

MODELS OF COMPETITIVE PRICE PROMOTIONS: SOME EMPIRICAL EVIDENCE FROM THE COFFEE AND SALTINE CRACKERS MARKETS

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I present tests of a competitive rationale for price promotions. In a model with a population of informed and uninformed customers, price competition yields a static equilibrium in which each seller draws a price from a specified density function. Price data on coffee and saltine crackers products are used to test whether the sample of prices on each product could have possibly come from the theoretically specified density function. The results suggest that some markets are indeed consistent with the marginal distributions of prices predicted by the model. Furthermore, in the process of testing this rationale for price promotions, estimates are obtained for the marginal cost of each product, the number of competing goods, and the percentage of informed consumers. The resulting excess variability of these estimates across competing brands can also raise questions with respect to the empirical validity of the model.

1. INTRODUCTION

The use of price promotions has become increasingly popular among brand managers in recent years. Temporary, and seemingly uncertain, price cuts occur frequently in almost all markets.

The purpose of this paper is to test empirically some of the competitive explanations for price promotions.¹ These explanations have

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1. Other explanations for price promotions include (1) models that do not rely on the existence of competition, but simply focus on some special characteristics of the market, such that a monopolist would also find in its best interest to price promote (Aghion, Bolton, Harris, and Jullien, 1991; Blattberg, Eppen, and Lieberman, 1981; Lazear, 1986; Jeuland and Narasimhan, 1985; Sobel, 1984), and (2) models that rely on some form of collusive behavior among the firms in the market (Green and Porter,

been presented by, among others, Narasimhan (1988); Raju, Srinivasan, and Lal (1990); Rao (1991); Rosenthal (1980); Salop (1977); Salop and Stiglitz (1977); Shilony (1977); Stiglitz (1979); Tellis and Wernerfelt (1987); and Varian (1980). These models explain price promotions as a result of the competitive behavior of firms and the advantages they derive from price discriminating between different types of consumers.

Although the predictions of these models are reasonable and sound, some of their most important implications are yet to be tested fully. Some supportive evidence has been presented by Raju, Srinivasan, and Lal (1990) for their results. Their approach is to test some general messages from their results but not to make direct tests on the density functions of prices that they derive. This approach does not depend on the specific functional forms assumed on the derivation of the results, but, on the other hand, the complete equilibrium promotion structure of the model remains untested.

This study follows up on Raju, Srinivasan, and Lal by being a first step in testing the complete equilibrium structure of the competitive promotions models, a much more stringent test on these models. The focus is on the Varian model² to test the price density functions resulting from the competitive explanations of temporal and spatial price dispersion.³ The special appeal of this type of models lies in the fact that price promotions, although unpredictable, result from both the competitive behavior of the firms and their advantages in price

1984; Lal, 1990; Rotemberg and Saloner, 1986). These explanations, although relevant in some types of markets, do not consider competitive behavior as the source of price promotions, contrary to what marketing executives often believe. In the second group of explanations, price competition (in contrast with price collusion), in fact, does not occur, and price promotions are completely predictable, contrary to what casual empiricism makes us believe.

2. Narasimhan (1988) generalizes Varian (1980) by allowing for asymmetric firms in a duopoly.

3. The results presented in Salop (1977), Salop and Stiglitz (1977), Stiglitz (1979), and Tellis and Wernerfelt (1987) are relatively implausible in frequently purchased products (or in products where the word-of-mouth effect is important) in the sense that they specify static pure strategies equilibria, and through time consumers might learn which stores have the lower prices. Rao (1991) allows the retailers to be able to commit to a frequency of price promotions. Raju, Srinivasan, and Lal (1990) and Shilony (1977) require all consumers to be informed completely about all prices being charged (an unlikely event), but allow the informed consumers (or switchers, depending on the interpretation) to have at least a bimodal distribution of the preferences parameter (an important feature that warrants some further empirical investigation). Rosenthal (1980) generalizes the Varian model in a direction that might have little empirical relevance: He allows both the "informed" and "uninformed" demand to be a function of price, but, according to Strang (1976), the level of promotions does not seem to empirically affect the total category sales.

discriminating between "informed" and "uninformed" consumers (alternatively, "switchers" and "loyals").

The major purpose of this work is to investigate the extent to which the competitive explanations do a good job of predicting the way marketing managers actually behave. Moreover, the estimates obtained may allow practitioners to grasp what competitors believe in terms of the percentage of informed demand, the variable costs, the reservation prices, and the relevant number of competing firms.

The test is applied to retail coffee and saltine crackers data. Coffee and saltine crackers price data from several supermarkets are used to check whether the distribution of prices observed for each brand during a certain period is close to the theoretical distribution derived in Varian. In order to do this, a maximum likelihood estimation technique of the density function parameters is used to obtain the marginal cost, the reservation price, and the percentage of informed consumers (switchers). In these product categories, the monopolistic reasons for price promotions (see references in footnote 1) do not seem to be very significant: Inventory costs do not seem to play a very important role⁴; coffee and saltine crackers are not durable products; and, being relatively stable product categories, the sellers in these categories are likely to know the tastes of the consumers and the demand structure relatively well.

Another general important characteristic of this research is the fact of looking at the market equilibrium conditions, that is, accounting for all the interactions among the different agents in the market. Failure to account for these interactions (e.g., "naive" response function estimations) can result in bias in the parameters being estimated.

Section 2 briefly explains the theoretical model. Section 3 examines the data that were used. The estimation procedure is described in Section 4, and its results are discussed in Section 5. Section 6 goes on to discuss the reasonableness of the models' assumptions in the coffee and saltine crackers product categories. Finally, Section 7 concludes with implications and suggestions for future research.

2. THE THEORETICAL FRAMEWORK

In the Varian model, there is a large number of consumers, each of whom wants to buy one unit per period (price promotions do not affect category sales; Strang, 1976; Gupta, 1988) and has a reservation

4. Gupta (1988) decomposes the effects of the increase in sales due to promotions for ground coffee. He finds that 84% comes from brand switching, and only 16% may come from inventory effects or a simple increase in consumption.

price, r , which can be seen as the "acceptable price" in Monroe and Venkatesan (1969) and Monroe (1971). There are two types of consumers: informed and uninformed.⁵ The informed consumers buy from the seller that posts the lowest price. The uninformed consumers buy at random and uniformly through all the stores. This dichotomous representation of the market might be relatively reasonable, given that consumers have relatively high up-front costs for searching for information (i.e., the important decision might be whether or not to search for information). Another interpretation of the model calls the informed consumers "switchers" and the uninformed consumers "loyals."

