Abstract. We estimate a model of college admissions wherein students endogenously accrue pre-college human capital (HC) as part of a contest for enrollment at high quality colleges. We use methods from the empirical auctions literature to separately identify the roles of school quality, HC, and students' privately known learning costs on post-college household income. Conditional on graduating, college quality is the most important factor in determining income, while unobserved student characteristics play a nontrivial secondary role. Pre-college HC drives college placement and graduation probability, but not post-college income. We conduct counterfactual experiments comparing the status quo to a color-blind admissions rule and a proportional quota for minority students. Color-blind admissions results in fewer (more) minority students enrolling at the best (worst) schools with a corresponding reduction in household incomes and graduation rates. The signs and magnitudes of changes to HC investment and graduation rate depend on the learning cost of the particular student in question, and accounting for the endogeneity of HC is crucial for predicting the effect of each admissions rule.

1. INTRODUCTION

For many Americans, the competition to be admitted to a high quality college is one of the highest stakes contests of their lives. The reason is simple: there is a high degree of college heterogeneity in terms of both educational inputs and the outcomes realized by students. For example, in 1988 the interquartile range of spending per student is $5,931 to $9,551, while the interquartile ranges of the graduation rates and household income 10 years after graduation are 38% to...
Separating the causes of these disparate outcomes requires disentangling the influence of college quality, human capital (HC) investment, and privately-known type on the returns to attending college. Previous empirical work has attempted to address the issue by instrumenting for the influence of college quality while subsuming unobserved student characteristics into the unexplained error term in the model (see Brewer, Eide, and Ehrenberg [15]; Dale and Krueger [24]; Black and Smith [10, 11]; and Long [43]). Moreover, this literature does not account for how affirmative action (AA) shapes HC accumulation incentives.

We take a novel approach to this problem by explicitly modeling the separate influences of school quality, pre-college HC investment, and unobserved student characteristics in determining post-college economic outcomes. A structural approach is necessary to identify the students’ private information about their unobserved characteristics. We estimate a model wherein a continuum of college applicants with differing unobserved learning costs compete for admission to college through the accrual of HC in a contest. Each college occupies a different point on the quality spectrum and uses a distinct, endogenous admissions cutoff. AA programs cause these cutoffs to be different for different demographic groups. The contest structure allows us to leverage techniques from the empirical auctions literature to identify the privately known learning costs of the students.

Within our model, there are two incentives to invest in HC. First, HC is a productive asset, which creates a productive channel of incentives. The strength of this channel (i.e., the marginal productivity of HC) may be a function of the quality of the student’s college or the student’s type. This channel is present in the complete-information assortative matching model of Becker [9]. Second, since colleges wish to admit the best students possible, they rank students using the students’ academic achievement (i.e., HC). Therefore, HC investment plays a second role as a rank-order index that allows better students to out-compete their peers and enroll into better schools. These indirect returns create a competitive channel of incentives that drives students to use achievement as a means of establishing their position above less-accomplished competitors. This effect resembles the costly signaling phenomenon analyzed by Spence [58], although our model captures signaling between students and colleges rather than college students and firms. The competitive channel creates a strategic interaction between one’s own actions and the actions of others: if rivals never study mathematics and science, one might study less, consume more leisure time, and still place into a top college. The important difference between our setting and traditional applications of signaling is that it is not altogether wasteful—study generally produces more useful knowledge—but rather, results in over-production of HC in response to competitive pressure. Importantly, our model views HC accumulation as an endeavor spanning

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1The graduation rate data comes from Integrated Postsecondary Education Data System, while the income data comes from the Baccalaureate and Beyond (B&B) survey. Income quartiles have been rounded to the nearest thousand dollars to protect B&B respondents’ anonymity.

2Throughout this paper, when we use the term AA we refer to the practice of granting preferential treatment in college admissions on the basis of race, holding academic merit (grades and exam scores) fixed.
years of the student’s life prior to the college application process. A full understanding of a student’s HC choice must take into account both channels of incentives.  

The first goal of our analysis is to tease apart the effects of college quality, HC, and unobservable characteristics on post-college household income. Our analysis uses individual-level data from the Baccalaureate and Beyond (B&B) survey conducted by the U.S. Department of Education on student demography, academic achievement, and post-college household income. These data allow us to provide an analysis that fully accounts for the strategic interactions at work in the nation-wide admissions contest, which would be impossible with data from a single school or even an entire state.

For our purposes there are two key sources of variation in the B&B data. First, although the market is highly assortative in that better colleges tend to attract more accomplished students, it is not perfectly so, and each university’s student body exhibits a distribution of HC within each demographic group. Therefore, college quality and HC achievement are imperfectly correlated, which allows us to separate the influence of these two factors on post-college earnings. The second source of variation is AA practices, which create different investment incentives for students in different demographic groups by altering their strategic incentives to compete on academics for college seats. Crucially, our model allows us to apply techniques from the empirical auctions literature in a novel way to identify unobserved student learning costs. Recognizing a connection to auctions is useful because that body of work has developed a wealth of empirical methods for identifying private information from strategic bidding data (e.g., see Athey and Haile [6] and Guerre, Perrigne, and Vuong [32] (GPV)). The key insight from our model is that the academic competition between future college applicants provides an inverse mapping from students’ observable academic achievement—analogous to a bid—into the underlying learning cost types that rationalize their HC choices as best responses to their rivals’ achievement levels. Since AA causes the incentives facing members of distinct demographic groups to differ, the inverse mapping from HC choice to learning-cost type will differ across groups. Therefore, HC choices and unobserved characteristics are imperfectly correlated, which allows us to identify their distinct effects on post-college outcomes.

Unlike in standard auction settings, the value of winning a college seat is a function of the exogenous quality of the college, the endogenous HC choice of the student, and her privately-known learning cost. Therefore, we need to simultaneously estimate the determinants of match utility while using the GPV approach to estimate the learning cost of each student. In effect, the inverse mapping from HC choices into privately-known learning costs derived from the GPV method is embedded into a household income regression as a non-linear control function that separates the effects of HC and student type on post-college income.
We find that college quality and pre-college HC both have a significant effect on the probability that a student graduates. On the other hand, pre-college HC has very little influence on household income 10 years following graduation, where college quality and unobserved student characteristics play dominant roles. This is consistent with a view of the world in which a student’s pre-college HC helps the student pass the curriculum of the college in which she enrolls, but post-college household income is determined by the level of HC accrued in college, which is generated by college quality and unobserved student characteristics (i.e. learning costs).

We also find that the distribution of learning costs for minority students stochastically dominates the distribution for nonminority students. In other words, our results suggest minority students in general have higher learning costs. We do not believe that there is any reason to think that the learning cost differences are biologically innate. For example, the higher socioeconomic strain that lower-income (on average) minority families face makes the direct costs and opportunity costs of early childhood investments more difficult than for the typical, relatively more affluent, nonminority family. In Appendix D we show that childhood socioeconomic factors are highly predictive of our estimated learning costs, while race, conditional on affluence, parents’ education, etc., is not.

Once the drivers of post-college outcomes are identified, it is then interesting to understand how affirmative action programs affect outcomes for members of different demographic groups. We use our estimates to conduct a counterfactual analysis of the effects of AA schemes on enrollment, graduation rates, and household income. AA has a long history in the United States that stretches back to the Kennedy Administration. It is motivated by its proponents on grounds of racial disparities in college placement and a desire to achieve greater diversity within student bodies at top universities. For example, in 1988 16% of all new college freshmen were underrepresented minorities (Black, Hispanic, or Native American) who accounted for only 10% of new enrollees within the top fifth of US colleges, despite substantial considerations for race in the admissions process. This disparity is in turn driven by gaps in pre-college academic achievement: in that same year median minority GPAs and SAT scores were slightly below the 25th percentile for Whites and Asians.

A structural approach to the AA counterfactuals is required since changes to admissions rules have complex effects on the productive and competitive channels of incentives, and in turn, on HC choices. Our first result is that admissions would be quite different under a color-blind admissions system. Under the status quo AA regime generating the data, minorities are underrepresented at the best schools and heavily over-represented at the worst ones, but a color-blind college admissions scheme would result in a shift of minority students into lower ranked schools (e.g., minority enrollment in the bottom quintile of colleges increases by 50% and decreases in the top quintile by 25%). Under a proportional quota scheme, the opposite occurs: minorities enroll in higher quality programs with (mechanically) proportional representation in each college quality quintile.

5We remain agnostic about whether diversity is an end itself or whether it is instrumental in providing a high quality college education (see Regents of the University of California v. Bakke).
Color-blind and proportional systems, which can be interpreted through the auction-theoretic lens of a bid subsidy, have opposite effects on the HC investments of minority students (relative to the status quo). In a color-blind scheme, minority students with high and medium learning costs reduce their HC choices. This is because a high learning cost minority student must out-compete a larger number of other students to significantly improve her college assignment. In other words, the competitive channel is weaker under a color-blind system for these students, which in turn reduces their HC choices. Students with low learning costs find that the quality of the available college seat improves rapidly as she out-competes her rivals, which strengthens the competitive channel and increases her HC investments. For nonminority students, the effects of shifting from the status quo to color-blind are the opposite, but of smaller and generally inconsequential magnitude. The changes in HC investment from a shift toward a proportional quota from the status quo are larger in magnitude and reversed in sign. The reasoning is essentially symmetric to the color-blind case: the competitive channel is stronger (weaker) for high (low) learning cost minority students, which leads to more (less) HC accumulation.

The impact of AA programs on graduation rates are complicated by the joint effects of changing enrollment and altered HC investments. In a color-blind system, the lowest learning-cost minority students have higher graduation rates because of the increased HC investments despite enrolling in worse schools. In contrast, the highest learning-cost minority students both have less HC and enroll in worse colleges, which reduces graduation rates. In a proportional quota, low learning-cost students invest less, but enroll in better schools, yielding no net effect. The highest learning-cost students both enroll in better colleges and invest more in HC, which provides a large boost to their graduation rates. This analysis emphasizes the importance of understanding changing HC investment incentives in these counterfactuals.

Since household income 10 years after graduation is primarily driven by college quality and student type under our empirical estimates, the counterfactual results are simpler to explain. Color-blind systems place minority students in worse colleges, so their average household incomes drop by $818 relative to the status quo. Under a proportional system, minority student household income increases by an average of $1,936 due to better college placements. The changes for nonminority students are in the opposite direction and less than 1/5th as large. In terms of equivalent variation, the net utility effect on minorities of a color-blind (proportional) system is an decrease (increase) of roughly 2%-3% of household income.

We close our empirical analysis by studying the relative magnitudes of the productive and competitive channels of incentives. To the best of our knowledge, ours is the first paper to separate the marginal benefit of pre-college HC accumulation into a competitive channel and a productive channel. We find that the competitive channel is stronger than the productive channel for all but the highest-achieving 15% of students overall. The relative strength of the competitive channel increases for students with higher learning costs, being roughly three times as powerful as the productive channel for the middle 50% of the learning cost distribution.

The remainder of this paper has the following structure: we first briefly summarize the previous literature on AA and discuss its relation to the current model. In Section 2, we describe
the US college market structure and the data that will be used. Section 3 outlines the theoretical model on which the econometric exercise is based. In Section 4.1 we formally outline our semi-parametric identification results, and in Section 4.2 we describe our estimator. In Section 5, we discuss the results of estimation and in Section 6 we present the counterfactual exercises. Section 7 concludes and briefly describes directions for future research.

1.1. Related Literature. Our paper forms part of a new literature focusing on endogeneity of academic achievement with respect to college admissions rules. Coate and Loury [21] and [22] were among the first to explore a theory of endogenous educational achievement as a function of AA rules. Unlike our project, both of these papers assume homogenous firms, binary HC investment, and no strategic interaction between the students/workers. Recent work by Olszewski and Siegel [50] and Bodoh-Creed and Hickman [13] introduce contest models that are amenable to the college admissions setting as they include heterogeneous school quality and endogenous HC choices. Olszewski and Siegel [50] does not include AA or noisy human capital, both of which are crucial for our identification strategy. For that reason, we base our empirical model on Bodoh-Creed and Hickman [13].

Several recent empirical studies have also explored endogenous high-school achievement. The two most closely related papers to ours are Cotton, Hickman, and Price [23] and Akhtari and Bau [2]. Cotton, et. al. [23] seeks to replicate the Bodoh-Creed and Hickman [13] framework in a field-experimental classroom setting involving competitive human capital investment (math learning) and shifts in (short-term) rank-order monetary incentives which mimic the difference between color-blind college admissions and AA in the form of representative quotas. Akhtari and Bau [2] use US high-school data and a 2003 Supreme Court decision, which restored race-based admission considerations to Texas colleges, in order to identify impacts of AA on minority students’ incentives to achieve academically. Both studies find evidence that pre-college HC investment is influenced by AA, and in particular, that it can play a significant, positive role in reducing achievement gaps and shaping investment incentives for its beneficiaries. Our paper takes a structural approach in order to contribute several new dimensions to the discussion on admissions rules. First, our model and methodology allow us to study pre-college achievement, enrollment, and post-college outcomes on a market-wide basis, rather than just focusing on one section of the country or market. Second, our structural approach facilitates counterfactual studies of how an AA ban or other forms of AA would endogenously shape the college market.

Another closely related paper is Kapor [41], who estimates a structural counterfactual to analyze the effect of the “Texas Top 10%” program. This program guarantees admission at each Texas public university to Texas high school students ranking in the top decile of their class, and was used as a substitute for race-based AA in Texas universities after an AA ban by the US 5th Circuit Court in 1996. Kapor’s [41] model includes costly college application decisions and

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enrollment choices by students with different preferences over the universities, strategic admissions decisions on the part of colleges, and information frictions that interfere with the students’ decisions. Since the model does not include private information or an endogenous HC decision, Kapor [41] does not answer the questions we pose regarding the determinants of post-college income and the relative importance of the competitive and productive channels of incentives.

Our paper also contributes to an established literature on the returns to attending a more selective college. One of the first such papers was Brewer, Eide, and Ehrenberg [15] who estimated large income gains to attending an elite college after modeling college choice as a function of net costs; they also found that these gains persist after attempts to control for selection on unobservables. Dale and Krueger [24] (DK) attempted to control for selection on unobserved student characteristics by using overlap across 1970s college applicants in the sets of schools that accepted and rejected them. DK found small average returns to a more selective college, but that for low-income applicants the economic gains were meaningful. Black and Smith [10] (BS04) re-examined this question with a propensity-score matching estimator which was designed to address potential failures of a common support condition used by linear-in-parameters models in previous studies. They argued that this problem can lead to under-estimation of the return to a better college, and they found evidence from their alternative approach that all students—rich and poor—benefit from an elite education. Long [43] re-visited the methodologies of DK and BS04 with a more recent dataset (NELS, 2000 wave), and compared them to results from ordinary least squares. He found robust evidence across methodologies for significant gains to college quality in terms of graduation rates and household income, with weaker evidence in terms of hourly wages. Finally, Black and Smith [11] use multiple proxies for college quality, rather than just measuring quality by median exam scores of enrollees as done in previous studies. They argue that using only a single measure of college quality will under-estimate the economic gains if that single measure imperfectly reflects actual quality. They utilize multiple additional proxies for quality (e.g., student-faculty ratios, retention rates, faculty salaries) and find evidence that the existing literature under-states the gains to attending a higher quality college.

Our paper draws insights from previous work and also contributes to the literature in some new ways. Like Black and Smith [11], we use a measure of college heterogeneity that incorporates information from multiple proxies for quality (see Section 2.1). Like Long [43], we focus on household income as our preferred post-college outcome, as it encapsulates the total economic gains from a more selective college, which may influence both one’s own earnings as well as earnings of one’s spouse. As a result, college quality may have a much larger effect on household income than on individual level measures such as salary or wages (see Section 2.2 for more details). In our case, using household income also allows us to retain data on both male and female college graduates, which is important since both genders play a large role in shaping the competition for college admissions.

Within the literature on returns to college quality, we are the first to explicitly model and estimate the role of unobserved student characteristics. Previous work has instrumented for unobservable characteristics in order to focus on the marginal impact of college quality, but
counterfactual analysis of alternative college admissions scenarios requires knowing how student characteristics contribute to match quality as well. We point identify students’ learning costs at the individual level and show that they play a secondary, but important, role to college quality. We also show that our estimated costs are strongly correlated with childhood home environment characteristics (see Appendix D). However, our results are difficult to compare to prior work. Since the prior work did not include private information explicitly as one of the determinants of income, the effect of the private information on learning costs could be loaded onto measures of HC. We do not view our results as refining previous results that parsed between the roles of college quality and HC. Instead, our work parses between the roles of college quality, HC, and the previously unmodeled role of privately known learning costs.

There is a sizable empirical literature examining the role of AA in determining college admissions, including Bowen and Bok [14], Kane [40], Chung and Espenshade [19], Chung, Espenshade, and Walling [20], Epple, Romano, and Sieg [26], Arcidiacono [4], and Howell [38]. All of these papers find that AA plays a significant role in shaping black educational outcomes, especially among selective institutions. Some of these studies lack nation-wide data on college admissions, and some others condense college-side heterogeneity to coarse terms for tractability. Moreover, exam scores are used as an exogenous proxy for student ability. If scores are jointly determined by both ability and market-based incentives for investment, then this is problematic. Our paper is the first one to combine market-wide data, rich supply-side heterogeneity, post-college outcomes, and endogenous pre-college HC.

We are able to measure the effect of affirmative action without making any a priori assumptions on the parametric form of the demographic admissions bonus for minority students. Since we find that affirmative action benefits minority students the least at the top and bottom of the college quality spectrum, it is plausible that the average effect over the quality spectrum is relatively weak. Due to this methodological difference, it is not obvious how to compare our results to the prior literature that assumed a parametric form for the effect of affirmative action (e.g., Hinrichs [37]).

Another vein of empirical literature on AA focuses on mismatch, the idea that AA may cause black students to be placed higher, but then graduate with lower probability or to self-select into less lucrative majors. Some empirical studies have found evidence of mismatch (e.g., Loury and Garman [45] and Arcidiacono, Aucejo, and Hotz [5]), while other empirical work has found evidence that the mismatch problem is small and likely outweighed by other benefits of higher-quality placement (e.g., Long [43], Rothstein and Yoon [55], and Chambers, Clydesdale, Kidder, and Lempert [18]). Other evidence suggests that all students generally benefit from attending higher-quality schools (e.g., Dillon and Smith [25] and Badge, Epple, and Taylor [8]). Our work contributes by modeling the endogenous adjustment of pre-college academic preparation under alternative admissions schemes. We find that there is no clear-cut ranking between AA schemes in general and color-blind admissions, since the sign and magnitude of the graduation rate

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The difference between our work and the prior literature is magnified by our focus on the effect of AA on final enrollment outcomes rather than college admissions that merely determine the enrollment choice set.
change varies with endogenous HC choices and college assignment.

While the status quo AA leads to a small reduction in average minority graduation rates (about 4.9%) relative to color-blind admissions, proportional quotas (a more generous form of AA) would lead to a significant increase in minority graduation (about 13.4%). On the other hand, we find that the total economic gains to minorities from AA, including graduation rates and post-college household income, are unambiguously positive.

Of course, the current paper is subject to its own limitations as well. We abstract from the intricacies of the admissions process, and concentrate on the link between achievement and final college placement outcomes in terms of matriculation. We do not explicitly model “supply-side” concerns—e.g., decisions on how many students to admit and how much to charge them—but instead we model college seats as fixed objects of known quality in order to concentrate on student HC investment. To the extent that supply-side competition plays a role, this work can be seen as complementary to models such as Epple et. al. [26], Chade, Lewis, and Smith [17], Fu [31], Azevedo and Leshno [7], and Fillmore [28] who treat these forces explicitly.

2. DATA AND MARKET STRUCTURE

In this section we describe the observables we use in our study. To conduct our analysis, we need enough data to effectively model the incentives facing college applicants when they make their HC accumulation choices. Our estimates require metrics for the HC of students that graduate from each college, measures of the quality and graduation rate of each college, and statistics on the post-college outcomes for each student. We begin by describing the combination of college-level and individual-level data we use.

We use US college data for academic year 1992-1993 for two main reasons. First, one can reasonably assume AA policies were stable and understood by decision-makers at that time. The only successful legal challenge prior to 1993 was in 1978, when the Supreme Court declared quotas unconstitutional in University of California v. Bakke [1]. The second reason for studying AY1992-1993 is that individual-level data on students graduating during this academic year are available from the Baccalaureate and Beyond (B&B) database linking college quality and HC choices to the household income of college graduates from that year.

2.1. Colleges. For a sample \( \mathcal{L} = \{1, 2, \ldots, L\} \) of 4-year colleges we have a vector \( \mathbf{Y}_l \) of school characteristics. The first is a quality measure derived from data and methodology by US News & World Report (USNWR) for their annual America’s Best Colleges rankings (see Morse [48]).

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We abstract from the endogeneity of HC, it is not obvious that the status quo system generates outcomes “in between” the color-blind and proportional quota systems.

USNWR computes its quality score as a weighted arithmetic mean of a school’s quantile rank in 15 quality indicators, falling into 6 different categories: Reputation Rank (based on survey data from college presidents and deans; 25% weight), Student Selectivity (acceptance rate for 1992 freshman class, yield rate for 1992 freshman class, % of enrollees in top 25% of high school class, and mean SAT/ACT score among enrollees; 25% weight), Faculty Resources (comprising student-faculty ratios, % of full-time faculty with terminal degrees, % of faculty on part-time status, average salary and benefits for full-time faculty, and proportion of classes with fewer than 20 students; 20% weight), Financial Resources (per-student education expenditures and per-student other expenditures; 15% weight), Graduation Rate (% of students in 1983-1986 freshman classes who graduated within 6 years; 10% weight), and Alumni Satisfaction (% of living alumni who donated to AY1991-1992 fund drives; 5% weight). USNWR computes the quality
adopt this measure as the college quality index $p_l$, and we argue that interpreting this index as a reflection of meaningful quality rankings is sensible for three reasons. First, USNWR removes information frictions by providing a wealth of data on many schools, along with advice on how to interpret the data. Consumers’ response to this service has been large enough that rankings are now the primary focus of USNWR’s business model. Second, the validity of USNWR rankings is reinforced in students’ minds by the enthusiasm with which universities advertise their status in America’s Best Colleges. Third, many previous studies have depicted college quality either with coarse, discrete measures (e.g., flagship state schools versus non-flagship schools) or with relatively simplistic ones such as mean student-body exam score alone. Our measure provides for a continuous transition from low quality to high, and it encompasses a host of factors influencing the college experience such as selectivity, per-student spending, and faculty quality.

