Influencers: To Grow or To Monetize*

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Abstract

Social media influencers can grow their number of followers by endorsing products that are authentic for their social media persona, or alternatively, monetize their followers by endorsing a wider variety of products. We develop a dynamic model in which an influencer continuously decides whether to be authentic as she balances increasing awareness with generating revenues from sponsored posts. We derive conditions in which the influencer is authentic during an early growth phase, but she becomes inauthentic once a large enough fraction of potential followers are aware of her. Celebrities become inauthentic at a lower awareness level than pure social media influencers. Rapid turnover of potential followers can cause the influencer to remain permanently authentic.

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1 Introduction

Companies pay social media influencers to make sponsored posts endorsing products on Instagram and other social media websites. Influencer marketing is growing rapidly, with companies spending about $16.4 billion in 2022 and $21.1 billion in 2023 on paid social media endorsements (Influencer Marketing Hub 2023).

Influencers post content, generate revenues by endorsing products, and try to grow their number of followers. Most influencers receive frequent messages from companies offering endorsement deals (Chiang 2018; Carufel 2021). If the product being endorsed is a good fit for the influencer's followers, then an endorsement deal allows an influencer both to generate revenue and to deliver additional useful content to her followers. By contrast, endorsing a product that is a poor fit reduces the average value of her content and may cause some current followers to unfollow her (Cheng and Zhang 2022). Thus, influencers face a trade off between growing their number of followers by accepting endorsement deals only for products that are a good fit, or alternatively, monetizing their followers by endorsing a wider variety of products.

Influencer marketing managers refer to an influencer as authentic if she only endorses products that she genuinely likes and that are consistent with her social media persona (Brown 2021). A common pattern is that influencers are authentic during an early period of rapid growth in followers, they become inauthentic after they attract a large following, and then the growth rate of their followers begins to slow. For example, an Instagram influencer with the user name shutthekaleup provides followers with advice on healthy eating. During her first three years on Instagram, most of her endorsements were for healthy foods such as Pressed Freeze Juicery frozen juice and Perfect Bar organic snack bars. During these three years, her number of followers grew rapidly to 250 thousand (Golub 2018). However, during the next five years, she began endorsing a variety of unrelated products such as Adidas track suits and Fossil wristwatches, and during this period her Instagram follower count increased by only 100 thousand. See Appendix A for screenshots of shutthekaleup's Instagram page and some of her early and more recent product endorsements.[1]

Consistent with this example, popular business articles have pointed out that small influencers are typically more authentic than large influencers (e.g., Ehlers 2021; Vogl 2022; Wiley 2023). A recent article in *Forbes* states, "One of the most important benefits smaller influencers often bring to the table is deceptively simple: authenticity. Mega-influencers aren't typically viewed as authentic and relatable, compared to their more 'everyday' counterparts" (Wiley 2023). Similarly, an article in *Brandwatch* says, "With micro-influencers you can reach people in a more authentic way. Social Media is becoming a more and more difficult place to cut through the noise and ads are often seen as untrustworthy and annoying" (Vogl 2022).

During interviews with the news media, micro-influencers often discuss the overwhelming number of endorsement offers they receive and how they decide which ones to accept. Many of them mention the importance of authenticity in their endorsement policy. For example, an interior design influencer with 80 thousand followers stated, "When brands approach me, I'd like to know that they respect me, my audience, and what I put out in the world. I can help brands reach my audience authentically and turn that engagement into new relationships, fans, followers, and customers – but it has to resonate with my audience, and I know them best" (Baklanov 2021). Given how selective small influencers are with their endorsements, public relations firms advise brands that would like to advertise with micro-influencers to contact many different influencers, sending each one a personally tailored message explaining why she would be a good fit for the brand (Chiang 2018; Carufel 2021). By contrast, larger influencers often endorse products they do not even use (Nephew 2020).

[1] During an interview in 2018, the interviewer asked, "How do you approach working with brands? I'm sure you get so many requests." Shutthekaleup replied, "I look at their product and figure out if it's a good fit. But I solely work with brands I absolutely love. I refuse to put crap on my feed for a few bucks. It's not worth it to me" (Shape Shift Report 2018). More recently, on a Reddit thread in 2023, users said they had stopped following shutthekaleup because she now endorses too many products. One user wrote, "In the beginning, she may have been authentic, but now she's only interested in making money, putting in links to further engage her bank account not even looking into what she's trying to sell us if it's even a good product. No one absolutely no one can use all these products at the same time and say how great they are" (Reddit 2023).

We build a model that captures the dynamics of an influencer's growth in continuous time, starting when she has a very small number of followers, for example, her friends and family, until she approaches full awareness among potential followers. Over time the influencer attracts new followers and receives offers for paid endorsement deals. She continuously decides her endorsement policy as she balances increasing awareness with generating revenues from sponsored posts. At any moment, she can choose either to be authentic by only accepting endorsement offers that are a good fit for her online persona or to be inauthentic by endorsing a wider variety of products. Being inauthentic causes some fraction of followers to unfollow her, but also results in higher revenues per follower. New followers become aware of the influencer at a rate that depends on the current number of followers. Thus, being authentic maximizes the current number of followers and allows the influencer to build awareness more quickly.

We derive conditions in which the influencer initially is authentic in order to grow awareness as quickly as possible, but she later becomes inauthentic once awareness is sufficiently large. The early growth rate is exponential, with awareness growing at a constant percentage rate when current awareness is near zero. Therefore, as long as the rate at which each follower attracts new followers exceeds the rate at which the influencer discounts the future, a small influencer prefers to be authentic in order to grow awareness quickly. In other words, as long as the influencer places any reasonable weight on future profits, the value of initial faster growth from authenticity exceeds the value of greater immediate profits from being inauthentic. However, as the influencer attracts more followers, the financial incentive for her to become inauthentic grows, as brands offer larger and larger payments for an endorsement deal. Meanwhile, her potential for future growth in followers diminishes as the pool of potential followers who are not yet aware of her becomes smaller. As a result, the influencer eventually decides to prioritize monetizing her current followers rather than attracting new followers. At this point, she becomes inauthentic in her endorsement policy, some of her current followers unfollow her, and her growth rate of awareness slows down.

Standard reputation models imply that large firms are more protective of their brand than small firms (Kreps and Wilson 1982; Diamond 1989; Chu and Chu 1994), and empirical evidence shows that customers trust big brands more than small brands for consumer products (Rajavi et al. 2019). By contrast, our model implies that small influencers are more authentic than large influencers. We show that small influencers have a stronger incentive to be authentic in order to grow awareness, so in our model, followers trust smaller influencers more.

Furthermore, the influencer in our model may have the same number of followers at two different points in her social media lifecycle. The first occurs during an early phase in which she is authentic and quickly builds awareness, and she again has the same number of followers during a later phase in which she already has widespread awareness and has become inauthentic. Thus, for a given number of followers, the influencer may be either authentic or inauthentic depending on her awareness level.

Our results imply that advertising managers should offer endorsement deals to a rapidly growing new influencer only if her organic social media content is a good fit for the product. Alternatively, managers can sign endorsement deals with a more established influencer even if the product is not a clear fit. Because new influencers appeal to young customers who are interested in the latest trends, finding an influencer with an authentic fit for the product is essential for targeting such customers.

We also present three model extensions. The first extension adapts the model to traditional celebrities who can generate awareness from their current followers and also directly through the activity that makes them famous. We show that an increase in the rate at which a celebrity directly generates awareness causes her to become inauthentic at a lower awareness level. Although it is intuitive that sports, music, and movie stars are inauthentic near the end of their careers, our model predicts that young rising stars also make inauthentic social media endorsements. We show that, unlike pure social media influencers, celebrities do not have a strong incentive to remain authentic early in their careers to build awareness. Therefore, celebrities are willing to endorse a wide variety of products on social media starting at a young age. For example, when they were still teenagers, rising tennis stars Carlos Alcaraz and Emma Raducanu signed endorsement deals with luxury car brands BMW and Porsche (Boon 2022; Jones 2022). They began making frequent Instagram posts endorsing these car brands, despite tennis fans complaining that these endorsements are not authentic (see screenshots in Appendix A).