The fraction of uninformed consumers for each store is α . The number of sellers is n . Therefore, the fraction of all the uninformed consumers is $n\alpha$, and the fraction of all the informed consumers is $(1 - n\alpha)$.

In each period, each seller chooses a price. If the price chosen is the lowest, that seller gets a market share equal to $[1 - (n - 1)\alpha]$. If a seller does not have the lowest price, it gets a market share equal to α . If two or more sellers charge the lowest price, each of them gets an equal share of the informed consumers plus α .

In each period, each seller fixes prices (P) drawn from a density function $f(P)$, the strategy of each of the sellers. Sellers maximize profits (choose $f(P)$) given the strategies of other sellers and the behavior of the consumers.

Each seller has the same constant marginal cost c .⁶ This seems relatively reasonable in the coffee and saltine crackers markets, since sellers have basically the same technology, and the linear cost function may be relatively close to the real one.

Given the structure of the model, notice that each seller can guarantee itself at least $(r - c)\alpha$ per consumer in the market. The highest market share a seller can ever have per period is $[1 - (n - 1)\alpha]$. Therefore, the lowest price a firm would consider charging associated with this market share is $P^* = c + (r - c)\alpha/[1 - (n - 1)\alpha]$.

Varian is then able to show that there are no pure strategies equilibria and that the unique mixed strategies equilibrium has each firm mixing between P^* and r according to the density function (no

5. I am considering here the information difference among consumers as exogenous. An equivalent method would be to make it dependent upon search costs that differ among consumers.

6. Fixed costs can be allowed in the model. If there are barriers to entry, fixed costs do not change the results of the model.

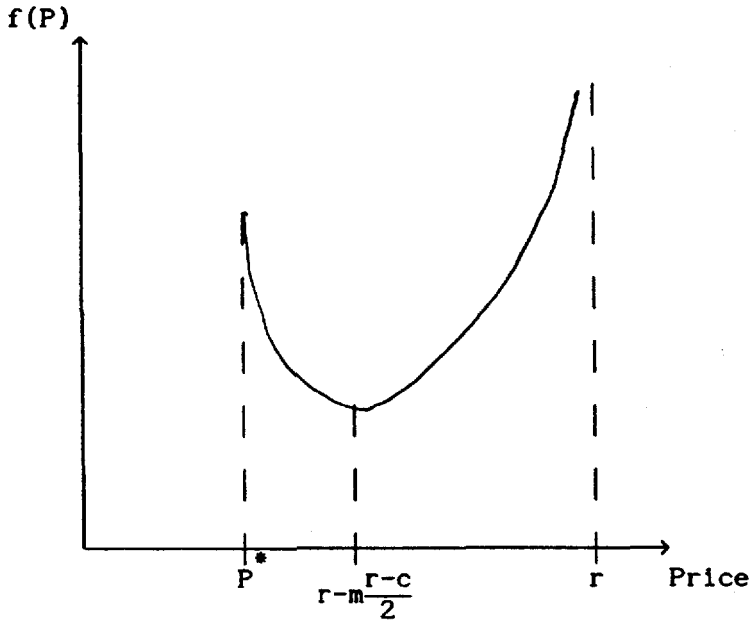


FIGURE 1. DENSITY FUNCTION OF PRICES.

mass points),

$$f(P) = \left(\frac{P^* - c}{r - P^*} \right)^{1/(n-1)} \frac{1}{n-1} \left(\frac{r - P}{P - c} \right)^{1/(n-1)-1} \frac{r - c}{(P - c)^2}. \quad (1)$$

Finally, notice that $\lim_{P \rightarrow r} f(P) = \infty$, which implies that the major portion of the price density is concentrated in the high price range.⁷ A typical graph of $f(P)$ is shown in Figure 1 for the case where P^* is sufficiently close to c ($P^* < r - (r - c)(n - 2)/[2(n - 1)]$). The graph also shows that there is greater density in the low price range than in the intermediate price range.

If the model has any predictive power, one should observe a large proportion of prices to be high, and there should exist random promotions through time. Furthermore, one should not observe many prices being charged between the high prices and the promoted ones (i.e., the intermediate price range).

7. Notice that $f(P)$ does not have mass points, contrary to the characteristics of the data. However, these characteristics of the data might be due to the firms mixing over a grid rather than over a continuum. If the grid is fine enough, the main features of $f(P)$ go through, and the Varian model cannot be ruled out exclusively on these grounds.

Expression (1) for each good is then estimated by maximum likelihood (using P^* , r , c , and n as parameters). Data on prices for each good are described in the next section. The estimation procedure is explained in Section 4, and the results are presented in Section 5.

3. THE DATA

The empirical testing of the model presented in Section 2 uses price data on coffee and saltine crackers brands. All data were provided by Information Resources Incorporated.

The coffee data set was obtained from a panel of families in Kansas City. It is composed of prices on all cover packages sold across six stores during 108 weeks (a price per week from mid-1985 to mid-1987).⁸ A cover package is defined by the brand (for example: Folgers, Tasters' Choice, Maxim, etc.), the type of coffee (for example: instant coffee, freeze-dried coffee, roast instant coffee, etc.) and the size of the package (for example: 2 oz., 4 oz., 8 oz., 16 oz., etc.). The physical good is defined by the brand and type of coffee. In reporting results, I place more emphasis on the brands with greater market share in the coffee market (Grover and Srinivasan, 1987, 1992; Guadagni and Little, 1983; Gupta, 1988; Urban, Johnson, and Hauser, 1984), and on a store from the chain with the greatest sales volume.

The six stores were part of three chains: Chain 1 had three stores, chain 2 had two stores, and chain 3 had one store. Table I presents the number of brands and packages carried by each chain.

