Technology, Uncertainty, and Unique Firm Assets*

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Abstract

As technology capabilities evolve, companies have to optimally reallocate their unique assets to the markets in which they are more productive. Technology evolution may lead to situations in which those unique assets may stop being especially productive in certain markets and may become very productive in other markets. There is uncertainty about the timing of when the unique assets become more and less productive in different markets, and there are switching costs for the unique assets to move from one market to another. This leads to uncertain payoffs for the company over time depending on the realization of uncertainty, with the possibility of lower profitability of the company when technology allows for other entrants to be functional without the unique assets, and there are no new markets available. This paper presents a model with these features, showing that the frequency of lower profitability periods is increasing in the rate at which markets disappear and in the rate at which technological advances allow other inputs to substitute the unique assets in production, and decreasing in the rate at which new potential markets appear. An increase in the rate at which new potential markets appear increases the likelihood of the unique assets being highly profitable but decreases the ratio of the company profits to total surplus. The likelihood of the profitability of the company and the total surplus not moving together is increasing in the rate at which new potential markets appear, and may also be increasing in the rate at which technological advances cause other inputs to be able to substitute the unique assets. Consideration of the future evolution of different markets makes the company more cautious about deploying unique assets to different markets than when the future evolution is not considered.

1. Introduction

Firms often need to adapt the deployment of their unique assets to remain profitable during technological advances. These assets may include a strong brand reputation, intellectual properties, skilled personnel's expertise, good relationships with suppliers, or a loyal customer base.

For example, consider a firm in a technology-intensive industry, that has built its success on its data analytics expertise, industry know-how, and customer insight. As technology progresses, this firm faces two distinct forms of technological shocks. First, new technologies emerge that enable the creation of new markets. For example, advances in artificial intelligence, cloud computing, blockchain, or other disruptive technologies might open up possibilities for the firm to leverage its unique assets to enter and compete in these nascent markets.

However, even after the firm has entered a new market, it faces a second form of technological shock: the emergence of new technologies that allow other inputs to substitute for the firm's unique assets. While the transition into the new market move may initially provide a competitive advantage, technology spillovers and the rise of alternative solutions, such as the development of advanced algorithms, open-source software, or decentralized networks, could eventually allow rivals to replicate the firm's offerings. This could erode the firm's profit margins and market position.

This scenario is not unique; firms across multiple industries face similar challenges as they strive to remain relevant and profitable with technological changes. From pharmaceutical companies adapting to data-driven drug discovery, to automotive manufacturers grappling with electrification and autonomous vehicles, to the media industry's shift from physical to digital distribution, it becomes important for firms to navigate technological shocks and adapt unique assets to new markets.

This paper develops a model of the uncertainty of technological developments in different markets, with the company having an interest in moving the unique assets across markets in response to those technological developments.[1] The paper includes two dimensions of technological developments. First, there are technological developments that allow the output in a market to be obtained not only with the firm's unique assets but with other inputs as well. For example, some developments in artificial intelligence could allow the output

[1]The uncertainty of technological developments (and their timing) is discussed, for example, in Mansfield et al. (1968), Pakes (1986), and Jaffe and Trajtenberg (2002).

in some markets not to need as much direct management involvement. These technological developments lead to a decreased firm profitability. As an alternative to this decreased profitability, the firm may want to switch the unique assets to another market where they will be more profitable, but this often imposes costs of transition, which may potentially include retraining costs.

Second, there are technological developments that permit the appearance of new markets, for which the unique assets are initially essential to generate output. These markets can be associated with technologies that substitute the unique assets in an existing market. For example, with the development of automation nowadays, there is now greater demand for software programmers with the associated creation of new markets in which software programmers are particularly productive. The appearance of these new markets can also be random in their timing, and either be readily available if the firm wants to move to the new market (at some costs of transition), or not be available for some time, with the resulting relatively lower profitability of the firm for some period of time.

The existence of these two types of technological developments can then lead to two different types of relative costs for the company having to move to new markets because of decreased profitability in the market in which they are employed. First, having to move to a new market imposes transition costs. Second, if there is no alternative market with high profitability to switch to, the unique assets can remain with low profitability during some (stochastic) period of time.

The paper presents a model that includes these two types of technological developments and the two possibilities of stochastic costs for a company. The model illustrates rich dynamics of growth, costs of transition, and periods of lower profitability for the firm.

We find that the frequency of periods with lower profitability is increasing in the rate at which markets disappear and in the rate at which technological advances allow other inputs to substitute the unique assets in production, and decreasing in the rate at which new markets appear. When other inputs can substitute for the unique assets in production in markets of higher firm profitability, the company is more likely to get to a state of lower profitability. When new markets appear, the company is better able to keep high profitability, by moving to a new market, even when other inputs become a substitute for the unique assets in the market in which the firm is.

Interestingly, although an increase in the rate at which new markets appear increases average profits (because there is a lower likelihood of the firm being stuck in a low-profitability market), it decreases the ratio of profits to total surplus, as the economy ends up with more markets, with greater consumer surplus. That is, technological advances that lead to greater profits benefit the firm but may benefit consumers even more. On the other hand, technological advances that allow other inputs to substitute for the unique assets in the high-profit markets reduce both the average profits and the ratio of profits to total surplus.

The model offers interesting dynamics, allowing for situations in which the firm profitability and the total surplus do not move together, with either technological advances that increase the general profitability of the unique assets but do not increase the total surplus, or technological advances that increase the total surplus and do not increase the firm's profitability (or can even hurt the firm's profits). The likelihood of the firm's profits and the total surplus not moving together is increasing in the rate at which new markets appear, and also can be increasing in the rate at which technological advances allow for the firm's profits to fall.

Consideration of the future evolution of different types of makes makes the company more cautious about moving across markets, compared to when the future evolution is not considered. That is, if the company is aware of the subsequent technological advances that may allow other inputs to substitute for the unique assets in a new market, the company may be less willing to incur the costs of transition to move to the new high-profit market.

There is substantial research on the effect of technological developments on firm profitability and on the resulting adjustments to the market structure, e.g., Golder, Shacham, and Mitra (2009). There has been some research on the uncertainty of technological progress, including Jensen (1983), which looks at adoption when firms are uncertain about the quality of innovation, and Balcer and Lippman (1984), which considers when a firm should adopt a new technology given a stochastic evolution of technology, and Robinson and Min (2002), which studies how technological uncertainties affect the survival rates of pioneers into a new market. In relation to Balcer and Lippman (1984), this paper presents cases in which the evolution of technologies is fully characterized, allows for a stochastic evolution in the current technology as well as future technologies, interprets the evolution of technologies as a process in which the firm moves to a new market, and obtains the effects on the firm's present value of profits depending on the different profitability across the existing and potential new markets, and on the costs of transition across markets. Moreover, Tambe and Hitt (2014), finds that technology innovations can diffuse to competitors through the mobility of technical workers. This can be seen as a possibility that other inputs can substitute the unique assets in a new market in our model. This paper contributes to this literature by exploring the interplay between the rates of different types of technological shocks (e.g., the emergence of new markets, and the substitution of unique assets by other inputs) and their impact on firm profitability and market structure over time.

There has also been considerable work on the interplay of unique firm assets and technology innovations. This view of unique firm assets is consistent with the resource-based view of firms, e.g., Wernerfelt (1984), Teece, Pisano, and Shuen (1997), Selove (2014). Moreover, the literature has identified various forms of unique assets, e.g., leadership positions in Bayus, Jain, and Rao (1997) and Ofek and Sarvary (2003), dynamic capabilities of renewing the technology base in Narasimhan, Rajiv, and Dutta (2006), demand signals precision in Gal-Or, Geylani, and Dukes (2008), data assets or being in labor markets where workers have acquired complementary technical skills in Tambe (2014), good relationships with business partners in Cao and Ke (2019), and Choi and Sayedi (2023), ability to create high-quality advertising content in Katona, Zhu, and Zhuang (2020), and Lin (2024), digital capital in Tambe, Hitt, Rock, and Brynjolfsson (2020), and the possibility of managerial lobbying in Bao, Shi, and Kalra (2022). Several studies also document that various unique assets have a premium on firm market value, e.g., patent citations in Hall, Jaffe, and Trajtenberg (2005), and organizational information technology capabilities in Saunders and Brynjolfsson (2016). This literature also finds that firms with unique assets can better adapt to new markets when technological innovations allow so. For example, Gao and Hitt (2012) show that firms with more information technology capital stock apply for more new trademarks, leading to greater product variety. Our model contributes to this literature by investigating how firms navigate the uncertainty of technological progress and reallocate their unique assets across markets.

The remainder of the paper is organized as follows: The next section presents a base model of the stochastic evolution of technologies in discrete levels. Section 3 considers the case in which the firm adjusts gradually across markets, and Section 4 allows for the possibility of multiple markets waiting to be activated. Section 5 considers the case in which the stochastic evolution of profitability of the unique assets across technologies is continuous, and Section 6 concludes.

2. The Model

Consider a company that could potentially participate in N markets where markets disappear at some hazard rate ζ, and where new markets appear at some other hazard rate

λ. Each market has a unit size. In all markets, consumers have the same willingness to pay for the product, $\widetilde{\Delta}$. The company has one unit of some unique assets that can deployed to one of the markets. In some markets, the output can only be obtained if the unique assets are deployed there, while in other markets, depending on the technology evolution, the output can be obtained with either the unique assets or with some inputs that can be obtained in a competitive market.

When markets appear, the output can only be produced with the unique assets, requiring one unit of the unique assets to produce one unit of the output in one of the markets. That is, in such markets, the elasticity of substitution between the unique assets and the other inputs is zero. Let us also assume that there are constant returns to scale in production. At some constant hazard rate η technological evolution allows output in a market which could be produced with the firm unique assets to be produced with other inputs as well. That is, when other inputs can substitute the unique assets in producing output in a market, we assume that the unique assets and the other inputs become perfect substitutes.[2] This hazard rate of when the other inputs can substitute the unique assets in the production in a market captures the uncertainty of technological evolution, i.e., uncertainty as to when the other inputs can replace the firm unique assets, as a way to lower production costs. We assume that one unit of the other inputs is enough to produce one unit of output in any of the markets and let $\widetilde{\zeta}$ be the price of the other inputs. We assume $\widetilde{\Delta} > \widetilde{\zeta}$.

A *new market* (a market for which output cannot be obtained with the other inputs) allows the firm to charge the monopoly price, which is the consumer willingness to pay, $\widetilde{\Delta}$. The profit of the firm operating in a new market is then $\widetilde{\Delta}$, without counting for any remuneration of the unique assets. In *mature markets* (markets for which the output can also be produced with the other inputs), perfect competition yields an equilibrium price of $\widetilde{\zeta}$. The profit of the firm operating in a mature market is then $\widetilde{\zeta}$. Let Δ be the differential in profits between these two possible types of states of the company, $\Delta = \widetilde{\Delta} - \widetilde{\zeta}$. Note that Δ represents also the consumer surplus in a mature market.

