PRE-COLLEGE HUMAN CAPITAL INVESTMENT AND AFFIRMATIVE ACTION: A STRUCTURAL POLICY ANALYSIS OF US COLLEGE ADMISSIONS*

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Abstract: We study a structural model of college admissions framed as a contest between a continuum of students for enrollment in a continuum of colleges where the contest outcome is decided by the students’ choice of human capital (HC). Students have private information about their learning costs, and colleges have heterogeneous, observable qualities. Our econometric model is inspired by methods from the empirical auctions literature and allows us to separately identify the roles of school quality, HC, and unobserved student characteristics on post-college household income. Conditional on graduating, we estimate that college quality is the most important factor in determining income, while unobserved characteristics play a secondary but significant role as well. Pre-college human capital significantly drives college placement and graduation probability, but plays only a negligible role in post-college income. We use our estimates to conduct counterfactual experiments comparing different college admissions rules including color-blind admissions, a proportional quota for minority students, and means-tested affirmative action (AA). An AA ban would result in a large migration of minority students out of the best schools and into the lowest quality schools with a corresponding reduction in household income and mean graduation rates. However, the signs and magnitudes of changes to HC investment and individual graduation rate depend on the demographics and learning cost type of the particular student in question. We also find evidence that a means-tested AA plan is a poor instrument for increasing racial diversity on college campuses. Finally, our estimates imply that the competitive incentive to accrue HC is stronger than the productive incentive for all but the top 14% achieving students.

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JEL subject classification: D44, C72, I20, I28, L53.

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1. INTRODUCTION

There are many economically salient features of the competition between high school students for seats at colleges and universities in the United States. An ideal model of this market would include heterogeneous college qualities, heterogeneous unobserved student abilities, and endogenous human capital choices.\(^1\) This already complex picture is complicated further by the roles of asymmetric distributions of student characteristics by race and affirmative action (AA) admissions policies that take into account student demographics. While prior studies often include some of these features, ours is the first econometric analysis to account for all of them in a single model. Our structural approach allows us to endogenize responses to counterfactual admissions rules changes as we explore their impact on human capital investment, college placement, and post-college outcomes.

For many Americans, the competition to be admitted to a high quality college is one of the highest stakes contests of their lives. The reason is simple: there is a high degree of college heterogeneity in terms of both educational inputs and the outcomes realized by students. For example, the interquartile range of spending per student is $5,931 to $9,551 for students in our data set, whom enroll in college in 1988. Turning to college outcomes, the interquartile range in graduation rates is 38% to 63%, and the interquartile range of household income 10 years after graduation is $54,000 to $113,000.\(^2\) It is well known that minority students systematically enroll in lower quality colleges and obtain worse educational outcomes. The difficulty is separating the causes of these worse outcomes, which are likely to be a function of each individual student’s privately-known characteristics and college preparation as well as the quality of the college in which he or she enrolls.

Affirmative action further complicates this market since it results in applicant demographics playing an important role in the admissions process. Affirmative action has a history in the United States that stretches back to the Kennedy Administration in the 1960s, and is now a pervasive, though controversial, fixture of American higher education.\(^3\) The US Supreme Court has ruled on legal challenges to racial considerations in college admissions in four separate cases, *Regents of the University of California v. Bakke* (1978)[1], *Gratz v. Bollinger* (2003)[2], *Grutter v. Bollinger* (2003)[3], and *Fisher v. Texas* (2016)[4]. Affirmative action is in general motivated by racial disparities in college placement and a desire to achieve greater diversity within student bodies at top universities. For example, in 1988 16% of all new college freshmen were underrepresented minorities (Black, Hispanic, or Native American) who accounted for only 10% of new enrollees within the top fifth of US colleges, despite substantial considerations for race in

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\(^1\)In the United States, the terms *college* and *university* are used synonymously to refer to bachelor’s degree granting institutions. We apologize if this differs from the reader’s familiar usage, but we maintain the American convention of using the two terms interchangeably for our empirical application.

\(^2\)The graduation rate data comes from Integrated Postsecondary Education Data System, while the income data comes from the Baccalaureate and Beyond (B&B). Income quartiles have been rounded to the nearest thousand dollars to protect B&B respondents’ anonymity.

\(^3\)Throughout this paper, when we use the term AA we refer to the practice of granting preferential treatment in college admissions on the basis of race, holding academic merit (grades and exam scores) fixed.
the admissions process. In a world of competitive admissions, this disparity is in turn driven by gaps in pre-college academic achievement: in that same year median minority GPAs and SAT scores were slightly below the 25th percentile for Whites and Asians. It is widely believed that racial achievement gaps are rooted in various stark racial disparities of access to resources that play a critical role in childhood development. Among these are income, healthcare, and high-quality schools, though the precise mechanisms through which these factors act are not yet fully understood.

Building on the theoretical framework of Bodoh-Creed and Hickman [13], we estimate a model wherein a continuum of college applicants with differing unobserved learning costs compete for admission to college through the accrual of productive human capital (HC). Colleges each occupy a different point on the quality spectrum and use distinct, endogenous admissions cutoffs for different demographic groups. One important concern that the previous literature has ignored is the extent to which AA alters students’ incentives to invest in pre-college HC. Within our model, these incentives are two-fold. First, HC is a productive asset because it prepares students to fully benefit from higher education. We refer to this as the productive channel of incentives, and our model controls for the fact that if HC and school quality are complementary inputs in the match utility function, then altering college assignments will change its magnitude. This channel is present in the complete-information assortative matching model of Becker [10]. Second, since colleges wish to admit the best students possible, they rank students using the available information, namely academic achievement. Therefore, HC investment (as measured by grades and exam scores) plays a second role as a rank-order index that allows better students to out-compete their peers and enroll into better schools. These indirect returns create a competitive channel of incentives that drives students to use achievement as a means of establishing their position above less-accomplished competitors, and it resembles the signaling phenomenon analyzed in Spence’s [57] seminal model. The competitive channel creates a strategic interaction between one’s own actions and the actions of others: if rivals never study mathematics and science, one might study less, consume more leisure time, and still place into a top college, and vice versa. In other words, a full understanding of a student’s HC choice cannot focus only on her individual production technology, but rather, it must also take into account strategic pressures she faces from the rest of the market.

The first goal of our analysis is to tease apart the effects of college quality, human capital, and unobservable characteristics on post-college income. Our structural approach is crucial for any counterfactual analysis of different AA schemes, since changes to admissions rules have complex effects on both the productive and competitive channels of incentives, and in turn, on choices. For example, would a stronger AA scheme increase minority HC levels due to complementarity between school quality and HC? Or would the prospect of easier admissions reduce the incentive to accrue HC? And, are these effects different for students with distinct underlying types?

Our model allows us to apply techniques developed by Guerre, Perrigne, and Vuong [34] (GPV) for first-price auctions in a novel way to identify student learning costs from the choice each student made given the strategic incentives she faced. Intuitively, our theory model provides
an inverse mapping from students’ observable academic achievement into the underlying cost types which rationalize their academic effort choices as best responses to prevailing competition. For example, under the model we infer from observed choices to accrue more human capital (i.e., achieve higher GPA/exam scores) that the student has a lower learning cost.

Unlike in standard auction settings, the value of earning a seat at a school in the college admissions contest is a function of the exogenous quality of the school, the endogenous human capital choice of the student, and the privately-known learning cost of the student. Therefore, we need to simultaneously estimate the determinants of the value of each college seat while using the GPV approach to estimate the learning cost of each student. In effect, the inverse mapping from human capital choices into privately-known learning costs derived from the GPV method is embedded into a wage regression as a non-linear control function that separates the effect of invested HC from one’s permanent type on post-college income.

Our analysis uses individual-level data from the Baccalaureate and Beyond (B&B) survey conducted by the U.S. Department of Education on student demography, academic achievement, and post-college household income. These data allows us to provide a market-wide analysis, whereas previous studies often focused on elite private colleges where application data are most readily available. There are two main sources of variation in the attributes of students on a given college campus that feed into our identification strategy. This data allows us to provide a market-wide analysis, whereas previous studies of AA often focused on elite private colleges where applications data is most readily available. First, although the market is highly assortative in that better colleges tend to attract more accomplished students, it is not perfectly so, and each university’s student body exhibits a non-degenerate distribution of human capital within race groups. This feature of the data implies that college quality and achievement are imperfectly correlated, which allows us to separate the influence of these two factors on post-college earnings. The second source of variation is AA practices, which create different investment incentives for students in different demographic groups. This means that the mapping from a student’s private information into her choice of HC will depend on her demographic group affiliation. This fact implies that two students from different demographic groups on the same campus with the same HC will have different (privately-known) learning costs. This second source of variation causes HC choices and unobserved characteristics to be imperfectly correlated, which allows us to identify their distinct effects on post-college outcomes. We combine this model structure with individual-level data on achievement, college of attendance, and post-graduation income to identify the wage regression function.

We also use the B&B data to identify the joint distributions of school quality and achievement for new college enrollees by race, which requires a novel sample-selection correction procedure. To correct for sample selection—college dropouts do not appear in B&B—we also use college-level data on school quality and race-specific graduation rates, provided by U.S. News & World Report and the US Department of Education’s Integrated Postsecondary Education Data System (IPEDS). Together, these data allow us to identify the form of status quo AA, the wage regression function, and unobserved student types.
We find a non-trivial role for AA to shape HC incentives and enrollment on all segments of the college quality spectrum above the 10% lowest-quality colleges. We find that college quality and pre-college HC both have a significant effect on the probability that a student graduates. On the other hand, HC has very little influence on household income 10 years following graduation, where college quality and unobserved student characteristics play dominant roles. We also find that the distribution of learning costs for minority students stochastically dominates the distribution of nonminority students. In other words, our empirical evidence suggests minority students in general have higher learning costs. We discuss the connection between this result and a large body of empirical work documenting substantial racial disparity in childhood developmental and educational resources.

We then conduct a counterfactual analysis to study the effects of different AA schemes on enrollment of minority and nonminority students, graduation rates, and household income. Our first result is that admissions would be quite different in an alternative color-blind world. Under the status quo AA regime we estimate from the data, minorities are under-represented at the best schools and heavily over-represented at the worst ones, with 28% enrolling in each of the lowest two quintiles. However, a color-blind college admissions scheme would result in a large shift of minority students into lower ranked schools, with 42% in the bottom quintile and 19% in the second to last quintile. Under a proportional quota scheme, the opposite occurs: minorities enroll in higher quality programs with (mechanically) 20% of them in each college quality quintile.

In terms of the HC accumulation decisions of minority students, color-blind and proportional systems have roughly symmetric, but opposite, effects. In a color-blind scheme, minority students with high learning costs reduce their HC choices relative to the status quo. The primary driver of the decrease in investment is that shifting to a color-blind system increases the number of seats at low-quality colleges for high learning cost minority students. This means such a student would have to exert a great deal of effort to earn a better college seat since she would have to out-compete all of the rivals that, in equilibrium, are assigned to the numerous seats at low quality colleges. Symmetrically, minority students with low learning costs find themselves facing a restricted supply of high-quality seats in a color-blind system. This means that the quality of the available college seat improves rapidly as she out-competes her rivals, which increases her incentive to invest in HC. For nonminority students, the effects of shifting from the status quo to color-blind are the opposite, but of smaller and generally inconsequential magnitude.

The changes in HC investment from a shift toward a proportional quota from the status quo are larger in magnitude and reversed in sign. We find that the 90% of minority students with the highest learning costs increase their HC invesments, while the 10% of minority students with the lowest learning costs reduce their HC investment. The reasoning is essentially symmetric.

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4We do not believe that there is much if any reason to think that these learning costs are “innate” in a biological sense. For example, the higher socioeconomic strain that lower-income (on average) minority families tend to be under can make the actual and opportunity cost of early childhood monetary investments more difficult than for the typical nonminority family. This is compounded by institutional features such as the fact that minority students tend to be enrolled in primary and secondary schools with a high level of poverty and racial segregation and relatively low achievement.
to the color-blind case—high learning cost minority students now face a more favorable (i.e., lower) ratio of college seats to competitors, while low learning cost minority students confront the opposite change.

We find that the effects of the AA programs on enrollment translate into intuitive effects on graduation rates and household incomes 10 years after graduation. More generous AA schemes generally lead to higher average graduation rates for minorities with little effect on non-minorities. Since minority students are assigned to worse colleges under a color-blind admissions system, the average household income of minority students drops by $818, relative to the status quo, while nonminority students have $158 more annual income on average. Under a proportional system, minority students increase their household income 10 years after graduation by an average of $1,936 due to their better college placements, while nonminority student income drops by an average of $363.

Due to the legally contentious nature of AA programs, particularly at public universities, proposals have emerged for means-tested forms of AA that might have the benefit of encouraging racial diversity on college campuses in a less legally problematic way. To test this conjecture, we analyze several different proportional quotas that reserve a fixed fraction of the seats at each university for the most socioeconomically disadvantaged applicants, which we measure in terms of the Expected Family Contribution (EFC) to college expenses, a measure based on wealth and income used by the US Department of Education. We find that under an AA scheme based purely on socioeconomic status, the outcome is very close to that which obtains under a color-blind admissions system. The reason for this negative result is that poor and affluent students are more similar in their learning cost distributions than are minority and nonminority students.5

We close our empirical analysis by studying the relative magnitudes of the productive and competitive channels of incentives. To the best of our knowledge, ours is the first paper to separate the marginal benefit of pre-college HC accumulation into a competitive channel and a productive channel. We find that the competitive channel is stronger than the productive channel for all but the highest-achieving 15% of students overall. The relative strength of the competitive channel increases for students with higher learning costs, being roughly three times as powerful as the productive channel for the middle 50% of the learning cost distribution.

The remainder of this paper has the following structure: we first briefly summarize the previous literature on AA and discuss its relation to the current model. In Section 2, we describe the US college market structure and the data that will be used. Section 3 outlines the theoretical model on which the econometric exercise is based, and in Section 4 we formally outline our semi-parametric identification results based on methods pioneered by Guerre, et. al. [34] in the empirical auctions literature. In Section 4.2 we define a two-stage estimator for the structural model that falls within the class of Generalized Method of Moments estimators. In Section 5,

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5This result would seem to corroborate a relatively new body of evidence which suggests that educational differences by race in the US cannot be simply decomposed as differences by wealth and income, due in part to persistent racial segregation in housing and schooling (e.g., see Reardon [51]).
we discuss the results of estimation and in 6 we present the counterfactual exercise. Section 7 concludes and briefly describes directions for future research.