TABLE I.
PHYSICAL GOODS AND COVER
PACKAGES ACROSS CHAINS FOR
COFFEE MARKET

Chain	No. of Physical Goods	No. of Cover Packages
1		96
2		87
3		85
Total	87	136

8. In terms of the model presented earlier, a good is defined here by the physical characteristics, package size, and location of sale: The same physical good in different package sizes corresponds to different goods.

The saltine crackers data set was obtained from a panel of families in Williamsport, Pennsylvania, in 1984 and 1985 (103 weeks). It is composed of prices on all cover packages sold across four stores. I restricted the attention to the four most important brands in the two largest stores (Nabisco, 16 oz.; Sunshine, 16 oz.; Keebler, 16 oz.; and the Private Label, 16 oz., of each of the two stores; together these brands account for approximately 90% of the sales volume in each store).

4. TESTING PROCEDURE

This section presents the estimation and testing procedure. The first step is to estimate the theoretical density function for each brand.⁹ Expression (1) was used to accomplish this. The parameters to estimate are r , P^* , c , and n for each week. Given that we only have one observation per good per week, some restrictions have to be imposed on the evolution of these parameters through time (which was apparent from the plots of the prices of the different brands—see representative plots in Figs. 2–3).

r and P^* were assumed to be constant through time except for changes in weeks B_1 and B_2 , which are also estimated.¹⁰ This approach was consistent with the plots of the prices of the different brands,¹¹ and it seemed to be important when I checked the value of the likelihood function with the constraints $B_1 = 0$, or $B_1 = B_2 = 0$. (For 87% of the coffee brands in Table II and all the saltine crackers brands, the log-likelihood decreased by more than 4.0 when the constraint $B_1 = 0$ was imposed.¹²)

9. An alternate way (and a more efficient one) would be to estimate the joint density of all the prices. The problem with this approach is that I would then have to define which are the sets of goods that were in direct competition, a task neither easy nor straightforward. In the method presented here, each good belongs to a certain submarket in the coffee, or the saltine crackers category. I do not study the composition of each submarket and, therefore, do not impose any constraints on the relation between parameters of different brands.

10. Being $r(t)$ the reservation price in week t , $r(t) = \bar{r}_1$, for $t \leq B_1$, $r(t) = \bar{r}_2$, for $B_1 < t \leq B_2$, and $r(t) = \bar{r}_3$, for $t > B_2$; the same for P^* .

11. These plots seem to indicate that the values of r and P^* are constant during some intervals of time. More general evolutions of r and P^* are also difficult to implement with the current computer capabilities. The problem is that r (respectively P^*) for each week has to be greater than (smaller than) the price actually set in that week. This requires the estimation procedure to have a number of constraints at least equal to the number of observations. If r (or P^*) is constant within a certain subperiod, then there is only a constraint per subperiod: r greater or equal to the highest price in that subperiod (or P^* smaller or equal to the lowest price in that subperiod).

12. Though not standard, the use of the likelihood ratio statistic to test $B_1 = 0$ (or $B_1 = B_2 = 0$) has often good properties (Quandt, 1960; Hinkley, 1970; Poirier, 1976,

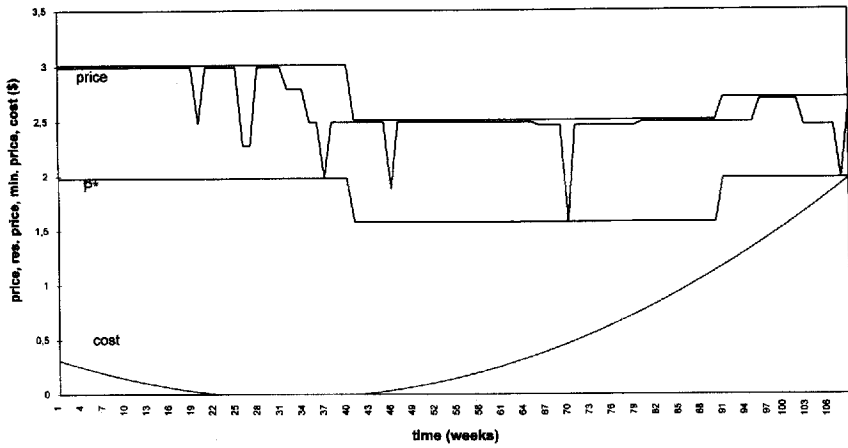


FIGURE 2. FOLGERS REGULAR—STORE 1 (GROUND CAFFEINATED).

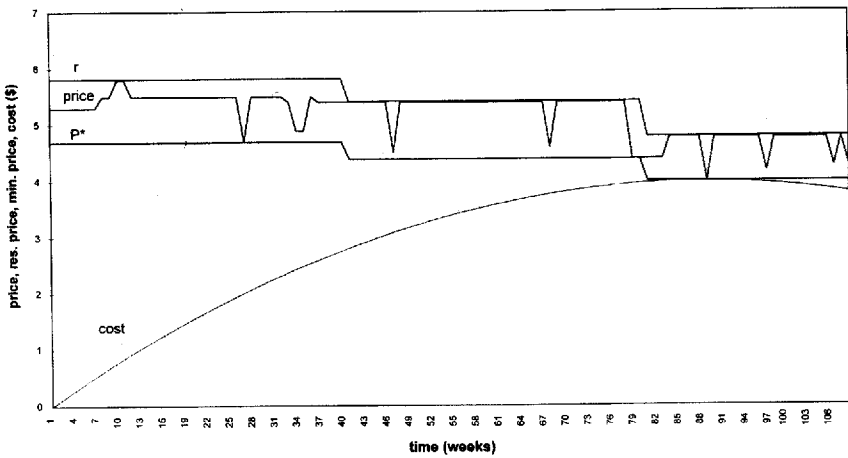


FIGURE 3. SANKA REGULAR—STORE 1 (INSTANT COFFEE DECAFFEINATED).