This difference in these profits can be seen as representing the cost to the firm when technologies are developed such that the output can also be produced with the other inputs, and if no other new market is available for the firm to move to.[3] That is, when the other

[2]This no substitution in production between the unique asset and the other inputs in the new markets, and perfect substitution occurring when technology evolves represents in the model the main effect of technology evolution in a simple way. In some cases, this could be the result of automation.

[3]We will say that the firm moves to a new market when it deploys the firm's unique assets to that new market. Literally, the firm does not need to abandon the old market to move to the new market, as it could

inputs can substitute the unique assets in production in a new market and no other new market is available for the unique assets to move to, the firm's profit falls from $\widetilde{\Delta}$ to $\widetilde{\zeta}$.

In addition to these potential costs to the firm when no new market is available, we also consider the direct costs that the firm incurs in moving the unique assets from one market to another, which we denote by F. These costs can represent the costs of retraining or the costs of studying the characteristics of the new market. We consider that the firm can move across markets immediately if it so wishes. We assume that F is sufficiently low such that, if the firm is in a low-profit market, it wants to incur these costs F and move to a high-profit market. When the firm is risk-neutral, which we assume, this condition is $F < \frac{1}{r+\zeta+\eta}\Delta$, where r is the interest rate. We discuss below how this condition arises and its implications. In Section 3, we consider the case in which the firm's transition from one market to another is gradual over time.

As noted above, the possibility of a new market appearing occurs with hazard rate λ. The output in a new market can only be produced with the unique assets. If the unique assets are tied up in another new market in which there is still no technology that allows output to be also produced with the other inputs, the firm will not deploy the unique assets to the new potential market given the costs of transition, and the demand in the new potential market will not be satisfied. We say the new potential market *stays in reserve* for the firm to move to when a new technology develops in the market in which the unique assets are currently deployed, to allow for the other inputs to substitute for the unique assets in production. In the base model, we allow for at most one market to stay in reserve. Moreover, for simplicity, we assume that when a market stays in reserve, it will not disappear. This possibility of a market being or not being in reserve captures the idea that when technology substitutes the unique assets in a market, the unique assets may or may not have an alternative market to move to.

The firm has one unit of the unique assets, and cannot acquire more of those assets. The other inputs can be obtained competitively, and have to be remunerated at $\widetilde{\zeta}$, and noted above. With N markets, $N-1$ of them will use only the other inputs as an input (one unit per market), and one market will use only the unique assets as an input. The supply of the other inputs is assumed as infinitely elastic.

If the other inputs can be used for production in all the N markets, the firm's profit will be $\widetilde{\zeta}$, and the total consumer surplus will be $N\Delta$. If the other inputs can only be used for

still be in the old market by using the other inputs for production and obtaining zero profit in the old market.

production in $(N-1)$ markets, the firm's profit will be $\widetilde{\Delta}$, and the total consumer surplus will be $(N-1)\Delta$. In both cases, the total surplus will be $N\Delta+\widetilde{\zeta}$. Note that the ratio of the firm's profit to total surplus could easily be considered greater by allowing further units of the unique assets in the model.

In the model considered, automation allows the other inputs to substitute all the unique assets alike, while in reality, automation may substitute some of the unique assets and complement others, even if no new markets appear. The assumption here simplifies the analysis to capture the effects of stochastic profit crises on the equilibrium path but does not consider all the effects that occur with automation. Another dimension of heterogeneity not considered here is that the costs for the unique assets to switch markets may be different across different unique assets, which may lead to only some unique assets to switch markets. This effect is considered in some way in Section 3 by allowing different components of the unique assets to change markets at different times.

The existence of uncertainty about the appearance of new markets and about when automation technologies will appear leads to successive and stochastic periods of profit crises on the equilibrium path. Consideration of the costs of moving markets allows for a better calibration of the incentives and of the costs for the firm of moving across markets. The possibility of existing markets in reserve allows for variability in the intensity of demand for the unique assets, such that automation does not necessarily lead to periods of low profits.

Finally, there is some evidence that technological developments are targeted at reducing the costs of production (e.g., Acemoglu, 2002). In the context of the model, this is captured by the existence of the hazard rate of technological developments that allow the other inputs to substitute the unique assets in the high-profit market, η. One could also consider an extension of the model in which η is greater when there is a market in reserve, so that there is a likelihood of greater savings on the use of the unique assets.

2.1. Market Evolution

Considering the set of all active markets in which the firm could be, with N markets, the full market structure can be in one of three states: (i) We can be in a state in which the output in all markets can be produced with the other inputs, such that the firm profits are $\widetilde{\zeta}$ and the total consumer surplus from these markets is $N\Delta$. Let $\overline{N}$ denote this state when it has N markets, and $\overline{S}$ denote that we are in this state for any number of markets. (ii) We can be in a state in which the output in one of the markets can only be produced with the firm unique assets, such that the firm profits are $\widetilde{\Delta}$, and there is no new potential market in reserve. Let N_0 denote this state with N markets, and S_0 denote that we are in this state for any number of markets. (iii) Finally, we can be in a state in which the output in one of the markets can only be produced with the firm's unique assets, such that the firm's profit is $\widetilde{\Delta}$, and there is one new potential market in reserve. Let N_1 denote this state with N markets, and S_1 denote that we are in this state for any number of markets.

We can then construct a transition matrix from one state to another during a period of time $\mathrm{d}t$. If we are in state $\overline{N}$ we can move to state $(N+1)_0$ with probability $\lambda\,\mathrm{d}t$, we can move to state $\overline{N-1}$ with probability $N\zeta\,\mathrm{d}t$, and stay in the same state with the complementary probability. If we are in state N_0 we can go to state N_1 with probability $\lambda\,\mathrm{d}t$, we can go to state $\overline{N}$ with probability $\eta\,\mathrm{d}t$, we can go to state $\overline{N-1}$ with probability $\zeta\,\mathrm{d}t$, we can go to state $(N-1)_0$ with probability $(N-1)\zeta\,\mathrm{d}t$, and stay in the same state with the complementary probability. Finally, if we are in state N_1 we can go to state $(N+1)_0$ with probability $\eta\,\mathrm{d}t$, we can go to state N_0 with probability $\zeta\,\mathrm{d}t$, we can go to state $(N-1)_1$ with probability $(N-1)\zeta\,\mathrm{d}t$, and stay in the same state with the complementary probability.

We can then obtain that the expected growth in the number of markets depending on the state can be obtained as $E[\frac{\mathrm{d}N}{\mathrm{d}t}|\overline{S}] = \lambda - N\zeta$, $E[\frac{\mathrm{d}N}{\mathrm{d}t}|S_0] = -N\zeta$, and $E[\frac{\mathrm{d}N}{\mathrm{d}t}|S_1] = \eta - (N-1)\zeta$.

In steady-state, we can obtain the probabilities of being in states $\overline{S}, S_0$, and S_1. The transition matrix among these states can be obtained by noting that from state $\overline{S}$, we can go to state S_0 with probability $\lambda\,\mathrm{d}t$ during period of time $\mathrm{d}t$, and stay at $\overline{S}$ with the complementary probability, from state S_0 we can go to state S_1 with probability $\lambda\,\mathrm{d}t$, to state $\overline{S}$ with probability $(\zeta+\eta)\mathrm{d}t$, and remain at state S_0 with the complementary probability, and from state S_1 we can go to state S_0 with probability $(\zeta+\eta)\mathrm{d}t$, and stay at state S_1 with the complementary probability. From this one can obtain the steady-state probabilities of being at states $\overline{S}, S_0$, and S_1, $p_{\overline{S}}, p_{S_0}$, and p_{S_1}, respectively, as

$$p_{\overline{S}} = \frac{(\zeta+\eta)^2}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2}, \tag{1}$$

$$p_{S_0} = \frac{\lambda(\zeta+\eta)}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2}, \tag{2}$$

$$p_{S_1} = \frac{\lambda^2}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2}. \tag{3}$$

One can then obtain the following result about the frequency with which the firm profits are low (state $\overline{S}$).

Proposition 1. *The frequency of low profits is increasing in the hazard rate at which markets disappear (ζ) and in the hazard rate at which the other inputs can become a substitute in production for the firm's unique assets (η), and decreasing in the hazard rate at which new markets appear (λ).*

As markets disappear at a greater hazard rate, there is a greater likelihood that the market with high profits disappears, and therefore the firm can only be in a low-profit market. When the other inputs can become a substitute in production for the firm's unique assets at a greater hazard rate (the likelihood of automation), there is a higher probability that a high-profit market will become a low-profit market, leading to a higher frequency of low profits. As new markets appear at a greater hazard rate, there is a greater likelihood of there being a high-profit market to which the firm's unique assets can be allocated, leading to a lower frequency of periods of time with low profits.

Using these steady-state probabilities on the different states we can then compute the expected growth in the number of markets as

$$E\left[\frac{\mathrm{d}N}{\mathrm{d}t}\middle| N\right] = \frac{\lambda(\zeta+\eta)(\zeta+\eta+\lambda)}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2} - N\zeta, \tag{4}$$

which is increasing in λ and η and decreasing in ζ. The effect of λ is by allowing new markets to appear, and the effect of η is to release the firm's unique assets from old markets to be deployed in new markets. We can consider that the number of markets is small compared to the steady state, such that expected growth in the number of markets is positive until the number of markets converges stochastically, which would make the expected growth in the number of markets equal to zero. Note that if $\zeta = 0$, then there is always positive expected growth in the number of markets, with declining growth rates as the number of markets increases.

Interestingly, while an increase in the hazard rate at which the other inputs can substitute the unique assets in production in a market (the automation rate) increases growth in the number of markets, it affects profits negatively. On the other hand, both an increase in the rate at which new markets appear and a decrease in the rate at which markets disappear, have a positive effect on both consumer surplus and on profits. In the next subsection, after considering the dynamic effects on the firm profits, we study the extent to which consumer surplus and the overall firm payoffs can move in different directions over time.

Let N_{ss} be the number of markets such that the expected growth in the number of markets is zero, i.e., the number of markets that makes (4) equal zero. We then can also compute the average ratio of profits to total surplus as

$$\frac{p_{\overline{S}}\widetilde{\zeta} + (p_{S_0} + p_{S_1})\widetilde{\Delta}}{N_{\text{ss}}\Delta + \widetilde{\zeta}} = \frac{\zeta[\lambda(\zeta+\eta) + \lambda^2 + (\zeta+\eta)^2]\widetilde{\zeta} + \lambda\zeta(\zeta+\eta+\lambda)\Delta}{\zeta[\lambda(\zeta+\eta) + \lambda^2 + (\zeta+\eta)^2]\widetilde{\zeta} + \lambda(\zeta+\eta)(\zeta+\eta+\lambda)\Delta}. \tag{5}$$

Of course, this ratio can be made greater just by considering further units of unique assets in the model. Note also that the model as presented converges to a steady state of no expected growth in total surplus, but positive expected growth in total surplus can be easily obtained by allowing $\widetilde{\Delta}$ for the new markets to increase over time.