The US postsecondary education industry is vast and diverse, with thousands of institutions offering students at least one type of 4-year degree, but many of these specialize in vocational training. Thus, we adopt the USNWR universe of schools as our definition of “the college market.” This leaves us with 1,245 non-profit colleges and universities specializing primarily in liberal arts education leading up to a bachelor’s degree. This set of schools accounts for the majority of 4-year degree production in the United States. Between the late 1980’s and early 1990’s the total size of the incoming freshman class for these schools was roughly 1.6 million students.

The other college-level data are provided by the National Center for Education Statistics (NCES) through their Integrated Postsecondary Education Data System (IPEDS), which includes school-level enrollment for all first-time freshmen (including full-time and part-time) by race for Whites, Blacks, Hispanics, Asians or Pacific Islanders, and American Indians/Alaskan Natives. For the 1988 incoming class, we have freshman headcount, denoted $M_l$, for underrepresented minorities—Blacks, Hispanics, and Native Americans—and a headcount, denoted $N_l$, for all others—Whites and Asians. Aggregating this information across schools allows us to compute

$$
\mu = \frac{\sum_{l=1}^{L} M_l}{\sum_{l=1}^{L} (M_l + N_l)}.
$$

IPEDS also allows the researcher to compute race-specific, 6-year graduation rates for each college campus, which we denote by $\Gamma_{jl}$, where $j \in \{M, N\}$ refers to $\mathcal{M}$inority and $\mathcal{N}$onminority students. In total then, the data representing schools $l = 1, \ldots, L$ are denoted $Y_l = \{p_l, M_l, N_l, \Gamma_{Ml}, \Gamma_{Nl}\}_{l=1}^{L}$.

Colleges are separated into five tiers with Tier I representing the top quality quintile of college seats. Entries in Table 1 represent the fraction of all students within a tier that are of a given demographic group. In a hypothetical world with no under-representation, each cell would be the same as the overall share of each race group. “Minorities” are demographic groups that are under-represented in the top three tiers and over-represented in the bottom two. Similar patterns hold as well for race sub-categories: Blacks, Hispanics, and Native Americans are individually under-represented in top tiers. The reverse is true for Whites and Asians who comprise the
Table 1. Racial Representation Within Academic Quality Quintiles

<table>
<thead>
<tr>
<th>Group</th>
<th>Total Share</th>
<th>Tier I Share</th>
<th>Tier II Share</th>
<th>Tier III Share</th>
<th>Tier IV Share</th>
<th>Tier V Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minority:</td>
<td>15.81%</td>
<td>10.23%</td>
<td>13.09%</td>
<td>11.75%</td>
<td>22.16%</td>
<td>21.81%</td>
</tr>
<tr>
<td>Non-Minority:</td>
<td>84.19%</td>
<td>89.77%</td>
<td>86.91%</td>
<td>88.25%</td>
<td>77.84%</td>
<td>78.19%</td>
</tr>
</tbody>
</table>

“Nonminority” group. Minority and nonminority students are defined as they are because AA in college admissions specifically targets under-represented minority groups.

2.2. Students. Our individual-level data on the student population comes from the 1993 Baccalaureate and Beyond Survey (B&B), which randomly samples colleges and then samples students graduating in AY1992-1993 within each college. The data contain several variables pertaining to pre-college investment for student \( i \in \{1, 2, \ldots, I\} \). The first variable we draw from the B&B survey is annual household income after 10 years in the workforce, denoted \( w_i \). Household income has two major advantages relative to individual-level measures of income (e.g., wages or salary). First, it captures the effect of educational assortative mating, which enhances a student’s welfare by increasing the income of his or her spouse. Second, the distribution of household income in our data is essentially identical for men and women.\(^{10}\) Previous work has focused on employed men to avoid confounding the effects of gender on individual salary/wages with other features of the data. Dropping women from the data is impossible in our context since the market-wide contest involves competition between the entire pool of prospective college students. However, retaining women in the sample is less problematic since household income is largely independent of gender in our data. The top panel of Table 2 provides summary statistics for college-level variables and the bottom panel summarizes student-level data.

The second piece of information we require from the data is a measure of pre-college academic preparation. Two outcome variables which researchers and college admissions officers focus on most for assessing achievement are exam scores, denoted \( e_i \), and academic record as measured by grade point average (GPA), denoted \( a_i \). In the B&B data, \( e_i \) takes the form of either the ACT or the SAT, both of which are standardized college entrance exams.\(^{11}\)

B&B does not contain high-school GPA directly, but it has other information including GPA from all non-major and non-minor courses. We adopt college non-major/non-minor GPA (NMGPA) as a proxy for high school GPA.\(^{12}\) One might worry, however, that NMGPA at institutions with

\(^{10}\)In addition, under our metric of HC (see Section 4), the average HC of women is only 0.08 standard deviations higher than the population mean.

\(^{11}\)The companies which develop these exams produce concordance tables which allow one to relate ACT scores into SAT scale and vice versa.

\(^{12}\)More specifically, we assume that exam scores plus high-school GPA together contain roughly the same information about a student’s pre-college academic preparation as exam scores plus NMGPA together. These two variables will be used to construct a single index for the HC choice within the model.
different qualities might not be comparable\[13\] but there are several reasons to believe it is a meaningful reflection of pre-college academic preparation and also more comparable across schools than major or overall GPA would be\[14\]. First, NMGPA primarily consists of coursework during the first one or two years of college, meaning the student has not had much time to accrue college-specific HC. Second, under the liberal arts model typical of American higher education, students take a wide variety of non-major courses focusing on general education much as they did in high school. These courses are not within the student’s specific area of interest or idiosyncratic talent, so the fact that better students are matched with better universities is of less importance. In addition, content covered in general education courses is largely independent of a student’s major, with English and Economics students being required to take many of the same non-major courses. Third, non-major courses are typically introductory in nature and use textbooks that are standardized across schools. In a familiar example to economists, Lopus and Paringer\[44\] reported in 2011 that the two most popular Principles of Economics textbooks—by Mankiw\[46\] and McConnell, Brue and Flynn\[47\]—had a combined 40% market share\[15\]. Fourth, in another dataset containing similar information, the National Longitudinal Survey of Youth-1997 wave, or NLSY97, we find that NMGPA and high school GPA are highly correlated.

Of course, this last observation begs the question of why we did not choose to execute our study using wage and college attendance data from NLSY97 instead. Although the NLSY97 has the virtue of containing direct observations of high-school GPA, there are three reasons why we believe that B&B is superior on the whole for our purposes. The first is sample size: only 30% of NLSY97 respondents satisfy the criterion of having high school GPA available and having enrolled in college. Of these, only 38% (1,040 in total) obtain a college degree, and, after dropping respondents engaged in post-graduate studies, the NLSY97 sample contains only about a quarter of the viable observations (875 total) as B&B for estimating our wage equation. The second major problem is that NLSY97 respondents are spread across five separate cohorts of students—being between the ages of 12 and 18 in calendar year 1997—who graduated college and entered the workforce across various years. This means that payoffs would have to be estimated using an unbalanced panel of earnings data. The problem further exacerbates the data shortage problem, as it would require estimation of additional parameters to control for the potential effects of workforce experience and/or other unobservable differences across college graduation cohorts. By contrast, the B&B database contains only students who graduated college in the 1992-1993 academic year and entered the workforce at the same time under the same general market conditions. Third, the NLSY97 respondents had the misfortune of experiencing

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13Previous research on high school achievement has frequently used high-school GPA, but the interpretation of high school GPAs likely suffer the same problem. While our metric of pre-college HC is imperfect due to potential difficulties comparing courses and grades between colleges, this is a problem common to HC metrics based on educational data. For example, high school GPA is also difficult to compare between schools due to different grading standards and course offerings.

14In Section 5.6, we show that using exam scores as the only measure of HC yields estimates with implausibly low levels of assortativity between HC and school quality.

15Lopus and Paringer\[44\] also found a remarkable degree of content overlap across 26 different principles textbooks.
Table 2. Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IPEDS/USNWR:</strong> (school-level data)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-Yr Graduation Rate: M</td>
<td>961</td>
<td>0.3427</td>
<td>0.2812</td>
<td>0.2159</td>
</tr>
<tr>
<td>6-Yr Graduation Rate: N</td>
<td>961</td>
<td>0.4831</td>
<td>0.4568</td>
<td>0.2295</td>
</tr>
<tr>
<td>Freshman Cohort Size: M</td>
<td>1,245</td>
<td>145.37</td>
<td>46</td>
<td>254.56</td>
</tr>
<tr>
<td>Freshman Cohort Size: N</td>
<td>1,245</td>
<td>660.86</td>
<td>378</td>
<td>788.68</td>
</tr>
<tr>
<td>College Quality Index</td>
<td>1,245</td>
<td>0.4842</td>
<td>0.4598</td>
<td>0.2132</td>
</tr>
<tr>
<td>*<em>BACCALAUREATE AND BEYOND</em>: (individual level data, college graduates only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAT/SAT equivalent scores: M</td>
<td>500</td>
<td>820</td>
<td>820</td>
<td>220</td>
</tr>
<tr>
<td>SAT/SAT equivalent scores: N</td>
<td>4,980</td>
<td>990</td>
<td>980</td>
<td>190</td>
</tr>
<tr>
<td>Academic Record (GPA): M</td>
<td>500</td>
<td>2.7</td>
<td>2.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Academic Record (GPA): N</td>
<td>4,980</td>
<td>3.0</td>
<td>3.0</td>
<td>0.6</td>
</tr>
<tr>
<td>College Quality: M</td>
<td>500</td>
<td>0.5174</td>
<td>0.5236</td>
<td>0.2140</td>
</tr>
<tr>
<td>College Quality: N</td>
<td>4,980</td>
<td>0.5846</td>
<td>0.6051</td>
<td>0.2024</td>
</tr>
<tr>
<td>10-Year Household Income: M</td>
<td>260</td>
<td>$100,700</td>
<td>$89,500</td>
<td>$52,500</td>
</tr>
<tr>
<td>10-Year Household Income: N</td>
<td>2,800</td>
<td>$108,300</td>
<td>$92,700</td>
<td>$77,300</td>
</tr>
</tbody>
</table>

*As per USDOE data security requirements, in order to protect anonymity of B&B respondents, sample sizes have been rounded to the nearest 10, dollar figures have been rounded to the nearest $100, and GPAs have been rounded to the nearest 0.1.

the worst economic downturn (unrelated to war) since the great depression during the sample period. This further complicates matters and exacerbates the previous two problems: not only was the financial meltdown of 2008-2009 a significant shock for all respondents, but it likely affected the various waves of college graduates in different ways by interrupting career paths with involuntary unemployment spells at different points during the formative, early career years. Adequately controlling for these systematic differences would require allowing for the economic returns to college quality, pre-college HC, and unobserved student characteristics to vary by college graduation cohort. In contrast, since the B&B contains 3.5 times as many viable income observations, and is not subject to the same complicated cohort effects as the NLSY97, we believe the B&B survey is the most appropriate data source for our study on the links between pre-college effort, college quality, and post-college outcomes.

3. THEORETICAL MODEL

Following Bodoh-Creed and Hickman [13], we model college admissions as a Bayesian game where high-school students are characterized by a privately-known type that governs the costliness of HC production and the payoff from enrolling in college. On the other side of the market are colleges that have preferences over students based on their HC and race. Colleges mechanically admit the best students from each demographic group that the school can attract, and
the trade-off between diversity and pre-college academic achievement is captured by the colleges’ AA plans. In this section we outline the model and summarize technical results proven in Bodoh-Creed and Hickman [13]. We summarize the technical assumptions required in Appendix A although the empirical model we estimate satisfies all of these assumptions.

3.1. Agents. There are two demographic subgroups within the student population, minority students (\(M\)) and nonminorities (\(N\)), and the demographic class of each student is observable. There is a continuum of students of total mass 1 with mass \(\mu\) of them in the minority group. We often refer to our model with a continuum of students as a limit model to emphasize the fact that it can be viewed as a limit of a model with a finite number of students as their number approaches infinity. We elaborate on this point in Section 3.4.

Each student has a privately-known learning cost type \(\theta \in [\theta, \bar{\theta}]\), and the distribution of \(\theta\) in group \(j \in \{M, N\}\) follows a cumulative distribution function (CDF) \(F_j(\theta)\). For convenience, we denote the unconditional type distribution by \(F_K(\theta) \equiv \mu F_M(\theta) + (1-\mu) F_N(\theta)\). Each agent’s strategy space, \(S = [s, \infty)\), is the set of attainable HC levels. We view the HC choice as the outcome of a long-run plan for the accumulation of HC over several years leading up to the college admissions process. These are observable (e.g., through standardized exam scores and high school GPAs) and \(s\) is the minimum required to attend some college. In order to produce \(s\) units of HC, a student incurs cost \(C(s; \theta)\) which is increasing in both \(s\) and \(\theta\). Investment costs can arise in various ways, such as from a consumption–investment tradeoff or psychic costs from difficult learning activities. Mathematically, the difference between types \(\theta\) and investment \(s\) is that the former reflects the exogenous portion of HC costs and the latter arises from a costly decision under the control of the agent.

Learning cost type \(\theta\) encapsulates both cognitive and non-cognitive characteristics, and they may be influenced by forces both internal to the individual (e.g., innate ability or natural curiosity) and external (e.g., home environment, primary/secondary school quality, parental education and financial resources). For example, consider the case of a parent who may enrich his daughter’s educational experience by spending time reading or doing schoolwork with her. In a low-income household where the parent must work two jobs, his time may be more constrained than in an affluent household where the parent has a single, high-paying job. In this example, the parent’s and child’s choice of how much time to spend on learning activities is encapsulated in \(s\), whereas the pre-existing opportunity cost of time is reflected in \(\theta\). Asymmetric type distributions reflect these factors, many of which are correlated with race. When we condition outcomes on unobserved types in our empirical framework, \(\theta\) controls for many environmental correlates of race even if they do not appear in our model explicitly. We remain agnostic on the exact interpretation of \(\theta\), but we assume that it is fixed from the perspective of a student when she chooses her level of HC investment.

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16One might have imagined a more complex model where the students can generate distinct kinds of HC through different costly activities. In fact, this is essentially what we will do with our empirical exercise, taking account of both exam scores and GPA. All that is essential about our modeling choice is that the schools reduce each student’s portfolio of HC to a single index and that the schools do this in roughly the same way. It is not necessary to understand exactly how a student created their HC to answer the questions we have posed.
One empirical regularity of American college data is a nontrivial degree of academic heterogeneity among students on a given college campus (even within demographic groups). To rationalize these deviations from perfect assortativity between school quality and student HC, our model includes market frictions in the form of a random matching shock to the colleges’ perceptions of each student’s HC choice. This shock is observable to colleges at the time the student applies, but not to the student while she is investing in her HC. The unobservability of the shock to the student is consistent with an interpretation of HC choice as the gradual accumulation of learning over years, while the shock is the result of events that occur within a short time before her application to college and that are out of the student’s control. The matching shock is applied to each student’s choice of HC, \( s \), to generate a noisy HC (NHC) value, \( t \), that is commonly observed by all of the colleges. We assume the noise enters additively, so if student \( i \) chooses \( s \), the associated NHC is \( t = s + \epsilon \). \( \epsilon \) has CDF \( F_\epsilon \), and the draws of \( \epsilon \) are independent across students. The assignment of students to colleges becomes more assortative as the variance of \( \epsilon \) shrinks, and the market becomes a lottery as the variance grows.

3.2. Payoffs. On the other side of the market there is a continuum of colleges with total mass 1. Each college’s quality is described by an index \( p \in [p, \overline{p}] \) that is distributed \( P \sim F_p(P) \). By assuming the measure of students and college seats are the same, we abstract from the extensive margin of college attendance and focus on the competition for admission to the best colleges conditional on entering the market. Both college quality and HC are intrinsically valued: match utility \( U(p, s, \theta) \) results from a student with type \( \theta \) having HC \( s \) and enrolling in a college with quality \( p \). The ex post payoff to agent \( i \) in group \( j \in \{M, N\} \) is the match utility minus the cost of achievement, \( U(p, s, \theta) - C(s, \theta) \).

3.3. Allocation Mechanisms. In an admissions contest, the quality of the seat allocated to a student is a function of how her NHC realization compares to the distribution of NHC across the population of college applicants. Affirmative action schemes cause the NHC realizations of minority and nonminority students to be compared to the total distribution of NHC differently, which makes the contest asymmetric between the two demographic groups. We consider color-blind (cb), proportional quota (q), and admissions preference (ap) AA systems\(^{17}\)

The allocation mechanism is an assignment mapping \( P_{r_j}^r : \mathbb{R} \to [p, \overline{p}], j \in \{M, N\} \) and \( r \in \{ap, cb, q\} \) indicates the allocation mechanism and \( j \in \{M, N, K\} \) indicates a demographic group or the unconditional population. \( P_{r_j}^r \) is an endogenous object that depends on the form of AA at work in the market, the distribution of college qualities, and the equilibrium distribution of NHC in the population, which is in turn determined by the endogenous choices of HC by the students. To facilitate discussion, we denote the equilibrium CDFs of HC and NHC as \( G_{r_j}^r \) and \( H_{r_j}^r \), respectively. \( H_{r_j}^r \) is a convolution of \( G_{r_j}^r \) and \( F_\epsilon \) with density \( h_{r_j}^r(t) = \int_{-\infty}^{\infty} f_\epsilon(\epsilon) g_{r_j}^r(t - \epsilon) d\epsilon \).

\(^{17}\)One can also view the contest as a form of all-pay auction in which the admission scheme gives rise to an endogenous pricing rule that dictates the amount of HC an individual must “pay” to win a given seat.
We define AA as the difference between the level of NHC required for a minority and nonminority applicant to be admitted to a school. We do not take a stand on why schools choose to implement AA schemes. The fraction of minority students on the campus of a particular college is the result of the interaction of the school’s AA policy with the densities of the distributions of NHC for each group. The assignment mappings describe how admissions practices work at different points in the college quality spectrum.

In the context of our model, the problem facing the students is rather simple. Instead of requiring common knowledge of the strategies of the other agents, we require that the students understand the assignment mapping. In order to do this, the students merely need to understand the admissions standards of colleges of a quality similar to those to which they will eventually enroll. This information can be gleaned from public data about (for example) the typical SAT and high school GPA of freshman enrolling at different colleges.\footnote{See Bodoh-Creed and Hickman\cite{13} for further discussion of the relationship between the continuum and finite-agent games.} To the extent that college students make errors in this process, it would be reflected in a lack of assortativity in the match, an issue that is discussed further in Sections 5.1 and 5.6.

The color-blind allocation mechanism is the simplest to define since it assigns students to schools assortatively with higher NHC realizations leading to assignment at higher quality colleges. Since demographics do not affect the assignment, \( P^{cb}_M(t) = P^{cb}_N(t) = P^{cb}(t) = F^{-1}_P(H^{cb}_K(t)) \).

In words, a student with NHC realization \( t \) at quantile rank \( H^{cb}_K(t) \) of the NHC distribution (for all students) is placed at a school with the same quantile rank in the college quality distribution, \( F^{-1}_P(H^{cb}_K(t)) \). Since the assignment mapping is the same for both groups, marginal investment incentives, conditional on type \( \theta \), are also identical across groups.

Under a quota scheme, students from group \( j \in \{M,N\} \) are reserved sets of seats with qualities distributed \( P \sim Q_j(P) \), where \( \mu Q_M(p) + (1 - \mu)Q_N(p) = F_P(p) \) is required for feasibility. Members of each demographic group compete in disjoint contests for prizes only against the other members of their same group. The resulting assignment map is \( P^{pl}_j(t) = Q^{-1}_j(H^{pl}_j(t)) \), which means a student with NHC \( t \) at quantile rank \( H^{pl}_j(t) \) is placed at a school with the same quantile rank in the distribution of colleges allocated to her demographic group, \( Q^{-1}_j(H^{pl}_j(t)) \). The most familiar member of this class is a proportional quota where \( Q_M = Q_N = F_P \), meaning that a fraction \( \mu \) of seats at each point in the college quality spectrum are reserved for minorities.

Although proportional quotas are not directly applicable in the US as a policy instrument, they provide a useful benchmark relative to a color-blind mechanism.\footnote{Although currently used in various parts of the world, the US Supreme Court declared it unconstitutional in the college admissions context (Regents of the University of California v. Bakke\cite{11}).} When racial asymmetries in learning costs exist—because race is correlated with childhood school quality, for example—proportional quotas are designed so that these differences are not reflected in the fraction of
students from each demographic group present on each college campus. A color-blind mechanism allows asymmetries in learning costs between the demographic groups to be maximally reflected in the fraction of minority students enrolling in high-quality colleges.

Finally, an admission preference system refers to one in which the NHC values of both groups are compared to the distribution of the total student population, but the demographic status of a given student determines how this comparison is made. To the extent that American colleges engage in race-based AA at present, the only legally permissible form is an admissions preference system where race is taken as a “plus factor” among other considerations like grades and test scores. Formally, an admission preference is defined by a markup function \( \tilde{T} : \mathbb{R} \to \mathbb{R} \) that transforms the NHC levels of the minority students. The resulting assignment mappings are

\[
\begin{align*}
    P^p_M(t) &= F_p^{-1} \left( \mu H^p_M(t) + (1 - \mu) H^p_N \left[ \tilde{T}(t) \right] \right) \\
    P^p_N(t) &= F_p^{-1} \left( \mu H^p_M \left[ \tilde{T}^{-1}(t) \right] + (1 - \mu) H^p_N(t) \right)
\end{align*}
\]

In words, a minority student’s NHC level \( t \) is compared to the raw NHC of other minority students and marked up when compared to other nonminority students. Conversely, NHC for a nonminority student is compared to the raw NHC levels of other nonminority students and it is effectively “de-subsidized” for comparisons to NHC levels of minority students. Note that a color-blind system is a special case of an admissions preference system.

