Following the Customers: Dynamic Competitive Repositioning*

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Abstract

We consider dynamic repositioning when competing firms try to follow the evolution of consumer preferences, while taking into account the competitive interaction, both in terms of static market competition, and the dynamic effects of different firm positionings. We fully characterize the dynamic market equilibrium, which includes the timing of the firms' repositionings depending on consumer preferences. As consumer preferences evolve away from where both firms are located, one firm first moves to follow consumer preferences, with the second firm only moving if the consumer preferences continue evolving away from that firm. The model predicts rich market dynamics, where firms stay for some period in different positionings if consumer preferences are in a relatively middle ground, or where a firm repositions to follow consumer preferences, but then repositions back to the original position, if consumer preferences return. We find that, when the variability of the consumer preferences or the discount rate is greater, or when the importance of the repositioning attribute is smaller, firms are less likely to follow consumer preferences. Firms are more heterogeneous in their responses, which leads to longer periods of differentiation, when the variability of the consumer preferences, the discount rate, or the importance of the repositioning attribute increases. We also find that competing firms reposition less frequently than what is socially optimal and than what collusion would imply, and we find more differentiation under collusion than under competition.

1. Introduction

Consumer preferences evolve over time which can make existing products obsolete. Armstrong and Kotler (2009, p.250) point out that “[s]ales decline for many reasons, including ... shifts in consumer tastes,” and when that happens, “management may decide to reposition or reinvigorate the brand in hopes of moving it back into the growth stage of the product life cycle.” Firms have to keep adjusting their positioning to follow consumer preferences. Repositioning a brand, which may require adjustments to products, packages, logos, or communication, is costly, and firms have to decide when to incur those costs depending on how far the consumer preferences are from the current offering, as well as their competitors’ own positioning and potential decision to reposition. Managing product positioning under changing consumer preferences and competition is a critical problem in marketing. Anderson and Shugan (1991) point out “a need for analytical and empirical understanding of how changes in consumer preferences can interact with the ability to reposition. This is particularly urgent in a world where change comes rapidly and the ability of firms to adapt to the competitive environment can be of critical importance.”

Firms may decide to reposition when consumer preferences are sufficiently far away from what the product is offering, but at the same time have to take into account the strategic interactions with the competitors. For example, in the autombile market, when consumer preferences evolved for a greater appreciation for sport utility vehicles or for electric vehicles, the automobile manufacturers had to decide to make the move to this type of vehicle, depending on both consumer preferences and the potential strategic investments of their competitors.[1] Automobile manufacturers also make major re-designs of their models every few years (e.g., BMW, Honda, Mercedes, Toyota).[2] Companies also adjust their logo every few years (e.g., Apple, IBM, Oracle). Some of these changes can be seen as making the products or communications adjust to the changing consumer preferences. Competing fashion designers also adjust over time their clothing, for example, changing the length of skirts, or, the width of ties, likely also to accommodate perceived variations in consumer preferences. In 2016, man’s fragrance brand AXE “underwent a total 180-degree repositioning,” with new advertising campaigns and packaging, after Unilever’s research found that the brand’s legacy image of masculinity no longer resonated with young consumers (Wischhover 2017).

[1] Obviously, automobile manufacturers can also offer multiple products to target different consumer segments, but there is still a sense of the dimension on which they put the major effort.

[2] In the automobile manufacturing industry, model updates often include the latest technological innovations (which could be related to a greater preference for technology), but also include significant redesign changes.

Whether and when a firm should adjust its position to keep pace with changing consumer preferences is a common dilemma for established brands, such as Pepsi, Gillette, Porsche, Harley-Davidson, and Marc Jacobs, among others that have undergone repositioning (Avery and Gupta 2014).

We consider a dynamic model under competition, where firms have to decide when to invest in repositioning their products, given the evolution of preferences and the repositioning decisions of the competitors. That is, firms follow consumer preferences while keeping in mind the market interaction with competitors.

The timing of repositioning is a critical, strategic decision. One crucial consideration of when to reposition in the face of evolving consumer preferences and competition is that at some stage it does not pay off for all the competing firms to reposition simultaneously, so that the market outcome involves sometimes firms differentiated in a particular dimension, and other times firms having the same positioning in that dimension. We characterize these possibilities as a market equilibrium behavior. The model then allows us to quantify the fraction of time when firms are differentiated versus when firms are co-located, and compare firms' repositioning decisions and the long-run distribution of positionings under different competitive settings.

The model predicts rich market dynamics, where firms stay for some period in different positionings if consumer preferences are in a relatively middle ground, or where a firm repositions to follow consumer preferences, but then repositions back to the original position, if consumer preferences return back to the original position. As consumer preferences evolve in a direction, a firm that repositions ahead of its competitor has a first-mover advantage if consumer preferences continue to evolve in that direction, by being closer to consumers' ideal point while pushing the competitor to strategically delay its own repositioning. At the same time, because consumer preferences can return to where they were, a firm that repositions first could also end up with a first-mover disadvantage. The equilibrium outcome results from a balance of these market forces, with each firm considering what the competitor could do. We concentrate the analysis for the case in which the costs of repositioning are small, but we could not find conditions under which the comparative statics that we present did not hold for larger repositioning costs.

We find that firms are less likely to follow consumer preferences if there is greater variability over time in consumer preferences. With greater variability of consumer preferences, firms wait longer to reposition, as it is more likely that the consumer preferences will return to where the firms are positioned. Similarly, if firms' discount rate is higher, firms are less willing to reposition because the present value of the gains from repositioning are now lower.

We also investigate the likelihood of one firm following a competitor in repositioning when consumer preferences change. This can also be seen as related to the likelihood of the firms having different positions in the market. We find that the greater the variability of consumer preferences, the less likely is a firm to follow the competitor in repositioning, which yields a greater likelihood of firms having a different positioning in the market. A greater variability in consumer preferences makes the firms try to pick locations of unique influence with the hope that consumer preferences go in their direction, while being aware that the competitor is less likely to reposition. Similarly, if the discount rate is greater, a firm is also less likely to follow the competitor's repositioning, which leads to a greater likelihood of firms being positioned differently in the market. Additionally, firms are less likely to be differentiated when the repositioning attribute is more important, as competition intensifies. These effects present interesting managerial insights and testable empirical implications on when firms should be differentiated and not differentiated under competition, as a function of the variability of consumer preferences, discount rate, and importance of the repositioning attribute.

We also compare the firm repositioning in competition with what would be socially optimal. We find that competing firms reposition less frequently than what is socially optimal, as the competing firms only account for the private benefits of repositioning. A competing firm only repositions when the profits it obtains from the repositioning decision are greater than the cost of repositioning. But the firm is not able to appropriate all the social benefits of repositioning, as some of those benefits go to the consumers because they have now products that better fit their preferences. We then have that competing firms reposition less frequently and are less differentiated in the long run than what is optimal from a social welfare point of view.

We also find that competing firms reposition less frequently than collusive firms, if we allow for collusion on prices, as collusive firms are better able to appropriate the benefits of having products that better fit the consumer preferences (although still not all the social benefits of repositioning). As collusive firms can be seen as a firm carrying both products, this argues for the merger of two competing firms to be more active in repositioning their products in response to changes in consumer preferences, offer better products on average, and result in higher social welfare, than the competing firms would do without the merger. Our model also predicts that collusive or merged firms are more differentiated than competing firms, which is different from a static Hotelling model, and offers a theoretical support for previous empirical findings that market consolidation increases differentiation in some markets (Berry and Waldfogel 2001, George 2007, Sweeting 2010).

We also find that a deterministic trend in the evolution of consumer preferences leads to an acceleration of the repositioning decisions, and to there being less differentiation in the market.

There has been substantial research on static positioning in markets (e.g., Hotelling 1929, Hauser 1988, Moorthy 1988, Sayman et al. 2002, Lauga and Ofek 2011), with particular focus on the competitive interaction. One critical question in that literature is how dynamic repositioning decisions in response to changes in consumer preferences would work out, and this paper addresses that question.[3] There is also work on the effects of the resources of the firms on their strategic positioning (e.g., Wernerfelt 1989). With dynamics, there is work on investments in R&D that generate with a certain probability success in the repositioning of the product (e.g., Harris and Vickers 1987, Ofek and Sarvary 2003). In contrast, this paper allows for the decision to reposition to have immediate effects, and therefore the timing of when to reposition a product in competition becomes the crucial decision.[4] Furthermore, here the consumer preferences move in non-predictable ways, so it becomes important to understand when firms follow consumer preferences. Another related paper is Budd, Harris, and Vickers (1993), where firms compete dynamically to gain market share, where market share evolves stochastically and continuously, depending on the firms' efforts. Here market share can evolve discontinously because of repositioning decisions by firms.[5]

A similar decision to the one considered here is whether and when to adopt new technologies, which is considered in a two-state version in Villas-Boas (1992). This paper considers a richer, uncertain environment, where the decision on when to reposition is investigated in greater depth. Another related stream of work considers richer environments of dynamic competition in R&D among firms that is presented for empirical work and which can be solved with numerical methods (e.g., Ericson and Pakes 1995, and, in particular with dynamic repositioning, Sweeting 2013, Jeziorski 2014). In relation to that work, this paper

[3]There is also a literature on static differentiation on multiple dimensions, where one general message is that firms may want to fully differentiate on one dimension and co-locate on the other dimensions (e.g., Vandenbosch and Weinberg 1995, Ansari, Economides, and Steckel 1998, Irman and Thisse 1998). See also Kuksov (2004), Guo and Zhang (2012), and Ke, Shin, and Yu (2020) for effects of consumer search costs on positioning decisions.

[4]See also Zhang (2016) on the uncertainty of the product development process.

[5]See also Selove (2014), where initial identical firms develop different capabilities over time, and Cabral and Riordan (1994), where firms compete on learning-by-doing, with actions in each period affecting the future costs.

presents sharper analysis of when to reposition, and how that decision depends on the degree of uncertainty in the market and on the discount rate.[6]

The remainder of the paper is organized as follows. The next section presents the model of competition and consumer preferences. Section 3 presents the competitive market equilibrium. Section 4 presents analysis on the expected duration when firms are differentiated, and on the deterministic trend case, and studies the collusion and social welfare optima. Section 5 concludes.

2. The Model

Consider a market with two competing firms selling to a unit mass of consumers. Each consumer buys from one of the two firms at each moment in time and time is continuous. Consumer preferences are characterized by $(x, z) \in [0, 1]^2$, where all consumers have the same x and are uniformly distributed on the dimension z.