We can obtain that the average ratio of profits to total surplus at the steady-state number of markets is increasing in the rate at which markets disappear. That is, although the frequency of low profits is greater when the rate at which markets disappear is greater, the overall average ratio of profits to total surplus is greater because a greater rate at which markets disappear is associated with a smaller number of markets around the steady state. Also, we can obtain that the average ratio of profits to total surplus is decreasing in the rate at which other inputs can substitute the unique assets. That is, when the rate at which other inputs can substitute the unique assets is greater, both the frequency of low profits and the number of markets are greater, so the average ratio of profits to total surplus falls. Furthermore, we can obtain that the expected ratio of profits to total surplus is decreasing in the rate at which new markets appear. That is, even though the firm's profitability increases with a greater rate of appearance of new markets, a greater fraction of the gains in the longer term go elsewhere (to consumer surplus), and the expected ratio of profits to total surplus falls with increases in the rate at which markets appear. We state these results in the following proposition.

Proposition 2. *An increase in the hazard rate at which markets disappear (ζ), decreases the expected profits but increases the ratio of profits to total surplus over time. An increase in the rate at which other inputs can substitute the unique assets (η) decreases both the expected profits and the ratio of profits to total surplus over time. An increase in the rate at which markets appear (λ) increases the expected profits but decreases the ratio of profits to total surplus over time.*

Note that one could have an increased firm profit because of the appearance of new markets such that an increase in the rate at which new markets appear would increase the ratio of firm profits to total surplus. This effect is not present here because the unique assets and the other inputs are perfect substitutes when the other inputs can also be used in the production in a market. Alternatively, we could have the firm profits increase with the new markets, and that would also be a force for the ratio of firm profits to total surplus to increase with the rate at which new markets appear.

It is also interesting to consider the full distribution of how the number of markets can evolve over time. In the steady-state on the probability distribution on states S, during the period of time $\mathrm{d}t$, the total number of markets can grow by one market with probability $(p_{\overline{S}}\lambda + p_{S_1}\eta)\mathrm{d}t$, and decrease by one market with probability $[(N-1)\zeta + (p_{\overline{S}} + p_{S_0})\zeta]\mathrm{d}t$.

This structure captures interesting effects of technological developments. The appearance of new markets (λ) can increase the size of the total surplus either immediately if the firm's unique assets are underused, or in the future when the unique assets have to switch markets as they become underused in another market. The technological developments that allow for the other inputs to substitute for the unique assets (η) increase the total surplus immediately if there is a new market in reserve, or in the future when a new market appears.

Note that the firm is hurt by the transition costs both when it has to switch markets (when the unique assets are more profitable in a new market than in an existing market), and when technological developments allow the other inputs to substitute for the unique assets in a market and there is no new market in reserve, such that profits drop during that period from $\widetilde{\Delta}$ to $\widetilde{\zeta}$. The next subsection explores the effects on the firm's profit.

2.2. Effects on Firm's Profit

The firm is guaranteed to have a profit of $\widetilde{\zeta}$ and can earn at most a profit of $\widetilde{\Delta}$. Let us then consider the firm's profit in addition to getting $\widetilde{\zeta}$ continuously. Let $V(S)$ be the present value of profits as a function of the state S where $S \in \{\overline{S}, S_0, S_1\}$ in addition to getting $\widetilde{\zeta}$ continuously, $\frac{\widetilde{\zeta}}{r}$.[4]

[4] We assume that the costs of transition of changing markets are only incurred when the firm switches to a high-profit market, and are not incurred when the firm switches from a market with low profits to another market with low profits, for example because the initial market was extinguished. We assume that in the small probability event of there being zero markets being considered, the firm is able to independently obtain a profit of $\widetilde{\zeta}$.

We can then obtain

$$
\begin{aligned}
V(\overline{S}) &= \lambda \,\mathrm{d}t\, e^{-r\,\mathrm{d}t}[V(S_0) - F] + (1-\lambda\,\mathrm{d}t)e^{-r\,\mathrm{d}t}V(\overline{S}) && (6)\\
V(S_0) &= \Delta\,\mathrm{d}t + (\eta+\zeta)\mathrm{d}t\, e^{-r\,\mathrm{d}t}V(\overline{S}) + \lambda\,\mathrm{d}t\, e^{-r\,\mathrm{d}t}V(S_1) + (1-\eta\,\mathrm{d}t \\
&\quad -\zeta\,\mathrm{d}t - \lambda\,\mathrm{d}t)e^{-r\,\mathrm{d}t}V(S_0) && (7)\\
V(S_1) &= \Delta\,\mathrm{d}t + (\eta+\zeta)\mathrm{d}t\, e^{-r\,\mathrm{d}t}[V(S_0)-F] + (1-\eta\,\mathrm{d}t - \zeta\,\mathrm{d}t)e^{-r\,\mathrm{d}t}V(S_1). && (8)
\end{aligned}
$$

Making $\mathrm{d}t \to 0$, these expressions reduce to

$$
\begin{aligned}
(r+\lambda)V(\overline{S}) &= \lambda[V(S_0) - F] && (9)\\
(r+\eta+\zeta+\lambda)V(S_0) &= \Delta + (\eta+\zeta)V(\overline{S}) + \lambda V(S_1) && (10)\\
(r+\eta+\zeta)V(S_1) &= \Delta + (\eta+\zeta)[V(S_0)-F]. && (11)
\end{aligned}
$$

From this, we can obtain

$$
\begin{aligned}
V(S_0) &= \frac{\Delta(r+\lambda)(r+\eta+\zeta+\lambda) - \lambda(\eta+\zeta)(2r+\eta+\zeta+\lambda)F}{r(r+\eta+\zeta+\lambda)^2 - r\lambda(\eta+\zeta)} && (12)\\
V(\overline{S}) &= \frac{\lambda}{r+\lambda}[V(S_0)-F] && (13)\\
V(S_1) &= \frac{\Delta}{r+\eta+\zeta} + \frac{\eta+\zeta}{r+\eta+\zeta}[V(S_0)-F]. && (14)
\end{aligned}
$$

This yields that $V(\overline{S}) < V(S_0) < V(S_1)$. When technological developments allow the other inputs to substitute for the firm's unique assets in production and there is no new potential market in reserve, the present value of profits falls from $V(S_0)$ to $V(\overline{S})$, and that loss remains for an expected period of time of length $1/\lambda$. The steady-state probability of being in that state is $p_{\overline{S}}$, which is low when $(\zeta+\eta)/\lambda$ is small. For example, take $\lambda = 1$. For $\zeta+\eta = .4$, we have $p_{\overline{S}} \approx 10\%$, while for $\zeta+\eta = .65$, we have $p_{\overline{S}} \approx 20\%$. Considering the unit of time in years, these latter parameters suggest that, on average, there would be a one-year low-profit period every five years due to technological developments depressing profits.

For the proposed equilibrium to hold we must have that $V(S_0) - F > 0$, because otherwise, the firm would prefer not to switch the unique assets to the high-profit market from the low-profit market. This condition reduces to $F < \frac{1}{r+\zeta+\eta}\Delta$, which can be seen as relatively intuitive. This condition is more likely to be satisfied the less the firm discounts the future, and the less likely the market is to disappear or for the other inputs to become a substitute for the firm's unique assets. It is interesting to compare how this condition would compare with the case in which the firm did not foresee that the other inputs could at some point become a substitute for the unique assets in the high-profit market. That condition is $F < \frac{1}{r+\zeta}\Delta$. That is, the firm is more likely to switch the unique assets to the high-profit market if it is not aware that the other inputs may become a substitute for the firm's unique assets in the high-profit market at some point in the future. A model that does not consider this possibility may overestimate the likelihood of the firm switching to the high-profit market, and therefore overestimate the costs for the unique assets to switch markets if it tries to fit the likelihood of the unique assets switching markets to what is observed in the real world.

When technological developments allow the other inputs to substitute for the firm's unique assets in production, and there is a new market in reserve, the present value of profits falls from $V(S_1)$ to $V(S_0) - F$, which represents the transition costs F of the unique assets having to switch markets and the risk that further technological developments in the future will allow the other inputs to substitute for the unique assets without a new potential market appearing.

Note also that considering only a one-time shock of the other inputs substituting for the firm's unique assets in the high-profit market, and its effects on the firm incurring the transition costs of switching markets to the market in reserve, does not account for the fact that the firm is now in a potentially worse situation of facing a new potential shock that will again allow the other inputs to substitute for the unique assets in the new high-profit market, and the firm now not having a market in reserve to switch to.

Similarly, if there is no market in reserve, considering only a one-time shock of the other inputs being able to substitute the unique assets in the high-profit market overestimates the negative effects on the firm of this shock, without taking into account that in the future new high-profit markets may appear.

Figures 1-3 illustrate the evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as function of r, η, and λ.

The model offers rich dynamics about the evolution of the number of markets, profits, and total surplus. Given that there is a change in the market structure, the ratio of the firm's present value of profits to consumer surplus could either increase or decrease. One possibility is that this ratio decreases. Let us denote this event as event RD. This can occur when (i) the high-profit market disappears, or when (ii) it becomes possible for the other inputs to substitute the unique assets in the high-profit market. In the former case, the consumer surplus stays constant, and the expected present value of profits falls from $V(S_0)$ to $V(\overline{S})$ if there is no market in reserve or falls from $V(S_1)$ to $V(S_0) - F$ if there is no market

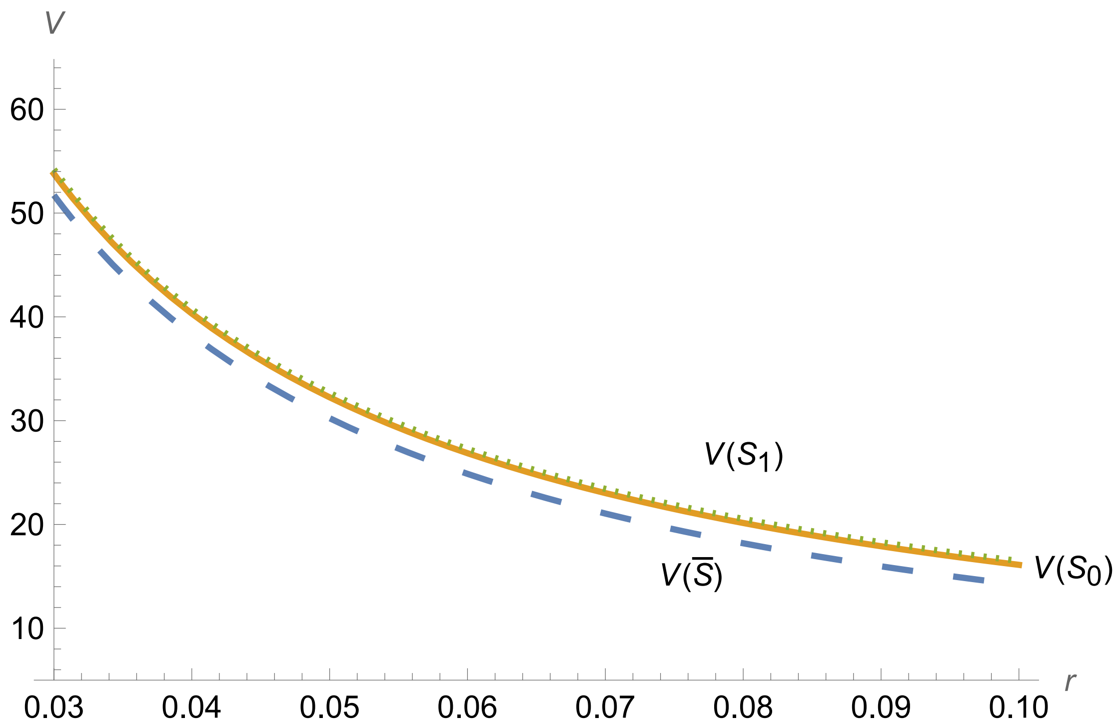

Figure 1: Evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as a function of r for $\Delta = 2$, $\lambda = 1, \eta{+}\zeta = .4$, and $F = .5$.