1.1. Related Literature. Bowen and Bok [14] and Kane [42] were among the first to study how actual AA policies change college admissions. They found that it plays a substantial role in allocating high-quality college seats to minorities. Chung and Espenshade [19] and Chung, Espenshade, and Walling [21] undertake a similar study using matched student-college applications data from a small set of elite private universities to similar results. Several other papers have taken a structural approach to studying the impact of AA. Epple, Romano, and Sieg [26] model colleges setting tuition and admissions standards for different races. They calibrate the model to the US market and find that the data are broadly consistent with diversity preferences on the part of colleges. Two additional papers structurally estimate the college admissions process in order to control for changes in application behavior from counterfactual policy shifts. Arcidiacono [6] models applications, admissions, enrollment, major choice, and entrance into the labor market, with the aim of identifying counterfactuals when AA is eliminated from college admissions. Howell [39] performs a similar exercise with a more recent data set, but focusing solely on the admissions process. Both papers find that AA plays a significant role in shaping black educational outcomes, especially among the most selective institutions.

Another vein of empirical literature on AA focuses on mismatch, or the idea that AA may cause black students to be placed higher, but then graduate with lower probability, or to self-select into less lucrative majors. Some empirical studies have found evidence of mismatch (e.g., Loury and Garman [46], Arcidiacono, Aucejo, Fang, and Spenner [50], and Arcidiacono, Aucejo, and Hotz [7]), while other empirical work has found evidence that the mismatch problem is small and likely outweighed by other benefits of higher-quality placement (e.g., Long [45], Rothstein and Yoon [54], and Chambers, Clydesdale, Kidder, and Lempert [18]). Other evidence suggests, to the contrary, that all students generally benefit from attending higher-quality schools (e.g., Dillon and Smith [25] and Badge, Epple, and Taylor [9]). Our work contributes to this debate in a unique way: by allowing for pre-college academic preparation to endogenously adjust to alternative admissions schemes, we allow for graduation rates to either rise or fall, counterfactually. We find that more generous AA schemes raise average graduation rates for minorities, but that the sign and magnitude of the change varies with student unobservable characteristics.

Along those lines, in most of the papers above, exam scores are used as a proxy for student ability, and assumed to be fixed with respect to variations in admissions criteria. However, if scores are jointly determined by both ability and market-based incentives for investment, then this is problematic. Our paper forms part of a new literature focusing on endogeneity of academic achievement with respect to admissions rules. Other such papers include Ferman and Assunção [29] who leverage a natural experiment in top Brazilian university admissions policies to uncover reduced-form evidence that pre-college academic achievement varies by admissions rules. Cotton, Hickman, and Price [22] seeks to replicate the Bodoh-Creed and Hickman [13] framework in a field experimental classroom setting in order to directly measure shifts of investment incentives. Cotton, et. al. set up a math competition among middle school students,
with AA in place to assist students from lower grades who have less math preparation. These studies find evidence, in both observational data and in the field, that HC investment is influenced by AA, and therefore a model must take this into account when producing counterfactual estimates. Our paper takes a structural approach to control for behavioral shifts induced by changes to market allocation rules. This facilitates other market design questions such as, how do American allocation rules compare to other untested mechanisms?

Kapor [43] estimates a counterfactual exercise that is related to our socioeconomic AA counterfactuals. The author analyzes the effect of the “Texas Top 10%” (TTT) program on admissions to universities in Texas. The TTT guaranteed admission at each Texas public university to all students ranking in the top decile of their high school class, and the program was enacted in response to Supreme Court rulings limiting the scope of race-based AA programs. Kapor’s [43] model includes costly college application decisions and enrollment choices by students as well as strategic admissions decisions on the part of colleges. In addition, students can disagree about the relative quality of different universities, and the model includes a variety of plausible information frictions that interfere with the students’ decisions. Since the model does not include private information or an endogenous HC decision, Kapor [43] does not seek to answer the questions we pose regarding the determinants of post-college income and the relative importance of the competitive and productive channels of incentives.

Of course, the current paper is subject to its own limitations as well. We do not explicitly model “supply-side” concerns—e.g., decisions on how many students to admit and how much to charge them—but instead we model college seats as fixed objects of known quality in order to concentrate just on student HC investment. To the extent that supply-side competition plays a role, this work can be seen as complementary to models such as Epple, et. al. [26], Chade, Lewis, and Smith [17], Fu [33], Azevedo and Leshno [8], and Fillmore [30] who treat these forces explicitly.

2. DATA AND MARKET STRUCTURE

In this section we describe the observables we use in our study. To conduct our analysis, we need enough data to effectively model the incentives facing college applicants when they make their HC accumulation choices. Our estimates require metrics for the HC of students that graduate from each college, measures of the quality and graduation rate of each college, and statistics on the post-college outcomes for each student. We begin by describing the combination of college-level and individual-level data we use. In Section 2.3, we provide descriptive analysis of these data that motivates the use of the contest model described in Section 3.

We use US college data for academic year 1992-1993 for two main reasons. First, one can reasonably assume AA policies were stable and understood by decision-makers at that time. The only successful legal challenge prior to 1993 was in 1978, when the Supreme Court declared quotas unconstitutional in University of California v. Bakke [1]. The second reason for studying AY1992-1993 is that individual-level data on students graduating during this academic year are available from the Baccalaureate and Beyond (B&B) database linking college quality and HC
choices to the household income of college graduates from that year. Thus, our empirical application can be interpreted as a case study of how AA shaped the college landscape for the parents of today’s high-school students.

2.1. Colleges. For a sample \( L = \{1, 2, \ldots, L\} \) of 4-year colleges we have a vector \( Y_l \) of school characteristics. The first is a quality measure derived from data and methodology by US News & World Report (USNWR) for their annual *America’s Best Colleges* rankings (see Morse [47]). We adopt this measure as the college quality index \( p_l \), and we argue that interpreting this index as a reflection of meaningful quality rankings is sensible for three reasons. First, USNWR solves market information frictions by providing a wealth of data on many schools, along with advice on how to interpret the data. Consumers’ response to this service has been large enough that rankings are now the primary focus of USNWR’s business model. Second, the validity of USNWR rankings is undoubtedly reinforced in students’ minds by the enthusiasm with which universities advertise their status in *America’s Best Colleges*. Third, many previous studies have depicted college quality either with coarse, discrete measures or with relatively simplistic ones such as mean student-body exam score alone. Our measure provides for a continuous transition from low quality to high, and it encompasses a host of factors influencing the college experience such as selectivity, per-student spending, and faculty quality.

The US postsecondary education industry is vast and diverse, with thousands of institutions offering students at least one type of 4-year degree, but many of these specialize in vocational training. Thus, we adopt the USNWR universe of schools as our definition of “the college market.” This leaves us with 1,245 non-profit colleges and universities specializing primarily in liberal arts education leading up to a bachelor’s degree. This set of schools accounts for the majority of 4-year degree production in the United States. Between the late 1980’s and early 1990’s the total size of the incoming freshman class for these schools was roughly 1.6 million students.

The other college-level data are provided by the National Center for Education Statistics (NCES) through their Integrated Postsecondary Education Data System (IPEDS), which includes school-level enrollment for all first-time freshmen (including full-time and part-time) by race for Whites, Blacks, Hispanics, Asians or Pacific Islanders, and American Indians/Alaskan Natives. For the 1988 incoming class, we have freshman headcount, denoted \( M_l \), for underrepresented

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6USNWR computes its quality score as a weighted arithmetic mean of a school’s quantile rank in 15 quality indicators, falling into 6 different categories: Reputation Rank (based on survey data from college presidents and deans; 25% weight), Student Selectivity (comprising acceptance rate for 1992 freshman class, yield rate for 1992 freshman class, % of enrollees in top 25% of high school class, and mean SAT/ACT score among enrollees; 25% weight), Faculty Resources (comprising student-faculty ratios, % of full-time faculty with terminal degrees, % of faculty on part-time status, average salary and benefits for full-time faculty, and proportion of classes with fewer than 20 students; 20% weight), Financial Resources (comprising per-student education expenditures and per-student other expenditures; 15% weight), Graduation Rate (% of students in 1985-1986 freshman classes who graduated within 6 years; 10% weight), and Alumni Satisfaction (% of living alumni who donated to AY1991-1992 fund drives; 5% weight). USNWR computes the quality metric separately within 4 different groups of schools (National universities, national liberal arts colleges, regional universities, and regional liberal arts colleges, see Morse [47]), but we modify their method slightly and define quality indicator quantile ranks across the entire universe of schools. This produces a single index ranking that applies to all colleges in the sample. We used the 1994 edition because several of its quality indicators are on a 2-year lag, and we will be combining these data with student-level observations from the graduating class of AY1992-1993.
minorities—Blacks, Hispanics, and Native Americans—and a headcount, denoted \(N_l\), for all others—Whites and Asians. Aggregating this information across schools allows us to compute \(\mu = \sum_{l=1}^{L} M_l / \sum_{l=1}^{L} (M_l + N_l)\). IPEDS also allows the researcher to compute race-specific, 6-year graduation rates for each college campus, which we denote by \(\Gamma_{jl}, j \in \{M, N\}\). In total, then, the data representing schools \(l = 1, \ldots, L\) are denoted \(Y_l = \{p_l, M_l, N_l, \Gamma_{Ml}, \Gamma_{Nl}\}\).

Colleges are separated into five tiers with Tier I representing the top quality quintile of college seats. Entries in Table 1 represent the fraction of all students within a tier that are of a given demographic group. In a hypothetical world with no under-representation, each cell would be the same as the overall share of each race group. “Minorities,” denoted \(M\), as defined in the table are under-represented in the top three tiers, and over-represented in the bottom two. Similar patterns hold as well for race sub-categories: Blacks, Hispanics, and Native Americans are individually under-represented in top tiers. The reverse is true for Whites and Asians who comprise the “Nonminority” group, denoted \(N\). Minority and nonminority students are defined as they are because AA in college admissions specifically targets under-represented minority groups.

### Table 1. Racial Representation Within Academic Quality Quintiles

<table>
<thead>
<tr>
<th>Group</th>
<th>Total Share</th>
<th>Tier I Share</th>
<th>Tier II Share</th>
<th>Tier III Share</th>
<th>Tier IV Share</th>
<th>Tier V Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minority</td>
<td>15.81%</td>
<td>10.23%</td>
<td>13.09%</td>
<td>11.75%</td>
<td>22.16%</td>
<td>21.81%</td>
</tr>
<tr>
<td>Non-Minority</td>
<td>84.19%</td>
<td>89.77%</td>
<td>86.91%</td>
<td>88.25%</td>
<td>77.84%</td>
<td>78.19%</td>
</tr>
</tbody>
</table>

2.2. **Students.** Our individual-level data on the student population comes from the 1993 Baccalaureate and Beyond Survey (B&B), which randomly samples colleges and then samples students graduating in AY1992-1993 within each college.\(^7\) The data contain several variables pertaining to pre-college investment for student \(i \in \{1, 2, \ldots, I\}\). The two outcome variables which researchers and college admissions officers focus on most for assessing academic achievement are exam scores, denoted \(e_i\), and academic record as measured by grade point average (GPA), denoted \(a_i\). In the B&B data, \(e_i\) takes the form of either the ACT or the SAT, both of which are standardized college entrance exams. The companies which develop these exams also produce concordance tables which allow one to relate ACT scores into SAT scale and vice versa.

Unfortunately, B&B does not contain high-school GPA directly, but it has other information including combined GPA from declared major courses, combined GPA from declared minor courses, and combined GPA from all other courses. We adopt college non-major/non-minor GPA (NMGPA) as a proxy for high school GPA, and we argue this is a plausible substitution for several reasons. First, NMGPA primarily consists of coursework during the first one or two years of college immediately following high school. Second, non-major college coursework is

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\(^7\)There are other data sets that provide information on student background and career outcomes (e.g., the U.S. Bureau of Labor’s National Longitudinal Survey of Youth). However, to the best of our knowledge, the B&B survey is the only data set which contains all variables which are needed for our analysis, including pre-college achievement, the identity of each student’s college, and post-college income.
Table 2. Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IPEDS/USNWR:</strong> (school-level data)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6-Yr Graduation Rate: $M$</td>
<td>961</td>
<td>0.3427</td>
<td>0.2812</td>
<td>0.2159</td>
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<tr>
<td>6-Yr Graduation Rate: $N$</td>
<td>961</td>
<td>0.4831</td>
<td>0.4568</td>
<td>0.2295</td>
</tr>
<tr>
<td>Freshman Cohort Size: $M$</td>
<td>1,245</td>
<td>145.37</td>
<td>46</td>
<td>254.56</td>
</tr>
<tr>
<td>Freshman Cohort Size: $N$</td>
<td>1,245</td>
<td>660.86</td>
<td>378</td>
<td>788.68</td>
</tr>
<tr>
<td>College Quality Index</td>
<td>1,245</td>
<td>0.4842</td>
<td>0.4598</td>
<td>0.2132</td>
</tr>
<tr>
<td><strong>BACCALAUREATE AND BEYOND</strong>: (individual level data, college graduates only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAT/SAT equivalent scores: $M$</td>
<td>500</td>
<td>820</td>
<td>820</td>
<td>220</td>
</tr>
<tr>
<td>SAT/SAT equivalent scores: $N$</td>
<td>4,980</td>
<td>990</td>
<td>980</td>
<td>190</td>
</tr>
<tr>
<td>Academic Record (GPA): $M$</td>
<td>500</td>
<td>2.7</td>
<td>2.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Academic Record (GPA): $N$</td>
<td>4,980</td>
<td>3.0</td>
<td>3.0</td>
<td>0.6</td>
</tr>
<tr>
<td>College Quality: $M$</td>
<td>500</td>
<td>0.5174</td>
<td>0.5236</td>
<td>0.2140</td>
</tr>
<tr>
<td>College Quality: $N$</td>
<td>4,980</td>
<td>0.5846</td>
<td>0.6051</td>
<td>0.2024</td>
</tr>
<tr>
<td>10-Year Household Income: $M$</td>
<td>260</td>
<td>$100,700</td>
<td>$89,500</td>
<td>$52,500</td>
</tr>
<tr>
<td>10-Year Household Income: $N$</td>
<td>2,800</td>
<td>$108,300</td>
<td>$92,700</td>
<td>$77,300</td>
</tr>
</tbody>
</table>

*As per USDOE data security requirements, in order to protect anonymity of B&B respondents, sample sizes have been rounded to the nearest 10, dollar figures have been rounded to the nearest $100, SAT scores have been rounded to the nearest 10, and GPAs have been rounded to the nearest 0.1.