The location of x evolves over time, and we denote by x_t the location of the consumer preferences at time t. The position x_t evolves over time as a Brownian motion with zero drift and variance σ^2. We consider in the main model that the Brownian motion has reflecting boundaries at 0 and 1, and then discuss in subsection 3.5 how all the results of the main model generalize, under some conditions, to the case when there are no reflecting boundaries, and the consumer preferences x_t can evolve freely on the real line. Note that with reflecting boundaries, in the long run x_t is uniformly distributed between 0 and 1 (see, e.g., Dixit 1993, pp. 58-60), which is the standard assumption on consumer heterogeneity in a static Hotelling model. We focus the presentation on the case in which there is only a random component in the evolution of preferences. In subsection 4.4 we consider a case when there is a deterministic trend in the evolution of the consumer preferences. We note that the composition of the zero-drift Brownian motion on the real line and the deterministic trend captures several main features of any continuous consumer preference dynamics in one dimension.[7]

[6]Also related to this paper, but here with competition, is the literature on portfolio choice with transaction costs, where an investor only adjusts the portfolio once in a while because of transaction costs and the portfolio evolves stochastically (e.g., Magill and Constantinidis 1976), and the literature on (S,s) economies from inventory problems (e.g., Scarf 1959, Sheshinski and Weiss 1983). See also Villas-Boas (2018) for the dynamic repositioning problem in a monopoly setting. See also Shen (2014), Gardete (2016) and Bonatti, Cisternas, and Toikka (2017) for dynamic competition with asymmetric information and/or uncertainty.

[7]More generality can be obtained by allowing the variance and drift of the Brownian motion to be state-dependent, or if the consumer preference dynamics are allowed to have occasional random jumps. It would be interesting to explore those possibilities in future research.

Firms can reposition on the dimension x but they cannot reposition on the dimension z. On dimension z, firms are positioned on the opposite extremes of the segment $[0,1]$. On dimension x, firms can be positioned at one, and only one, of the two ends of a segment $[0,1]$. At any moment, a firm can choose to reposition to the other side of the segment.[8] Note that, depending on the repositioning decisions, firms on dimension x can have the same positioning or different positionings.[9] In subsection 3.5 we discuss how all the results generalize, under some conditions, to the case where firms can reposition to any adjacent integer, and consumer preferences x_t evolve on the real line. When we consider the deterministic trend case in subsection 4.4 this more flexible repositioning is also allowed.

Repositioning is modeled as instantaneous but with a cost of K. Firms are symmetric with a discount rate r. We assume without loss of generality that Firm 1 is positioned at location 0. If Firm 1 repositions, we relabel its new position as 0 and the opposite side as 1. Figure 1 illustrates how consumer preferences could evolve over time and the possible repositioning of the two firms.

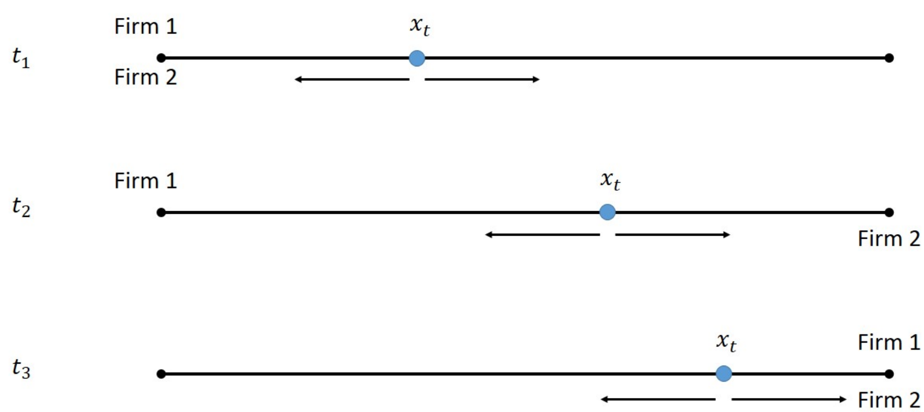

Figure 1: Possible positions of consumers and firms over time

A consumer gets utility of $v - \delta x_t - z - p_1$ when she purchases from Firm 1, where p_1 is the price charged by Firm 1. We assume v is high so that the market is fully covered.

[8]Each firm can carry only one product to focus on the repositioning decisions.

[9]We consider a one–dimension model of consumer preferences and repositionings which can be seen as somewhat simplified for the potential multidimensional changes in the examples mentioned in the introduction, for example, the automobile market examples. One could also think of projecting several potential dimensions into one dimension. For example, in the automobile example, it could be a dimension of sporty versus functional.

Dimension z represents the idea that firms never have exactly the same positioning, and always have some market power; firms still have positive profits if they are positioned at the same point on dimension x. Note that if dimension z did not exist,firms would get zero profits under price competition, when positioned at the same location, and, therefore, firms would always be differentiated, and there would never be repositioning on the equilibrium path.[10] The assumed uniform distribution of z allows us to obtain sharper results.[11] The parameter δ measures the importance of dimension x relative to dimension z in consumer preferences. If δ is very high, consumers care mostly about the dimension on which firms can reposition. If δ is very low, consumers do not care too much about the dimension on which the firms can reposition.[12] In general, firms may have some attributes that are easier to reposition (represented by dimension x in the model), and some attributes that are more difficult, or impossible, to reposition (represented by dimension z). For example, BMW and Mercedes can redesign their car models from year to year, but the brand image associated with them may be more difficult to adjust, and be seen from the perspective of consumers as differentiated. For a sharper example, Mercedes being positioned as coming from Germany, and Lexus being positioned as coming from Japan are difficult, if not impossible, to adjust.

Firms engage in price competition. Let π_s denote a firm's instantaneous profit when firms are located on the same side. Let $\pi_d(x_t)$ denote a firm's instantaneous profit when firms are located on different sides, where x_t is the distance between the firm and the consumers' ideal location. Under price competition the instantaneous profits are thus $\pi_s = \frac{1}{2}$ and $\pi_d = \frac{1}{2}p_1^{*2}$ where $p_1^* = 1 + \frac{\delta(1-2x)}{3}$.

We look for a symmetric Markov perfect equilibrium of this game. That is, strategies depend on the payoff-relevant state variables. In this case, the payoff-relevant state variables are the current positioning of the firms, and the distance x_t of the consumer preferences in dimension x to the positioning of Firm 1. We allow firms to play behavioral strategies, that

[10] Alternatively, we could consider a model without dimension z, where consumers are heterogenous on dimension x and firms reposition to locations in the real line while remaining differentiated. Such a model becomes more complex to fully solve as firms have to choose the degree of differentiation at each repositioning decision, with an increased number of choices and states for the firms. That issue of the degree of differentiation is partially fixed here with dimension z.

[11] We expect that other probability distributions on z would generate similar results.

[12] The case of the dimension z not existing can be seen as the case of $\delta \to \infty$ (with also $v \to \infty$). Note also that if $\delta = 0$ there is no gain in firms repositioning. Note also that on the static differentiation literature over multiple dimensions (e.g., Vandenbosch and Weinberg 1995, Ansari, Economides, and Steckel 1998, Irman and Thisse 1998) there is a result that firms can maximally differentiate on one dimension and co-locate on the other dimensions. The model here can be seen as fixing maximal differentiation on one dimension, z, and to check when the firms want to differentiate on the other dimension depending on the evolution of the consumer preferences.

is, mixed strategies at each moment in time. A behavioral strategy at a given state x_t is represented by a function $\mu(x_t)$, which gives the hazard rate at which a firm repositions.

Let us conclude this Section by briefly discussing some of the model assumptions. The model considers two specific positionings for the product (and a discrete number in subsection 3.5). It would be interesting to consider more flexible repositioning (for example, to a continuum of positionings on the segment $[0, 1]$, but such possibility would increase the dimensionality of the state space, and the complexity of the potential repositioning strategies. The current set-up still captures the trade-offs faced by firms deciding whether to reposition when faced with competition and changing consumer preferences. The model also restricts attention to the two-firm case, and it would be interesting also to investigate what happens with more than two firms. Finally, the model restricts attention to the case of the market being fully covered. This allows us to focus on the competitive interaction effects, but a more general model could allow for less-than-full market coverage.

3. Market Equilibrium

3.1. Form of the Equilibrium

We now study the symmetric dynamic market equilibrium of this market. We first discuss the potential form of the equilibrium, and then construct the market equilibrium.

3.1.1. Differentiated Firms

If firms are positioned on different sides, their equilibrium strategy should feature a threshold x^* such that a firm repositions when its distance to consumers, x_t, hits x^*. The opposing firm repositions when x_t hits $1 - x^*$. That is, if firms are positioned on different sides of dimension x, they continue to be so differentiated as long as $x_t \in [1 - x^*, x^*]$, where we expect x^* to be greater than 1/2.

3.1.2. Non-Differentiated Firms

Now suppose that firms are positioned on the same side. Consider first that firms use pure strategies. Suppose that the strategy of one firm is to reposition for some $x_t < x^*$. The competitor has two potentially profitable deviations. It can let the opponent move and wait until x^* to reposition itself. By definition of x^*, this is always preferred to moving together at x_t. Alternatively, it can preempt the opponent's repositioning by some ε, which forces the opponent to wait until x^* to reposition. One of these two strategies must be strictly preferred to relocating together at x_t. Then the only possible pure strategy equilibrium would be an asymmetric equilibrium where one firm moves at some $x_t < x^*$, and the other firm follows at x^*. We present such a possible equilibrium in subsection 3.6.

Given our focus on the symmetric equilibrium, we look for behavioral strategy equilibria in which firms mix their repositioning decisions at each node. Firms on the same side reposition with some hazard rate $\mu(x)$. The hazard rate can only be positive for a range of x between 1/2 and x^*, because $\pi_d(1-x) > \pi_d(x)$ if and only if $x > 1/2$. Firms have no incentive to move if they end up farther away from consumers after moving. In the remainder of the paper, we study this equilibrium characterized by x^* and $\mu(x)$.

To complete the equilibrium characterization, we need to consider what happens when firms are positioned on the same side with $x_t \geq x^*$. From previous analysis, we know that in equilibrium firms cannot be located on different sides with $x_t \geq x^*$ because the firm farther away from consumers should relocate. Suppose one firm moves at x_t so they become differentiated, then the other firm strictly prefers to follow immediately because $x_t \geq x^*$. They both incur repositioning cost K but do not gain in flow profit, because firms are still positioned on the same side. Repositioning only increases total industry revenue if it results in differentiation. Thus if one firm does not reposition at x, then the other firm also does not have incentive to do so. In this range, repositioning becomes a coordination problem. For example, one equilibrium could be that firms never reposition for $x_t \geq x^*$. Another equilibrium could be that they both reposition for all $x_t \geq x^*$. The former equilibrium gives a higher payoff to both firms by reducing total repositioning costs. The latter equilibrium is better for consumer welfare, as consumers have now products available that are closer to their preferences, and could be seen as being more reasonable as a representation of the real-world. Furthermore, if the market is not fully covered, the firms would have a further incentive to reposition at $x_t \geq x^*$. In this paper, we focus on this equilibrium that is better for consumer welfare. In any case, as we will see below, the strategy profile for $x_t \geq x^*$ is payoff irrelevant after some possible initial repositioning. Because the equilibrium involves one firm repositioning with probability one for some $x_t < x^*$, the case where $x_t > x^*$ and both firms are on the same side is never on the equilibrium path, except, potentially, for some initial period of time if the market starts with both firms on the same side and $x_0 > x^*$.