in reserve. In the latter case, the consumer surplus increases by Δ, and the present value of profits falls from $V(S_0)$ to $V(\overline{S})$ if there is no market in reserve or falls from $V(S_1)$ to $V(S_0) - F$ if there is a market in reserve. The probability of the total surplus moving in a positive direction in relation to the firm's profit given that there is a change in the market structure can be computed to be

$$\gamma_{\text{RD}} = \frac{(\zeta + \eta)(p_{S_0} + p_{S_1})}{\zeta N_{\text{ss}} + \eta(p_{S_0} + p_{S_1}) + \lambda(p_{\overline{S}} + p_{S_0})}. \tag{15}$$

Another possibility is that the ratio of the firm's present value of profits to consumer surplus increases, given that there is a change in the market structure. Let us denote this event as event RI. This can occur when (i) a low-profit market disappears, or when (ii) we are in a state with low profits or with high profits but no markets in the reserve (states $\overline{S}$ or S_0), and a new market appears. In the former case, the consumer surplus falls, but there is no impact on the firm's profits. In the latter case, in which the consumer surplus and profits stay constant, and the present value of profits increases from $V(\overline{S})$ to $V(S_0) - F$ in a state with low profits or increases from $V(S_0)$ to $V(S_1)$ in a state with high profits but no markets in the reserve. The probability of the total surplus moving in a negative direction

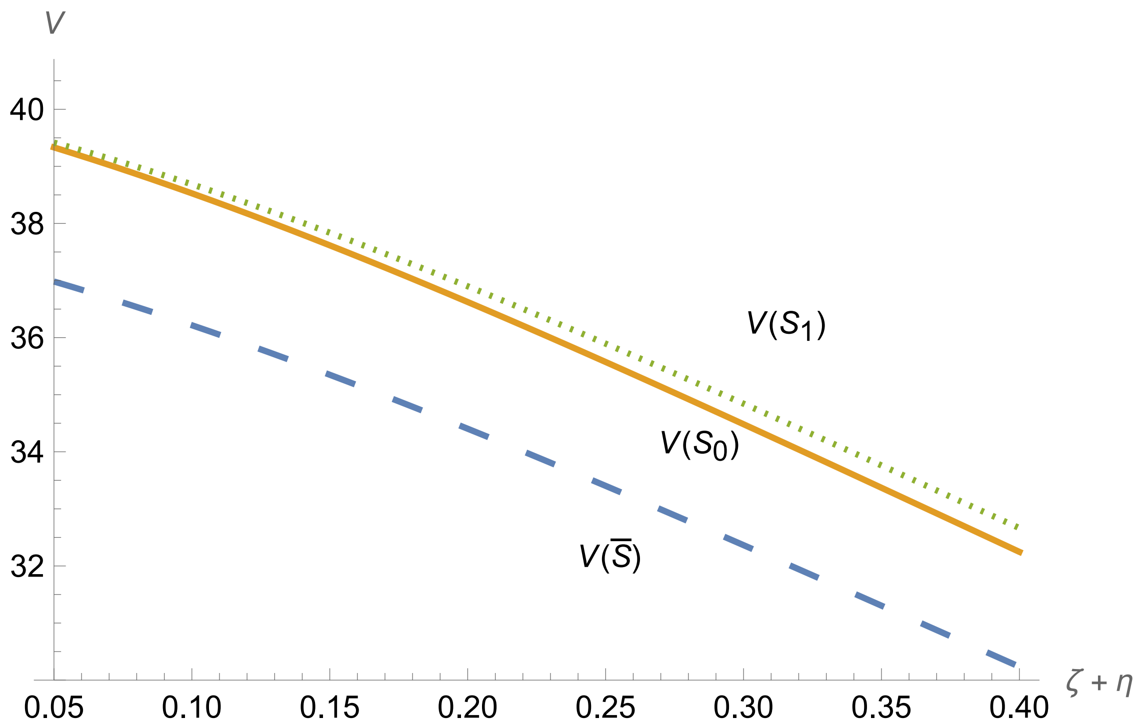

Figure 2: Evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as a function of $\eta + \zeta$ for $\Delta = 2$, $\lambda = 1, r = .05$, and $F = .5$.

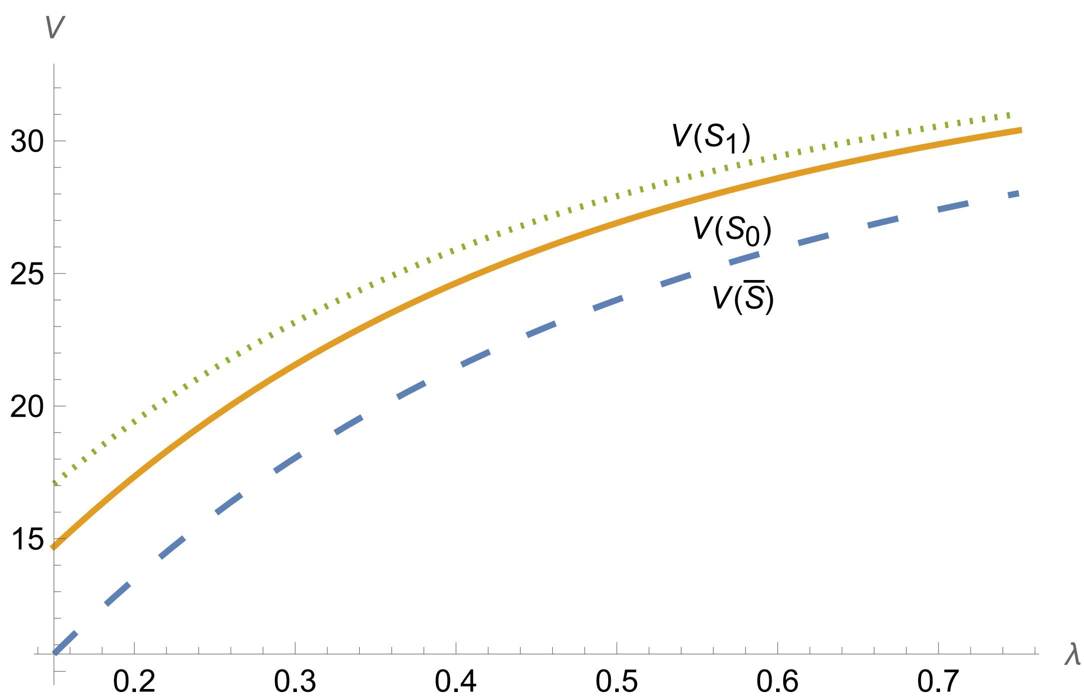

Figure 3: Evolution of $V(\overline{S}), V(S_0)$, and $V(S_1)$, as a function of λ for $\Delta = 2$, $\eta + \zeta = .4, r = .05$, and $F = .5$.

in relation to the firm's present value of profits given that there is a change in the market structure can be computed to be

$$\gamma_{\mathrm{RI}} = \frac{\zeta(N_{\mathrm{ss}} - p_{S_0} - p_{S_1}) + \lambda(p_{\overline{S}} + p_{S_0})}{\zeta N_{\mathrm{ss}} + \eta(p_{S_0} + p_{S_1}) + \lambda(p_{\overline{S}} + p_{S_0})}. \tag{16}$$

We can then obtain the following results about the joint evolution of the total surplus and of the present value of profits.

Proposition 3. *Given that there is a change in the market structure, the likelihood that the ratio of the firm's present value of profits to consumer surplus increases is greater than the likelihood that this ratio decreases ($\gamma_{RI} > \gamma_{RD}$). The likelihood that the ratio of the firm's present value of profits to consumer surplus increases is decreasing in the hazard rate at which markets disappear and increasing in the hazard rate at which the other inputs can substitute the unique assets in production in a high-profit market, and is independent of the hazard rate of the appearance of new markets.*

A greater hazard rate at which markets disappear has three effects: Markets disappear more frequently, the steady-state number of markets decreases, and the set of markets considered is more frequently in the low-profit state (state $\overline{S}$). An increase in the hazard rate of the disappearance of markets leads to proportional increases in each of the likelihood of the events RD and RI(ii) considered above. However, it will lead to a lower rate of increase in the likelihood of the events RI(i) because of the second effect (the steady-state number of markets decreases). As a consequence, the likelihood that the ratio of the firm's present value of profits to consumer surplus increases is decreasing in the hazard rate at which markets disappear.

A greater hazard rate at which the other inputs can substitute the unique assets in production in the high-profit market has three effects: The steady-state number of markets increases, other inputs can substitute the unique assets in production in the high-profit market more frequently when we are in a high-profit state (states S_0 or S_1), and the set of markets considered is more frequently in the low-profit state (state $\overline{S}$). An increase in the hazard rate of the appearance of new markets leads to proportional increases in each of the likelihood of the events RD and RI(ii) considered above. However, it will lead to a higher rate of increase in the likelihood of the event RI(i), because both the first effect (the steady-state number of markets increases) and the third effect (the set of markets considered is more frequently in the low-profit state) appear there. As a consequence, the likelihood that the ratio of the firm's present value of profits to consumer surplus increases is increasing in the hazard rate at which the other inputs can substitute the unique assets in production in a high-profit market.

A greater hazard rate of the appearance of new markets has three effects: The steady-state number of markets increases, the set of markets considered is more frequently in the state in which it has a market in reserve (state S_1), and markets appear more frequently when we are in a state with low profits or with high profits but no markets in the reserve (states $\overline{S}$ or S_0). An increase in the hazard rate of the appearance of new markets leads to proportional increases in each of the likelihood of the four events RD(i)(ii) and RI(i)(ii). As a consequence, the likelihood that the ratio of the firm's present value of profits to consumer surplus increases is independent of the hazard rate of the appearance of new markets.

Note that these effects on the firm's present value of profits consider only the effect through profits and through the costs of switching markets. One could also consider that the firm may also keep some market power in the markets in which the other inputs substitute for the unique assets, such that it also benefits when the technology changes, alleviating somewhat the negative effects on profits, or the transition costs incurred when switching markets. That is, the effects considered assume no market power by the firm in the old markets. If the firm keeps some market power in the old markets, then the technology that allows the other inputs to substitute the unique assets in production could also be beneficial to the firm.

