Technology Surfing, Uncertainty, and Costs of Transition*

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September, 2021

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Abstract

Technology evolution often leads to situations where both jobs may disappear in some sectors, and other jobs may be created in other sectors. There is uncertainty about the timing of when some jobs disappear and others appear, and there are switching costs for employees to move from one sector to another. This then yields economic growth through technology evolution, costs for workers for the switching costs when having to take on different jobs, and additional costs when alternative technologies are not yet available. This leads to uncertain payoffs to workers over time depending on the realization of uncertainty, with the possibility of particularly poor outcomes when technology replaces existing jobs, without creating new opportunities. This paper presents a model with these features, showing that the frequency of wage crises is increasing in the rate at which industries disappear and in the rate in technological advances that allow capital to substitute labor, and decreasing in the rate at which new jobs are created. A increase in the rate at which jobs are created increases average wages, but decreases the labor factor share. The likelihood of the labor payoffs and economic output not moving together is increasing in the rate at which new jobs are created, and can also be increasing in the rate at which technological advances allow for high paying jobs to disappear. The consideration of the future evolution on the different types of jobs makes labor be more cautious about switching industries than when the future evolution is not considered. The paper discusses the application of the framework to several new technologies, including the recent expansion of the use of robots to replace a variety of jobs.

1. Introduction

Technology evolution often leads to situations where both jobs may disappear in some sectors, and other jobs may be created in other sectors. There is uncertainty about the timing of when some jobs disappear and others appear, and there are switching costs for employees to move from one sector to another.

These developments have occurred since the technological changes in agriculture over the centuries, to the technological changes during the industrial revolution with textile machines or train transportation, for example, to the recent developments in robot automation. These developments seem to occur with various intensities over time which suggests uncertainty over the timing of their occurrence.

This paper develops a model of uncertainty of technological developments with labor having an interest of switching industries in response to those technological developments.[1] The paper includes two dimensions of technological developments. First, there are technological developments that allow machines/capital to substitute for labor in the production of some output. This is, for example, the case of textile machines in the 19th century to substitute for labor (and the resulting Luddite movement), or the recent automation developments.[2] See, for example, Acemoglu (2002) for a discussion of the presence of these technological developments to save on labor costs. These technological developments lead to decreased productivity of labor in those industries, with the resulting reduction in wages if employees continue to choose to stay in those jobs, with jobs disappearing in some cases. As an alternative to the lower wages, labor may also want to switch to jobs that offer greater wages, but this often makes labor incur costs of transition, which may likely include retraining costs.

Second, there are technological developments that permit the appearance of new industries, for which labor can have a high productivity with the associated higher wages. These jobs can be associated with technologies that substituted labor in an existing industry. For example, with the developments of trains in the 19th century, new jobs appeared for the production of railway materials, or for train conductors, With the development of automation nowadays, there is now greater demand for software programmers with the associated higher wages in those positions, But there could also be jobs created by the increased wealth or

[1]The uncertainty of technology developments (and their timing) is discussed, for example, in Mansfield (1968), Pakes (1986), Trajtenberg and Jaffe (2002).

[2]See, for example, Mokyr, Vickers, and Ziebarth (2015), for a discussion of the labor disruptions in the 19th century, and an argument of why those disruptions may have been limited, and especially in the longer term.

productivity in the economy, such as increased productivity in entertainment (for example, performers or sports athletes) or in health related positions. The appearance of these new industries can also be random in its timing, and be either readily available if labor wants to switch industries at some costs of transition, or not be available for some time, with the resulting low wages for some period of time.

The existence of these two types of technological developments can then lead to two different types of costs for labor when having to switch industries because of decreased productivity in the industry in which it is employed. First, having to switch industries makes employees incur costs of transition. Second, if there is no alternative industry with high wages to switch to, labor can be during some (stochastic) period of time with low wages.

The paper presents a model which includes these two types of technological developments and the two possibilities of labor costs resulting from technological developments to illustrate how labor can have stochastically the different types of transaction costs with different duration, The model illustrates rich dynamics of growth, costs of transition for labor, and periods of low wages for labor. We can also generate for various parameter values different expected periods of time with low wages, which could potentially be matched with real-world data. The model can also be seen as presenting a perspective on the recent automation developments as a part of a sequence of successive technological developments that economies have faced over the years, and then trying to evaluate the availability of alternative high-wage industries to effectively measure the effects on labor of the automation effects.

The paper finds that the frequency of wage crises (low wages) is increasing in the rate at which industries disappear and in the rate in technological advances that allow capital to substitute labor, and decreasing in the rate at which new jobs are created. When capital can substitute for labor in high-wage industries, the economy is more likely to get to a state of low wages. When new jobs are created labor is better able to keep high wages, by moving to a new industry, even when capital becomes a substitute for labor in the industry in which labor is currently employed.

Interestingly, although an increase in the rate at which jobs are created increases average wages as there is a lower likelihood of workers being stuck in a low-wage industry, it decreases the labor factor share, as the economy ends up with more industries, with greater payoff for non-labor factors. That is, technological advances that lead to new high-paying jobs benefit labor, but may benefit non-labor factors even more. On the other hand, technological advances that allow capital to substitute for labor in the high-wage industries lower both the average wages in the economy, and the labor factor share.

The model offers interesting dynamics, allowing for situations in which the labor payoffs and the economic output do not move together, with either technological advances which benefit labor but do not increase the economic output, or technological advances which increase the economic output and do not improve the labor payoffs (or can even hurt the labor payoffs). The likelihood of the labor payoffs and economic output not moving together is increasing in the rate at which new jobs are created, and can also be increasing in the rate at which technological advances allow for high paying jobs to disappear.

The consideration of the future evolution on the different types of jobs makes labor be more cautious about switching industries than when the future evolution is not considered. That is, if labor is aware of the subsequent technological advances that may allow capital to substitute for labor in a new industry, labor may be less willing to incur the costs of transition to switch to the new high-paying industry.

There is substantial research on the effect of technological developments on economic growth, and adjustments to the structure of the economy. There has been some research on the uncertainty of the technological progress, including Jensen (1983), which looks at adoption when firms are uncertain about the quality of an innovation, and Balcer and Lippman (1984), which considers when a firm should adopt a new technology given a stochastic evolution of technology. In relation with Balcer and Lippman, this paper present cases in which the evolution of technologies is fully characterized, allows for a stochastic evolution also in the current technology, and interprets the evolution of technologies as labor switching industries, and the effects on labor payoffs depending on the different labor productivity across the existing and potential new industry, and on the costs of transition across industries.

There has also been work on the strategic issues under competition of technology adoption (e.g., Spence 1984) and on the effects of technology spillovers and learning by doing (e.g., Jovanovic and Lach 1989). There has been considerable work on how the heterogeneity of abilities and skill acquisition can adjust to changes in technologies. Chari and Hopenhayn (1991) consider an overlapping generations model where technologies appear over time deterministically, and workers choose the technology that they will use during their lifetime in their first period in the economy. In relation to Chari and Hopenhayn, this paper considers uncertainty in the development of technologies with stochastic payoffs for workers over time depending on how technology evolves. Acemoglu and Restrepo (2018) consider a dynamic model where capital (automation) can be used for some set of tasks, characterizing the equilibrium number of tasks that use capital, and growth, and considering also the effect on the dynamic evolution of the economy after an unanticipated and permanent increase in the relative number of tasks which can be done with capital (tasks which can be automated). In relation to that paper, here we consider successive shocks on new industries (new tasks), and on industries which allow automation, studying the uncertain transition labor dynamics across industries, with effects on the equilibrium wage dynamics across multiple technologies. Adão, Beraja, and Pandalai-Nayar (2019) consider the labor market adjustment to a change in technologies by considering the skill acquisition decisions by workers, and investigating the effects in terms of payoffs for workers with different skills. In relation to that paper, here we investigate successive technology innovations, characterizing the stochastic nature of the hardships for workers. Guerreiro, Rebelo, and Teles (2020) study the optimal taxation of the new technology (interpreted as robots). There is also considerable work on how workers of different abilities sort themselves into different industries, and how automation affects that sorting and workers' payoffs (e.g., Acemoglu and Autor 2011, Acemoglu and Restrepo 2020). Several studies have also studied the effect of automation on the wage structure and employment, such as Krueger (1993), Autor, Levy, and Murnane (2003), Akerman, Gaarder, and Mogstad (2015), Frey and Osborne (2017), Brynjolfsson, Mitchell, and Rock (2018), Graetz and Michaels (2018), and Bessen et al. (2019),

The remainder of the paper is organized as follows. The next Section presents a base model of stochastic evolution of technologies in discrete levels. Section 3 considers the case in which labor adjusts gradually across industries, and Section 4 allows for the possibility of there existing multiple industries waiting to be activated. Section 5 considers the case in which the stochastic evolution of labor productivity across technologies is continuous, and Section 6 concludes.

2. The Model

Consider an economy with N industries where industries disappear at some hazard rate ζ, and where new industries appear at some other hazard rate λ. All industries produce the same output $\widetilde{\Delta}$. So, when there are N industries in the economy, the total output in the economy is $N\widetilde{\Delta}$. Industries are perfectly substitutes in consumption, and the consumption good is the numeraire.