The second model extension allows for turnover in followers, for example, because users continuously enter and leave the social media platform, or because the influencer focuses on an activity that is relevant to each follower for only a limited amount of time, like caring for babies or applying to colleges. If the turnover rate is sufficiently high, the influencer permanently stays authentic. Thus, our model implies that influencers become inauthentic in product categories with long-term followers like cooking and sports, but they remain authentic in product categories with short-term followers like baby care and college application advice. Furthermore, an influencer may always remain authentic on a platform like TikTok that targets a youthful audience with rapid turnover of users (Maheshwari 2023). We predict that influencers stay authentic if there is rapid turnover, unlike tourist traps that sell low quality products if there is rapid customer turnover. In this respect, our model contrasts with traditional reputation models, which imply that short-term customers cause firms to make low quality investment (Shapiro 1982; Fudenberg et al. 1990).

The third model extension allows for intermediate levels of authenticity. In other words, at any given time, the influencer may choose to accept some but not all endorsement offers for products with poor fit. In some cases, the influencer is totally authentic at first (she rejects all poorly targeted offers), but she eventually converges to a level of partial authenticity for which she accepts a positive fraction of poorly targeted offers.

Section 2 discusses related literature. Section 3 presents the model. Section 4 presents conclusions. Appendix A contains examples of sponsored Instagram posts. Appendix B contains formal proofs of all results.

2 Related Literature

There is a literature that has studied how influencer marketing affects product line design (Kuksov and Liao 2019), competition among firms (Katona 2020), and competition for sponsorships (Fainmesser and Galeotti 2021). Mitchell (2021) studies an influencer's optimal trade-off between good advice and advertising, and Pei and Mayzlin (2022) study the optimal affiliation between a firm and influencers to persuade consumers to purchase the product. Berman et al. (2023) compare the benefits of influencer marketing with targeted advertising when consumers can react by liking a post. Nistor and Selove (2023) show how informative and uninformative comments from followers affect an influencer's endorsement policy. Liu and Liu (2023) study the effect of artificial intelligence matching algorithms on influencer and platform profits. The current paper focuses instead on how an influencer balances growing awareness with monetizing followers over time.

Previous research has also studied the dynamics of reputation (e.g., Kreps and Wilson 1982; Rob and Fishman 2005; Cabral and Hortaçsu 2010; Board and Meyer-ter-Vehn 2013). In contrast with our results, most of these earlier papers find that larger firms make greater effort to protect their reputation. Board and Meyer-ter-Vehn (2013) show that, under some conditions, larger firms make lower investment in quality, although the mechanism for that result is different than in this paper. In that model, a large firm shirks if consumers learn about the firm through stochastically arriving signals that can provide good news about quality, but works harder if the signals can provide bad news. By contrast, in this paper, a large influencer endorses products with poor fit because she has less potential to grow awareness. A key feature that distinguishes the model presented here from most reputation models is that, in the model, endorsing products with poor fit does not reduce the value of the influencer's reputation but instead reduces its growth rate. In particular, the state variable in our model is the number of people who are aware of the influencer. The decision to become inauthentic reduces the growth rate of awareness but does not reduce the large stock of awareness the influencer has already built.

Our paper uses a growth equation similar to the model of new product adoption by Bass (1969), which is also similar to susceptible-infected-recovered models used in public health research (e.g., Liu et al. 2020). Previous research has extended the Bass model to include price and advertising decisions (Bass et al. 1994; Krishnan et al. 2012; Cosguner and Seetharaman 2022). Our paper introduces a new control variable, the influencer's authenticity level, which affects both profits and growth. We then solve for the influencer's optimal policy. Whereas most previous research on the Bass model focuses on fitting the model empirically, we adapt this model to derive theoretical insights and derive conditions in which an influencer shifts from authentic to inauthentic endorsement policies.

Empirical research has also studied related topics. Consistent with our model, Cheng and Zhang (2022) find that YouTube influencers lose subscribers after a sponsored post, but this effect is mitigated if the sponsored video is a good fit for the influencer's organic content. Yalcin et al. (2020) document that influencers can act as both educators and pure advertisers. Bentley et al. (2021) find that smaller influencers have deeper engagement with their followers, which is consistent with our finding that influencers who are in the growth phase and thus have a smaller following provide sponsored content with better fit for followers.

3 Model

An influencer builds a network of followers and generates profits continuously over time $t \in [0, \infty)$. There is a unit mass of potential followers. In reality, the number of potential followers may depend on the type of organic content the the influencer posts, her geographic location, and other personal characteristics, but for simplicity of notation and without loss of generality, we scale the number of potential followers to one. The number of followers who are aware of the influencer at time t is denoted by A_t, the number who choose to follow her is F_t, and her instantaneous profits are π_t.

3.1 Profits and Followers

Given the influencer's current awareness level, we now model her profits and number of followers at time t.

Endorsement offers arrive according to a Poisson process with rate μ, so during a time period of small length dt, the probability that a company offers the influencer an endorsement deal is $\mu\, dt$. Each endorsement offer has independent probability θ of good fit and $1-\theta$ of poor fit with the influencer's organic (non-sponsored) content. The probability that a randomly chosen follower is interested in a product with good fit is normalized to one, and the probability of him being interested in a product with poor fit is ω, where $0 < \omega < 1$.

At each time t, the influencer can choose to be authentic and endorse only products with good fit, or inauthentic and endorse all products. This endorsement policy is observed continuously by followers and binding. In reality, influencers receive frequent endorsement offers, so followers learn almost immediately if an influencer changes her endorsement policy, and they can then choose to unfollow her.[2]

An endorsement makes followers aware of the product. The value of consuming an endorsed product is U if the follower is interested and zero if he is not interested, where $U > 0$. For the company selling a product that the influencer endorses, the optimal strategy is to set product price U. Let $\widehat{U}$ denote the amount of profit that goes to the influencer for each unit sold as a result of the endorsement deal, which could be based, for example, on a bargaining process in which each party receives a fraction of the profits generated from the deal. Thus, the influencer receives profits $\widehat{U}F_t$ from endorsing a product with good fit and $\omega\widehat{U}F_t$ from endorsing a product with poor fit. If the influencer is authentic, her instantaneous expected profits are $\mu\theta\widehat{U}F_t$, and if she is inauthentic, these profits are $\mu(\theta+(1-\theta)\omega)\widehat{U}F_t$. For notational

[2] We could also derive formal conditions in which reputation concerns prevent the influencer from deviating from any given policy. Suppose the influencer receives endorsement offers for a third category of products with very poor fit. The influencer's endorsement policy could then be sustained by the threat that, if she deviates from her stated policy at any time t, customers then believe she will always endorse all products, even those with very poor fit.

simplicity and without loss of generality, we rescale the profit parameter $\widehat{U}$ such that $\mu\theta\widehat{U} = 1$ and we define the parameter $\phi \equiv \mu(\theta + (1-\theta)\omega)\widehat{U}$, so the profits from being authentic and inauthentic are F_t and ϕF_t, respectively, where $\phi > 1$.

Potential followers incur cost c from seeing a sponsored post, so the instantaneous expected cost of seeing sponsored posts is $\mu\theta c$ if the influencer is authentic and μc if she is inauthentic. Followers also derive positive utility from the influencer's organic social media content. The instantaneous expected utility from this organic content is greater than $\mu\theta c$ for all potential followers and greater than μc for a fraction γ of potential followers. Therefore, everyone who is aware of the influencer follows her if she is authentic but only a fraction γ follow her if she is inauthentic.

Thus, if the influencer is authentic at time t, her number of followers and instantaneous profits both equal her awareness level A_t. If she is inauthentic, her number of followers is γA_t and her profits are $\gamma\phi A_t$, where $0 < \gamma < 1$ and $\phi > 1$. Below we derive the influencer's optimal dynamic policy given parameters γ and ϕ.

In the interest of analytical tractability, we have allowed potential followers to follow and unfollow the influencer at no cost and to learn immediately when the influencer changes her endorsement policy. This modeling framework is well suited to real world social media platforms, given that people can follow or unfollow an influencer simply by clicking a button, and most influencers post content multiple times per week, so followers quickly observe any change in endorsement policies.[3]

3.2 Growth in Awareness

The influencer begins with a small level of awareness A_0, as her friends and family follow her on social media, where $0 < A_0 < 1$. At any time t, a pool of $1 - A_t$

[3]In principle, we could modify our model set-up and derive similar results. If we included a cost of following or unfollowing the influencer, then forward-looking users may anticipate the time when she will change her policy, and some users may choose not to follow her when they expect a future increase in the rate of endorsements. If we included a delay in followers learning the endorsement policy, then the influencer would maintain her base of followers for a brief period of time after changing policies, which would cause her to become inauthentic sooner and at a lower awareness level.

potential followers are not yet aware of her. The probability of any given person becoming aware of the influencer increases with her number of followers, as social media algorithms are more likely to recommend following an influencer if a user's friends are already following her. The parameter β reflects the rate at which each current follower increases the probability of a new potential follower becoming aware of the influencer, where $\beta > 0$. Formally, awareness grows according to the following equation of motion:

$$\frac{dA_t}{dt} = \beta F_t(1 - A_t) \tag{1}$$

Note that the growth rate in awareness increases with the number of followers, which implies that the influencer builds awareness more rapidly if she is authentic.