Bodoh-Creed and Hickman [13, Theorem 4] proves that quota and admissions preference systems have the same set of equilibria. Our empirical model is based on a (non-proportional) quota system that is outcome equivalent to the real-world admissions preference. Since the allocation of students to schools is known \textit{ex ante} under a quota, this simplifies estimation.

\textbf{Theorem 3.1.} [Theorem 4, Bodoh-Creed and Hickman [13]] \( P^q_j(t) : \mathbb{R} \to \mathcal{P}, j \in \{M,N\} \), is the result of an equilibrium of some quota system with \((Q_M, Q_N)\) if and only if there exists an admissions preference system with some \( \tilde{T} \) that has the same equilibrium assignment mappings and strategies.

\textbf{3.4. Equilibrium.} The distribution of college seats, the form of the admission system, and the measures and distributions of student competitors are common knowledge prior to individual choices of HC investment. When combined with the equilibrium strategies of the minority and nonminority students, denoted \( \sigma_M(\theta) \) and \( \sigma_N(\theta) \) respectively, the students can forecast the form of the assignment mapping \( P^r_j \) arising in equilibrium. Each student solves the following optimization problem where \( j \in \{M,N\} \) and \( r \in \{cb, pq, ap\} \)

\[
\sigma_j(\theta) = \arg \max_s \left\{ E_{\varepsilon} \left[ U \left( P^r_j(s + \varepsilon), s, \theta \right) \right] - C(s, \theta) \right\}.
\]

In equilibrium, students’ beliefs about \( \sigma_M(\theta) \) and \( \sigma_N(\theta) \) must be consistent with the solution to equation (2). Bodoh-Creed and Hickman [13, Theorem 6] proves that an equilibrium exists.

\textbf{Theorem 3.2.} [Theorem 6, Bodoh-Creed and Hickman [13]] There exists a monotone, pure strategy Nash equilibrium of our limit model in the color-blind, quota, or admissions preference systems.
The first-order condition can be written as
\[ E_\epsilon \left[ U_\sigma (P_j(s + \epsilon), s, \theta; \alpha) \right] + E_\epsilon \left[ U_P (P_j(s + \epsilon), s, \theta; \alpha) P_j'(s + \epsilon) \right] = \theta^c(s), \]
with the left-hand side describing marginal benefits and the right-hand side reflecting marginal costs. The productive channel represents the direct effect of HC on utility, while the competitive channel captures the marginal effect of HC on utility through its role in college placement.

Actual college markets one might study empirically have only finitely many students and colleges, but on the other hand, the finite version of this model is computationally intractable. In this finite model, \( K_M \) minority students draw types from the distribution \( F_M(\theta) \), \( K_N \) nonminority students draw types from the distribution \( F_N(\theta) \), and \( K_M + K_N \) college seats draw their qualities from \( F_P \). In the limit as \( K_M + K_N \to \infty \) and \( K_M/(K_M + K_N) \to \mu \), the primitives of the finite games approach those of the continuum model. Bodoh-Creed and Hickman [13] show that the limit games approach those of the continuum model. Bodoh-Creed and Hickman [13] show that the finite model, \( K_M \) minority students draw types from the distribution \( F_M(\theta) \), \( K_N \) nonminority students draw types from the distribution \( F_N(\theta) \), and \( K_M + K_N \) college seats draw their qualities from \( F_P \). In the limit as \( K_M + K_N \to \infty \) and \( K_M/(K_M + K_N) \to \mu \), the primitives of the finite games approach those of the continuum model. Bodoh-Creed and Hickman [13] show that the finite model approximates the continuum model in the following sense.

**Definition 3.3.** Given \( \epsilon > 0 \), an \( \epsilon \)-approximate equilibrium of the \( K \)-agent game is a \( K \)-tuple of strategies \( \sigma^e = (\sigma^e_1, \ldots, \sigma^e_K) \) such that for all agents, almost all types \( \theta \), and all HC choices \( s' \) we have
\[ U \left( P_j'(\sigma^e_1(\theta), \sigma^e_i(\theta)), \sigma^e_i(\theta), \theta_j \right) - C(\sigma^e_i(\theta), \theta_i) + \epsilon \geq U \left( P_j'(s', \sigma^e_i(\theta)), s', \theta_i \right) - C(s', \theta_i) \]

Definition 3.3 describes an approximate equilibrium in terms of incentives: agents that follow an \( \epsilon \)-approximate equilibrium can gain at most \( \epsilon \) by deviating. Intuitively, students lose little utility if they base their actions on the easy-to-compute limit game equilibrium. Our final theorem shows that we can choose \( \epsilon > 0 \) to be arbitrarily small as the size of the market increases. This result justifies our use of the continuum model as an approximation of the more realistic, but intractable, finite model.

**Theorem 3.4.** [Theorem 7, Bodoh-Creed and Hickman [13]] Let \( \sigma^e_j, j \in \{M, N\} \) and \( r \in \{cb, q, ap\} \), denote an equilibrium of the game with a continuum of agents. We can choose \( K^* \) such that \( \sigma^e_j \) is an \( \epsilon \)-approximate equilibrium of the \( K \)-agent game for any \( K > K^* \).

3.5. **Motivating Our Model.** Our contest model assumes that students agree on the ranking of schools and that schools agree on the quality of each student within each demographic group. As a result, the outcome predicted by our contest model will be nearly assortative in terms of school quality and student HC choice. Violations of assortativity are caused both by AA policies and by an exogenous matching shock. We also assume students are motivated by the expected household income premium obtained by college graduates. We now motivate these modeling choices by describing how well the raw data fit these assumptions.

First let us consider whether a contest structure fits the patterns we observe in our data. We in fact find that the correlation between SAT scores and college quality for college graduates in the B&B survey is 0.449\textsuperscript{20} In addition, we estimate a reduced-form single index for HC in the form

\[ \text{Productive Channel} \]

\[ \text{Competitive Channel} \]

\[ \text{Marginal Cost} \]

\[ \text{Definition 3.3.} \]

\[ \text{Theorem 3.4.} \]

\[ \text{Motivating Our Model.} \]

\[ \text{We find a similar correlation between college quality and median within-campus SAT score in the USNWR data.} \]
of a polynomial in student exam scores and GPA (see Section 4). We find that this single index of academic achievement has a Spearman rank correlation of 0.88 with our metric of college quality. These high correlations imply a high degree of assortativity, in line with a contest model. The fact that students do not seem willing to compromise significantly on college quality is not surprising given the stark quality heterogeneity among U.S. colleges. The interquartile range of spending per student is $5,931 to $9,551, the interquartile range in graduation rates is 38% to 63%, and the interquartile range of the average household income of graduates from each college 10 years after graduation is $78,000 to $102,000.

Our model also assumes a national market for college admissions as has been done in previous work (e.g., Epple, Romano, and Sieg [27], Chade, Lewis, and Smith [17], and Fu [31]). The key requirement in our context is that all prospective college enrollees have roughly the same access to the college quality spectrum regardless of where they attend high school. One might be concerned about this assumption given the large fraction of state-funded colleges in the market. Our raw college-level data exhibit empirical patterns consistent with the national market view. For example, the median college enrolls students from over one third of the states in the country. Moreover, 76% of colleges in our sample enroll students from at least 10 different states, while 23% of them enroll students from 30 or more states. We interpret these figures as being consistent with an integrated national college market where the incentives to enroll in a quality college are high enough that geography is not a significant barrier.

As discussed in detail in Section 5.1, deviations from the contest structure would imply a low degree of assortativity (i.e., small correlation) between college quality and academic achievement, which would be expressed as a large estimate of the variance of the matching shock $\varepsilon$. In fact, we find a low variance for the matching shock, which is not surprising given the high assortativity. This suggests that (1) aspects of colleges other than quality (e.g., geography, student demography) are not strong drivers of student preferences; (2) students have access to the full spectrum of college qualities; and (3) information frictions are not causing students to enroll in lower quality colleges than would admit them.\footnote{We test for demographic specific matching shock variances or trends in the matching shock variance with respect to HC and find no significant effects. See Section 5.6}

We also assume that there is a continuum of college seats, meaning no school enrolls a significant fraction of the students. In 1988 (four years prior to the graduating class of AY1992-1993) there were a total of 1,644,340 freshman seats in the market with any single school having only a negligible market share. The largest college in 1988 (Ohio State) had a total market share of only 0.76% of new freshman seats. The mean, median, and standard deviation of market shares for individual universities were 0.091%, 0.047%, and 0.102%, respectively.

Finally, we assume that students are motivated by the expected college income premium. Although there are other non-pecuniary benefits to college, it is reasonable to assume that the impact on lifetime income is the largest benefit and best reflects the primary arguments made for the importance of a high-quality college education given the wide interquartile of range household income per year and the significant raw correlation between college quality and the

\footnote{We test for demographic specific matching shock variances or trends in the matching shock variance with respect to HC and find no significant effects. See Section 5.6}
average household income of graduates. As we will see in Section 5.5, although our model does not require college quality to have any effect on graduation probability or post-college household income, our estimates imply college quality is a strong driver of both.

4. MODEL IDENTIFICATION AND ESTIMATION

Our basic identification challenge is to disentangle the influence of college quality, HC investment, and privately-known type on the returns to attending a higher quality college. The empirical auctions literature has developed a set of tools specifically designed to identify private information in game-theoretic models. This literature was pioneered by Paarsch [51] and then revolutionized by Guerre, Perrigne, and Vuong [32, GPV] who proposed a non-parametric estimator for mapping observed bids into underlying private valuations in first-price auctions. We combine this approach with common techniques from labor econometrics to parse between the influences of student and school characteristics in producing post-college household income.

Intuitively, because AA changes the marginal investment incentives across race groups during high school, one can surmise that two students having the same GPA/exam scores but different race must have distinct underlying types. This fact breaks what would otherwise be perfect rank correlation between achievement and unobserved types, and the matching shock breaks what would otherwise be perfect rank correlation between college placement and achievement. Together, these components of the empirical model provide a full-rank condition that allows us to separate the influences of achievement, college placement, and permanent types in our household income regression.

4.1. Identification. We begin this section by coupling some additional assumptions with the theoretical framework above in order to complete the formal definition of our empirical model. Some of these merely serve the purpose of tractability, and some are crucial for model identification; we make these distinctions clear in our discussion. Overall our identification/estimation strategy is semi-parametric, although we do not require restrictions on the functional forms of the type distributions or the equilibrium college assignment mappings.

Assumption 4.1. (Single Index) Human capital $S$ is a single index function of exam scores $E$ and academic record $A$,

\[ S_i = S(E_i, A_i) = \beta_s^1 E_i + \beta_s^2 E_i^2 + \beta_s^3 A_i + \beta_s^4 A_i^2 + \beta_s^5 E_i A_i, \]

with $S^c(E_i, A_i) > 0$, $S^a(E_i, A_i) > 0 \forall (E_i, A_i)$, and $\max_{(E, A) \in \mathbb{R}^2} \{S(E, A)\} = 1$.

Assumption 4.2. (Separable Exponential Costs) $C(s; \theta) = \theta c(s)$ with $c(s) = \exp(s)$.

Assumption 4.1 imposes a quadratic form on the single index equation for HC with regularity conditions and a scale normalization to fix the units of $S$. This single index equation condenses the two margins of achievement—GPA and exam scores—into one measure of HC. Implicit here is the idea that individuals/households optimize their portfolio of investment activities $(e_i, a_i)$

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22 We experimented with a cubic form $S(E_i, A_i) = \beta_s^0 + \beta_s^1 E_i + \beta_s^2 E_i^2 + \beta_s^3 E_i^3 + \beta_s^4 A_i + \beta_s^5 A_i^2 + \beta_s^6 A_i^3 + \beta_s^7 E_i A_i + \beta_s^8 E_i^2 A_i + \beta_s^9 E_i A_i^2$, but this did not produce a statistically or economically meaningful change in our estimates.
Assumption 4.3. Match Utility

**Cobb-Douglas Income Production:**

\[ u(P_i, S_i, \theta_i) = E[W_i|P_i, S_i, \theta_i] = \alpha_0 P_i^{\alpha_p} S_i^{\alpha_s} \theta_i^{-\alpha_\theta} \text{ where } 0 < \alpha_0, \quad \alpha_p, \alpha_s, -\alpha_\theta \in (0, 1) \]

**Graduation Probability:**

\[ \rho(P_i, S_i) = \Pr[i \text{ graduates college}|P_i, S_i] = \beta_0^P + \beta_1^P P_i + \beta_2^P P_i^2 + \beta_3^P P_i^3 + \beta_4^S S_i + \beta_5^S S_i^2 + \beta_6^\theta P_i S_i + \beta_7^\theta P_i^2 S_i + \beta_8^\theta S_i^2 + \beta_9^\theta P_i^2 S_i \]

**Expected Log-Utility:**

\[ U(P_i, S_i, \theta_i) = \rho(P_i, S_i) \log [u(P_i, S_i, \theta_i)] + [1 - \rho(P_i, S_i)] \log [\kappa u(P_i, S_i, \theta_i)] \text{ where } \kappa \in (0, 1) \]

Assumption 4.3 assumes Cobb-Douglas household income production from a match. We further assume that a college dropout’s income is \( \kappa < 1 \) times what it would have been had she graduated, and we adopt a flexible, cubic complete polynomial form for the graduation probability function. This formulation allows the household income of college dropouts to vary by the student’s underlying ability, HC, and college placement. We adopt a log utility form for the student’s preferences over household income, as it is a benchmark choice for lifetime consumption models.

There is a long history of estimating Mincerian regressions to identify the causal effect of college quality on earnings (Brewer, Eide, and Ehrenberg [15]; Dale and Krueger [24]; Black and Smith [11]; and Long [43]), and our household income equation lies firmly within this tradition. This approach abstracts away from the particulars of how/why college produces these effects. Rather, our empirical model views college quality, pre-college HC, and individual characteristics as the raw materials which, over the course of one’s college career, give rise to post-college outcomes. Our model allows for counterfactuals wherein individual types \( \theta \) choose different pre-college investment \( S \) and are matched to different colleges \( P \), while holding fixed the production technology, \( u(P_i, S_i, \theta_i) \), that maps matches into post-college outcomes. One might wonder whether the estimated household income equation still applies under counterfactual equilibria. For example, sorting students into different colleges might alter college quality, although peer effects have proven small or hard to detect on college campuses (Winston and Zimmerman [59]). It would be easy to write down a model where the household income production function is invariant across counterfactuals. For example, suppose students mechanically accrue human capital during college as a function of \((P_i, S_i, \theta_i)\) through a combination of the resources of the college (i.e., \(P_i\)), the student’s pre-college preparation (i.e., \(S_i\)), and effort during college as determined by the student’s learning-cost (i.e., \(\theta_i\)). Post-college household incomes are then set by a competitive
labor market based on observations of post-college HC, which provides utility \( u(P_i, S_i, \theta_i) \). The household income could also potentially be based on inferences firms make about the students’ learning-costs from the joint distribution of \((P_i, S_i)\) and post-college HC.

Assumption 4.3 did not condition utility on demographics. This is formalized in the following exclusion restriction.

**Assumption 4.4. (Exclusion Restriction)** Race does not directly affect match utility conditional on type \( \theta \), investment \( s \), and college assignment \( p \): if \( D_{Mi} \equiv 1(i \in M) \) is an indicator for minority status then \( U(P_i, S_i, \theta_i, D_{Mi}) = U(P_i, S_i, \theta_i) \).

The exclusion restriction 4.4 is central to our identification strategy, and implies minority status plays no direct role in match utility. Although this assumption rules out some forms of racial discrimination on the labor market (e.g., taste-based racial animus), our model is still compatible with a world in which statistical discrimination occurs. In the equilibrium data generating process, statistical discrimination would appear as minority and non-minority students with the same realizations of \((s, p)\) receiving different wages because of differing values of \( \theta \) that employers infer from the joint distributions of \((p, s, \theta)\) in conjunction with any other information employers possess (e.g., HC mechanically acquired during college) conditional on race.

It is important to emphasize here that our exclusion restriction does not rule out influences by a host of other important factors that are highly correlated with race and influence learning costs. Recall that \( \theta \) implicitly subsumes environmental factors such as parental education and income, availability of early-life developmental resources, and K-12 school quality, as well as cognitive and non-cognitive ability which are also known to be influenced by childhood experience. Therefore, our exclusion restriction requires only that racial affiliation plays no direct role in post-college income conditional on achievement, college placement, and the various environmental and idiosyncratic factors contained in \( \theta \).

This assumption is supported by the existing labor economics literature. Beginning with Neal and Johnson [49], a body of empirical work over the past two decades has emphasized non-animus factors for explaining the majority of Black-White wage differentials. More recently, Fryer, Pager, and Spenkuch [30] has found that roughly 89% of the Black-White wage gap is attributable to differentials in observable controls other than race. They then test various different theories—including racial animus, differences between blacks and whites in job-search strategies, search intensity, bargaining, and discount factors—to explain the remaining wage gap. They find empirical evidence inconsistent with these various theories for explaining the remaining 11% wage gap; rather, they find that blacks are systematically willing to accept lower reservation wage offers at the hiring stage, but that the returns to tenure on the job, post-hire, are larger for blacks than for whites. The authors conclude that these patterns in their data are most naturally rationalized by a labor-search model exhibiting matching frictions and statistical discrimination.

If our exclusion restriction were violated, then the impact of animus on household income would be loaded onto our estimates of \( \theta \) and \( \alpha_\theta \) as these are the only components of the household income equation that can reflect systematic variation between the minority and nonminority
groups conditional on fixed \((p, s)\). If minority status predicts \(\theta\), then Assumption 4.4 might be violated. Appendix D tests this idea by regressing our estimated \(\theta\)'s on a rich set of controls for childhood home environmental factors, a race dummy, and a full set of race interactions with other controls. We find that, while home background and childhood socioeconomic controls are highly significant, minority status is not a meaningful predictor of \(\theta\) since a joint F-test on the race dummy and race interaction terms fails to reject the null hypothesis that all the corresponding regression coefficients are zero. Since there is nothing in our empirical model forcing this to be true, we interpret this result as indicating that there is nothing in our data which reveals a clear violation of the exclusion restriction.

The main function of our exclusion restriction is to identify the wage equation parameters by parsing out the influences of endogenous achievement and exogenous type. Holding type fixed, AA changes the marginal benefit (in terms of college placement) of an increase in achievement for blacks, relative to whites. This results in different race-specific mappings between unobserved types and observed achievement levels, and therefore AA allows the econometrician to effectively use race-specific pre-college investment as non-linear control functions under our exclusion restriction. In order to completely lose identifying power, the racial animus would have to exactly offset the incentive effects of affirmative action (i.e., \(\tilde{T}\)). Since the AA markup function, \(\tilde{T}\)—which we recover from raw data without imposing equilibrium assumptions—is highly non-linear, it follows that a non-generic, college-specific racial-animus penalty would be required to offset the AA incentive effects.

Finally, note that if the effect of racial animus on household income was known ex ante and did not exactly offset the AA incentive effects, then our identification result would continue to hold. As an example of this, we estimated a model wherein we imposed a uniform 11% animus-based household income penalty on minority students to explore the possible effects of violations of our exclusion restriction.\textsuperscript{23} We found that our estimates of the household income equation parameters did not qualitatively change, which suggests that our model and identification strategy are relatively robust to uniform deviations from our exclusion restriction. See Appendix E.4 for more details.

**Assumption 4.5. (Unique Investment)** \(U(p_j(s), s, \theta) - \theta c(s)\) is strictly concave in \(s\).

**Assumption 4.6. (Normal Shocks)** Matching shocks \(\varepsilon \sim N(0, \sigma^2)\) are normally distributed with zero mean and variance \(\sigma^2\) and are independent of HC \(s\) and demographic status \(j = \in \{M, N\}\).

Assumption 4.5 ensures that the agent’s decision problem (equation 2) has a unique solution, a needed property for mapping \(s\) into a corresponding \(\theta\). Note that this assumption is testable: the theory model requires strict supermodularity of the student’s HC choice problem (see Appendix A), which implies the investment strategy is strictly monotone, so there would be jumps in the student’s HC accumulation strategy if Assumption 4.5 failed to hold. Assumption 4.6 assumes normal matching shocks with zero mean. Since the NHC index is scale-free, the assumption of

\textsuperscript{23}The choice of 11% was motivated by the magnitude of the black-white wage gap studied by Fryer et al.\textsuperscript{30}.
a 0 mean distribution is without loss of generality, and the choice of a normal distribution was made solely for tractability.

The structural objects to identify are the type distributions, \( f_j(\theta) \), the matching shock variance, \( \sigma^2 \), the assignment functions, \( P_j^\rho \), and the match utility parameters, \( (\beta^\rho, \alpha) \). The single index parameters \( \beta^\rho \) and the joint distributions of \((P, S)\) across race groups are intermediate model components to be identified along the way.

