Dynamic Competition with Experience Goods

J. Miguel Villas-Boas
Haas School of Business
University of California at Berkeley
Berkeley, CA 94720-1900
villas@haas.berkeley.edu

This paper considers dynamic competition in the case in which consumers are only able to learn about their preferences for a certain product after experiencing it. After trying a product a consumer has more information about that product than about untried products. When competing in such a market firms with more sales in the past have an informational advantage because more consumers know their products. If products provide a better-than-expected fit with greater likelihood, taking advantage of that informational advantage may lead to an informational disadvantage in the future. This paper considers this competition with an infinite horizon model in a duopoly market with overlapping generations of consumers. Two effects are identified: On one hand marginal forward-looking consumers realize that by purchasing a product in the current period will be charged a higher expected price in the future. This effect results in reduced price sensitivity and higher equilibrium prices. On the other hand, forward-looking firms realize that they gain in the future from having a greater market share in the current period and compete more aggressively in prices. For similar discount factors for consumers and firms, the former effect is more important, and prices are higher the greater the informational advantages. The paper also characterizes oscillating market share dynamics, and comparative statics of the equilibrium with respect to consumer and firm patience, and the importance of the experience in the ex post valuation of the product.

1. Introduction

This paper considers dynamic competition in the case in which consumers are only able to learn about their preferences for a certain

I thank Drew Fudenberg, Dmitri Kuksov, and seminar participants at Cornell University, Harvard University, New York University, Northwestern University, and Universidad Torcuato Di Tella for helpful suggestions on an earlier version of this paper.

© 2006, The Author(s)
Journal Compilation © 2006 Blackwell Publishing
product after experiencing it. This type of products has been classified as experience goods (Nelson, 1970). Furthermore, in several experience goods markets an important part of the different valuations for the different products is idiosyncratic to each consumer, that is, consumers have different relative valuations (after experiencing them) of the available products. In addition, firms may not have any significant private information regarding which consumers value more or less their own product.

When purchasing a product, a consumer learns about its valuation. Then, in future periods this product has an informational advantage in the sense that a consumer knows more about the products that the consumer has tried than about the products that she has not tried. This informational advantage may benefit the products that were bought first. The idea is that after trying a product and understanding its valuation, a consumer may prefer the product whose valuation she knows better than the product whose valuation remains mostly uncertain. With risk neutrality this can be obtained with products offering a better-than-expected fit with greater likelihood. In this sense, firms may compete fiercely for consumers to first try their products. Similarly, Bain (1956) argued that this informational advantage may work as a barrier to entry because consumers tend to be loyal to the pioneering brands.

This paper examines the competitive effects of these informational advantages in an infinite horizon model with overlapping generations of consumers. In an infinite horizon firms have to trade off exploiting any informational advantages today with having lower informational advantages in the future. Similarly, the marginal consumers realize that by purchasing a product today they will be charged a higher expected price in the future. That is, forward-looking consumers become less price sensitive. Furthermore, one has to account for the fact that information advantages may also lead to some consumers finding that the product that they tried first is not very valuable for them, and therefore are more likely to try another product.

These informational advantages could potentially be seen as consumers having switching costs of changing products, and have, in fact,

1. This is in contrast with search goods where a consumer can fully evaluate the fit with a product prior to purchase. In some markets, consumer search, or pre-purchase inspection, may be costly (see, for example, Diamond, 1971, Kuksov, 2004, 2005, for the impact of consumer search costs in markets), and not be enough to evaluate product fit.

2. Other possible important dimensions, not explored here, are that there may be some common effects across consumers on how they value different brands, and some possible private information of the firms regarding these common effects. These issues are further discussed below.

3. See also Schmalensee (1982).
been used as an important justification for the existence of switching costs, "the uncertainty about the quality of untested brands" (Klemperer, 1995, p. 517). In such markets with switching costs firms gain in the future from having a higher market share today, because consumers have a preference for the products that they buy first (Beggs and Klemperer, 1992; Klemperer, 1995). This paper can then be seen as endogeneizing one central explanation for switching costs, the informational advantages of the products tried first. Endogeneizing this explanation is important for several reasons. First, it is not clear what is measured by the switching cost parameter in a market with experience goods. Second, by having the model fully specified one is able to completely determine the role played by each of the primitive parameters in the experience goods framework. For example, one can show how the prior distribution of valuations plays a crucial role in whether the market behaves as if there are "switching costs." In fact, as argued below, in order for the market to behave as in a switching costs framework the prior distribution of the experienced valuations has to be negatively skewed, with a likelihood greater than one half of getting a valuation above the mean valuation. Also interestingly, the consumers gaining more information on the products that they try changes the level of differentiation between the products in the market, which does not have a direct equivalent in a switching costs model. The consumers that get a bad draw of the product that they try first, always go and try the competitor’s product, also unlike in a switching costs model. Forward-looking consumers also realize that their purchases today may influence the prices in the future, and become less price sensitive.

The paper finds that steady-state prices, for similar discount factors for firms and consumers, are higher the greater the informational differentiation effects. In other words, the effect of increased prices because of lower price sensitivity of the forward-looking consumers dominates the effect of lower prices caused by firm competition for market share for future gains. The intuition is that in an infinite horizon firms realize that they should take advantage for any informational advantages when they have them, because in the future they also have to compete for a new

4. Padilla (1995) considers a model of competition for a homogenous good where consumers have switching costs, that yields a mixed-strategy equilibrium where firms benefit from switching costs. Chen and Rosenthal (1996) consider a dynamic competition model in which a fixed fraction of consumers changes loyalties in each period from the current high-price firm to the current low-price firm. This captures some of the switching cost effects of consumers being slow to change suppliers in response to a price change. Both of these papers do not consider the strategic effects of consumers being forward looking, which will be important in the discussion below.

5. See also Caminal and Matutes (1990) for endogeneizing switching costs through loyalty programs. See also Wernerfelt (1991) on the dynamic effects of brand loyalty.

6. The cases of no skewness and positive skewness are also discussed below.
generation of consumers. The steady-state prices are found to increase both in consumer patience and in the importance of the experience in the \emph{ex post} valuation of the product, and decrease in firm patience.

One also finds that the market share dynamics of the first-time consumers are oscillating, so that the firm with a greater market share of new consumers in one period has the smaller market share in the next period. This is because the firm with the greater market share ends up pricing higher to take advantage of the greater number of consumers that experience a good fit with their product. This higher price then yields a lower market share from the new generation of consumers in the market. The convergence to the steady state is slower when the information differentiation effects are greater (greater likelihood of finding a good fit in the first experience), when firm patience is greater, when consumer patience is smaller, and when the importance of the experience in the \emph{ex post} valuation of the product is greater.

Related to this paper, and following Bain (1956), it has been argued that informational differentiation is a barrier to entry for potential entrants (which is translated in this paper to a potential advantage of having a greater initial market share).\footnote{Golder and Tellis (1993) discuss several studies that provide some empirical support to this argument.} Schmalensee (1982) shows that a high-quality incumbent may deter entry because of the existing informational differentiation. Farrell (1986) argues that moral hazard on the part of the entrant may also create a barrier to entry even for a low-quality incumbent. Bagwell (1990) shows the same result for the case of adverse selection. These two later papers rely on the entrant having private information about some dimension of quality that affects all consumers equally, which is not considered in the current paper, and which may not be too important with well-established firms. Riordan (1986) considers the case in which competing firms choose, under private information and both price and quality commitment, that common quality dimension.\footnote{Liebeskind and Rumelt (1989) investigate quality choice with uncertain outcomes and without price commitment by the firms. Gale and Rosenthal (1994) look at quality choice without price or quality commitment by the firms.} Villas-Boas (2004) considers consumer learning in a two-period model, without considering competition for both first-time and experienced consumers simultaneously. In the first period, prices are lower the greater the informational advantages, contrary to the result for the steady-state prices in this paper, because of the existence here of consumers with experience in every period. Without assuming that competing firms have any private information, Bergemann and Välimäki (1996) look at the case of homogeneous consumers where all firms are able to observe the results of the consumers' experiences, and
focus on the consumer experimentation problem. In contrast, here, I look at heterogeneous consumers (also without private information by the firms) but get away from the consumer experimentation issues by limiting consumers to be in the market for only two periods.\textsuperscript{9}

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents some preliminary results. Section 4 solves the model and presents the main messages of the paper. Section 5 discusses extensions, and Section 6 concludes.

\section{The Model}

Two firms, \(A\) and \(B\), produce at zero marginal cost nondurable goods \(A\) and \(B\), respectively. In each period there is a new generation of a continuum of consumers coming into the market. Each generation lives for two periods, and has mass normalized to one.\textsuperscript{10} Therefore, in each period there is a mass of two consumers in the market, a mass of one composed of young consumers, and a mass of one composed of old consumers. In each period, each consumer can use one unit of product \(A\), or one unit of product \(B\), or neither. No consumer has any additional gain from using more than one unit from either product in each period.