When industries appear they can only produce with labor, requiring one unit of labor to produce $\widetilde{\Delta}$. At some constant hazard rate η technology evolution allows an industry for which output can only be produced with labor to be produced with one unit of capital as well. This captures the uncertainty of technology evolution of when new technologies can replace labor, as a way to lower the production costs. There is some evidence of technology developments targeted at reducing labor costs (for example, see Acemoglu 2002). An example of a technological development that allowed the same output to be produced by either capital or labor is the appearance of robots in the last few decades to replace labor in several industries. Capital once used in an industry, stays associated with that industry (e.g., robots are industry specific) and is destroyed once that industry disappears.

The productivity of labor in a new industry is the output produced $\widetilde{\Delta}$, and supply is perfectly competitive. Therefore the wage obtained in that industry is $w = \widetilde{\Delta}$. In mature technologies, labor has to compete with capital, and there the wage is the rental cost of capital $r+\zeta, w = r+\zeta$, where r is the economy interest rate. We assume $\widetilde{\Delta} > r+\zeta$, and let Δ be the wage differential between the two possible types of industries, $\Delta = \widetilde{\Delta} - (r+\zeta)$. This difference represents the cost for labor when technologies are developed such that capital can substitute for labor in the production of an industry, and if no other new industry is available for labor to switch to. That is, when capital can substitute for capital in a new industry and no other new industry is available for labor to switch to, the wage falls from $\widetilde{\Delta}$ to $r+\zeta$. The technology that allows capital to substitute labor in a certain industry is held by a monopolist and can be copied by a competitive fringe at cost per unit of time of $\widetilde{\Delta} - (r+\zeta)$. That allows the technology monopolist for each industry to have profits of $\widetilde{\Delta} - (r+\zeta)$ per unit of time. The existence of the fringe competitive industry determines the price and profit by the monopolist in the mature industries.

In addition to these potential costs for labor if no new industry is available, we consider also direct costs of transition of labor switching from one industry to another, which we denote by F. These costs can represent the costs of retraining, or the costs for looking for a job in a new industry. We consider that labor can switch industries immediately if it so wishes. We assume that F is sufficiently low such that labor wants to incur these costs F and switch to a high-wage industry when it is in a low-wage industry. This condition is $F < \frac{1}{r+\zeta+\eta}\Delta$. We discuss below how this condition arises and its implications. In Section 3 we consider the case in which labor switching from one industry to another is gradual over time.

As noted above, the possibility of a new industry appearing occurs with hazard rate λ. A new industry only produces with labor. If labor is tied up in another industry in which there is still no technology that allows capital to substitute labor, labor will not switch to the new industry given the costs of job transition, and the new industry does not produce any output. The new industry stays in reserve for labor to switch to when a new technology develops in the industry in which labor is, to allow for capital to substitute for labor. In the base model we allow for at most one industry to stay in reserve. This possibility of an industry being or not being in reserve captures the idea that when technology substitutes labor in an industry, labor may or may not have an alternative industry to go to.

The economy has one unit of labor and capital can be obtained in the capital markets, and has to be remunerated, when used, at r. With N industries, $N-1$ of them will have only as input capital, one unit per industry, and one industry will have only labor as input. Labor supply is inelastic. The treatment of the interest rate r as fixed, determined by the capital markets, can be justified considering the set-up of a small open economy open to capital flows, with a perfectly elastic capital supply. This substantially simplifies the analysis and allows us to concentrate on the dynamics in the labor market. Alternatively, the analysis presented here could be done by replacing capital with a non-labor intermediate input which costs $r+\zeta$ to obtain, and then we would need to adjust the interpretation of output and of some comparative statics. Alternatively, although outside the scope of this paper, it would be interesting to fully consider the capital market inside the economy, endogeneizing the interest rate. There could be interesting effects of the dynamics of the interest rates in that set-up.

If capital can be used for production in all the N industries, labor total costs will be $r+\zeta$, capital total costs will be $(r+\zeta)(N-1)$, and the total profits of the technology monopolists will be $(\widetilde{\Delta}-r-\zeta)N$. If capital can only be used in $(N-1)$ industries, labor total costs will be $\widetilde{\Delta}$, capital total costs will be $(r+\zeta)(N-1)$, and the total profits of the technology monopolists will be $(\widetilde{\Delta}-r-\zeta)(N-1)$. Note that the share of labor costs could easily be considered greater by allowing further units of labor in the model of the economy.

It is interesting to consider the relation of the stylized model considered here to Acemoglu and Restrepo (2018). The new tasks there are the new industries which appear at hazard rate λ here, and the new automation tasks there are the possibility of capital being able to substitute for labor in a new industry here, which occurs at hazard rate η. The existence of a unit mass of tasks in Acemoglu and Restrepo correspond here to the possibility of industries disappearing, which occurs at a hazard rate ζ. Similarly, the intermediary technology monopolists there correspond to the technology monopolists here. Aspects considered here that are not considered in Acemoglu and Restrepo are (1) the uncertainty by which new industries appear and disappear, and by which automation technologies appear, (2) the costs for labor of switching industries, and (3) the possibility of existing industries in reserve.

As noted above, there is some evidence that technology developments are targeted at reducing the costs of production (e.g., Acemoglu 2002). In the context of the model this is being captured by the existence of the hazard rate of technology developments that allow capital to substitute labor in the high-wage industry, η. One could also consider an extension of the model in which η is greater when there is an industry in reserve, so that there is a likelihood of greater savings on labor costs.

2.1. Economic Growth

An economy with N industries can be in one of three states: (i) It can be in a state in which the output in all industries can be produced with capital, such that $w = r + \zeta$ and the total profits in the economy are $N\Delta$. Let $\overline{N}$ denote this state, when it has N industries, and $\overline{S}$ denote that the economy is in this state for any number of industries. (ii) The economy can be in a state in which the output in one of the industries can only be produced with labor, such that $w = \widetilde{\Delta}$, and there is no new potential industry in reserve. Let N_0 denote this state with N industries, and S_0 denote that the economy is in this state for any number of industries. (iii) Finally, the economy can be in a state in which the output in one of the industries can only be produced with labor, such that $w = \widetilde{\Delta}$, and there is one new potential industry in reserve in the case technology allows the industry in which labor is the only possible input, for capital to be also a possible input. Let N_1 denote this state with N industries, and S_1 denote that the economy is in this state for any number of industries.

We can then construct a transition matrix from one state to another during a period of time dt. If the economy is in state $\overline{N}$ it can move to state $(\overline{N}+1)_0$ with probability $\lambda\, dt$, it can move to state $\overline{N-1}$ with probability $\overline{N}\zeta\, dt$, and stay in the same state with the complementary probability. If the economy is in state N_0 it can go to state N_1 with probability $\lambda\, dt$, it can go to state $\overline{N}$ with probability $\eta\, dt$, it can go to state $\overline{N-1}$ with probability $\zeta\, dt$, it can go to state $(N_0-1)_0$ with probability $(N_0-1)\zeta\, dt$, and stay in the same state with the complementary probability. Finally, if the economy is in state N_1 it can go to state $(N+1)_0$ with probability $\eta\, dt$, it can go to state N_0 with probability $\zeta\, dt$, it can go to state $(N_1-1)_1$ with probability $(N_1-1)\zeta\, dt$, and stay in the same state with the complementary probability.

We can then obtain that the expected growth depending on the state can be obtained as $E[\frac{dN}{dt}|\overline{S}] = \lambda - \overline{N}\zeta$, $E[\frac{dN}{dt}|S_0] = -N_0\zeta$, and $E[\frac{dN}{dt}|S_1] = \eta - (N_1-1)\zeta$.

In steady-state, we can obtain the probabilities of being in states $\overline{S}, S_0$, and S_1. The transition matrix among these states can be obtained by noting that from state $\overline{S}$, the economy can go to state S_0 with probability $\lambda\, dt$ during period of time dt, and stay at $\overline{S}$ with the complementary probability, from state S_0 the economy can go to state S_1 with probability $\lambda\, dt$, to state $\overline{S}$ with probability $(\zeta+\eta)dt$, and remain at state S_0 with the complementary probability, and from state S_1 the economy can go to state S_0 with probability $(\zeta+\eta)dt$, and stay at state S_1 with the complementary probability. From this one can obtain the steady-state probabilities of being at states $\overline{S}, S_0$, and S_1, $p_{\overline{S}}, p_{S_0}$, and p_{S_1}, respectively, as

$$p_{\overline{S}} = \frac{(\zeta+\eta)^2}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2}, \tag{1}$$

$$p_{S_0} = \frac{\lambda(\zeta+\eta)}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2}, \tag{2}$$

$$p_{S_1} = \frac{\lambda^2}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2}. \tag{3}$$

One can then obtain the following result about the frequency with which wages are low (state $\overline{S}$).

Proposition 1. *The frequency of low wages is increasing in the hazard rate at which industries disappear (ζ) and in the hazard rate at which capital can become a substitute for labor (η), and decreasing in the hazard rate at which new industries appear (λ).*

As industries disappear at a greater hazard rate, there is a greater likelihood that the industry with high wages disappears, and therefore labor can only be in a low-wage industry. As capital can become a substitute for labor at a greater hazard rate (the likelihood of automation), the higher the probability that a high-wage industry becomes a low-wage industry, leading to a higher frequency of low wages. As new industries appear at a greater hazard rate, the greater the likelihood of there being a high-wage industry in which labor can be employed, leading to a lower frequency of periods of time with low wages.