TABLE IIA.
COFFEE MARKET—FIT TO VARIAN MODEL

	r	P^*	c	n	"Loyals" (%)	Log (L)	Q	d.f.	Result
Ground caffeinated									
Folgers Regular	2.72	1.85	0.49	14.2	4.3	971.9	17.7	9	Reject
Folgers Drip	2.72	1.85	0.49	14.1	4.3	971.1	16.4	9	Not rej.
Folgers Perc	2.72	1.85	0.49	14.1	4.3	986.9	13.8	9	Not rej.
Maxwell Hse Regular	2.82	2.49	2.48	9.3	0.3	829.7	17.1	9	Reject
Maxwell Hse Drip	2.79	2.49	2.48	10.2	0.3	910.1	15.9	9	Not rej.
Maxwell Hse Perc	2.82	2.49	2.48	9.3	0.3	829.7	17.1	9	Reject
Hills Bros. Regular	2.66	1.80	1.19	12.0	3.5	746.4	76.4	9	Reject
Chock-Full-o'-Nuts	2.56	2.36	2.29	9.7	2.6	1011.9	15.7	9	Not rej.
Ground decaffeinated									
Sanka Regular	3.72	3.14	2.66	15.2	3.0	1052.6	60.3	9	Reject
Sanka Drip	3.72	3.14	2.44	14.8	3.7	1022.4	87.5	9	Reject
Brim Regular	3.79	3.15	2.40	10.3	4.3	630.2	31.8	9	Reject
Brim Drip	3.79	3.15	2.66	10.7	4.0	483.8	15.1	6	Reject
Brim Perc	3.79	3.15	2.47	9.3	5.6	574.9	64.3	9	Reject
Instant caffeinated									
Folgers Regular	4.51	3.71	3.41	16.0	1.7	1006.6	19.6	9	Reject
Maxwell Hse Regular	4.75	3.99	2.92	11.9	4.9	721.8	32.8	9	Reject
Nescafe Regular	4.68	4.34	3.52	11.8	5.9	861.9	34.4	9	Reject
Tasters Freeze Dr.	5.31	4.62	1.49	7.9	10.4	426.3	11.7	8	Not rej.
Maxim Freeze Dr.	5.21	4.47	3.26	13.1	4.7	893.6	59.6	9	Reject
Instant decaffeinated									
High Point Regular	5.14	4.38	3.95	18.6	1.9	1164.3	15.8	8	Reject
Nescafe Regular	5.14	4.49	4.23	9.4	3.0	574.3	48.3	9	Reject
Sanka Regular	5.32	4.36	2.88	12.7	4.8	803.4	8.2	9	Not rej.
Tasters Freeze Dr.	5.78	5.04	3.87	12.1	5.1	849.9	229.4	9	Reject
Sanka Freeze Dr.	5.84	2.70	1.74	19.9	1.2	1307.1	15.5	9	Not rej.
Brim Freeze Dr.	5.84	2.70	1.74	19.9	1.2	1307.1	15.5	9	Not rej.

Notes: All data for store 1 in chain 1. The ground-coffee brands are for 16-oz. packages. The instant-coffee brands are all for 8-oz. packages except for the regular coffee, which are for 10-oz. packages. r is reservation price, P^* is the minimum price being charged, c is marginal cost, and n is number of firms. The data for r , P^* , c , n , and the percentage of "loyals" is the average across all weeks. Q is the value of the test statistic (described in the text), and d.f. are the degrees of freedom of the test.

TABLE IIb.
COFFEE MARKET—FIT TO VARIAN MODEL

	r	P^*	c	n	"Loyals" (%)	Log (L)	Q	d.f.	Result
Constant marginal costs									
Ground caffeinated									
Folgers Regular	2.72	1.85	0.53	14.1	4.3	970.4	39.6	9	Reject
Folgers Drip	2.72	1.85	0.61	14.0	4.2	969.6	37.3	9	Reject
Folgers Perc	2.72	1.85	0.43	14.0	4.4	985.4	39.6	9	Reject
Maxwell Hse Regular	2.82	2.49	2.48	8.9	0.3	697.2	77.9	9	Reject
Maxwell Hse Drip	2.79	2.49	2.48	9.8	0.3	774.8	41.1	9	Reject
Maxwell Hse Perc	2.82	2.49	2.48	8.9	0.3	697.2	77.9	9	Reject
Hills Bros. Regular	2.66	1.80	1.47	12.2	2.3	744.9	105.9	9	Reject
Chock-Full-o'-Nuts	2.56	2.36	2.29	8.8	2.9	939.9	24.1	9	Reject
Marginal costs as cubic expression of time									
Folgers Regular	2.72	1.85	0.96	14.5	3.5	973.2	16.4	9	Not rej.
Folgers Drip	2.72	1.85	0.94	14.3	3.6	972.3	20.3	9	Reject
Folgers Perc	2.72	1.85	0.89	14.3	3.7	987.7	17.8	9	Reject
Maxwell Hse Regular	2.82	2.49	2.48	9.7	0.3	839.2	13.5	9	Not rej.
Maxwell Hse Drip	2.79	2.49	2.48	10.7	0.3	922.4	16.4	9	Not rej.
Maxwell Hse Perc	2.82	2.49	2.48	9.7	0.3	839.2	13.5	9	Not rej.
Hills Bros. Regular	2.66	1.80	1.40	12.2	2.6	752.5	79.9	9	Reject
Chock-Full-o'-Nuts	2.56	2.36	2.29	9.7	2.6	1011.9	15.7	9	Not rej.

Notes: All data for store 1 in chain 1. The ground-coffee brands are for 16-oz. packages. The instant-coffee brands are all for 8-oz. packages except for the regular coffee, which are for 10-oz. packages. r is reservation price, P^* is the minimum price being charged, c is marginal cost, and n is number of firms. The data for r , P^* , c , n , and the percentage of "loyals" is the average across all weeks. Q is the value of the test statistic (described in the text), and d.f. are the degrees of freedom of the test.

The evolution of c was assumed to be a flexible algebraic expression of the second degree on time ($c(t) = a_0 + a_1t + a_2t^2$, where t represents the week number). The evolution of n was fixed by restricting $m = 1 - 1/(n - 1)$ to evolve also as an algebraic expression of the second degree ($m(t) = b_0 + b_1t + b_2t^2$). The relevance of allowing a_1 , a_2 , b_1 , and b_2 to be different from zero was confirmed through a log-likelihood ratio test. (Sixty-two percent of the coffee brands and 50% of the saltine crackers brands reject the null hypothesis $a_2 = b_2 = 0$ at the 5% significance level.)