Each consumer's preferences are characterized by the triple \((\mu_A, \mu_B, x)\), which is fixed throughout their life. The three elements of the triple are independent in the population. The elements \(\mu_A\) and \(\mu_B\) measure the gross benefits received from products \(A\) and \(B\), respectively, that are learned through experience. The consumer only learns \(\mu_i\) for product \(i\) after trying (and buying) product \(i\). The marginal prior cumulative distribution function for \(\mu_i\) is common across all consumers, with support \([\mu, \bar{\mu}]\).\textsuperscript{11} Except for the possible mass points at the extremes of the distribution \((\mu\) and \(\bar{\mu})\), \(\mu\) is assumed to be uniformly distributed.

\textsuperscript{9} Bergemann and Välimäki (1997) consider heterogeneous consumers, only one generation of consumers, and uncertain valuation regarding only one of the two competing firms. Vettas (1998) considers homogeneous consumers and equal valuation for all firms in a free-entry industry. Shapiro (1983) studies the monopoly case. Cambini and Vives (1996, 1999) consider the competition case in which consumers learn about the quality of the products through their market shares, and where consumers live only for one period. Moscarini and Ottaviani (2001) consider the case of competition for a buyer without horizontal differentiation. Another important economic application of the framework presented here is matching models of the labor market with wage renegotiation, and where human capital is seen as information (see Felli and Harris, 1996).

\textsuperscript{10} The role of this assumption, and the case where consumers live for more than two periods, is discussed in Section 5.2.

\textsuperscript{11} The assumption of common priors for all consumers is taken for simplicity, but may not hold in several markets. In fact, in this model it generates a greater differentiation between the products after than prior to the experience with one of the products. The idea that the degree of differentiation changes with experience is robust; the idea that differentiation increases with experience depends critically on the common priors assumption.
In Section 4, I consider first the case with no mass points. It turns out, as shown below, that this case does not have any dynamic effects as the consumers that try the product first and have a good fit exactly cancel out with the consumers that try the product first and have a poor fit. I then consider the case with a mass point at the top of the distribution, \( \bar{\mu} \). This is the case where the probability distribution over the experience attribute is negatively skewed. This negative skewness could be justified by risk aversion with respect to the physical fit of the experience attribute.\(^1\) This case is also the one in which this experience goods framework becomes similar to a switching costs model, and therefore, this framework can then be seen as endogenizing the switching costs. One possible interpretation of this distribution is that with some probability \( \alpha \) the product perfectly fits the consumer, that is, it works at the maximum and certain level (\( \bar{\mu} \)). When the product does not fit the consumer then there are several degrees of misfit that are uniformly distributed between close-to-perfect fit (\( \bar{\mu} \)) to serious misfit (\( \mu \)). The cumulative distribution over valuations is then \( F(\mu) = (1 - \alpha) \frac{\mu - \mu}{\bar{\mu} - \mu} \) for \( \mu \leq \mu < \bar{\mu} \) and \( F(\mu) = 1 \) for \( \mu = \bar{\mu} \). Suppose also that \( \alpha < \frac{1}{2} \); then the median is strictly smaller than \( \bar{\mu} \). In Section 5, I briefly describe what happens when the mass point is at the bottom of the distribution \( \mu \). In what follows I concentrate on the case with a mass of \( \alpha \) at the top of the distribution (except for Section 4.1); the case with no mass points is just making \( \alpha = 0 \).

Let \( k = \frac{1 - \alpha}{\bar{\mu} - \mu} \) be the density of \( \mu \) for \( \mu \in (\mu, \bar{\mu}) \). The mean of \( \mu_i \) is \( E\mu_i = \mu + \frac{1 + \alpha}{2} (\bar{\mu} - \mu) \) that is lower than the median, \( m = \mu + \frac{\bar{\mu} - \mu}{2(1 - \alpha)} \). This feature captures the idea that more than one half of the consumers experience first a product with a value exceeding the expected value of the alternative product. If \( \alpha = 0 \), exactly half of the consumers experience first a product with value exceeding the expected value of the alternative product.

The element \( x \) is known by each consumer before purchasing any product and represents a preference between products A and B. This is related to the characteristics of a product that can be inspected before purchase. The cumulative distribution of \( x \) is uniform on \([0, 1]\), where \( x \) can represent the distance from product A and \((1 - x)\) the distance from product B.

\(^{12}\) If \( \mu \) represents a utility of the physical fit of the product, then it can be shown that greater risk aversion over the physical fit (i.e., greater concavity of a function \( \mu \) of the physical fit) decreases the skewness of \( F(\mu) \). Furthermore, for infinitely large risk aversion over the physical fit, any skewness measure is at its lowest level, which is negative.
The net benefit of buying product $A$ in period $t$ is defined by
$U(\mu_A, x) = \mu_A - \tau x - p_t^A$. The net benefit of buying product $B$ is defined by
$U(\mu_B, x) = \mu_B - \tau (1 - x) - p_t^B$. The parameter $\tau$ can be seen as representing a per unit cost of “traveling” to the product being purchased. The variables $p_t^A$ and $p_t^B$ are the prices charged in period $t$ by firms $A$ and $B$, respectively.

The relative size of $\bar{\mu} - \mu$ with respect to $\tau$ helps determine the relative importance of the experience of the product in the total consumer valuation in relation to the product characteristics that can be inspected before purchase. If $\tau$ is small in comparison to $\bar{\mu} - \mu$, the most important part of consumer valuation of a product has to do with what is learned when trying it. Throughout the paper it is assumed that $\bar{\mu} - \mu > 2\tau$ so that what is learned through experiencing a product can always be more important than what can be inspected prior to purchase, but $\frac{\bar{\mu} - \mu}{1 - \tau}$ is not much greater than $2\tau$ such that it is never profitable for a firm to deviate and charge a high price yielding zero demand from the new consumers.\footnote{This possibility would generate equilibria in mixed strategies. The conditions for the pure-strategy equilibrium analyzed are satisfied for $\alpha$ close to zero.}

If a consumer has a very poor experience with a product he chooses to try the other product. Similarly, if a consumer has a very good experience with a product, he chooses to purchase that product in the next period. The consumers are assumed to be risk neutral with respect to their net benefit of buying either product.\footnote{If the net benefit of the purchase of this product is small in comparison to the consumers’ income, then the consumers are locally risk-neutral with respect to the purchase of this product (see Rabin, 2000). All the messages of this paper should also carry through in a model in which consumers are risk averse with respect to their net benefit of buying a product. In such a model there would be an even greater advantage of the product that was bought first. Note also that, even for specific utility functions, such a model would become too complex in order to formally obtain several of the results presented here.} The expected value of the gross benefit of either product, $\mu_A$ or $\mu_B$, is assumed high enough so that in equilibrium all consumers purchase one of the products in each period.

The role of the heterogeneity in $x$ is for one to be able to compare the relative impact of a search attribute with the experience attribute. Note also that with no heterogeneity in $x$ (or $\tau = 0$), the model becomes less tractable with mixed strategy equilibria, as the product is homogeneous for the new consumers. In fact, in order for there to be a pure-strategy equilibrium we have to assume, as argued above, $\tau$ to be bounded away from zero such that it is never optimal to deviate and to charge a high price such that the demand from new consumers is zero.

The assumption of independence between the gross benefits $\mu_A$ and $\mu_B$ in the population is just to guarantee that a consumer does
not learn about one brand while trying the other brand. This is the extreme case where experience with a product does not give any information about the benefit provided by other products. The independence between $x$ and either $\mu_A$ or $\mu_B$ in the population is just to have no interaction between the observable characteristics of a product and the characteristics that are only learned through experience. The main messages of the results in the paper would still go through with some interaction between these two types of product characteristics.

The lifetime net benefit of a consumer is the discounted sum of the net benefits of the two periods in which the consumer is in the market with discount factor $\delta_C$, with $0 \leq \delta_C < 1$.

In each period $t$, firms choose simultaneously the prices to be charged, $p_t^A$ and $p_t^B$. Firms want to maximize the expected discounted value of their profits, using a discount factor $\delta_F$, with $0 \leq \delta_F < 1$. The discount factors $\delta_C$ and $\delta_F$ are considered distinct in order to be able to study the role of each of them in the market equilibrium. The case of $\delta_C = \delta_F$ is immediate from the results below.

I am interested in the Markov perfect equilibria (Fudenberg and Tirole, 1991, p. 513) of this game, that is, equilibria in which each firm’s strategy in each period depends only on the payoff-relevant state variables in that period. In this particular game the payoff-relevant state variables in each period are the stocks of previous customers of each of the firms still in the market in that period. As we argue below, this reduces to type $x$ of a consumer that is still in the market and was indifferent between products $A$ and $B$ in the previous period.

For possible benchmarks consider the case where products (or tastes) change completely from period to period and the case where consumers are fully informed about the gross benefits of the competing products.