4.1.1. Identification: Single Index Parameters and Graduation Probabilities. For simplicity of discussion, assume at first that the single index parameters \( \beta^\rho \) are known. The first hurdle to overcome is a problem of sample selection: because the B&B survey only contains information for college graduates, we do not observe \((P, S)\) pairs for anyone who failed to graduate. Thus, at first we can only treat the conditional school quality and HC distributions condition on graduation, \( f_{RS}(p, s|\mathcal{M}, \text{grad}) \) and \( f_{RS}(p, s|\mathcal{N}, \text{grad}) \), as observables. Given the graduation probability function \( \rho(p, s) \), from Bayes’ law we know that \( f_{RS}(p, s|\mathcal{M}, \text{grad}) \) and \( f_{RS}(p, s|\mathcal{N}, \text{grad}) \) relate to the unconditional densities as follows:

\[
(4) \quad f_{RS}(p, s|j) = \frac{f_{RS}(p, s|j, \text{grad}) \Gamma_j}{\rho(p, s)}, \quad j \in \{\mathcal{M}, \mathcal{N}\},
\]

where \( \Gamma_j \) is a constant that equals the total fraction of enrollees from group \( j \) who graduate college and normalizes the joint density to integrate to one. Although we do not know \( \rho(p, s) \) ex ante, the graduation parameters are pinned down by the graduation rate of each demographic group at each college. The model-generated graduation rate at each college is computed by averaging over graduation parameters \( \beta^\rho \) and normalizes the joint density to integrate to one. Although we do not know \( \rho(p, s) \) we can only treat the conditional school quality and HC distributions condition on graduation, \( f_{RS}(p, s|\mathcal{M}, \text{grad}) \) and \( f_{RS}(p, s|\mathcal{N}, \text{grad}) \), as observables. Given the graduation probability function \( \rho(p, s) \), from Bayes’ law we know that \( f_{RS}(p, s|\mathcal{M}, \text{grad}) \) and \( f_{RS}(p, s|\mathcal{N}, \text{grad}) \) relate to the unconditional densities as follows:

\[
(5) \quad \Gamma_{jl} = Z_{jl} \beta^\rho + \epsilon_{jl},
\]

where \( Z_{jl} = [1, p_l, p_l^2, p_l^3, \bar{S}_{jl}, \bar{S}_{jl}^2, \bar{S}_{jl}^3, p_l \bar{S}_{jl}, p_l^2 \bar{S}_{jl}, p_l^3 \bar{S}_{jl}] \) contains the regressors for group \( j \) at school \( l \), \( \epsilon_{jl} \) is random and arises from finite sampling within campus \( l \),

\[
(6) \quad \bar{S}_{jl}^k = \int_s s^k f_{S|P}(s|j, p_l) ds = \int_s s^k \frac{f_{RS}(p_l, s|j)}{f_{P_l}(p_l)} ds
\]

is the conditional expectation, across both graduates and non-graduates, of the \( k \)th power of \( s \) given \( p_l \), and

\[
(7) \quad f_{P_l}(p_l) = \int_s f_{RS}(p_l, s|j) ds, \quad j \in \{\mathcal{M}, \mathcal{N}\}
\]

is the unconditional marginal distribution of \( P \) for group \( j \). Equations (4) – (7) provide a sample selection correction to identify graduation probability parameters \( \beta^\rho \) as long as the single index parameters \( \beta^\rho \) are known. One appealing characteristic of our proposed is that it does not require parametric restrictions on the form of the unconditional joint distributions of \((P, S)\).
Now we require a further condition to pin the single index parameters down. Recall that the HC index $S$ represents all observable information about the student prior to the application process that predicts where he/she will place. Therefore, we assume that the parameters $\beta^s$ are such that college placement predictive power is maximized. Since the mapping between $P$ and $S$ arises from a rank-order contest, we adopt the Kendall’s $\tau$ measure of rank correlation to formalize our notion of predictive power.

Kendall’s $\tau$ is defined in terms of concordance of random variables; we say that two ordered pairs $(p_1, s_1)$ and $(p_2, s_2)$ are concordant if the ordering of the first coordinate agrees with the ordering of the second, or $p_1 < p_2$ if and only if $s_1 < s_2$. Likewise, we say the two pairs are discordant when this condition is violated. For a joint distribution of $(P, S)$, Kendall’s $\tau$ is defined as the probability of concordance minus the probability of discordance for two iid random variables $(P_1, S_1), (P_2, S_2)$: $\tau_{PS} \equiv Pr[(P_1 - P_2)(S_1 - S_2) > 0] - Pr[(P_1 - P_2)(S_1 - S_2) < 0]$. Within a rank order contest, Kendall’s $\tau$ is the probability that the ordering of two students’ college assignments (within the same demographic group) respects the ordering of their pre-college achievement minus the probability that it does not. In the absence of a matching shock, our model predicts a perfectly assortative match between $p$ and $s$ within each demographic group. This suggests that the single index should be estimated to maximize assortativity as measured by Kendall’s $\tau$, and deviations from assortativity are reflected in our estimate of the matching shock.

With the joint distribution $(P, E, A)$ known, we formalize our assumption on $\beta^s$ as:

$$\beta^s = \arg \max \left\{ \mu \left( Pr[(P_1 - P_2)(S_1 - S_2) > 0|\mathcal{M}] - Pr[(P_1 - P_2)(S_1 - S_2) < 0|\mathcal{M}] \right) \right\}$$

$$+(1 - \mu) \left( Pr[(P_1 - P_2)(S_1 - S_2) > 0|\mathcal{N}] - Pr[(P_1 - P_2)(S_1 - S_2) < 0|\mathcal{N}] \right) \right\}.$$  

(8)

From the above arguments, the first part of our identification result follows:

**Proposition 4.7.** There is a unique configuration of the single index and graduation probability parameters $(\beta^s, \beta^p)$ that is consistent with the joint distributions of the observables $\{ (Y_i, p, e_i, a_i)_{i=1}^l \}$ and equations (4), (5) and (8).

4.1.2. Identification: Matching Shock Variance. At this point several important equilibrium objects can be treated as known, including the unconditional joint distribution of HC and school assignments. Identifying the matching shock variance parameter $\sigma_\epsilon$ is simple since it uniquely determines the degree to which the joint distribution of $P$ and $S$ deviates from full rank correlation within each race group. Intuitively, the larger is the variance of the matching shock, the more latitude there is for students with lower HC levels to place above students with more HC. Thus, it is easy to see that Kendall’s $\tau$ within each race group is decreasing in $\sigma_\epsilon$. We use Theorem 3.1 to treat the data-generating process as equivalent to a quota mechanism that reserves a distribution of college seats for each group equal to $Q_j(p) = F_{P_j}(p)$, $j \in \{\mathcal{M}, \mathcal{N}\}$, which are the marginal distributions of the selection-corrected $F_{PS}(p, s|j)$’s from the previous section.
For each group \( j \in \{M, N\} \), let \( \tau_{PS}(\sigma_{i}|j) \) denote the rank correlation between HC and school assignment implied by shock variance parameter \( \sigma_{i} \) holding \( G_{j}(s) \) and \( F_{\tilde{p}}(p) \) fixed. Since school assignment within each group is determined by the rank ordering of perturbed HC levels and the perturbations are independent, the following must be true: (i) \( \tau_{PS}(0|M) = \tau_{PS}(0|N) = 1 \); (ii) \( \tau'_{PS}(\sigma_{i}|j) < 0, j \in \{M, N\} \); and (iii) \( \lim_{\sigma_{i} \to \infty} \tau_{PS}(\sigma_{i}|M) = \lim_{\sigma_{i} \to \infty} \tau_{PS}(\sigma_{i}|N) = 0 \). The following result directly follows from these facts:

**Proposition 4.8.** There is a unique value of \( \sigma_{i} \) that is consistent with perturbed, rank-order allocations and the joint distributions \( F_{PS}(p, s|M) \) and \( F_{PS}(p, s|N) \).

4.1.3. Identification: Admission Preference Markups. With the distribution of the matching shock known, we can now consider the equilibrium distributions of noisy HC as known objects since they are a convolution of HC and the shock, \( H_{j}(t) = (G_{j} \circ F_{\tilde{e}})(t) \). These CDFs enter into the assignment mappings described in Section 3.3 to determine the allocation of college seats. One key observation about the admission preference mechanism relevant to identification is the following:

\[
(9) \quad P_{M}(t) = F_{P_{M}}^{-1}[H_{M}(t)] = F_{P_{N}}^{-1}[H_{N}(\tilde{T}(t))] = P_{N}(\tilde{T}(t)).
\]

In other words, a minority student with perturbed investment \( t = s + \varepsilon \) matched to the same college as a non-minority student with HC level \( \tilde{T}(s + \varepsilon) \). Manipulating equation (9), we find

\[
(10) \quad \tilde{T}(t) = H_{N}^{-1}\left[F_{P_{N}}\left(F_{P_{M}}^{-1}[H_{M}(s)]\right)\right].
\]

No restrictions are imposed on the form of the markup function. One need not even assume that the markup aids minorities (i.e., \( \tilde{T}(t) > t \)). These arguments imply the following result:

**Proposition 4.9.** Under assumptions A.1 and A.9 of Appendix A there exists a unique markup mapping \( \tilde{T}(\cdot) \) that is consistent with \( (G_{M}, G_{N}, F_{P_{M}}, F_{P_{N}}, \sigma_{\tilde{\varepsilon}}) \).

4.1.4. Identification: Utility Parameters and Cost Types. At this point, we can now treat the assignment mappings \( P_{M}^{pp}(t) \) and \( P_{N}^{pp}(t) \) as known. We drop the superscript in order to simplify notation unless it is needed for clarity.

An approach for estimating strategic models with private information was proposed by Guerre, Perrigne, and Vuong [32] for first-price auctions. Their idea was simple but powerful: since the equilibrium distributions of bids are observable, one can reverse engineer a bidder’s private valuation as that which rationalizes her bid as a best response to competitors’ bids. Our setting is similar in that each student’s investment choice is a best response to the distribution of HC choices given her type. The first-order condition for the student’s problem is:

\[
(11) \quad \theta = \frac{E_{\varepsilon} \left[ U_{p} \left( P_{j}(s + \varepsilon), s, \theta; \alpha \right) P_{j}'(s + \varepsilon) \right]}{c'(s)} + E_{\varepsilon} \left[ U_{s} \left( P_{j}(s + \varepsilon), s, \theta; \alpha \right) \right], \quad j \in \{M, N\},
\]
where \( \alpha = [\log(a_0), a_p, a_s, -a_0]^\top \) is the vector of utility parameters governing household income production from a match of a student to a school given her HC investment. Because the equilibrium strategies are strictly monotone in the agent’s type, equation (11) uniquely defines the production from a match of a student to a school given her HC investment. Because the equilibrium strategies must be monotone, which in turn implies there is a unique, monotone solution that prevents the regressors from being perfectly colinear. Note that achievement also govern the HC accumulation process during college as well.

Based on the idea that the same unobserved characteristics that govern a student’s pre-college characteristics in the production of income. The intuition behind structural identification is as follows: in order for the parameters of the household income regression to be identified, we need an orthogonality condition and a full rank condition. Assumption 4.10 establishes orthogonality condition and a full rank condition. Assumption 4.10 establishes orthogonality.

Thus, our approach is to embed the inverse equilibrium equations into the household income regression model:

\[
\log(w_i) = \log(a_0) + a_p \log(p_i) + a_s \log(s_i) - a_0 \psi(S_i, D_{Mi}; \alpha) + \varepsilon_{wi},
\]

where \( \psi(S_i, D_{Mi}; \alpha) \equiv \log[D_{Mi} \theta_{M}(S_i; \alpha) + (1 - D_{Mi}) \theta_{N}(S_i; \alpha)] \) is \( i \)'s log-cost type and \( \varepsilon_{wi} \) is a transitory shock to 10-year household income. We assume transitory shocks are exogenous.

Assumption 4.10. \( \mathbb{E}\{[\log(p_i), \log(s_i), \psi(S_i, D_{Mi}; \alpha)]^\top \varepsilon_{wi}\} = 0. \)

For the full-rank condition, there must be something present in the data-generating process that prevents the regressors from being perfectly colinear. Note that \( \sigma_\varepsilon > 0 \) implies a non-degenerate distribution of HC types on each college campus under any college admissions rule, so that rank correlation between \( \log(S) \) and \( \log(P) \) must be less than one in absolute value. Second, denote the expected household income of minority (nonminority) students graduating from college \( p \) with HC level \( s \) as \( U_M(p, s) \) \( (U_N(p, s)) \). Suppose we observe a positive measure of \( (p, s) \) such that \( U_M(p, s) = U(p, s, \theta_M(s; \alpha)) \neq U_N(p, s) = U(p, s, \theta_N(s; \alpha)) \). Our exclusion restriction implies that it must be the case that \( \theta_M(s; \alpha) \neq \theta_N(s; \alpha) \), which insures that the rank correlation between \( s \) and \( \theta \) is greater than \(-1\). Thus, in expectation the matrix of regressors

\[
X(\alpha) = \begin{bmatrix}
1 & \log(p_1) & \log(s_1) & \psi(s_1, D_{M1}; \alpha) \\
1 & \log(p_2) & \log(s_2) & \psi(s_2, D_{M2}; \alpha) \\
\vdots & \vdots & \vdots & \vdots \\
1 & \log(p_I) & \log(s_I) & \psi(s_I, D_{MI}; \alpha)
\end{bmatrix}
\]

will have full rank for any configuration of the parameters \( \alpha \). Essentially, \( \psi(S, D_{Mi}; \alpha) \), which is derived from economic theory of investment behavior, serves as a control function and allows the researcher to separate out the influence of unobserved student characteristics \( \theta \) from achievement \( s \) and school quality \( p \). Finally, the supermodularity of our model implies that the equilibrium strategies must be monotone, which in turn implies there is a unique, monotone solution \( \theta_j(s; \alpha) \) to equation (11). This in turn implies that the type distributions \( F_j(\theta) = G_j[\theta_j^{-1}(\theta; \alpha)] \), \( j \in \{M, N\} \), are known if \( \alpha \) is known. This logic yields our final result on structural identification:
Proposition 4.11. Under Assumptions 4.1 - 4.10 household income parameters $\alpha$ and cost type distributions $F_M(\theta)$, $F_N(\theta)$ are identified, provided that

1. $0 < \mu < 1$
2. $\sigma_\epsilon > 0$
3. $\exists (p, s)$ such that $U_M(p, s) \neq U_N(p, s)$
4. $\nabla_\alpha^2 Q(\alpha)$ is positive definite on $(0, \infty) \times (0, 1)^3$, where

$$Q(\alpha) = [D_w (W - X(\alpha)) \alpha] \top [D_w (W - X(\alpha)) \alpha],$$

$W = [\log(w_1), \log(w_2), \ldots, \log(w_I)] \alpha$ is the vector of corresponding observed incomes, and $D_w = [D_{w1}, D_{w2}, \ldots, D_{wi}]$ is a vector of sampling weights.

4.2. A Two-Stage, Semiparametric Estimator. We now construct a two-stage GMM estimator to implement our identification strategy. First, we recover the preliminary model parameters that do not directly depend on our strategic investment model, $\beta^s$, $\beta^p$, $P_M(s)$, $P_N(s)$, and $\sigma_\epsilon$. Then we use these estimated values and the first-order conditions to recover the utility parameters $\alpha$ and the learning cost distributions, $F_M(\theta)$ and $F_N(\theta)$. The technical details of the stage I and II estimators are in Appendices B.1 and B.2 respectively.

4.2.1. Stage I Estimation. The first hurdle to overcome is to find a computationally tractable way of representing the joint distribution of $(P, S)$ conditional on graduation. High-dimensional density estimation is a difficult problem both computationally and because of the rapid decay of optimal statistical convergence rates as the dimensionality of the underlying random variable grows. Recent work by Hubbard, Li, and Paarsch has employed parametric copula functions to solve this problem. We follow this dimension reduction strategy by adopting the Gumbel-Hougaard copula to represent the correlation structure in the joint distribution between human capital and college placement. For the marginal distributions of human capital and college placement, we use a flexible approach based on B-splines. B-splines are a class of finite-dimensional functional forms that can be made arbitrarily flexible. Their added benefit is that they are numerically much better behaved than global polynomials (e.g., Chebyshev).

We divide all model parameters into two sets, placing type distributions and household income equation parameters in one set and all other model components in the other set. The main difference between the two sets is that estimating the former requires imposition of equilibrium theory. The latter does not and can therefore be estimated separately in a first stage. With the above functional representations of the marginal and joint distributions in equations (4) – (7) and (10), it is relatively straightforward to understand how, in principle, one could build a GMM-style estimator for all stage I parameters. The single index parameters are chosen to maximize the rank-predictability (i.e., Kendall’s $\tau$) of human capital $s$ for college placement $p$ given the empirical joint distribution of $(E, A, P)$. The joint moments of graduation rates, achievement,
and college quality across different colleges pin down the graduation rate parameters $\beta^\rho$. The empirical joint distribution of $P$ and $S$ for graduates is used to pin down the B-spline-copula representation of the same conditional distribution, and then Bayes’ Rule in conjunction with the graduation probability parameters is used to recover the unconditional joint distribution of $(P, S)$ for enrollees. Finally, the matching shock parameter $\sigma_v$ is chosen to match the empirical rank correlation in the distribution of $(P, S)$ (for enrollees) as closely as possible.

While it is relatively straightforward to establish intuitive connections between the moments in the data and stage I model parameters, a formal definition of the estimator is notationally quite intense since the majority of these terms must be estimated simultaneously. Therefore, we leave a formal treatment of the GMM stage I estimator to Appendix B.1. In what follows, we represent the stage I parameters by $\hat{\pi} = [\hat{\gamma}_p^M, \hat{\gamma}_p^N, \hat{\nu}_M, \hat{\nu}_N, \hat{\beta}_s, \hat{\beta}_\rho]^\top$ and $\hat{\sigma}_v$, which includes estimates for the B-spline parameters for the race-specific marginal distributions of $P$ conditional on graduation, $(\gamma_p^M, \gamma_p^N)$; the B-spline parameters for the race-specific marginal quantile functions of $S$ conditional on graduation, $(\gamma_q^M, \gamma_q^N)$; the (unconditional) race-specific copula parameters $(\nu_M, \nu_N)$; the single-index parameters $\beta^s$; the graduation probability parameters $\beta^\rho$; and the matching shock parameter $\sigma_v$.

4.2.2. Stage II Estimation. Our stage I estimator was based on a set of intuitive moment conditions that were notationally intense to formalize. Stage II estimation is the reverse: notationally compact and with considerable computational complexity under the surface. In this stage we simultaneously estimate the parameters of the household income equation and an inverse strategy that maps HC choices into learning-cost types. The former is a vector $\hat{\alpha} = (\hat{\alpha}_0, \hat{\alpha}_p, \hat{\alpha}_s, \hat{\alpha}_\theta)$ and the latter is fit using B-splines.

We build our estimator on a model of the quota AA system that is equivalent to the admissions preference AA system that generates our data (see Theorem 4, Bodoh-Creed and Hickman [13]). Using the stage I estimates of the matching shock and the distributions of seats allocated to and HC choices of each demographic group, we compute the mapping from NHC to school assignment. This mapping remains fixed throughout the stage II estimation process. For any given value of $\alpha$, we can compute the marginal benefit of each HC choice (i.e., the right-hand side of equation 11). This provides a nonparametric estimate of the inverse strategy function.

The B-spline fit of the inverse strategy mapping is a flexibly parameterized control function

$$
\psi \left( s, D_{Mi}; \alpha, \hat{\pi}, \hat{\sigma}_v, \hat{\lambda}_M^t, \hat{\lambda}_N^t \right) = \log \left[ D_{Mi} \theta_M (s; \lambda_M^t (\alpha)) + (1 - D_{Mi}) \theta_N (s; \lambda_N^t (\alpha)) \right],
$$

with the extra parameter arguments emphasizing its implicit dependence on stage I objects. Moving forward we suppress the additional pre-determined parameter arguments for notational

\[26\text{Although we could nonparametrically estimate the distribution of } \theta, \text{ we use B-splines for computational ease.}\]
simplicity. We can now re-express the matrix of explanatory variables as
\[
X(\alpha) = \begin{bmatrix}
1 & \log(p_1) & \log(s_1) & \psi(s_1, D_{M1}; \alpha) \\
1 & \log(p_2) & \log(s_2) & \psi(s_2, D_{M2}; \alpha) \\
\vdots & \vdots & \vdots & \vdots \\
1 & \log(p_I) & \log(s_I) & \psi(s_I, D_{MI}; \alpha)
\end{bmatrix},
\]

The two moments we enforce are (1) a least squares condition insuring a good fit between the B-splines and the right-hand side of equation (11) and (2) a least squares condition for the household income equation
\[
\hat{\alpha} = \arg\min_{\alpha \in \mathbb{R} \times [0,1]^T} \left\{ [D_w (W - X(\alpha) \alpha)]^T [D_w (W - X(\alpha) \alpha)] \right\}.
\]

The type distributions are recovered by convolving \(\theta_j(s, \lambda_j(\alpha))\), \(j \in \{M, N\}\), with the CDFs of HC choices.

4.2.3. Asymptotics and Standard Errors. The empirical strategy we propose above falls within the broad class of GMM estimators. Our empirical implementation uses B-splines with a finite number of parameters to estimate the marginal distributions of \(P, S,\) and \(\theta\). One can view this as a parametric class assumption—though one with a considerable degree of flexibility—which is held fixed as the sample size grows. Under this view, standard GMM asymptotic theory (e.g., see Hayashi [33]) establishes consistency and asymptotic normality of the parameter estimates, with convergence at the standard rate of \(\sqrt{T}\).

In order to explore the role of sampling variability, we employ a block-bootstrap procedure which involves re-sampling 1000 times from the race-specific B&B subsamples. We separately resample \(I_M\) student observations (with replacement) from the minority subsample \(\{w_i, p_i, e_i, a_i\}_{i=1}^{I_M}\) and \(I_N\) student observations (with replacement) from the non-minority subsample \(\{w_i, p_i, e_i, a_i\}_{i=1}^{I_N}\). Because college-level variables represent the universe of four-year colleges and the full universe of students enrolled in these colleges, we hold these observables fixed during our bootstrap procedure. Moreover, we calibrate the minority mass \(\mu\) from the IPEDS college-level data and hold it fixed as well. Thus, all variation in our standard errors comes from the finite sampling of the student-level B&B data.

5. ESTIMATION RESULTS

We now discuss our model estimates. Goodness of fit metrics are provided in Appendix C.