3.3 Value and Policy Functions

The influencer's objective is to maximize discounted profits with discount rate r. Her value function is:

$$V_t = \int_{u=t}^{\infty} e^{-r(u-t)} \pi_u \, du \tag{2}$$

We also define $V(A_t)$ as the value function if the influencer starts with awareness level A_t and always follows the optimal policy.

If $\gamma\phi < 1$, then being authentic leads to both higher instantaneous profits and faster growth in awareness, so the influencer is always authentic. For the remainder of the paper, we focus on the case in which $\gamma\phi > 1$, so instantaneous profits are higher if the influencer is inauthentic, and the influencer faces a trade off between higher current profits and faster growth in awareness. Under this condition, we will first derive a sufficient condition that ensures awareness is large enough that the influencer's optimal policy is to be inauthentic.

Based on (1), we see that $\frac{dA_t}{dt} = \beta(A_t - A_t^2)$ if the influencer is authentic, and $\frac{dA_t}{dt} = \gamma\beta(A_t - A_t^2)$ if the influencer is inauthentic at time t. By differentiating with respect to A_t, we find that, for either policy, the growth rate of awareness is decreasing in awareness once $A_t > \frac{1}{2}$. Therefore, once more than half of the potential customers are aware of the influencer, a small increase in awareness of size ϵ at time t results in an increase in awareness at all future times of less than ϵ for any given policy starting at time t, which implies the following result. See the appendix for a formal proof.

Lemma 1. *$V(A_t + \epsilon) - V(A_t) < \frac{\epsilon}{r}$ for all $A_t > \frac{1}{2}$.*

Intuitively, the value of increasing awareness at time t by ϵ is less than the value of the additional revenues that would come if awareness were permanently higher by ϵ because higher current awareness results in slower future growth.

We can now compare the benefits of each endorsement policy. Being inauthentic results in instantaneous profits that are greater by $(\gamma\phi - 1)A_t$, and being authentic results in a growth rate of awareness that is greater by $(1-\gamma)\beta A_t(1-A_t)$. We therefore have the following result.

Lemma 2. *The influencer is inauthentic at time t if awareness is large enough that $A_t > \frac{1}{2}$ and $(\gamma\phi - 1) > \frac{1}{r}(1-\gamma)\beta(1-A_t)$.*

Once awareness is large enough, firms offer large payments for an endorsement deal and there is a relatively small pool of potential followers who are not yet aware of the influencer, so the benefits of generating greater profits from being inauthentic outweigh the benefits of faster awareness growth from being authentic.

We now solve for the influencer's optimal policy during the early growth phase of building awareness. Let $\underline{V}(A_t)$ denote the value function if the influencer is always inauthentic starting with awareness level A_t. For all time $u \geq t$, awareness then grows according to $\frac{dA_u}{du} = \gamma\beta A_u(1-A_u)$. The proof of the following lemma (see appendix) solves this differential equation and derives the resulting value function.

Lemma 3. *The value function given a policy of always being inauthentic is*

$$\underline{V}(A_t) = \int_{u=t}^{\infty} \frac{e^{-r(u-t)}\gamma\phi}{\left[1 + \left(\frac{1-A_t}{A_t}\right)e^{-\gamma\beta(u-t)}\right]}\, du$$

Lemma 2 guarantees that, when awareness is sufficiently large, the influencer does follow a policy of being inauthentic. We now use backward induction and determine whether there is an earlier point with lower awareness at which the influencer prefers to be authentic. To evaluate the value of faster awareness growth from being authentic, we compute the derivative of the the value function $\underline{V}(A_t)$ with respect to awareness.

$$\frac{d\underline{V}(A_t)}{dA_t} = \int_{u=t}^{\infty} \frac{e^{-(\gamma\beta+r)(u-t)}\gamma\phi}{\left[A_t + \left(1 - A_t\right)e^{-\gamma\beta(u-t)}\right]^2} \, du \tag{3}$$

It is not generally possible to solve this integral with a closed-form expression because the function being integrated contains an exponential in the numerator and a constant plus a different exponential in the denominator. However, as the awareness level A_t approaches zero, the constant in the denominator approaches zero, so we can derive a closed for expression for the integral.

$$\lim_{A_t \to 0} \left[\frac{d\underline{V}(A_t)}{dA_t}\right] = \int_{u=t}^{\infty} e^{(\gamma\beta-r)(u-t)}\gamma\phi \, du \tag{4}$$

If $\gamma\beta \geq r$ this integral diverges to infinity, which guarantees that there is a point at which the influencer prefers to be authentic for A_0 sufficiently small. If $\gamma\beta < r$ the integral equals $\frac{\gamma\phi}{r-\gamma\beta}$. In this case, if A_0 is sufficiently small, there is a point at which the influencer prefers to be authentic if $\left[\frac{\gamma\phi}{r-\gamma\beta}\right](1-\gamma)\beta A_0 > (\gamma\phi - 1)A_0$. The left side of this inequality is the limit (as A_0 approaches zero) of the derivative of the value function times the additional growth in awareness if the influencer is authentic at time zero. The right side is the additional profits from being inauthentic at time zero. Rearranging terms, we find that the influencer prefers to be authentic for small levels of awareness if the following condition holds:[4]

Condition 1. $\left[\frac{\beta(1-\gamma)}{r-\gamma\beta}\right]\gamma\phi > \gamma\phi - 1$

To help understand this condition, suppose the influencer has a policy of always being inauthentic, and consider her decision whether to deviate from this policy by being authentic for a short time at a very low level of awareness. The right side of

[4]Formally, we define Condition 1 to hold if either $\gamma\beta \geq r$ or the stated inequality holds.

the above inequality, $\gamma\phi - 1$, is the increase in profits per unit of current awareness that come from being inauthentic.

On the left side, the term $\beta - \gamma\beta$ is the increase in awareness (per unit of current awareness) that comes from being authentic when awareness is near zero. We need to compute the value of the resulting increase in awareness. Once the influencer switches to being inauthentic, awareness initially grows at a rate that is approximately exponential with growth rate $\gamma\beta$. As current awareness approaches zero, the duration of this initial period of exponential growth becomes larger without bound. Thus, the term $\frac{\gamma\phi}{r-\gamma\beta}$ is the limit of the value of an early unit increase in awareness as the period of exponential growth becomes arbitrarily large.

In practical terms, if an influencer has very few followers, then her potential for growth is so great, and her period of exponential growth will last long enough, that she should prioritize increasing awareness over current profits as long as she places any reasonable weight on future profits.

We can now fully characterize the influencer's optimal policy, which is stated in the following proposition.

Proposition 1. *If $\gamma\phi \leq 1$, the influencer is always authentic. If $\gamma\phi > 1$ and Condition 1 does not hold, the influencer is always inauthentic. If $\gamma\phi > 1$, Condition 1 holds, and initial awareness A_0 is sufficiently small, the influencer is authentic until awareness is large enough that the value function derivative stated in* (3) *equals* $\frac{\gamma\phi-1}{(1-\gamma)\beta(1-A_t)}$, *and then permanently switches to being inauthentic.*

Note that Condition 1 is guaranteed to hold if $\beta > r$. Therefore, if the rate at which each follower attracts new followers exceeds the rate at which the influencer discounts the future (for example, each current follower attracts .2 new followers per year and the yearly discount rate is less than 20%), a small influencer prefers to be authentic in order to grow awareness quickly.

However, once her level of awareness is sufficiently large, so firms offer her large payments for an endorsement deal and the pool of potential followers who are not yet aware of her is relatively small, the influencer becomes inauthentic to monetize her followers. In particular, $(1-\gamma)\beta A_t(1-A_t)\frac{dV(A_t)}{dA_t}$ represents the additional growth in awareness from being authentic times the marginal value of awareness (if the influencer becomes inauthentic) at awareness level A_t. Once this value falls below the additional profits from being inauthentic, represented by $(\gamma\phi - 1)A_t$, the influencer becomes inauthentic.