Using these steady-state probabilities on the different states we can then compute the expected economic growth as

$$E\left[\frac{dN}{dt}\right] = \frac{\lambda(\zeta+\eta)(\zeta+\eta+\lambda)}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2} - N\zeta, \tag{4}$$

which is increasing in λ and η and decreasing in ζ. The effect of λ is by allowing new industries to appear, and the effect of η is by releasing labor from old industries to work in new industries. We could consider that the number of industries is small in the economy compared to the steady-state, such that expected economic growth is positive until the number of industries converges stochastically to around a number of industries in the economy which would make the expected economic growth in the economy equal to zero. Note that if $\zeta = 0$, then there is always positive expected economic growth, with declining growth rates as the number of industries in the economy increases.

Note interestingly that, while an increase in the hazard rate at which capital can substitute labor in an industry (the automation rate) raises economic growth, it affects wages negatively. On the other hand either an increase in the rate at which new industries appear, or a decrease in the rate at which industries disappear, have a positive effect on both the economy and on wages. In the next subsection, after considering the dynamic effects on labor, we study the extent by which the size of the economy and the labor overall payoffs can move in different directions over time.

Let N_{ss} be the number of industries such that the expected economic growth is zero, the number of industries that make (4) equal to zero. We can also then compute the average factor shares in the economy. For example, the average labor factor share is

$$\frac{w}{N_{ss}\widetilde{\Delta}} = \zeta \left[\frac{(\zeta+\eta)(r+\zeta)}{\lambda\widetilde{\Delta}(\zeta+\eta+\lambda)} + \frac{1}{\zeta+\eta} \right]. \tag{5}$$

Of course, this labor share can be made greater by just considering further units of labor in the economy. Note also that the model as presented converges in steady-state to no expected economic growth, but positive expected economic growth can be easily obtained by allowing $\widetilde{\Delta}$ for the new industries to be increasing over time.

We can obtain that the average labor share (and wage) at the steady-state number of industries is increasing in the the rate at which industries disappear. That is, although the frequency of low wages is greater when the rate at which industries disappear is greater, the overall expected wages and labor share are greater because a greater rate at which industries disappear is associated with higher low wages (higher $r+\zeta$). Note that this result on the labor share can also be interpreted with the assumption that the capital in an industry is destroyed when that industry disappears. The role of the rate at which an industry disappears (and the destruction of the associated capital) can also be seen as the role of the depreciation rate of capital in Acemoglu and Restrepo (2018). Furthermore, we can obtain that the average labor share is decreasing in both the rate at which capital can substitute labor and in the rate at which new industries appear. That is, even though the wage bill increases with a greater rate of appearance of new industries, a greater fraction of the gains in the longer term go elsewhere in the economy (technology monopolists), and the average labor share falls with increases in the rate at which industries appear. We state these results in the following proposition.

Proposition 2. *An increase in the hazard rate at which industries disappear (ζ), while increasing the frequency at which low wages occur, increases both the average wages and the labor factor share over time. An increase in the rate at which capital can substitute labor (η) decreases both the average wages and the labor factor share over time. An increase in the rate at which industries appear (λ) increases the average wages but decreases the labor factor share over time.*

It is also interesting to consider the full distribution of how the number of industries can evolve over time. In the steady-state on the probability distribution on states S, during the period of time dt, the economy can grow by one industry with probability $(p_{\overline{S}}\lambda + p_{S_1}\eta)dt$, and decrease by one industry with probability $[(N-1)\zeta + (p_{\overline{S}} + p_{S_0})\zeta]dt$.

This structure captures interesting effects of technological developments. The appearance of new industries (λ) can increase the size of the economy either immediately if labor is underused, or in the future when labor has to switch industries as it becomes underused in another industry. The technology developments that allow for capital to substitute for labor (η) increase economic output immediately if there is a new industry in reserve, or in the future when a new industry appears.

Note that labor is hurt with the transition costs both when it has to switch industries (when its productivity is higher in a new industry than in an existing industry), and when technology developments allow capital to substitute for labor in an industry and there is no industry in reserve, such that wages drop during that period from $\widetilde{\Delta}$ to $r+\zeta$. The next subsection explores the effects on labor.

2.2. Effects on Labor

From the point of view of labor, it is guaranteed to be paid $r+\zeta$ and can earn at most a wage of $\widetilde{\Delta}$. Let us then consider labor's payoff in addition to getting $r+\zeta$ continuously. Let $V(S)$ be the present value of payoffs for labor as function of the state S where $S \in \{\overline{S}, S_0, S_1\}$ in addition to getting $r+\zeta$ continuously, $\frac{r+\zeta}{r}$.[3]

[3] We assume that the costs of transition of changing jobs are only paid when labor switches to a high productivity industry, and are not incurred when labor switches from an industry with low productivity to

We can then obtain

$$
\begin{aligned}
V(\overline{S}) &= \lambda\, dt e^{-r\,dt}[V(S_0) - F] + (1-\lambda\,dt)e^{-r\,dt}V(\overline{S}) && (6)\\
V(S_0) &= \Delta\, dt + (\eta+\zeta)dte^{-r\,dt}V(\overline{S}) + \lambda\, dte^{-r\,dt}V(S_1) + (1-\eta\, dt \\
&\quad -\zeta\, dt - \lambda\, dt)e^{-r\,dt}V(S_0) && (7)\\
V(S_1) &= \Delta\, dt + (\eta+\zeta)dte^{-r\,dt}[V(S_0) - F] + (1-\eta\, dt - \zeta\, dt)e^{-r\,dt}V(S_1). && (8)
\end{aligned}
$$

Making $dt \to 0$, these expressions reduce to

$$
\begin{aligned}
(r+\lambda)V(\overline{S}) &= \lambda[V(S_0) - F] && (9)\\
(r+\eta+\zeta+\lambda)V(S_0) &= \Delta + (\eta+\zeta)V(\overline{S}) + \lambda V(S_1) && (10)\\
(r+\eta+\zeta)V(S_1) &= \Delta + (\eta+\zeta)[V(S_0) - F]. && (11)
\end{aligned}
$$

From this we can obtain

$$
\begin{aligned}
V(S_0) &= \frac{\Delta(r+\lambda)(r+\eta+\zeta+\lambda) - \lambda(\eta+\zeta)(2r+\eta+\zeta+\lambda)F}{r(r+\eta+\zeta+\lambda)^2 - r\lambda(\eta+\zeta)} && (12)\\
V(\overline{S}) &= \frac{\lambda}{r+\lambda}[V(S_0) - F] && (13)\\
V(S_1) &= \frac{\Delta}{r+\eta+\zeta} + \frac{\eta+\zeta}{r+\eta+\zeta}[V(S_0) - F]. && (14)
\end{aligned}
$$

This yields that $V(\overline{S}) < V(S_0) < V(S_1)$. When technology developments allow capital to substitute for labor and there is no new potential industry in reserve, the present value of payoffs of labor falls from $V(S_0)$ to $V(\overline{S})$, and that loss stays on for an expected period of time of length $1/\lambda$. The steady-state probability of being in that state is $p_{\overline{S}}$, which is low when $(\zeta+\eta)/\lambda$ is small. For example, take $\lambda = .4$. For $\zeta+\eta = .2$, we have $p_{\overline{S}} \approx 14\%$, while for $\zeta+\eta = .1$, we have $p_{\overline{S}} \approx 5\%$. Considering the unit of time in years, these latter parameters would suggest that on average there would be a 2.5-year labor crisis due to technological developments depressing wages every 50 years.

For the proposed equilibrium to hold we must have that $V(S_0) - F > 0$, because otherwise labor would prefer not to switch to the high-wage industry from the low-wage industry. This condition reduces to $F < \frac{1}{r+\zeta+\eta}\Delta$, which can be seen as relatively intuitive. This condition

another industry with low productivity, for example because the initial industry got extinguished. We assume that in the small probability event of the economy having zero industries labor is able to get independently an output of $r + \zeta$.

is more likely to be satisfied the less economic agents discount the future, and the less likely the industry is to disappear or for capital to become a substitute for labor. It is interesting to compare how this condition would compare with the case in which labor did not foresee that the capital could become at some point a substitute for labor in the high-wage industry. That condition is $F < \frac{1}{r+\zeta}\Delta$. That is, labor is more likely to switch to the high-wage industry if it is not aware that capital may become a substitute for labor in the high-wage industry at some point. A model that does not consider this possibility may overestimate the likelihood of labor switching to the high-wage industry, and therefore overestimate the costs for labor to switch industries if it tries to fit the likelihood of labor to switch industries to what is observed in the real-world.

When technology developments allow capital to substitute for labor and there is a new industry in reserve, the present value of payoffs of labor falls from $V(S_1)$ to $V(S_0) - F$, which represents the transition costs F of having to switch industries and the risk of further technology developments in the future of capital being able to substitute labor without a new potential industry appearing.

Note also that existing models that consider only a one-time shock of capital substituting for labor in the high-wage industry, and its effects on the workers incurring the transition costs of switching industries to the industry in reserve, do not account for the fact that workers are now in a potentially worse situation of a new potential shock that makes capital be able to substitute for labor in the new high-wage industry, and labor not having now an industry in reserve to switch to.

Similarly, if there is no industry in reserve, a model that only considers a one-time shock of capital being able to substitute labor in the the high-wage industry overestimates the negative effects on labor of this shock, without taking into account that in the future new high-wage industries will appear.

Figures 1-3 illustrate the evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as function of r, η, and λ.