Another effect not considered here is that the consumer surplus earned in the old markets could increase demand in markets only produced with the firm's unique assets (the new markets here) through an income effect, thus providing extra benefits for the firm. This effect could be important empirically, but is not considered here in order to obtain more directly the uncertain profit dynamics due to automation. Finally, in order to better obtain the intuition for the effects modeled, we consider that the firm is able to switch all the unique assets to the high-profit market with the same costs of transition F. In the real world, the firm may have different costs of transition for different components of the unique assets, with some potentially having very high costs of transition, such that they never switch to the high-profit market. The next section considers a variation on that set-up, in which the firm is only able to switch some of the unique assets to the high-profit market after some time, with that time of transition being heterogeneous across the components of the unique assets.

The assumption of constant returns to scale of the unique assets, although simplifying the analysis, may also limit the model's ability to fully present the benefits for the firms from the appearance of a new market. If there are diminishing returns to scale of the unique assets in a market, the creation of a new market would allow some assets to switch to the new market and increase overall profits. This could then have effects on the incentives to invest in automation if technological change is endogenous.

3. Gradual Adjustments

In the previous section, we considered that the unique assets could switch markets immediately by paying fixed costs F. In practice, the firm may not be able to change the deployment of the unique assets to new markets immediately, and, in fact, may only be able to switch the unique assets gradually across markets over time, potentially because of the need to be adapted to a new market. In this section, we consider the possibility that the unique assets are only able to switch markets gradually, without fixed costs of switching, at a constant hazard rate α. This captures the idea that the firm may only have occasional opportunities to have each infinitesimal unit of unique assets deployed to new markets when it has the interest in doing so. As noted above, the unique assets have mass one.

Let $x(t)$ be the mass of unique assets that switched to the new markets at time t after that new market appeared. Given the constant hazard rate α we have $x(t) = 1 - e^{-\alpha t}$. Note that in this case, all markets will have some unique assets, with the older markets having fewer unique assets over time, with all the unique assets in the old markets getting a return $\widetilde{\zeta}$.

A model with N markets, where one market's output can only be produced with the unique assets, will then have a total surplus of $[N - 1 + x(t)]\Delta + \widetilde{\zeta}$ at time t after that new market appeared. With that set of markets, the total surplus is growing by $\alpha(1 - x)\Delta$ per period. Note that, if a new technology is developed in such a model, such that output that previously could only be produced with the unique assets now can also be produced with other inputs (which occurs at a hazard rate η), the total surplus in the model would jump to $N\Delta + \widetilde{\zeta}$. This could be seen as representing the largest possible growth in total surplus resulting from the new technology being developed.

In this case, we can also compute the expected growth in total surplus. Let Y represent the total surplus. Then we can obtain

$$\begin{aligned} E\left[\frac{\mathrm{d}Y}{\mathrm{d}t}\middle|\overline{S}, N\right] &= -N\zeta\Delta \qquad (17)\\ E\left[\frac{\mathrm{d}Y}{\mathrm{d}t}\middle|S_0, N, x\right] &= [(\alpha+\eta)(1-x) - (N-1+x)\zeta]\Delta \qquad (18)\\ E\left[\frac{\mathrm{d}Y}{\mathrm{d}t}\middle|S_1, N, x\right] &= [(\alpha+\eta)(1-x) - (N-1+x)\zeta]\Delta. \qquad (19)\end{aligned}$$

We can then obtain the overall expected total surplus growth as

$$E\left[\frac{\mathrm{d}Y}{\mathrm{d}t}\middle|N, x\right] = \left[\frac{\lambda(1-x)(\alpha+\zeta+\eta)(\zeta+\eta+\lambda)}{\lambda(\zeta+\eta)+\lambda^2+(\zeta+\eta)^2} - N\zeta\right]\Delta. \qquad (20)$$

This yields that the expected growth in total surplus in the short-run is greater the lower the fraction of the unique assets in the high-profit market, or the greater the hazard rate at which the unique assets switch to the high-profit market, or the greater the rate at which markets appear. We state these results in the following proposition.

Proposition 4. *In the gradual adjustments case, the expected growth in total surplus in the short run is greater the lower the fraction of the unique assets in the high-profit market, x. This effect is greater the greater the hazard rate at which the unique assets switched to the high-profit market, α. Also, this effect is greater the rate at which markets appear, λ.*

The effect on the expected growth in total surplus of the fraction of unique assets in the high-profit market occurs through three mechanisms. First, a greater fraction of unique assets in the high-profit market means that there are fewer unique assets to be deployed from the low-profit to the high-profit market, and therefore the expected short-run growth in total surplus is smaller. Second, when the high-profit market disappears, the loss in total surplus is $x\Delta$, and a greater fraction of unique assets in the high-profit market means that this loss is larger. Third, the other inputs becoming a substitute for the unique assets in the high-profit market just increases growth in total surplus on the unused potential in that market, $(1-x)\Delta$, and a greater fraction of unique assets in that market means that this number is now smaller.

Similarly to the previous section, we can consider the steady-state probabilities of being in states $\overline{S}$, (S_0, x), and (S_1, x). Note that in state $\overline{S}$, the fraction of unique assets switched to new markets is always $x = 0$, so we can consider the state $\overline{S}$ as a whole; in contrast, in states S_0 and S_1, x can take values between 0 and 1, so we consider the states (S_0, x)

and (S_1, x) in each case respectively. To consider these steady-state probabilities, we first examine the evolution of each infinitesimal unit of the unique assets. Now, in state S_0, each unit of the unique assets can be deployed either in the high-profit market or in a low-profit market, which we denote by state S_{0Y} or state S_{0N}, respectively. In the same way, in state S_1, each unit of the unique assets can now be employed either in the high-profit market or in a low-profit market, which we denote by state S_{1Y} or state S_{1N}, respectively.

We first examine the amount of time that each infinitesimal unit of the unique assets ends up in each state. Let $\overline{p}$ be the row vector of the steady-state probability of being in each state, which has in the first element $p_{\overline{S}}$, in the second element $p_{S_{0N}}$, in the third element $p_{S_{0Y}}$, in the fourth element $p_{S_{1N}}$, and in the fifth element $p_{S_{1Y}}$. Let A be the transition matrix (5×5) between the five different states, ordered as in the vector $\overline{p}$, with general element a_{ij} representing the probability of transitioning from state i to state j during the period of time $\mathrm{d}t$. We then have

$$A = \begin{pmatrix} (1-\lambda\,\mathrm{d}t) & \lambda\,\mathrm{d}t & 0 & 0 & 0 \\ (\zeta+\eta)\,\mathrm{d}t & 1-(\alpha+\zeta+\eta+\lambda)\,\mathrm{d}t & \alpha\,\mathrm{d}t & \lambda\,\mathrm{d}t & 0 \\ (\zeta+\eta)\,\mathrm{d}t & 0 & 1-(\zeta+\eta+\lambda)\mathrm{d}t & 0 & \lambda\,\mathrm{d}t \\ 0 & (\zeta+\eta)\mathrm{d}t & 0 & 1-(\alpha+\zeta+\eta)\mathrm{d}t & \alpha\,\mathrm{d}t \\ 0 & (\zeta+\eta)\mathrm{d}t & 0 & 0 & 1-(\zeta+\eta)\mathrm{d}t \end{pmatrix}.$$

We know that $\overline{p}$ is defined by $\overline{p}A = \overline{p}$. This yields

$$p_{\overline{S}} = \frac{(\zeta+\eta)^2}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2} \tag{21}$$

$$p_{S_{0N}} = \frac{\lambda(\zeta+\eta)}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{\zeta+\eta+\lambda}{\alpha+\zeta+\eta+\lambda} \tag{22}$$

$$p_{S_{0Y}} = \frac{\lambda(\zeta+\eta)}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{\alpha}{\alpha+\zeta+\eta+\lambda} \tag{23}$$

$$p_{S_{1N}} = \frac{\lambda^2}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{\zeta+\eta+\lambda}{\alpha+\zeta+\eta+\lambda}\frac{\zeta+\eta}{\alpha+\zeta+\eta} \tag{24}$$

$$p_{S_{1Y}} = \frac{\lambda^2}{(\zeta+\eta)^2+\lambda(\zeta+\eta)+\lambda^2}\frac{\alpha+2(\zeta+\eta)+\lambda}{\alpha+\zeta+\eta+\lambda}\frac{\alpha}{\alpha+\zeta+\eta}. \tag{25}$$

It is worth mentioning that the probabilities $p_{\overline{S}}$, $p_{S_0} = p_{S_{0N}} + p_{S_{0Y}}$, and $p_{S_1} = p_{S_{1N}} + p_{S_{1Y}}$ equal the probabilities in (1), (2), and (3), respectively. Intuitively, this is because the state S_0 in the main model is equal to the combination of states S_{0N} and S_{0Y} in the model here, and the state S_1 in the main model is equal to the combination of states S_{01N} and S_{1Y} in the model here.

The likelihood of a unit of unique assets having a low profit can be obtained as $p_{\overline{S}} + p_{S_{0N}} + p_{S_{1N}}$ which is increasing in ζ and η, and decreasing in λ and α. For $\zeta + \eta = .65, \lambda = 1$, and $\alpha = .3$, the unique assets would have a low return for around 67% of the time. With $\alpha = .3$, the unique assets move to a high-profit market if available, on average, in a little over three years.

In order to investigate the steady-state probabilities of being in states $\overline{S}$, (S_0, x), and (S_1, x), and the firm's stationary profits, we now examine the stationary distribution of the aggregate quantity x. Conditional on the states S_0 and S_1 respectively, the stationary distributions of x can not have any point masses, so the conditional density functions $p(x|S_0)$ and $p(x|S_1)$ can be characterized by the following Kolmogorov forward equations:[5]

$$\begin{aligned} 0 &= -(\zeta+\eta+\lambda)p_{S_0}p(x|S_0) - \frac{\mathrm{d}}{\mathrm{d}x}[\alpha(1-x)p_{S_0}p(x|S_0)] && (26) \\ 0 &= -(\zeta+\eta)p_{S_1}p(x|S_1) + \lambda p_{S_0}p(x|S_0) - \frac{\mathrm{d}}{\mathrm{d}x}[\alpha(1-x)p_{S_1}p(x|S_1)], && (27) \end{aligned}$$

where p_{S_0} and p_{S_1} are given by (2) and (3) respectively. Solving (26)-(27) leads to

$$\begin{aligned} p(x|S_0) &= \frac{\zeta+\eta+\lambda}{\alpha}(1-x)^{\frac{\zeta+\eta+\lambda}{\alpha}-1} && (28) \\ p(x|S_1) &= \frac{\zeta+\eta}{\lambda}\frac{\zeta+\eta+\lambda}{\alpha}\left[(1-x)^{\frac{\zeta+\eta}{\alpha}-1} - (1-x)^{\frac{\zeta+\eta+\lambda}{\alpha}-1}\right]. && (29) \end{aligned}$$

We then have $P(x=0) = p_{\overline{S}}$, and the density for $x \in (0,1]$ is

$$p(x) = p_{S_0}p(x|S_0) + p_{S_1}p(x|S_1) = \frac{\lambda(\zeta+\eta)(\zeta+\eta+\lambda)}{\alpha[(\zeta+\eta)^2 + \lambda(\zeta+\eta) + \lambda^2]}(1-x)^{\frac{\zeta+\eta}{\alpha}-1}. \quad (30)$$

It can be obtained that the stationary distribution of x is stochastically increasing (in the sense of first-order stochastic dominance) in the rate of transition to a high-profit market (α) and in the rate at which new markets appear (λ), and stochastically decreasing in the rate at which markets disappear (ζ) and in the rate at which other inputs can substitute the unique assets (η). Moreover, when a mass x of unique assets has switched to the high-profit

[5]These Kolmogorov forward equations are also known as Fokker–Planck equations with jumps, and are used in modeling the evolution of customer brand choices or states (e.g., Montgomery, 1969; Hauser and Wisniewski, 1982), and the player payoff dynamics in mean-field games analysis (e.g., Li, Reppen, and Sircar, 2023).

market, the firm's profit is $(1-x)\widetilde{\zeta}+x\widetilde{\Delta}$. Thus, the above comparative statics results also apply to the stationary distribution of the firm's profit. The following proposition states these results.