3.4 Numerical Example

We now present a numerical example with the following parameter values. Note that $\gamma\phi > 1$ and $\beta > r$, which we have shown implies the influencer is initially authentic and later becomes inauthentic once awareness is sufficiently large.

Table 1. Parameter values used in the numerical example

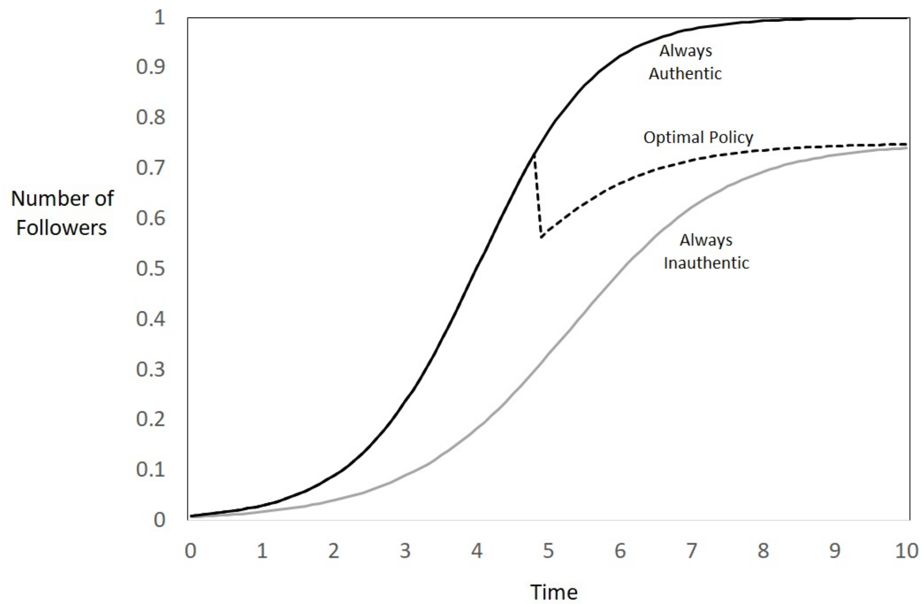
$\gamma = 0.75$	Fraction of people who are willing to follow the influencer if she is inauthentic
$\phi = 1.5$	Profits per follower if the influencer is inauthentic
$\beta = 1.2$	Coefficient on followers in awareness growth equation
$r = 0.1$	Discount rate
$A_0 = 0.01$	Initial awareness level

We use numerical integration to solve for the awareness level at which the influencer switches to being inauthentic based on Proposition 1, which occurs when 73% of potential followers are aware of the influencer.[5]

Figure 1 illustrates the number of followers over time for three possible policies. The top line represents a policy of always being authentic, the bottom line represents a policy of always being inauthentic, and the dashed line in the middle represents the optimal policy that maximizes discounted profits. Under the optimal policy, when the influencer becomes inauthentic, 25% of her current followers unfollow her, and her growth rate of awareness slows down.

[5] Alternatively, we can solve for the optimal policy with value function iteration, and obtain similar results.

Figure 1. Number of followers if influencer is always authentic, is always inauthentic, or follows the optimal policy to maximize discounted profits



3.5 Model Extension: Traditional Celebrities

The main version of the model focuses on influencers who develop a following primarily through social media, and who therefore depend on their current followers to attract new followers. We now extend the model to allow for traditional celebrities such as sports, music, and movie stars.

Traditional celebrities can attract followers through the activity that makes them famous, so we now allow the equation for awareness growth to include a constant term α. This constant α reflects the instantaneous probability of a potential follower becoming aware of the influencer by seeing her on television, for example. The expression for the equation of motion would then be

$$\frac{dA_t}{dt} = (\alpha + \beta F_t)(1 - A_t) \tag{5}$$

If the influencer is authentic, $\frac{dA_t}{dt} = \alpha + (\beta - \alpha)A_t - \beta A_t^2$, and if the influencer is inauthentic, $\frac{dA_t}{dt} = \alpha + (\gamma\beta - \alpha)A_t - \gamma\beta A_t^2$. By differentiating with respect to A_t, we find that the growth rate is guaranteed to be decreasing in awareness for $A_t > \frac{1}{2} - \frac{\alpha}{2\beta}$. Therefore, Lemmas 1 and 2 from the main version of the model still hold, and if $\gamma\phi > 1$, the influencer becomes inauthentic when awareness is sufficiently large.

We now show that, for a given awareness level A_t, the marginal value of an increase in awareness is decreasing in α. If awareness is growing quickly because of people who learn about the influencer directly (high α), the impact of authenticity on future awareness diminishes because the influencer rapidly approaches full awareness regardless of her current endorsement policy. Therefore, the derivative of the value function $V(A_t)$ with respect to awareness is also decreasing in α (see appendix for formal proof).

Lemma 4. $\frac{dV(A_t)}{dA_t}$ *is decreasing in* α*.*

For low values of α, if the influencer is authentic, then the resulting increase in current awareness may cause persistently higher awareness relative to the alternative case in which she is inauthentic. Therefore, an increase in current awareness has a large effect on the value of discounted profits. By contrast, for high values of α, being authentic increases profits in the short term, but there is little long-term effect because the influencer quickly approaches full awareness regardless of her current policy. Therefore, an increase in current awareness has a smaller effect on discounted profits when α is large.

We can now show how the constant growth term affects the optimal policy.

Proposition 2. *An increase in* α *causes the celebrity to become inauthentic at a lower level of awareness.*

This result implies that traditional celebrities who can generate awareness through the activity that makes them famous become inauthentic earlier than pure social media influencers who depend on their followers to generate awareness. Because celebrities can directly generate awareness even if they are inauthentic, they have an incentive to begin endorsing a wide variety of products early in their careers.

We now present a numerical example using the same parameters as in the previous example and a constant growth term $\alpha = .35$. We solve for the optimal policy with value function iteration and find the influencer is authentic until awareness is 57% and then becomes inauthentic.

Figure 2. Number of followers if celebrity is always authentic, is always inauthentic, or follows the optimal policy to maximize discounted profits

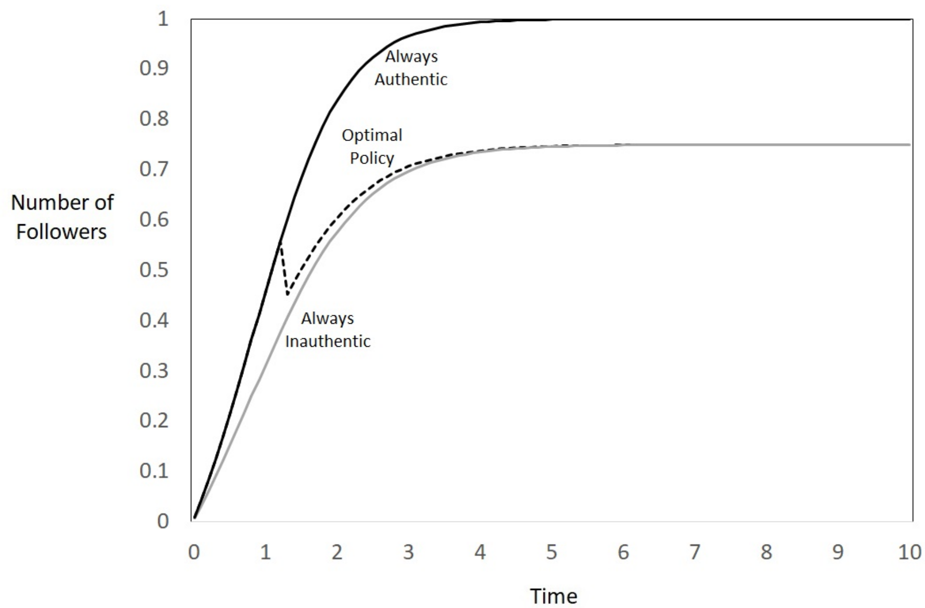


Figure 2 presents the number of followers over time if the celebrity is always authentic, is always inauthentic, or follows the optimal policy. The influencer's optimal policy is to become inauthentic at a much earlier time in this example than in the previous example, for two reasons. First, she becomes inauthentic at a lower awareness level (57% versus 73%). In addition, she reaches this awareness level quickly because of the additional constant in the growth equation.