The model offers rich dynamics about the evolution of the economy, wages, and the present values of employees' payoffs. In fact, we can have both situations in which the economy and the workers' payoffs move in the same direction, and situations in which the economy and the workers' payoffs move in opposite directions, or do not move together. One possibility is that the economic output moves in a positive direction in relation to the workers' payoffs. This can occur when (1) it becomes possible for capital to substitute labor (in the high-wage industry), or (2) when there is an industry in reserve, and the industry

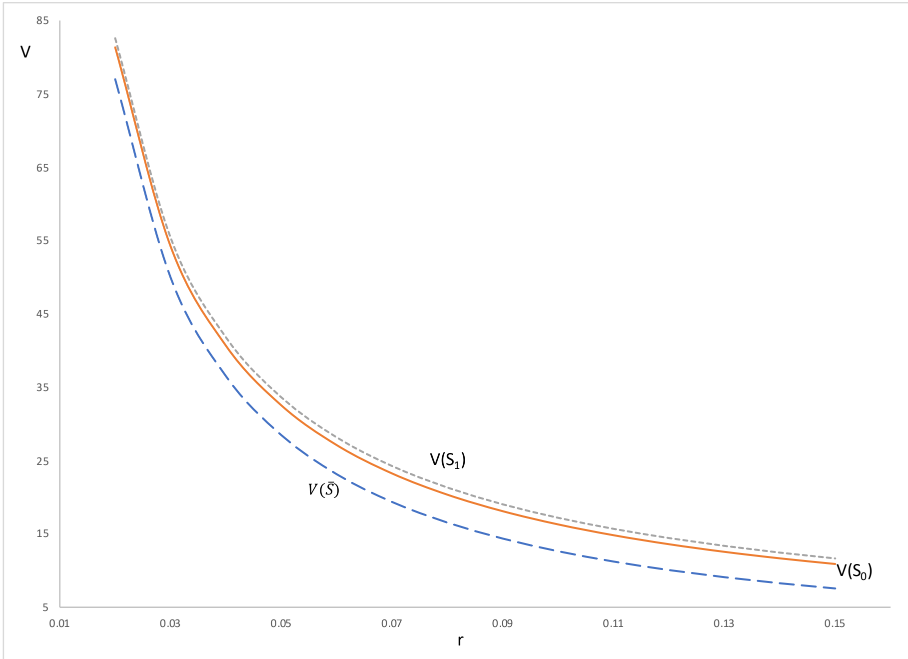

Figure 1: Evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as a function of r for $\Delta = 2$, $\lambda = .4, \eta = .2$, and $F = .5$.

with high wages disappears. In the former case, the economic output stays constant and wages fall if there is no industry in reserve, or the economic output increases with labor incurring the costs of transition and having lower expected present value of payoffs if there is an industry in reserve. In the latter case, the economic output and wages stay constant, but the expected present value of labor payoffs falls. The probability of the economic output moving in a positive direction in relation to the workers' payoffs given that there is a change in the structure of the economy can be computed to be

$$\gamma_{EW} = \frac{\eta(p_{S_0} + p_{S_1}) + \zeta p_{S_1}}{\zeta N_{ss} + \eta(p_{S_0} + p_{S_1}) + \lambda(p_{\overline{S}} + p_{S_0})}. \tag{15}$$

Note also that if wages are low, and a new industry appears, the economic output increases, and workers see their wages rise, although having to pay the costs of transition. Overall, the workers' present values of payoffs improve because $V(S_0) - F > V(\overline{S})$, given (13).

Another possibility is that the economic output moves in a negative direction in relation to the workers' payoffs. This can occur (1) if a low-wage industry disappears, or (2) we

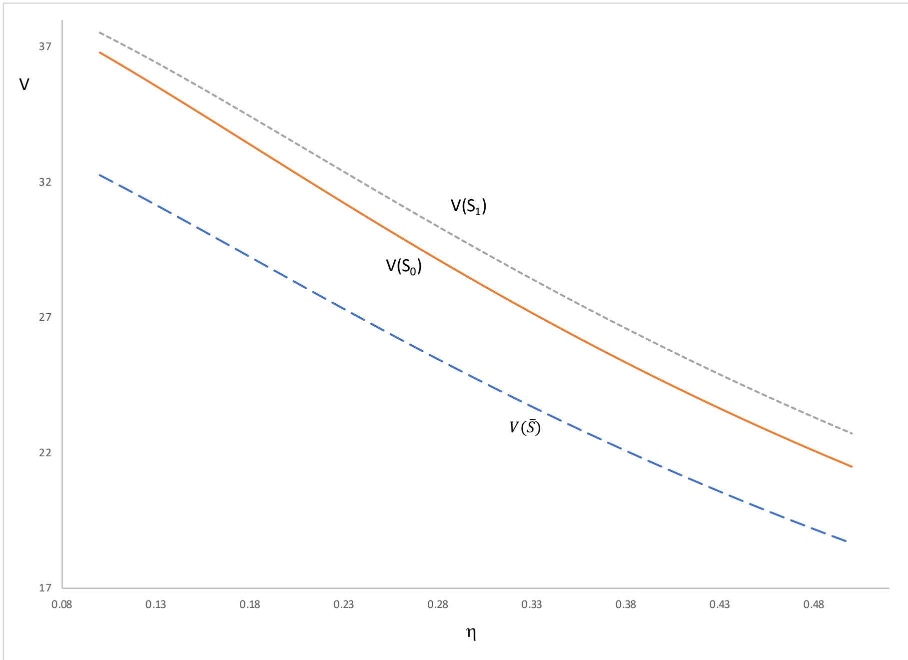

Figure 2: Evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as a function of η for $\Delta = 2$, $\lambda = .4, r = .05$, and $F = .5$.

are in a state with a high-wage industry and no industry in reserve (state S_0) and a new industry appears. In the former case, the economic output falls, but there is no impact on labor's payoffs. In the latter case, in which the economic output and wages stay constant, the expected present value of the workers' payoffs increases, $V(S_1) > V(S_0)$. The probability of the economic output moving in a negative direction in relation to the workers' payoffs given that there is a change in the structure of the economy can be computed to be

$$\gamma_{WE} = \frac{\zeta(N_{ss} - p_{S_0} - p_{S_1}) + \lambda p_{S_0}}{\zeta N_{ss} + \eta(p_{S_0} + p_{S_1}) + \lambda(p_{\overline{S}} + p_{S_0})}. \tag{16}$$

We can then obtain the following results about the joint evolution of the economy and of the workers' payoffs.

Proposition 3. *Given that there is a change in the structure of the economy, the likelihood of a relative shock in favor of labor payoffs in relation to the economic output is greater than the likelihood of a relative shock in favor of the economic output in relation to labor payoffs ($\gamma_{WE} > \gamma_{EW}$). The likelihood of the economy and the workers' payoffs not moving*

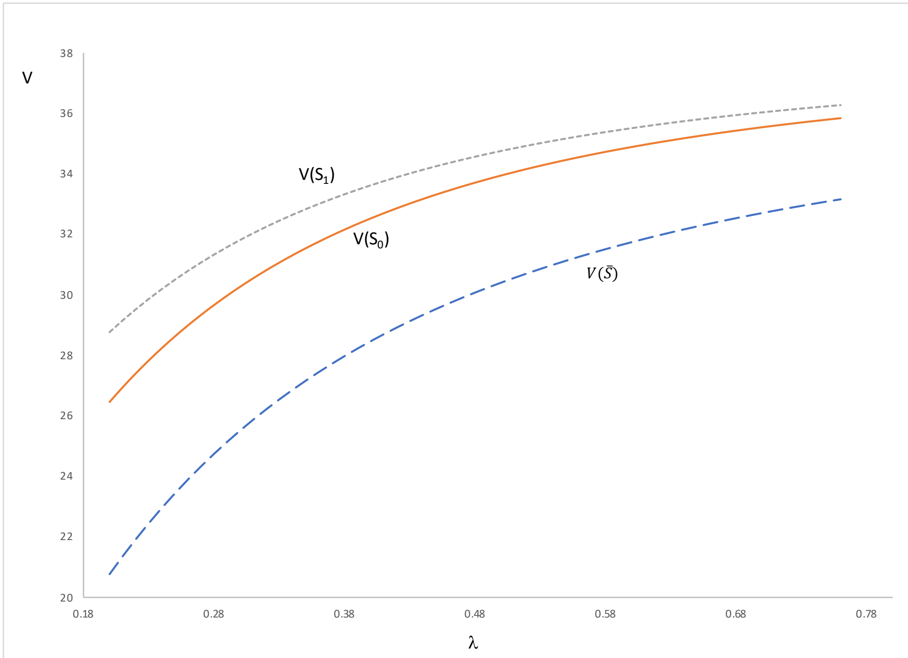

Figure 3: Evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as a function of λ for $\Delta = 2$, $\eta = .2, r = .05$, and $F = .5$.

together is increasing in the hazard rate of appearance of new industries, and is decreasing in the hazard rate at which industries disappear and increasing in the hazard rate at which capital can substitute labor in an high-wage industry if the hazard rate of appearance of new industries is not too high.

A greater hazard rate of the appearance of new industries means that the economy is more frequently in the state in which it has an industry in reserve (state S_1), which is the state in which it is more likely for changes in the economy to result in the economy and the workers' payoff not moving together, either because a low-wage industry disappears (in which case, the economic output falls, but the payoff of the workers remains stable), or the high-wage industry disappears (in which case, the economic output remains stable because of the industry in reserve, but the workers' present value of payoffs falls), or capital becomes a substitute for labor in the high-wage industry (in which case, the economic output increases, and the workers' present value of payoffs falls).