When products change completely from period to period, the expected gross benefits $E\mu_i$ cancel out, and the equilibrium becomes exactly like in the traditional Hotelling case with prices equal to $\tau$ and profit for each firm equal to $\tau$. When consumers are fully informed about the gross benefits of the competing products, demand for each firm as a function of the prices is less straightforward because one has to account for all possible combinations $(\mu_A, \mu_B, x)$ that choose either firm. We can obtain the equilibrium prices to be characterized by (proofs are presented in the Appendix).

$$p^i = \frac{\tau}{\alpha^2 + 2k\tau - \tau^2k^2}, \quad \text{for} \quad i = A, B. \quad (1)$$
Given $\tilde{\mu} - \mu \equiv \frac{1 - \alpha}{k} > 2 \tau$, as assumed above, it can be seen that the equilibrium price under full information is higher than the equilibrium price when the products change completely from period to period, $\tau$. Moreover, as expected, the equilibrium prices are increasing in the two differentiation measures, $\frac{1}{k}$ and $\tau$. Finally, when $\alpha$ increases the equilibrium prices decrease because there is a greater mass of consumers that value the firms equally (on the experience dimension) and at the top of the distribution, $\tilde{\mu}$.

3. Preliminaries

In this section, we start considering the case of experience goods where consumers learn about their fit with the products that they experience.

3.1 Consumer Behavior of the Old Consumers

Consider the case of a consumer in period $t$, after having bought product $A$ in the previous period (having bought product $B$ is the symmetric case). In this period $t$ this consumer compares the net benefit of purchasing product $A$, $\mu_A - \tau x - p^A_t$, with the expected net benefit of purchasing product $B$, which is $E \mu_B - \tau (1 - x) - p^B_t$, where $E$ is the expected value operator. Learning $\mu_i$ of the product being purchased in the previous period generates another dimension of differentiation between products. Note that if the experience in the first period was good, that is, if $\mu_A$ is high, the consumer will buy product $A$ in his second period in the market. If, on the other hand, there was a poor experience, low $\mu_A$, the consumer chooses product $B$. In fact, the marginal consumers are characterized by $\mu_A = E \mu_B - \tau (1 - 2x) + p^A_t - p^B_t$. Consumers with greater $\mu_A$ or lower $x$ choose product $A$ in the second period. Consumers with lower $\mu_A$ or greater $x$ choose product $B$.

Consider then the demands of the consumers that are in their second period in the market in period $t$. Suppose that the market share of firm $A$ in period $(t - 1)$ among the consumers that were in their first period in the market in period $(t - 1)$ is $\bar{x}_{t-1}$. As shown below, the consumers that chose product $A$ in period $(t - 1)$ are those with type $x < \bar{x}_{t-1}$. Because of the assumption that the market is covered, the market share of firm $B$ among those consumers was $(1 - \bar{x}_{t-1})$. Note also that $\bar{x}_{t-1}$ is the type of the marginal consumer born in period $(t - 1)$ for product $A$ (or for product $B$) in period $(t - 1)$.

The demand in period $t$ for firm $A$ from the consumers that bought product $A$ in period $(t - 1)$ is then composed of the consumers that had a good experience in period $(t - 1)$, which is $\int_{0}^{\bar{x}_{t-1}} [1 - F(E \mu + p^A_t - p^B_t - \tau (1 - 2x))] dx$. Similarly, the demand in period $t$ for firm $A$ from the old consumers that bought product $B$ in period $(t - 1)$ is composed of
the consumers that had a bad experience in period \((t-1)\), which is 
\[
\int_{x_{t-1}}^{1} F(E\mu + p^B - p^A - \tau(2x - 1)) \, dx.
\]

The total demand for firm A from the consumers that entered the market in period \((t-1)\) is then the sum of these two terms, which yields, as shown in the Appendix, 
\[
\bar{x}_{t-1} \alpha^2 + \frac{1-\alpha^2}{2} + k(p^B_t - p^A_t).
\]

The total demand for firm B from this type of consumers is 
\[
(1 - \bar{x}_{t-1}) \alpha^2 + \frac{1-\alpha^2}{2} + k(p^A_t - p^B_t).
\]

Note that if there is no skewness, \(\alpha = 0\), the demand from the old consumers does not depend on the previous period market shares, that is, there are no dynamic effects. This is because the fraction of consumers that have a product fit above the expected value is exactly equal to the fraction of consumers that have a product fit below the expected value. Note that for the experience goods model to recover the effect in the switching costs model, that a greater market share in the past leads to a greater demand in the future, we need that the probability distribution over the experience attribute be negatively skewed \((\alpha > 0\) in this case).

### 3.2 Consumer Behavior of the First-Time Consumers

When making the decision of which product to buy in their first period in the market, consumers are able to foresee the second period prices, and how these should affect the consumer decisions.

One useful result that can be obtained is that if one consumer of type \(x\) chose product A in his first period in the market, then any consumer of type \(\hat{x} < x\) also chose product A in that period. In order to see this consider the decisions by the consumers in their first period in the market. Suppose it is period \(t\). A consumer with type \(x\) has expected value of lifetime net benefits of buying product A of 
\[
E\mu_A - \tau x - p^A_{t+1} + \delta_c E[\max\{\mu_A - p^A_{t+1} - \tau x, E\mu_B - p^B_{t+1} - \tau(1-x)\}],
\]

where \(p^i_t\) represents the price charged by firm \(i\) in period \(t\). Similarly, the expected value of lifetime net benefits of buying product B is 
\[
E\mu_B - \tau(1-x) - p^B_{t+1} + \delta_c E[\max\{\mu_B - p^B_{t+1} - \tau(1-x), E\mu_A - p^A_{t+1} - \tau x\}].
\]

Subtracting the latter from the former, one obtains

\[
\begin{align*}
&\bar{p}^B_t - \bar{p}^A_t + \tau(2x - 1) + \delta_c F(E\mu - \tau(1-2x) + p^A_{t+1} - p^B_{t+1}) \\
&\times [E\mu - p^B_{t+1} - \tau(1-x)] + \delta_c \int_{E\mu-\tau(2x-1)+p^A_{t+1}}^{\bar{\mu}} k[\mu - p^A_{t+1} - \tau x] \, d\mu \\
&+ \delta_c \alpha(\bar{p}^A_t - \tau x) - \delta_c F(E\mu - \tau(2x-1) + p^B_{t+1} - p^A_{t+1}) \\
&\times E\mu - p^A_{t+1} - \tau x - \delta_c \int_{E\mu-\tau(2x-1)+p^B_{t+1}}^{\bar{\mu}} k \mu - p^B_{t+1} - \tau(1-x) \, d\mu \\
&- \delta_c \alpha \bar{p}^B_t - p^B_{t+1} - \tau(1-x) \\
&- \delta_c \alpha \bar{p}^A_t - p^A_{t+1} - \tau x
\end{align*}
\]

(2)
under the assumption that in equilibrium, as shown below, consumers with a sufficiently positive experience continue to buy the tried product, while consumers with a sufficiently negative experience try the alternative product.

Differentiating with respect to $x$ one obtains (as shown in the Appendix)

$$-2\tau + \delta c \tau \left[ 2F \left( E\mu - \tau (1 - 2x) + p_{t+1}^A - p_{t+1}^B \right) + 2F \left( E\mu - \tau (2x - 1) + p_{t+1}^B - p_{t+1}^A \right) - 2 \right]$$

which is negative. Therefore, if a consumer with type $x$ in his first period in the market chooses to purchase product $A$, then any other consumer with type $\tilde{x} < x$ also chooses to purchase product $A$.

In order to obtain the marginal consumer with type $\tilde{x}$ that is indifferent between buying product $A$ or product $B$ in period $t$ and entering the market in this period, we make equation (2) equal to zero, with $x$ being substituted by $\tilde{x}$ and where $p_{t+1}^A$ and $p_{t+1}^B$ are functions of $\tilde{x}$. This can then be reduced (see Appendix) to

$$p_t^B - p_t^A + \tau (1 - 2\tilde{x}) + \alpha^2 \delta c \left[ p_{t+1}^B - p_{t+1}^A + \tau (1 - 2\tilde{x}) \right] = 0. \quad (3)$$

Note that, if there is no skewness, $\alpha = 0$, the demand from the first-time consumers does not depend on the expectation about the future prices. This is because consumers are as likely to have an experience above or below the expected value, and therefore, are as likely to buy one product or the other in the next period.

In the computation of the Markov perfect equilibria I restrict attention to equilibria in affine strategies, that is, $p_t^A = p^A(\tilde{x}_{t-1})$ and $p_t^B = p^B(\tilde{x}_{t-1})$, where the functions $p^A()$ and $p^B()$ are linear functions of $\tilde{x}_{t-1}$, and, as noted above, $\tilde{x}_{t-1}$ summarizes the payoff-relevant state variables in period $t$. This focus on affine strategies is further discussed below.