To summarize, a market equilibrium is going to be characterized by an x^* and a $\mu(x)$.

When firms are positioned on different sides, a firm repositions if its distance to the consumers reaches x^*. When firms are positioned on the same side, a firm repositions with hazard rate $\mu(x)$. Figure 2 illustrates the market equilibrium depending on the positioning of the firms and the location of the consumer preferences.

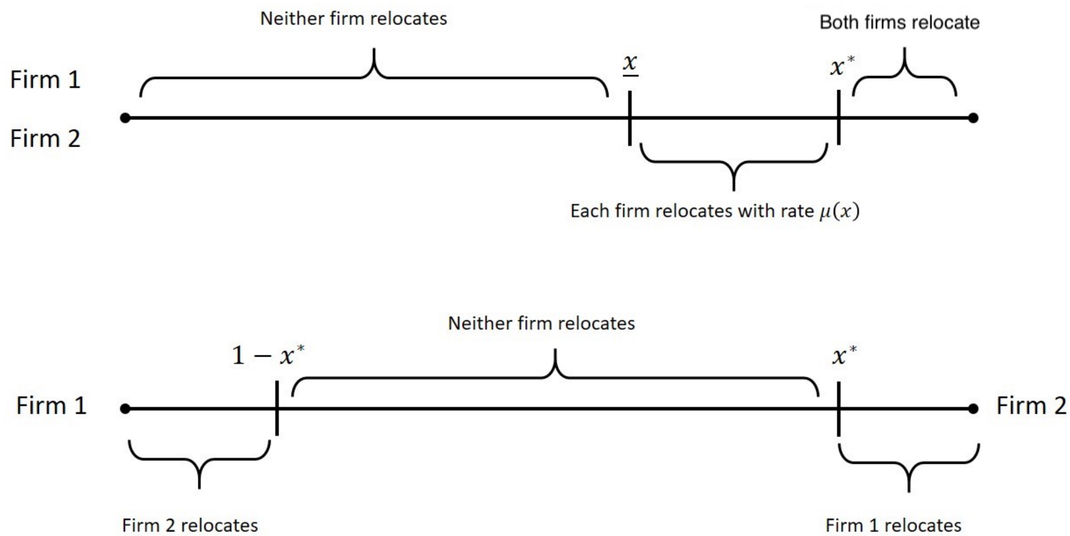

Figure 2: Equilibrium

3.2. Value Functions and Boundary Conditions

We now construct the equilibrium using the objective functions of the firms at the possible different situations.

When firms are at different locations, and $x \in [1 - x^*, x^*]$, a firm's value function as a function of its distance to consumers, x, can be written as:

$$
\begin{aligned}
V_d(x) &= \pi_d(x)dt + e^{-rdt}\mathbb{E}[V_d(x + dx)] && (1)\\
&= \frac{\pi_d(x)}{r} + \frac{1}{\lambda^2}V_d''(x)\\
&= \frac{\pi_d(x)}{r} + \frac{4\delta^2}{9r\lambda^2} + Ae^{\lambda x} + Be^{-\lambda x} \quad \text{where} \quad \lambda = \sqrt{\frac{2r}{\sigma^2}} && (2)
\end{aligned}
$$

for some constants A and B to be determined.

Let $\underline{x}$ denote the lowest x such that $\mu(x) > 0$. When firms are at the same location, and $x < \underline{x}$, a firm's value function as a function of its distance to consumers, x, can be written as:

$$V_s(x) = \pi_s dt + e^{-rdt}\mathbb{E}[V_s(x+dx)] \tag{3}$$
$$= \frac{\pi_s}{r} + Ce^{\lambda x} + De^{-\lambda x} \tag{4}$$

for some constants C and D to be determined. The reflecting boundary at $x = 0$ implies that $V_s'(0) = 0$, because at $x = 0$ the firm knows that x can only move in the positive direction.[13] This yields that:

$$\lambda Ce^{\lambda x} - \lambda De^{-\lambda x} = 0$$
$$C = D$$

So we can write $V_s(x) = \frac{\pi_s}{r} + C[e^{\lambda x} + e^{-\lambda x}]$ for $x < \underline{x}$.

When firms are on the same side, a firm must be indifferent between repositioning and not repositioning at $\underline{x}$, so that the firm is willing to mix between the two actions. So it is weakly optimal for the firm to reposition at $\underline{x}$. Then, we must have value-matching and smooth-pasting (e.g., Dixit 1993) at $\underline{x}$ for optimality, which yields

$$V_d(1-\underline{x}) - K = V_s(\underline{x}) \tag{5}$$
$$-V_d'(1-\underline{x}) = V_s'(\underline{x}). \tag{6}$$

When firms are on different sides, a firm repositions, in the proposed equilibrium, when its distance to consumers reaches x^*. The value-matching and smooth-pasting conditions at x^* are:

$$V_d(x^*) = V_s(1-x^*) - K \tag{7}$$
$$V_d'(x^*) = -V_s'(1-x^*). \tag{8}$$

Finally, when the firms are on different sides and the distance to consumers decreases to $1 - x^*$, the opponent repositions in the proposed equilibrium. Then we must have value

[13]See, Dixit 1993, for example.

matching at $1-x^*$ which yields[14]

$$V_d(1-x^*) = V_s(1-x^*) \tag{9}$$

3.3. Repositioning Rate $\mu(x)$

When firms are on the same side, let $\widetilde{V}(x)$ be a firm's value function for the range of x such that $\mu(x) > 0$. The firm should be indifferent between repositioning instantly, or waiting for an infinitesimal period of length dt, during which the opponent enters with probability $\mu(x)dt$. Repositioning instantly gives payoff

$$\widetilde{V}(x) = V_d(1-x) - K \tag{10}$$

Waiting for period dt gives payoff

$$\begin{aligned}\widetilde{V}(x) &= \pi_s dt + e^{-rdt}\mathbb{E}\Big[\mu(x)dtV_d(x+dx) + (1-\mu(x)dt)\widetilde{V}(x+dx)\Big] \\ &= \pi_s dt + \mu(x)dtV_d(x) + (e^{-rdt} - \mu(x)dt)\left[\widetilde{V}(x) + \frac{\sigma^2}{2}\tilde{V}''(x)dt\right],\end{aligned} \tag{11}$$

from which we can get

$$\mu(x) = \frac{\pi_s - r\widetilde{V}(x) + \frac{\sigma^2}{2}\widetilde{V}''(x)}{\widetilde{V}(x) - V_d(x)} \tag{12}$$

Combining (10) and (12), and using Itô's Lemma, which gives $rV_d(1-x) = \pi_d(1-x) + \frac{\sigma^2}{2}V_d''(1-x)$, we get

$$\mu(x) = \frac{rK - \pi_d(1-x) + \pi_s}{V_d(1-x) - V_d(x) - K}. \tag{13}$$

The denominator of (13) captures the benefit of preempting the opponent. There are two sources of benefit. First, if the firm repositions at x, it gets closer to consumers with a new distance of $1-x$. Second, by repositioning before its competitor does, it pushes the competitor to delay repositioning until consumer preferences reach a higher threshold, $\overline{x}$. The function $V_d(1-x)$ represents the value function for the first mover, and the function $V_d(x)$ represents the value function for the second mover, if the first mover repositions at x. Thus the denominator, $V_d(1-x) - V_d(x) - K$, is the value of being the first mover at x minus the cost of repositioning.

[14]Note that, because this is the decision of the competitor, we do not need to have smooth pasting in this case, as there is no optimality condition for the firm.

The numerator of (13) captures the benefit of delaying repositioning. By delaying for a length of time dt, the firm reduces the cost of repositioning by rK, but it receives a flow profit of π_s instead of $\pi_d(1-x)$. The equilibrium $\mu(x)$ has to balance the benefit of delaying paying the repositioning cost with the value of being the first mover.

Let us now discuss the values of x for which (13) applies. We established that a firm repositions with a positive hazard rate only for some x in the range of $(\underline{x}, x^*)$. Note that both the numerator and the denominator of $\mu(x)$ have to be positive in order for the firm to mix at x. If the denominator is negative, then the firm strictly prefers to stay put if the opponent has some probability of repositioning. In equilibrium, both firms should not relocate at that x. Such x should be below $\underline{x}$. If the numerator is negative, then the firm should reposition immediately if the opponent is mixing because there is no incentive to delay. But if both firms relocate at x, a firm should deviate if $x < x^*$. Thus, $\mu(x)$ is positive only in the region $(\underline{x}, x^*)$.

Note that by (7) and (9), $V_d(1-x^*) - K - V_d(x^*) = 0$. Thus $\mu(x) \to \infty$ as $x \to x^{*-}$. This implies that one firm always repositions before x hits x^*. Our selection of the equilibrium for $x > x^*$ is (mostly) payoff irrelevant. If the starting condition is not that both firms are on the same side with distance $x > x^*$ to consumers, then that condition never happens on the equilibrium path, as mentioned above. Even if the starting condition is on the same side with $x > x^*$, that condition never happens on the equilibrium path again once x goes below x^*.

3.4. Equilibrium

From (5)-(9) we can obtain (see Appendix)

$$e^{\lambda(x^*+\underline{x}-1)} = \frac{\pi_d(x^*) - \pi_s + \frac{\pi_d'(x^*)}{\lambda} + rK + \frac{4\delta^2}{9\lambda^2}}{\pi_d(1-\underline{x}) - \pi_s + \frac{\pi_d'(1-\underline{x})}{\lambda} - rK + \frac{4\delta^2}{9\lambda^2}} \tag{14}$$

and

$$e^{\lambda(x^*+\underline{x}-1)} = \frac{\pi_d(1-\underline{x}) - \pi_s - \frac{\pi_d'(1-\underline{x})}{\lambda} - rK + \frac{4\delta^2}{9\lambda^2}}{\pi_d(x^*) - \pi_s - \frac{\pi_d'(x^*)}{\lambda} + rK + \frac{4\delta^2}{9\lambda^2}}. \tag{15}$$

Equations (14) and (15) form a system of two equations from which one can obtain x^* and $\underline{x}$. One can then use x^* and $\underline{x}$ to solve for A, B, and C. Plugging everything into (13) gives $\mu(x)$. This fully characterizes the equilibrium if the cost of repositioning K is not too large, such that a firm wants to reposition if consumer preferences are sufficiently far away from the firm's positioning. We state this in the following proposition.