When the rate at which new industries appear is not too high, the likelihood of the economy being in states without an industry in reserve (states $\overline{S}$ and S_0) is higher, and then an increase in the hazard rate at which industries disappear, reduces the steady-state number of industries, and increases the likelihood of the industry output and the workers' payoffs moving together in the states in which there is no industry in reserve. In the state in which wages are low (state $\overline{S}$), the more likely change is the one in which the economic output and the workers' payoff move together through the appearance of a new industry. In the state in which wages are high but there is no industry in reserve, an increase in the hazard rate of industries disappearing, makes the likelihood of the industry of high wages disappearing greater, in which case both the economic output, the wages, and the present value of the workers' payoffs fall. When the rate at which new industries appear is high, the state in which there is an industry in reserve (state S_1) is now more likely, and then the effect of an increase in the rate at which industries disappear is now more affected by the possibilities in that state, which lead to the economy output and workers' payoffs not to move together.

The effect of an increase in the hazard rate at which capital can substitute labor in the high-wage industry can lead, as expected, to a greater likelihood that economic output and workers' payoffs do not move together, as the effect of capital substituting labor is to lower wages or for labor to incur costs of transition, while economic output can stay constant or actually grow. However, this only occurs if the rate at which industries appear is not too high. If the rate at which industries appear is very high, an increase in the rate at which capital can substitute labor increases the likelihood of the low-wage state (state $\overline{S}$), in which case economic output and workers' payoff can move together when a new industry appears.

Note that these effects on labor considered only the effect through wages and through the costs of switching industries. One could also consider that labor has shares on the technology monopolists such that it also benefits when the technology monopolists improve their payoffs, alleviating somewhat any negative effects on wages, or the transition costs incurred when switching industries. That is, the effects considered here are exclusively the ones of the labor owners. If workers also have shares on the technology monopolists (or on capital), then capitalization could also be beneficial to workers, as obtained in Acemoglu and Restrepo (2018) that the welfare of the representative agent increases with capitalization.

Another effect not considered here is that the extra income received by the technology monopolists could increase demand for industries only produced with labor (the new industries here), providing then extra benefits for labor. Finally, in order to better obtain the intuition for the effects modeled, we consider that all workers are able to switch to high-wage industry with the same costs of transition F. In the real-world, workers may have different costs of transition, with some potentially with very high costs of transition (which could be called low-skilled workers), such that they never switch to the high-wage industry. The next Section considers a variation on that set-up, in which workers are only able to switch to the high-wage industry after some time, with that time of transition being heterogeneous across workers. Workers that can only switch after a long period of time, are more hurt by the industry in which they are employed being capitalized.

The assumption of constant returns to scale of labor, although simplifying the analysis, may also limit in the model the benefits for labor of the creation of one industry. If there diminishing returns to scale of labor in an industry, the creation of a new industry would allow some labor to switch to the new industry and increase overall wages.

3. Gradual Labor Adjustments

In the previous Section we considered that labor could switch industries immediately by paying fixed costs F. In practice, labor may not be able to switch industries immediately, and, in fact, may only be able to switch industries gradually over time, potentially because of the need to develop human capital for a new industry. We consider in this Section the possibility of labor only being able to switch industries gradually, without fixed costs of switching, at a constant hazard rate α. This captures the idea that labor may only have occasional opportunities to change industries, when it has interest in doing so. We consider in this Section that labor consists of a continuum of workers with mass one.

Let $x(t)$ be the mass of workers who switched to the new industry at time t after that new industry appeared. Given the constant hazard rate α we have $x(t) = 1 - e^{-\alpha t}$. Note that in this case all industries will have some workers, with the older industries with fewer workers, all getting a wage $w = r + \zeta$.

An economy with N industries, where one industry's output can only be produced with labor, will then have an output of $[N-1+x(t)]\widetilde{\Delta}$ at time t after that new industry appeared. Note that with that set of industries the economy is growing at the rate $\alpha\frac{1-x}{N-1-x}$. Note that if a new technology is developed in such an economy, such that in that industry in which output could only produced with labor, output can also be produced with capital (which occurs at a hazard rate η), the total output in the economy would jump to $N\widetilde{\Delta}$. This could be seen as representing the large possible economic growth resulting from the new technology being developed.

We can also compute in this case the expected economic growth in this economy. Let Y represent the economy's output. Then we can obtain

$$
\begin{aligned}
E\left[\frac{dY}{dt}|\overline{S},N\right] &= -N\zeta\widetilde{\Delta} &\quad (17)\\
E\left[\frac{dY}{dt}|S_0,N,x\right] &= [\alpha(1-x)-N\zeta]\widetilde{\Delta} &\quad (18)\\
E\left[\frac{dY}{dt}|S_1,N,x\right] &= [\eta(1-x)-N\zeta]\widetilde{\Delta}. &\quad (19)
\end{aligned}
$$

We can then obtain the overall expected economic growth as

$$
E\left[\frac{dY}{dt}|N,x\right] = \left[\lambda\frac{\alpha(1-x)(\zeta+\eta)+\eta(1-x)\lambda}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2} - N\zeta\right]\widetilde{\Delta}. \quad (20)
$$

This yields that the expected economic growth in the short-run is greater the lower the fraction of workers in the high-wage industry, which we state in the following proposition.

Proposition 4. *In the gradual labor transition case, the expected economic growth in the short run is greater the lower the fraction of workers in the high-wage industry, x, and this effect is greater the greater the hazard rate at which workers transition to the high-wage industry, and the greater the hazard rate at which capital becomes able to substitute for labor in the high-wage industry.*

The effect on the expected economic growth of the fraction of workers in the high-wage industry occurs through two mechanisms. First, a greater fraction of workers on the high-wage industry means that there are fewer workers to transition from the low-wage to the high-wage industry, and therefore the expected short-run economic growth is smaller. Second, a greater fraction of workers on the high-wage industry means that capital becoming a substitute for labor in that industry just increases economic growth on the unused potential in that industry, $\eta(1-x)$, which is now smaller.

Similarly to the previous Section, we can consider the effects on labor of the different types of technology developments across different states. Note that now in state S_0 the worker can be either employed in the high-wage industry or in a low-wage industry, which we denote by state S_{0Y} or state S_{0N}, respectively. In the same way, in state S_1 the worker can now be either employed in the high-wage industry or in a low-wage industry, which we denote by state S_{1Y} or state S_{1N}, respectively.

Let $\overline{V}$ be a vector of dimension five, which has in the first element $V(\overline{S})$, in the second element $V(S_{0N})$, in the third element $V(S_{0Y})$, in the fourth element $V(S_{1N})$, and in the fifth element $V(S_{1Y})$. Let A be the transition matrix (5×5) between the five different states, ordered as in the vector $\overline{V}$, with general element a_{ij} representing the probability of transitioning from state i to state j during the period of time dt. We then have

$$A = \begin{pmatrix} (1-\lambda\,dt) & \lambda\,dt & 0 & 0 & 0 \\ (\zeta+\eta)\,dt & 1-(\zeta+\eta+\alpha+\lambda)\,dt & \alpha\,dt & \lambda\,dt & 0 \\ (\zeta+\eta)\,dt & 0 & 1-(\zeta+\eta+\lambda)dt & 0 & \lambda\,dt \\ 0 & (\zeta+\eta)dt & 0 & 1-(\zeta+\eta+\alpha)dt & \alpha\,dt \\ 0 & 0 & (\zeta+\eta)dt & 0 & 1-(\zeta+\eta)dt \end{pmatrix}.$$

Letting U be a vector of dimension five with the current wages (the premium over $r+\zeta$) per unit of time depending on the state, $U^T = [0\ \ 0\ \ \Delta\ \ 0\ \ \Delta]$, we can write[4]

$$\overline{V} = e^{-r\,dt} A\overline{V} + U\,dt. \tag{21}$$

We can then obtain $\overline{V} = [I - e^{-r\,dt}A]^{-1}U\,dt$, where I represents the identity matrix. Making $dt \to 0$ we can obtain

$$\overline{V} = \widetilde{A}^{-1}U, \tag{22}$$

which determines the value function at the different states, and where

$$\widetilde{A} = \begin{pmatrix} r+\lambda & -\lambda & 0 & 0 & 0 \\ -(\zeta+\eta) & r+\alpha+\zeta+\eta+\lambda & -\alpha & -\lambda & 0 \\ -(\zeta+\eta) & 0 & r+\zeta+\eta+\lambda & 0 & -\lambda \\ 0 & -(\zeta+\eta) & 0 & r+\alpha+\zeta+\eta & -\alpha \\ 0 & 0 & -(\zeta+\eta) & 0 & r+\zeta+\eta \end{pmatrix}.$$

We can then obtain that, as expected, $V(\overline{S}) < V(S_{0N}) < V(S_{1N}) < V(S_{1Y})$, and that $V(S_{0N}) < V(S_{0Y}) < V(S_{1Y})$ as workers benefit from the existence of higher wage industries, of being employed in higher wage industries, and of there being higher wage industries in reserve. The relation between $V(S_{0Y})$ and $V(S_{1N})$ depends on the parameter values. For example, if λ is small, or α is high, we can have $V(S_{1N}) > V(S_{0Y})$. That is, a worker can be better off if not employed in a high-wage industry, but be in a state where there is a new

[4]Where the notation X^T means the transpose of matrix X.

industry in reserve than if employed in a high-wage industry and be in a state in which there is no new industry in reserve, if the likelihood of new industries is relatively low (low λ), or if the transition to a high-wage industry is relatively fast (high α). In the case of low λ, having a new industry in reserve is very valuable as new industries are rare to appear. In the case of high α, being in a low-wage industry when a high-wage industry is available is not too costly as the worker understands that the transition to the high-wage industry will occur relatively quickly.