Then, we can write $p_{t+1}^A - p_{t+1}^B = a + b \tilde{x}$, where $a$ and $b$ are real numbers to be computed in the equilibrium. We can then rewrite (3) as

$$\tilde{x} = \frac{y + p_t^B - p_t^A}{2y}, \quad (4)$$

where $y \equiv \tau + \delta c (\tau - a) \alpha^2$ and $b = -2a$ because $p_{t+1}^A = p_{t+1}^B$ for $\tilde{x} = \frac{1}{2}$ in a symmetric equilibrium. Finally, note that the demand in period $t$ for firm $A$ from the consumers entering the market in this period is $\tilde{x}$, and the demand for firm $B$ from this type of consumers is $(1 - \tilde{x})$. 
Total demand in period $t$ for firm $A$ given $\bar{x}_{t-1}$ is then $\frac{y + p_t^B - p_t^A}{2y} + \bar{x}_{t-1} \alpha^2 + \frac{1 - \alpha^2}{2} + k(p_t^B - p_t^A)$.

### 3.3 Firm's Problem

The problem for firm $A$ in period $t$ can then written as the right-hand side of

$$W_A(\bar{x}_{t-1}) = \max_{p_t^A} p_t^A \left[ \frac{y + p_t^B - p_t^A}{2y} + \bar{x}_{t-1} \alpha^2 + \frac{1 - \alpha^2}{2} + k(p_t^B - p_t^A) \right] + \delta_F W_A \left( \frac{y + p_t^B - p_t^A}{2y} \right),$$

(5)

where $W_A(x)$ is the net present value of profits for firm $A$ from period $t$ on if the marginal consumer buying the product in $(t - 1)$ and living in period $t$ had type $x$. The solution to the right-hand side gives $p_t^A = p^A(\bar{x}_{t-1})$. Similar expressions can be written for firm $B$ with functions $W_B(x)$ and $p^B(\bar{x}_{t-1})$.

As stated above, I am looking for Markov perfect equilibria where the price strategies $p^A(\bar{x}_{t-1})$ and $p^B(\bar{x}_{t-1})$ are affine in $\bar{x}_{t-1}$, which is already assumed in the construction of the demand for product $A$ in (4). Note also that if $W_A(x)$ on the right-hand side of (5) is quadratic, then $W_A(x)$ on the left-hand side results indeed quadratic in $x$, and the equilibrium strategies are indeed affine in $x$, and the demand is indeed linear in $x$ and in the prices. There may be equilibria that are not affine, but all the equilibria of any finite-horizon version of this game are in affine strategies. Furthermore, when such a finite horizon goes to infinity, the equilibrium of the finite game have, as a limit, the equilibrium in affine strategies that is presented here (the infinite game).\(^{15}\)

Given that the firms are symmetric, we are looking for a symmetric equilibrium with $W_A(x) = W_B(1 - x) = W(x)$, $\forall x$. Denote also

$$W(x) = c + \bar{a} x + \bar{e} x^2,$$

(6)

$$p^A(x) = s + g x.$$  

(7)

From the solution to (5) and the corresponding problem for firm $B$, and from equalizing in (5) the constant term, the term in $\bar{x}_{t-1}$, and the term in $\bar{x}_{t-1}^2$, we can obtain $a, b, c, \bar{a}, \bar{e}, s$, and $g$. Throughout I also define $e \equiv \frac{\bar{y} \bar{c}}{y}$, $d \equiv \delta_F \bar{d}$, and $r \equiv 3 + 6yk - 2e$.

15. I could not find other equilibria in affine strategies in the infinite horizon model.
4. Results

4.1 Symmetric Distribution of Experience Attribute, $\alpha = 0$

Consider first the case in which the distribution over $\mu$ is symmetric, that is, $\alpha = 0$. As discussed above, in this case there are no dynamic effects, and the Markov perfect equilibrium reduces to a static Nash equilibrium in every period. In each period $t$, demand for firm $i$ from the first-time consumers is $\frac{1}{2} + \frac{p_i^i - p_i^j}{2\tau}$, with price sensitivity decreasing in $\tau$, and independent of $k$ or of the discount factors. Demand for firm $i$ from the old consumers is $\frac{1}{2} + k(p_i^j - p_i^i)$, with lower price sensitivity than the demand from the first-time consumers, given the assumption $k > \frac{1}{2\tau}$. The equilibrium prices can be computed to be $\bar{p} = \frac{2\tau}{1 + 2\tau k}$, this being also the equilibrium profits per period, as each firm has a demand of mass one in each period.

The equilibrium prices and profits are increasing in the differentiation parameters for observable ($\tau$), or experienced product characteristics ($\frac{1}{k}$). In each generation, a fraction $\frac{1 - k}{2}$ of consumers change products from their first to their second period in the market. Note that this fraction is decreasing in the importance of the search attribute ($\tau$), and increasing in the importance of the experience attribute ($\frac{1}{k}$). Note that this increased product differentiation due to experience does not have an immediate parallel in a switching costs model (e.g., Beggs and Klemperer, 1992). Here, experience with one product creates a greater heterogeneity in the relative preference for both products (independent of which product was tried first when $\alpha = 0$). In a switching costs model, consumers have a greater relative preference for the product that they try first.

4.2 Negatively Skewed Distribution of Experience Attribute, $\alpha > 0$

Consider now the case in which the probability distribution of the experience attribute $\mu$ is negatively skewed, $\alpha > 0$. This is the case where there are dynamic effects in the market that are similar to the effects of switching costs. We start by characterizing the Markov perfect equilibrium, and then present results on the price sensitivity of first-time consumers, on market dynamics, and on the equilibrium steady-state prices and profits.

4.2.1 Markov Perfect Equilibrium

The following proposition characterizes some of the equilibrium variables that are used below. The reader less interested in the technical aspects of the equilibrium can jump directly to the next subsection.
**TABLE 1.**

**Characterization of Equilibrium for Different \( \alpha \)**

\[
\begin{array}{cccc}
\alpha & r^* & y & \frac{2a^2}{r^*} & \bar{p} \\
0 & 3.60 & 1.00 & .000 & 1.667 \\
.1 & 3.61 & 1.01 & .006 & 1.671 \\
.2 & 3.62 & 1.04 & .022 & 1.685 \\
.3 & 3.65 & 1.09 & .049 & 1.712 \\
.4 & 3.69 & 1.17 & .087 & 1.755 \\
.5 & 3.75 & 1.28 & .133 & 1.820 \\
\end{array}
\]

**Proposition 1:** In the Markov perfect equilibrium, \( r \) is the only solution of \( r^4 - r^3 [2\alpha c^2 + 6k(1 + \delta_c \alpha^2) + 3] + 6\delta_c \alpha^2 r^2 + r[4\alpha^2 \delta_c + 8\delta_c \delta_F \gamma (1 + \delta_c \alpha^2)] = 0 \) that is greater than \( 2\alpha^2 \). We also have \( y = \frac{\tau}{1 + 2\delta_c \alpha^2}, e = \frac{3 + 6k - r^2}{2}, g = \frac{2y^2}{r}, a = -g, \tilde{x}_i - \frac{1}{2} = \frac{2y^2}{r}, \) and \( \mu^* = \frac{2y - d - yu}{1 + 2yk} + \frac{\alpha^2 y + 2y^2 \tilde{x}_{i-1}}{r} \). For \( \alpha \to 0 \), we have \( r \to 3 + 6k, y \to \tau, e \to 0, d \to 0, a \to 0, b \to 0, s \to \frac{2\tau}{1 + 2y}, \) and \( g \to 0 \).

See Table 1 for values of \( r^* \) and \( y \) as a function of \( \alpha \). Note that \( \mu = \frac{1}{k} \frac{u}{q} \) and \( \mu \) is assumed high enough such that the market is fully covered.

### 4.2.2 Price Sensitivity of First-Time Consumers

From the demand by the first-time consumers, \( \frac{y + p^B - p^A}{2y} \) for firm \( A \), it is clear that these consumers are less price sensitive the greater \( y \) is, where price sensitivity is defined as the absolute value of the derivative of the demand for one firm with respect to that firm’s price. The comparative statics of \( y \) with respect to the different parameters yields the following result.

**Proposition 2:** First-time consumers are less price sensitive the greater \( \frac{1}{k}, \tau, \delta_c, \) and \( \delta_f \) are. For small \( \alpha \) first-time consumers are less price sensitive the greater \( \alpha \) is.

Increasing the importance of the observable differentiation, \( \tau \), decreases, as expected, consumer price sensitivity. Increases in \( \alpha, \delta_c, \) and \( \frac{1}{k} \) make the first-time consumers less price sensitive.16 The marginal first-time consumers foresee that by choosing one product they get a higher expected price in the next period because they are more likely to buy the product that they bought first, and that the firm is going to charge a

---

16. I could not find parameter values where the result on \( \alpha \) would reverse when \( \alpha \) was not small.
higher price in the next period. Therefore, consumers become less price sensitive in the first period in which they are in the market. The effect is greater the greater consumers value the future, higher $\delta f$, and the greater the increase in the next period’s price as a result of an increase in the firm’s market share. This increase in the next period’s price is greater the greater the importance of the experience of the product (smaller $k$), and the greater the probability of a perfect product fit (greater $\alpha$). An interesting part of this result is that the firms, even though competing for a new generation of consumers, raise their prices when they have a greater demand in the previous period. Table I presents the value for $y$ as a function of $\alpha$.