Proposition 1. *Suppose that the cost of repositioning K is not too large. Then the symmetric market equilibrium has $\underline{x}$ and x^* obtained as solutions of the system (14) and (15), and hazard rate of repositioning $\mu(x)$ for $x \in (\underline{x}, x^*)$ given by (13) if firms have the same positioning.*

To get sharper results, we can consider the case when $K \to 0$. In that case, we can obtain that $\underline{x}, x^* \to 1/2$, as expected. When the cost of repositioning converges to zero, firms reposition right away to the side of the market that is closer to the consumer preferences.

Perhaps more interestingly, we can obtain the speed with which $\underline{x}$ and x^* converge to $1/2$ as K converges to zero. We can obtain that as $K \to 0$ we have

$$\frac{x^* - 1/2}{K^{1/3}} \to \sqrt[3]{\frac{9\sigma^2}{4\delta}} + \frac{1}{2}\sqrt[3]{\frac{\delta\sigma^4}{18}}K^{1/3}, \tag{16}$$

$$\frac{\underline{x} - 1/2}{K^{1/3}} \to \sqrt[3]{\frac{9\sigma^2}{4\delta}} - \frac{1}{2}\sqrt[3]{\frac{\delta\sigma^4}{18}}K^{1/3}, \tag{17}$$

$$\frac{x^* - \underline{x}}{K^{2/3}} \to \sqrt[3]{\frac{\delta\sigma^4}{18}}. \tag{18}$$

This yields the following result.

Proposition 2. *The thresholds x^* and $\underline{x}$ increase infinitely fast with K as $K \to 0$. For a small K, the thresholds x^* and $\underline{x}$ increase with the variability of consumer preferences σ^2, and decrease with the importance of the repositioning attribute, δ; the difference $(x^* - \underline{x})$ increases with both the variability of consumer preferences σ^2 and the importance of the repositioning attribute δ. As $K \to 0$, the effect of the discount rate r on the thresholds x^* and $\underline{x}$ approaches zero.*

When the variability of consumer preferences, σ^2, increases, and K is low, the firms realize that it is more likely that the consumer preferences will return to where the firm is currently positioned. As a result, they are slower to reposition with higher x^* and $\underline{x}$. Otherwise, firms have to incur repositioning costs more often while enjoying the benefit of repositioning for a shorter period as consumer preferences evolve quicker with a higher σ^2. When the relative importance of the repositioning attribute, δ, increases, the firms become now faster to reposition, as x^* and $\underline{x}$ decrease. Intuitively, firms are more eager to be close to consumers' ideal point on an attribute if consumers value that attribute more. This result can also be seen from the point of view of the attribute z. A lower δ means a relative increase in their degree of differentiation on attribute z, which leads to slower repositioning decisions. That is, less competition leads to less repositioning.

The difference between x^* and $\underline{x}$ captures how heterogeneous firms are in their responses to change in consumer preferences. With a greater $(x^* - \underline{x})$, it takes longer for a firm to follow its competitor after its competitor repositioned first. That is, the higher is the importance of the repositioning attribute in the consumer preferences, the more likely it is for the firms to remain differentiated on that attribute.

At $K \to 0$ the effect of discount rate r on the thresholds x^* and $\underline{x}$ approaches zero, as the rental cost of repositioning becomes zero.

Figures 3, 4, 5, and 6 illustrate how x^* and $\underline{x}$ evolve with K, δ, σ^2, and r, respectively. Note that Figure 4 illustrates that x^* and $\underline{x}$ decrease in δ, as suggested in the proposition for K small, at a decreasing rate. Similarly, Figure 5 illustrates how x^* and $\underline{x}$ increase with σ^2. Finally, Figure 6 illustrates that the effect of r on the thresholds x^* and $\underline{x}$ is rather small, with those thresholds increasing in r. Note also that we can obtain the values of x^* and $\underline{x}$ for $r \to 0$, and that x^* and $\underline{x}$ are increasing in r for $K > 0$. The analysis of such a case is presented in the Appendix. In such a case, we can similarly obtain analytically that x^*, $\underline{x}$, and $(x^* - \underline{x})$ increase in K, σ^2, and δ at the limit $r \to 0$. As an example, for the case with $K = .07, \delta = 4$, and $\sigma^2 = .2$, we can obtain $x^* \simeq .721$ and $\underline{x} \simeq .684$ at the limit $r \to 0$.

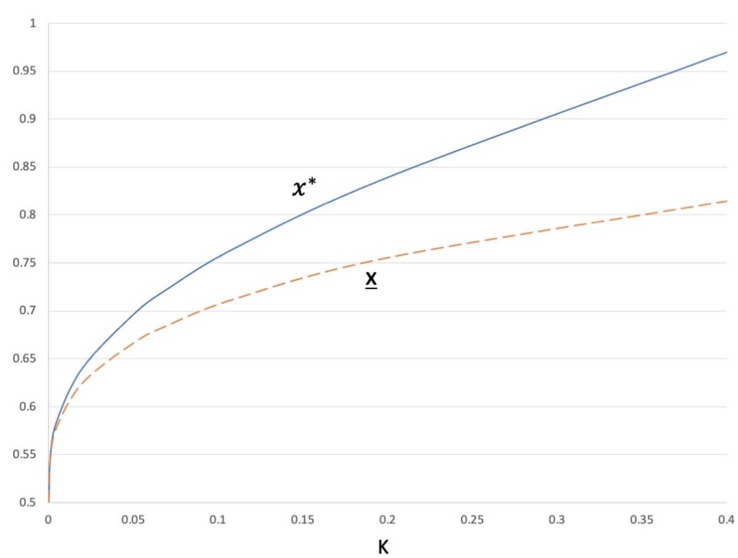

Figure 3: Evolution of x^* and $\underline{x}$ as a function of K for $\delta = 4$, $\sigma^2 = .2$, and $r = .1$

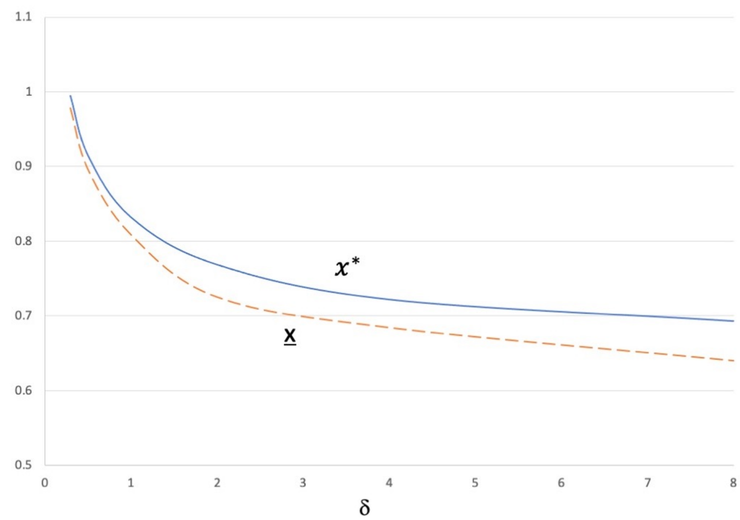

Figure 4: Evolution of x^* and $\underline{x}$ as a function of δ for $K = .07$, $\sigma^2 = .2$ and $r = .1$

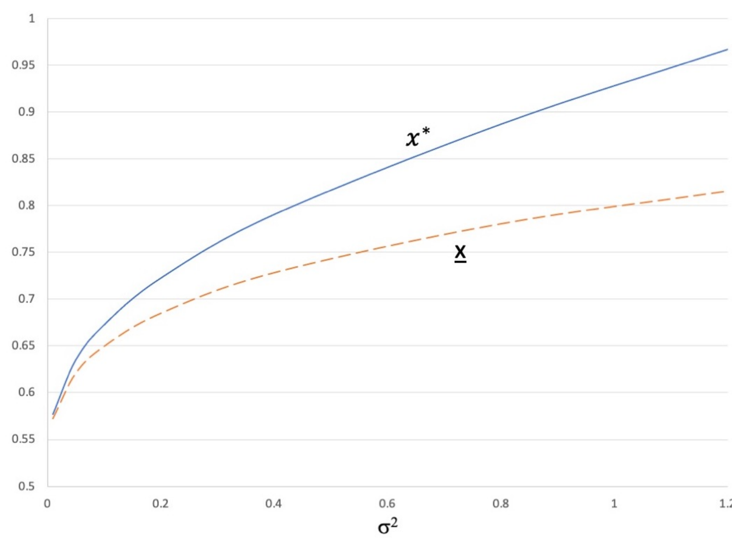

Figure 5: Evolution of x^* and $\underline{x}$ as a function of σ^2 for $K = .07$, $\delta = 4$ and $r = .1$

3.5. Unbounded Evolution of Preferences

Until now we constrained the evolution of preferences to be in the segment $[0, 1]$, with x_t evolving over time as Brownian motion, with reflecting boundaries at 0 and 1, and with the possible positionings of the firm at either 0 or 1. This was convenient and parsimonious as it limited the possible strategies of the firm, and allowed for a complete and tractable characterization of the market equilibrium.

Consider now that x_t evolves as a Brownian motion on the real line without reflecting boundaries, and that firms can reposition to any integer, positive or negative,

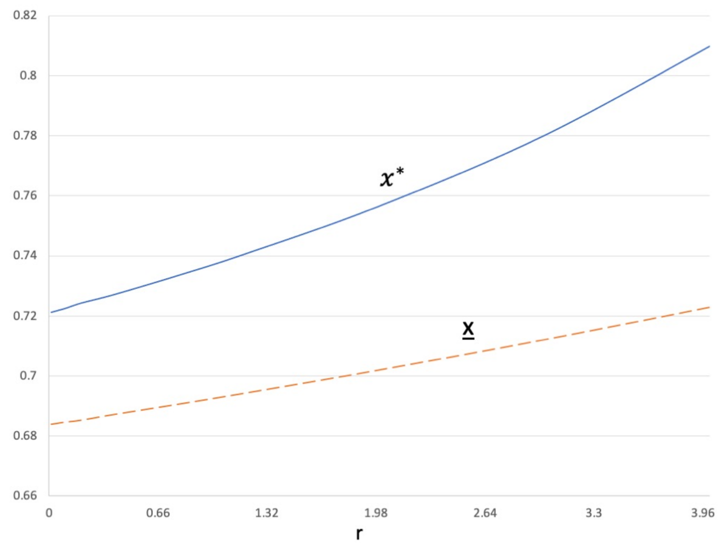

Figure 6: Evolution of x^* and $\underline{x}$ as a function of r for $K = .07$, $\delta = 4$ and $\sigma^2 = .2$

$... -2, -1, 0, 1, 2, ...$. Consider also that the fixed costs of repositioning, K, are sufficiently small. We should then expect that the market equilibrium would be such that no firm jumps over any integer in a repositioning decision, and that the market is always fully covered, with each firm with positive market share. That is, if a firm is positioned at some point n in the set of integers $\mathbb{Z}$, its next positioning will be at either $n+1$ or $n-1$, and the competitor's positioning is either $n-1, n,$ or $n+1$.