5.1. ESTIMATES: Single Index Function \(S(e, a)\) and Matching Shock Variance \(\sigma^2\). The parameter estimates for the quadratic HC single index equation are contained in Table 3.\footnote{Recall that the single index function is scale-free and normalized to attain a maximum value of one. Recall also that a more flexible cubic form did not improve our model fit (see footnote 22).} The HC index in convex in both \(e\) and \(a\) and admits significant complementarities between the arguments. The marginal effect of a one standard deviation change of \(e\) from the median is 0.0668, while the marginal effect of a one standard deviation change in \(a\) from the median is equal to 0.1520. In
Table 3. ESTIMATES: Single Index Function and Matching Shock Variance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
<th>P-Value</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1^e$ ($e$)</td>
<td>0.0506***</td>
<td>(9.63 x 10^{-4})</td>
<td>&lt; 0.001</td>
<td>[0.0485, 0.0524]</td>
</tr>
<tr>
<td>$\beta_2^e$ ($e^2$)</td>
<td>0.1567***</td>
<td>(0.0095)</td>
<td>0.006</td>
<td>[0.1399, 0.1662]</td>
</tr>
<tr>
<td>$\beta_3^a$ ($a$)</td>
<td>0.3039***</td>
<td>(0.0085)</td>
<td>&lt; 0.001</td>
<td>[0.2912, 0.3208]</td>
</tr>
<tr>
<td>$\beta_4^a$ ($a^2$)</td>
<td>0.2898***</td>
<td>(0.0096)</td>
<td>&lt; 0.001</td>
<td>[0.2738, 0.3067]</td>
</tr>
<tr>
<td>$\beta_5^{(e \cdot a)}$</td>
<td>0.2296***</td>
<td>(0.0090)</td>
<td>&lt; 0.001</td>
<td>[0.2114, 0.2421]</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.0275</td>
<td>(7.25 x 10^{-4})</td>
<td>—</td>
<td>[0.0260, 0.0286]</td>
</tr>
<tr>
<td>$\tau_{PS}(\sigma_\varepsilon</td>
<td>M)$</td>
<td>0.8723</td>
<td>(0.0072)</td>
<td>—</td>
</tr>
<tr>
<td>$\tau_{PS}(\sigma_\varepsilon</td>
<td>N)$</td>
<td>0.8952</td>
<td>(0.0034)</td>
<td>—</td>
</tr>
</tbody>
</table>

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

other words, $a$ is 2.5 times as important as $e$ for determining a median student’s HC single index. Appendix E contains an illustrative plot of the single index function with confidence bounds.

Table 3 includes estimates of the matching shock standard deviation, $\sigma_\varepsilon$, and group-specific Kendall’s $\tau$ at enrollment to provide different perspectives on the matching frictions. The noise-to-signal ratio of the matching shock (the ratio of $\sigma_\varepsilon$ to a standard deviation of HC) is 18.2%. Since Kendall’s $\tau$ for minority students is estimated at 0.872, this means for two randomly selected minority students there is a 93.6% chance that the student with higher HC will enroll in a higher quality college. Likewise, this probability for the nonminority group is 94.8%. Thus, while matching shocks play a nontrivial role, our empirical model suggests a high degree of assortativity in the college market.

If students are motivated by concerns other than quality or make mistakes in the college application process, then this would be reflected in a large value $\sigma_\varepsilon$. For example, if students were willing to make trade-offs between the geography or demography of a school and college quality, then the assortativity of our match would be lowered (i.e., $\sigma_\varepsilon$ would be large). If students were unable to identify the highest quality of college that they could be admitted to because of (for example) information frictions, then the match would be less assortative. The fact that $\sigma_\varepsilon$ is low leads us to conclude that, while these issues may play a role, they are not as powerful as the incentives our model captures. In addition, we test whether assortativity fails for lower HC and minority students explicitly in Section 5.6 and find no significant effects, which further lends confidence in our model.

5.2. ESTIMATES: Graduation Probability Function, $\rho(p,s)$. Point estimates and standard errors for the graduation probability parameters are displayed in Table 4. We again compute the marginal effect of a one standard deviation change in $p$ and $s$ (from the median values) on $\rho(p,s)$ as our metric of the relative importance of these variables for determining the probability of graduation. In this case, we compute the median and standard deviation statistics using the
Table 4. Graduation Probability Estimates $\hat{\rho}(p, s)$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
<th>P-Value</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1^p (p)$</td>
<td>0.0471</td>
<td>(0.0805)</td>
<td>0.4530</td>
<td>[−0.0901, 0.2283]</td>
</tr>
<tr>
<td>$\beta_2^p (p^2)$</td>
<td>−0.1991*</td>
<td>(0.1391)</td>
<td>0.0990</td>
<td>[−0.443, 0.0755]</td>
</tr>
<tr>
<td>$\beta_3^p (p^3)$</td>
<td>0.2160*</td>
<td>(0.1305)</td>
<td>0.0550</td>
<td>[−0.0070, 0.4891]</td>
</tr>
<tr>
<td>$\beta_4^s (s)$</td>
<td>0.8447***</td>
<td>(0.1658)</td>
<td>&lt; 0.001</td>
<td>[0.6102, 1.3090]</td>
</tr>
<tr>
<td>$\beta_5^s (s^2)$</td>
<td>−0.0179</td>
<td>(0.1421)</td>
<td>0.9640</td>
<td>[−0.0651, 0.0108]</td>
</tr>
<tr>
<td>$\beta_6^p (p^3)$</td>
<td>0.0029</td>
<td>(0.0129)</td>
<td>0.3020</td>
<td>[−0.0254, 0.0249]</td>
</tr>
<tr>
<td>$\beta_7^p (p \cdot s)$</td>
<td>0.0464</td>
<td>(0.3753)</td>
<td>0.9040</td>
<td>[−0.4627, 1.1309]</td>
</tr>
<tr>
<td>$\beta_8^p (p^2 \cdot s)$</td>
<td>0.0093</td>
<td>(0.1443)</td>
<td>0.8450</td>
<td>[−0.1216, 0.0309]</td>
</tr>
<tr>
<td>$\beta_9^p (p^2 \cdot s)$</td>
<td>0.0975</td>
<td>(0.2739)</td>
<td>0.4800</td>
<td>[−0.3067, 0.9115]</td>
</tr>
<tr>
<td>$\beta_0^p$ (const.)</td>
<td>−0.0532**</td>
<td>(0.0236)</td>
<td>0.065</td>
<td>[−0.0844, 0.0086]</td>
</tr>
</tbody>
</table>

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

selection-corrected distributions of the respective variables. The marginal effect of a one standard deviation change in $p$ is 0.026, while the marginal effect of a one standard deviation change in $s$ is 0.143. This means that $s$ is roughly 5.5 times as important as $p$ for determining college graduation probabilities. Appendix E contains an illustrative plot of the graduation probability function with confidence bounds.

5.3. ESTIMATES: Selection-Corrected Joint Distributions $f_j(p, s)$. Figure 1a displays the distributions of HC levels for each demographic group, including the selected sample of graduates from the raw data (dashed lines) and the selection-corrected distributions for all enrollees (solid lines). Figure 1b displays the distributions of college seats allocated to each group, including the selected raw samples (dashed lines) and the selection corrected distributions for all enrollees (solid lines). The CDF plots also include 95% confidence bounds at the deciles of the population-wide distributions. The first plot illustrates the achievement gap, a stochastic dominance relationship between minority HC and non-minority HC. The second plot illustrates the enrollment gap, a similar stochastic dominance relationship between minority and non-minority college quality. Figure 1b does not have any large jumps in the CDFs, which is consistent with our model’s assumption that there is a continuum of college seats.

5.4. ESTIMATES: Minority Markup Function, $\tilde{T}$. Figure 2 describes the estimated markup function, $\tilde{T}(t)$. The horizontal axes of both panels display quantile ranks of NHC for nonminority students. The left panel describes the shape of $\tilde{T}$. The vertical axis displays the quantile rank of subsidized NHC (within the non-minority NHC distribution) with 95% confidence bounds at the deciles of NHC. If a minority student has an NHC at the quantile rank marked on the
horizontal axis, the student gets the same college assignment as a nonminority student with an NHC at the quantile rank denoted on the vertical axis. For example, a minority student with an NHC equal to the median of the nonminority population gets the same college assignment as a nonminority student at the 64th percentile of the nonminority population. The dashed line denotes the 45° line for reference.

The right panel of Figure 2 describes the markup function in terms of school quality. The vertical axis denotes the gap in the quantile rank of college quality between a minority student and a nonminority student at each NHC quantile with 95% confidence bounds at the deciles of NHC. For example, the plot shows that if two students from different groups both have an NHC value equal to the median of the nonminority population, then the minority student is assigned to a school whose quantile rank is 0.13 higher in the school quality distribution.

Our plot reveals that the effect of the status quo admissions preference scheme is insignificant at colleges in the bottom decile, but is statistically and economically significant across the rest of the college quality spectrum. Several papers have estimated a substantial impact of AA, including Bowen and Bok [14]; Chung and Espenshade [19]; and Chung, Espenshade, and Walling [20], but these studies used data from elite colleges, whereas ours provides a market-wide picture.
The most similar previous study is Kane [40], which also used a nationally representative sample (the High School and Beyond (HS&B) survey), but estimated a significant role for AA only in the top quintile of the market. Several differences exist between Kane [40] and our study. First, we use measures of final market allocations (enrollment data), whereas Kane [40] uses applications data which may not fully reflect final enrollment decisions. Second, the HS&B data contain potentially important sources of sample selection that could affect probit regression results in unpredictable ways. HS&B respondents were asked their two top choices (sample truncation) among the schools to which they applied (endogenous selection in student-school pairs) and whether they were accepted.

At the end of the day, our estimates of the markup function are the most directly data-driven component of the empirical model: they hinge on a stage I reduced-form sample-selection correction to map our observed set of college graduates into the original set of college enrollees. The intuition behind our result is that a color-blind world implies a very specific form for the distribution of \((p, s)\). Our stage I reduced-form data products deviate from this form, and in such a way that more generous admissions practices toward minorities must exist on the majority of the market in order to rationalize observed allocations from observed achievement.

5.5. **ESTIMATES: Match Utility and Learning Cost Type Distributions.** The type distributions are presented in Figure 3. The top panel displays the type CDFs \(F_M(\theta)\) and \(F_N(\theta)\) with 95% confidence bounds represented by the shaded areas. The type distribution of the minority students stochastically dominates the type distribution of nonminority students, which is consistent with minority students having higher learning costs than nonminority students. The right panel of Figure 3 plots the pointwise difference between the CDFs with 95% confidence bounds, and the difference between the type distribution is significant at all learning-cost quantiles.

This result conforms with a large body of empirical evidence on stark racial differences in access to resources that affect childhood development in the US. For example, Black and Hispanic children are nearly three times as likely to live below the poverty line (see Kena et. al. [42]). They
Table 5. Estimates of the Wealth Equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_p$</td>
<td>0.1349***</td>
<td>(0.0260)</td>
<td>[0.0894, 0.1902]</td>
</tr>
<tr>
<td>$a_s$</td>
<td>1.181 × 10^{-6}</td>
<td>(0.0306)</td>
<td>[-1.000 × 10^{-5}, 2.412 × 10^{-4}]</td>
</tr>
<tr>
<td>$a_\theta$</td>
<td>0.0574**</td>
<td>(0.0386)</td>
<td>[0.009, 0.1478]</td>
</tr>
<tr>
<td>$a_0$</td>
<td>$79,108***$</td>
<td>($4,735)</td>
<td>[$69,105, $87,499]</td>
</tr>
</tbody>
</table>

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

There are also much less likely to be covered by health insurance (see Smith and Medalia [57]) or to be raised by parents with bachelor degrees (see Fox, Kewal, and Ramani [29]). Holding income level fixed, Blacks and Hispanics are also much more likely to attend under-performing schools that serve poorer student bodies relative to their White and Asian counterparts of similar incomes (see Reardon, Townsend, and Fox [54], Reardon [52], and Reardon, Kalogrides, and Shores [53]). In our data we find direct evidence consistent with this view as well. In Appendix D we show that various factors relating to childhood family background—e.g., parents’ education and wealth among others—are highly predictive of the learning cost types we estimate. With these empirical facts in mind, the estimated stochastic dominance relationship in learning costs would seem a natural, though unfortunate, consequence of resource stratification by race.

Table 5 presents our estimates of the returns to various inputs of household income production including a more selective college, more HC, and lower learning costs. The first takeaway is that pre-college HC choice, while important for determining the graduation probability, has very little effect on household income conditional on graduation. The second takeaway is that both school quality and the agent’s unobservable type have economically and statistically significant effects on household income production. The marginal effect of a one standard deviation increase in $\log(p)$ is 3.44 times larger than the marginal effect of a one standard deviation reduction in $\log(\theta)$, which is in turn $5.37 \times 10^4$ times larger than the marginal effect of a one standard deviation increase in $\log(s)$. 29

There is a sizable literature on the returns to a higher quality college. In general, the empirical evidence is clear that more selective colleges benefit poorer students significantly, which our results support, but there is some disagreement as to the magnitude of the return for more affluent students. Dale and Krueger [24] find negligible returns to a more selective college for affluent students, but Brewer, Eide, and Ehrenberg [15]; Black and Smith [11]; Long [43]; and Andrews, Li, and Lovenheim [3] estimate significant returns on the investment in a more selective college for all students. Our results strongly support the latter view that a higher quality college is a resource from which all students benefit. Our new contribution to this discussion is in quantifying the role of unobserved student characteristics in determining the returns to a higher quality college.

29 All of the bootstrap confidence intervals are biased-corrected and accelerated.
30 As a robustness check on the relative importance of $S$ with respect to the other factors, we experimented with estimating an alternative household income equation with quadratic terms in $P$ and $S$. See Section 5.6.
quality college. Holding $s$ and $\theta$ fixed at their relative medians, a move from being placed at the 25th percentile college to the 75th percentile college induces an estimated shift of $7,500 in annual household income. If we hold fixed $\theta$ at the 25th percentile, we find the same shift in college quality yields a $7,600 increase in annual household income, demonstrating the (modest) complementarities that exist between $p$ and $\theta$. We are also the first paper to directly quantify the role of unobserved heterogeneity in the returns to an education, which we find is significant: reducing the learning cost from the upper quartile of $\theta$ to the lower quartile, holding $p$ fixed at the median, induces an increase of roughly $2,100 in annual household income. The same shift made while holding $p$ fixed at the 75th percentile of the college quality spectrum yields an increase of $2,200, again due to $p$ and $\theta$ complementarities.

On the other hand, while a student’s pre-college HC choices do not significantly affect the household income conditional on graduation, these investments have an influence on the probability of graduating college. This is consistent with a view of the world in which a student’s pre-college HC helps the student pass the curriculum of the college in which she enrolls, but post-college household income is determined by the level of HC accrued in college, which is generated primarily by college quality and learning cost.

5.6. ROBUSTNESS CHECKS. First, we experimented with adding an additional parameter to the cost model, $c(s) = \exp(\nu(s - \bar{s}))$, for added flexibility. Identification would then require additional conditions, for example, an incentive compatibility assumption for marginal market participants who are indifferent to college attendance versus entering the workforce. In practice the extra parameter adds little explanatory power. The estimate of $\nu$ is most influenced by the behavior of very low achieving college students where data are sparse. Elsewhere, changes in the curvature parameter $\nu$ are compensated for by corresponding changes to the scale of $\theta$ with little change to the economic implications of the point estimates. We also estimated the model with an alternative power law cost function of the form $C(s, \theta) = \theta(s + 1)^2$ and found that model estimates were qualitatively very similar, suggesting that our functional form assumption for costs is not unduly driving the results.

Second, we estimated a model wherein the matching shock was linear in the HC choice of the agents. This would allow, for example, the matching shock to be larger for students with lower HC choices, which might reflect higher effort costs that impede searches for potential colleges to apply to. If we include a linear trend in $s$, then we estimate that $\sigma_\varepsilon(s) = 0.0279 - 0.0006s$, which is essentially the same as the result in Table 3. We also experimented with allowing a distinct matching shock parameter for minority and nonminority students and requiring that each group’s parameter best rationalize the correlation of $P$ and $S$ amongst students in that group. We did not find that the parameters differed significantly.

Third, we considered alternative forms for the household income equation. For example, we explored including the estimated matching shock, $\varepsilon$, as an additional driver of household income by estimating a household income equation with the form $u(P_i, S_i, \theta_i) = \alpha_0 P_i^\alpha S_i^\beta \theta_i^{-\beta_0} \varepsilon^{\alpha_\varepsilon}$. Our estimates imply that the marginal effect of a one standard deviation change of $p$ and $\theta$ are almost
equal, the effect of $s$ is negligible, and the effect of a one standard deviation change of $\varepsilon$ is 10% as important as that of either $p$ or $\theta$. In short, the estimates are qualitatively the same as in our baseline model.

Fourth, we estimated a model where the household income equation is $u(P_i, S_i, \theta_i) = \alpha_0(P_i + \gamma_p P_i^2)^{\alpha_p}(S_i + \gamma_s S_i^2)^{\alpha_s} \theta_i^{-\alpha_{\theta}}$. Our estimates with this more flexible form of the model also indicated $P$ and $\theta$ are the primary drivers of household income, whereas $S$ is an insignificant driver of household income production except for the best students placing at the best colleges. Even for these students, $S$ is only marginally important.

Finally, we experimented with models that use a HC index based only on exam scores, which would address potential GPA measurement issues (see Section 2.2). We found that this model produced an implausibly low degree of assortativity between HC and college quality. For example, this model yields Kendall $\tau$ statistics of 0.343 and 0.336 for minority and nonminority students. This would imply that in any randomly chosen pair of minority (nonminority) students, the higher HC student enrolls in the higher quality college only 67.2% (66.8%) of the time. This indicates that our GPA measure carries significant amounts of HC information, which is why we maintain it in our analysis.

6. COUNTERFACTUAL POLICY EXPERIMENTS

We now explore the economic implications of changes to the AA system we estimated from the data-generating process. For each of the admissions schemes (color-blind and proportional quota), we solved for the equilibrium of the model holding our structural point estimates fixed (for technical details see Appendix F), which reveals how HC accumulation strategies change under each regime. We also discuss the extent to which controlling for the changing HC incentives is necessary for accurate predictions of the effects of AA.

6.1. Effects of AA on HC Investment. It is theoretically ambiguous whether the type-specific minority HC choices will increase or decrease under a particular AA scheme. Figure 4 presents the change in minority HC investment under our two counterfactual admissions schemes, so positive (negative) values indicate increases (decreases) relative to the status quo. For ease of presentation, we describe the student’s type in terms of quantiles of the minority cost distribution, and changes in HC are displayed as a fraction of a standard deviation in the status quo level.

The decisions of high and low learning-cost students are pushed in opposite directions, which means the effect on average HC is relatively small. The HC decisions of high and low learning-cost students are pushed farther apart in a color-blind world. The lowest 25% of the cost distribution increase their HC by more than 0.1 standard deviations, but the highest 25% of the learning cost distribution decrease their HC by about 0.05 standard deviations. In contrast, the HC choices

$31$Note that the form of the nonlinear control function $\psi(S_i, D_M; \alpha, \gamma_p, \gamma_s)$, which stands in for $\theta_i$, is already influenced by the additional terms in $P_i$ and $S_i$. Since the form of $\psi$ is determined endogenously within the model, we do not add a quadratic term in $\theta$.

$32$As discussed in Appendix F, we used multiple restarts to test for multiple equilibria and never found multiplicity.
of the students are brought together under a proportional quota system. A proportional quota drives down the HC choices of the bottom 10% of the learning cost distribution, while the upper 50% of the learning cost distribution increases their HC by 0.05-0.1 standard deviations.

The incentive effects of changing the AA system can be understood by examining the first order condition. We use the equivalence of all three systems to a quota to simplify the FOC expression, which is

\[
E_{\varepsilon} \left[ U_p \left( P_j(s + \varepsilon), s, \theta; \alpha \right) \frac{h_j(s + \varepsilon)}{q_j(P_j(s + \varepsilon))} \right] + E_{\varepsilon} \left[ U_s \left( P_j(s + \varepsilon), s, \theta; \alpha \right) \right] = \theta c'(s), \ j \in \{M, N\}. 
\]

\(h_j\) is the endogenous PDF of the NHC distribution for group \(j\), and \(q_j\) is the quota of seats for group \(j\) (or the quota that is equivalent to the color-blind or status-quo equilibrium). The influence of the AA system on the students’ strategic investment incentives is captured by the student-to-seat ratio, \(h_j(s + \varepsilon)/q_j(P_j(s + \varepsilon))\). Recall that the fraction of competitors surpassed is equal to the change in the enrolled school quality percentile. When \(h_j(s + \varepsilon)\) is large, small increases in NHC result in the student surpassing a large fraction of her competitors. Likewise, when \(q_j(P_j(s + \varepsilon))\) is low, small changes in the school quality percentile generate large changes in realized school quality.

Under a color-blind scheme, the lowest-cost minority students suddenly face more competitors (i.e., \(h_M/q_M\) increases) and the marginal benefit from HC accumulation rises, while the highest-cost minority students compete for a glut of seats at low quality colleges (i.e., \(h_M/q_M\) drops) and the marginal benefit to HC accumulation drops. Symmetrically, under a proportional quota, the lowest learning-cost students face a lower student-to-seat ratio, which depresses the marginal benefit of HC and their HC investment. The highest learning-cost students face a higher student-to-seat ratio, which increases the marginal benefit of HC and their HC investment. The effect on nonminority students is much more limited since the distribution of college seats and the types of their competitors closely resembles the corresponding distributions under the status quo.

\[33\] The quotas equivalent to the endogenous outcome of each AA system are presented in Table 6.
6.2. Effects of AA on Minority Enrollment. Table 6 describes counterfactual enrollment in terms of the fraction of each demographic group enrolled in each college quality quintile. The benchmark of full representation of each group in each quintile, 20%, is mechanically achieved by a proportional quota. Numbers below this imply under-representation and vice versa. For example, 13% of minority students enroll in colleges in the top quintile of the quality distribution under the status-quo AA scheme, but only 10% of them enroll in top colleges under a color-blind scheme. While the status-quo AA is less generous to minorities at top colleges than a proportional quota would be, a ban of race-based AA would reduce minority enrollment in the top tier by nearly one quarter.

The status quo AA scheme has the intended result in that there is a first-order stochastic dominance (FOSD) shift in the distribution of the quality of colleges in which minority students enroll, relative to the color-blind case. Interestingly, the largest shift is from the lowest quintile of college quality into the second lowest quintile: in a color-blind world, minority enrollment in the bottom quintile would increase by almost one half. A proportional quota yields an even stronger FOSD shift toward minority enrollment in better colleges, relative to the status quo. For completeness, Table 6 also provides the effects of AA on nonminorities. As expected, changing the AA scheme has the mirror opposite effect on the nonminority students, but the magnitudes of the shifts are small since the non-minority mass is five times larger.

The final column of Table 6, denoted fixed HC, describes the enrollment outcomes if we compute the enrollment under a color-blind system while holding fixed the status quo HC choices.\[\text{\footnote{The proportional quota mechanically will result in a fifth of the students in each college quality quintile regardless of the strategy used by the students.}}\]
Table 7. Counterfactual Minority Graduation Probability...