When doing comparative statics on changes in $\alpha$ one wants to restrict the price sensitivity of demand from the consumers in the second period in the market to remain constant. The density of the marginal consumers is $k$, and therefore one has to consider decreases in $(\bar{\mu} - \mu)$ when $\alpha$ is increased such that this density does not change. In the distribution of $\mu$ this means that when $\alpha$ is increased there is a transfer of the distribution probability from the lower values (increase in $\mu$) or higher values (reduction of $\bar{\mu}$) to the mass at the top, $\bar{\mu}$.\footnote{The case in which $\alpha$ increases while keeping $\bar{\mu} - \mu$ constant can be directly obtained from the results below. In the distribution of $\mu$ this means that when $\alpha$ is increased there is a transfer of the distribution probability uniformly from all the values of $\mu$ to the mass at the top, $\bar{\mu}$. However, because in this case the price sensitivity of demand from old consumers is reduced, there is a greater force for equilibrium prices to increase with $\alpha$. As it is clear from the results below, this yields a higher net present value of profits for the firms.}

The role of how firms value the future, $\delta f$, is quite interesting. Firms being more forward looking, makes them compete more for the new generations of consumers in anticipation of future gains, which is a force toward lower prices. However, the effect of whether more forward-looking firms leads to higher or lower price sensitivity of first-time consumers, depends on the impact of $\delta f$ on how firms change their prices as a function of having had in the past a greater market share, the equilibrium value of $g$ in equation (7). When firms value the future more, they realize that the potential gains from having had a large market share in the previous period are to be taken advantage of when possible, because the competition for future gains is intense. This makes firms charge higher prices when they have higher market shares, higher $g$, which causes first-time consumers to be less price sensitive. This idea that first-time consumers are less price sensitive is also present in a switching costs model (as in Beggs and Klemperer, 1992), exactly for the same reason that consumers foresee being more likely to buy the same product in the following period (which leads to higher prices,
and greater importance of the observable product characteristic). In this setting of experience goods this effect is captured by the parameter $\alpha$. Note also that in experience goods there is an interesting interaction with the degree of differentiation of the experienced attribute ($\frac{1}{k}$), which does not have a parallel in a switching costs model.

### 4.2.3 Market Share Dynamics

From Proposition 1 the market share dynamics can be written as

$$\dot{x}_t - \frac{1}{2} = -\frac{2\alpha^2}{r} \left( \frac{x_{t-1} - 1}{2} \right),$$

from which we can derive several implications.

**Proposition 3:** The market shares converge to a steady state with a 50–50 division of the market for all starting points. The convergence to steady state goes through oscillating market shares of first-time consumers. The convergence is slower the greater $\delta_F$ is, and the smaller $\tau$, $k$, and $\delta_C$ are. For a small $\alpha$, the convergence is slower the greater $\alpha$ is.

The 50–50 steady state shows that any initial advantage of a firm disappears through time. That is, the dominance of one firm disappears through time. This lack of a dominance effect can be seen as due to entry of new generations of consumers. The convergence to the steady state goes through oscillating market shares of the first-time consumers, with the firm with a larger market share in a given period being the firm with the smaller market share in the next period. The oscillating nature of the market share dynamics is because the firm that just had a high market share will price higher to take advantage of the consumers that got a positive experience with the firm’s product, and this results in a smaller market share of the new generation of consumers. See Figure 1 for an example of the continuous version of the oscillating market shares toward the steady state when firm $B$, the incumbent, starts with a 100% market share.\(^{18}\)

The convergence to steady state becomes slower when either the probability of a perfect product fit is greater (greater $\alpha$) or the experience of the product becomes more important (greater $\frac{1}{k}$). In either of these cases a firm gains more from charging a higher price following a period with a large market share, and this results in a slower convergence to the 50–50 division of the market. Table I presents values for $\frac{2\alpha^2}{r}$ as a function of $\alpha$.

When firms value the future more, greater $\delta_F$, they realize that the potential gains from having had a large market share in the previous

\(^{18}\) See also the discussion in section 9.2 of Vives (1999) on convergence to steady state, speed of convergence, and dominance in other types of dynamic oligopoly models.
period are to be had when they are possible because the competition for future gains is very intense. This makes firms charge higher prices when they have higher market shares, which again causes the convergence to steady state to become slower.

When the importance of the observable characteristics of the products is greater, higher \( \tau \), the convergence to steady state is faster, because relatively, firms have less incentive to price higher when they had a higher market share in the previous period. Similarly, when consumers value the future more, greater \( \delta_C \), they become less price sensitive, as discussed above. Then, a firm charging a higher price in a certain period does not lose too many consumers, which means that we are going to get faster to the 50–50 division of the market.

### 4.2.4 Steady-State Prices and Profits

Consider now the steady-state prices and profits. Note first that because in steady state the demand for each firm has mass one (\( \frac{1}{2} \) from each generation of consumers) the profit per period is equal to the steady-state price, call it \( \bar{p} \). Differentiating (5) at the steady state we can obtain, using the envelope theorem,

\[
\frac{d W(\frac{1}{2})}{d \bar{x}_{t-1}} = \bar{p} \quad \alpha^2 + \frac{1}{2y} + k \quad \frac{dp_i^B}{d \bar{x}_{t-1}} + \delta_F \frac{d W(\frac{1}{2})}{d \bar{x}_t} \frac{\partial p_i}{\partial p_i} \frac{dp_i^B}{d \bar{x}_{t-1}}. \tag{9}
\]
Using \( \frac{d\tilde{x}_t}{dy} = \frac{1}{2} \) and \( \frac{d\tilde{p}_t}{d\tilde{x}_t} = -\frac{2\alpha^2}{r} \), one can obtain
\[
d\tilde{W}(\frac{1}{2}) = \frac{\alpha^2}{r + \delta_F \alpha} [r - 2ky - 1]
\]
The first-order condition for prices at \( \tilde{x}_{t-1} = \frac{1}{2} \) is
\[
\frac{y - \tilde{p}}{2y} + 1 - k\tilde{p} + \delta_F \frac{d\tilde{W}(\frac{1}{2})}{d\tilde{x}_{t-1}} \left[ -\frac{1}{2y} \right] = 0.
\]
Substituting for \( \frac{d\tilde{W}(\frac{1}{2})}{d\tilde{x}_{t-1}} \) one can then obtain the steady-state prices as
\[
\tilde{p} = \frac{2y}{1 + 2yk + \frac{\delta_F \alpha}{r + \delta_F \alpha} [r - 2ky - 1]}. \tag{10}
\]
For \( \alpha \) small one can easily check that the steady-state prices are smaller than the full-information equilibrium prices. Note that when \( \alpha \to 0 \) then \( \tilde{p} \to \frac{2\tau(\mu - \tilde{\mu})}{2\tau + \mu - \tilde{\mu}} \), which is greater than \( \tau \) for \( \mu - \tilde{\mu} > 2\tau \), as assumed above. From (10) one can also derive the following result.

**Proposition 4:** Consider \( \alpha \) small. Then the steady-state prices and profits increase in \( \tau \) and \( \delta_C \), and decrease in \( \delta_F \) and \( k \). The steady-state prices and profits increase in \( \alpha \) if and only if \( \delta_C > \frac{2}{\delta_F} \).

As expected, greater differentiation in either observable (\( \tau \)) or experienced product characteristics (\( \frac{1}{2} \)) result in higher prices.

When the probability of a perfect product fit is greater (greater \( \alpha \)) steady-state prices increase if \( \delta_C \) is close to \( \delta_F \). There are two conflicting effects of the probability of a perfect product fit on the equilibrium prices: On one hand, when consumers are forward-looking (reflected in \( \delta_C \)) they become less price sensitive because they realize that they will be charged a higher price in the next period (see Proposition 2). This is a force toward higher equilibrium prices. On the other hand, when firms are forward-looking (reflected in \( \delta_F \)) firms compete more for the future profits of having a higher market share today, which is a force toward lower equilibrium prices. This effect is the Bertrand supertrap effect discussed in Cabral and Villas-Boas (2005). In this model, the effect toward higher prices dominates for similar discount factors because firms realize that in the next period they will not only be able to take advantage of the market share gained in the current period, but they will also have to compete for market share in future periods. This reduces the impact of forward-looking firms competing for market share, and allows firms to end up with higher prices when the probability of a perfect product fit is greater. Note that if consumers are less forward-looking than the firms,
\(\delta_C < \frac{2}{3} \delta_F\) in the model, then the effect of \(\alpha\) is reversed, and by the effects argued above, a greater informational advantage leads to lower profits for the firms.