Furthermore, in the model, followers are a percentage of the total number of potential followers. A traditional celebrity may have a much higher number of potential followers than a pure social media influencer, so the actual follower count and profits would be scaled up by a higher factor for the traditional celebrity.

3.6 Model Extension: Follower Turnover

In the main version of the model, the same unit mass of potential followers always remains in the market. In this model extension, we allow for turnover in potential followers, for example, because new users join and old users leave the social media platform, or because the influencer focuses on a topic that is only relevant to each user for a limited amount of time.

In particular, there is always a unit mass of potential followers, as old potential followers leave and new potential followers arrive at a rate x per unit of time, where $x \in (0, \beta)$. This turnover implies the rate of change of awareness is now:

$$\frac{dA_t}{dt} = \beta F_t(1 - A_t) - xA_t \tag{6}$$

This equation implies that, if the influencer did not build any additional awareness, the number of potential followers who are aware of her would decay at a constant percentage rate x.

Once turnover is allowed, the influencer never approaches full awareness. If she is always authentic, awareness grows at a rate $\frac{dA_t}{dt} = A_t(\beta - x - \beta A_t)$, and awareness converges to $\frac{\beta - x}{\beta}$. At this level of awareness, the rate at which new potential followers become aware of the influencer exactly equals the rate at which followers who are currently aware of her leave the platform or otherwise leave the influencer due to turnover.

We now derive a condition in which the influencer always stays authentic. Let

$\overline{V}(A_t)$ denote the value function if the influencer is always authentic starting with awareness level A_t. The proof of the following lemma solves the growth differential equation with follower turnover and derives the resulting value function.

Lemma 5. *The value function given turnover rate x and a policy of always being authentic is the following:*

$$\overline{V}(A_t) = \int_{u=t}^{\infty} \frac{e^{-r(u-t)}}{\left[\frac{\beta}{\beta-x} + \left(\frac{1-A_t\frac{\beta}{\beta-x}}{A_t}\right)e^{-(\beta-x)(u-t)}\right]}\, du$$

Consider the influencer's decision whether to deviate from this policy and become inauthentic at the maximum possible level of awareness, $A_t = \frac{\beta-x}{\beta}$. Taking the derivative of the value function at this awareness level, we have:

$$\left.\frac{d\overline{V}(A_t)}{dA_t}\right|_{A_t=\frac{\beta-x}{\beta}} = \int_{u=t}^{\infty} e^{-(r+\beta-x)(u-t)}\, du = \frac{1}{r+\beta-x} \tag{7}$$

Given this derivative of the value function, we show the influencer remains authentic at the maximum feasible awareness level if the following condition holds.

Condition 2. $\frac{(1-\gamma)x}{r+\beta-x} > \gamma\phi - 1$

The right side of this inequality is the additional profit per unit of current awareness from being inauthentic. On the left side of this inequality, the term $(1-\gamma)x$ is the additional awareness generated (per unit of current awareness) from being authentic, and $\frac{1}{r+\beta-x}$ is the marginal value of awareness. The left side of this condition is increasing in the turnover rate x for two reasons. First, fast turnover leads to a large pool of potential followers who are not yet aware of the influencer, as reflected by the term x in the numerator. Second, fast turnover implies a temporary period of inauthenticity causes a long-lasting drop in awareness because awareness grows more slowly with high turnover, as reflected by the term $-x$ in the denominator. For both of these reasons, with fast turnover, the influencer prefers to remain authentic at the maximum feasible awareness level.

The proof of Proposition 3 shows that, if this condition holds, the influencer is authentic for all feasible levels of awareness, $A_t \in \left(0, \frac{\beta - x}{\beta}\right)$.

Proposition 3. *If the rate of turnover is high enough that Condition 2 holds, the influencer always stays authentic.*

For the parameter values used in the previous numerical example (in section 3.4), the condition of this proposition holds if $x \geq 0.43$, which implies at least $1 - e^{-0.43}$, or about 35%, of potential followers leave and are replaced by other users per unit of time. In this case, the influencer's optimal policy is always to be authentic. Thus, our model predicts that an influencer remains authentic if she endorses products on a platform with rapid turnover of users, or if she focuses on a product category with short-term followers.

3.7 Model Extension: Partial Authenticity

In the main version of the model, the influencer could be authentic and endorse only products with good fit or could be inauthentic and accept all endorsement offers. In this model extension, we allow for intermediate levels of authenticity.

As in the main version of the model, endorsement offers arrive according to a Poisson process with rate μ, and each endorsement offer has independent probability θ of good fit and $1-\theta$ of poor fit with the influencer's organic posts. At each time t, the influencer chooses a level of inauthenticity y_t, which is the probability of accepting any given offer with poor fit, where $y_t \in [0, 1]$. Note this implies $\theta + (1 - \theta)y_t$ is the overall acceptance probability across all offers. In the main version of the model, the influencer chose either $y_t = 0$ (complete authenticity) or $y_t = 1$ (complete inauthenticity), whereas this extension allows intermediate values of y_t. Similar to the literature on Bayesian persuasion (e.g., Kamenica and Gentzkow 2011; Jerath and Ren 2021; Pei and Mayzlin 2022; Shulman and Gu 2023), we allow the influencer to commit to an offer acceptance probability at each time t.

Using the same notation and similar derivations as in the main version of the model, expected profits given inauthenticity level y_t are equal to $[1 + (\phi - 1)y_t]F_t$. Recall that $\phi > 1$, which implies that profits per follower increase with y_t because the influencer accepts more endorsement offers as y_t increases.

Furthermore, because potential followers incur cost c from seeing a sponsored post, the instantaneous expected cost of seeing sponsored posts is $\mu[\theta + (1 - \theta)y_t]c$. For this model extension, we let followers' positive utility from the influencer's organic social media content be uniformly distributed on $[\mu\theta c, \mu(\theta + \frac{1-\theta}{\gamma})c]$, which implies that the fraction of those who are aware of the influencer who choose to follow her is $1 + (\gamma - 1)y_t$. Recall that $\gamma < 1$, which implies that the influencer's number of followers decreases with y_t because her followers have a higher total cost from seeing sponsored posts if she accepts more endorsement offers.

Thus, given the influencer's choice of y_t, her number of followers is given by $F_t = [1+(\gamma-1)y_t]A_t$ and her instantaneous profits are $\pi_t = [1+(\gamma-1)y_t][1+(\phi-1)y_t]A_t$. The growth rate of awareness is given by the same equation of motion (1) as in the main model, that is, $\frac{dA_t}{dt} = \beta F_t(1 - A_t)$.

As the influencer decides y_t, which is the probability she accepts an endorsement offer conditional on receiving an offer with poor fit at time t, she faces the following trade-off. Accepting more endorsement offers reduces her number of followers, which causes awareness to grow more slowly, but it also leads to greater profits per follower.

The change in the growth rate of awareness that results from a marginal increase in y_t can be found by differentiating the equation of motion with respect to y_t:

$$\frac{d\left[\frac{dA_t}{dt}\right]}{dy_t} = \beta\frac{dF_t}{dy_t}(1 - A_t) = \beta(\gamma - 1)A_t(1 - A_t) \tag{8}$$

Because $\gamma < 1$, this equation implies that an increase in y_t causes awareness to grow more slowly.

The change in instantaneous profits that results from an increase in y_t is:

$$\begin{aligned}\frac{d\pi_t}{dy_t} &= [(\gamma - 1)(1 + (\phi - 1)y_t) + (\phi - 1)(1 + (\gamma - 1)y_t)]A_t \\ &= [\gamma + \phi - 2 + 2(\gamma - 1)(\phi - 1)y_t]A_t\end{aligned} \tag{9}$$

This derivative can be either positive or negative, that is, profits may increase or decrease with y_t, depending on the values of γ, ϕ, and y_t. Furthermore, the second derivative is $\frac{d^2\pi_t}{dy_t^2} = 2(\gamma - 1)(\phi - 1)A_t$. Because $\gamma < 1$ and $\phi > 1$, this second derivative is negative, that is, profits are strictly concave in y_t. The intuition for this result is that an increase in endorsement frequency causes profits per follower to increase, which implies it is more costly to lose additional followers by further increasing the endorsement frequency. Because profits are concave and may either increase or decrease in y_t, the value of y_t that maximizes current profits may lie on the interior of the interval $[0, 1]$. In other words, partial authenticity, which involves accepting some but not all poorly targeted endorsements, may maximize current profits.