<table>
<thead>
<tr>
<th>Learning-Cost Type Tier</th>
<th>Status Quo</th>
<th>Proportional Quota</th>
<th>Color-Blind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Achiever Quintile</td>
<td>0.562</td>
<td>0.562</td>
<td>0.580</td>
</tr>
<tr>
<td>Second Achiever Quintile</td>
<td>0.389</td>
<td>0.402</td>
<td>0.399</td>
</tr>
<tr>
<td>Third Achiever Quintile</td>
<td>0.299</td>
<td>0.311</td>
<td>0.302</td>
</tr>
<tr>
<td>Fourth Achiever Quintile</td>
<td>0.236</td>
<td>0.249</td>
<td>0.229</td>
</tr>
<tr>
<td>Bottom Achiever Quintile</td>
<td>0.159</td>
<td>0.165</td>
<td>0.151</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>College Quality Tier</th>
<th>Status Quo</th>
<th>Proportional Quota</th>
<th>Color-Blind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top College Quintile</td>
<td>0.612</td>
<td>0.560</td>
<td>0.661</td>
</tr>
<tr>
<td>Second College Quintile</td>
<td>0.439</td>
<td>0.401</td>
<td>0.481</td>
</tr>
<tr>
<td>Third College Quintile</td>
<td>0.353</td>
<td>0.310</td>
<td>0.391</td>
</tr>
<tr>
<td>Fourth College Quintile</td>
<td>0.270</td>
<td>0.251</td>
<td>0.307</td>
</tr>
<tr>
<td>Bottom College Quintile</td>
<td>0.178</td>
<td>0.166</td>
<td>0.195</td>
</tr>
</tbody>
</table>

By comparing the color-blind equilibrium outcomes with the fixed HC results, we see that controlling for the endogeneity of the HC choices blunts the effect of the change to a color-blind system for high learning-cost students and exaggerates it for the low learning-cost students. Accounting for endogenous HC yields more (less) diversity at high (low) quality colleges than a fixed HC analysis would predict, which is due to the fact that HC investment is higher (lower) for low (high) learning-cost students in the equilibrium of the color-blind system.

6.3. Effects of AA on Household Income, Graduation Probability, and Welfare. In order to give the reader a full sense for how AA shapes minority outcomes, we present graduation and income changes in two different ways. The first separates students by learning-cost quintiles, which represent the same set of individuals under each counterfactual scenario. We also present these outcomes in terms of college quality quintiles in order to depict the effect on aggregate, college-level statistics, but note that the types of individuals enrolling in each quality quintile change across counterfactual scenarios. We do not display the quintile-specific effects for nonminority students as they are much smaller.

The top panel of Table 7 displays graduation probability changes by achievement quintile. Two main forces govern the results here. First, any change in the AA system alters investment incentives. The second force is the counterfactual change in college assignments. The effects we see in the upper panel largely mirror what we found in Figure 4. High-achieving minority students have a stronger (weaker) incentive to make HC investments under a color-blind (proportional quota) system, and the extra investment is largely reflected in graduation rates, though mitigated somewhat by countervailing college placement shifts for these students. The opposite incentives and outcome effects occur for the lowest achieving minority students. The average effect of a change in AA systems on graduation rates across the student population is small.
(i.e., under 0.5%). In addition, because of the counterveiling effects of the HC and enrollment changes, it need not be the case that the status-quo outcome is “in between” the color-blind and proportional quota outcomes.

The bottom panel of Table 7 breaks out the graduation rate by quintile of the college quality distribution. We see an increase in graduation rates for minority students who enroll at top colleges under a color-blind scheme, due mostly to composition effects: a smaller number of minority students with lower cost types enroll in the best colleges, relative to the status quo. Due to their lower learning costs and stronger HC accrual incentives in a color-blind system (Figure 4), the graduation probability of minority students at the best schools rises. Interestingly, the graduation rate of minority students at low quality colleges also rises. Since minority students are assigned to low quality colleges at a higher rate under a color-blind system, the average learning-cost of minority students at these colleges falls. This in turn causes the level of HC accrued by minority students at low quality colleges to rise, which raises the average graduation rates of these students. A proportional quota has the opposite effect. For example, under a proportional scheme minority students enrolling at the top quintile of schools have a lower average rate of graduation because on average they have higher learning costs and their investment incentives (for top minority achievers) are weakened.

The contrast between the upper an lower panels of Table 7 shows the importance of taking into account the composition of minority students enrolling in each college quality quintile when assessing the effects of AA. The differences highlight the importance of taking a market-wide perspective when investigating the impact of AA on the rates at which minorities graduate college: flows of heterogeneous students to alternative segments of the market can create a misleading picture if one focuses only on minorities who enroll within a narrow band of the quality spectrum under alternative admissions systems. For example, a naive reading of the bottom portion of Table 7 would suggest that the mean minority graduation rate is highest under a color-blind system. However, this reasoning ignores compositional shifts in where minority students enroll, which generates the contrast between the top and bottom portions of Table 7.

Table 8 describes the graduation rate by learning-cost quintile when we hold fixed the status quo HC choices of the students. For comparison, the equilibrium graduation rates from Table 7...
Table 9. Counterfactual Minority Household Income by Achievement Quintile

<table>
<thead>
<tr>
<th>Learning Cost Type Tier</th>
<th>Status Quo</th>
<th>Proportional Quota</th>
<th>Color-Blind</th>
<th>Color-Blind, Fixed HC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Achiever Quintile</td>
<td>$80,873</td>
<td>$81,560</td>
<td>$80,336</td>
<td>$79,825</td>
</tr>
<tr>
<td>Second Achiever Quintile</td>
<td>$75,709</td>
<td>$77,223</td>
<td>$74,714</td>
<td>$74,236</td>
</tr>
<tr>
<td>Third Achiever Quintile</td>
<td>$71,308</td>
<td>$73,350</td>
<td>$69,856</td>
<td>$69,846</td>
</tr>
<tr>
<td>Fourth Achiever Quintile</td>
<td>$67,360</td>
<td>$69,143</td>
<td>$65,264</td>
<td>$65,723</td>
</tr>
<tr>
<td>Bottom Achiever Quintile</td>
<td>$61,606</td>
<td>$62,673</td>
<td>$60,010</td>
<td>$61,006</td>
</tr>
</tbody>
</table>

are presented as well. The difference between the equilibrium and fixed HC cases for the proportional quota are small, which is due to the fact that the effect on the investment incentives is modest for most student types and enrollment is (mechanically) identical in the two cases. The exception is the top achievers, students that significantly reduce their equilibrium HC investment under a proportional quota. The reduced HC explains why the graduation rate for this group under a fixed HC analysis is higher than in an equilibrium analysis. A fixed HC analysis predicts that shifting to a color-blind system will have a smaller impact (relative to an equilibrium analysis). Two effects drive this result. First, the equilibrium analysis predicts more (less) enrollment in high (low) quality colleges (an effect that is mechanically absent in a proportional quota). Second, equilibrium HC accumulation in a color-blind system is higher (lower) for low (high) learning-cost students.

Table 9 depicts the effect of AA on minority household income by achievement quintiles. Since HC investment has only negligible influence on household income, conditional on graduation, the associated counterfactual impact of AA is mediated entirely by college assignment shifts. An AA ban would reduce household income across all five quintiles of the type distribution, whereas a proportional quota would increase it across all quintiles. These effects are weakest for the highest achievers, where the difference between a color-blind system and a quota is around $1,200/year. The strengths of the effects increase until the fourth quintile, where the difference between the two peaks at almost $3,900/year of household income. The changes for nonminorities tend to have the opposite sign but are small and relatively inconsequential, so we do not report them at the quintiles.

The last column of Table 9 compares the income predictions of equilibrium and fixed HC analyses in the color-blind case. Low (high) learning cost students make more (less) money under an equilibrium analysis, and this is due to the fact that an equilibrium analysis predicts higher (lower) enrollment by minority students in the best (worst) colleges. For the lowest and highest learning-cost quintiles, where the HC incentive changes are most pronounced, not accounting for these incentive changes result in predictions that are off by $500 and $1,000 in household income.

In the proportional case the effect is under $30 for all quintiles because the enrollment and learning-costs are identical under both analyses and the differences in HC have only a small effect. The calculations for the proportional case are presented in Appendix E.
Table 10. Household Income (HHI) 10 Years After Graduation

<table>
<thead>
<tr>
<th></th>
<th>Average HHI of Minority Students</th>
<th>... Relative to Status Quo</th>
<th>Average HHI of Nonminority Student</th>
<th>... Relative to Status Quo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status Quo</td>
<td>$70,854</td>
<td>—</td>
<td>$74,130</td>
<td>—</td>
</tr>
<tr>
<td>Color-Blind</td>
<td>$70,036</td>
<td>$-818</td>
<td>$74,288</td>
<td>$158</td>
</tr>
<tr>
<td>Proportional</td>
<td>$72,790</td>
<td>$1,936</td>
<td>$73,768</td>
<td>$-363</td>
</tr>
</tbody>
</table>

Figure 5. Minority Equivalent Variations

income per year. For low learning-cost students, the fixed HC analysis reflects less than 40% of the equilibrium loss in household income.

Table 10 provides the average effect across the population for both groups. The effect of any change in AA policy averaged across the total population is on the order of ±$20, meaning AA entails a very small loss of total income production. In terms of the inequality in household income between the groups, a proportional quota results in the smallest gap and a color-blind system causes the largest gap.

We evaluate the net effect of changes to graduation rate, household income, and net learning cost from changing HC choices in terms of equivalent variation, which is the proportional increase in status quo household income that would make the student as well off as under a counterfactual admissions system. Values above 1 indicate utility increases, and the effect will be different for different types of students. For example, from Figure 5, if we multiply the median minority student’s status quo annual household income by a factor of 1.027, then his utility would rise to the same level as in equilibrium under a proportional quota regime. Minority equivalent variation for a proportional quota ranges from 0% to 2.8% of annual household income. The equivalent variation for a color-blind system is around −2.5%, and the effect remains significant for all but the lowest cost minority students. The welfare effect for non-minorities is less than one fifth as large.
6.4. The Relative Force of the Competitive and Productive Channels of Investment. Equation (15) decomposes the marginal benefit of HC investments into two components (left-hand side), which are equated to marginal cost (right-hand side) in equilibrium

\[
\begin{align*}
\text{Productive Channel:} & \quad E_{\varepsilon} \left[ U_p \left( P_j(s+\varepsilon), s, \theta; \alpha \right) \right] + E_{\varepsilon} \left[ U_p \left( P_j(s+\varepsilon), s, \theta; \alpha \right) P_j'(s+\varepsilon) \right] = \theta c'(s) .
\end{align*}
\]

The productive channel can be thought of as the “Beckerian” incentives that represent direct marginal benefits of holding an additional unit of productive HC. In a first-best world of complete information, students would be assortatively assigned to colleges according to their types, and students would then accrue the ideal amount of HC given their assignment (i.e., the amount dictated by the productive channel). The competitive channel is the indirect, “Spencerian” incentive to invest in order to obtain a seat at a better college. Investments undertaken purely due to the competitive channel represent wasteful overinvestment (from the perspective of the students).

To get a sense for the relative magnitudes of these forces as a function of the student’s cost type, Figure 6 displays the ratio of the competitive channel incentives to the total marginal benefit. The horizontal axis describes the quantile rank of the respective cost type in the group-specific type distribution. Whenever the line is above a benchmark of 0.5 the competitive channel is dominant, and whenever it is below, the productive channel is most important. The competitive channel is stronger than the productive channel for all but the lowest-cost agents. By extension, most students’ academic achievement levels would be significantly lower in a complete-information world where the competitive channel is turned off. Of course, this discussion does not necessarily have bearing on optimal policy—for example, if HC spillovers are important in the economy at large, then the social planner may wish to use any means to maximize HC production—but it casts light on the motivations underlying pre-college academic achievement.
7. CONCLUSION

This paper has developed identification and estimation results for a college assignment market based on contest models. By using individual-level data from the B&B survey, rather than focusing only on elite private colleges, we can provide a market-wide analysis of how admissions rules impact incentives and how changes to one’s college placement impact the returns to a college education, conditional on individual characteristics. Our analysis adapts auction-theoretic empirical techniques that allow us to identify the unobserved student characteristics that influence pre-college investment and post-college outcomes. We find that the household income conditional on graduation is determined almost entirely by college quality and the student’s unobservable type.

AA is a prominent feature of the entire college market and plays a significant role in investment, redistribution, and welfare. A strong AA regime such as a proportional quota results in minority students enrolling at better colleges, while a color-blind admissions rule results in minority students predominantly enrolling in the bottom two quintiles of the college quality distribution. Interestingly, the effect on human capital investment incentives is more ambiguous. A color-blind (quota) rule results in the best minority students increasing (reducing) their human capital investments, while high learning cost minority students reduce (increase) them. The effect of AA on graduation rates similarly varies by students’ types, the colleges in which they enroll, and HC investment. We also find that failing to account for changes to HC investment incentives causes significant errors in the estimates of the effect of AA.

Finally, we analyzed the strength of the incentive to accrue HC solely for its productive value relative to the incentive to choose higher levels of HC to compete for access to a better college. We find a surprisingly prominent role for the competitive channel of incentives, which is stronger for all but the best students. Moreover, there is a stark contrast in the strength of these two channels incentives for most students: the competitive channel is roughly twice as strong as the productive channel for half of college-bound students.

There remain many unanswered questions that are the subject of ongoing research. For example, are the income effects of college quality, HC accumulation, and learning costs different for students in STEM fields relative to those in the humanities? Is it possible to design a better AA scheme than the prototypical examples we study? However, the biggest and most obvious question is whether one could use our model to say anything about the long-run impacts of different college admissions systems on the evolution of distributional inequalities over time. Our analysis, which was static by design, can only be the first step in such a research agenda. Providing a serious answer to this question will require considering the inter-generational effects of these programs. We leave these questions for future work which will build on the insights gained here.

References


SUPPLEMENTAL ONLINE APPENDIX

TO ACCOMPANY

Pre-College Human Capital Investment and Affirmative Action: A Structural Policy Analysis of US College Admissions

by Aaron L. Bodoh-Creed and Brent R. Hickman

Appendix A. Model Assumptions

We do not get existence of an equilibrium without some assumptions. Although these assumptions are not used directly in our estimation, we reproduce them from Bodoh-Creed and Hickman [13] for completeness. Assumptions A.1 - A.3 require that the type, college quality, and matching shock distributions admit differentiable probability density functions (PDFs) with a connected support.

Assumption A.1. \( F_j(\theta) \in C^2, j \in \{M,N\} \) and densities \( f_M(\theta) \) and \( f_N(\theta) \) are strictly positive on a common compact support \( [\theta, \bar{\theta}] \) with non-empty interior.

Assumption A.2. \( F_P(p) \in C^2 \) and the prize density \( f_P(p) \) is strictly positive on a compact support \( [\underline{p}, \bar{p}] \) with non-empty interior.

Assumption A.3. The distribution of matching shocks is absolutely continuous with full support: \( \epsilon \sim F_\epsilon(\epsilon), F_\epsilon \in C^2, \) and \( f_\epsilon(\epsilon) > 0, \forall \epsilon \in (\underline{\epsilon}, \bar{\epsilon}) \subseteq \mathbb{R} \).

Assumption A.4 imposes regularity conditions on the cost function. We associate high values of one’s permanent type \( \theta \) with high HC production costs, and low values of \( \theta \) with low costs. Assumption A.5 imposes regularity conditions on the match utility function \( U(p, s, \theta) \). First we require that students benefit from enrolling in a high quality college (\( U \) is increasing in \( p \)), high levels of HC (\( U \) increasing in \( s \)), and we allow for permanent types to play a role as well (\( U \) decreasing in \( \theta \)). Moreover, we require utility to be monotone in \( s \), with convex costs and concave (in \( s \)) match utility.

Assumption A.4. \( C_s(s, \theta) > 0, C_{ss}(s, \theta) \geq 0, \) and \( C_\theta(s, \theta) > 0 \).

Assumption A.5. \( U_p(p, s, \theta) > 0, U_s(p, s, \theta) \geq 0, U_\theta(p, s, \theta) \leq 0 \) and \( U_{ss}(p, s) \leq 0 \).

Assumption A.6 implies that the highest cost students find it optimal to choose the lowest level of HC that qualifies the student to attend college. From a formal perspective, this assumption provides a boundary condition for solution of the model equilibrium.

Assumption A.6. \( \sigma^r_X(\bar{\theta}) = \arg \max_s E_\epsilon [U(P^r_X(s + \epsilon), s, \theta)] - C(s, \bar{\theta}) = s, r \in \{cb, q, ap\} \).

The following assumption ensures existence of a monotone equilibrium:

Assumption A.7. \( C(s, \theta) \) is strictly supermodular in \((s, \theta)\) and \( U(p, s, \theta) \) is supermodular in \((p, s, -\theta)\).

Finally, we require that there is a highest possible HC that any student is willing to choose, which means that the effective action space is compact. Our assumption requires that this upper
bound, denoted $\bar{s}$, will not be chosen by any type of student even if such a choice would result in enrollment into the best possible school.

**Assumption A.8.** There exists $\bar{s}$ such that for all $\theta$ we have:

\[ U(p, \bar{s}, \theta) - C(\bar{s}, \theta) \leq U(p, s, \theta) - C(s, \theta). \]

Finally, we require the following regularity condition on the markup function used in the admission preference system. This assumption is that $\tilde{T}$ is strictly increasing (i.e., the mechanism respects rank ordering within demographic groups) and that the markup function does not increase so steeply that students have an arbitrarily strong incentive to increase $s$.

**Assumption A.9.** There exists $0 < \lambda_1 < \lambda_2 < \infty$ such that for all $t$ we have $\lambda_1 < \tilde{T}'(t) \leq \lambda_2$. 
APPENDIX B. ESTIMATION TECHNICAL DETAILS

B.1. Stage I GMM Estimator. The first hurdle to overcome is to find a computationally tractable way of representing the selected joint distribution of \((P, S)\) conditional on graduation. High-dimensional density estimation is a difficult problem both computationally and because of the rapid decay of optimal statistical convergence rates as the dimensionality of the underlying random variable grows.\(^{36}\) Recent work in the auctions literature by Hubbard, Li, and Paarsch \(^{39}\) has employed parametric copula functions to solve this problem. Sklar’s Theorem states that any absolutely continuous joint distribution can be represented as a composition \(F_{PS}(p, s|\text{grad}) = C_j\left[F_p(p|\text{grad}), G_j(s|\text{grad})|\text{grad}\right], j \in \{M, N\}^{\nu}\), where \(C_j(\cdot, \cdot|\text{grad})\) is a unique copula function. This implies that the rapidly increasing computational cost and data-hungriness of nonparametric estimators come from the complexity of the correlation structure \(C_j\) since the complexity of the marginal distributions does not increase with the dimension of the joint distribution. Hubbard, Li, and Paarsch \(^{39}\) therefore propose a flexible approach to estimating the marginal distributions, while simplifying the copula with parametric assumptions for tractability. This allows the econometrician to maintain the familiar \(\sqrt{T}\) convergence rate when estimating a multi-dimensional joint distribution. We follow this dimension reduction strategy by adopting the Gumbel-Hougaard copula, \(C(r, q; \nu) = \exp \left[-\left((-\log(r))^{\nu} + (-\log(q))^{\nu}\right)^{1/\nu}\right], \nu \geq 1^{37}\) One advantage of the Gumbel-Hougaard copula is that it implies a closed-form expression for the Kendall’s \(\tau\) rank correlation index: \(\tau_{PS}^{j} = \frac{v_j-1}{v_j}, j = M, N.\)

For the selected marginal distributions we propose a flexible, semi-nonparametric approach based on B-splines. Like orthogonal polynomials, B-splines are defined as a linear combination of global basis functions, and B-splines can be made arbitrarily flexible while remaining much better behaved than global polynomials.\(^{38}\) For the selected marginal distributions of school assignment, we begin by specifying knot vectors \(k_p^j = \left\{ p = k_{j1}^p < k_{j2}^p < \cdots < k_{jk_p^p+1}^p = \bar{p} \right\}\) that uniquely define a set of \(k_p^j + 3\) cubic B-spline basis functions \(B_{jk}^p(p) : [\underline{p}, \bar{p}] \rightarrow \mathbb{R}, k = 1, \ldots, k_p^j + 3\), which in turn define our parameterization of the CDFs:

\[
F_{P} \left( p|\text{grad}; \gamma_{jk}^p \right) = \sum_{k=1}^{k_p^j+3} \gamma_{jk}^p B_{jk}^p(p), j \in \{M, N\}.
\]

\(^{36}\)See Silverman \(^{56}\) and Campo, Perrigne, and Vuong \(^{16}\) for a lengthy discussion on this concept.

\(^{37}\)We also experimented with several other copula functions including the Frank copula, \(C(r, q; \nu) = -\frac{1}{\nu} \log \left[ 1 + \frac{\exp(-r^{\nu}) - 1}{\exp(-q^{\nu}) - 1} \right], v \in \mathbb{R}\setminus\{0\}\); the Clayton copula, \(C(r, q; \nu) = \max\{r^{-\nu} + q^{-\nu} - 1; 0\}^{-1/\nu}, v \in [-1, \infty]\setminus\{0\}\); and the Gaussian copula, \(C(r, q; \nu) = \Phi_{\nu} \left[ \Phi^{-1}(r), \Phi^{-1}(q) \right] \) where \(\Phi\) is a standard normal CDF and \(\Phi_{\nu}\) is a bivariate normal CDF with correlation matrix \(R = \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix}, \nu \in [-1,1].\) All produced very similar results, which is consistent with the assumption that our parametric restriction of the copula function provides a robust approximation to the nonparametric correlation structure.

\(^{38}\)For a brief primer on B-splines and their advantages in empirical auctions models, see Hickman, Hubbard, and Paarsch \(^{55}\) and Bodoh-Creed, Boehnke, and Hickman \(^{12}\).
For the marginal distribution of the HC index, \( S \), we have an additional challenge: since its units (and therefore the relevant domain to span) are unknown ex ante, we instead parameterize the selected marginal quantile functions, whose domain is always \([0, 1]\). Let the knot vectors and basis functions for selected HC quantile functions be \( k_j^q = \left\{ 0 = k_{j1}^q < k_{j2}^q < \cdots < k_{jk_j^q}^q = 1 \right\} \) and \( B_j^q(r) : [0, 1] \rightarrow \mathbb{R}, k = 1, \ldots, k_j^q + 3, \) respectively, with B-spline marginal quantile functions parameterized similarly as above by \( Q_{S_j}^1(r|\text{grad}; \gamma_j^q) = \sum_{k=1}^{k_j^q+3} \gamma_{jk}^q B_j^q(r), j \in \{\mathcal{M}, \mathcal{N}\} \).