Qualitatively similar to high-school coursework in several ways. The American post-secondary education system follows a liberal arts model where students must progress through a structured, standardized learning regimen largely out of their control, unlike major and minor courses. This regimen spans many academic disciplines with a primary focus on introductory instruction. For these reasons we assume that exam scores plus high-school GPA together contain the same information about a student’s pre-college academic preparation as exam scores plus NMGPA together. These two variables will be used to construct a single index for the HC choice within the model.

The other variable we draw from the B&B survey is annual household income after 10 years in the workforce, denoted $w_i$. The top panel of Table 2 provides summary statistics for college-level variables and the bottom panel summarizes student-level data.

2.3. Motivating Our Model. The model defined in Section 3 is a contest wherein the students compete for admission to college via their choice of HC. Our contest model assumes that students agree on the ranking of schools and that schools agree on the quality of each student within each demographic group. As a result, the outcome predicted by our contest model will be nearly (but not perfectly) assortative in terms of school quality and student HC choice. Violations of assortativity are caused both by affirmative action policies and by an exogenous matching shock. We also assume students are motivated by the expected household income premium obtained by
college graduates. The goal of this section is to motivate these modeling choices by describing how well the raw data described above fit these assumptions.

First let us consider whether a contest structure fits the patterns we observe in our data. As we show in Section 3, our contest model predicts that the student enrollment will be a roughly assortative match in terms of student HC and college quality, and the deviations from assortativity are caused by AA as well as a matching shock. We in fact find that the correlation between SAT scores and college quality for college graduates in the B&B survey is 0.449. In addition, we estimate a reduced-form single index for HC in the form of a polynomial in student exam scores and GPA (see Section A.1 in the appendix). We find that this single index of academic achievement has a Spearman rank correlation of 0.88 with our metric of college quality.

Our model also assumes a national market for college admissions as has been done in previous work (e.g., Epple, Romano, and Sieg [27], Chade, Lewis, and Smith [17], and Fu [33]). Specifically, the key requirement in our context is that all prospective college enrollees have roughly the same access to the college quality spectrum, regardless of where they attend high school. One might be concerned about this assumption given the large fraction of state-funded colleges in the market. Our raw college-level data exhibit empirical patterns consistent with the national market view. We can count the number of states that are represented within each college campus in our sample. For example, the median college enrolls students from over one third of the states in the country. Moreover, 76% of colleges in our sample enroll students from at least 10 different states, while 23% of them enroll students from 30 or more states. We interpret these figures as being consistent with an integrated national college market where the incentives to enroll in a quality college are high enough that geographic variation across students is not a significant barrier.

Finally, we assume that students are motivated by the expected college income premium. Although there are other non-pecuniary benefits to college, it is reasonable to assume that the impact on lifetime income is the largest benefit and best reflects the primary arguments made for the importance of a high-quality college education. The interquartile of range household income per year in the B&B data set is $54,000 to $113,000, and there is a significant raw correlation between household income and college quality. Although this correlation conffates the joint effect of college quality, student HC, and unobservable characteristics, it suggests that the college premium may provide strong incentives for students to focus on college quality over other attributes of universities. In summary, we assume students care about post-college income (or at least that their match utility is well proxied by income), though we allow the data to determine the strength of the link between college placement and income.

3. THEORETICAL MODEL

Following Bodoh-Creed and Hickman [13], we model college admissions as a Bayesian game where high-school students are characterized by a privately-known type that governs the costliness of HC production, as well as the payoff from enrolling in college. Students compete for

---

8 We find a similar correlation between college quality and median within-campus SAT score in the USNWR data.
9 Income quartiles have been rounded to the nearest thousand dollars to protect B&B respondents’ anonymity.
enrollment at colleges of differing quality through their choice of HC level, which we view as a long-run plan for the accumulation of HC over several years leading up to the college admissions process. On the other side of the market are colleges that have preferences over students based on their HC and race. In general, colleges wish to attract the most academically advanced students, but they also prefer to have a demographically diverse student body as well, and their trade-off between race and pre-college academic achievement is reflected in their AA plan.

One empirical regularity of American college data is a nontrivial degree of academic heterogeneity among students on a given college campus (even within demographic groups). To capture this heterogeneity, our model includes market frictions in the form of a random matching shock to the colleges’ perceptions of a particular student’s HC choice. This shock is observable to colleges at the time the student applies, but not to the student while she is investing in her HC. The unobservability of the shock to the student is consistent with an interpretation of HC choice as the gradual accumulation of learning over years, while the shock is the result of events that occur within a short time before her application to college, and that are by and large out of the student’s control. A student’s ex-post payoff is her match utility minus her HC investment cost. Her match utility is a function of the student’s college assignment, her HC level, and her privately known learning cost type, while her cost of HC is a function of her type and the HC level she has chosen.

3.1. Agents. There are two demographic subgroups within the student population, minority students (M) and nonminorities (N), and the demographic class of each student is observable. There is a continuum of students of total mass 1 with mass \( \mu \) of them in the minority group. We often refer to our model with a continuum of students as a limit model to emphasize the fact that it can be viewed as a limit of a model with a finite number of students, as their number approaches infinity. We elaborate on this point in Section 3.5. Each student has a privately-known learning cost type \( \theta \in [\theta, \bar{\theta}] \), and the distribution of \( \theta \) in group \( i \in \{M, N\} \) follows a cumulative distribution function (CDF) \( F_i(\theta) \). For convenience, we denote the unconditional type distribution by \( F(\theta) \equiv \mu F_M(\theta) + (1 - \mu) F_N(\theta) \). Each agent’s strategy space, \( S = [s, \infty) \), is the set of attainable HC levels. These are observable (e.g., through standardized exam scores and high school GPAs) and \( s \) is the minimum required to attend some college. In order to produce \( s \) units of HC, a student incurs cost \( C(s; \theta) \) which is increasing in both \( s \) and \( \theta \).

Investment costs can arise in various ways, such as from a consumption–investment tradeoff or psychic costs from difficult learning activities. Learning cost types \( \theta \) may encapsulate both cognitive and non-cognitive characteristics, and they may be influenced by forces both internal to the individual (e.g., innate ability or natural curiosity) and external (e.g., home environment, primary/secondary school quality, parental education and financial resources). Asymmetric type

\[10\] One might have imagined a more complex model where the students can generate distinct kinds of HC through different costly activities. In fact, this is essentially what we will do with our empirical exercise, taking account of both exam scores and GPA. All that is essential about our modeling choice is that the schools reduce each student’s portfolio of HC to a single index and that the schools do this in roughly the same way. It is not necessary to understand exactly how a student created their HC to answer the questions we have posed.
distributions reflect these factors, many of which are correlated with race. When we condition outcomes on unobserved types in our empirical framework, \( \theta \) controls for many environmental correlates of race even if they do not appear in our model explicitly. We remain agnostic on the exact interpretation of \( \theta \), but we assume that it is fixed from the perspective of a student when she chooses her level of HC investment.

Mathematically, the difference between types \( \theta \) and investment \( s \) is that the former reflects the exogenous portion of HC costs and the latter arises from a costly decision under the control of the agent. For example, consider the case of a parent who may enrich his daughter’s educational experience by spending time reading or doing schoolwork with her. In a low-income household where the parent must work two jobs, his time may be more constrained than in an affluent household where the parent has a single, high-paying job. In this example, the parent’s and child’s choice of how much time to spend on learning activities is encapsulated in \( s \), whereas the pre-existing opportunity cost of time is reflected in \( \theta \).

A matching shock is applied to each student’s choice of HC, \( s \), to generate a noisy HC (NHC) value that is commonly observed by all of the colleges. We assume the noise enters additively, so if student \( i \) chooses \( s \), the associated NHC is \( t = s + \epsilon \). We assume that \( \epsilon \) is not observed by the student until after she has chosen \( s \), so from her perspective it is a random variable with CDF \( F_{\epsilon} \). The shock allows the market to deviate from perfect assortativity so as to rationalize within-campus HC variation in the data. As the variance of \( \epsilon \) gets very small it becomes perfectly assortative, and as the variance gets very large the market becomes a lottery.

### 3.2. Payoffs.

On the other side of the market there is a continuum of colleges with total mass 1. Each college’s quality is described by an index \( p \in [\underline{p}, \bar{p}] \) that is distributed \( P \sim F_{\theta}(P) \). By assuming the measure of students and college seats are the same, we abstract from the extensive margin of college attendance, focusing only on the intensive margin of competition for admission to the best colleges, conditional on entering the market. Both college quality and HC are intrinsically valued: match utility \( U(p, s, \theta) \) results from a student with type \( \theta \) having HC \( s \) and enrolling in a college with quality \( p \). The ex post payoff to agent \( i \) in group \( j \in \{M, N\} \) is the match utility minus the cost of achievement, \( U(p, s, \theta) - C(s, \theta) \).

### 3.3. Allocation Mechanisms.

In an admissions contest, students are allocated seats at colleges of varying quality, and the quality of the seat allocated to a student is a function of how her NHC realization compares to the distribution of NHC across the population of college applicants. Affirmative action schemes cause the NHC realizations of minority and nonminority students to be compared to the total distribution of NHC differently, which makes the contest asymmetric between the two demographic groups. We consider color-blind (cb), proportional quota (q), and admissions preference (ap) AA systems, which are described formally below. Despite the richness of the model, our contest mechanism provides a parsimonious characterization of endogenous HC investment within a complex market setting where HC plays dual roles of a productive asset and determining one’s match prospects. One can also view the contest as a form of all-pay
The auction in which the admission scheme gives rise to an endogenous pricing rule that dictates the amount of HC an individual must “pay” to win a given seat.

The equilibrium allocation mechanism is an assignment mapping \( P^r_j \colon \mathbb{R} \rightarrow [\underline{p}, \overline{p}], j \in \{\mathcal{M}, \mathcal{N}\} \) and \( r \in \{ap, cb, q\} \), that maps a student’s NHC realization into an enrollment at some college. \( P^r_j \) is an endogenous object that depends on the form of AA at work in the market, the distribution of college qualities, and the equilibrium distribution of NHC in the population, which is in turn determined by the endogenous choices of HC by the students. To facilitate discussion we denote the equilibrium CDFs of HC and NHC as \( G^r_j \) and \( H^r_j \), respectively, where \( r \in \{ap, cb, pr\} \) indicates the allocation mechanism and \( j \in \{\mathcal{M}, \mathcal{N}, \mathcal{K}\} \) indicates a demographic group or the unconditional population. Note that the distribution \( H^r_j \) is a convolution of \( G^r_j \) and \( F_\varepsilon \) with a density given by the familiar formula,

\[
\begin{align*}
    h^r_j(t) &= \int_0^\infty f_\varepsilon(\varepsilon) g^r_j(t-\varepsilon) d\varepsilon.
\end{align*}
\]

We define affirmative action as the difference between the level of NHC required for a minority and nonminority applicant to be admitted to a school. We do not take a stand on why schools choose to implement affirmative action schemes. The fraction of minority students on the campus of a particular college is the result of the interaction of the school’s affirmative action policy with the densities of the distributions of NHC for each group. The assignment functions, which we turn to now, operationalize how exactly admissions practices work at different points in the college quality spectrum.

The color-blind allocation mechanism is the simplest to define since it assigns students to schools assortatively with higher NHC realizations leading to assignment at higher quality colleges. Since demographics do not affect the assignment, \( P^cb_M(t) = P^cb_N(t) = P^cb(t) = F_p^{-1}(H^cb_K(t)) \). In words, a student with NHC realization \( t \) at quantile rank \( H^cb_K(t) \) of the NHC distribution (for all students) is placed at a school with the same quantile rank in the college quality distribution, \( F_p^{-1}(H^cb_K(t)) \). Since the assignment mapping is the same for both groups, marginal investment incentives, conditional on type \( \theta \), are also identical across groups.

The next simplest mechanism is a quota scheme, which is currently used in various parts of the world, although the US Supreme Court declared it unconstitutional in the college admissions context in 1978 [1]. Under an arbitrary quota scheme, students from group \( j \in \{\mathcal{M}, \mathcal{N}\} \) are reserved sets of seats with qualities distributed \( P \sim Q_j(P) \), where \( \mu Q_M(p) + (1-\mu) Q_N(p) = F_p(p) \), \( \forall p \) is required for feasibility. Under a quota mechanism, members of each demographic group compete for prizes only against the other members of their same demographic group, in disjoint contests. The resulting assignment map is \( P^{pq}_j(t) = Q_j^{-1}(H^{pq}_j(t)) \), which means a student with NHC realization \( t \) at quantile rank \( H^{pq}_j(t) \) of the NHC distribution of her own demographic group is placed at a school with the same quantile rank in the distribution colleges allocated to her demographic group, \( Q_j^{-1}(H^{pq}_j(t)) \). The most familiar member of this class is a proportional quota where \( Q_M = Q_N = F_p \), meaning that fraction \( \mu \) of seats at each point in the college quality spectrum are reserved for minorities.
Although proportional quotas are not directly applicable in the US as a policy instrument, they provide a useful benchmark relative to a color-blind mechanism. When racial asymmetries in learning costs exist—because race is correlated with childhood school quality, for example—proportional quotas are designed so that these differences are not reflected in the fraction of students from each demographic group present on each college campus, particularly the best ones that would otherwise be out of reach in equilibrium. A color-blind mechanism allows asymmetries in learning costs between the demographic groups to be maximally reflected in the fraction of minority students enrolling in high-quality colleges.

Finally, an admission preference system refers to one in which the NHC values of both groups are compared to the distribution of the total student population, but the demographic status of a given student determines how this comparison is made. To the extent that American colleges engage in race-based AA at present, the only legally permissible form is an admissions preference system where race is taken as a “plus factor” among other considerations like grades and test scores. Formally, an admission preference is defined by a markup function \( T : S \rightarrow R \) that transforms the NHC levels of the minority students. The resulting assignment mappings are

\[
P_{M}^{pp}(t) = F_{p}^{-1} \left( \mu H_{M}^{pp}(t) + (1 - \mu) H_{N}^{pp} \left( T(t) \right) \right)
\]

\[
P_{N}^{pp}(t) = F_{p}^{-1} \left( \mu H_{M}^{pp} \left( T^{-1}(t) \right) + (1 - \mu) H_{N}^{pp}(t) \right)
\]

In words, a minority student’s NHC level \( t \) is compared to the raw NHC of other minority students and marked up when compared to other nonminority students. Conversely, NHC for a nonminority student is compared to the raw NHC levels of other nonminority students and it is effectively “de-subsidized” for comparisons to NHC levels of minority students.