A maximum likelihood estimation technique was performed on the log-likelihood function

$$\log(L) = \sum_{t=1}^T \log \left\{ 1[P^*(t) \leq P_t \leq r(t)] \left(\frac{P^*(t) - c(t)}{r(t) - P^*(t)} \right)^{1/[n(t)-1]} \frac{1}{n(t) - 1} \right. \\ \left. \times \left(\frac{r(t) - P_t}{P_t - c(t)} \right)^{1/[n(t)-1]-1} \frac{r(t) - c(t)}{[P(t) - c(t)]^2} \right\},$$

where T is the total number of weeks in the sample, $r(t) = \bar{r}_1$ and $P^*(t) = \bar{P}_1^*$ if $t \leq B_1$, $r(t) = \bar{r}_2$ and $P^*(t) = \bar{P}_2^*$ if $B_1 < t \leq B_2$, $r(t) = \bar{r}_3$, and $P^*(t) = \bar{P}_3^*$ if $t > B_2$, $c(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2$, $m(t) = b_0 + b_1 \cdot t + b_2 \cdot t^2$, and $n(t) = [2 - m(t)]/[1 - m(t)]$.^{13,14}

The estimation of B_1 and B_2 was done through a grid search. It can also be easily confirmed that the maximum likelihood estimators for \bar{r}_i and \bar{P}_i^* for $i = 1, 2, 3$ are, respectively, the maximum and the minimum values of the series $\{P(t)\}$ in subperiod i .¹⁵ Moreover, while the maximum likelihood estimators of a_i , b_i for $i = 0, 1, 2$ converge to their true values at rate \sqrt{N} , the maximum likelihood estimators of \bar{r}_i , \bar{P}_i^* for $i = 1, 2, 3$ converge at rate N , as shown in Kendall and Stuart (1961), p. 424. (N is the number of observations in the subperiod

p. 111; Zacks, 1983). In fact, simulations for the sample sizes and the changes in parameters across subperiods that are used in the analysis suggested that the use of this test was relatively accurate: For example, if one generates samples from the specified model with $B_1 = 0$, then the log-likelihood ratio test at the 5% significance level rejects the null hypothesis $B_1 = 0$ for 5% of the generated samples. An important question is then why, for example, restrict $B_3 = 0$? The problem is one of sample size. r and P^* are estimated in each subperiod. Then, if the number of weeks in each subperiod is smaller, the estimates are, in expectation, further away from the true values.

13. The parameters to be estimated are \bar{r}_i , \bar{P}_i^* , for $i = 1, 2, 3$, B_1 , B_2 , and a_i , b_i for $i = 0, 1, 2$.

14. $1[\text{condition}]$ is the indicator function. It takes the value of one if the condition is satisfied and the value of zero otherwise.

15. Subperiod 1 has $t \leq B_1$, subperiod 2 has $B_1 < t \leq B_2$, and subperiod 3 has $t > B_2$.

for which \bar{r}_i or \bar{P}_i^* is being estimated.¹⁶) One can then use the maximum likelihood estimates of \bar{r}_i and \bar{P}_i^* for $i = 1, 2, 3$ as the true values and obtain the usual estimate for the variance matrix of a_i, b_i for $i = 0, 1, 2$ (see, e.g., Judge et al., 1985).

After having estimated $f(P; t)$, I tested if the data rejected $f(P; t)$ as the theoretical distribution from which came the observation $\{P(t), \forall t\}$. In order to do this test, the range of $f(P; t)$ was divided into intervals that depended on t . The intervals were defined in each week such that the theoretical probability in the i th interval was the same across weeks, whatever i . In each interval i , f_i was defined as the proportion of P 's in that interval and Π_i the theoretical probability of P being in that interval.

It is well known that

$$Q \equiv \sum_{i=1}^q \frac{(Tf_i - T\Pi_i)^2}{T\Pi_i} \rightarrow G \equiv \chi^2(q - 1),$$

where q is the number of intervals,^{17,18} and G is some distribution. The null hypothesis is that the observed P 's come from the estimated theoretical distribution $f(P; t)$. The statistic takes the value zero if the fit is perfect and is greater the further away the real distribution of the P 's is from $f(P; t)$. The null hypothesis is rejected if Q is greater than the critical value on the distribution $\chi^2_{(q-1)}$ (in all the cases considered with a 5% significance level).

16. Notice also that even though the maximum likelihood (ML) estimators for B_1 and B_2 are not consistent, the ML estimators for the other parameters in the model remain consistent (Hinkley, 1970).

17. In fact, this is not the standard case for the application of this test. In the standard case, Π_i is known with certainty, while in this case Π_i is estimated. If Π_i was estimated with a maximum likelihood procedure on the q intervals used in the test, Q would still be distributed asymptotically as χ^2 , but with $q - s - 1$ degrees of freedom, where s is the number of estimated parameters that converge at \sqrt{T} (in this case $s = 6$). This fact is shown in Kendall and Stuart (1961), p. 425. But here, Π_i was estimated with a maximum likelihood procedure on the T observations of the random variable (the price). In this case, Q does not have an asymptotic χ^2 distribution. "However, the distribution of Q is bounded between a $\chi^2_{(q-1)}$ and a $\chi^2_{(q-s-1)}$ variable, and since q becomes large, these are so close together that the difference can be ignored" (Kendall and Stuart, 1961, p. 430). Moreover, simulations suggested that this test was relatively accurate for this model and the sample sizes used in the analysis: series of prices generated from the theoretical model (with changes through time in r, P^*, c , and n according to the assumptions presented earlier) rejected the null hypothesis that they came from the theoretical model at the 5% significance level, in 5% of the simulations.

18. In principle, the choice of the number of intervals might have impact on the null hypothesis being rejected or not. For all the estimations that are presented, the number of intervals chosen was the highest integer less than or equal to $T/10$. Other rules did not seem to affect the results significantly.