In such a case, the conditions for the equilibrium would then be exactly the same ones as the conditions derived above,[15] with exactly the thresholds of repositioning as related to the current repositionings of the firms. For example, if both firms are positioned at n, then both firms start considering repositioning if $x_t = n + \underline{x}$ or $x_t = n - \underline{x}$. If one firm is positioned at n and the other firm is positioned at $n+1$, the firm positioned at n would reposition to $n+1$ when x_t reaches $n+x^*$, and the firm positioned at $n+1$ would reposition to n when x_t reaches $n+1-x^*$, considering that firms are on the equilibrium path, $x_t \in [n+1-x^*, n+x^*]$.

The results obtained above would then carry over to this more generalized setting, as well the results on the duration of the differentiation, comparison with collusion, and comparison with the social welfare optimum, discussed in Section 4.

Figure 7 presents an example of a sample path of the consumer preferences x and of the

[15]Note that the reflecting boundary condition $V_s'(0) = 0$, would now be replaced by the condition $V_s'(n) = 0$ as the value function has to be symmetric around any integer n. This then generates exactly the same conditions as presented above.

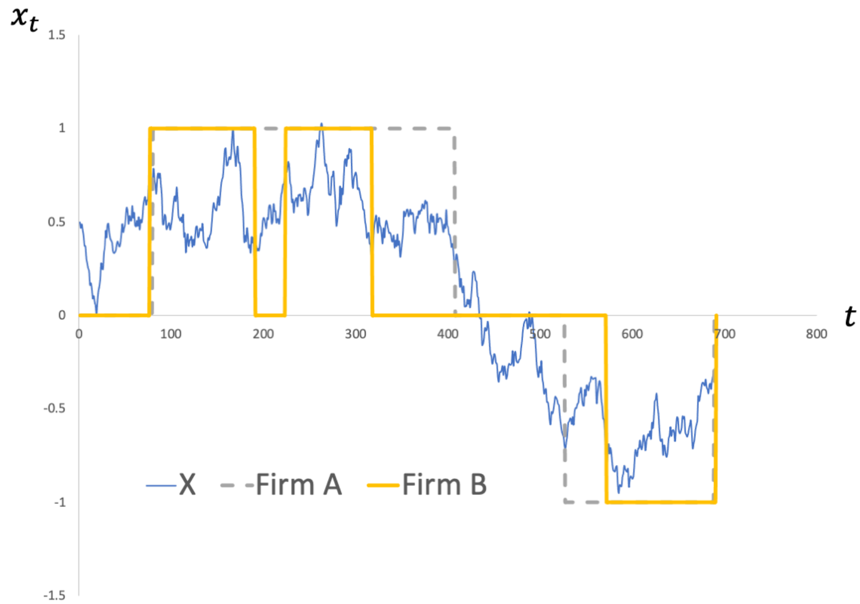

Figure 7: Example of sample path of x and of the repositioning decisons for $K = .07, \delta = 4, \sigma^2 = .2$, and $r = .1$, which yields $x^* \simeq .72$, and $\underline{x} \simeq .68$.

repositioning decisions of the two firms, starting from $x = .5$ with both firms positioned at $x = 0$, illustrating periods where firms are differentiated and periods where firms have the same positioning, depending on the evolution of the consumer preferences. In this example, the two firms co-locate on dimension x for about 75% of the time and differentiate for about 25% of the time.

3.6. Asymmetric Equilibria

As discussed earlier, there could be potentially asymmetric equilibria in pure strategies. A pure strategies asymmetric equilibrium could be for Firm 1 to be the one moving first if Firms 1 and 2 have the same positioning.[16] We look for an equilibrium where, if firms have the same positioning, Firm 1 moves to the other extreme when the consumer preferences are at a distance $\underline{x}$ from the original positioning, and if firms have different positioning, Firm 1 moves to the other extreme if consumer preferences are at a distance x^* from its original positioning, and Firm 2 moves to the other extreme if consumer preferences are at a distance

[16] Another potential possibility for an asymmetric equilibrium would be for Firm 1 to move first if both firms are positioned at 0, and for Firm 2 to be the one moving first if both firms are positioned at 1. Note however that such an equilibrium would not be Markov-perfect, as the payoff relevant state variable has the same value when both firms are at 0, or both firms are at 1, and the consumer preferences are at the same distance from the firms' positioning, and then the strategies by both firms have to be the same.

x^{**} from its original positioning. The existence of this additional threshold x^{**} makes the asymmetric equilibrium more complex to analyze.

Note first that the functional form of the value functions for both firms is still represented by (2) when firms have different positioning and by (4) when firms have the same positioning. The constants A, B, C, and D would now be different for both firms. Let A, B, C, and D represent the constants for Firm 1, and let $\widetilde{A}, \widetilde{B}, \widetilde{C}$, and $\widetilde{D}$ represent the corresponding constants for Firm 2. By the same arguments as presented in subsection 3.2, we have $C = D$ and $\widetilde{C} = \widetilde{D}$. Finally, let V represent the value functions of Firm 1, and $\widehat{V}$ represent the value functions of Firm 2.

Consider now the decisions of Firm 1. By the same arguments as presented in subsection 3.2, the value function of Firm 1 must satisfy (5), (6), (7), and (8). By the presentation for the symmetric equilibrium case, these equations determine x^* and $\underline{x}$, which therefore have the same values as in the symmetric equilibrium case.

Regarding Firm 1, note finally that the value function should be continuous when the firms are in different positionings, and Firm 2 decides to move, which occurs when the consumer preferences are at a distance $(1 - x^{**})$ from Firm 1. That condition is

$$V_d(1 - x^{**}) = V_s(1 - x^{**}) \tag{19}$$

which replaces condition (9) in the symmetric equilibrium case.

Consider now the conditions on the value functions for Firm 2. Similarly to Firm 1, it must be optimal for Firm 2 to reposition at x^{**}, which yields

$$\widehat{V}_d(x^{**}) = \widehat{V}_s - K \tag{20}$$

$$\widehat{V}_d'(x^{**}) = -\widehat{V}_s'(1 - x^{**}). \tag{21}$$

Similarly, Firm 2 should have value matching when Firm 1 moves at $\underline{x}$ and $(1 - x^*)$ which yields

$$\widehat{V}_s(\underline{x}) = \widehat{V}_d(\underline{x}) \tag{22}$$

$$\widehat{V}_d(1 - x^*) = \widehat{V}_s(1 - x^*). \tag{23}$$

Conditions (20)-(23) determine uniquely $x^{**}, \widetilde{A}, \widetilde{B}$, and $\widetilde{C}$, as we already have x^* and $\underline{x}$ from the symmetric equilibrium conditions and the arguments above. The derivation of the equilibrium is presented in the Appendix.

Note further that the mixed strategies considered in the symmetric equilibrium could be purified with private information by the firms on some dimension, as is well-known (Harsanyi 1973).[17]

3.7. Advantages in Repositioning Locations

One interesting possibility is that each firm has an advantage when repositioning to one of the locations, for example, with one firm having a relative advantage in repositioning from 1 to 0 (lower repositioning costs), and the other firm having a relative advantage in repositioning from from 0 to 1. This could be explained by each firm having a natural advantage at a particular location.

The analysis above would carry over to this case, but we would now have to keep track of a value function for each firm. As above, there would also be an equilibrium where firms follow a behavioral mixing strategy when both firms are in the same location and the consumer preferences are in a range away from the firms, but now each firm would have a different mixing strategy with the firm that has an advantage in repositioning being the one more likely to reposition. As above, there would also be pure strategy equilibria in repositioning, with the potentially nicer of those equilibria being the one in which the firm that moves first is the one that has an advantage in repositioning.

Another related possibility could be a situation where a firm has lower repositioning costs, in both directions of repositioning. This could be justified for example by a firm having a stronger marketing capability which makes repositioning easier for that firm.

In that case, we would again need to keep track of one value function per firm, and have, as above, both a behavioral mixed strategy equilibrium (where the firm with lower repositioning costs would be more likely to reposition first), and pure strategy equilibria (where the potentially nicer of those equilibria being again the one in which the firm with lower costs of repositioning repositions first).

[17] Note also we could construct an equilibrium with a public randomizing device which picks which firm to move first when both firms are in the same location. We concentrate here on the symmetric equilibrium without a public randomizing device to focus on the equilibrium of the basic problem. Given the symmetric problem we also focus on the symmetric equilibrium, which is also simpler to characterize in terms of thresholds for repositioning. We thank an anonymous reviewer for this suggestion.

4. Further Analysis and Model Extensions

The analysis presented above allows us to study the duration of differentiation in the model, and the set-up allows us to consider the possibility of there existing a deterministic trend in the evolution of consumer preferences. To further understand the implications of the main results, we compare the result of the competitive outcome with the collusive outcome, and the outcome that maximizes social welfare. We describe here the results of these further analyses, with details available in the online appendix.

4.1. Expected Duration of Differentiation and Industry Profits

Consider the duration of differentiation and co-positioning in the market. Starting from a point of differentiation we can compute the expected duration going forward until when the firms choose to co-position, which occurs when the consumer preferences x reach either x^* or $1 - x^*$. Similarly, starting from a position of co-positioning we can consider the expected duration going forward until when a firm chooses to differentiate, which can start occurring when the consumer preferences x reach a distance $\underline{x}$ from where the firms are positioned.

From this we can obtain that a measure of the fraction of time during which firms are differentiated to be $x^* - \underline{x}$. This then yields the following proposition.

Proposition 3. *Consider K small. Then, the fraction of time with differentiated firms increases and the net present value of industry profits decreases in the repositioning costs, K, in the variability of consumer preferences, σ^2, and in the importance of the attribute on which the firms can reposition, δ.*

Considering the difference between the x^* and $\underline{x}$ curves, Figures 3-6 illustrate how the fraction of time during which firms are differentiated depends on K, δ, σ^2, and r. For example, when $K = .4, \delta = 4, r = .1$, and $\sigma^2 = .2$, the fraction of time when firms are differentiated is close to 16%.