It is obvious that the admission preference mechanism nests the color-blind as a special case when \( T(t) = t \). Bodoh-Creed and Hickman [13, Theorem 4] proves that that all of the AA systems described above are equivalent under the conditions of our model in the following sense:

**Theorem 3.1.** [Theorem 4 of Bodoh-Creed and Hickman (2017)] \( P_{j}^{p}(t) : R \rightarrow P, j \in \{M,N\} \), is the result of an equilibrium of some quota system with \((Q_{M},Q_{N})\) if and only if there exists an admissions preference system with some \( \bar{T} \) that has the same equilibrium assignment mappings and strategies.

Because the allocation of students to schools is known ex ante under a quota scheme, our empirical approach will rely heavily on this useful equivalence result. We base our structural model on a (non-proportional) quota system that is outcome equivalent to the real-world admissions preference. This greatly simplifies identification and the construction of our estimator.

### 3.4. Model Assumptions.

We do not get existence of an equilibrium without some assumptions. Although these assumptions are not used directly in our estimation, we reproduce them from Bodoh-Creed and Hickman [13] for completeness. Assumptions 3.2 - 3.4 require that the type, college quality, and matching shock distributions admit differentiable probability density functions (PDFs) with a connected support.
Assumption 3.2. $F_j(\theta) \in C^2$, $j \in \{M,N\}$ and densities $f_M(\theta)$ and $f_N(\theta)$ are strictly positive on a common compact support $[\theta, \overline{\theta}]$ with non-empty interior.

Assumption 3.3. $F_P(p) \in C^2$ and the prize density $f_P(p)$ is strictly positive on a compact support $[p, \overline{p}]$ with non-empty interior.

Assumption 3.4. The distribution of matching shocks is absolutely continuous with full support: $\varepsilon \sim F_{\varepsilon}(\varepsilon)$, $F_{\varepsilon} \in C^2$, and $f_{\varepsilon}(\varepsilon) > 0$, $\forall \varepsilon \in (\varepsilon, \overline{\varepsilon}) \subseteq \mathbb{R}$.

Assumption 3.5 imposes regularity conditions on the cost function. We associate high values of one’s permanent type $\theta$ with high HC production costs, and low values of $\theta$ with low costs.

Assumption 3.6 imposes regularity conditions on the match utility function $U(p, s, \theta)$. First we require that students benefit from enrolling in a high quality college ($U$ is increasing in $p$), high levels of HC ($U$ increasing in $s$), and we allow for permanent types to play a role as well ($U$ decreasing in $\theta$). Moreover, we require utility to be monotone in $s$, with convex costs and concave (in $s$) match utility.

Assumption 3.7. The highest cost students find it optimal to choose the lowest level of HC that qualifies the student to attend college. From a formal perspective, this assumption provides a boundary condition for solution of the model equilibrium.

Assumption 3.8. $C(s, \theta)$ is strictly supermodular in $(s, \theta)$ and $U(p, s, \theta)$ is supermodular in $(p, s, -\theta)$.

Finally, we require that there is a highest possible HC that any student is willing to choose, which means that the effective action space is compact. Our assumption requires that this upper bound, denoted $s$, will not be chosen by any type of student even if such a choice would result in enrollment into the best possible school.

Assumption 3.9. There exists $\overline{s}$ such that for all $\theta$ we have:

$$U(\overline{p}, \overline{s}, \theta) - C(\overline{s}, \theta) \leq U(p, \overline{s}, \theta) - C(s, \theta)$$

Finally, we require the following regularity condition on the markup function used in the admission preference system. This assumption is that $\tilde{T}$ is strictly increasing (i.e., the mechanism respects rank ordering within demographic groups) and that the markup function does not increase so steeply that students have an arbitrarily strong incentive to increase $s$.

Assumption 3.10. There exists $0 < \lambda_1 < \lambda_2 < \infty$ such that for all $t$ we have $\lambda_1 < \tilde{T}'(t) \leq \lambda_2$. 


3.5. Equilibrium. The distribution of college seats, the form of the admission system, and the measures and distributions of student competitors are common knowledge prior to individual choices of HC investment. When combined with the equilibrium strategies of the minority and nonminority students, denoted \( \sigma_M(\theta) \) and \( \sigma_N(\theta) \) respectively, the students can forecast the form of the assignment mapping \( P_j \) rising in equilibrium. Each student solves the following optimization problem where \( j \in \{M, N\} \) and \( r \in \{cb, pq, ap\} \):

\[
\sigma_j(\theta) = \arg \max_s \left\{ E_\epsilon \left[ U \left( P_j'(s + \epsilon), s, \theta \right) \right] - C(s, \theta) \right\}
\]

In equilibrium, students’ beliefs about \( \sigma_M(\theta) \) and \( \sigma_N(\theta) \) must be consistent with the solution to Equation 1 so that the students’ beliefs about \( P_j'(t) \) are correct. Given the assumptions described in Subsection 3.4, Bodoh-Creed and Hickman [13, Theorem 6] proves that such an equilibrium exists, and for completeness we reproduce the statement of that result here without proof.

**Theorem 3.11.** [Theorem 6, Bodoh-Creed and Hickman [13]] There exists a monotone, pure strategy Nash equilibrium of our limit model in the color-blind, quota, or admissions preference systems.

Actual college markets one might study empirically have only finitely many students and colleges, but on the other hand, the finite version of this model is computationally intractable. However, Bodoh-Creed and Hickman [13] establish empirical relevance of the continuum model by showing that it provides an accurate representation of a finite model with a large number of players. In this finite model, \( K_M \) minority students draw types from the distribution \( F_M(\theta) \), \( K_N \) nonminority students draw types from the distribution \( F_N(\theta) \), and \( K_M + K_N \) college seats draw their qualities from \( F_P \). In the limit as \( K_M + K_N \to \infty \) and \( K_M/(K_M + K_N) \to \mu \), the primitives of the finite games approach those of the continuum model. Bodoh-Creed and Hickman [13] show that the continuum model approximates the finite game in the following sense:

**Definition 3.12.** Given \( \epsilon > 0 \), an \( \epsilon \)-approximate equilibrium of the \( K \)-agent game is a \( K \)-tuple of strategies \( \sigma^\epsilon = (\sigma_1^\epsilon, \ldots, \sigma_K^\epsilon) \) such that for all agents, almost all types \( \theta \), and all HC choices \( s' \) we have

\[
U \left( P_j'(\sigma_j^\epsilon(\theta), \sigma_j^\epsilon(\theta)), \sigma_j^\epsilon(\theta), \theta_j \right) - C(\sigma_j^\epsilon(\theta), \theta_j) + \epsilon \geq U \left( P_j'(s', \sigma_j^\epsilon(\theta)), s', \theta_j \right) - C(s', \theta_j)
\]

Definition 3.12 describes an approximate equilibrium in terms of incentives: agents that follow an \( \epsilon \)-approximate equilibrium can gain at most \( \epsilon \) by deviating. Intuitively, students lose little utility if they base their actions on the easy-to-compute limit game equilibrium. Our final theorem shows that we can choose \( \epsilon > 0 \) to be arbitrarily small as the size of the market increases.

**Theorem 3.13.** [Theorem 7, Bodoh-Creed and Hickman [13]] Let \( \sigma_j^*, j \in \{M, N\} \) and \( r \in \{cb, pq, ap\} \), denote an equilibrium of the game with a continuum of agents. We can choose \( K^* \) such that \( \sigma_j^* \) is an \( \epsilon \)-approximate equilibrium of the \( K \)-agent game for any \( K > K^* \).

4. MODEL IDENTIFICATION AND ESTIMATION

Our theoretical model produces a rich set of implications concerning investment behavior and equilibrium outcomes under different AA mechanisms and market conditions. We now use it
to construct an empirical model to address our main research questions. These include 1) the effect of college quality, pre-college investment, and permanent type on the returns to a college education; 2) the link between AA and racial inequality in achievement, college placement, and post-college outcomes; and 3) the role of relative incentives in driving pre-college human capital decisions.

Our basic identification challenge is to disentangle the influence of college quality, HC investment, and privately-known type on the returns to attending college. If the students who tend to enroll at more selective colleges have more advantageous characteristics to begin with, then raw correlations between college quality and earning power cannot be viewed as causal. Previous empirical work has attempted to address the issue essentially by instrumenting for the influence of college quality while subsuming unobserved student characteristics into the unexplained error term in the model (see Brewer, Eide, and Ehrenberg [15]; Dale and Krueger [23]; Black and Smith [11]; and Long [45]). We take a novel approach to this problem by explicitly modeling the separate influences of school quality, pre-college HC investment, and unobserved student characteristics in determining post-college economic outcomes. A structural approach is necessary to identify the students’ private information about their unobserved characteristics. Accounting for these unobservables is crucial in order to control for endogenous investment responses to the counterfactual changes in admissions criteria we consider in Section 6.

The empirical auctions literature has developed a set of tools specifically designed to identify private information in game-theoretic models. This literature was pioneered by Paarsch [49] and then revolutionized by Guerre, Perrigne, and Vuong [34, GPV] who proposed a non-parametric estimator for mapping observed bids into underlying private valuations in first-price auctions. We combine this approach with common techniques from labor econometrics to parse between the influences of student and school characteristics in producing post-college income. Essentially, using a GPV-inspired approach we can recover the Bayes-Nash equilibrium mapping between (observed) endogenous achievement and (unobserved) private types using raw data on the distributions of college quality and market-wide achievement. These race-specific mappings, whose shapes are directly determined by a combination of economic theory and raw data, then serve as control functions to account for the influence of unobserved types within a wage regression. Intuitively, because AA changes the marginal investment incentives across race groups during high school, one can surmise that two students having the same GPA/exam scores but different race must have distinct underlying types. This fact breaks what would otherwise be perfect rank correlation between achievement and unobserved types, and the matching shock breaks what would otherwise be perfect rank correlation between college placement and achievement. Together, these components of the empirical model provide a full-rank condition that allows us to separate the influences of achievement, college placement, and permanent types in our wage regression.

Section 4.1 outlines our identification strategy in more rigorous detail, and Section 4.2 describes the control-function estimator which follows from our identification strategy.
4.1. **Identification.** We begin this section by coupling some additional assumptions with the general theoretical framework above in order to complete the formal definition of our empirical model. Some of these merely serve the purpose of tractability, and some are crucial for model identification; we attempt to make these distinctions clear in our discussion below. Overall our identification/estimation strategy is semi-parametric, although we do not require direct restrictions on the functional forms of the type distributions or the equilibrium college assignment mappings.

**Assumption 4.1. (Single Index)** Human capital $S$ is a single index function of exam scores $E$ and academic record $A$,

$$
S_i = S(E_i, A_i) = \beta_1^s E_i + \beta_2^s E_i^2 + \beta_3^s A_i + \beta_4^s A_i^2 + \beta_5^s E_i A_i, 
$$

with $S_e(E_i, A_i) > 0$, $S_a(E_i, A_i) > 0 \forall (E_i, A_i)$, and $\max_{(E, A) \in \mathbb{R}^2} \{S(E, A)\} = 1$.\(^{11}\)

**Assumption 4.2. (Separable Exponential Costs)** $C(s; \theta) = \theta c(s)$ with $c(s) = \exp(s)$.

Assumption 4.1 above imposes a quadratic form on the single index equation for HC with regularity conditions and a scale normalization to fix the units of $S$. This single index equation condenses the two margins of achievement—GPA and exam scores—into one measure of HC for the purposes of ranking the desirability of different college applicants. Assumption 4.2 imposes separability and a functional form on the cost function. The exponential functional form is primarily for computational tractability, and we found through experimentation with alternative functional forms that it does not appear to be driving our empirical results in any meaningful way (see Section 5.6 for further discussion). The assumption that costs are separable in the idiosyncratic component $\theta$ and the common component $c(s)$ is more fundamental to identification. Implicit here is also an assumption that individuals/households optimize their portfolio of investment activities $(e, a)$ to generate the highest composite output $s$ at the lowest possible cost. Recall that a student’s type $\theta$ represents aspects of cost and match utility that are exogenous, whereas composite achievement $S$ represents the component of costs and match utility under her control.

**Assumption 4.3. (Exclusion Restriction)** Race does not directly affect match utility conditional on type $\theta$, investment $s$, and college assignment $p$: if $D_{Mi} \equiv 1(i \in M)$ is an indicator for minority status then $U(p, s, \theta, D_{Mi}) = U(p, s, \theta)$.

The exclusion restriction 4.3 is central to our identification strategy, and implies minority status plays no direct role in match utility. Although this assumption rules out some forms of racial discrimination on the labor market (e.g., taste-based racial animus), our model is still compatible with a world in which statistical discrimination occurs. In the data, statistical discrimination would appear as minority and non-minority students with the same realizations of $(s, p)$.

\(^{11}\) For the single index function, we also experimented with a more flexible, cubic, complete-polynomial form $S(E_i, A_i) = \beta_0^s + \beta_1^s E_i + \beta_2^s E_i^2 + \beta_3^s E_i^3 + \beta_4^s A_i + \beta_5^s A_i^2 + \beta_6^s E_i A_i + \beta_7^s E_i^2 A_i + \beta_8^s E_i A_i^2$. This merely increased computational cost without producing a statistically or economically meaningful change in our estimates, relative to the quadratic form above.
receiving different wages because of differing values of $\theta$ that employers infer from the joint distributions of $(p, s, \theta)$ conditional on race.

It is important to emphasize here that our exclusion restriction does not rule out influences by a host of other important factors that are highly correlated with race and influence learning costs. Recall that $\theta$ implicitly subsumes environmental factors such as parental education and income, availability of early-life educational interventions, and K-12 school quality, as well as cognitive and non-cognitive ability which are also known to be influenced by childhood experience. Therefore, our exclusion restriction requires only that racial affiliation plays no direct role in post-college income, conditional on achievement, college placement, and the various environmental and innate idiosyncratic factors contained in $\theta$. In Appendix B we regress $\theta$ on home environment data available in the B&B survey. We find that these variables are highly predictive of the learning costs we recover, and we find no conditional correlation between minority status and learning cost given these home environment factors.

This assumption is supported by the existing labor economics literature. Beginning with Neal and Johnson [48], a body of empirical work over the past two decades has emphasized non-animus factors for explaining the majority of Black-White wage differentials. More recently, Fryer, Pager, and Spenkuch [32] has found that roughly 89% of the Black-White wage gap is attributable to differentials in observable controls other than race. They then test various different theories—including racial animus, differences between blacks and whites in job-search strategies, search intensity, bargaining, and discount factors—to explain the remaining wage gap. They find empirical evidence inconsistent with these various theories for explaining the remaining 11% wage gap; rather, they find that blacks are systematically willing to accept lower reservation wage offers at the hiring stage, but that the returns to tenure on the job, post-hire, are larger for blacks than for whites. The authors conclude that these patterns in their data are most naturally rationalized by a labor-search model with matching frictions where employers use statistical discrimination at the hiring stage but then learn more about the worker’s type over time so that black and white wages on the same job eventually converge.