The influencer's optimal policy depends on the effect of authenticity on both awareness growth and current profits. In the long run, as awareness approaches one, the effect of authenticity on awareness growth approaches zero, as can be seen from (8). Therefore, as in the main version of the model, the influencer's optimal policy eventually converges to the policy that maximizes current profits. From (9), we see that if $\gamma + \phi - 2 < 0$ profits are maximized by rejecting all endorsement offers with poor fit, if $\gamma+\phi-2+2(\gamma-1)(\phi-1)>0$ profits are maximized by accepting all endorsement offers, and otherwise profits are maximized by accepting a fraction of endorsement offers with poor fit given by $y_t = \frac{\gamma+\phi-2}{2(1-\gamma)(\phi-1)}$. Thus, the influencer's authenticity level eventually converges to the level that maximizes current profits.

The optimal policy can be found by solving the Hamilton-Jacobi-Bellman equation for the influencer's optimization problem, which is a standard approach for solving continuous-time dynamic control problems (Kamien and Schwartz 2012; Bertsekas 2017). For a given value function, the optimal policy is to choose the value of y_t that maximizes:

$$\pi_t + \frac{dV(A_t)}{dA_t}\frac{dA_t}{dt} \tag{10}$$

The first term in this expression represents instantaneous profits, and the second term represents the rate at which the value of future profits is growing. If we differentiate (10) with respect to y_t and insert the values from (8) and (9), we find that (10) is maximized by setting y_t equal to:

$$y_t^* = \frac{\gamma+\phi-2+\beta(\gamma-1)(1-A_t)\frac{dV(A_t)}{dA_t}}{2(1-\gamma)(\phi-1)} \tag{11}$$

Because y_t must lie in the interval $[0,1]$, the optimal policy is to set $y_t=0$ if $y_t^*<0$, to set $y_t=1$ if $y_t^*>1$, and to set $y_t=y_t^*$ otherwise. This policy accounts for the effect of endorsement deals on both current profits and awareness growth.

We now present a numerical example using the same parameters as in the main version of the model, allowing for partial authenticity. We solve for the optimal policy with value function iteration.

Figure 3. Number of followers if influencer is completely authentic, is completely inauthentic, or follows the optimal policy to maximize discounted profits allowing partial authenticity

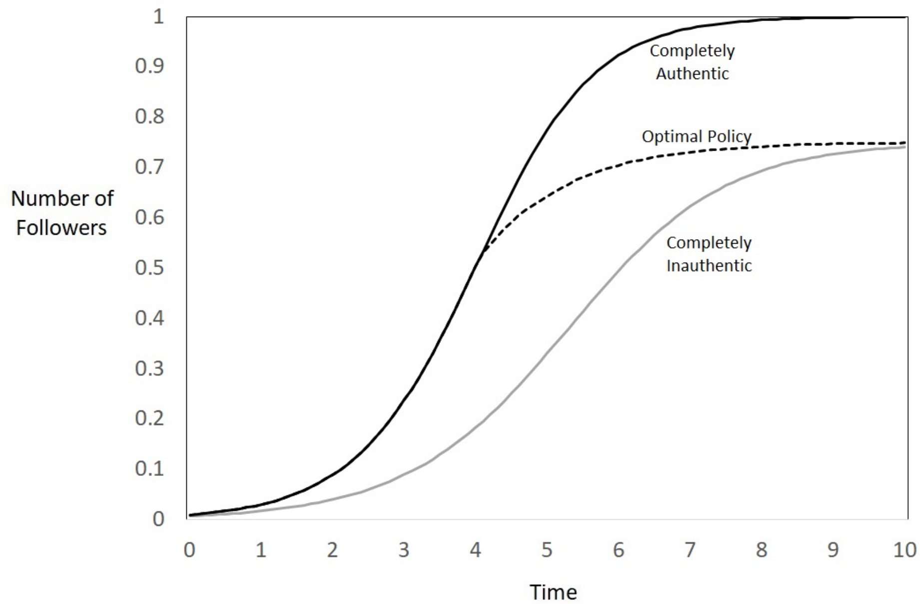


Figure 3 illustrates the number of followers over time if the influencer is always completely authentic, is always completely inauthentic, or follows the optimal policy. The influencer's optimal policy is to remain completely authentic until she reaches about 53% awareness. She then begins accepting a small fraction of poorly targeted endorsement offers. This fraction y_t increases until the influencer eventually converges to a policy of accepting all endorsement offers. Whereas Figure 1 in the main version of the model shows a sudden drop in followers when the influencer switches from complete authenticity to complete inauthenticity, Figure 3 does not show a drop in followers, but instead illustrates that the growth rate of followers slows as the influencer gradually becomes less authentic.

4 Conclusion

We develop a model in which an influencer balances faster growth from an authentic endorsement policy with greater current revenues from endorsing a wider variety of products. Our model helps explain the real world observation that small influencers are more authentic than large influencers. Whereas most models of reputation find that larger firms make greater effort to protect their brand (e.g., Kreps and Wilson 1982; Rob and Fishman 2005), in the model presented here, the optimal policy is to be authentic during an early growth phase and later to become inauthentic to monetize followers.

For most consumer products, customers trust large brands more than small brands (Rajavi et al. 2019). A key difference is that, whereas consumer product brands can build awareness though traditional advertising, social media influencers depends on current followers to generate awareness among new followers. Small influencers need to be authentic to attract early followers who then make other followers aware of them. Once the influencer has already generated widespread awareness, firms offer large payments for an endorsement deal and the pool of potential followers who are not aware of the influencer is relatively small, so it may be optimal to become inauthentic and endorse many products. Thus, we show that influencers have a stronger incentive to be authentic when they have few followers and low awareness, which leads consumers to trust small influencers more than large influencers.

Furthermore, firms often use social media advertising to target young customers who follow the latest fashion trends, and such customers seek out and follow new influencers who are growing rapidly. Therefore, our results imply that firms trying to attract young and trendy customers should make endorsement deals with influencers who have a rapidly growing number of followers and who post organic social media content that is a good fit for the firm's products. Alternatively, firms may target older and less trendy customers using endorsement deals with established influencers, even if the influencer's content is not an authentic fit for the product.

Future research could test the model empirically. Our results imply influencers initially endorse only products that are consistent with their organic content, and later begin endorsing other types of products as their growth rate slows. Empirical research could document such a pattern and estimate the model parameters based on observed follower growth rates and endorsement policies for influencers. Future research could also model related problems, such as how influencers adapt their content to new technology platforms. Content that appeals to a customer segment on Instagram may have less appeal for a younger generation on TikTok, for example, and an important challenge for influencers is how to attract followers from new segments while continuing to generate revenues from endorsement deals. An influencer who has become inauthentic may want to begin a new period of authenticity to attract the next generation of followers.

The choice between growing and monetizing also occurs in other business contexts. For example, social media platforms can focus on attracting more users with free services and minimal ads or focus on generating revenues from fees and advertising. More generally, technology products and other products with network effects initially try to increase the size of their user base and later try to profit from their users. Future research could adapt the modeling framework developed in this paper to study these related dynamic optimization problems.