In summary, when consumers value the future more, greater \(\delta_C\), they become less price sensitive in their first period in the market, as shown above, and this leads to higher steady-state prices and profits. When firms value the future more, greater \(\delta_F\), steady-state prices decrease because firms value more the future gains of having greater market share, and therefore, compete more for market share. These effects of the consumers’ and firms’ discount factors are also present in a model with switchings, as shown, for example, in Beggs and Klemperer (1992). Note, however, that the switching cost parameter in a model with switching costs does not have an immediate corresponding parameter in the experience goods model. The parameter in the experience goods model that plays the closest role in switching costs is the degree of negative skewness of the distribution of the experience attribute, \(\alpha\), which yields the effect that greater market shares in the past lead to a greater market share today. In the experience goods model this parameter interacts with the degree of differentiation of the experience attribute (\(\xi\)), for which there is no corresponding parameter in a switching costs model.

5. Extensions

5.1 Positively Skewed Distribution of Experience Attribute

For completeness, consider now the case in which the probability distribution of the experience attribute \(\mu\) is positively skewed, in particular with a mass point now at the bottom of the distribution, \(\mu\). The analysis of this case can be done similarly to the discussion above. Contrary to the case of a mass point at the top, now the demand for a product from the old consumers is greater the smaller the market share among those consumers in the previous period. That is, contrary to the switching costs framework, a greater market share in the past leads to a smaller market share today. This is because more than one half of the consumers trying one product have an experience below the expected experience of the untried product. This then leads to a less intensive competition for market share, and higher prices. As above, first-time consumers are less price sensitive the greater the degree of differentiation in the market (from observable or experienced attributes), the more consumers are forward-looking, and the greater is the degree of skewness in the probability distribution of the experience attribute. Because it is now the firm with a smaller previous market share that charges the higher
price, the convergence of market shares of the first-time consumers to the steady state is now monotonic (instead of oscillating).

5.2 Longer-Lived Consumers

Because in the model above consumers live only for two periods (this could also be seen as consumers completely changing tastes after two periods) there is no role for experimentation, that is, for trying different products in order to choose in the future the product that provides the best fit. Note also that because of the consumer life being only for two periods there is no room for consumers being dropped off the market for not having good experiences with both products. However, the current structure, being quite tractable, still captures several important aspects of markets with experience goods: First, after experiencing a good fit, consumers find it too costly to experiment further. Second, because tastes and products being offered change through time, any possible gains from experimentation can be greatly diminished. Third, after a large number of consumers have a positive experience with a firm, that firm charges a higher price to exploit those consumers’ positive experience.

It is, however, important to think about what would happen in the market if consumers lived for more than two periods. First, in such a market some consumers would purchase after having experimented both products. This means that there would be more information in the market, and consequently, more product differentiation (as in the benchmark above of complete information), which is a force toward higher prices. Second, some consumers when trying the first product have such a good experience, so that they choose not to try any other product and stick with the first product tried through their lives. This may give some incentive for firms to charge higher prices to take advantage of the existence of these customers. Third, several consumers will try both products to better learn which product may fit them better. That is, this demand for “learning” may give the firms some extra market power, which is another force toward higher prices. It is as if the products became complements through time: If a consumer buys one product in one period, the consumer “must” buy the other product in the next period. This would lead firms with a large market share in the past among the first-time consumers to have a lower market share today, as

19. See Bergemann and Välimäki (1996) for the case of experimentation with homogeneous consumers and constant tastes, and where all firms are able to observe the consumers’ experiences. For the case of one-sided experimentation (learning about demand) see Aghion et al. (1991) and Keller and Rady (1999).

20. See Iyer and Soberman (2000) for the case, without experimentation, where product attributes change through time.
in the case of positive skewness discussed above. In summary, allowing for a longer-lived consumer may lead to higher equilibrium prices, and one may need the probability distribution of the experience attribute to be more negatively skewed in order to generate the result that greater market shares in the past yield greater future market shares.

5.3 Market Growth

An interesting extension of the model is to see what would be the implication of market growth, a generation entering the market in period \( t + 1 \) having more consumers than a generation entering the market in period \( t \), for all \( t \). With market growth the advantage of charging higher prices on the previous generation is reduced, and of firms gaining market share today becomes more important. This then becomes a force toward lower prices.

6. Conclusion

This paper considers the dynamic competition implications of experience goods. I investigate an infinite-horizon model with overlapping generations of consumers.

For similar discount factors for firms and consumers, one finds that steady-state prices and profits are higher the greater the informational differentiation effects, that is, the greater the probability of perfect product fit. The steady-state prices are also found to increase in consumer patience, and in the importance of the experience in the \textit{ex post} valuation of the product. Prices decrease in the degree to which firms value the future. One also finds that the market share dynamics for the first-time consumers are oscillating: the firm with the greater market share in one period has the smaller market share in the next period. Several comparative statics with respect to the speed of convergence to the steady state are also presented. The paper also illustrates the importance of considering several possible shapes of the probability distribution of the experience attribute if one is considering doing empirical analysis in a market where experience effects are important.

One important issue not considered above is that firms have increasingly the ability to charge different prices depending on what the consumers purchased in the past. This ability to price discriminate based on the consumers’ past behavior could have important implications here, as firms could potentially charge higher prices from the consumers having an informational advantage for the firm’s product. It would be interesting to investigate what would happen to a market with experience effects if the firms have this ability to price discriminate,
and compare the results of such analysis with the case considered in Chen (1997) and Taylor (2003), where firms can practice behavior-based price discrimination in a market with switching costs.²¹

APPENDIX

Proof of Equilibrium Prices under Full Information. Demand for firm A is composed of all consumer types \((\mu_A, \mu_B, x)\) satisfying \(\mu_A - \tau x - p^A > \mu_B - \tau (1 - x) - p^B\). For \(x\) satisfying \(p^A - p^B > \tau (1 - 2x)\), which is equivalent to \(x > \frac{\tau + \mu_B - \mu_A}{2\tau}\), demand for firm A is

\[
1 - \int_\mu^{\mu_A - p^A + p^B + \tau (1 - 2x)} k \int_{\mu_A - p^A + p^B + \tau (1 - 2x)}^{\mu_B + p^B - p^A} k \, d\mu_A \, d\mu_B.
\]

Integrating over \(x\) one obtains the total demand for firm A, \(D^A\), as

\[
D^A = \int_{\mu}^{\bar{\mu}} \int_{\mu + p^A + p^B + \tau (1 - 2x)}^{\bar{\mu} + p^B - p^A} k^2 \, d\mu_A \, d\mu_B \, dx
+ \left[ 1 - \int_\mu^{\mu_A - p^A + p^B + \tau (1 - 2x)} \, d\mu_B \right] \left[ \int_{\mu_A - p^A + p^B + \tau (1 - 2x)}^{\mu_B + p^B - p^A} \, d\mu_A \right] \int_{\mu}^{\bar{\mu}} \, dx.
\]

(A1)

Firm A then maximizes its profit, \(\max_{p^A} p^A D^A\). Using the first-order condition of this maximization and the symmetry \(p^A = p^B\) one obtains the equilibrium price to be as stated in (1).

Derivation of Demand of Old Consumers. The demand of the old consumers is

\[
\int_0^{\bar{\mu}} \left[ 1 - F(E\mu + p^A - p^B - \tau (1 - 2x)) \right] \, dx
+ \int_{\bar{\mu}}^1 F(E\mu + p^B - p^A - \tau (2x - 1)) \, dx.
\]

Given that \(F(\mu) = k(\mu - \bar{\mu})\) and \(E\mu - \bar{\mu} = \frac{1 + \tau}{2} (\bar{\mu} - \mu)\) we have that this demand is equal to

²¹ For competition in markets with behavior-based price discrimination, but without switching costs, see Villas-Boas (1999) and Fudenberg and Tirole (2000).
Dynamic Competition with Experience Goods

\[ \ddot{x}_{t-1} - k \left[ \left( \frac{1 + \alpha}{2} (\bar{\mu} - \mu) + p_t^A - p_t^B - \tau \right) \dot{x}_{t-1} + \tau \ddot{x}_{t-1} \right] \]

\[ + k \left[ \left( \frac{1 + \alpha}{2} (\bar{\mu} - \mu) + p_t^B - p_t^A + \tau \right) (1 - \dot{x}_{t-1}^2) - \tau (1 - \dot{x}_{t-1}^2) \right], \]

which is equal to

\[ \ddot{x}_{t-1} + k \frac{1 + \alpha}{2} (\bar{\mu} - \mu)(1 - 2\dot{x}_{t-1}) + k(p_t^B - p_t^A), \]

which reduces to the expression in the text after noting that \( k = \frac{1 - \alpha}{\bar{\mu} - \mu}. \)

**Differentiation of (2) with Respect to \( x \).** Denoting \( p_{t+1}^A - \tau x \) as \( \bar{A} \), and \( p_{t+1}^B - \tau (1 - x) \) as \( \bar{B} \), the derivative of (2) can be written as

\[
-2\tau + 2\tau \delta_c k(E_{\mu} - \bar{B}) + \delta_c \tau F(E_{\mu} + \bar{A} - \bar{B}) - \delta_c \tau[1 - F(E_{\mu} + \bar{A} - \bar{B})] \\
-2\delta_c \tau k(E_{\mu} - \bar{B}) + 2\delta_c \tau k(E_{\mu} - \bar{A}) + 2\tau \delta_c \tau F(E_{\mu} + \bar{B} - \bar{A}) \\
-\delta_c \tau[1 - F(E_{\mu} + \bar{B} - \bar{A})] - 2\delta_c \tau k[E_{\mu} - \bar{A}],
\]

(A2)

from which one can obtain the expression in the text.