Figures 4-8 illustrate how the value function at the different states evolve as a function of Δ, r, η, λ, and α. Note that Figures 7 and 8 illustrate that $V(S_{1N}) > V(S_{0Y})$ for λ small or α large.

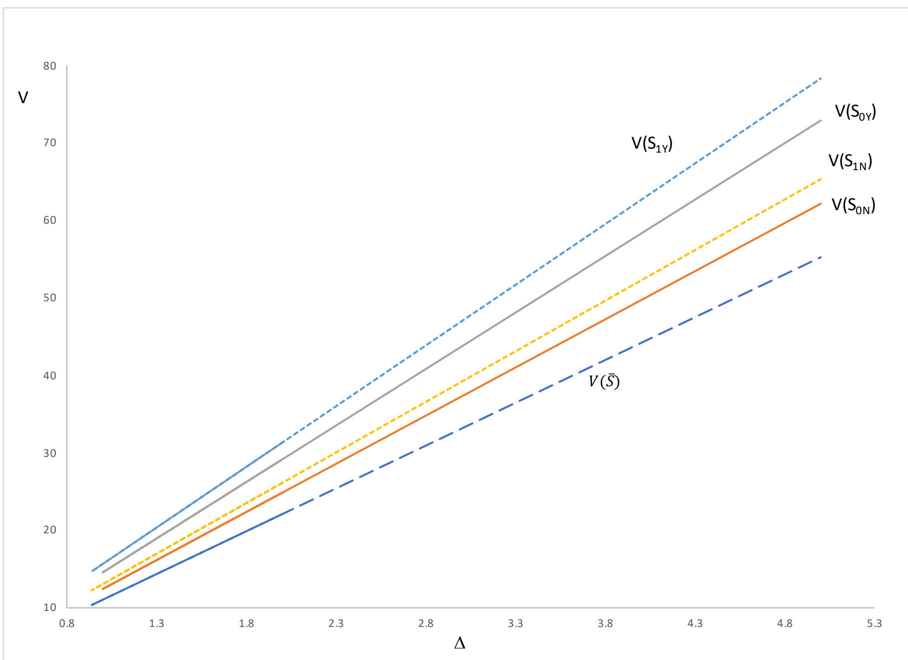

Figure 4: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(_{S1N}$, and $V(S_{1Y})$, as a function of Δ for $r = .05$, $\lambda = .4, \eta = .2$, and $\alpha = .3$.

It is also interesting to investigate the extent of time that workers end up in each state. Letting $\overline{p}$ be the row vector of steady-state probability of being in each state (with same order of states in the different elements of $\overline{p}$ as in $\overline{V}$), we know that $\overline{p}$ is defined by $\overline{p}A = \overline{p}$. This yields

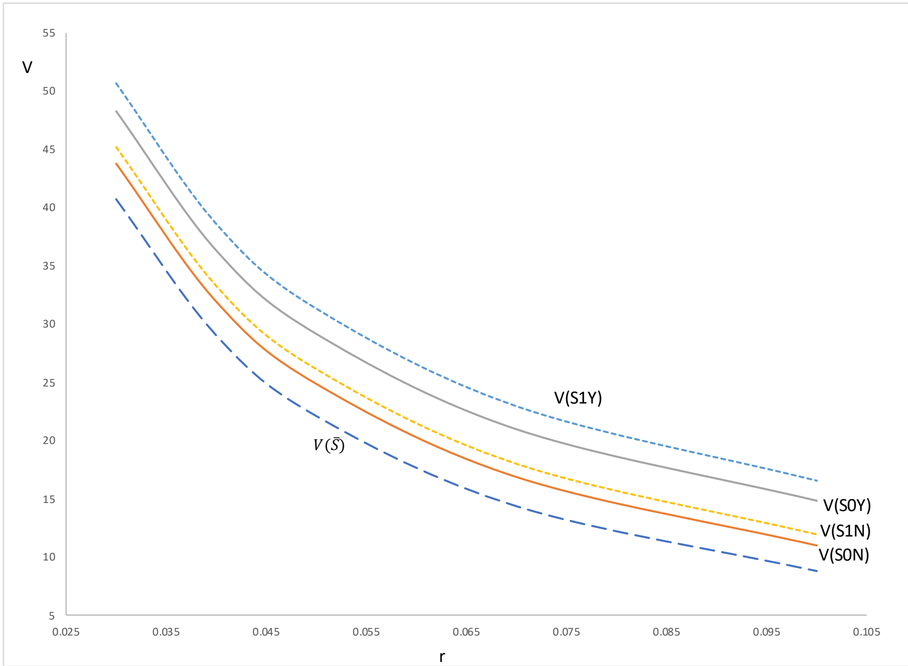

Figure 5: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(_{S1N}$, and $V(S_{1Y})$, as a function of r for $\Delta = 2$, $\lambda = .4, \eta = .2$, and $\alpha = .3$.

$$
\begin{aligned}
p_{\overline{S}} &= \frac{(\zeta+\eta)^2}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2} && (23)\\
p_{S_{0N}} &= \frac{\lambda(\zeta+\eta)}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{(\zeta+\eta)(\alpha+\zeta+\eta)}{(\zeta+\eta)^2+\alpha[\lambda+\alpha+2(\zeta+\eta)]} && (24)\\
p_{S_{0Y}} &= \frac{\lambda(\zeta+\eta)}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{\alpha(\alpha+\lambda+\zeta+\eta)}{(\zeta+\eta)^2+\alpha[\lambda+\alpha+2(\zeta+\eta)]} && (25)\\
p_{S_{1N}} &= \frac{\lambda^2}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{(\zeta+\eta)^2}{(\zeta+\eta)^2+\alpha[\lambda+\alpha+2(\zeta+\eta)]} && (26)\\
p_{S_{1Y}} &= \frac{\lambda^2}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{\alpha[\alpha+\lambda+2(\zeta+\eta)]}{(\zeta+\eta)^2+\alpha[\lambda+\alpha+2(\zeta+\eta)]}. && (27)
\end{aligned}
$$

The likelihood of the worker having a low wage can be obtained to be $p_{\overline{S}} + p_{S_{0N}} + p_{S_{1N}}$ which is increasing in ζ and η, and decreasing in α. For $\zeta + \eta = .1, \lambda = .4$, and $\alpha = .3$, we would have that workers would have a low wage for around 10% of the time. With $\lambda = .3$, workers move to a high-wage industry if available, on average, in a little over three years.

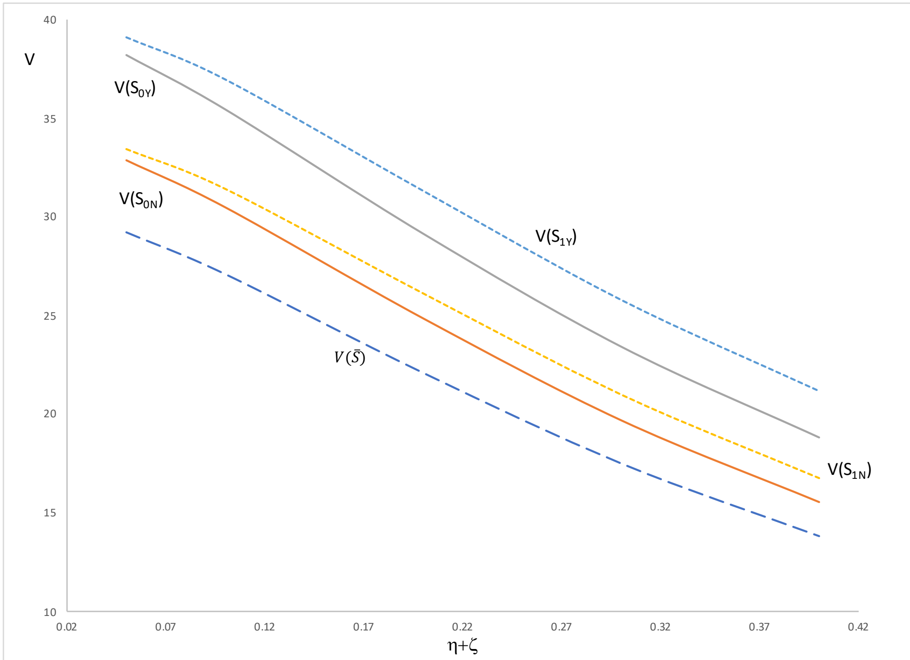

Figure 6: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(_{S1N}$, and $V(S_{1Y})$, as a function of η for $r = .05$, $\lambda = .4, \Delta = 2$, and $\alpha = .3$.

4. Multiple Industries in Waiting

The analysis above considered that there is at most one industry in reserve when there is a high-wage industry in the economy. This could be potentially justified by the technology efforts potentially decreasing when there is already a potential new industry in reserve, and by capturing the main effects of having or not having industries in reserve. In this Section we return to the model of Section 2 and consider now the possibility of there being multiple industries in reserve.