When K is small, both x^* and $\underline{x}$ are low, so firms reposition often in order to follow consumer preferences closely. Note that co-located firms each makes a constant flow profit of $1/2$ regardless of their distance to consumers. Thus as the cost of repositioning increases, firms that are co-located have to expect a longer period of differentiation after repositioning to justify for paying the higher cost. Similarly, in the online appendix we show that firms pay the repositioning costs more often with a higher variability of consumer preferences or a higher importance of the repositioning attribute. Thus in equilibrium, firms remain differentiated longer to compensate for paying more repositioning costs. When K is small, the negative effect of higher repositioning costs dominate the positive effect of longer differentiation; thus, the net present value of industry profit decreases in the repositioning costs, in the variability of consumer preferences, and in the importance of the repositioning attribute.

4.2. Collusion

We consider now the optimal collusive behavior of the two firms and compare it with the competitive case. We analyze first the case where collusion is only on the repositioning decisions, keeping the price equilibrium as competitive, and then consider when firms collude on both repositioning and prices. Note that the collusive case can also be seen as the case in which both products are carried by the same firm which is an useful benchmark to consider. This can also be seen as the effect of a merger between two competing firms. The case where collusion is only on repositioning decisions, keeping the price equilibrium as competitive, can be interpreted as the case in which a firm carries both products, but the pricing decisions are controlled by two separate (and competing) divisions within the company. This could be a case of a firm that has separate brand managers for each of their brands, who manage the tactical decisions (such as price), but where the strategic decisions (such as repositioning) are coordinated at the firm level. This case is also a benchmark where we study first the effect of just coordinating on the repositioning decisions, and then study the effect of coordinating on both repositioning and pricing.

As noted above, when the price equilibrium is competitive and firms are positioned in the same location, the profit for each firm is 1/2, independent of the location of the consumer preferences. When the price equilibrium is competitive and firms are positioned at different locations, the profit of a firm when the consumers are at a distance x from that firm is $\frac{1}{2}\left[1+\frac{\delta(1-2x)}{3}\right]^2$. We can then obtain that the industry profits when firms price competitively are greater when firms are differentiated than when firms are co-positioned, with strict inequality almost everywhere. We then have that when firms collude on repositioning and compete on prices, they will never reposition again once firms are in different locations (that is, once they are differentiated). This captures the idea that firms like to be differentiated when competing on price, as known from the static differentiation literature. However, under competitive repositioning, as obtained in Section 3, firms are not able to remain differentiated, and can remain in the same location for some period, depending on the evolution of consumer preferences. That is, there is more repositioning in the competitive market than if firms collude on repositioning but compete on prices.

Consider now the case of collusion on both repositioning and prices. In this case we can obtain that both products will be optimally repositioned as the consumer preferences evolve, with periods in which the products are differentiated and periods in which the products are co-positioned. This allows the industry to appropriate better the benefits generated by the products. To get sharper results we can consider the case when $K \to 0$. In that case, we can obtain that $\underline{x}, x^* \to 1/2$. When the cost of repositioning converges to zero, the full collusion outcome also involves repositioning right away to the side of the market that is closer to the consumer preferences.

We can also obtain the speed of convergence when $K \to 0$. We can obtain that as $K \to 0$ we have

$$\frac{x^* - 1/2}{K^{1/3}} \to \sqrt[3]{\frac{3\sigma^2}{2\delta}} + \frac{1}{2}\sqrt[3]{\frac{\delta\sigma^4}{12}}K^{1/3}, \tag{24}$$

$$\frac{\underline{x} - 1/2}{K^{1/3}} \to \sqrt[3]{\frac{3\sigma^2}{2\delta}} - \frac{1}{2}\sqrt[3]{\frac{\delta\sigma^4}{12}}K^{1/3}, \tag{25}$$

$$\frac{x^* - \underline{x}}{K^{2/3}} \to \sqrt[3]{\frac{\delta\sigma^4}{12}}. \tag{26}$$

This structure of the limits of $\underline{x}$ and x^* is similar to the one in the competitive case considered in the previous section, and all the results stated in Proposition 2 also apply for the full collusion case considered in this section. More interestingly, we can compare the thresholds of repositioning in the competitive case with the ones in the collusion case.

Let $\underline{x}_{comp}$ and x^*_{comp} be the values of $\underline{x}$ and x^*, respectively, from the competitive equilibrium in the previous Section. Let $\underline{x}_{coll}$ and x^*_{coll} be the values of $\underline{x}$ and x^*, respectively, in the full collusion case. We can then obtain:

Proposition 4. *For K small, we obtain $\underline{x}_{coll} < \underline{x}_{comp}, x^*_{coll} < x^*_{comp}$, and $x^*_{coll} - \underline{x}_{coll} > x^*_{comp} - \underline{x}_{comp}$.*

In the online appendix, we also illustrate how x^* and $\underline{x}$ compare between the collusion and competitive case and evolve as a function of K, suggesting that the results in Proposition 4 seem to hold for larger K.

Proposition 4 shows that, under full collusion, when the cost of repositioning is small, firms reposition more frequently than in the competitive equilibrium case. In the competitive case, a firm repositions because of its private incentives to reposition. In the full collusion case, one firm repositions because of the incentives for the industry, which is able to capture the whole value of the repositioning. The whole value of the repositioning under collusive pricing is greater than the private incentives under price competition, and, therefore, the full collusion case results in more repositioning than the competitive case. Thus a monopoly owning both products would on average provide better products than two competing firms would.

Also interestingly, as $x^* - \underline{x}$ is greater in this case than in the case of competition, we have, by the previous subsection, that there is more product differentiation under collusion than in the competitive market case. On the other hand, static models of product positioning with price competition often imply that two products owned by competing firms are more differentiated than two products owned by a single firm. In a Hotelling model with quadratic transportation costs, the principle of maximum differentiation suggest that differentiation is higher under competition than under consolidation (d'Aspremont, Gabszewicz, and Thisse 1978). Similarly, Moorthy (1988) find that in a vertically differentiated market, two competing firms position farther apart than two products owned by a monopoly. Empirically, some papers find, in contrast, that after merger products are repositioned to being more differentiated from each other (Berry and Waldfogel 2001, George 2007, Sweeting 2010). Our model with evolving consumer preferences offer a potential explanation for such an outcome.

4.3. Social Welfare

Consider now the optimal social welfare and compare it with the competitive and collusive cases. We can consider a case in which social welfare is only optimized on the repositioning decisions, keeping the price equilibrium as competitive, and the case of the full social welfare optimum. The question of what is socially optimal in terms of repositioning when firms continue to price competitively would be relevant when the social planner could implement some regulation on the repositioning behavior, but would have to allow firms to price competitively.

In both cases the optimum is also characterized by an $\underline{x}$ and an x^* (different $\underline{x}$ and x^*) such that, when firms have the same positioning, and consumer preferences are at a distance $\underline{x}$, one of the firms repositions, and when firms have different positionings, and when the firm farther away from the consumer preferences is at a distance x^* from those consumer preferences, that firm repositions.

To get sharper results, we can consider the case when $K \to 0$. In that case, we can obtain again in both cases that $\underline{x}, x^* \to 1/2$. When the cost of repositioning converges to zero, the social welfare optima in both cases also involve repositioning right away to the side of the market that is closer to the consumer preferences.

We can also obtain that as $K \to 0$ we have the speeds of convergence of x^* and $\underline{x}$ to $1/2$, and of $x^* - \underline{x}$ to zero, as in the collusion case, and faster than in competition. We can also obtain a comparison with the competition and collusion thresholds.

Let $\underline{x}_{SW-p}$ and x^*_{SW-p} be the values of $\underline{x}$ and x^*, respectively, in the case of the social welfare optimum subject to competitive pricing, and let $\underline{x}_{SW}$ and x^*_{SW} be the values of $\underline{x}$ and x^*, respectively, in the case of the social welfare optimum. We can then obtain.

Proposition 5. *For small K, we obtain $\underline{x}_{SW} < \underline{x}_{SW-p} < \underline{x}_{coll} < \underline{x}_{comp}$, $x^*_{coll} < x^*_{SW-p} < x^*_{SW} < x^*_{comp}$, and $x^*_{SW} - \underline{x}_{SW} > x^*_{SW-p} - \underline{x}_{SW-p} > x^*_{coll} - \underline{x}_{coll} > x^*_{comp} - \underline{x}_{comp}$.*

This shows that, under both social optima cases, when the cost of repositioning is small, firms reposition more frequently than in the competitive equilibrium case. In the competitive case, a firm repositions because of its private incentives to reposition. In the cases of the social welfare optimum, one firm repositions because of the incentives for social welfare, which includes the whole value of the repositioning (except for the mis-allocation resulting from competitive pricing, in the case of a competitive pricing constraint). The whole value of the repositioning for social welfare is greater than the private incentives under price competition, and, therefore, the cases of the social welfare optimum result in more repositioning than the competitive case.

The relationship of these cases to the full collusion case is also interesting. First, note that the thresholds of the social optimum cases are closer to the thresholds under full collusion than to the thresholds in the competitive market equilibrium. The difference between the thresholds of the cases of the social welfare optimum and the full collusion case are on the order of $K^{2/3}$, while the difference between the thresholds of the cases of the social welfare optimum and the competitive market equilibrium case are on the order of $K^{1/3}$, which is larger for K small. This would suggest that the outcome of full collusion is close to the social welfare optimum, with the difference that the consumers would get a much larger surplus under the social welfare optimum subject to competitive pricing than in the full collusion case.

The fact that the collusive thresholds approach the social optimum at a faster rate than the competitive thresholds do implies that, when K is small, social welfare is higher under collusion than under competition. One can show that, when firms are co-located, flow social welfare is the same whether firms are collusive or competitive. When firms differentiate, flow social welfare is higher under collusion than under competition, because more consumers will go to the firm that is closer on attribute x under collusion due to less price competition from the farther away firm. Thus, social welfare is higher under collusion even if collusive firms follow competitive firms' repositioning strategies.

Second, the case of the social welfare optimum subject to competitive pricing has longer periods of firms being differentiated than in the full collusion case. In the full collusion case, the industry is not able to appropriate the utility generated to the infra-marginal consumers due to differentiation among products, which leads to the result where the firms are not differentiated enough from each other. In the case of the social welfare optimum subject to competitive pricing, the social planner includes the utility generated to the infra-marginal consumers due to the products being differentiated, and therefore keeps the products differentiated for a longer period.