**Derivation of Equation (3).** Making (2) equal to zero, and denoting \( p_{t+1}^A - \tau x \) as \( \bar{A} \), and \( p_{t+1}^B - \tau (1 - x) \) as \( \bar{B} \), we obtain, after solving for the integrals,

\[
p_t^B - p_t^A + \tau(1 - 2x) + \delta_c[E_{\mu} - \bar{B}](1 - \alpha) \frac{E_{\mu} - \mu + \bar{A} - \bar{B}}{\bar{\mu} - \mu} \\
+ \delta_c k \left[ \frac{\mu^2}{2} - \bar{A}\mu \right]_{E_{\mu} + \bar{A} - \bar{B}} + \delta_c \alpha(\bar{\mu} - \bar{A}) \\
- \delta_c [E_{\mu} - \bar{A}](1 - \alpha) \frac{E_{\mu} - \mu + \bar{B} - \bar{A}}{\bar{\mu} - \mu} - \delta_c \alpha(\bar{\mu} - \bar{B}) \\
- \delta_c k \left[ \frac{\mu^2}{2} - \bar{B}\mu \right]_{E_{\mu} + \bar{B} - \bar{A}} = 0.
\]

(A3)

Using \( \frac{E_{\mu} - \mu}{\bar{\mu} - \mu} = \frac{1 + \alpha}{2} \) and \( k = \frac{1 - \alpha}{\bar{\mu} - \mu} \) one can then obtain

\[
p_t^B - p_t^A + \tau(1 - 2x) + \delta_c[E_{\mu} - \bar{B}] \frac{1 - \alpha^2}{2} - \delta_c k[E_{\mu} - \bar{B}](\bar{B} - \bar{A}) \\
- \delta_c [E_{\mu} - \bar{A}] \frac{1 - \alpha^2}{2} - \delta_c k[E_{\mu} - \bar{A}](\bar{B} - \bar{A}) - \delta_c k\bar{B}[E_{\mu} + \bar{B} - \bar{A}] \\
- \delta_c \frac{k}{2} [(E_{\mu})^2 + (\bar{A} - \bar{B})^2 + 2(\bar{A} - \bar{B})E_{\mu}] + \delta_c \frac{k}{2} [(E_{\mu})^2 + (\bar{A} - \bar{B})^2] \\
- 2(\bar{A} - \bar{B})E_{\mu} = 0.
\]

(A4)
Noting that \( \bar{\mu} - E\mu = (\bar{\mu} - \mu) \frac{1-u}{2} \), we can then obtain
\[
\rho_t^B - \rho_t^A + \tau(1-2x) + \delta_c(\bar{A} - \bar{B}) \frac{1-\alpha^2}{2} + \delta_c k(\bar{B} - \bar{A})(\bar{\mu} - \mu) \frac{1-\alpha}{2} = 0
\]
from which one can directly obtain (3).

**Proof of Proposition 1.** From the first-order conditions of the problem (5) and the corresponding problem for firm B one can obtain
\[
\rho_t^A - \rho_t^B = \frac{-2x^2 y_0 + 4x^2 y_1}{3 + 6yk - 2e'},
\]
which yields
\[
a = \frac{-2x^2}{3 + 6yk - 2e}'.
\]
By the definition of \( y \) we can then obtain
\[
\frac{\delta_c \tau \alpha^2 + \tau - y}{\delta_c \alpha^2} = \frac{-2y\alpha^2}{3 + 6yk - 2e'}
\]
which gives a relation between \( y \) and \( e \).

From \( \bar{x}_t = \frac{y + p_t^B - p_t^A}{2y} \), we can then obtain the equation of motion of market shares as
\[
\bar{x}_t - \frac{1}{2} = -\frac{2\alpha^2}{3 + 6yk - 2e'} \left( \bar{x}_{t-1} - \frac{1}{2} \right).
\]
The equilibrium price in period \( t \) for firm A is
\[
\rho_t^A = \frac{2y - d - ey}{1+2yk} + \frac{-\alpha^2 y + 2y\alpha^2 \bar{x}_{t-1}}{3 + 6yk - 2e'},
\]
from which one can directly obtain \( f = \frac{2y \cdot d + ey}{1+2yk} + \frac{-\alpha^2 y}{3 + 6yk - 2e'} \) and \( g = \frac{2y^2}{3 + 6yk - 2e'} \).

In order to compute \( y \) and \( e \), we can use the equality in the terms in \( \bar{x}^2_{t-1} \) in (5) to obtain
\[
\frac{ey}{\delta_c} = \frac{2y\alpha^2}{3 + 6yk - 2e'} \left[ -\frac{2\alpha^2}{3 + 6yk - 2e'} + \alpha^2 - k \frac{4\alpha^2 y}{3 + 6yk - 2e'} \right]
+ ey \frac{4\alpha^4}{(3 + 6yk - 2e')^2}.
\]
Using \( r = 3 + 6yk - 2e \) one can write (A6) and (A9) as
\[
y = h_1(r) = \tau \frac{(1 + \delta_c \alpha^2)r}{r - 2\delta_c \alpha^4}
\]
\[ y = h_2(r) = \frac{r^3 - 3r^2 + 4\alpha^4\delta_F}{6r^2 - 8\alpha^4\delta_F k}, \] (A11)

respectively. Substituting (A10) into (A11) one obtains the following quartic equation on \( r \):

\[
h_3(r) = r^4 - r^3 \left[2\delta_C \alpha^4 + 6\tau k \left(1 + \delta_C \alpha^2\right) + 3\right] + 6\delta_C \alpha^4 r^2 \\
+ r \left[4\alpha^4 \delta_F + 8\delta_F \alpha^4 k \left(1 + \delta_C \alpha^2\right)\right] - 8\delta_C \delta_F \alpha^8 = 0. \] (A12)

After solving for this equation one can then obtain directly \( y, e, a, b, \) and \( g \).

In order to find the appropriate solution \( r^* \) for \( r \), note that \( h_2(r) \) is increasing in \( r \) if and only if \(|r| > 2\sqrt{\delta_F} \alpha^2\). For \( r < -2\sqrt{\delta_F} \alpha^2 \) there is no solution of (A12) because \( h_2(r) \) is negative while \( h_1(r) \) is positive. For \( r > 2\sqrt{\delta_F} \alpha^2 \) there is only one solution because \( h_1(r) \) is decreasing but positive, while \( h_2(r) \) increases from a negative number to infinity. Because for the Markov perfect equilibrium we need that the market share dynamics is mapping into \([0, 1]\), we have \( r \) satisfying \(|r| > 2\alpha^2\), which allows us to conclude that the appropriate solution \( r^* \) for \( r \) satisfies \( r^* > 2\alpha^2 \). See Figure A1 for a graphic representation of \( h_1(r) \) and \( h_2(r) \). In order to obtain the explicit expression for \( r^* \) we follow Birkhoff and MacLane (1996, pp. 107-108). Define \( \gamma_3 \equiv -2\delta_C \alpha^4 + 6\tau k \left(1 + \delta_C \alpha^2\right) + 3 \), \( \gamma_2 \equiv 6\delta_C \alpha^4 \), \( \gamma_1 \equiv [4\alpha^4 \delta_F + 8\delta_F \alpha^4 k \left(1 + \delta_C \alpha^2\right)] \), and \( \gamma_0 \equiv -8\delta_C \delta_F \alpha^8 \). Define also \( Q \equiv \frac{3\gamma_1 \gamma_3 - 4\gamma_0 - \gamma_2^2}{9} \), \( R = \frac{-9\gamma_2(3\gamma_3 - 4\gamma_0) - 27(4\gamma_2 \gamma_0 - \gamma_1^2 - \gamma_0^3) + 2\gamma_2^3}{81} \), and \( Y \equiv \frac{\gamma_3}{3} + 2\sqrt{-Q} \cos \left(\frac{\arccos \frac{R}{\sqrt{Q}}}{3}\right) \). Then,

\[
r^* = -\frac{\gamma_3}{4} + \frac{1}{2\sqrt{4\gamma_2^2 - \gamma_2 + Y}} + 1 \sqrt{\frac{\gamma_3^2}{2} - \gamma_2 - Y + \frac{4\gamma_3 \gamma_2 - 8\gamma_1 - \gamma_3}{4} \left(1 - \gamma_2 - Y\right)}.
\] (A13)