Let M be the number of industries available such that labor is the only input. The case of $M = 0$ corresponds to state $\overline{S}$, the case of $M = 1$ corresponds to state S_0, and $M = 2$ corresponds to state S_1, and we also now allow for $M > 2$ as well.

We concentrate on investigating the effects on labor. Suppose that $M \geq 2$. Then, the

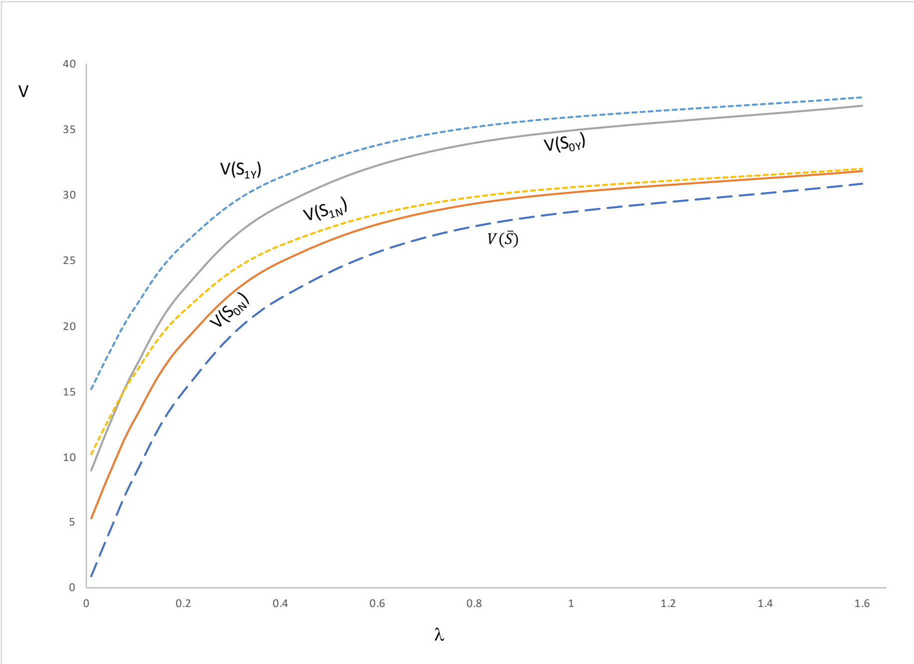

Figure 7: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(_{S1N}$, and $V(S_{1Y})$, as a function of λ for $r = .05$, $\Delta = 2, \eta = .2$, and $\alpha = .3$.

expected present value of payoffs for labor has to satisfy

$$V(M) = \Delta\, dt + (\zeta+\eta)dte^{-r\,dt}[V(M-1)-F] + \lambda\, dte^{-r\,dt}V(M+1) + [1-(\zeta+\eta+\lambda)dt]e^{-r\,dt}V(M), \tag{28}$$

which, when making $dt \to 0$, reduces to

$$(r+\zeta+\eta+\lambda)V(M) = \Delta - (\zeta+\eta)F + (\zeta+\eta)V(M-1) + \lambda V(M+1). \tag{29}$$

Similarly, we can obtain

$$(r+\lambda)V(0) = \lambda V(1) - F \tag{30}$$

$$(r+\zeta+\eta+\lambda)V(1) = \Delta + (\zeta+\eta)V(0) + \lambda V(2). \tag{31}$$

From (29), we can get obtain the steady-state as $V^* = \frac{\Delta-(\zeta+\eta)K}{r}$. The characteristic

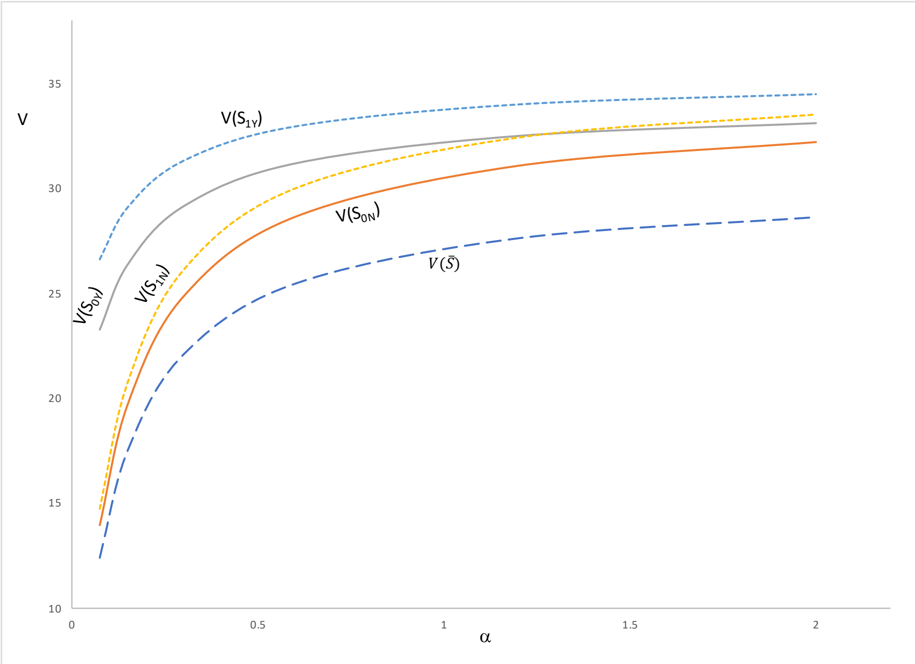

Figure 8: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(_{S1N}$, and $V(S_{1Y})$, as a function of α for $r = .05$, $\lambda = .4, \eta = .2$, and $\Delta = 2$.

equation associated with the recursive equation (29) has solutions

$$\phi_1 = \frac{1}{2} + \frac{r+\zeta+\eta}{2\lambda} + \frac{\sqrt{[\lambda-(r+\zeta+\eta)^2]+4r\lambda}}{2\lambda} \tag{32}$$

$$\phi_2 = \frac{1}{2} + \frac{r+\zeta+\eta}{2\lambda} - \frac{\sqrt{[\lambda-(r+\zeta+\eta)^2]+4r\lambda}}{2\lambda}. \tag{33}$$

For $V(N)$ to converge, we have

$$V(N) = V^* + C\phi_2^N \tag{34}$$

for $N \geq 1$ where C is a constant to be determined. Using (30) and (31) we can obtain

$$C = (\zeta+\eta)\frac{\Delta-(r+\lambda+\zeta+\eta-1)F}{\lambda(r+\lambda)\phi_2^2-\phi_2[(r+\lambda)^2+r(\zeta+\eta)]}, \tag{35}$$

from which we can obtain $C < 0$, and

$$V(0) = \frac{\lambda(V^*+C\phi_2)-F}{r+\lambda}. \tag{36}$$

It is now interesting to investigate the likelihood to be in each state in steady-state. For there to be a steady-state we need $\zeta + \eta > \lambda$, which is assumed in the remainder of this Section. Letting p_M represent the steady-state probability of being in state M, we have

$$
\begin{aligned}
(1 - \lambda\, dt)p_0 + (\zeta + \eta)dt p_1 &= p_0 && (37)\\
\lambda\, dt p_{M-1} + [1 - (\lambda + \zeta + \eta)dt]p_M + \eta\, dt p_{M+1} &= p_M \text{ for } M \geq 1, && (38)
\end{aligned}
$$

which yields

$$
p_M = \frac{\zeta + \eta - \lambda}{\zeta + \eta} \left(\frac{\lambda}{\zeta + \eta} \right)^M \tag{39}
$$

for all M. The fraction of time in which labor would be with low wages would then be $\frac{\zeta+\eta-\lambda}{\zeta+\eta}$, from which we can obtain the relatively expected comparative statics that the fraction of time with lower wages increases with the likelihood of an industry disappearing (ζ) and the likelihood of technological advances permitting capital to substitute labor in the output of the industry (η), and decreases with the likelihood of technological advances generating the appearance of new possible industries (λ).

This model further illustrates that the situation of the economy and potential growth and wage dynamics do not depend only on the state of the economic output or wages, but may depend crucially on the number of industries in reserve. If the extent of cumulative R&D expenditures, or unused patents, is a proxy for the number of industries in reserve, those measures could be used to predict future medium term economic growth and the likelihood of entering a period of low wages.

5. Continuous Technological Innovations

In the previous Sections we considered that technological advances generated discrete changes in the productivity of labor, for either new industries or for the appearance of new industries. We consider here the possibility that technological advances are smoother, which generates the question of when labor decides to switch industries depending on the relative wages across industries. The presentation in this Section is focused on the question of when labor wants to switch industries and the payoff for labor, and we can see the industry from which labor switches as the industry that becomes then automated.

In particular, consider that the productivity of the industry where labor is employed follows a Brownian motion with negative drift, given the technological advances to reduce dependence on labor in that industry (Acemoglu, 2002). Furthermore, consider that the productivity of the potential new industry follows a Brownian motion with positive drift, given the technological advances allowing for greater productivity in new industries. Let x_0 be the wage in the industry where labor is employed, and x_1 be the wage in the industry to which labor could switch to. Let the labor cost of transition from one industry to the other be F. We have

$$\begin{aligned} dx_0 &= -h\,dt + \sigma\,dW_0 && (40) \\ dx_1 &= g\,dt + s\,dW_1, && (41) \end{aligned}$$

where $h, g > 0$, $\sigma > 0$ represents the variability in the stochastic process of the productivity in the existing industry, $s > 0$ represents the variability in the stochastic process of the productivity of the new potential industry, and W_0 and W_1 are independent standardized Brownian motions.