Proposition 5. *Consider the gradual adjustments model. Then, the stationary distribution of the share of the unique assets in the high-profit market is increasing (in the sense of first-order stochastic dominance) in the rate of transition to a high-profit market (α) and in the rate at which new markets appear (λ), and stochastically decreasing in the rate at which markets disappear (ζ) and in the rate at which other inputs can substitute the unique assets (η).*

Moreover, we can consider the effects on the firm's present value of profits of the different types of technological developments across different states. Let $\overline{V}$ be the vector of the net present value for each state (with the same order of states in the different elements of $\overline{V}$ as in $\overline{p}$). Let U be a vector of dimension five with the current return on the unique assets (the premium over $\widetilde{\zeta}$) per unit of time depending on the state, $U^T = [0 \quad 0 \quad \Delta \quad 0 \quad \Delta]$, we can write[6]

$$\overline{V} = e^{-r\,\mathrm{d}t} A\overline{V} + U\,\mathrm{d}t. \tag{31}$$

We can then obtain $\overline{V} = [I - e^{-r\,\mathrm{d}t}A]^{-1}U\,\mathrm{d}t$, where I represents the identity matrix. Making $\mathrm{d}t \to 0$, we can obtain

$$\overline{V} = \widetilde{A}^{-1}U, \tag{32}$$

which determines the value function at the different states, and where

$$\widetilde{A} = \begin{pmatrix} r+\lambda & -\lambda & 0 & 0 & 0 \\ -(\zeta+\eta) & r+\alpha+\zeta+\eta+\lambda & -\alpha & -\lambda & 0 \\ -(\zeta+\eta) & 0 & r+\zeta+\eta+\lambda & 0 & -\lambda \\ 0 & -(\zeta+\eta) & 0 & r+\alpha+\zeta+\eta & -\alpha \\ 0 & -(\zeta+\eta) & 0 & 0 & r+\zeta+\eta \end{pmatrix}.$$

We can then obtain that, as expected, $V(\overline{S}) < V(S_{0N}) < V(S_{1N}) < V(S_{1Y})$, and that $V(S_{0N}) < V(S_{0Y}) < V(S_{1Y})$ as the unique assets benefit from the existence of higher-profit markets, from being deployed in higher-profit markets, and from there being higher-profit markets in reserve (see the Appendix for the analytical expressions). The relation between

[6] The notation X^T means the transpose of matrix X.

$V(S_{0Y})$ and $V(S_{1N})$ depends on the parameter values. Specifically, $V(S_{1N}) > V(S_{0Y})$ if and only if

$$\alpha > 2r + \zeta + \eta + \lambda + \frac{(r+\lambda)^2}{\zeta + \eta}. \tag{33}$$

That is, if the transition to a high-profit market is relatively fast (high α), the unique assets can have higher present values if not deployed in a high-profit market, but in a state where there is a new market in reserve, than if employed in a high-profit market, but in a state in which there is no new market in reserve. In the case of high α, being in a low-profit market when a high-profit market is available is not too costly because the transition to the high-profit market will occur relatively quickly. Moreover, the condition is easier to satisfy when the discount rate is low (low r) or when the likelihood of new markets appearing is relatively low (low λ). In the case of low r, the firm is patient enough so that being in a low-profit market when a high-profit market is available still yields higher present values. In the case of low λ, having a new market in reserve is very valuable because new markets rarely appear.

Figures 4-7 illustrate how the value function at the different states evolve as a function of r, η, λ, and α. Note that Figure 7 illustrates that $V(S_{1N}) > V(S_{0Y})$ when α is large.

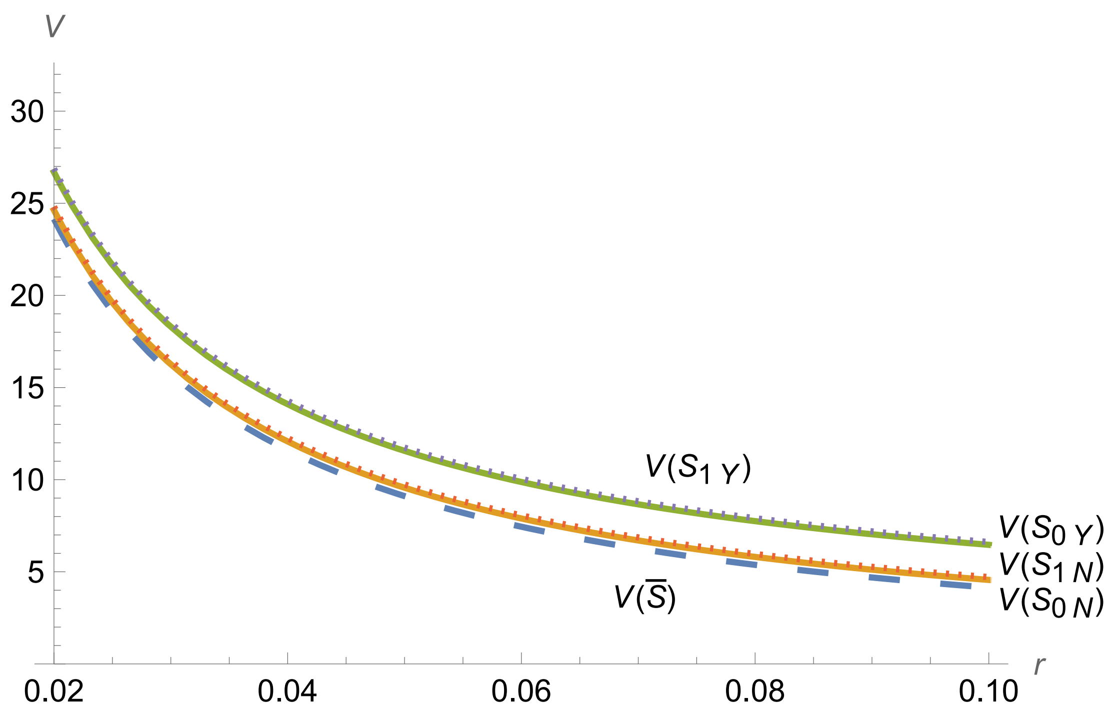

Figure 4: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(S_{1N})$, and $V(S_{1Y})$, as a function of r for $\Delta = 2$, $\lambda = 1, \eta + \zeta = .65$, and $\alpha = .3$.

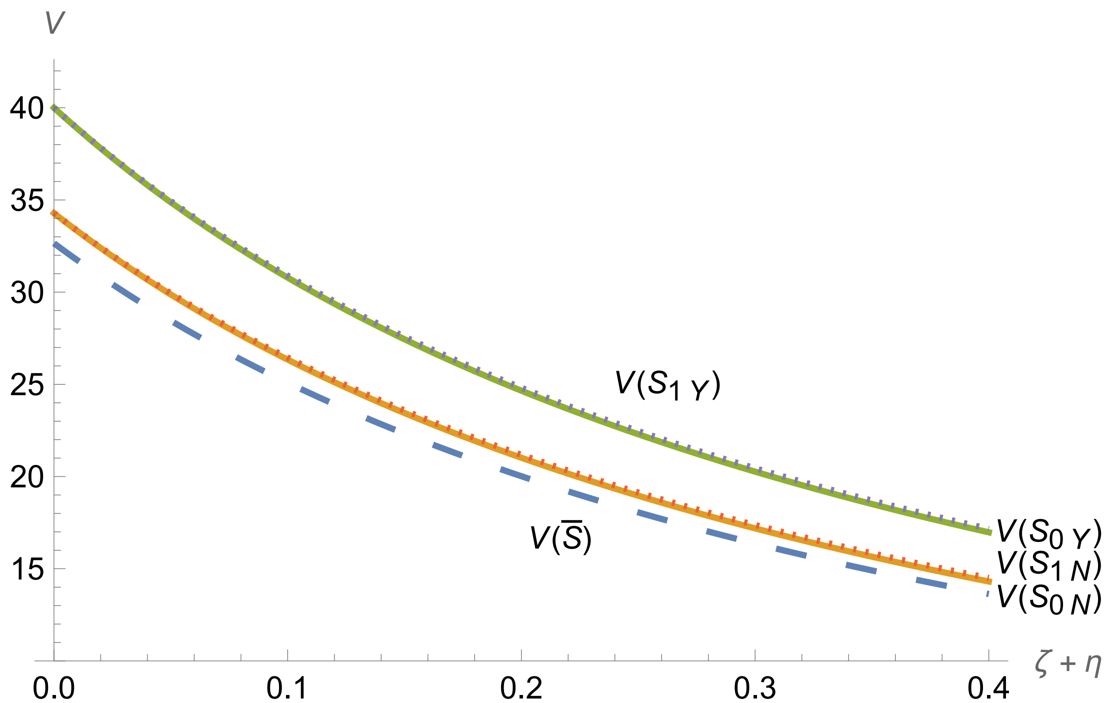

Figure 5: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(S_{1N})$, and $V(S_{1Y})$, as a function of $\eta + \zeta$ for $r = .05$, $\lambda = 1, \Delta = 2$, and $\alpha = .3$.

