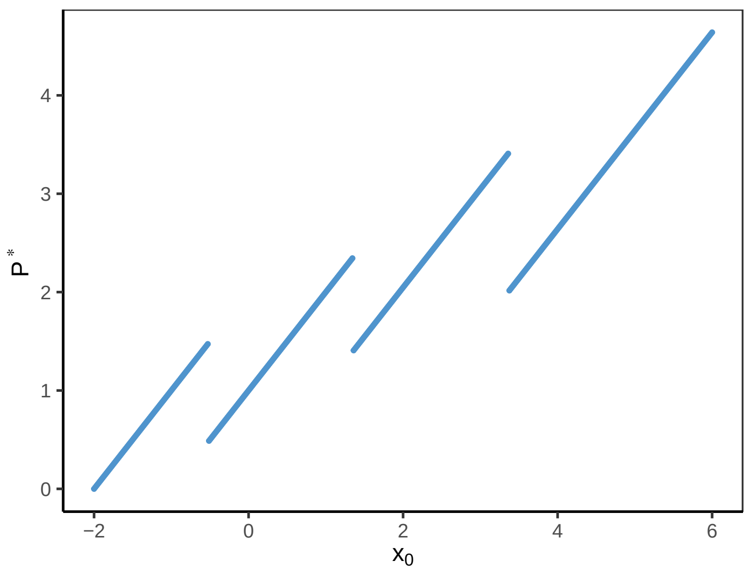



Figure A.2: Example of the optimal price $P^*$ as a function of $x_0$ for $r = .95, \lambda = .5$, and $\Delta = 1$.

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