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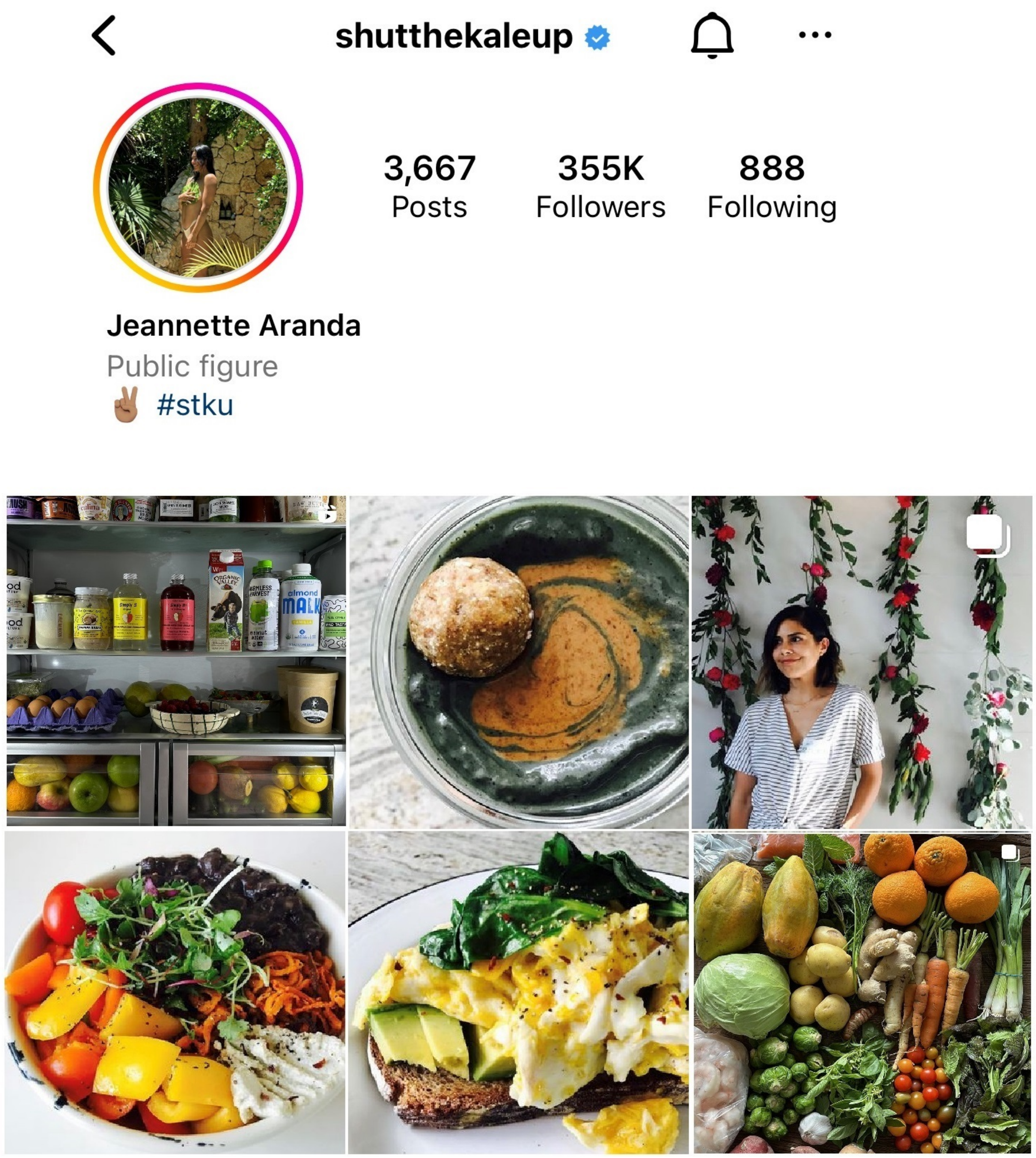
Appendix A: Influencer Examples

Shutthekaleup Instagram Content (Non-sponsored Posts)

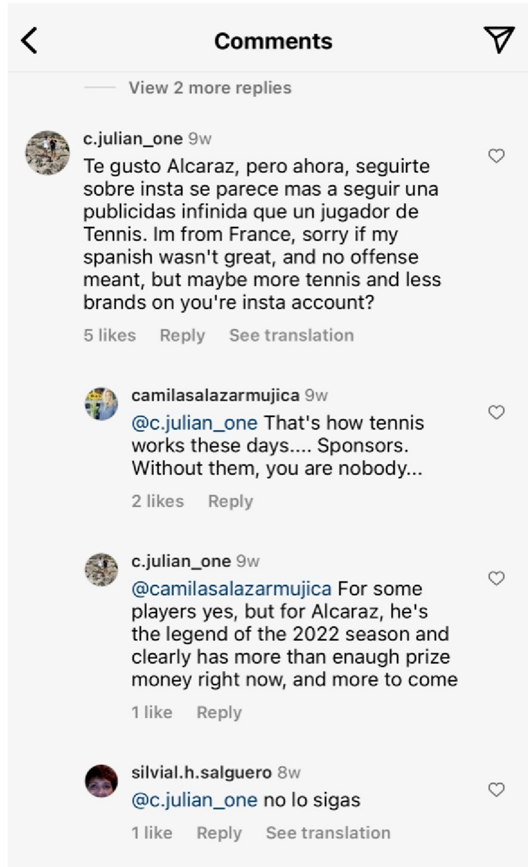


Shutthekaleup Food Endorsements (posted in 2017)

Shutthekaleup Non-food Endorsements (posted in 2018 and 2023)

Retrieved May 14, 2023, https://www.instagram.com/shutthekaleup/

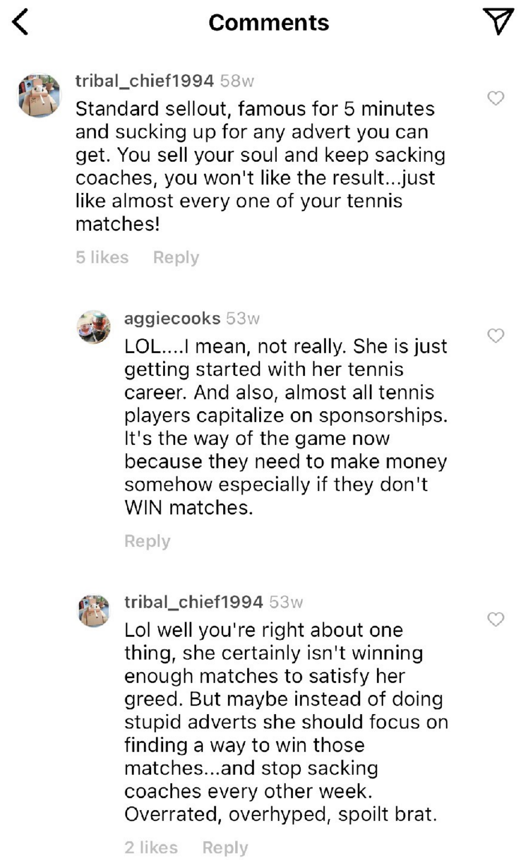
Carlos Alcaraz BMW Endorsement (posted in 2023)



(The last comment in Spanish: "Do not follow him.")

Retrieved May 19, 2023, https://www.instagram.com/carlitosalcarazz/

Emma Raducanu Porsche Endorsement (posted in 2022)



Retrieved June 12, 2023, https://www.instagram.com/emmaraducanu/

Appendix B: Proofs

Proof of Lemma 1

We will compute an upper bound on the difference between $V(A_t + \epsilon)$ and $V(A_t)$ when $A_t > \frac{1}{2}$. For $u > t$, define $\widehat{A}_u$ and $\overline{A}_u$ as the awareness at time u if the influencer starts at time t with awareness $A_t + \epsilon$ and A_t, respectively, and follows the policy that would be optimal starting with awareness $A_t + \epsilon$. For any given policy, awareness is strictly increasing over time, which ensures $\widehat{A}_u > \overline{A}_u$ for all $u > t$. Furthermore, the growth rate is decreasing in awareness for $A_t > \frac{1}{2}$, which implies $\widehat{A}_u - \overline{A}_u$ decreases over time. Because the initial difference in awareness is ϵ, $\widehat{A}_u - \overline{A}_u < \epsilon$ for all $u > t$. Because π_u equals either A_u or γA_u, the value increase from permanently increasing awareness by ϵ would be less than or equal to $\frac{\epsilon}{r}$, so the actual value increase from increasing awareness by ϵ is less than this amount. QED

Proof of Lemma 2

Being inauthentic at time t increases instantaneous profits by $(\gamma\phi - 1)A_t$. Being authentic at time t increases the growth rate of awareness by $(1-\gamma)\beta A_t(1-A_t)$. Lemma 1 guarantees the first derivative of the value function with respect to awareness is less than $\frac{1}{r}$ for $A_t > \frac{1}{2}$. Therefore, the influencer prefers to be inauthentic if $A_t > \frac{1}{2}$ and $(\gamma\phi - 1) > (1-\gamma)\beta(1-A_t)\frac{1}{r}$. QED

Proof of Lemma 3

If the influencer is always inauthentic starting at time t, then for time $u \geq t$, awareness grows according to $\frac{dA_u}{du} = \gamma\beta A_u(1-A_u)$. This differential equation has the following solution:

$$A_u = \frac{1}{1 + \left(\frac{1-A_t}{A_t}\right) e^{-\gamma\beta(u-t)}} \tag{12}$$

$$\frac{dA_u}{du} = \frac{\gamma\beta\left(\frac{1-A_t}{A_t}\right) e^{-\gamma\beta(u-t)}}{\left[1 + \left(\frac{1-A_t}{A_t}\right) e^{-\gamma\beta(u-t)}\right]^2} \tag{13}$$

To verify this solution, one can differentiate (12) and check that (13) is the derivative, and note these equations also satisfy $\frac{dA_u}{du} = \gamma\beta A_u(1 - A_u)$. Finally, if we set $u = t$, we can confirm that (12) equals A_t.