Although it is impossible to directly test the validity of our exclusion restriction with our dataset, we interpret Fryer, Pager, and Spenkuch [32] as providing some level of confidence that it is not an unreasonable assumption. The main function of our exclusion restriction is to identify the wage equation parameters by parsing out the influences of endogenous achievement and exogenous type. Holding type fixed, AA increases the marginal benefit in terms of college placement of an increase in achievement for blacks (relative to whites). This results in different race-specific mappings between unobserved types and observed achievement levels, and therefore AA allows the econometrician to effectively use race as a non-linear instrument under our exclusion restriction. In order to completely lose identifying power, the racial animus would have to alter the incentives exactly enough that the endogenous investment strategy is the same across demographic groups. Stated more formally, a racial-animus penalty on black post-college wages would have to exactly offset the increase in the marginal benefit caused by improved college placements due to AA. In other words, such a wage offset would have to equalize the
pre-college marginal HC investment incentives for blacks and whites of any given type. Since the markup function \(\tilde{T}\)—which determines college placement and which we can recover from raw data without imposing equilibrium assumptions (see Figure 5 below)—is highly non-linear in achievement, and since college placement then enters as an argument in a non-linear match utility function, it follows that a non-generic, college-specific racial-animus penalty would be required to achieve this. In Section 5.6 below we explore various robustness analyses to probe for violations of the exclusion restriction—including imposition of a uniform 11% racial animus penalty and inclusion of our fitted matching shocks as an additional argument in the utility function—and we find no evidence that it is obviously inconsistent with our data.

Assumption 4.4. **Match Utility**

**Cobb-Douglas Income Production:**

\[
    u(P_i, S_i, \theta_i) = E[W_i|P_i, S_i, \theta_i] = \alpha_0 P_i^{\alpha_p} S_i^{\alpha_s} \theta_i^{-\alpha_\theta},
\]

\(0 < \alpha_0, \quad \alpha_p, \alpha_s, -\alpha_\theta \in (0,1)\)

**Graduation Probability:**

\[
    \rho(P_i, S_i) = \Pr[i \text{ graduates college}|P_i, S_i] = \beta_0 + \beta_1 P_i + \beta_2 P_i^2 + \beta_3 P_i S_i + \beta_4 S_i + \beta_5 S_i^2 + \beta_6 P_i S_i + \beta_7 P_i S_i^2 + \beta_8 P_i + \beta_9 P_i S_i^2
\]

**Expected Log-Utility:**

\[
    U(P_i, S_i, \theta_i) = \rho(P_i, S_i) \log[u(P_i, S_i, \theta_i)] + [1 - \rho(P_i, S_i)] \log[ku(P_i, S_i, \theta_i)],
\]

\(\kappa \in (0,1)\).

Assumption 4.4 adopts a form for the match utility function. We assume a Cobb-Douglas income production from a match. We further assume that a college dropout’s income is \(\kappa < 1\) times what it would have been had she graduated, and we adopt a flexible, cubic complete polynomial form for the graduation probability function. This formulation allows the wage of college dropouts to vary by the student’s underlying ability, HC, and college placement. We adopt a log utility form for the student’s preferences over wage income, as it is a benchmark choice for lifetime consumption models. Since Assumption 4.4 explicitly conditions \(U\) on \((P_i, S_i, \theta_i)\), we will need to take an expectation over the matching shock to find the expected utility function describing the students’ incentives.

Assumption 4.5. **(Unique Investment)** \(U(P_j(s), s, \theta) - \theta c(s)\) is strictly concave in \(s\).

Assumption 4.6. **(Normal Shocks)** Matching shocks \(\epsilon \sim N(0, \sigma_\epsilon)\) are normally distributed with zero mean and variance \(\sigma_\epsilon^2\) and are independent of HC s and demographic status \(j \in \{M, N\}\).

Assumption 4.5 ensures that the agent’s decision problem (Equation 1) has a unique solution, a needed property for mapping \(s\) into a corresponding \(\theta\).\(^{12}\) Note that this assumption is

\(^{12}\)Assumptions 3.5 and 3.6 imply that \(U(p, s, \theta) - \theta c(s)\) is concave in \(s\), but without making assumptions on \(P(s + \epsilon)\) we cannot ensure the decision problem is concave. In lieu of this, we simply assume that Equation 1 has a unique solution.
testable: given the supermodularity of the problem (Assumption 3.8), there would be jumps in the student’s HC accumulation strategy if Assumption 4.5 failed to hold. Finally, Assumption 4.6 assumes normal matching shocks with zero mean. Ultimately, the structural objects to identify are the type distributions, \( F_i(\theta) \), the matching shock variance, \( \sigma^2_\epsilon \), the assignment functions, \( F^p_j \), and the match utility parameters, \((\beta', \alpha)\). The single index parameters \( \beta^s \) and the joint distributions of \((P, S)\) across race groups are intermediate model components to be identified along the way.

### 4.1.1. Identification: Single Index Parameters and Graduation Probabilities

For simplicity of discussion, assume at first that the single index parameters \( \beta^s \) are known. Then the first hurdle to overcome is a problem of sample selection: because the B&B Survey only contains information for college graduates, we do not observe \((P, S)\) pairs for anyone who failed to graduate.\(^{13}\) Thus, at first we can only treat the conditional school quality and HC distributions condition on graduation, \( f_{PS}(p, s|M, \text{grad}) \) and \( f_{PS}(p, s|N, \text{grad}) \), as observables. Given the graduation probability function \( \rho(p, s) \), from Bayes’ law we know that \( f_{PS}(p, s|M, \text{grad}) \) and \( f_{PS}(p, s|N, \text{grad}) \) relate to the unconditional densities as follows:

\[
f_{PS}(p, s|j) = \frac{f_{PS}(p, s|j, \text{grad}) \Gamma_j}{\rho(p, s)}, \quad j \in \{M, N\},
\]

where \( \Gamma_j \) is a constant that equals the total fraction of enrollees from group \( j \) who graduate college, and normalizes the joint density to integrate to one. Although we do not know \( \rho(p, s) \) ex ante, the graduation parameters are pinned down by the graduation rate of each demographic group at each college. We compute the model-generated graduation rate at each college by averaging over the graduation rates of the individual students at the respective college, and these students have a distribution of HC choices described by \( f_{PS}(p, s|j) \). Written formally, we compute:

\[
\Gamma_j = \beta_0^p + \beta_1^p p_l + \beta_2^p p_l^3 + \beta_3^p p_l^3 \\
+ \beta_4^p \overline{S}_j + \beta_5^p \overline{S}_j^2 + \beta_6^p \overline{S}_j^3 + \beta_7^p p_l \overline{S}_j + \beta_8^p p_l \overline{S}_j^2 + \beta_9^p p_l^2 \overline{S}_j + \epsilon_j \\
= Z_j \beta^p + \epsilon_{jl},
\]

where \( Z_j = [1, p_l, p_l^3, \overline{S}_j, \overline{S}_j^2, \overline{S}_j^3, p_l \overline{S}_j, p_l^2 \overline{S}_j, p_l^3 \overline{S}_j] \) contains the regressors for group \( j \) at school \( l \), \( \epsilon_{jl} \) is random and arises from finite sampling within campus \( l \),

\[
\overline{s}_j = \int_{s}^{\overline{s}} s f_{S|P}(s|j, p_l) ds = \int_{s}^{\overline{s}} s \frac{f_{PS}(p_l, s|j)}{f_{P_l}(p_l)} ds
\]

\(^{13}\)One might imagine collecting data directly from colleges that reveal the characteristics of nongraduates, which would obviate the need to perform a selection correction. Although possible, we are skeptical that a wide enough selection of colleges in the B&B data would be willing to reveal this information due to its sensitive nature and the related federal privacy laws.
is the conditional expectation, across both graduates and non-graduates, of the \( k \)th power of \( s \) given \( p_l \), and

\[
  f_{p_l}(p_l) = \int_{\mathcal{E}} f_{PS}(p_l, s \mid j) \, ds, \ j \in \{\mathcal{M}, \mathcal{N}\}
\]

is the unconditional marginal distribution of \( P \) for group \( j \). As mentioned before, since \( \Gamma_{jl} \) is an aggregate, college-level variable, to compute its model-generated analog, \( Z_{jl}\beta^0 \), we compute the average of the individual-level graduation probabilities for group \( j \) on campus \( l \). This is why equation (5) averages over individual-level powers of \( S \). Equations (3) – (6) provide a sample selection correction to identify graduation probability parameters \( \beta^0 \) as long as the single index parameters \( \beta^s \) are known. One appealing characteristic of this sample selection proposal is that it does not impose direct parametric restrictions on the form of the unconditional joint distributions of \((P, S)\).

Now we require a further condition to pin the single index parameters down. Recall that one of the roles of \( S \) is to determine a student’s access to a high-quality match partner. In other words, the HC index \( S \) represents all observable information about the student prior to the application process that predicts where he/she will place. Therefore, we adopt as our final condition the convention that the parameters \( \beta^s \) are such that college placement predictive power is maximized. Since the mapping between \( P \) and \( S \) arises from a rank-order contest, we adopt the Kendall’s \( \tau \) measure of rank correlation to formalize our notion of predictive power.

Briefly, Kendall’s \( \tau \) is defined in terms of concordance of random variables; we say that two ordered pairs \((p_1, s_1)\) and \((p_2, s_2)\) are concordant if the ordering of the first coordinate agrees with the ordering of the second, or \( p_1 < p_2 \) if and only if \( s_1 < s_2 \). Likewise, we say the two pairs are discordant when this condition is violated. For a joint distribution of \((P, S)\), Kendall’s \( \tau \) is defined as the probability of concordance minus the probability of discordance for two iid realizations \((P_1, S_1), (P_2, S_2)\): \( \tau_{PS} \equiv \Pr[(P_1 - P_2)(S_1 - S_2) > 0] - \Pr[(P_1 - P_2)(S_1 - S_2) < 0] \). It is easy to see why this measure is directly relevant to our model with its perturbed rank-order contest structure. Within our context, Kendall’s \( \tau \) is directly interpretable as the probability that the ordering of two students’ college assignments (within the same demographic group) respects the ordering of their pre-college achievement, minus the probability that it does not.

With the joint distribution \((P, E, A)\) known, we formalize our assumption on \( \beta^S \) as:

\[
  \beta^s = \arg \max \left\{ \mu \left( \Pr[(P_1 - P_2)(S_1 - S_2) > 0|\mathcal{M}] - \Pr[(P_1 - P_2)(S_1 - S_2) < 0|\mathcal{M}] \right) \right. \\
  + (1 - \mu) \left. \left( \Pr[(P_1 - P_2)(S_1 - S_2) > 0|\mathcal{N}] - \Pr[(P_1 - P_2)(S_1 - S_2) < 0|\mathcal{N}] \right) \right\}.
\]

From the above arguments, the first part of our identification result follows:

**Proposition 4.7.** There is a unique configuration of the single index and graduation probability parameters \((\beta^s, \beta^0)\) that is consistent with the joint distributions of the observables \( \{Y_i\}_{i=1}^T, \{p_i, e_i, a_i\}_{i=1}^T \) and equations (3), (4) and (7).
4.1.2. **Identification: Matching Shock Variance.** At this point several important equilibrium objects can be treated as known, including the unconditional joint distribution of HC and school assignments. From this starting point, identifying the matching shock variance parameter \( \sigma \) is simple since it uniquely determines the degree to which the joint distribution of \( P \) and \( S \) deviates from full rank correlation within each race group. Intuitively, the larger is the variance of the matching shock, the more latitude there is for students with lower HC levels to place above students with more HC. Thus, it is easy to see that Kendall’s \( \tau \) within each race group is decreasing in \( \sigma \). We use Theorem 3.1 to treat the data-generating process as equivalent to a quota mechanism that reserves a distribution of college seats for each group equal to \( Q_j(p) = F_p(p), j \in \{M, N\} \), which are the marginal distributions of the selection-corrected \( F_{PS}(p, s|j) \)’s from the previous section.

For each group \( j \in \{M, N\} \), let \( \tau_{PS}(\sigma_j|j) \) denote the rank correlation between HC and school assignment implied by shock variance parameter \( \sigma \) holding \( G_j(s) \) and \( F_p(p) \) fixed. Since school assignment within each group is determined by the rank ordering of perturbed HC levels and the perturbations are independent, the following must be true: (i) \( \tau_{PS}(0|M) = \tau_{PS}(0|N) = 1 \); (ii) \( \tau'_{PS}(\sigma_j|j) < 0, j \in \{M, N\} \); and (iii) \( \lim_{\sigma \to \infty} \tau_{PS}(\sigma_j|M) = \lim_{\sigma \to \infty} \tau_{PS}(\sigma_j|N) = 0 \). The following result directly follows from these facts:

**Proposition 4.8.** There is a unique value of \( \sigma \) that is consistent with perturbed, rank-order allocations and the joint distributions \( F_{PS}(p, s|M) \) and \( F_{PS}(p, s|N) \).

4.1.3. **Identification: Admission Preference Markups.** With the distribution of the matching shock known, we can now consider the equilibrium distributions of noisy HC as known objects since they are a convolution of HC and the shock, \( H_j(t) = (G_j \circ F_t)(t) \). These CDFs enter into the assignment mappings through Equations (3.3) to determine the allocation of college seats. One key observation about the admission preference mechanism relevant to identification is the following:

\[
(8) \quad P_M(t) = P_N(\bar{T}(t)).
\]

In other words, a minority student with perturbed investment \( t = s + \epsilon \) is matched to the same college as a non-minority student with perturbed investment \( \bar{T}(s + \epsilon) \). This observation allows us to recover \( \bar{T} \) from the observables by determining what rule could have produced allocations \( F_{PM} \) and \( F_{PN} \) from the investment distributions \( G_M \) and \( G_N \).