5. RESULTS

In the case of the coffee market, the results are presented in Table IIa and only for the main brands in the market and for store 1 of chain 1 (the largest chain).¹⁹ The results for the saltine crackers market are presented in Table III and only for stores 1 and 2 (the largest ones).

The results can be divided into two parts: estimates of the parameters of the model and estimates of the value of the statistic Q .

For the coffee market, the estimates of the number of firms in the market show that the managers of all the brands seem to believe that the relevant submarket they compete in has a relative large number of firms. (Estimates range from 7.9 to 19.9.) The estimates for the number of competitors in the saltine crackers market are also relatively high. (They range from 14.4 to 17.6.²⁰)

The estimates for the marginal costs vary somewhat across the coffee brands in all the subcategories except for ground decaffeinated. Furthermore, the estimates vary substantially through time. The estimates of the marginal costs for the saltine crackers market also vary across stores. This variability of the cost estimates across brands and stores, and through time, raises some problems with respect to the fit of the model to reality. However, one should also be aware that these costs should be interpreted as the marginal costs of the pair manufacturer-retailer (and not of the retailer alone) as suggested in Subsection 6.1. Furthermore, these costs can vary somewhat through time and across sellers because of different inventory positions. (They are opportunity costs and not accounting costs.²¹)

For the coffee market, the estimates of the "loyals" (uninformed consumers)²² segment also vary somewhat across brands (ranging from 0.2% to 10.4%). The low values for the estimates reflect the large number of competitors perceived by the managers. For example, the manager of the Ground Caffeinated Folgers Drip brand behaves as if 4.3% of her potential customers are "loyal" to her, as if she is facing 14.1 competitors, and as if 39% of her potential customers are "switchers" (i.e., buy the brand that has the lowest price). The estimates of

19. The results for other stores are not significantly different. The same applies for the saltine crackers market in relation to the results that are presented here.

20. Notice that the estimates of the number of firms in the market should not be taken at face value. They estimate the number of equal competitors that have perfectly substitute brands, such that the level of competition would be similar to the existing one.

21. Table IIb presents results for the ground caffeinated coffee brands for the cases of constant marginal costs and marginal costs as a cubic function of time. Results for the other subcategories are similar.

22. These estimates can be obtained from the other parameters in the model (see Section 2).

TABLE III.
SALTINE CRACKERS MARKET—FIT TO VARIAN MODEL

	r	P^*	c	n	"Loyals" (%)	Log (L)	Q	d.f.	Result
Store 1									
Nabisco 16 oz.	1.26	0.85	0.62	17.0	2.1	1193.3	7.2	9	Not rej.
Sunshine 16 oz.	1.26	0.75	0.47	16.9	2.1	1145.4	18.1	9	Reject
Keebler 16 oz.	1.28	0.95	0.83	14.4	1.9	1003.3	8.3	9	Not rej.
Private Label 16 oz.	0.72	0.59	0.58	17.6	0.4	1199.5	31.1	9	Reject
Store 2									
Nabisco 16 oz.	1.26	0.85	0.61	16.8	2.2	1177.8	7.3	9	Not rej.
Sunshine 16 oz.	1.26	0.71	0.63	15.1	0.8	961.3	17.2	9	Reject
Keebler 16 oz.	1.28	0.95	0.81	14.8	2.0	1050.3	10.4	9	Not rej.
Private Label 16 oz.	0.72	0.59	0.58	17.1	0.4	1163.6	51.8	9	Reject

Notes: r is reservation price, P^* is the minimum price being charged, c is marginal cost, and n is number of firms. The data for r , P^* , c , n , and the percentage of "loyals" is the average across all weeks. Q is the value of the test statistic (described in the text), and d.f. are the degrees of freedom of the test.

the "loyal" segment for the Ground Maxwell House brands seem too low for such an important umbrella of brands in the coffee market. There is, in fact, a negative correlation between the estimates presented here and the estimates of the "loyal" segments presented in Grover and Srinivasan (1992). As discussed there, the explanation for this might be the higher loyalty of the Maxwell House brands in the particular market they considered (Pittsfield, Massachusetts) due to these brands having been early entrants. On the other hand, the estimates for the instant coffee subcategory have a positive correlation with the estimates of the "loyal" segment presented in Grover and Srinivasan (1987).²³

The estimates for the saltine crackers market reflect very small loyalty in the market (the switchers range from 63% to 93%) and might be the result of in-store search. These results are also consistent with the high degree of information consumers seem to have about the brands that are being promoted (if we interpret α as the percentage of uninformed consumers per store; see Krishna, Currim, and Shoemaker, 1991). In the saltine crackers market, as expected, the estimates for the private label brands are lower than for the national brands.

Figures 2 and 3 present representative plots of the prices, reservation prices, minimum prices, and marginal costs of brands in the coffee market.

Finally, the chi-square test on the estimated distribution enables us to reject the Varian model for 66% of the coffee brands and for 50% of the saltine crackers brands. The rejection pattern is not uniform across subcategories in the coffee market and across national brands and private labels in the saltine crackers market.

In the coffee market, the ground caffeinated and the instant decaffeinated subcategories are the ones for which the most brands (50% of the brands) do not reject the model. This is especially interesting, because the ground caffeinated subcategory is the largest one in the data set being used.

In the saltine crackers market, the Varian model is not rejected for 66% of the national brands (the ones with greater market share), but it is rejected for 100% of the private label brands considered.

The results show that the Varian model is not rejected for an important number of the brands in the coffee and the saltine crackers markets. This is relatively surprising given the considerations presented in Section 6 and, therefore, suggests that the assumptions of

23. The correlation is equal to 0.15 and is not significantly different from zero. Notice also that Grover and Srinivasan only use demand data while I only use price data.

the model might be a relatively good approximation of the real world for important market situations. However, the excess variability of the parameter estimates across seemingly competing brands can raise questions with respect to the empirical validity of the model.