Third, the case of the full social welfare optimum has longer periods of firms being differentiated than under the constraint of competitive pricing. In the case of the social welfare optimum subject to competitive pricing, there was an incentive for firms to be in the same location because of the distortion in the allocation of demand due to competitive pricing. Because this incentive disappears without the constraint of competitive pricing, the optimum then has longer periods of differentiation between firms.

In the online appendix we present numerically how x^* and $\underline{x}$ compare across the different cases, and how that comparison evolves as a function of K, illustrating that the results in Proposition 5 seem to hold for larger K. However, note that the effect of longer periods with differentiation gets stronger with larger K, such that at some point we have $x^*_{SW} > x^*_{comp}$.

4.4. Deterministic Trend

Consider the possibility of existing a deterministic trend in the evolution of the dimension x, the dimension in which the firm can reposition. We consider the case of unbounded x_t presented in subsection 3.5, with the fixed costs of reposition, K, small enough such that if a firm is positioned at n, it repositions to $n+1$ at some point, the market is fully covered, and both firms have always a strictly positive market share in equilibrium.

Consider first the case in which there is only a deterministic trend in the evolution of the consumer preferences (and there is no random component), and we then briefly discuss the case in which there is both a deterministic trend and a random component in the evolution of the consumer preferences.

We can obtain that, for the costs of repositioning, K, small, the thresholds for repositioning are farther away from $1/2$ in the only random component case than in the only deterministic trend case. This can be understood by the fact that in the only deterministic trend case the evolution of preferences is moving away from 0 for sure, which makes the firms reposition sooner, while in the only random component case, it is possible that the consumer preferences return to 0. We can also obtain that there is less differentiation in the only deterministic trend case than in the only random component case.

Finally, we can also obtain that firms are slower to reposition, and stay differentiated for a longer period, when the discount rate is greater, and when the importance of the repositioning attribute, δ, is lower. We summarize these results in the following proposition.

Proposition 6. *Consider that the costs of repositioning K are small. Then, the only deterministic trend case results in lower differentiation and lower thresholds for repositioning than the only random component case. Furthermore, in the only deterministic trend case the threshold to reposition are increasing in the repositioning costs, K, and in the discount rate, r, and decreasing in the importance of the repositioning attribute, δ.*

We could also consider that the evolution of consumer preferences has both a random component and a deterministic component. That case would get the composition of the effects of the only random component case considered in the previous section, and of the only deterministic case considered in this subsection. In particular, as in the only random component case the thresholds for firms to reposition move more quickly away from $1/2$ than in the only deterministic case as the repositioning costs K increase from zero, we have then that the effects of the only random component case over the only deterministic trend case would be to increase the thresholds at which firms want to reposition.

5. Conclusion

We have studied a model where competing firms follow consumer preferences, and choose when to reposition. We characterize how the competitive market equilibrium behaves, illustrating rich dynamics where firms are differentiated for some time, and have the same positioning during other periods. We can characterize how the market behaves depending on different market factors, showing that greater uncertainty in consumer preferences and greater importance of the repositioning attribute lead to less repositioning and more differentiation.

We compare the market equilibrium with the cases of collusion and social welfare optimization. If there is collusion on the repositioning decisions, but firms continue to price competitively, we get to a situation where there is differentiation forever. On the other hand, if there is collusion on both the repositioning decisions and pricing, as in a merger or a monopoly owning both brands, or if we want to maximize social welfare, we can obtain that there is more repositioning than in the competitive market equilibrium when the costs of repositioning are not too large. This is because the collusive firms, or the social planner, can better appropriate all the benefits of repositioning, which leads to more repositioning. Firms are also differentiated more often under collusion than under competition.

We can also obtain that under social welfare maximization there are longer periods of product differentiation than under collusion. This is because under social welfare maximization we take into account the utility of the infra-marginal consumers, which is a force towards more differentiation. Furthermore, the case of social welfare maximization subject to competitive pricing leads to shorter periods of differentiation than under unrestricted social welfare maximization, as competitive pricing creates a distortion (in favor of the less desirable product) when products are differentiated, which makes product differentiation less advantageous.

The analysis in the paper is constrained to a model of limited repositioning, only two firms, and a fully covered market. It would be intesting to consider extensions in either of these dimensions. The model allows for the firms to have only two (or a discrete number of) positionings. It would be interesting to understand what would happen if a continuum of positionings were allowed. The case of a large number of discrete positionings is also considered under the case in which only adjacent positionings are possible at each stage, and it would be interesting to consider the case where jumps to non-adjacent repositionings are possible. It would also be interesting to investigate what happens in a market with more than two firms. Finally, it would be interesting to study a model where the market is not necessarily fully covered, to have a more complete characterization of the welfare effects, and to consider the possibility of consumers having also heterogeneous preferences on the dimension where there are changing preferences.

APPENDIX

Intermediate steps in derivation of market equilibrium:

Equations (5)-(9) form the following system of equations:

$$\frac{\pi_s}{r} + Ce^{\lambda \underline{x}} + Ce^{-\lambda \underline{x}} = \frac{\pi_d(1-\underline{x})}{r} + \frac{4\delta^2}{9r\lambda^2} + Ae^{\lambda(1-\underline{x})} + Be^{-\lambda(1-\underline{x})} - K \quad \text{(i)}$$

$$\lambda Ce^{\lambda \underline{x}} - \lambda Ce^{-\lambda \underline{x}} = -\frac{\pi'_d(1-\underline{x})}{r} - \lambda Ae^{\lambda(1-\underline{x})} + \lambda Be^{-\lambda(1-\underline{x})} \quad \text{(ii)}$$

$$\frac{\pi_s}{r} + Ce^{\lambda(1-x^*)} + Ce^{-\lambda(1-x^*)} - K = \frac{\pi_d(x^*)}{r} + \frac{4\delta^2}{9r\lambda^2} + Ae^{\lambda x^*} + Be^{-\lambda x^*} \quad \text{(iii)}$$

$$-\lambda Ce^{\lambda(1-x^*)} + \lambda Ce^{-\lambda(1-x^*)} = \frac{\pi'_d(x^*)}{r} + \lambda Ae^{\lambda x^*} - \lambda Be^{-\lambda x^*} \quad \text{(iv)}$$

$$\frac{\pi_s}{r} + Ce^{\lambda(1-x^*)} + Ce^{-\lambda(1-x^*)} = \frac{\pi_d(1-x^*)}{r} + \frac{4\delta^2}{9r\lambda^2} + Ae^{\lambda(1-x^*)} + Be^{-\lambda(1-x^*)} \quad \text{(v)}$$

Subtracting (ii) divided by λ from (i) gives

$$\frac{\pi_s}{r} + 2Ce^{-\lambda \underline{x}} = \frac{\pi_d(1-\underline{x})}{r} + \frac{\pi'_d(1-\underline{x})}{r\lambda} + 2Ae^{\lambda(1-\underline{x})} - K + \frac{4\delta^2}{9r\lambda^2}$$

$$2Ce^{-\lambda} = 2A + \left[\frac{\pi_d(1-\underline{x}) - \pi_s}{r} + \frac{\pi'_d(1-\underline{x})}{r\lambda} - K + \frac{4\delta^2}{9r\lambda^2}\right] e^{-\lambda(1-\underline{x})} \quad \text{(vi)}$$

Adding (iv) divided by λ to (iii) gives:

$$\frac{\pi_s}{r} + 2Ce^{-\lambda(1-x^*)} - K = \frac{\pi_d(x^*)}{r} + \frac{\pi'_d(x^*)}{r\lambda} + 2Ae^{\lambda x^*} + \frac{4\delta^2}{9r\lambda^2}$$

$$2Ce^{-\lambda} = 2A + \left[\frac{\pi_d(x^*) - \pi_s}{r} + \frac{\pi'_d(x^*)}{r\lambda} + K + \frac{4\delta^2}{9r\lambda^2}\right] e^{-\lambda x^*} \quad \text{(vii)}$$

Similarly, adding (ii) divided by λ to (i) gives

$$\frac{\pi_s}{r} + 2Ce^{\lambda \underline{x}} = \frac{\pi_d(1-\underline{x})}{r} - \frac{\pi'_d(1-\underline{x})}{r\lambda} + 2Be^{-\lambda(1-\underline{x})} - K + \frac{4\delta^2}{9r\lambda^2}$$

$$2Ce^{\lambda} = 2B + \left[\frac{\pi_d(1-\underline{x}) - \pi_s}{r} - \frac{\pi'_d(1-\underline{x})}{r\lambda} - K + \frac{4\delta^2}{9r\lambda^2}\right] e^{\lambda(1-\underline{x})} \quad \text{(viii)}$$

Subtracting (iv) divided by λ from (iii) gives

$$\frac{\pi_s}{r} + 2Ce^{\lambda(1-x^*)} - K = \frac{\pi_d(x^*)}{r} - \frac{\pi_d'(x^*)}{r\lambda} + 2Be^{-\lambda x^*} + \frac{4\delta^2}{9r\lambda^2}$$

$$2Ce^{\lambda} = 2B + \left[\frac{\pi_d(x^*) - \pi_s}{r} - \frac{\pi_d'(x^*)}{r\lambda} + K + \frac{4\delta^2}{9r\lambda^2}\right]e^{\lambda x^*} \tag{ix}$$

We can then obtain (14) from (vi) and (vii), and obtain (15) from (viii) and (ix), as they appear in the main text.

Limit as $K \to 0$: Let $p^* = \frac{3+\delta-2\delta x^*}{3}$ denote price at x^*, and $\underline{p} = \frac{3-\delta+2\delta\underline{x}}{3}$ denote price at $1-\underline{x}$. Then we have $\pi_d(x^*) = \frac{1}{2}p^{*2}$, $\pi_d'(x^*) = -\frac{2\delta}{3}p^*$, $\pi_d(1-\underline{x}) = \frac{1}{2}\underline{p}^2$, and $\pi_d'(1-\underline{x}) = -\frac{2\delta}{3}\underline{p}$. Furthermore, let $G = e^{\lambda(x^*+\underline{x}-1)} = e^{\frac{3\lambda}{2\delta}(\underline{p}-p^*)}$. We can re-write equations (14) and (15) as:

$$G(\underline{p}^2 - 1) - (p^{*2} - 1) + \frac{4\delta}{3\lambda}(p^* - G\underline{p}) + \frac{8\delta^2}{9\lambda^2}(G-1) = 2rK(1+G) \tag{x}$$

$$G(p^{*2} - 1) - (\underline{p}^2 - 1) + \frac{4\delta}{3\lambda}(Gp^* - \underline{p}) + \frac{8\delta^2}{9\lambda^2}(G-1) = -2rK(1+G) \tag{xi}$$

Subtracting equation (xi) from (x), and dividing by $(1+G)(p^*+\underline{p})$, we get:

$$(\underline{p} - p^*) - \frac{4\delta}{3\lambda}\frac{G-1}{G+1} = \frac{4rK}{p^*+\underline{p}}$$

or

$$\log G - 2\frac{G-1}{G+1} = \frac{3\lambda}{2\delta}\frac{4rK}{p^*+\underline{p}}. \tag{xii}$$

As $K \to 0$ in (xii), $G \to 1$, which implies $\underline{p} = p^*$, or $\underline{x} + x^* = 1$, in the limit.