Let $V(x_0, x_1)$ be the expected present value of payoffs for labor if it is employed in an industry with productivity x_0, and the productivity of the new potential industry is x_1. When labor switches at (x_0^*, x_1^*) from the old industry to the new industry, we then have that $x_0 = x_1^*$, and $x_1 = x_1^* - \beta$, for the next problem of when to switch industries.

In this case labor gets hurt stochastically if technological advances lead both to capital being a good substitute for labor in the old industry (x_0 falls), and not enough productivity growth in the new industry (x_1 stays low).

The Bellman equation for $V(x_0, x_1)$ can be represented by

$$V(x_0, x_1) = x_0 + e^{-r\,dt} EV(x_0 + dx_0, x_1 + dx_1). \tag{42}$$

Applying Itô's Lemma this reduces to the partial differential equation

$$rV(x_0, x_1) = x_0 - hV_1(x_0, x_1) + gV_2(x_0, x_1) + \frac{\sigma^2}{2} V_{11}(x_0, x_1) + \frac{s^2}{2} V_{22}(x_0, x_1), \tag{43}$$

where V_i and V_{ii} represent the first and second derivative of the function V with respect to the i-th argument of V.

Let $u(x_1)$ be the value of x_0 at which labor decides to switch to the new industry when the productivity of the new potential industry is at x_1. Then value matching and smooth pasting at $x_0 = u(x_1)$ yields

$$
\begin{aligned}
V(u(x_1), x_1) &= V(x_1, x_1 - \beta) - F & (44)\\
V_1(u(x_1), x_1) &= 0 & (45)\\
V_2(u(x_1), x_1) &= V_1(x_1, x_1 - \beta) + V_2(x_1, x_1 - \beta) - F. & (46)
\end{aligned}
$$

It turns out that in this case we can have that the form of $V(x_0, x_1)$ satisfies $V(x_0, x_1) = x_0/r + W(x_1 - x_0)$. Using this we can obtain that (43) reduces to

$$
rW(a) = -\frac{h}{r} + (h+g)W'(a) + \frac{\sigma^2 + s^2}{2} W''(a), \tag{47}
$$

where $a = x_1 - x_0$, that (44)-(45) reduce to

$$
\begin{aligned}
W(\overline{a}) &= W(-\beta) + \frac{\overline{a}}{r} - F & (48)\\
W'(\overline{a}) &= \frac{1}{r}, & (49)
\end{aligned}
$$

respectively, where $x_1 - u(x_1) = \overline{a}$, for all x_1, and that (46) is always satisfied given (49). Note that this means that labor switches jobs when $x_1 - x_0$ reaches the threshold $\overline{a}$, and that this can occur at either low or high wages, x_1 and x_0.

Let $\mu = \frac{1}{\sigma^2 + s^2}\left(\sqrt{(h+g)^2 + 2r(\sigma^2 + s^2)} - (h+g)\right)$, which is positive. Note further that $\lim_{a \to -\infty} W(a) = 0$, as when x_0 is infinitely greater than x_1 labor only switches to the new industry at infinity. We can then obtain, using (47)-(49) that

$$
W(a) = \frac{1}{r\mu} e^{\mu(a - \overline{a})}, \tag{50}
$$

where $\overline{a}$ is obtained by

$$
\mu\overline{a} + e^{-\mu(\beta + \overline{a})} = 1 + r\mu F. \tag{51}
$$

This then yields comparative statics of the threshold $\overline{a}$ with respect the different model parameters, which are stated in the following proposition.

Proposition 5. *Consider the gradual evolution of technologies model. Then the threshold $\overline{a}$ of the difference of the productivity of industries for labor to decide to switch industries is increasing in the costs of transition F, the lag of the new technology β, the interest rate r, the absolute value of the drifts of the old and new technology, h and g, and in the uncertainty of the evolution of the old and new technology, σ^2 and s^2.*

As expected, when the cost of transition increases labor requires a bigger difference in salaries for labor to decide to switch industries. Similarly, when the lag of the new technology, β, is greater, labor realizes that it will take longer to move on to the technology after the next one, and requires a further difference is salaries to be willing to switch industries. A greater interest rate makes also the present value of the benefits of switching lower, which then leads labor to only switch industries if the difference between wages is greater.

Potentially more interesting, the difference of wages required to switch industries increases in the speed with which the new technology improves in relation to the old technology (greater h and g), as then labor will not have to switch industries as frequently. Similarly, as the uncertainty in the evolution of technologies increases (greater σ^2 and s^2), labor requires a greater difference in wages in order to switch industries, as there is a greater chance that the difference in wages gets reduced.

We can also obtain that $V(x_0, x_1)$ is increasing in both the labor productivity obtained in the current industry and the labor productivity that could be obtained in the new industry. That is, labor can be in periods of low present value of payoffs if both x_0 and x_1 get a sufficient number of negative shocks. That is, the situation of occasional labor hardship could be explained by negative shocks on the labor productivity of both the current industry and the potential new industry. This would also indicate that given that we may know that labor productivity in the current industries in which labor in employed received a negative shock, the true state of labor hardship would depend on the evaluation of labor productivity in potential new industries.

Note that if $x_1 - x_0$ is close to $\overline{a}$, the present value of payoffs for labor is more increasing in the productivity of the new industry, x_1, than in the productivity of the current technology, x_0. That is, for $x_1 - x_0$ close to $\overline{a}$ we can obtain $\frac{\partial V}{\partial x_1} > \frac{\partial V}{\partial x_0}$. As we get closer to a situation when labor may want to switch industries, labor actually benefits more in the productivity in the new industry, as it accelerates the move to the industry, than in the productivity of the current industry, which delays the switch to the new industry.

Using standard methods (see Appendix) we can also obtain the expected time period until the next time that labor switches industries given the difference of productivities a, $T(a)$, as

$$T(a) = \frac{\overline{a}}{h+g} e^{\frac{2(h+g)}{\sigma^2+s^2}(\overline{a}-a)} - \frac{a}{h+g}. \tag{52}$$

We can then also obtain the expected time period between labor industry switches as $T(-\beta)$. Considering the empirical distribution of time period of labor industry switches, we can potentially compare it with the model prediction.

6. Conclusion

This paper presents a stochastic perspective on the evolution of technologies, with the resulting effects for the costs of transition and wages. Technology evolution can have two dimensions: First, it can lead to the ability to use capital as a production input to substitute labor. This can be seen as an example of the recent automation developments. Second, technology may allow new industries to appear, that have high labor productivity with the associated high wages. The uncertainty of these two dimensions of the evolution of technologies leads to the potential for stochastic hardships for labor. There are two forms of the labor negative effects considered. On one hand, there could be direct costs of transition from one industry to another, which can often involve the cost of development new skills. On the other hand, there could be the possibility of new industries with high wages not being yet available to switch to once jobs in the existing industries start having lower wages because of new technologies that allow labor input to be substituted with capital.

The model permits an interpretation of the recent automation developments as a stochastic event that is part of a sequence of successive technology developments, and not be a unique event per se. The model also indicates that costs for labor of these automation developments depend on the extent to which new alternative high paying jobs are created, which again may depend on the evolution of technologies.

The model is quite stylized without considering heterogeneity of abilities among workers, or the skill acquisition investments that workers can potentially engage in. This leads to the dramatic effect that labor can move as a whole from one industry to another, without including the possibility that all industries have labor (except for the analysis in Section 3). It would be interesting to consider in future research heterogeneity of abilities across workers and the modeling of skill acquisition investments,

It would also be interesting to measure the effects of different technological disruptions in the last two centuries to better characterize the stochastic processes of the input substitution technological innovations and of the appearance of new industries or occupations, and how the realization of these uncertainties led to different effects on overall wages.

APPENDIX

Proof of Proposition 3: We can obtain

$$\gamma_{EW} = \frac{(\zeta+\eta)(\eta+\lambda)}{\zeta(\zeta+\eta)+(\zeta+3\eta)(\zeta+\eta+\lambda)} \tag{i}$$

$$\gamma_{WE} = \frac{(\zeta+\eta)(\eta+\lambda)+\eta\lambda}{\zeta(\zeta+\eta)+(\zeta+3\eta)(\zeta+\eta+\lambda)} \tag{ii}$$

which immediately yields $\gamma_{WE} > \gamma_{EW}$. Differentiating γ_{WE} and γ_{EW} with respect to λ, ζ, and η yields the remaining results in the proposition.

Derivation of Expected Time Period to Next Labor Industry Switch:

Note that

$$da = (h+g)dt + \sqrt{\sigma^2+s^2}dW \tag{iii}$$

where W is a standardized Brownian motion. Let $T(a)$ represent the expected time period until the next time that labor switches industries. We know that evolution of $T(a)$ satisfies

$$T(a) = dt + E[T(a+da)]. \tag{iv}$$

Using Itô's Lemma this reduces to the differential equation

$$\frac{\sqrt{\sigma^2+s^2}}{2}T''(a) + hT'(a) + 1 = 0. \tag{v}$$

Given that $T(\overline{a}) = 0$, we can obtain from (v) that

$$T(a) = \frac{\overline{a}}{h+g}e^{\frac{2(h+g)}{\sigma^2+s^2}(\overline{a}-a)} - \frac{a}{h+g}. \tag{vi}$$

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