The parameterized, selected joint distributions are given by

\[
F_{PS}^j(p, s|j, \text{grad}; \gamma_j^p, \gamma_j^q, \nu_j) = C \left[ F_{P_j}^j(p|\text{grad}; \gamma_j^p), Q_{S_j}^1(s|\text{grad}; \gamma_j^q); \nu_j \right], j \in \{\mathcal{M}, \mathcal{N}\}.
\]

Going forward, one important detail to note is that the parameters \( \nu_\mathcal{M} \) and \( \nu_\mathcal{N} \) reflect the empirical correlation structure for the selected joint distribution of \( P \) and \( S \) for college graduates only. Below we will define other notation for separate copula parameters that apply to the selection-corrected joint distribution for all college enrollees (i.e., including dropouts).

In order to complete our GMM estimator, we also need to construct empirical analogs to the joint and marginal distributions of \((p, s)\). In the case of CDFs, we use the standard Kaplan-Meier empirical distribution functions

\[
\hat{F}_{P_j}(p|\text{grad}) = \frac{\sum_{i=1}^{I_j^p} \mathbb{1}(p_i \leq p) \mathbb{1}(i \in j)}{\sum_{i=1}^{I_j} \mathbb{1}(i \in j)}, \quad \text{and}
\]

\[
\hat{F}_{PS}(p, s|j, \text{grad}) = \frac{\sum_{i=1}^{I_j^p} \mathbb{1}(p_i \leq p) \mathbb{1}(s_i \leq s) \mathbb{1}(i \in j)}{\sum_{i=1}^{I_j} \mathbb{1}(i \in j)}, j \in \{\mathcal{M}, \mathcal{N}\}.
\]

For the empirical marginal quantiles of \( S \), we use a new method developed by Hedblom, Hickman, and List [35] for smooth nonparametric quantile estimation. For a random sample \( S_j = \{S_{ji}\}_{i=1}^{I_j} \) of size \( I_j \equiv \sum_{i=1}^{I_j} \mathbb{1}(i \in j) \), this estimator exists as a weighted average of the ordered data \( \{S_j(1) \leq S_j(2) \leq \cdots \leq S_j(I_j)\} \). Specifically, for \( k \in \{1, 2, \ldots, I_j\} \), the \((k/I_j)\)th empirical quantile is estimated as

\[
\hat{Q}_{S_j}(k/I_j) = \sum_{i=1}^{I_j} \Pi_{ik}^j S_j(i),
\]

where the weights \( \Pi_{ik}^j \) are known and mimic the limiting behavior of a re-sampled quantile estimator as the number of simulated samples approaches infinity. Specifically, \( \Pi_{ik}^j \) gives the probability that the \( i \)th order statistic of \( S_j \) will occupy the \( k \)th position in an ordered, randomly generated bootstrap sample from the raw data.\(^{39}\) However, an important constraint for this estimator is that, for a fixed sample size \( I_j \), it can only be evaluated at quantile ranks on the

\(^{39}\)A benefit to this method is that it is differentiable and more efficient than the traditional empirical quantile estimator, \( \inf \left\{ s : \frac{1}{I_j} \sum_{i=1}^{I_j} \mathbb{1}(S_{ji} \leq s) \geq (k/I_j) \right\} \). Intuitively, the empirical quantile estimator, being the nearest-neighbor inverse of the Kaplan-Meier empirical CDF, incorporates cardinal information from only a single datum, with all other data providing only ordinal information. In contrast, \( \hat{Q}_{S_j}(k/I_j) \) as defined above uses both ordinal and cardinal information from the entire sample. See Hedblom, Hickman, and List [35] for additional details.
First, for notational ease let \( r_j \) be evaluated, where \( r_j = \{r_{j0}, r_{j1}, r_{j2}, \ldots, r_{j100} \} \) as a vector of quantile ranks at which the empirical quantile function is to be evaluated, where \( r_{j0} = 1/I_j \) and \( r_{jk} = \text{round}(kI_j/100)/I_j, 1 \leq k \).

Finally, in order to estimate \( \sigma_e \) we need to introduce four additional ancillary parameters. First, for notational ease let \( \pi = [\gamma^p_M, \gamma^p_N, \gamma^q_M, \gamma^q_N, v_M, v_N, \beta^s, \beta^w] \) summarize all primary Stage I parameters except for the shock variance. Note that if \( \pi \) is known, Equation 4 defines the selection-corrected joint distribution of \( (P, S) \), \( F_{PS}(p, s|j; \pi) \), for group \( j \in \{M, N\} \) as a function of the entire parameter vector \( \pi \) along with its two marginal distributions, \( F_p(p; \pi) \) and \( G_j(s; \pi) \). Let \( v^*_j \) denote the best-fit copula parameter of the selection-corrected joint distribution:

\[
v^*_j \equiv \arg\min_{n \in [1, \infty]} \left\{ \int_S \int_P \left[ C \left( F_p(p; \pi), G_j(s; \pi); n \right) - F_{PS} (p, s|j; \pi) \right]^2 dp \right\}.
\]

Implicitly, \( v^*_j \) is a function of \( \beta^w \) since these directly control how well the ranks of \( S(e_i, a_i; \beta^w) \) predict the ranks of \( i \)'s placement in \( P \) space.

Now, let \( F_{PS}^\circ (p, s|j; \pi, \sigma_e) \) denote the joint distribution of \( (P, S) \) implied by the marginal distributions \( F_p(p; \pi) \) and \( G_j(s; \pi) \), but assuming rank-order allocations with respect to NHC are generated by mean-zero, normal shocks with variance \( \sigma_e^2 \). Let \( v^\circ_j (\sigma_e) \) denote the best-fit copula parameter for that joint distribution, in the following sense:

\[
v^\circ_j (\sigma_e) \equiv \arg\min_{n \in [1, \infty]} \left\{ \int_S \int_P \left[ C \left( F_p(p; \pi), G_j(s; \pi); n \right) - F_{PS}^\circ (p, s|j; \pi, \sigma_e) \right]^2 dp \right\}
\]

Subject to:

\[
F_{PS}^\circ (p, s|j; \pi, \sigma_e) = \int_S \int_P f_{PS}^\circ (p|s, j; \pi, \sigma_e) g_j(s; \pi) dp \right\}.
\]

\[
= \int_S \int_P f_\pi \left( P_j^{-1}(p; \pi, \sigma_e) - s; \sigma_e \right) \frac{dP_j^{-1}(p; \pi, \sigma_e)}{dp} g_j(s; \pi) dp \right\}.
\]

\[
H_j (t; \pi, \sigma_e) = \int_{-\infty}^t \int_{-\infty}^{\infty} f_\pi(x; \sigma_e) g_j(x - \varepsilon; \pi, \sigma_e) d\varepsilon dx, j = M, N.
\]

The ancillary parameters \( v_M^*, v_N^*, v_M^\circ(\sigma_e) \), and \( v_N^\circ(\sigma_e) \) are used below to define moment conditions for estimation of \( \beta^w \) and \( \sigma_e \). Intuitively, \( v^*_j \) is the copula parameter that best reflects the correlation structure between \( P \) and \( S \) implied by the data (post-selection-correction), and \( v^\circ_j (\sigma_e) \) is the copula parameter that best reflects the correlation structure between \( P \) and \( S \) generated endogenously by our structural model of a noisy, rank-order college admissions contest given \( \sigma_e \) and the empirical marginal distributions of \( P \) and \( S \). With the above definitions, we can now formalize our Stage I GMM estimator:
\[
\begin{aligned}
\hat{\pi} \\
\hat{\sigma}
\end{aligned}
= \arg \min \left\{ \sum_{i=1}^{I} \left( D_{M_i} \right) \left[ F_{P,M} (p_i | \text{grad}; \gamma^p_{M_i}) - \bar{F}_{P,M} (p_i | \text{grad}) \right]^2 \\
+ (1 - D_{M_i}) \left[ F_{P,\n} (p_i | \text{grad}; \gamma^\n_{\n_i}) - \bar{F}_{P,\n} (p_i | \text{grad}) \right]^2 \\
+ \sum_{k=0}^{100} \left( [Q_{S,M} (r_{M_k}, \text{grad}; \gamma^q_{M_k}) - \bar{Q}_{S,M} (r_{M_k})]^2 + [Q_{S,\n} (r_{\n_k}, \text{grad}; \gamma^q_{\n_k}) - \bar{Q}_{S,\n} (r_{\n_k})]^2 \right) \\
+ \sum_{i=1}^{I} \left( D_{M_i} \right) \left[ F_{P,S} (p_i, s_i | M, \text{grad}; \gamma^p_{M_i}, \gamma^q_{M_i}, v_{M_i}) - \bar{F}_{P,S} (p_i, s_i | M, \text{grad}) \right]^2 \\
+ (1 - D_{M_i}) \left[ F_{P,S} (p_i, s_i | \n, \text{grad}; \gamma^p_{\n_i}, \gamma^q_{\n_i}, v_{\n_i}) - \bar{F}_{P,S} (p_i, s_i | \n, \text{grad}) \right]^2 \\
+ \sum_{i=1}^{I} \left( [I_{M_l} - Z_{M_l} \beta^p]^2 + [I_{\n_l} - Z_{\n_l} \beta^\n]^2 \right) \\
+ \left( \frac{v^p_{M} - 1}{v^p_{M}} - 1 \right)^2 + \left( \frac{v^\n - 1}{v^\n} - 1 \right)^2 \\
+ \left( \frac{v^p_{M} - v^p_{M}(\sigma_{\epsilon}) - 1}{v^p_{M}(\sigma_{\epsilon}) - 1} \right)^2 + \left( \frac{v^\n - v^\n(\sigma_{\epsilon}) - 1}{v^\n(\sigma_{\epsilon}) - 1} \right)^2 \right\}.
\]

**Subject to:**

\[
\begin{aligned}
s_i &= S(e_i, a_i; \beta), \quad i = 1, \ldots, I \\
\frac{\partial S(e_i, a_i; \beta)}{\partial e} &> 0, \quad \frac{\partial S(e_i, a_i; \beta)}{\partial a} > 0 \quad \forall (e_i, a_i; \beta), \quad \text{and} \quad \max_{(e,a) \in \mathbb{R}^2} \{S(e,a; \beta^*)\} = 1 \\
\gamma^p_{j,k-1} &\leq \gamma^p_{j,k}, \quad k = 2, \ldots, K^p, \quad v = p, s, \quad j \in \{M, \n\} \\
\min\{\Gamma_{M1}, \ldots, \Gamma_{ML}, \Gamma_{N1}, \ldots, \Gamma_{NL}\} &\leq \rho(p,s; \beta^p) \leq 1 \quad \forall (p,s) \\
\frac{\partial \rho(p,s; \beta^p)}{\partial p} &> 0, \quad \frac{\partial \rho(p,s; \beta^p)}{\partial s} > 0, \quad \frac{\partial^2 \rho(p,s; \beta^p)}{\partial s^2} \leq 0 \quad \forall (p,s) \\
Z_{il} &= [1, p_l, p^2_l, p^3_l, S^2_{jl}, S^3_{jl}, p_l S^2_{jl}, p_l S^3_{jl}, p_l^2 S^2_{jl}, p_l^2 S^3_{jl}], \quad j \in \{M, \n\}, \quad l = 1, \ldots, L \\
S^k_{jl} &\text{ agrees with equations } [4] [6] [7], \quad k = 1, 2, 3, \quad j \in \{M, \n\}, \quad l = 1, \ldots, L \\
v_{M_i}, v_{\n_i}, v^p_{M_i}, v^\n_{\n_i}(\sigma_{\epsilon}), v^p_{M_i}(\sigma_{\epsilon}), v^\n_{\n_i}(\sigma_{\epsilon}) &\in [1, \infty); \quad \text{and} \quad \sigma_{\epsilon} > 0.
\end{aligned}
\]

The first two summations in the objective function are the moment conditions for the selected marginal distributions of \( P \) and the selected marginal quantile functions of \( S \). The third summation contains moment conditions for the copula of the selected joint distribution of \((P,S)\). The fourth summation contains the selection-corrected regression equations for graduation probabilities. The second to last line of the objective function contains moment conditions for the single-index parameters: they minimize the distance between the selection-corrected rank correlations, \( \tau^p_{PS}, \tau^\n_{PS} \), and their theoretical maxima of one. Recall from the discussion in Section 4.1.1.
that we define the parameters \( \beta^s \) within our model as those which maximize the Kendall’s \( \tau \) rank correlation between the implied single index \( S \) and \( P \), which is why the estimate \( \hat{\beta}^s \) is chosen in this way. The final line of the objective function contains moment conditions for the matching shock variance: it is chosen to minimize the distance between the empirical rank correlations, \( \tau_{PS}^{M_s} \) and \( \tau_{PS}^{N_s} \), and their model-generated analogs, \( \tau_{PS}^{M_0}(\sigma_t) \) and \( \tau_{PS}^{N_0}(\sigma_t) \).

As for the constraints, the last line imposes natural bounds on the shock variance and copula parameters. The two lines above this establish the selection correction procedure for the graduation probability regressions, and the two lines above that impose regularity conditions on the graduation probability parameters. The third constraint from the top imposes monotonicity on the B-spline CDFs and quantile functions, and the first two lines define the \( s_i \)'s as a single index in \((e_i, a_i)\), and they impose monotonicity and a scale normalization to fix its units.

B.2. Stage II Type Distribution GMM Estimator. Amending notation somewhat, for computational convenience we first parameterize the assignment functions and inverse equilibrium strategies for group \( j \in \{ M, N \} \) as flexible B-splines, similarly as we did for other functionals in stage I. B-spline functions are defined as linear combinations of basis functions \( B_{jk}^s(t), k = 1, \ldots, K_j^s + 3 \) and \( B_{jk}^s(s), k = 1, \ldots, K_j^s + 3 \). When we combine these with weights \( \lambda^t_{jk} \in \mathbb{R}^{K_j^t+3} \) and \( \lambda^s_{jk} \in \mathbb{R}^{K_j^s+3} \) our parameterized B-spline functions have the form \( P_j(t; \lambda^t_j) = \sum_{k=1}^{K_j^t+3} \lambda^t_{jk} B_{jk}^t(s) \), and \( \theta_j(s; \lambda^s_j) = \sum_{k=1}^{K_j^s+3} \lambda^s_{jk} B_{jk}^s(s) \). Given a pre-specified grid of points \( \{t_1, \ldots, t_{K_j^t}\} \) spanning \([L, T]\), the assignment function weights are chosen to satisfy

\[
\hat{\lambda}^t_j = \arg \min_{\lambda \in \mathbb{R}^{K_j^t}} \left\{ \sum_{k=1}^{K_j^t} \left( P_j(t_k; \lambda) - F_{\hat{\pi}_j}^{-1} \left[ H_j(t_k; \hat{\pi}, \hat{\sigma}_\epsilon); \hat{\pi} \right] \right)^2 \right\}
\]

Subject to: \( \lambda_k < \lambda_{k+1}, k = 1, \ldots, K_j^t + 2. \)

The assignment mappings are a function of Stage I parameters, which are taken as fixed in Stage II, which is why we express the relevant B-spline weights using hat notation. On the other hand,
the inverse equilibrium strategies solve
\[
\hat{\lambda}_j^s(\alpha) = \arg\min_{\lambda \in \mathbb{R}^{K_j^s}} \sum_{i=1}^{I} D_{wi} \left( \theta_j(s_i; \lambda) - \bar{\theta}_{ji} \right)^2
\]

Subject to:
\[
\dot{\theta}_{ji} c'(s_i) = E_{\varepsilon} \left[ U_p \left( P_j(s_i + \varepsilon; \lambda^j, s_i, \hat{\theta}_{ji}; \alpha, \hat{\pi} \right) P_j(s_i + \varepsilon; \lambda^j) \right] 
\]
\[
+ E_{\varepsilon} \left[ U_s \left( P_j(s_i + \varepsilon; \lambda^j, s_i, \hat{\theta}_{ji}; \alpha, \hat{\pi} \right) P_j(s_i + \varepsilon; \lambda^j) \right] 
\]
\[
\lambda_k > \lambda_{k+1}, \; k = 1, \ldots, K_j^s + 2, \; j \in \{M, N\},
\]
and are not fixed during Stage II. The B-spline strategies are a function of the Stage I parameters as well as the Stage II utility parameters $\alpha$, and therefore must be adjusted each time these are updated during estimator runtime.

The control function, takes the form
\[
\psi(s, D_{M_i}; \alpha, \hat{\pi}, \hat{\sigma}_c, \lambda_{M_j}^s, \lambda_N^s) = \log \left[ D_{M_i} \theta_{M} (s; \lambda_{M_j}^s(\alpha)) + (1 - D_{M_i}) \theta_{N} (s; \lambda_{N_j}^s(\alpha)) \right],
\]
with the extra parameter arguments emphasizing its implicit dependence on Stage I objects. Moving forward we suppress the additional pre-determined parameter arguments for notational simplicity. We can now re-express the matrix of explanatory variables as
\[
X(\alpha) = \begin{bmatrix}
1 \log(p_1) & \log(s_1) & \psi(s_1, D_{M_1}; \alpha, \lambda_{M}^s, \lambda_N^s) \\
1 \log(p_2) & \log(s_2) & \psi(s_2, D_{M_2}; \alpha, \lambda_{M}^s, \lambda_N^s) \\
\vdots & \vdots & \vdots \\
1 \log(p_I) & \log(s_I) & \psi(s_I, D_{M_I}; \alpha, \lambda_{M}^s, \lambda_N^s)
\end{bmatrix},
\]
from which our utility parameter estimator is defined by
\[
\hat{\alpha} = \arg\min_{\alpha \in \mathbb{R}_+ \times [0,1]^3} \left\{ D_w \left( W - X(\alpha) \alpha \right) \right\} \cdot [D_w \left( W - X(\alpha) \alpha \right)]^\top.
\]
In words, Stage II consists of estimating the household income regression, where the third explanatory variable is a nonlinear function of all model parameters with a form that is derived from the equilibrium conditions of our theory of HC investment. Also, note that problems 16 and 18 are solved simultaneously.

The final step of estimation is to recover the type distributions. We recover the distribution of types by convolving the inverse strategy function for each demographic group with the CDFs of the distribution of HC choices for the respective group.

B.3. Practical Issues for Implementation of the Estimator. In the implementation of our GMM estimator of Stage I parameters, we adopt a simplification for computational convenience. First, during solver runtime we normalize $\beta_i^s = 1$ instead of constraining the objective function so that the maximal single index value is one. This simplifies the problem by reducing the number of parameters to choose and constraints to satisfy. After the estimator has run we re-scale the HC
single index so that it’s maximal possible value is one, and we accordingly make adjustments to the graduation probability parameters and HC distributions to reflect the re-scaling.

There are several model tuning parameters that we must specify, among which are knot vectors \(k^p_j, k^q_j, k^t_j, \) and \(k^s_j\). We adopt the convention that knots are to be chosen uniformly in empirical quantile space, as this evenly spreads the statistical power of the data across all basis functions and simplifies the decision to a choice of the number of knots. Specifically, we chose \(K^p_M = K^p_N = K^t_M = K^t_N = K^s_M = K^s_N = 5\) (i.e., knots at the quintiles with 8 total B-spline basis functions) for the selected school quality distributions, assignment functions, and inverse strategies, respectively; and \(K^q_M = K^q_N = 10\) (i.e., knots at the deciles with 13 basis functions) for the selected HC quantile functions. Additional knots did not appreciably improve model fit. When approximating the assignment mappings, we imposed a truncation \(t = \min_i \{s_i\} - 5\hat{\sigma}_e, \quad \bar{t} = \max_i \{s_i\} + 5\hat{\sigma}_e\) on the support of NHC, and we chose a set of evaluation points that included the modes of the B-spline basis functions and the midpoints between the modes.

We calibrated \(\kappa\) from the U.S. Census Bureau’s Current Population Survey (CPS), which is consistent with the structural estimates of Heckman, Humphries, and Veramendi [34]. Recall that the students in our sample were initially surveyed after graduation in 1993, and the household income data we use was collected in the 2003 follow-up survey. To get a benchmark for the fraction of college graduate incomes garnered by dropouts, we computed the ratio of the average household income of 33-year-olds in the 2003 CPS survey with some college to the average household income of 33-year-old college graduates (with no additional post-graduate education). The result is a value of \(\kappa = 0.714\). As a robustness check we also repeated our estimates assuming \(\kappa\) values ranging from 0.5 to 0.9. Only two aspects of our analysis change. First, the choice of \(\kappa\) affects the estimated values of \(\alpha_\theta\) and the distribution of types, \(F_j(\theta; \lambda^p_j), j \in \{M, N\}\). However, the overall role of \(\theta\) in the wealth equation, which we measure as \(\alpha_\theta\) times the standard deviation of \(\log(\theta)\), is stable to changes in \(\kappa\). The second thing that is affected is the productive channel of incentives, which we estimate to be weaker as \(\kappa\) rises. This is intuitive since, as we see in Section 5.5, \(s\) only affects the graduation probability, so anything that makes the utility gap between completing college and dropping out shrink will weaken the direct, productive benefit of HC.