Adding (x) and (xi), and dividing by $p^* - \underline{p}$, we obtain:

$$\frac{G-1}{p^*-\underline{p}}(\underline{p}^2 + p^{*2} - 2 + \frac{16\delta^2}{9\lambda^2}) + \frac{4\delta}{3\lambda}(G+1) = 0. \tag{xiii}$$

With $\frac{G-1}{p^*-\underline{p}} \to -\frac{3\lambda}{2\delta}$ as $G \to 1$ and $p^* - \underline{p} \to 0$, we obtain

$$\underline{p}^2 + p^{*2} = 2 \tag{xiv}$$

which implies that, as $K \to 0$, $\underline{p}, p^* \to 1$. So, we then have that both $\underline{x}$ and x^* approach 1/2 in the limit.

Consider now the question of the speed of convergence. Let $y = p^* + \underline{p}$. Then, we can write

$$x^* = \frac{1}{2} + \frac{3(2-y)}{4\delta} + \frac{1}{2\lambda}\log(G) \tag{xv}$$

$$\underline{x} = \frac{1}{2} + \frac{3(y-2)}{4\delta} + \frac{1}{2\lambda}\log(G) \tag{xvi}$$

$$x^* - \underline{x} = \frac{3(2-y)}{2\delta}. \tag{xvii}$$

Note now that

$$\lim_{G\to 1} \frac{\log(G) - 2\frac{G-1}{G+1}}{(G-1)^3} = \frac{1}{12}. \tag{xviii}$$

Then, from (xii), we can obtain

$$\lim_{K\to 0} \frac{(G-1)^3}{K} = \frac{36r\lambda}{\delta}. \tag{xix}$$

Noting that $p^* = \frac{y}{2} - \frac{\delta}{3\lambda}\log(G)$ and $\underline{p} = \frac{y}{2} + \frac{\delta}{3\lambda}\log(G)$, we can obtain from (xiii) that

$$\lim_{K\to 0} \frac{y-2}{(G-1)^2} = -\frac{\delta^2\sigma^2}{54r}. \tag{xx}$$

Using this plus (xix) in (xv), (xvi), and (xvii), and using the definition of λ, we can obtain (16), (17), and (18).

Limit as $r \to 0$: Noting that $\frac{\log(G)}{\sqrt{r}} = \frac{3\sqrt{2}}{2\delta\sigma}(\underline{p} - p^*)$ and using (xviii), we can obtain from (xii) that at the limit $r \to 0$ we have

$$(\underline{p} - p^*)^3(\underline{p} + p^*) = H \tag{xxi}$$

where $H = 32K\delta^2\sigma^2/3$. Similarly, we can obtain from (xiii) that at the limit

$$3(p^{*2} + \underline{p}^2 - 2) = (\underline{p} - p^*)^2. \tag{xxii}$$

We can then solve (xxi)-(xxii) to obtain

$$\lim_{r\to 0} x^* = \frac{1}{2} + \frac{3}{2\delta}\left[1 - \frac{1}{2}\left(\frac{H}{w^{3/2}} - \sqrt{w}\right)\right] \tag{xxiii}$$

$$\lim_{r\to 0} \underline{x} = \frac{1}{2} - \frac{3}{2\delta}\left[1 - \frac{1}{2}\left(\frac{H}{w^{3/2}} + \sqrt{w}\right)\right] \tag{xxiv}$$

where $w = (\underline{p} - p^*)^2$ solves $w^4 - 12w^3 + 3H^2 = 0$. For example, for $\sigma^2 \to 0$ we can obtain

$H \to 0$, which yields $w \to 0$ and $\underline{p}, p^* \to 1$, which then gives $x^*, \underline{x} \to \frac{1}{2}$.

Asymmetric Equilibrium:

Conditions (20)-(23) can be written as

$$\frac{\pi_s}{r} + \widetilde{C}e^{\lambda(1-x^{**})} + \widetilde{C}e^{-\lambda(1-x^{**})} - K = \frac{\pi_d(x^{**})}{r} + \frac{4\delta^2}{9r\lambda^2} + \widetilde{A}e^{\lambda x^{**}} + \widetilde{B}e^{-\lambda x^{**}} \tag{xxv}$$

$$-\lambda\widetilde{C}e^{\lambda(1-x^{**})} + \lambda\widetilde{C}e^{-\lambda(1-x^{**})} = \frac{\pi_d'(x^{**})}{r} + \lambda\widetilde{A}e^{\lambda x^{**}} - \lambda\widetilde{B}e^{-\lambda x^{**}} \tag{xxvi}$$

$$\frac{\pi_s}{r} + \widetilde{C}e^{\lambda\underline{x}} + \widetilde{C}e^{-\lambda\underline{x}} = \frac{\pi_d(\underline{x})}{r} + \frac{4\delta^2}{9r\lambda^2} + \widetilde{A}e^{\lambda\underline{x}} + \widetilde{B}e^{-\lambda\underline{x}} \tag{xxvii}$$

$$\frac{\pi_s}{r} + \widetilde{C}e^{\lambda(1-x^*)} + \widetilde{C}e^{-\lambda(1-x^*)} = \frac{\pi_d(1-x^*)}{r} + \frac{4\delta^2}{9r\lambda^2} + \widetilde{A}e^{\lambda(1-x^*)} + \widetilde{B}e^{-\lambda(1-x^*)}. \tag{xxviii}$$

Solving (xxvii) and (xxviii) for $(\widetilde{C} - \widetilde{A})$ and $(\widetilde{C} - \widetilde{B})$, we can obtain

$$\widetilde{A} = \widetilde{C} - \frac{1}{rG - r/G}\left[-e^{-\lambda\underline{x}}\left(\pi_d(1-x^*) - \pi_s + \frac{4\delta^2}{9\lambda^2}\right) + e^{-\lambda(1-x^*)}\left(\pi_d(\underline{x}) - \pi_s + \frac{4\delta^2}{9\lambda^2}\right)\right] \tag{xxix}$$

$$\widetilde{B} = \widetilde{C} - \frac{1}{rG - r/G}\left[e^{\lambda\underline{x}}\left(\pi_d(1-x^*) - \pi_s + \frac{4\delta^2}{9\lambda^2}\right) - e^{\lambda(1-x^*)}\left(\pi_d(\underline{x}) - \pi_s + \frac{4\delta^2}{9\lambda^2}\right)\right] \tag{xxx}$$

where $G = e^{\lambda(x^* + \underline{x} - 1)}$. Note now that dividing (xxvi) by λ, and adding it to, and subtracting it from, (xxv), we can obtain

$$2\widetilde{C}e^{-\lambda} = 2\widetilde{A} + e^{-\lambda x^{**}}\left[\frac{\pi_d(x^{**}) - \pi_s}{r} + \frac{\pi_d'(x^{**})}{r\lambda} + K + \frac{4\delta^2}{9r\lambda^2}\right] \tag{xxxi}$$

$$2\widetilde{C}e^{\lambda} = 2\widetilde{B} + e^{\lambda x^{**}}\left[\frac{\pi_d(x^{**}) - \pi_s}{r} - \frac{\pi_d'(x^{**})}{r\lambda} + K + \frac{4\delta^2}{9r\lambda^2}\right], \tag{xxxii}$$

respectively. Using (xxix) in (xxxi), and (xxx) in (xxxii), one obtains

$$2\widetilde{C}(e^{-\lambda}-1) = e^{-\lambda x^{**}}\left[\frac{\pi_d(x^{**})-\pi_s}{r}+\frac{\pi_d'(x^{**})}{r\lambda}+K+\frac{4\delta^2}{9r\lambda^2}\right] - \frac{2}{rG-r/G}\left[-e^{-\lambda\underline{x}}\left(\pi_d(1-x^*)-\pi_s+\frac{4\delta^2}{9\lambda^2}\right)+e^{-\lambda(1-x^*)}\left(\pi_d(\underline{x})-\pi_s+\frac{4\delta^2}{9\lambda^2}\right)\right] \tag{xxxiii}$$

$$2\widetilde{C}(e^{\lambda}-1) = e^{\lambda x^{**}}\left[\frac{\pi_d(x^{**})-\pi_s}{r}-\frac{\pi_d'(x^{**})}{r\lambda}+K+\frac{4\delta^2}{9r\lambda^2}\right] - \frac{2}{rG-r/G}\left[e^{\lambda\underline{x}}\left(\pi_d(1-x^*)-\pi_s+\frac{4\delta^2}{9\lambda^2}\right)-e^{\lambda(1-x^*)}\left(\pi_d(\underline{x})-\pi_s+\frac{4\delta^2}{9\lambda^2}\right)\right]. \tag{xxxiv}$$

Dividing (xxxiii) by (xxxiv), we can then obtain

$$\frac{e^{-\lambda}-1}{e^{\lambda}-1} = \left\{e^{-\lambda x^{**}}(G-1)\left[\pi_d(x^{**})-\pi_s+\frac{\pi_d'(x^{**})}{\lambda}+rK+\frac{4\delta^2}{9\lambda^2}\right] - \frac{2Ge^{-\lambda\underline{x}}}{G+1}\left[G\pi_d(\underline{x})-\pi_d(1-x^*)-(G-1)\pi_s+(G-1)\frac{4\delta^2}{9\lambda^2}\right]\right\}\Bigg/ \left\{e^{\lambda x^{**}}(G-1)\left[\pi_d(x^{**})-\pi_s-\frac{\pi_d'(x^{**})}{\lambda}+rK+\frac{4\delta^2}{9\lambda^2}\right] - \frac{2Ge^{\lambda(1-x^*)}}{G+1}\left[G\pi_d(1-x^*)-\pi_d(\underline{x})-(G-1)\pi_s+(G-1)\frac{4\delta^2}{9\lambda^2}\right]\right\} \tag{xxxv}$$

which determines x^{**} given that we already have x^* and $\underline{x}$. For example, for $K=.07, \delta = 4, \sigma^2=.2$, and $r=.1$, we can obtain $x^*=.721, \underline{x}=.684$, and $x^{**}=.730$.

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