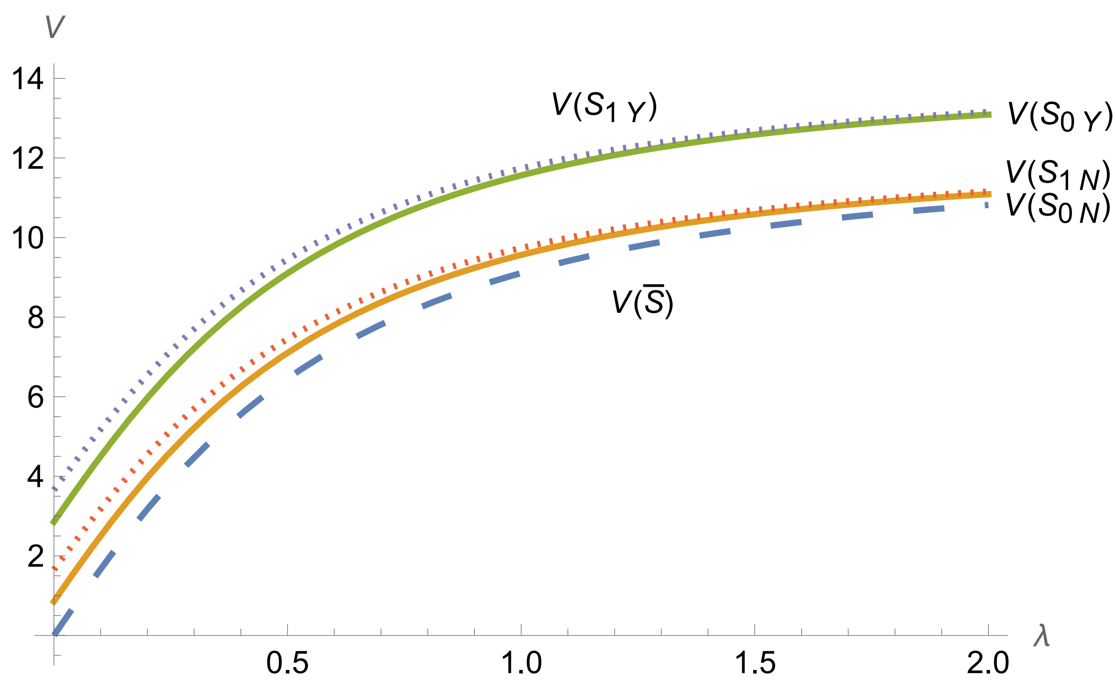

Figure 6: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(S_{1N})$, and $V(S_{1Y})$, as a function of λ for $r = .05$, $\Delta = 2, \eta + \zeta = .65$, and $\alpha = .3$.

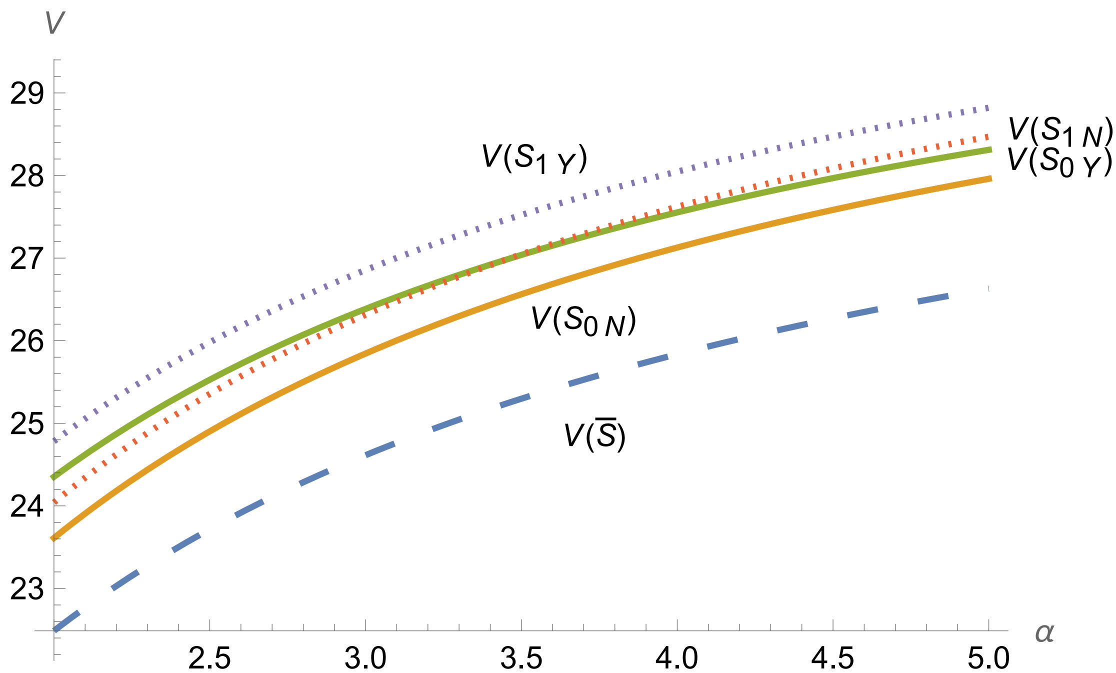

Figure 7: Evolution of $V(\overline{S}), V(S_{0N}), V(S_{0Y}), V(S_{1N})$, and $V(S_{1Y})$, as a function of α for $r = .05$, $\lambda = 1, \eta + \zeta = .65$, and $\Delta = 2$.

From the firm's aggregated view, conditional on that a mass x of unique assets have switched to the high-profit market, the present values of the profits in state S_0 and S_1 are given by

$$V(x|S_0) = (1-x)V(S_{0N}) + xV(S_{0Y}) \tag{34}$$

$$V(x|S_1) = (1-x)V(S_{1N}) + xV(S_{1Y}). \tag{35}$$

These value functions are linear in x. This is expected because we assume the unique assets to be homogeneous. We can also obtain that $V(\overline{S}) < V(S_0, x) < V(S_1, x)$ for any given x. This is consistent with the results in Section 2.

4. Multiple Markets in Waiting

The analysis above considered that there is at most one market in reserve when there is a high-profit market in the model. This could be potentially justified if the efforts to develop new technologies potentially decrease when there is already a potential new market in reserve, and by the model already capturing the main effects of having or not having markets in reserve. In this section, we return to the model of Section 2 and consider now the possibility of there being multiple markets in reserve.

Let M be the number of markets available such that the unique asset is the only input. The case of $M = 0$ corresponds to state $\overline{S}$, the case of $M = 1$ corresponds to state S_0, and $M = 2$ corresponds to state S_1, and we also now allow for $M > 2$ as well.

We concentrate on investigating the effects on the firm's present value of profits. Suppose that $M \geq 2$. Then, the expected present value of profits from the unique assets has to satisfy

$$V(M) = \Delta\,\mathrm{d}t + (\zeta+\eta)\mathrm{d}t\,e^{-r\,\mathrm{d}t}[V(M-1)-F] + \lambda\,\mathrm{d}t\,e^{-r\,\mathrm{d}t}V(M+1) + [1-(\zeta+\eta+\lambda)\mathrm{d}t]e^{-r\,\mathrm{d}t}V(M), \tag{36}$$

which, when making $\mathrm{d}t \to 0$, reduces to

$$(r+\zeta+\eta+\lambda)V(M) = \Delta - (\zeta+\eta)F + (\zeta+\eta)V(M-1) + \lambda V(M+1). \tag{37}$$

Similarly, we can obtain

$$(r+\lambda)V(0) = \lambda[V(1) - F] \tag{38}$$

$$(r+\zeta+\eta+\lambda)V(1) = \Delta + (\zeta+\eta)V(0) + \lambda V(2). \tag{39}$$

From (37), we can get obtain the steady-state as $V^* = \frac{\Delta-(\zeta+\eta)F}{r}$. The characteristic equation associated with the recursive equation (37) has solutions

$$\phi_1 = \frac{1}{2} + \frac{r+\zeta+\eta}{2\lambda} + \frac{\sqrt{[\lambda-(r+\zeta+\eta)^2]+4r\lambda}}{2\lambda} \tag{40}$$

$$\phi_2 = \frac{1}{2} + \frac{r+\zeta+\eta}{2\lambda} - \frac{\sqrt{[\lambda-(r+\zeta+\eta)^2]+4r\lambda}}{2\lambda}. \tag{41}$$

For $V(M)$ to converge, we have

$$V(M) = V^* + C\phi_2^M \tag{42}$$

for $M \geq 1$ where C is a constant to be determined. Using (38) and (39) we can obtain

$$C = -\frac{\Delta - (r+\zeta+\eta)F}{(r+\lambda) - \phi_2\lambda}, \tag{43}$$

from which we can obtain $C < 0$, and

$$V(0) = \frac{\lambda(V^* + C\phi_2 - F)}{r + \lambda}. \tag{44}$$

It is now interesting to investigate the likelihood of being in each state around the steady state. For there to be a steady state, we need $\zeta + \eta > \lambda$, which is assumed in the remainder of this section. Letting p_M represent the steady-state probability of being in state M, we have

$$\begin{aligned} (1 - \lambda\,\mathrm{d}t)p_0 + (\zeta + \eta)\mathrm{d}t\,p_1 &= p_0 & (45)\\ \lambda\,\mathrm{d}t\,p_{M-1} + [1 - (\lambda + \zeta + \eta)\mathrm{d}t]p_M + (\zeta + \eta)\,\mathrm{d}t\,p_{M+1} &= p_M \text{ for } M \geq 1, & (46)\end{aligned}$$

which yields

$$p_M = \frac{\zeta + \eta - \lambda}{\zeta + \eta}\left(\frac{\lambda}{\zeta + \eta}\right)^M \tag{47}$$

for all M. The fraction of time in which the unique assets would have low profits would then be $\frac{\zeta+\eta-\lambda}{\zeta+\eta}$, from which we can obtain the relatively expected comparative statics that the fraction of time with lower profits increases with the likelihood of a market disappearing (ζ) and with the likelihood of technological advances permitting the other inputs to substitute the unique assets (η), and decreases with the likelihood of technological advances generating the appearance of new markets (λ).

This model further illustrates that the situation of the markets and potential growth and unique assets return dynamics do not depend only on the state of the markets (whether there is a high-profit market and whether there is a market in reserve), but may depend crucially on the number of markets in reserve. If the extent of cumulative R&D expenditures, or unused patents, is a proxy for the number of markets in reserve, those measures could be used to predict future medium-term growth in total surplus and the likelihood of entering a period of low profits.

5. Continuous Technological Innovations

In the previous sections, we considered that technological advances generate discrete changes in the profits from the unique assets, either by changing the technology of production in new markets or by the appearance of new markets. We consider here the possibility that technological advances are smoother, which generates the question of when the firm decides to switch the unique assets to new markets depending on the relative profitability across markets. This section is focused on the question of when the firm wants to switch the unique assets to other markets and the present value of profits of the firm. We can see the market from which the unique assets switch as the market that then becomes automated.

In particular, consider that the profitability for the firm of the market where the unique assets are deployed follows a Brownian motion with negative drift, given technological advances to reduce dependence on the unique assets in that market. Furthermore, consider that the profitability for the firm of the potential new market follows a Brownian motion with positive drift, given the technological advances allowing for greater profitability in new markets. Suppose also that the firm gets zero profits from markets where it does not use the unique assets.[7] Let x_0 be the profit in the market where the unique assets are deployed, and let x_1 be the profit in the market to which the unique assets could be switched. Let the cost of transition from one market to the other be F. We have

$$\begin{aligned} \mathrm{d}x_0 &= -h\,\mathrm{d}t + \sigma\,\mathrm{d}W_0 && (48) \\ \mathrm{d}x_1 &= g\,\mathrm{d}t + s\,\mathrm{d}W_1, && (49) \end{aligned}$$

where $h, g > 0$, $\sigma > 0$ represents the variability in the stochastic process of the profitability in the existing market, $s > 0$ represents the variability in the stochastic process of the profitability of the new potential market, and W_0 and W_1 are independent standardized Brownian motions.