Finally, two sources of sampling weights were used in our empirical implementation, but in order to avoid further complicating notation we left them out of the formal definition above. The first is cross-sectional sampling weights contained in the B&B data. In Stage I these were used for group \(j \in \{M, N\}\) to calculate the empirical analogs of the joint distributions \(\hat{F}_{p,s}(p,s|j,\text{grad})\), the marginal quantile functions \(\hat{Q}_{S_j}(r)\), and the marginal distributions \(\hat{F}_{p_j}(p|\text{grad})\). In simple terms, each of these functions at a point \((p,s)\) is a sample mean of indicator functions evaluated at each datapoint, and we converted them into weighted sample means (in the usual way) using the B&B cross-sectional weights. In Stage II they were also used to weight each component of the household income regression (in the usual way that weighted regressions are constructed) and to calculate the empirical analog of the type distributions \(\hat{F}_j(\theta)\). The second source of sampling
weights came from IPEDS. In Stage I, the graduation probability regression (the fourth summation in the definition of \([\hat{\pi}, \hat{\sigma}_e]^\top\) above) is converted into a weighted regression in the usual way by using the number of individuals in each school-race freshman cohort in 1988 as weights.
Table 11. Household Income Equation Fit

<table>
<thead>
<tr>
<th>Model Component</th>
<th>Fit Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graduation Rate Fit</td>
<td>$R^2 = 0.5396$</td>
</tr>
<tr>
<td>$F_{PS}$ Fit</td>
<td>$R^2 = 0.9335$</td>
</tr>
<tr>
<td>Single-Index Fit</td>
<td>Pooled Kendall’s $\tau = 0.8897$</td>
</tr>
<tr>
<td></td>
<td>$1 - \text{noise/signal-ratio under } \sigma = 0.8178$</td>
</tr>
</tbody>
</table>

Appendix C. Goodness of Fit Metrics

C.1. **Stage I Estimates.** Table [11] contains goodness-of-fit metrics for stage I estimates, which include the graduation rate equation, the single-index equation (that condenses exam scores $E$ and NMGPA $A$ into a single HC index $S$ for predicting college placement), and the fit for the parametric representation of the joint distribution between $P$ and $S$. Since each of these three things was estimated via a least-squares moment-matching routine, in all three cases it is possible to compute an $R^2$ or something like it to gauge model fit. The estimated graduation rate model is able to account for 54% of the variation in observed college-race group graduation rates. Using a parametric Gumbel-Hougaard copula for the correlation structure between $P$ and $S$ and B-spline forms for the marginal CDFs does not appear to unduly restrict the data-generating process. The resulting joint distribution achieves an $R^2$ of 0.93 for predicting the values of the empirical joint CDF of $P$ and $S$.

For the single index equation, there are two alternative measures of fit to consider. First, recall that the single index parameters are chosen so as to maximize the the power of one’s rank in $S$ to predict one’s rank in $P$. The pooled Kendall’s $\tau$ measure in the table—a weighted average of the Kendall’s $\tau$’s for the minority and non-minority joint distributions—demonstrates that the single index parameters achieve a high degree of predictive power for college placement, since the probability of concordance minus the probability of discordance is nearly 90%. An alternative measure of fit for the single HC index model that more closely resembles an $R^2$ measure is 1 minus the noise-signal ratio, or in other words, one minus the standard deviation of the matching shock, $\sigma$, divided by the standard deviation of the single index $S$, roughly 0.82.

C.2. **Stage II Estimates.** Now we turn to the goodness of the fit of the household income equation. Since the household income equation includes a component identified through the structure of our model, $\theta$, we think the goodness of fit can be best assessed by comparing how well our model predicts the household income of students from different quartiles of the college quality distribution 10 years after college. Because of a significant positive skew in the household income distribution, we think the a comparison of the empirical and model generated medians is most revealing. For reference, the median household income income in the data is $78,000, and the median model generated household income is $77,449.

In order to get a more granular look at the fit of our household income model, Table [12] provides both the median income within each quartile as well as the model generated median
Table 12. Household Income Equation Fit

<table>
<thead>
<tr>
<th>College Quality Tier</th>
<th>Quality Upper Bound</th>
<th>Median Household Income</th>
<th>Model Predicted Median Household Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top College Quartile</td>
<td>0.955</td>
<td>$80,000</td>
<td>$80,917</td>
</tr>
<tr>
<td>Second College Quartile</td>
<td>0.660</td>
<td>$77,500</td>
<td>$77,212</td>
</tr>
<tr>
<td>Third College Quartile</td>
<td>0.473</td>
<td>$75,932</td>
<td>$73,129</td>
</tr>
<tr>
<td>Bottom College Quartile</td>
<td>0.312</td>
<td>$65,000</td>
<td>$68,272</td>
</tr>
</tbody>
</table>

Figure 7. Goodness of Fit of the Type Distributions

Now we consider the fit of the inverse strategy functions. We use the group-specific inverse strategy functions to directly impute a value of \( \log(\theta) \) for each HC choice. When this is done exactly from the first order condition, there is no issue of goodness of fit. However, we parameterize the inverse strategy functions with B-splines for computational convenience. When we compare the B-spline and direct imputations of type, we find that the difference is less than or equal to 0.0055, which is roughly 1% of the standard deviation of \( \log(\theta) \). The resulting differences in the type distributions are shown in Figure 7. The fact that the CDFs derived from the B-spline (e.g., Minority Type, B-Spline) lie directly on top of those derived from direct imputation (e.g., Minority Type, Direct) verifies the exceptionally good fit provided by the B-splines.
Appendix D. Determinants of $\theta$

If our exclusion restriction is violated and race directly influences household income (i.e., racial animus has a significant effect), then our analysis will load the effects of this animus onto the estimates values of $\theta$ for the minority students. If this is the case, then it may be possible to predict $\theta$ using minority status in combination with other demographic traits, which we reference in Section 5.6 on robustness checks of our empirical method. Of course, understanding the determinants of $\theta$ is of independent interest, but a broad investigation of this question is outside of the scope of this paper. One method for assessing the drivers of $\theta$ is to regress $\theta$ on other plausibly exogenous demographic traits contained in the B&B data.

For the duration of this section we work with $\ln(\theta)$ as the outcome variable of interest. Given our interpretation of $\theta$ as student characteristics that are influenced by forces internal to the individual (e.g., innate ability) and external conditions (e.g., primary/secondary school quality, parental education and financial resources), the most plausibly exogenous explanatory variables describe properties of the students’ parents or family. The variables we consider are described in Table 13. The data set includes the state of residence of the student at enrollment, and this has been condensed into dummy variables for the northeast, southeast, midwest, southwest, mountain west, and west coast regions. We consider only students that have nonmissing values for all of these variables.

The variable risk index is the sum of seven binary risk factors for failure to complete college (e.g., GED recipient). Mother college graduate and father college graduate are dummies that indicate whether the respective parent graduated from college. Parents married is a dummy variable indicating whether both parents are married (but perhaps not to each other). Parental cash savings refers to the amount of cash and savings reported on financial aid forms (submitted at college enrollment).

We include the variables in Table 13 as well as a dummy for minority status and a full set interactions with the minority status dummy in our regressions. Our regression results are contained in Table 14. The point estimates on the minority variables are nontrivial, but they are not precisely estimated. To provide a reasonably powerful test of the significance of the minority
variable, we ran an F test reveals that the minority dummy and the minority interaction terms are not jointly significant at the 10% confidence level. In addition, the coefficient on the minority dummy is either of comparable magnitude or smaller than several other demographic variables such as the gender of the student, the parent’s marital status, whether the father has graduated from college, and whether the parents have liquid savings.

It is interesting to note that in Section 5.5 we estimate a substantial race gap in learning costs, whereas through the current exercise we can conclude that the race gap in costs disappears when home environment controls are used. Our analysis implies that the black-white household income gap in our data is due to the role of learning-costs in determining household income combined with a black-white gap in the distribution of learning-costs. If racial animus played a significant role in the household income gap (contrary to Assumption 4.4 and Fryer et al. [30]), then one might expect to find that the minority dummy is a significant predictor of learning-costs. However, our analysis suggests that inter-group differences in learning costs are not driven by minority status once one controls for other demographic traits. In short, we do not find evidence that our exclusion restriction is violated and causing the effect of racial animus to be loaded onto our estimates of $\theta$.  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minority Dummy:</td>
<td>0.0894</td>
<td>0.419</td>
</tr>
<tr>
<td>Gender Dummy:</td>
<td>-0.0855***</td>
<td>0.0240</td>
</tr>
<tr>
<td>Gender Dummy x Minority Dummy:</td>
<td>-0.0224</td>
<td>0.0787</td>
</tr>
<tr>
<td>Risk Index&gt;0:</td>
<td>0.113***</td>
<td>0.0273</td>
</tr>
<tr>
<td>Risk Index x Minority Dummy:</td>
<td>0.0792</td>
<td>0.829</td>
</tr>
<tr>
<td>Parent Adj. Gross Income:</td>
<td>5.42*10^{-4}</td>
<td>5.62*10^{-4}</td>
</tr>
<tr>
<td>Parent Adj. Gross Income x Minority Dummy:</td>
<td>1.39*10^{-4}</td>
<td>2.20*10^{-3}</td>
</tr>
<tr>
<td>Mother College Grad.:</td>
<td>7.75*10^{-3}</td>
<td>0.0314</td>
</tr>
<tr>
<td>Mother College Grad. x Minority Dummy:</td>
<td>-7.60*10^{-3}</td>
<td>0.102</td>
</tr>
<tr>
<td>Father College Grad.:</td>
<td>-0.152***</td>
<td>0.028</td>
</tr>
<tr>
<td>Father College Grad. x Minority Dummy:</td>
<td>-0.0407</td>
<td>0.0841</td>
</tr>
<tr>
<td>Parents Married:</td>
<td>-0.0803***</td>
<td>0.0341</td>
</tr>
<tr>
<td>Parents Married x Minority Dummy:</td>
<td>0.144</td>
<td>0.0932</td>
</tr>
<tr>
<td># Members of Family:</td>
<td>2.65*10^{-3}</td>
<td>0.0104</td>
</tr>
<tr>
<td># Members of Family x Minority Dummy:</td>
<td>-0.0258</td>
<td>0.0824</td>
</tr>
<tr>
<td>Parental Cash Savings&gt;0:</td>
<td>-0.0894***</td>
<td>0.0382</td>
</tr>
<tr>
<td>Parental Cash Savings&gt;0 x Minority Dummy:</td>
<td>-0.0144</td>
<td>0.0816</td>
</tr>
<tr>
<td>Region Dummies</td>
<td>Included</td>
<td></td>
</tr>
<tr>
<td>Region Dummies x Minority Dummies</td>
<td>Included</td>
<td></td>
</tr>
<tr>
<td>N:</td>
<td>1196</td>
<td></td>
</tr>
<tr>
<td>$R^2$:</td>
<td>0.451</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.
Appendix E. Additional Tables and Figures

E.1. Point Estimates: HC Single Index Function. Figure 8 illustrates our single index equation. Each line depicts the effect of academic record \( a \) on the HC index while holding exam score \( e \) fixed at one of it’s quartiles. The lesser importance of \( e \) relative to \( a \) is reflected in the fact that the difference between the 75\textsuperscript{th} and 25\textsuperscript{th} percentile lines is less than 0.1, while the difference between the 75\textsuperscript{th} and 25\textsuperscript{th} quintiles on the line describing \( S(e_{\text{median}}, a) \) is close to 0.2. The upward curve of the lines is a result of the convexity of the single index with respect to the student’s GPA. We include 95% confidence intervals on the line describing \( S(e_{\text{median}}, a) \) at each decile of the distribution of \( a \). The distance between the line representing the 75\textsuperscript{th} percentile of \( e \) and the maxima of \( e \) is due to the convexity of the single index in that variable. The large distance between the line representing the 25\textsuperscript{th} percentile of \( e \) and the minima of \( e \) is due to a long tail of low value of \( e \) within the data.
Table 15. Counterfactual Minority Household Income by Achievement Quintile

<table>
<thead>
<tr>
<th>Learning-Cost Type Tier</th>
<th>Proportional Quota</th>
<th>Proportional Quota, Partial Eqm</th>
<th>Color-Blind</th>
<th>Color-Blind, Partial Eqm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Achiever Quintile</td>
<td>$81,560</td>
<td>$81,563</td>
<td>$80,336</td>
<td>$79,896</td>
</tr>
<tr>
<td>Second Achiever Quintile</td>
<td>$77,223</td>
<td>$77,212</td>
<td>$74,714</td>
<td>$74,351</td>
</tr>
<tr>
<td>Third Achiever Quintile</td>
<td>$73,350</td>
<td>$73,378</td>
<td>$69,856</td>
<td>$70,258</td>
</tr>
<tr>
<td>Fourth Achiever Quintile</td>
<td>$69,143</td>
<td>$69,143</td>
<td>$65,264</td>
<td>$66,154</td>
</tr>
<tr>
<td>Bottom Achiever Quintile</td>
<td>$62,673</td>
<td>$62,654</td>
<td>$60,010</td>
<td>$61,209</td>
</tr>
</tbody>
</table>

Figure 10. Noisy Human Capital Markup Function Equivalent to a Proportional Quota, $\tilde{T}$

E.2. **Point Estimates: Graduation Probability Function.** Figure 9 illustrates the relative impact of $p$ and $s$ on graduation probabilities. Each line of the plot depicts the effect of HC on graduation probability holding that college quality fixed at one of its quartiles. We have also plotted 95% confidence bounds on graduation rates at a college of median quality at the deciles of the distribution of HC. The convexity of $\rho(p, s)$ is evident from the increasing difference between the lines as the college quality improves, meaning that college quality matters most above the 75th percentile of college quality. The complementarity between $p$ and $s$ results in an increasing spread between the lines as $s$ increases, meaning that college quality is more important for higher achieving students.

E.3. **Noisy Human Capital Markup Function Equivalent to a Proportional Quota, $\tilde{T}$**. Figure 10 describes the estimated markup function, $\tilde{T}(t)$, that is equivalent to a proportional quota. In other words, an admissions preference system using the markup function would yield 20% of the minority students in each quintile of the college quality spectrum. The horizontal axes of both panels display quantile ranks of NHC for nonminority students. The markup of minority
Table 16. Estimates of the Wealth Equation with Minority Household Income Penalty

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p$</td>
<td>0.1300</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>$1.122 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_\theta$</td>
<td>0.0426</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>$80,769$</td>
</tr>
</tbody>
</table>

student NHC required to mimic the effects of a proportional quota is, as intuition would suggest, larger than in the status quo admissions admissions preference system (see Figure 2).

The left panel describes the shape of $\tilde{T}$. The vertical axis displays the quantile rank of subsidized NHC within the non-minority NHC distribution. If a minority student has an NHC at the quantile rank marked on the horizontal axis, the student gets the same college assignment as a nonminority student with an NHC at the quantile rank denoted on the vertical axis. For example, a minority student with an NHC equal to the median of the nonminority population gets the same college assignment as a nonminority student at the 73rd percentile of the nonminority population. The dashed line denotes the 45° line for reference. The right pane of Figure 2 describes the markup function in terms of school quality. The vertical axis denotes the gap in the quantile rank of college quality between a minority student and a nonminority student at each NHC quantile. For example, the plot shows that if two students from different groups both have an NHC value equal to the median of the nonminority population, then the minority student is assigned to a school whose quantile rank is 0.23 higher in the school quality distribution.

E.4. Robustness Check: Minority Household Income Penalty. In this section we present both the estimation results and the associated counterfactuals with imposing an immediate 11% household income penalty for minority students as found in Fryer et al. [30]. Our log utility structure means that such a penalty will primarily have an effect of shifting the levels of the utility of minority students. To the extent that this has an effect on the learning-cost estimates, it can only be through the change in the income equation parameters $\alpha_p$ and $\alpha_s$, which as shown in Table 16 is rather small. Therefore, the learning costs attributed to each member of our dataset are essentially identical to those in the benchmark analysis.

The enrollment statistics are identical to the third decimal point, so we omit this table. We include the graduation probability statistics due to small differences that we observe for the proportional system. The household income counterfactuals for the minority students differ from the baseline model primarily due to the mechanical imposition of the 11% penalty. These differences suggest our model is robust to the inclusion of a fixed household income penalty.
Table 17. Counterfactual Minority Graduation Probability with Minority Household Income Penalty

<table>
<thead>
<tr>
<th>Learning-Cost Type Tier</th>
<th>Status Quo</th>
<th>Proportional Quota</th>
<th>Color-Blind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Achiever Quintile</td>
<td>0.562</td>
<td>0.558</td>
<td>0.577</td>
</tr>
<tr>
<td>Second Achiever Quintile</td>
<td>0.389</td>
<td>0.415</td>
<td>0.398</td>
</tr>
<tr>
<td>Third Achiever Quintile</td>
<td>0.299</td>
<td>0.329</td>
<td>0.300</td>
</tr>
<tr>
<td>Fourth Achiever Quintile</td>
<td>0.236</td>
<td>0.265</td>
<td>0.228</td>
</tr>
<tr>
<td>Bottom Achiever Quintile</td>
<td>0.159</td>
<td>0.183</td>
<td>0.150</td>
</tr>
</tbody>
</table>

...BY COLLEGE QUALITY QUINTILE

<table>
<thead>
<tr>
<th>College Quality Tier</th>
<th>Status Quo</th>
<th>Proportional Quota</th>
<th>Color-Blind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top College Quintile</td>
<td>0.612</td>
<td>0.557</td>
<td>0.658</td>
</tr>
<tr>
<td>Second College Quintile</td>
<td>0.439</td>
<td>0.415</td>
<td>0.480</td>
</tr>
<tr>
<td>Third College Quintile</td>
<td>0.353</td>
<td>0.328</td>
<td>0.390</td>
</tr>
<tr>
<td>Fourth College Quintile</td>
<td>0.270</td>
<td>0.267</td>
<td>0.305</td>
</tr>
<tr>
<td>Bottom College Quintile</td>
<td>0.178</td>
<td>0.184</td>
<td>0.193</td>
</tr>
</tbody>
</table>

Table 18. Counterfactual Minority Household Income by Achievement Quintile with Minority Household Income Penalty

<table>
<thead>
<tr>
<th>Learning Cost Type Tier</th>
<th>Status Quo</th>
<th>Proportional Quota</th>
<th>Color-Blind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Achiever Quintile</td>
<td>$72,411</td>
<td>$72,980</td>
<td>$71,945</td>
</tr>
<tr>
<td>Second Achiever Quintile</td>
<td>$68,301</td>
<td>$69,632</td>
<td>$67,430</td>
</tr>
<tr>
<td>Third Achiever Quintile</td>
<td>$64,679</td>
<td>$66,490</td>
<td>$63,403</td>
</tr>
<tr>
<td>Fourth Achiever Quintile</td>
<td>$61,396</td>
<td>$62,925</td>
<td>$59,558</td>
</tr>
<tr>
<td>Bottom Achiever Quintile</td>
<td>$56,601</td>
<td>$57,562</td>
<td>$55,195</td>
</tr>
</tbody>
</table>

Appendix F. Solving the Model Numerically

Counterfactuals that compare different AA regimes necessarily require solving the model to determine the student behavior (i.e., human capital investments), which in turn determines college enrollments and final outcomes. There are two endogenous objects that need to be computed for each demographic group: the student strategies and the assignment functions that map NHC to college assignment.

Although we prove an equilibrium exists (Theorem 3.2), we do not have a result that the equilibrium is unique. We used multiple restarts to test for the existence of multiple equilibria, but we never found multiplicity. This is perhaps not terribly surprising since Bodoh-Creed and Hickman [13] prove that the equilibrium of an analogous color-blind or quota model without a matching shock has a unique equilibrium since the FOC defines an ODE that has a unique solution. The difficulty of proving uniqueness for models with a matching shock is that the ODE...
defined by the FOC is nonlocal. To the best of our knowledge, the study of nonlocal ODEs and proofs of properties like uniqueness are limited to special cases that our model does not fall under. Please see Bodoh-Creed and Hickman [13] for details.

We numerically approximate all four objects: two group-specific strategies and two group-specific inverse assignment functions. B-splines were used for the numerical approximations because these functions allow for accurate approximations of both a function and its derivatives using relatively few parameters. We chose to approximate the inverse assignment function because while the range of these functions (the colleges) are known ex ante, the domain (the noisy human capitals) is endogenous. We used six knots in our approximation, but found that using up to 20 knots had a negligible effect on our results. We insist that the strategies be consistent with the first order conditions for the student’s decision problem in the AA regime of interest (Equation 2). The assignment functions (i.e., the inverse of the inverse assignment function approximated by our B-splines) are required to be consistent with the human capital choices of the students.

We solved the model using an optimization algorithm that tries to minimize the inconsistencies between the approximated objects and the theoretical analogs described above. The optimization algorithm adjusts the variables describing all four of the numerical approximations simultaneously. Our first step within each iteration is to compute the induced inverse assignment function for each group, which is simply the inverse of the assignment function generated by the approximate student strategies and the contest structure generated by the form of AA we are studying. We then use an $L_2$ penalty function for the distance between the B-spline fit of the inverse assignment function and the induced inverse assignment function, which is computed at 50 points evenly spaced across the support of the distribution of college qualities. The measure applied in the $L_2$ norm is the estimated CDF of the distribution of college qualities available to that demographic group.

We now turn to our metric of the inconsistency between the approximate and exact equilibrium strategies. Our third step within each iteration is to compute the assignment function implied by our B-spline of the inverse assignment function. The fourth step of our algorithm is to calculate the first order condition given the assignment function for each group implied by the associated B-spline of the inverse assignment function and the B-spline of the strategy for that group. Our error function was an $L_2$ penalty function enforced at 50 evenly gridded values over the support of $\theta$. The measure used in the $L_2$-norm is the type distribution of the respective group.

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41 By nonlocal, we mean that the derivative of the solution at a given point depends on the value of the solution at many other points. This occurs in our setting because of the expectations in the FOC. The derivative of a local ODE, the most familiar form, only depends on the value of the function at that point.

42 We solved for the status-quo system as a quality assurance check on our algorithms. Because quota and admissions preference schemes are outcome equivalent (Bodoh-Creed and Hickman [13]), we treated the status quo admissions preference scheme as quota wherein each group competed separately for the distribution of seats allocated to members of that group in the status quo.

43 In the color-blind and proportional quota counterfactuals, this is the total measure of college qualities. In the status-quo model, the distribution of school available to students in a given demographic group is equal to the distribution of college qualities in which those students enroll.
The complete objective function for the optimization problem is the sum of the penalty functions for the inverse assignment mapping and the first order condition, which we chose for simplicity and the fact that the weighting did not seem to significantly affect the optimization results. If an optimal value of 0 is found, then the approximated inverse assignment functions and the approximate strategies are consistent with the first order conditions of the decision problem. In addition, an optimal value of 0 implies that the approximation strategies and inverse assignment functions are also consistent with each other as required by equilibrium.

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