**Proposition 4.9.** Under assumptions 3.2 and 3.10 there exists a unique markup mapping \( \bar{T}(\cdot) \) that is consistent with \( (G_M, G_N, F_{PM}, F_{PN}, \sigma) \)

**Proof:** For \( \varphi \in (0,1) \) define \( t_X(\varphi) \equiv H_X^{-1}(\varphi) \) as the \( \varphi^\text{th} \) quantile in the non-minority noisy HC distribution. For minorities, let \( \varphi_M(\varphi) \equiv H_M\left(\bar{T}^{-1}(t_X(\varphi))\right) \) denote the quantile rank of the de-subsidized version of \( t_X(\varphi) \) within the minority noisy HC distribution. By Equation (8), it follows that \( F_{PM}^{-1}(\varphi_M(\varphi[i])) = F_{PN}^{-1}(\varphi), \forall \varphi \). By substituting in \( \varphi_M \) and rearranging, we get

\[
H_N^{-1}(\varphi) = \bar{T}\left(H_M^{-1}\left[F_{PM}\left(F_{PN}^{-1}(\varphi)\right)\right]\right), \text{ from which it follows that}
\]

\[
(9) \quad \bar{T}(t) = H_N^{-1}\left[F_{PN}\left(F_{PM}^{-1}[H_M(s)]\right)\right].
\]
The proof is constructive since the right-hand side of equation (9) is a composition of distribution and quantile functions that can be estimated directly from data. Aside from basic regularity conditions, no a priori restrictions are imposed on the form of the markup function. In particular, one need not even assume that the markup aids minorities, since whether or not \( \bar{T}(t) \geq t \) is left for the data to indicate.

4.1.4. Identification: Utility Parameters and Cost Types. At this point, we can now treat the assignment mappings \( P_{ap}^{M}(t) \) and \( P_{ap}^{N}(t) \) as known. Since the remainder of our discussion on identification and estimation assumes the admission preference mechanism, we drop the superscript in order to simplify notation unless it is needed for clarity.

A classic empirical approach for estimating strategic models with private information was proposed by Guerre, Perrigne, and Vuong [34] for first-price auctions. Their idea was simple but powerful: since the equilibrium distributions of bids is observable, one can reverse engineer a bidder’s private valuation as that which rationalizes her bid as a best response to competitors’ bids. Our setting is similar to an auction in that each student’s investment choice is a best response to the distribution of HC choices given her type. The first-order conditions for the student’s decision problem depicted in equation (1) are:

\[
\theta = \frac{E_x \left[ U_p \left( P_j(s + \varepsilon), s, \theta; \alpha \right) P'_j(s + \varepsilon) \right] + E_x \left[ U_s \left( P_j(s + \varepsilon), s, \theta; \alpha \right) \right]}{c'(s)}, \quad j \in \{M, N\},
\]

where \( \alpha = [\log(a_0), a_p, a_s, -\alpha_\theta]^\top \) is the vector of utility parameters governing wage production from a match of a student to a school given her HC investment.

Because the equilibrium strategies are strictly monotone in the agent’s type, equation (10) uniquely defines the inverse strategy mapping if the utility parameter vector \( \alpha \) is known. Let \( \theta_j(s; \alpha) \) denote the implicit solution to the first-order condition given utility parameters \( \alpha \). Intuitively, equation (10) indicates that with our knowledge of the distribution of college seats and the equilibrium HC distribution, we can reverse-engineer a cost type as a best response to the rest of the market once we can pin down the mapping between actions, college assignment, and income. To do this, we need an additional moment condition, which we get from the following wage regression model:

\[
\log(w_i) = \log(a_0) + a_p \log(p_i) + a_s \log(s_i) - \alpha_\theta \psi(S, D_{Mi}; \alpha) + \varepsilon_{wi},
\]

where \( \psi(S_i, D_{Mi}; \alpha) \equiv \log \left[ D_{Mi} \theta_M(S; \alpha) + (1 - D_{Mi}) \theta_N(S; \alpha) \right] \) is \( i \)'s log-cost type and \( \varepsilon_{wi} \) is a transitory shock to 10-year household income. We assume transitory shocks are exogenous.

**Assumption 4.10.** \( E \{[\log(p_i), \log(s_i), \psi(S_i, D_{Mi}; \alpha)]^\top \varepsilon_{wi} \} = 0. \)

Thus, our approach is to embed the inverse equilibrium equations into the wage regression as nonlinear control functions to account for the role of students’ unobserved characteristics in the production of income. The intuition behind structural identification is as follows: in order for the parameters of the wage regression to be identified, we need an orthogonality condition and a full rank condition. Assumption 4.10 establishes orthogonality based on the idea that the same
unobserved characteristics that govern a student’s pre-college achievement also govern the HC accumulation process during college as well.

For the full-rank condition, there must be something present in the data-generating process that prevents the regressors from being perfectly colinear. The matching shock breaks colinearity between college quality and HC. AA plus the exclusion restriction breaks the colinearity between HC and learning cost types because AA implies a minority student and a non-minority student that have different household incomes (despite possessing the same observable HC and college assignment) must have different underlying cost types.

More formally, note that \( \sigma_v > 0 \) implies a non-degenerate distribution of HC types on each college campus under any college admissions rule, so that rank correlation between \( \log(S) \) and log\((P)\) must be less than one in absolute value. Second, denote the expected wage of minority (nonminority) students graduating from college \( p \) with HC level \( s \) as \( U_M(p,s) (U_N(p,s)) \). Suppose we observe a positive measure of \((p,s)\) such that \( U_M(p,s) = U(p,s,\theta_M(s;a)) \neq U_N(p,s) = U(p,s,\theta_N(s;a)) \). Our exclusion restriction implies that it must be the case that \( \theta_M(s;a) \neq \theta_N(s;a) \), which insures that the rank correlation between \( s \) and \( \theta \) is greater than \(-1\). Thus, in expectation the matrix of regressors

\[
X(\alpha) = \begin{bmatrix}
1 & \log(p_1) & \log(s_1) & \psi(s_1, D_{M1}; \alpha) \\
1 & \log(p_2) & \log(s_2) & \psi(s_2, D_{M2}; \alpha) \\
\vdots & \vdots & \ddots & \vdots \\
1 & \log(p_1) & \log(s_1) & \psi(s_1, D_{M1}; \alpha)
\end{bmatrix}
\]

will have full rank for any configuration of the parameters \( \alpha \). Essentially, \( \psi(S, D_M; \alpha) \), which is derived from economic theory of investment behavior, serves as a control function and allows the researcher to separate out the influence of unobserved student characteristics \( \theta \) from achievement \( s \) and school quality \( p \). Finally, recall that by supermodularity (equations (10)) there is a unique, monotone solution \( \theta_j(s; \alpha) \), which implies that the type distributions

\[
F_j(\theta) = G_j \left[ \theta_j^{-1}(\theta; \alpha) \right], \quad j \in \{ \mathcal{M}, \mathcal{N} \}
\]

are known if \( \alpha \) is known. This logic yields the following, final result on structural identification:

**Proposition 4.11.** Under Assumptions 3.2 – 4.10, wage parameters \( \alpha \) and cost type distributions \( F_M(\theta) \), \( F_N(\theta) \) are identified, provided that

(i) \( 0 < \mu < 1 \)

(ii) \( \sigma_v > 0 \),

(iii) \( \exists (p,s) \) such that \( U_M(p,s) \neq U_N(p,s) \)

(iv) \( \nabla_\alpha Q(\alpha) \) is positive definite on \((0, \infty) \times (0,1)^3\), where

\[
Q(\alpha) = \left[ D_w \ (W - X(\alpha)\alpha) \right]^T \left[ D_w \ (W - X(\alpha)\alpha) \right],
\]

\( W = [\log(w_1), \log(w_2), \ldots, \log(w_1)]^T \) is the vector of corresponding observed incomes, and \( D_w = [D_{w1}, D_{w2}, \ldots, D_{w1}] \) is a vector of sampling weights.
4.2. A Two-Stage, Semiparametric Estimator. We now construct a two-stage GMM estimator to implement our identification strategy. First, we recover the preliminary model parameters that do not directly depend on our strategic investment model, $\beta^s$, $\beta^p$, $P_M(s)$, $P_N(s)$, and $\sigma_e$. Then we use these estimated values and the first-order conditions to recover the utility parameters $\alpha$ and the learning cost distributions $F_M(\theta)$ and $F_N(\theta)$.

4.2.1. Stage I Estimation. The first hurdle to overcome is to find a computationally tractable way of representing the selected joint distribution of $(P, S)$ conditional on graduation. High-dimensional density estimation is a difficult problem both computationally and because of the rapid decay of optimal statistical convergence rates as the dimensionality of the underlying random variable grows. Recent work in the auctions literature by Hubbard, Li, and Paarsch [41] has employed parametric copula functions to solve this problem. Sklar’s Theorem states that any absolutely continuous joint distribution can be represented as a composition $F_{P,S}(p,s|j, \text{grad}) = C_j[F_P(p|\text{grad}), F_S(s|\text{grad})|\text{grad}], j \in \{M,N\}$, where $C_j(\cdot, \cdot|\text{grad})$ is a unique copula function. This implies that the rapidly increasing computational cost and data-hungriness of nonparametric estimators come from the complexity of the correlation structure $C_j$ since the complexity of the marginal distributions does not increase with the dimension of the joint distribution. Hubbard, Li, and Paarsch [41] therefore propose a flexible approach to estimating the marginal distributions, while simplifying the copula with parametric assumptions for tractability. This allows the econometrician to maintain the familiar $\sqrt{T}$ convergence rate when estimating a multi-dimensional joint distribution. We follow this dimension reduction strategy by adopting the Gumbel-Hougaard copula, $C(r,q;v) = \exp \left[ - (\log(r))^v + (\log(q))^v \right]^{1/v}, v \geq 1$ to represent the correlation structure in the joint distribution between human capital $s$ and college placement $p$. One advantage of the Gumbel-Hougaard copula is that it implies a closed-form expression for the Kendall’s $\tau$ rank correlation index: $\tau_{PS}^j = \frac{v_j - 1}{v_j}, j = M,N$.

For the marginal distributions of human capital and college placement, we propose a flexible approach based on B-splines. Like orthogonal polynomials, B-splines are a class of finite-dimensional functional forms that can be made arbitrarily flexible. Their added benefit is that they are numerically much better behaved than global polynomials (e.g., Chebyshev). We divide all model parameters into two sets, placing type distributions and wage equation parameters in one set and all other model components in the other set. The main difference

\[\text{See Silverman [55] and Campo, Perrigne, and Vuong [16] for a lengthy discussion on this concept.}\]

\[\text{We also experimented with several other copula functions including the Frank copula, } C(r,q;v) = \frac{-1}{v} \log \left[ 1 + \frac{\exp(-rv) - 1}{\exp(-rv) + \exp(-qv) - 1} \right], v \in \mathbb{R} \setminus \{0\}; \text{ the Clayton copula, } C(r,q;v) = \left[ \max\left\{ r^{-v} + q^{-v} - 1; 0 \right\} \right]^{-1/v}, v \in [-1,\infty) \setminus \{0\}; \text{ and the Gaussian copula, } C(r,q;v) = \Phi_N^{-1}(r), \Phi_N^{-1}(q) \right\} \text{ where } \Phi \text{ is a standard normal CDF and } \Phi_N \text{ is} \]

\[\text{a bivariate normal CDF with correlation matrix } R = \begin{bmatrix} 1 & v \\ v & 1 \end{bmatrix}, v \in [-1,1]. \text{ All produced very similar results, which is consistent with the assumption that our parametric restriction of the copula function provides a robust approximation to the nonparametric correlation structure.}\]

\[\text{For a brief primer on B-splines and their advantages in empirical auctions models, see Hickman, Hubbard, and Paarsch [38] and Bodoh-Creed, Boehnke, and Hickman [12].}\]
between the two sets is that estimating the former requires imposition of Bayes-Nash equilibrium theory. The latter does not and can therefore be estimated separately in a first stage. With the above functional representations of the marginal and joint distributions in equations (3) – (6) and (9), it is relatively straightforward to understand how, in principle, one could build a GMM-style estimator for all stage I parameters. The single index parameters are chosen to maximize the rank-predictability (i.e., Kendall’s $\tau$) of human capital $s$ for college placement $p$ given the empirical joint distribution of $(E, A, P)$. The joint moments of graduation rates, achievement, and college quality across different colleges are used to pin down the graduation rate parameters $\beta^p$. The empirical joint distribution of $p$ and $s$ for graduates is used to pin down the B-spline-copula representation of the same conditional distribution, and then Bayes’ Rule in conjunction with the graduation probability parameters is used to recover the unconditional joint distribution of $(P, S)$ for enrollees. Finally, the matching shock parameter $\sigma_e$ is then chosen to match the empirical rank correlation in the distribution of $(P, S)$ (for enrollees) as closely as possible.

While it is relatively straightforward to establish intuitive connections between the moments in the data and stage I model parameters, a formal definition of the estimator is notationally quite intense since the majority of these terms must be estimated simultaneously. Therefore, we leave the data and stage I model parameters, a formal treatment of the GMM stage I estimator to an appendix. In what follows, for notational convenience we represent the collective stage I parameters by $\hat{\pi} = [\hat{\gamma}_M^p, \hat{\gamma}_N^p, \hat{\gamma}_M^q, \hat{\gamma}_N^q, \hat{v}_M, \hat{v}_N, \hat{\beta}^s, \hat{\beta}^p]^\top$ and $\hat{\sigma}_e$, which includes, respectively, estimates for the B-spline parameters for the race-specific marginal distributions of $P$ conditional on graduation, $(\gamma_M^p, \gamma_N^p)$; the B-spline parameters for the race-specific marginal quantile functions of $S$ conditional on graduation, $(\gamma_M^q, \gamma_N^q)$; the (unconditional) race-specific copula parameters $(v_M, v_N)$; the single-index parameters $\beta^p$; the graduation probability parameters $\beta^s$; and the matching shock parameter $\sigma_e$.