6. DISCUSSION OF ASSUMPTIONS

There are several reasons why the models presented in Section 2 may not describe the coffee and saltine crackers retail markets (in addition to the alternative explanations for price promotions presented in footnote 1):

(1) *Prices are the outcome of both retailer and manufacturer decisions.* In the markets being considered, prices are an outcome of two decision makers (retailer and manufacturer), and this was not accounted for in the model presented in Section 2: However, if there is no asymmetric information, decisions that are made by several decision makers might, under some incentive schemes (which are assumed to exist and to be implemented), be the same as the ones taken by a unique decision maker, that is, be efficient decisions (Coase, 1960).²⁴ The test of the models of Section 2 could, in this perspective, be interpreted as a test of whether or not the asymmetric information problems between retailers and manufacturers are really important.

(2) *Multiproduct sellers.* In several cases the same seller (interpreted here as a pair manufacturer-retailer) contributes for the decision making on different goods, and this was not allowed for in the Varian model: Retailers sell several brands, manufacturers sell through more than one retailer. (See earlier definition of a good.) However, given the relative large number of competitors, the impact on the model equilibrium might be relatively small. The test of the model of Section 2 could, in this perspective, be interpreted as a test of whether the impact of multiproduct sellers is really important.

Table IV presents the average correlations between prices of the brands in the coffee market. (Results for the saltine crackers market are very similar.) Notice that the correlation between prices of goods with the same cover package but sold in different stores is higher than the correlation between prices of goods with different cover packages (0.74 vs. 0.52), that is, two goods from the same manufacturer are priced differently than if they were from different manufacturers. The same can be said with respect to the retailer when one compares

24. The retailer and manufacturer could contract, for example, in a share rule of the total profits of the brand. The retailer sets the retail price, and the wholesale price is set so that the share rule is satisfied.

TABLE IV.
PRICE VARIABILITY AND CORRELATION BETWEEN PRICES
FOR COFFEE MARKET

	Range	Average	Standard Deviation
Prices	\$.89–\$7.25		
Coefficient of variation of the prices (standard deviation/mean)—541 goods		21.45%	11.88%
Correlation between prices of cover packages from different manufacturers and sold in different stores— ≈11,000 correlations		0.52	0.45
Correlation between prices of the same cover package across stores belonging to different chains— ≈250 correlations		0.74	0.30
Correlation between prices of the same cover package across stores of the same chain— ≈300 correlations		0.97	0.01

correlations between prices of the same brand across store chains and between prices of the same brand across stores within the same chain (0.74 vs. 0.97).

(3) *Menu costs and the definition of the period.* The model considered in Section 2 assumes that the cost of changing prices is zero. This is not realistic, and we rarely observe price promotions occurring for only one week. Typically the price promotions extend themselves during two, four, or six weeks. This raises the question of the definition of the correct period of analysis: In the model, a period is the minimum lapse of time during which a price cannot be changed; in the data set, in spite of having weekly prices, the minimum lapse of time between price changes was often two weeks or longer (and this might be justified by the existence of positive costs on launching a price promotion). This yields a strong positive autocorrelation in every price series (significantly different from zero for 90% of the brands in the coffee and saltine crackers markets), which is a violation of the results of the models. (The models predict independence of prices through time.) As hinted above, this problem would be solved with a correct defini-

tion of the period, but this important issue was not an object of the analysis of this paper. The estimation used weekly data, but because of the existing autocorrelation in the observations, the test statistics have less power than if the observations were truly independent. (Everything is as if we had, in fact, fewer observations than we really have.)

An alternative explanation for the observed autocorrelation in the price series is the set of collusion theories cited in footnote 1. The relative high average correlation between prices of different cover packages sold in different stores that is reported in Table IV seems also to corroborate the collusion stories.

(4) *Parameter shifts.* The plots of the price series suggest the existence of changes in the parameters of the model through time. This possibility was incorporated into the model to be estimated in the way described in Sections 4: three different levels of the reservation price and minimum price and expressions of the second degree on a trend variable for the marginal cost and the number of firms. (Statistical tests confirmed the importance of considering parameter changes in the reservation price and minimum price, and of considering expressions of the second degree for the evolution of the marginal cost and the number of firms; see Section 4.) The data were allowed to detect the timings of the changes in the levels of the reservation price and the minimum price. However, if the form of the parameter shifts is different from the one assumed here, the models can be misestimated and rejected, although they may actually be accurate. (This could also explain the observed autocorrelation in the price series.²⁵) In this sense, a test of the model is a test at the particular form of parameter shifts that was assumed.

Notice also that the estimates for the marginal costs, in particular, not only vary somewhat across brands (coffee and saltine crackers) and across stores (saltine crackers) but vary substantially through time as well. This variability of the cost estimates across brands and stores, and through time, raises some problems with respect to the fit of the model to reality.

(5) *Market structure.* The Varian model makes strong assumptions with respect to the market structure. It assumes all the firms in the relevant market to be in the same conditions: same "loyal" demand, same reservation price of consumers, same marginal cost. This is clearly unrealistic. However, if the conditions for the different firms

25. It could also happen that, even though the reality does not conform with the models being considered, the excess of parameter shifting might allow the theoretical models not to be rejected by the model.

are relatively similar, then the price equilibrium is close to the Varian results. (This can be easily checked for the two-firm case: Narasimhan [1988] presents the equilibrium for the two different firms case; this converges to the Varian model when the firms get more and more equal.) In this sense, a test of the model is a test of how similar the conditions faced by different firms are.

Another approach is to test directly the equilibrium for the two different firms case (Narasimhan, 1988): (1) The fraction of loyal consumers varies across firms ($\alpha_1 \neq \alpha_2$, where the subscript refers to the firm—firm 1 and firm 2). (2) The two products are valued differently by consumers such that (a) the reservation price for product 1 is r_1 , the reservation price for product 2 is r_2 , and $r_1 \neq r_2$ is possible, and (b) the switchers buy from product 1 if $P_1 < P_2 + (r_1 - r_2)$, buy from product 2 if $P_1 > P_2 + (r_1 - r_2)$, and distribute themselves equally between the two products if $P_1 = P_2 + (r_1 - r_2)$ (where P_1 and P_2 are, respectively, the prices being charged for products 1 and 2). The equilibrium is in mixed strategies and is described in Narasimhan (1988).

Given the equilibrium strategies, one can then estimate the joint distribution of prices²⁶ and proceed to test the fit of the model with a chi-square test as described in Section 4, with the adjustment that now we have a matrix with intervals in the price ranges of both brands.²⁷

Tables V and VI present respectively some results for the coffee and saltine crackers markets. In each "submarket," the choice of the two competitors for the analysis was made based on the market share of the different brands.