Let $V(x_0, x_1)$ be the expected present value of profits from the unique assets if it is deployed in a market with a profit of x_0, and the profit in the new potential market is x_1. When the unique assets switch at (x_0^*, x_1^*) from the old market to the new market, we then have that $x_0 = x_1^*$, and $x_1 = x_1^* - \beta$, for the next problem of when to switch markets, with $\beta > 0$.

In this case, the firm gets hurt stochastically if technological advances lead both to the other inputs being a good substitute for the unique assets in the old market (x_0 falls) and not enough profitability growth in the new market (x_1 stays low).

The Bellman equation for $V(x_0, x_1)$ can be represented by

$$V(x_0, x_1) = x_0 + e^{-r\,\mathrm{d}t} E V(x_0 + \mathrm{d}x_0, x_1 + \mathrm{d}x_1). \tag{50}$$

[7]This would occur if there is perfect competition in the market with other inputs.

Applying Itô's Lemma, this reduces to the following partial differential equation

$$rV(x_0,x_1) = x_0 - hV_1(x_0,x_1) + gV_2(x_0,x_1) + \frac{\sigma^2}{2}V_{11}(x_0,x_1) + \frac{s^2}{2}V_{22}(x_0,x_1), \tag{51}$$

where V_i and V_{ii} represent the first and second derivative of the function V with respect to the i-th argument of V.

Let $u(x_1)$ be the value of x_0 at which the firm decides to switch the unique assets to the new market when the profitability of the new potential market is at x_1. Then value matching and smooth pasting at $x_0 = u(x_1)$ yield

$$\begin{aligned} V(u(x_1),x_1) &= V(x_1, x_1-\beta) - F & (52)\\ V_1(u(x_1),x_1) &= 0 & (53)\\ V_2(u(x_1),x_1) &= V_1(x_1,x_1-\beta) + V_2(x_1,x_1-\beta) - F. & (54) \end{aligned}$$

In this case, the form of $V(x_0,x_1)$ satisfies $V(x_0,x_1) = x_0/r + W(x_1 - x_0)$. Using this, we can obtain that (51) reduces to

$$rW(a) = -\frac{h}{r} + (h+g)W'(a) + \frac{\sigma^2+s^2}{2}W''(a), \tag{55}$$

where $a = x_1 - x_0$, that (52)-(53) reduce to

$$\begin{aligned} W(\overline{a}) &= W(-\beta) + \frac{\overline{a}}{r} - F & (56)\\ W'(\overline{a}) &= \frac{1}{r}, & (57) \end{aligned}$$

respectively, where $x_1 - u(x_1) = \overline{a}$, for all x_1, and that (54) is always satisfied given (57). Note that this means that the firm switches the unique assets when $x_1 - x_0$ reaches the threshold $\overline{a}$, and that this can occur when the profitability is either low or high, x_1 and x_0.

Let $\mu = \frac{1}{\sigma^2+s^2}\left(\sqrt{(h+g)^2+2r(\sigma^2+s^2)} - (h+g)\right)$, which is positive. Note further that $\lim_{a\to-\infty} W(a) = 0$, as when x_0 is infinitely greater than x_1 the firm only switches the unique assets to the new market at infinity. Using (55)-(57), we can then obtain that

$$W(a) = \frac{1}{r\mu}e^{\mu(a-\overline{a})} - \frac{h}{r^2}, \tag{58}$$

where $\overline{a}$ is obtained by

$$\mu\overline{a} + e^{-\mu(\beta+\overline{a})} = 1 + r\mu F. \tag{59}$$

This yields comparative statics of the threshold $\overline{a}$ with respect to different model parameters, which are stated in the following proposition.

Proposition 6. *Consider the continuous technological evolution model. Then the threshold* $\overline{a}$ *of the difference in the profitability of markets at which the firm decides to switch the unique assets to the new market is increasing in the costs of transition* F*, the lag of the new technology* β*, the interest rate* r*, the absolute value of the drifts of the old and new technology,* h *and* g*, and in the uncertainty of the evolution of the old and new technology,* σ^2 *and* s^2*.*

As expected, when the cost of transition increases, the firm requires a greater difference in profitability for the firm to decide to switch markets. Similarly, when the lag of the new technology, β, is greater, it will take longer for the firm to decide to move the unique assets to the new potential market and requires a further difference in profitability to be willing to switch markets. A higher interest rate also reduces the present value of the benefits of switching, which then leads the firm to switch markets only if the difference in profitability is greater.

Potentially more interesting, the difference in the profitability required to induce the firm to switch the unique assets to the new market increases in the speed with which the new technology improves in relation to the old technology (greater h and g), because then the firm will not have to switch the unique assets to the new market as frequently. Similarly, as the uncertainty in the evolution of technologies increases (greater σ^2 and s^2), the firm requires a greater difference in profitability in order to switch the unique assets to the new market, because there is a greater chance that the difference in profitability will be reduced.

We can also obtain that $V(x_0, x_1)$ is increasing in both the profitability of the current market and the profitability that could be obtained in the new market. That is, the firm can be in periods of low present value of payoffs if both x_0 and x_1 receive a sufficient number of negative shocks. That is, the situation of occasional low profitability could be explained by negative shocks to the profitability of both the current market and the potential new market. This would also indicate that, given that we know about a negative shock to profitability in the current markets in which the unique assets are deployed, the true state of low profitability would depend on the evaluation of profitability in potential new markets.

Note that if $x_1 - x_0$ is close to $\overline{a}$, the present value of payoffs for the unique assets is increasing more in the profitability of the new market, x_1, than in the profitability of the current market, x_0. That is, for $x_1 - x_0$ close to $\overline{a}$ we can obtain $\frac{\partial V}{\partial x_1} > \frac{\partial V}{\partial x_0}$. As we get closer to a situation when the firm may want to switch the unique assets to the new market, it actually benefits more from the profitability in the new market (because this accelerates its move to the market) than from the productivity of the current market (which delays the switch to the new market).

Using standard methods (see the Appendix) we can also obtain the expected time period until the next time that the firm switches the unique assets to the new market given the difference of profitability a, $T(a)$, as

$$T(a) = \frac{\overline{a}}{h+g} e^{-\frac{2(h+g)}{\sqrt{\sigma^2+s^2}}(\overline{a}-a)} - \frac{a}{h+g}. \tag{60}$$

We can then also obtain the expected time period between when the firm switches the unique assets to new markets as $T(-\beta)$. Considering the empirical distribution of time periods of the firm switching the unique assets to new markets, we can potentially compare it with the model prediction.

6. Conclusion

This paper presents a stochastic perspective on the evolution of technologies, with the resulting effects on the costs that a firm experiences when re-deploying its unique firm assets during a transition between technologies, and the effects on profitability. Technology evolution can have two dimensions. First, it can allow other inputs to be used in production to substitute the unique assets. An example can be seen in the recent automation developments. Second, technology may allow the appearance of new markets that have high profitability. The uncertainty of these two dimensions of the evolution of technologies leads to the potential for stochastic low profitability periods for the firm. We consider two forms of negative effects on the firm. On one hand, there could be direct costs of transition from one market to another, which can often involve the cost for the firm of adapting the unique assets to the new market. On the other hand, there could be the possibility of new markets with high profits not yet being available to switch to once the unique assets in existing markets start having lower profits because of new technologies that allow the unique assets to be substituted with the other inputs.

The model permits an interpretation of the recent automation developments as a stochastic event that is part of a sequence of successive technology developments, and not a unique event. The model also indicates that the costs imposed on the firm by these automation developments depend on the extent to which new alternative high-profit markets are created, which again may depend on the evolution of technologies.

Moreover, the model presented here can be adapted to characterize the transition of labor across different sectors, as a response to automation shocks. This would then lead to wage crises once in a while, occurring on the equilibrium path, with some stochastic pattern.

The model is quite stylized and does not consider the heterogeneity of returns among different types of unique assets, or the investments that the firm potentially can make in revenue enhancement and R&D. This leads to the dramatic effect that the unique assets can move as a whole from one market to another, without including the possibility that the unique assets are deployed in all markets (except for the analysis in Section 3). In future research, it would be interesting to consider the heterogeneity of returns across different types of unique assets and to model investment in revenue enhancement and R&D.

APPENDIX

Proof of Proposition 3: We can obtain

$$\gamma_{\text{EW}} = \frac{\zeta+\eta}{2\zeta+3\eta} \tag{i}$$

$$\gamma_{\text{WE}} = \frac{\zeta+2\eta}{2\zeta+3\eta} \tag{ii}$$

which immediately yields $\gamma_{\text{WE}} > \gamma_{\text{EW}}$. Differentiating γ_{WE} and γ_{EW} with respect to ζ and η yields the remaining results in the proposition.

Expressions for Present Values in Section 3:

Let $R = \frac{\Delta}{r+\alpha+\zeta+\eta}$ be the present value of the unique asset's premium when deployed to a new market, then (32) leads to

$$V(\overline{S}) = \frac{\alpha\lambda R(r+\zeta+\eta+\lambda)}{r[(r+\zeta+\eta+\lambda)^2-\lambda(\zeta+\eta)]} \tag{iii}$$

$$V(S_{0N}) = \frac{\alpha R(r+\lambda)(r+\zeta+\eta+\lambda)}{r[(r+\zeta+\eta+\lambda)^2-\lambda(\zeta+\eta)]} \tag{iv}$$

$$V(S_{0Y}) = V(S_{0N}) + R \tag{v}$$

$$V(S_{1N}) = \frac{\alpha R[(r+\lambda)(r+\zeta+\eta+\lambda)+r(\zeta+\eta)]}{r[(r+\zeta+\eta+\lambda)^2-\lambda(\zeta+\eta)]} \tag{vi}$$

$$V(S_{1Y}) = V(S_{1N}) + R. \tag{vii}$$

Derivation of Expected Time Period to Unique Assets' Next Market Switch:

Note that

$$\mathrm{d}a = (h+g)\mathrm{d}t + \sqrt{\sigma^2+s^2}\,\mathrm{d}W \tag{viii}$$

where W is a standardized Brownian motion. Let $T(a)$ represent the expected time period until the next time that the unique assets switch markets. We know that evolution of $T(a)$ satisfies

$$T(a) = \mathrm{d}t + E[T(a+\mathrm{d}a)]. \tag{ix}$$

Using Itô's Lemma this reduces to the differential equation

$$\frac{\sqrt{\sigma^2+s^2}}{2}T''(a) + (h+g)T'(a) + 1 = 0. \tag{x}$$

Given that $T(\overline{a}) = 0$, we can obtain from (x) that

$$T(a) = \frac{\overline{a}}{h+g} e^{-\frac{2(h+g)}{\sqrt{\sigma^2+s^2}}(\overline{a}-a)} - \frac{a}{h+g}. \tag{xi}$$

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