4.2.2. Stage II Estimation. Our Stage I estimator was based on a set of intuitive moment conditions that were notationally intense to formalize. Stage II estimation is the reverse: notationally compact and with considerable computational complexity under the surface. Amending notation somewhat, for computational convenience we first parameterize the assignment functions and inverse equilibrium strategies for group $j \in \{M,N\}$ as flexible B-splines, similarly as we did for other functionals in stage I. B-spline functions are defined as linear combinations of basis functions $B_{jk}(t)$, $k = 1, \ldots, K_j^1 + 3$ and $B_{jk}(s)$, $k = 1, \ldots, K_j^3 + 3$.\footnote{As explained in the appendix, the fundamental building block of B-spline functions are knot vectors $k^1_j = \{t = k^1_{j1} < \cdots < k^1_{jk_j+1} = T\}$ for the assignment functions, and $k^3_j = \{s = k^3_{j1} < \cdots < k^3_{jk_j+1} = S\}$, for equilibrium strategies. These knot vectors partition the domains of the assignment functions and inverse strategies into $K_j^1$ and $K_j^3$ smaller segments, respectively. On each of these sub-intervals, the function fit is governed by a strict subset of the overall basis functions; this locality property is what makes B-splines better behaved than Chebyshev polynomials. The knot vectors also uniquely determine the shapes of the basis functions themselves (see appendix for further detail).} When we combine these with weights $\lambda^1_j \in \mathbb{R}^{K_j^1+3}$ and $\lambda^3_j \in \mathbb{R}^{K_j^3+3}$ our parameterized B-spline functions have the form $P_j(t; \lambda^1_j) = \sum_{k=1}^{K_j^1+3} \lambda^1_{jk} B^1_{jk}(s)$ and $\theta_j(s; \lambda^3_j) = \sum_{k=1}^{K_j^3+3} \lambda^3_{jk} B^3_{jk}(s)$. Given a pre-specified grid of points $N_{	ext{grid}}$ for each assignment function $M 	imes N$ and $N 	imes M$, we can use these B-splines to build joint probability distributions for $(E_j, A_j, P_j)$ and $(E_j, A_j, S_j)$, which we then use to estimate the joint moments of graduation parameters $\hat{\beta}^s$ and $\hat{\beta}^p$.
{t_1, \ldots, t_{K^T}} \text{spanning } [L, T], \text{ the assignment function weights are chosen to satisfy}

\hat{\lambda}_j^t = \arg\min_{\lambda \in \mathbb{R}^{K^j}} \left\{ \sum_{k=1}^{K^T} \left( P_j(t_k; \lambda) - F^{-1}_{p_i} [H_j(t_k; \hat{\pi}, \hat{\sigma}_e); \hat{\pi}] \right)^2 \right\}

\text{Subject to : } \lambda_k < \lambda_{k+1}, \ k = 1, \ldots, K_j^s + 2.

The assignment mappings are a function of Stage I parameters, which are taken as fixed in Stage II, which is why we express the relevant B-spline weights using hat notation. On the other hand, the inverse equilibrium strategies solve

\lambda_j^s(\alpha) = \arg\min_{\lambda \in \mathbb{R}^{K^j}} \left[ D_{wi} \left( \theta_1(s_i; \lambda) - \hat{\theta}_j^t \right) \right]^2 \]

\text{Subject to :}

\hat{\theta}_{ji} c'(s_i) = E_c \left[ U_p \left( P_j(s_i + \varepsilon; \lambda_j^s), s_i, \hat{\theta}_{ji}; \alpha, \hat{\pi} \right) P_j'(s_i + \varepsilon; \lambda_j^s) \right] + E_c \left[ U_s \left( P_j(s_i + \varepsilon; \lambda_j^s), s_i, \hat{\theta}_{ji}; \alpha, \hat{\pi} \right) \right]

\lambda_k > \lambda_{k+1}, \ k = 1, \ldots, K_j^s + 2, \ j \in \{M, N\},

and are not fixed during Stage II. The B-spline strategies are a function of the Stage I parameters as well as the Stage II utility parameters \alpha, and therefore must be adjusted each time these are updated during estimator runtime.

Essentially, the vectors \hat{\lambda}_j^t and \lambda_j^s(\alpha) are ancillary parameters that will be embedded inside the wage regression equation. The latter defines the control function for unobservable types and allows the econometrician to enforce monotonicity of the strategies as required by our theory. With this in mind, we adopt a shorthand notation for our flexibly parameterized control function,

(12) \psi \left( s, D_{Mi}; \alpha, \hat{\pi}, \hat{\sigma}_e, \hat{\lambda}_M^t, \hat{\lambda}_N^s \right) = \log [D_{Mi}\theta_M (s; \lambda_M^s(\alpha)) + (1 - D_{Mi})\theta_N (s; \lambda_N^s(\alpha))],

with the extra parameter arguments emphasizing its implicit dependence on Stage I objects. Moving forward we suppress the additional pre-determined parameter arguments for notational simplicity. We can now re-express the matrix of explanatory variables as

\[ X(\alpha) = \begin{bmatrix} 1 & \log(p_1) & \log(s_1) & \psi(s_1, D_{M1}; \alpha) \\ 1 & \log(p_2) & \log(s_2) & \psi(s_2, D_{M2}; \alpha) \\ \vdots & \vdots & \vdots & \ddots \\ 1 & \log(p_1) & \log(s_1) & \psi(s_1, D_{M1}; \alpha) \end{bmatrix}, \]

from which our utility parameter estimator is defined by

\[ \hat{\alpha} = \arg\min_{\alpha \in \mathbb{R}^+ \times [0, 1]^p} \left\{ [D_w (W - X(\alpha) \alpha)]^T [D_w (W - X(\alpha) \alpha)] \right\}. \]

In words, Stage II consists of estimating the wage regression, where the third explanatory variable is a nonlinear function of all model parameters with a form that is derived from the equilibrium conditions of our theory of HC investment.
The final step of estimation is to recover the type distributions. Note that with the utility parameter estimates in hand, we can recover an estimate of the private type for each student in the sample using equation (12) above. Therefore, we estimate the type distributions via GMM by fitting the parameters of a flexible B-spline representation of the type CDFs to the empirical distributions of

\[ \hat{\psi}(s, D_M; \hat{\alpha}, \hat{\pi}, \hat{\sigma}_e, \hat{\lambda}_M, \hat{\lambda}_N) = \log [D_M \theta_M(s; \lambda_M^\theta(\hat{\alpha})) + (1 - D_M) \theta_N(s; \lambda_N^\theta(\hat{\alpha}))]. \]

See the appendix for a complete explanation of this step of estimation.

A final comment on various practical issues is in order before moving on. The estimator as defined above (and in the appendix) includes various ancillary components, such as choices of knot vectors (which is equivalent to choosing the number of basis functions and hence, the flexibility of the B-splines), sampling weights, and \( \kappa \). We include a complete discussion of these issues in an appendix.

4.2.3. Asymptotics and Standard Errors. It is evident that the empirical strategy we propose above falls within the broad class of Generalized Method of Moments estimators. In terms of asymptotic theory, there are two alternative views one could assume. Our empirical implementation uses 5 knots each (and therefore 8 basis functions and 8 parameters) to estimate the marginal distributions of \( P, S, \) and \( \theta \). One could potentially view this as a parametric class assumption which is held fixed as the sample size grows. Under this view, standard GMM asymptotic theory (e.g., see Hayashi [35]) establishes consistency and asymptotic normality of the complete set of parameter estimates, with convergence at the standard rate of \( \sqrt{T} \). For the current data set, the 8-parameter parametric restrictions allow for a remarkably tight fit to the raw empirical CDFs they represent.

Alternatively, if one wished to consider gradually increasing the flexibility of the B-spline distributional forms as the sample size grows, then our estimator belongs to a broad class of semiparametric M-estimators for which large-sample properties have also been explored. In particular, since B-splines are mathematically equivalent to piecewise splines with differentiability conditions imposed at the interior knots (see de Boor [24]), our Stage I estimators fit within a subset of this broad class–spline series least squares estimators–for which consistency and pointwise asymptotic normality are known (see Chen [20] and Huang [40]). Here, pointwise normality refers to the estimated functionals from Stage I, \( G_B(b; \hat{\alpha}_b) \) and \( G_R(r; \hat{\alpha}_r) \), rather than to the parameters themselves. Since Stage II empirical objects (including the B-spline type distributions) are all smooth transformations of the functionals and other parameters estimated in Stage I, they are also asymptotically pointwise normal. Of course, the optimal rate at which curvature restrictions on the B-spline functional form—i.e., the rate at which the number of knots should increase—is an open question and beyond the scope of the current exercise.

Regardless of which view one prefers, one can conclude that a well-defined limiting distribution exists for our estimates. In order to explore the role of sampling variability we employ a block-bootstrap procedure which involves re-sampling 1000 times from the race-specific B&B subsamples. Following the notation above, we separately re-sample \( I_M \) student observations
Table 3. ESTIMATES: Single Index Function and Matching Shock Variance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
<th>P-Value</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1^s(e)$</td>
<td>0.0507***</td>
<td>(9.63 x 10^{-4})</td>
<td>&lt; 0.001</td>
<td>[0.0488, 0.0527]</td>
</tr>
<tr>
<td>$\beta_2^s(e^2)$</td>
<td>0.1580***</td>
<td>(0.0095)</td>
<td>0.006</td>
<td>[0.1415, 0.1695]</td>
</tr>
<tr>
<td>$\beta_3^s(a)$</td>
<td>0.3023***</td>
<td>(0.0085)</td>
<td>&lt; 0.001</td>
<td>[0.2877, 0.3179]</td>
</tr>
<tr>
<td>$\beta_4^s(a^2)$</td>
<td>0.2906***</td>
<td>(0.0096)</td>
<td>&lt; 0.001</td>
<td>[0.2754, 0.380]</td>
</tr>
<tr>
<td>$\beta_5^s(e \cdot a)$</td>
<td>0.2229***</td>
<td>(0.0090)</td>
<td>&lt; 0.001</td>
<td>[0.2105, 0.2410]</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>0.0275</td>
<td>(7.25 x 10^{-4})</td>
<td>—</td>
<td>[0.026, 0.029]</td>
</tr>
<tr>
<td>$\tau_{PS}(\sigma_e</td>
<td>M)$</td>
<td>0.867</td>
<td>(0.01458)</td>
<td>—</td>
</tr>
<tr>
<td>$\tau_{PS}(\sigma_e</td>
<td>N)$</td>
<td>0.893</td>
<td>(0.0021)</td>
<td>—</td>
</tr>
</tbody>
</table>

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

5. ESTIMATION RESULTS

5.1. ESTIMATES: Single Index Function $S(e, a)$ and Matching Shock Variance $\sigma_e^2$. Recall that the HC single index function was specified as a quadratic polynomial in SAT score $e$ and GPA $a$. The values for these parameters along with their standard errors are contained in Table 3. Recall also that a more flexible cubic form did not improve our model fit (see footnote 11). The HC index is convex in both $e$ and $a$ and admits significant complementarities between the arguments. To help the reader understand the marginal effects of $e$ and $a$ on the HC function, we present marginal effect statistics at the median values found in the B&B dataset. The marginal effect of a one standard deviation change of $e$ is 0.411, while the marginal effect of a one standard deviation change in $a$ is equal to 0.131. In other words, $a$ is roughly 2.5 times as important as $e$ for determining a median student’s HC single index. Due to convexity of $s(e, a)$, these marginal effects are maximized for the highest values of $(e, a)$ and minimized for the lowest values.

Figure 1 illustrates our single index equation. Each line depicts the effect of academic record $a$ on the HC index while holding exam score $e$ fixed at one of its quartiles. The lesser importance of $e$ relative to $a$ is reflected in the fact that the difference between the 75th and 25th percentiles lines is less than 0.1, while the difference between the 75th and 25th quintiles on the line describing $S(e_{median}, a)$ is close to 0.2. The upward curve of the lines is a result of the convexity of the
single index with respect to the student’s GPA. We include 95% confidence intervals on the line describing $S(e_{\text{median}}, a)$ at each decile of the distribution of $a$.

Table 3 also presents our estimate of the matching shock standard deviation, $\sigma_\varepsilon$, and group-specific Kendall’s $\tau$ in order to provide different perspectives on the matching frictions in the college admissions market. The matching shock variance provides insight into how informative the NHC realizations are for students’ underlying HC choices. Our estimates imply that the noise-to-signal ratio of the matching shock (the ratio of $\sigma_\varepsilon$ to a standard deviation of HC) is 18.9%. Kendall’s $\tau$ provides an alternative way to interpret the economic meaning of the matching shock variance. Our rank-order contest structure implies Kendall’s $\tau$ can never be negative, but a value near 0 would mean that the matching shock almost entirely determines the allocation of students to schools. On the other hand, a value close to 1 means the matching shock plays only a negligible role in assignment of students to colleges. Since Kendall’s $\tau$ for minority students is estimated at 0.867, this means for two randomly selected minority students there is a 93.4% chance that the student with higher HC will enroll in a higher quality college. Likewise, this probability for the minority group is 94.6%. Thus, while matching shocks play a nontrivial role, our empirical model suggests a high degree of assortativity in the college market.

5.2. ESTIMATES: Graduation Probability Function, $\rho(p, s)$. Point estimates and standard errors for the graduation probability parameters are displayed in Table 4. We again compute the marginal effect of a one standard deviation change in $p$ and $s$ (from the median values) on $\rho(p, s)$ as our metric of the relative importance of these variables for determining the probability of graduation. In this case, we compute the median and standard deviation statistics using the selection-corrected distributions of the respective variables. The marginal effect of a one standard deviation change in $p$ is 0.026, while the marginal effect of a one standard deviation change

---

We experimented with more flexible matching shock models, but we found that our results were unchanged. For example, if we include a linear trend in $s$, then we estimate that $\sigma_\varepsilon(s) = 0.0279 - 0.0006s$, which is essentially the same as the result in Table 3.
### Table 4. Graduation Probability Estimates $\hat{\rho}(p, s)$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
<th>P-Value</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1^p (p)$</td>
<td>0.0500</td>
<td>(0.0805)</td>
<td>0.415</td>
<td>[-0.087, 0.2377]</td>
</tr>
<tr>
<td>$\beta_2^p (p^2)$</td>
<td>-0.2042*</td>
<td>(0.1391)</td>
<td>0.097</td>
<td>[-0.4568, 0.0689]</td>
</tr>
<tr>
<td>$\beta_3^p (p^3)$</td>
<td>0.2190*</td>
<td>(0.1305)</td>
<td>0.054</td>
<td>[-0.0028, 0.4972]</td>
</tr>
<tr>
<td>$\beta_4^s (s)$</td>
<td>0.8445***</td>
<td>(0.1658)</td>
<td>&lt; 0.001</td>
<td>[0.6097, 1.3085]</td>
</tr>
<tr>
<td>$\beta_5^s (s^2)$</td>
<td>-0.0938***</td>
<td>(0.1421)</td>
<td>&lt; 0.001</td>
<td>[-0.9295, -0.0136]</td>
</tr>
<tr>
<td>$\beta_6^s (s^3)$</td>
<td>0.0077</td>
<td>(0.0129)</td>
<td>0.146</td>
<td>[-0.0172, 0.036]</td>
</tr>
<tr>
<td>$\beta_7^p (p \cdot s)$</td>
<td>0.0735</td>
<td>(0.3753)</td>
<td>0.936</td>
<td>[-0.4381, 1.2508]</td>
</tr>
<tr>
<td>$\beta_8^p (p \cdot s^2)$</td>
<td>0.0732</td>
<td>(0.1443)</td>
<td>0.476</td>
<td>[-0.0795, 0.1990]</td>
</tr>
<tr>
<td>$\beta_9^p (p^2 \cdot s)$</td>
<td>0.0760</td>
<td>(0.2739)</td>
<td>0.633</td>
<td>[-0.3497, 0.8685]</td>
</tr>
<tr>
<td>$\beta_0^p$ (const.)</td>
<td>-0.0497*</td>
<td>(0.0236)</td>
<td>0.065</td>
<td>[-0.0814, 0.0048]</td>
</tr>
</tbody>
</table>

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and $$$, respectively.