We can also find out that \( 3 - \alpha < r^* \leq 3 + \frac{6\tau k}{1 - \alpha} \) because \( h_b(3 - \alpha) < 0 \) and \( h_3(3 + \frac{6\tau k}{1 - \alpha}) \geq 0 \). Furthermore, we then know that \( \frac{\dot{\gamma}_3}{\dot{\gamma}_r} > 0 \). This shows the existence of this equilibrium for \( \alpha \) close to zero and \( \tilde{\mu} - \mu \) close to \( 2\tau \), because for these values both local and nonlocal deviations are not profitable, prices are close to each other, and consumers with a sufficiently positive experience continue purchasing the tried product, while consumers with a sufficiently bad experience try the alternative product. For \( \alpha \to 0 \), we have \( r \to 3 + 6\tau k, \ y \to \tau, \ e \to 0, \ d \to 0, \ a \to 0, \ b \to 0, \ s \to \frac{2\tau}{1 + 2\tau k}, \) and \( g \to 0. \) \[\square\]
FIGURE A1. (A) CURVES $h_1(T)$ AND $h_2(T)$ DETERMINING $T^*$. (B) CURVES $h_1(T)$ AND $h_2(T)$ CLOSE TO $T = 0$
Proof of Proposition 2. In order to prove the proposition we have to check the comparative statics of $y$ with respect to $\alpha$, $k$, $\tau$, $\delta_C$, and $\delta_F$. First remember that $\frac{\partial h_3(r^*)}{\partial r} > 0$, from the proof of Proposition 1, so that the sign of the derivative of $r^*$ with respect to any parameter is equal to minus the sign of the derivative of $h_3()$ with respect to that parameter.

(i) Consider first the comparative statics with respect to $\tau$. We have

$$\frac{\partial h_3(r^*)}{\partial \tau} = -k \left( 1 + \delta_C \alpha^2 \right) r^* \left( 6r^* - 8\delta_F \right) < 0. \quad (A14)$$

Then, $\frac{dr^*}{d\tau} > 0$.

Now, note that $\frac{dy}{d\tau} = \frac{\partial h_3(r^*)}{\partial \tau} + \frac{\partial h_3(r^*)}{\partial r} \frac{dr^*}{d\tau}$. One can directly check that $\frac{\partial h_3(r^*)}{\partial \tau} = 0$, and $\frac{\partial h_3(r^*)}{\partial r} = \frac{3r^* (2r^* - 4\delta_F \alpha \beta)}{2k(3r^* - 4\delta_C \alpha \beta)} > 0$. This yields $\frac{dy}{d\tau} > 0$.

(ii) Consider now the comparative statics with respect to $\delta_F$. We have

$$\frac{\partial h_3(r^*)}{\partial \delta_F} = 4\alpha^4 \left[ r^* (1 + 2\tau k (1 + \delta_C \alpha^2)) - 2\delta_C \alpha^4 \right] > 0. \quad (A15)$$

Then we know that $\frac{dr^*}{d\delta_F} < 0$.

Note also that $\frac{dy}{d\delta_F} = \frac{\partial h_3(r^*)}{\partial \delta_F} + \frac{\partial h_3(r^*)}{\partial r} \frac{dr^*}{d\delta_F}$. One can directly check that $\frac{\partial h_3(r^*)}{\partial \delta_F} = 0$, and $\frac{\partial h_3(r^*)}{\partial r} = -\tau \frac{2\delta_C \alpha^4 (1 + \delta_C \alpha^2)}{(r^* - 2\delta_C \alpha^2)} < 0$. Therefore, we obtain $\frac{dy}{d\delta_F} > 0$.

(iii) Consider now the comparative statics with respect to $\delta_C$. We have

$$\frac{\partial h_3(r^*)}{\partial \delta_C} = 2\alpha^2 r^* [3\alpha^2 - (2\alpha^2 + 3\tau k) r^*] + 8\alpha^6 [r^* k \tau - \alpha^2]. \quad (A16)$$

Note that the first term is decreasing in $r^*$, and the second term in increasing in $r^*$. Therefore, (A16) is smaller than the expression where we substitute $r^* = 3 - \alpha$ in the first term and $r^* = 3 + \frac{6\tau k}{1-\alpha}$ in the second term, which yields $(\tau k)^2 \frac{2\alpha^4}{1-\alpha} + \tau k [12\alpha^4 - 3(3 - \alpha^3)] + 3\alpha^2 (3 - \alpha^2) - 2(3 - \alpha^3) \alpha^2 - 4\alpha^4$, which is negative because we assumed $\tau k < \frac{1}{2} \cdot \frac{\alpha}{2}$. Then $\frac{\partial h_3(r^*)}{\partial \delta_C} < 0$, and $\frac{dr^*}{d\delta_C} > 0$.

Note also that $\frac{dy}{d\delta_C} = \frac{\partial h_3(r^*)}{\partial \delta_C} + \frac{\partial h_3(r^*)}{\partial r} \frac{dr^*}{d\delta_C}$. One can directly check that $\frac{\partial h_3(r^*)}{\partial \delta_C} = 0$, and we already know that $\frac{\partial h_3(r^*)}{\partial r} > 0$. Therefore, we obtain $\frac{dy}{d\delta_C} > 0$.

(iv) Consider now the comparative statics with respect to $k$. We have

$$\frac{\partial h_3(r^*)}{\partial k} = -\tau \left[ 1 + \delta_C \alpha^2 \right] 6r^* - 8\delta_F \alpha^4 < 0. \quad (A17)$$
Note now that \( \frac{dy}{dk} = \frac{\partial h_3(r*)}{\partial r} + \frac{\partial h_2(r*)}{\partial k} \). We have \( \frac{\partial h_3(r*)}{\partial r} = 0 \) and we showed above that \( \frac{\partial h_2(r*)}{\partial k} < 0 \), which yields \( \frac{dy}{dk} < 0 \).

(v) Consider now the comparative statics with respect to \( \alpha \). We have

\[
\frac{\partial h_3(r*)}{\partial \alpha} = r^* \left[ 24\deltaC \alpha^3 + r^* \left[ -12\deltaC \alpha \tau k - 8\deltaC \alpha^3 \right] \right] + 4\alpha^3 \deltaF r^* \left[ 4 + 8(1 + \deltaC \alpha^2) \tau k + 4\deltaC \alpha^2 \tau k \right] - 64\deltaF \deltaC \alpha^7.
\]

(A18)

For \( \alpha \) close to zero this derivative is negative, which yields \( \frac{d\alpha}{d\alpha} > 0 \).

Finally, note that \( \frac{dy}{d\alpha} = \frac{\partial h_3(r*)}{\partial \alpha} + \frac{\partial h_2(r*)}{\partial \alpha} \). One can directly check that \( \frac{\partial h_2(r*)}{\partial \alpha} > 0 \) and we already found out above that \( \frac{\partial h_2(r*)}{\partial \alpha} > 0 \). Therefore, we obtain \( \frac{dy}{d\alpha} > 0 \).

Proof of Proposition 3. Noting that \( r^* \in [3 - \alpha, 3 + \frac{6r_k}{1 - \alpha}] \), from the proof of Proposition 1, yields convergence to a steady state through oscillating market shares. Direct differentiation, using \( \lim_{\alpha \to 0} \frac{1}{\alpha} \frac{dr^*}{d\alpha} = 12\deltaC \tau k \), \( \frac{dr^*}{d\delta} < 0 \), \( \frac{dr^*}{d\delta} > 0 \), \( \frac{dr^*}{d\delta} > 0 \), and \( \frac{dy}{d\alpha} > 0 \), yields the comparative statics results.

Proof of Proposition 4. When \( \alpha \to 0 \), one can obtain \( \frac{1}{\alpha} \frac{dy}{d\alpha} \to 12\deltaC \tau k \), \( \frac{dr^*}{d\tau} \to 6\tau \), \( \frac{dr^*}{d\delta} \to 6\delta \), \( \frac{1}{\alpha} \frac{dy}{d\delta} \to \frac{4}{(1 + 2\tau k)^2} \), \( \frac{1}{\alpha} \frac{dy}{d\delta} \to \frac{8\delta \tau}{81(1 + 2\tau k)^2} \). Differentiating (10) one can then obtain when \( \alpha \to 0 \) that \( \frac{1}{\alpha} \frac{d\delta}{d\alpha} \to \frac{4\delta (1 - 2\delta k)}{3(1 + 2\tau k)^2} > 0 \), \( \frac{d\tau}{d\delta} \to \frac{4r^2}{(1 + 2\tau k)^2} < 0 \), \( \frac{d\tau}{d\delta} \to \frac{2r}{(1 + 2\tau k)^2} > 0 \), \( \frac{1}{\alpha} \frac{d\delta}{d\delta} \to \frac{2r}{(1 + 2\tau k)^2} > 0 \), and \( \frac{1}{\alpha} \frac{d\tau}{d\delta} \to \frac{4r}{3(1 + 2\tau k)^2} < 0 \). By continuity for \( \alpha \) small we get the result in the proposition.

References

Dynamic Competition with Experience Goods