If the influencer is inauthentic, then $\pi_u = \gamma\phi A_u$. Therefore, the value function is:

$$\underline{V}(A_t) = \int_{u=t}^{\infty} \frac{e^{-r(u-t)}\gamma\phi}{\left[1 + \left(\frac{1-A_t}{A_t}\right) e^{-\gamma\beta(u-t)}\right]} du \tag{14}$$

QED

Proof of Proposition 1

If $\gamma\phi < 1$, being authentic leads to higher current profits and faster growth, so the influencer is always authentic.

If $\gamma\phi > 1$, Lemma 2 guarantees the influencer eventually becomes inauthentic. However, the derivations in the body of the paper show that, if A_0 is sufficiently small and Condition 1 holds, the influencer cannot always be inauthentic starting at awareness A_0 because she would prefer to deviate from this policy and to be authentic for a period starting at time zero.

The only remaining step is to show the second derivative of the value function is negative, which implies the influencer may start by being authentic to grow quickly and then switch to being inauthentic to generate more profits, but can never change policies in the other direction and go from being inauthentic to authentic.

The derivative of the value function if the influencer is always authentic, denoted by $\frac{d\underline{V}(A_t)}{dA_t}$ in equation (3), is decreasing in A_t, which implies the second derivative of $\underline{V}(A_t)$ is negative.

Now suppose the influencer is authentic starting at time t and then switches to being inauthentic at time u^*, which is chosen as the time at which it is optimal to become inauthentic. Similar derivations to those in the proof of Lemma 3 show that, on the time interval $u \in [t, u^*]$, awareness is given by $A_u = \frac{1}{1+\left(\frac{1-A_t}{A_t}\right)e^{-\beta(u-t)}}$. The value function is then:

$$V(A_t) = \int_{u=t}^{u^*} \frac{e^{-r(u-t)}}{\left[1 + \left(\frac{1-A_t}{A_t}\right)e^{-\beta(u-t)}\right]} du + e^{-r(u^*-t)}\underline{V}(A_{u^*}) \tag{15}$$

When differentiating this value function, the envelope theorem implies that the change in the optimal u^* has only a second-order effect, so we can simply differentiate each component. Taking second derivatives shows that the profits on the interval $[t, u^*]$ are concave in A_t, and A_{u^*} is also concave in A_t. We have already shown that the function $\underline{V}$ is concave. Therefore, the second derivative of $V(A_t)$ is negative.

Finally, if it is optimal to be authentic for awareness level A_t, that implies $(1-\gamma)\beta(1-A_t)V'(A_t) > (\gamma\phi - 1)$. Given that $V'(A_t)$ is decreasing in A_t, this inequality must also hold for all awareness levels less than A_t, so it must also be optimal to be authentic at lower levels of awareness. Thus, if it is ever optimal to be authentic, then the optimal policy is to be authentic starting at awareness level A_0 and then switch to being inauthentic at the awareness level stated in the proposition. QED

Proof of Lemma 4

We will compare $V(A_t + \epsilon)$ with $V(A_t)$ for small ϵ. For time $u > t$, let A_u denote awareness at time u if the firm starts with awareness A_t and follows the optimal policy, and let Δ_u denote the resulting increase in awareness at time u if the firm starts with awareness $A_t+\epsilon$ instead of A_t. If the influencer is authentic, $\frac{dA_u}{du} = \alpha+(\beta-\alpha)A_u-\beta A_u^2$. Therefore, as $\epsilon \to 0$, the change of the gap in awareness approaches the following if the influencer is authentic at time u:

$$\frac{d\Delta_u}{du} = (\beta - \alpha)\Delta_u - \beta[(A_u + \Delta_u)^2 - A_u^2] = (\beta - \alpha)\Delta_u - 2\beta A_u \Delta_u - \beta \Delta_u^2 \qquad (16)$$

If the influencer is inauthentic, each term β is replaced by $\gamma\beta$. In either case, the derivative $\frac{d\Delta_u}{du}$ is decreasing in A_u. Because an increase in α implies a higher value of A_u, the derivative $\frac{d\Delta_u}{du}$ is also decreasing in α, and thus the resulting future gap in awareness based on an ϵ increase in awareness at time t is decreasing in α. Furthermore, the envelope theorem implies the effect of an increase in α on the optimal policy has only a second order effect on the value function, so the derivative of the value function with respect to awareness is also decreasing in α. QED

Proof of Proposition 2

As in the main version of the model, it is optimal for the influencer to be inauthentic if $(\gamma\phi - 1) > (1-\gamma)\beta(1 - A_t)V'(A_t)$. Lemma 4 shows that $V'(A_t)$ is decreasing in α. Therefore, when α increases, the right side of this inequality decreases, which causes the influencer to become inauthentic at a lower level of awareness. QED

Proof of Lemma 5

If the influencer is always authentic starting at time t, then for time $u \geq t$, awareness grows according to $\frac{dA_u}{du} = \beta A_u(1 - A_u) - xA_u$. This differential equation has the following solution:

$$A_u = \frac{1}{\left[\frac{\beta}{\beta-x} + \left(\frac{1-A_t\frac{\beta}{\beta-x}}{A_t}\right)e^{-(\beta-x)(u-t)}\right]} \tag{17}$$

$$\frac{dA_u}{du} = \frac{(\beta-x)\left(\frac{1-A_t\frac{\beta}{\beta-x}}{A_t}\right)e^{-(\beta-x)(u-t)}}{\left[\frac{\beta}{\beta-x} + \left(\frac{1-A_t\frac{\beta}{\beta-x}}{A_t}\right)e^{-(\beta-x)(u-t)}\right]^2} \tag{18}$$

To verify this solution, one can differentiate (17) and check that (18) is the derivative, and note these equations also satisfy $\frac{dA_u}{du} = \beta A_u(1 - A_u) - xA_u$. Finally, if we set $u = t$, we can confirm that (17) equals A_t.

If the influencer is authentic, then $\pi_u = A_u$. Therefore, the value function is:

$$\overline{V}(A_t) = \int_{u=t}^{\infty} \frac{e^{-r(u-t)}}{\left[\frac{\beta}{\beta-x} + \left(\frac{1-A_t\frac{\beta}{\beta-x}}{A_t}\right)e^{-(\beta-x)(u-t)}\right]}\, du \tag{19}$$

QED

Proof of Proposition 3

We will show Condition 2 ensures the influencer never has an incentive to deviate from the policy of being authentic.

To compute the marginal value of awareness given a policy of authenticity, we differentiate the value function (19) with respect to awareness:

$$\frac{d\overline{V}(A_t)}{dA_t} = \int_{u=t}^{\infty} \frac{e^{-(r+\beta-x)(u-t)}}{\left[A_t \frac{\beta}{\beta-x} + \left(1 - A_t \frac{\beta}{\beta-x}\right) e^{-(\beta-x)(u-t)}\right]^2} \, du \tag{20}$$

The influencer has an incentive to maintain her policy of authenticity at awareness level A_t if:

$$(1-\gamma)\beta(A_t)(1-A_t)\frac{d\overline{V}(A_t)}{dA_t} > (\gamma\phi - 1)A_t \tag{21}$$

We first show the influencer stays authentic at the maximum feasible awareness level $A_t = \frac{\beta-x}{\beta}$. Evaluating (20) at this level of awareness, we have $\frac{d\overline{V}(A_t)}{dA_t} = \frac{1}{r+\beta-x}$. Furthermore, $1 - A_t = \frac{x}{\beta}$. Inserting these values into (21), we find this inequality is equivalent to Condition 2, so the influencer stays authentic at the maximum feasible awareness if this condition holds.

Because $(1 - A_t)$ and $\frac{d\overline{V}(A_t)}{dA_t}$ are both decreasing in A_t, Condition 2 ensures (21) holds and the influencer also stays authentic for all lower levels of awareness. QED