The results for both the coffee and the saltine crackers market seem to reject the joint distribution of the two different-firms case. For these markets, one gets the relatively surprising result of the estimates of the marginal cost being very close to zero. For the ground-coffee submarket, the estimates of the relative size of the "loyal" segments are consistent with Grover and Srinivasan (1992). However, for the instant coffee submarket, this is not the case in relation to Grover and Srinivasan (1987). For the saltine crackers market, the loyalty estimates show greater loyalty for the brand that has greater market share—Nabisco.

There are several explanations for the somewhat different results between the Varian and the Narasimhan models: (1) The Narasimhan model restricts the number of firms in the market to two, and this

26. Details on the estimation procedure are available from the author upon request.

27. I defined three intervals per brand that yield nine cells for the chi-square test.

TABLE V.
COFFEE MARKET—FIT TO NARASIMHAN MODEL

	<i>r</i>	<i>P</i> *	<i>c</i>	"Loyals"	<i>Q</i>	d.f.	Result
Ground caffeinated							
Folgers Regular	2.94	1.69	0.0	0.22	407.3	8	Reject
Maxwell Hse Regular	3.06	1.81		0.44			
Folgers Drip	2.94	1.69	0.0	0.21	409.5	8	Reject
Maxwell Hse Drip	3.06	1.81		0.45			
Ground decaffeinated							
Sanka Regular	3.80	3.10	0.0	0.72	356.2	8	Reject
Brim Regular	3.96	3.26		0.13			
Instant caffeinated							
Folgers Regular	4.79	3.55	0.0	0.55	95.4	8	Reject
Maxwell Hse Regular	4.77	3.53		0.21			
Maxwell Hse Regular	4.88	3.70	0.0	0.55	49.6	8	Reject
Tasters Freeze Dried	5.75	4.63		0.29			
Instant decaffeinated							
High Point Regular	5.26	4.04	0.0	0.62	13.6	8	Not reject
Sanka Regular	5.54	4.32		0.15			
Sanka Regular	5.45	4.22	0.0	0.46	64.9	8	Reject
Tasters Freeze Dried	5.97	4.74		0.32			

Notes: All data for store 1 in chain 1. The ground-coffee brands are for 16-oz. packages. The instant-coffee brands are all for 8-oz. packages except for the regular coffee, which are for 10-oz. packages. *r* is reservation price, *P** is the minimum price being charged, *c* is marginal cost, and *n* is number of firms. The data for *r*, *P**, *c*, *n*, and the percentage of "loyals" is the average across all weeks. *Q* is the value of the test statistic (described in the text), and d.f. are the degrees of freedom of the test.

TABLE VI.
SALTINE CRACKERS MARKET—FIT TO NARASIMHAN MODEL

	<i>r</i>	<i>P</i> *	<i>c</i>	"Loyals"	<i>Q</i>	d.f.	Result
Store 1							
Nabisco 16 oz.	1.33	0.78	0.00	0.41	77.0	8	Reject
Sunshine 16 oz.	1.30	0.75		0.16			
Nabisco 16 oz.	1.31	0.86	0.38	0.47	386.3	8	Reject
Keebler 16 oz.	1.37	0.92		0.00			
Nabisco 16 oz.	1.33	0.87	0.00	0.40	153.2	8	Reject
Private Label 16 oz.	0.90	0.44		0.12			
Store 2							
Nabisco 16 oz.	1.31	0.73	0.00	0.56	240.4	8	Reject
Sunshine 16 oz.	1.31	0.73		0.00			
Nabisco 16 oz.	1.31	0.86	0.38	0.47	378.6	8	Reject
Keebler 16 oz.	1.37	0.92		0.00			
Nabisco 16 oz.	1.33	0.87	0.00	0.46	137.1	8	Reject
Private Label 16 oz.	0.94	0.48		0.09			

Notes: *r* is reservation price, *P** is the minimum price being charged, *c* is marginal cost, and *n* is number of firms. The data for *r*, *P**, *c*, *n*, and the percentage of "loyals" is the average across all weeks. *Q* is the value of the test statistic (described in the text), and d.f. are the degrees of freedom of the test.

may be relatively important. (In the Varian model, the null hypothesis of $n = 2$ at the 5% significance level was rejected for all brands.) (2) As presented here, the test for the Narasimhan model was a test of the joint distribution, while the test for the Varian model was a test of the marginal distributions. (If, e.g., there is any correlation between prices of different brands, the Narasimhan model is rejected while the Varian model might, incorrectly, not be rejected.) (3) The test of the Varian model allowed the marginal costs and the definition of the subperiods to be different across brands, while the test for the Narasimhan model restricted the subperiods to be the same across brands.

7. CONCLUDING REMARKS

Price data on coffee and saltine crackers products are used to test whether the sample of prices on each product could have possibly come from the density functions specified in the equilibrium of a model of competitive price promotions. The results suggest that some markets are indeed consistent with the prices' marginal distributions predicted by the model. The important proportion of brands that do not reject the marginal distribution prediction might be looked at as relatively surprising given the considerations of the previous section. These considerations suggest also the changes that these models of competitive price promotions might need in order to fit reality better.²⁸

In the process of testing the model, estimates were obtained of the marginal cost for each product, the number of perceived competing goods, and the percentage of informed ("switchers") demand to potential demand. The excess variability of these estimates across seemingly competing brands can also raise questions with respect to the empirical validity of the model.

Being one of the first attempts at testing the theoretical models of competitive price promotions, this work is still incomplete. A natural continuation of this line of research would be to try to develop a more general way for including parameter changes in the model, to operationalize and confirm the explanatory power of the collusion type of models of price dispersion that were cited earlier, and to try to identify the factors that influence equilibrium selection. Another interesting line of work is to test the competitive models of price promotions that allow for at least bimodal densities of the switchers' preferences (Shilony, 1977; Raju, Srinivasan, and Lal, 1990).

28. However, almost all of the brand pairs reject the joint distribution of prices for the two asymmetric-firms versions of the model.

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