Figure 2 illustrates the relative impact of $p$ and $s$ on graduation probabilities. Each line depicts the effect of HC on graduation probability holding that college quality fixed at one of its quartiles. We have also plotted bootstrapped 95% confidence bounds on graduation rates at a college of median quality at the deciles of the distribution of HC. The convexity of $\rho(p, s)$ is evident from the increasing difference between the lines as the college quality improves, meaning that college quality matters most above the 75th percentile of college quality. The complementarity between $p$ and $s$ results in an increasing spread between the lines as $s$ increases, meaning that college quality is more important for higher achieving students.

in $s$ in 0.143. This means that $s$ is roughly 5.5 times as important as $p$ for determining college graduation probabilities.
5.3. **ESTIMATES: Selection-Corrected Joint Distributions** $f_j(p,s)$. Figure 3 displays the distributions of HC levels for each demographic group, including the selected sample of graduates from the raw data (dashed lines) and the selection-corrected distributions for all enrollees (solid lines). Figure 4 displays the distributions of college seats allocated to each group, including the selected raw samples (dashed lines) and the selection corrected distributions for all enrollees (solid lines). The CDF plots also include 95% confidence bounds at the deciles of the population-wide distributions. The first plot illustrates the achievement gap, a stochastic dominance relationship between minority HC and non-minority HC. The second plot illustrates the enrollment gap, a similar stochastic dominance relationship between minority and non-minority college quality.

Figure 4 does not have any large jumps in the CDFs, which is consistent with our model’s assumption that there is a continuum of college seats. In 1988 (four years prior to the graduating class of AY1992-1993) there were a total of 1,644,340 freshman seats in the market with any single school having only a negligible market share. The largest college in 1988 (Ohio State) had a total market share of only 0.76% of new freshman seats, and the next ten largest schools (in descending order of size: UT-Austin, Michigan State, Akron, BYU, Purdue, Minnesota, Northeastern, Toledo, Texas A&M, and Pittsburgh) combined for only 5.4% of total market supply. The mean, median, and standard deviation of market shares for individual universities were 0.091%, 0.047%, and 0.102%, respectively. Thus, a large, atomless market approximation appears reasonable.

5.4. **ESTIMATES: Minority Markup Function, $\bar{T}$**. Our estimate of the markup function is based on equation 9. Figure 5 describes the markup function in two separate ways. The horizontal axes of both panels display quantile ranks of NHC for nonminority students. The top panel describes the shape of $\bar{T}$. Its vertical axis displays the quantile rank of subsidized NHC within the non-minority NHC distribution. If a minority student has an NHC at the quantile rank marked on the horizontal axis, the student gets the same college assignment as a nonminority student with an NHC at the quantile rank denoted on the vertical axis. For example, the plot shows that a minority student with an NHC equal to the median value of the nonminority population gets
CDF ESTIMATES

\begin{align*}
\hat{F}_{ap}^{P_m}(p | \text{Graduate}) \\
\hat{F}_{ap}^{P_n}(p | \text{Graduate}) \\
\hat{F}_{ap}^{P_m}(p) \\
\hat{F}_{ap}^{P_n}(p)
\end{align*}

Figure 4. School Quality Distribution

\begin{align*}
H_N(t) \\
H_N(\tilde{T}(t)) \\
\text{School Quality Gap}
\end{align*}

\begin{align*}
F_P(P_{ap}^m(t)) - F_P(P_{ap}^n(t))
\end{align*}

Figure 5. Noisy Human Capital Markup Function, \(\tilde{T}\)
the same college assignment as a nonminority student at the 64\textsuperscript{th} percentile of the nonminority population. The dashed line denotes the 45° line for reference.

The bottom pane of Figure 5 describes the effect of the admissions preference schemes in terms of school quality. Again, the horizontal axis denotes quantile ranks of the distribution of nonminority NHC levels. The vertical axis denotes the gap in the quantile rank of college quality between a minority student and a nonminority student at each NHC quantile. For example, the plot shows that if two students from different groups both have an NHC value equal to the median of the nonminority population, then the minority student is assigned to a school whose quantile rank is 0.13 higher in the school quality distribution.

We display 95% confidence bounds at the deciles of NHC. What our plot reveals is that the effect of the status quo admissions preference scheme is insignificant at colleges in the bottom decile, but is statistically and economically significant across the rest of the college quality spectrum. This result improves upon previous empirical work in various ways. Several papers have estimated a substantial impact of AA, including Bowen and Bok [14]; Chung and Espenshade [19]; and Chung, Espenshade, and Walling [21], but these studies used data from elite colleges, whereas ours uncovers a market-wide picture. The most similar previous study is Kane [42], which also used a nationally representative sample (the High School and Beyond (HS&B) survey), but estimated a significant role for AA only in the top quintile of the market.

Several differences exist between Kane [42] and our study. First, we use measures of final market allocations (enrollment data), whereas Kane [42] uses applications data which may not fully reflect final enrollment decisions. Second, the HS&B data contain potentially important sources of sample selection that could affect probit regression results in unpredictable ways. HS&B respondents were asked their two top choices (sample truncation) among the schools to which they applied (endogenous selection in student-school pairs), and whether they were accepted. Third, HS&B focuses on students who entered college in 1980 and 1982, while the B&B focuses largely on the freshman class of 1988.

Given the above factors, it is hard to pinpoint the source of the differences between our results and Kane [42]. At the end of the day, our estimates of the markup function are the most directly data-driven component of the empirical model: they do not depend on Bayes-Nash equilibrium theory, but instead hinge only on a Stage I reduced-form sample-selection correction to map our observed set of college graduates into the original set of college enrollees. The intuition behind our result is that we see marginal distributions of HC and college quality by race, and a color-blind world implies a very specific form to which the latter must conform, given the former. However, our Stage I reduced-form data products deviate from this specific form, and in such a way that more generous admissions practices toward minorities must exist on the majority of the market in order to rationalize observed allocations from observed achievement.

5.5. **ESTIMATES: Match Utility and Learning Cost Type Distributions.** The type distributions are presented in Figure 6. The top panel displays the type CDFs $F_M(\theta)$ and $F_N(\theta)$ with 95% confidence bounds represented by the shaded areas. The type distribution of the minority students stochastically dominates the type distribution of nonminority students, which is consistent
with minority students having higher learning costs than nonminority students. The bottom panel of Figure 6 plots the pointwise difference between the CDFs with 95% confidence bounds. Our estimates strongly suggest that the difference between the distribution of types between the two groups is significant at all cost quantiles.

This result would seem to conform with a large body of empirical evidence on stark racial differences in access to resources that affect childhood development in the US. For example, Black and Hispanic children are nearly three times as likely to live below the poverty line (see Kena, et. al. [44]). They are also much less likely to be covered by health insurance (see Smith and Medalia [56]) or to be raised by parents with bachelor degrees (see Fox, Kewal, and Ramani [31]). Moreover, holding income level fixed, Blacks and Hispanics are also much more likely to attend under-performing schools that serve poorer student bodies relative to their White and Asian counterparts of similar incomes (see Reardon, Townsend, and Fox [53], Reardon [51], and Reardon, Kalogrides, and Shores [52]). With these empirical facts in mind, the estimated stochastic dominance relationship in learning costs would seem a natural, though unfortunate, consequence of resource stratification by race.

Table 5 presents our estimates of the returns to various inputs of income production including a more selective college, more HC, and lower learning costs.\(^\text{20}\) The first takeaway is that pre-college HC choice, while important for determining the graduation probability, has very little

\(^{20}\text{All of the bootstrap confidence intervals are biased-corrected and accelerated.}\)
Table 5. Estimates of the Wealth Equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p$</td>
<td>0.1349***</td>
<td>(0.0260)</td>
<td>[0.0894, 0.1902]</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>$1.181 \times 10^{-6}$</td>
<td>(0.0306)</td>
<td>[$-1.000 \times 10^{-5}$, 2.412 $\times 10^{-4}$]</td>
</tr>
<tr>
<td>$\alpha_\theta$</td>
<td>0.0574**</td>
<td>(0.0386)</td>
<td>[0.009, 0.1478]</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>$79,108***$</td>
<td>($4,735$)</td>
<td>[$69,105$, $87,499$]</td>
</tr>
</tbody>
</table>

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

Effect on wages conditional on graduation. The second takeaway is that both school quality and the agent’s unobservable type have economically and statistically significant effects on income production. To get a sense for the scale of the relative importance of these variables, we compute the marginal effect of a one standard deviation change in the inputs to the income production function. The marginal effect of a one standard deviation increase in $\log(p)$ is 3.44 times larger than the marginal effect of a one standard deviation reduction $\log(\theta)$, which is in turn $5.37 \times 10^4$ times larger than the marginal effect of a one standard deviation increase in $\log(s)$.

In summary, we estimate that household income 10 years following graduation is driven primarily by the quality of the college attended, although an agent’s unobservable type also plays a significant role. There is a sizable literature on the returns to a higher quality college, including Brewer, Eide, and Ehrenberg [15]; Dale and Krueger [23]; Black and Smith [11]; Long [45]; and Andrews, Li, and Lovenheim [5]). In general, the evidence is clear that more selective colleges benefit poorer students significantly. Other than that, there is some disagreement as to the magnitude of the return for more affluent students. Our results support the view that a higher quality college is a resource from which all students benefit. Holding $s$ and $\theta$ fixed at their relative medians, a move from being placed at the 25th percentile college to the 75th percentile college induces an estimated shift of roughly $7,500 in annual household income. We are also the first paper to directly quantify the role of unobserved heterogeneity in the returns to an education, which we find is significant though not dominant: reducing the learning cost from the upper quartile of $\theta$ to the lower quartile, holding $p$ fixed at the median, induces an increase of roughly $2,100 in annual household income. On the other hand, while a student’s HC choices prior to college do not significantly affect the wage conditional on graduation, these investments have a strong influence on the probability of graduating college.

5.6. ROBUSTNESS CHECKS. We conducted a number of robustness checks to stress-test the assumptions underlying our model.

First, we considered adding an additional parameter to the cost model, $c(s) = \exp(\nu(s - s))$, for added flexibility. Identification would then require additional conditions, for example, individual rationality and incentive compatibility assumptions for marginal market participants who are nearly indifferent to college attendance versus entering the workforce. We experimented with this approach in our empirical implementation, and in practice the extra parameter adds
little explanatory power within the exponential family of cost functions. It affects the behavior of the model most significantly for very low achieving college students where data are sparse. Elsewhere, changes in the curvature parameter $\nu$ are largely compensated for by corresponding changes to the scale of $\theta$ with little change to the economic implications of the point estimates. Thus, we focus on a simplified version of the cost model where $\nu = 1$. As a robustness check in our empirical application, we also estimated the model with an alternative power law cost function of the form $C(s, \theta) = \theta(s + 1)^2$ and found that model estimates were qualitatively very similar, suggesting that our functional form assumption for costs is not unduly driving the results.

Second, we estimated a model wherein the matching shock was linear in the HC choice of the agents. This would allow, for example, the matching shock to be larger for students with lower HC choices, which might reflect higher effort costs that impedes HC accumulation and searches for potential colleges to apply to. If we include a linear trend in $s$, then we estimate that $\sigma_{\epsilon}(s) = 0.0279 - .0006s$, which is essentially the same as the result in Table 3. We conclude that the matching shock does not significantly vary with HC choice. We also experimented with allowing a distinct matching shock parameter for minority and nonminority students and requiring that each group’s parameter best rationalize the correlation of $p$ and $s$ amongst students in that group. We did not find that the parameters differed significantly, which is not surprising given our inability to detect a trend in the matching shock parameter. For example, if the matching shock was larger for minorities, then this ought to have caused us to detect a decreasing trend in the matching shock since the minority students disproportionately choose to accrue less HC.

Third, we considered alternative forms for the wage equation. For example, we explored including the estimated matching shock as an additional parameter in the wage equation, in effect estimating a wage equation with the form: $u(P_i, S_i, \theta_i) = \alpha_0 P_i^{\alpha_p} S_i^{\alpha_s} \theta_i^{-\alpha_0} \epsilon^{\alpha_\epsilon}$ where $\epsilon$ denotes the matching shock realization. Our estimates of this model imply that the marginal effect of a one standard deviation change of $p$ and $\theta$ are almost equal, the effect of $s$ is negligible, and the effect of a one standard deviation change of $\epsilon$ is roughly 10% as important as that of either $p$ or $\theta$. In short, the estimates are qualitatively the same as in our baseline model.

We also estimated a model wherein we imposed a uniform 11% animus-based household income penalty on minority students to explore the possible effects of violations of our exclusion restriction (Assumption 4.3). The particular choice of 11% was motivated by the magnitude of the black-white wage gap studied by Fryer et al. [32], who found that this gap is explained by factors other than animus. We found that our estimates of the wage equation parameters did not qualitatively change, which suggests that our model and identification strategy are relatively robust to uniform deviations from our exclusion restriction.21 This is also consistent with our failure in Appendix B to find that minority status predicts $\theta$, conditional on controls for childhood home environment factors. We interpret these findings as showing that there is nothing in our data that reveals an obvious violation of this assumption.

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21 Recall from the discussion in Section 4.1 above that it would require a nonlinear, college-specific, racial-animus penalty function to completely wipe out the identifying power of racial incentive differences from AA.
6. COUNTERFACTUAL POLICY EXPERIMENTS

The issue of demographic diversity on college campuses has been controversial, and the subject of repeated judicial scrutiny in the United States. The cornerstone of Supreme Court jurisprudence regarding AA in college admissions is the 1978 case *University of California Regents v. Bakke* [1]. Justice Powell’s opinion established that the government has a compelling interest in encouraging diversity in university admissions founded on principles of academic freedom and a university’s right to take what actions it feels necessary to provide a high quality education to its students, but using racial admission quotas is an illegal method to do so. With ongoing legal challenges to race-based AA in college admissions its future may be uncertain, and in recent years some state education systems have begun to experiment with alternative forms of AA that use proxies for race, such as socioeconomic status.

We turn our attention now to an exploration of the economic implications of various changes to the status quo AA we estimated from the data-generating process. We will compare how the 1988 AA admissions policies shaped college diversity relative to the benchmarks of a color-blind system (*i.e.*, a hypothetical AA ban) and a proportional quota system (an even more generous form of AA). For each of the three admissions schemes, we numerically solved for the equilibrium of the model holding our structural point estimates fixed (for technical details see online appendix). Given the importance of school quality in determining outcomes of college enrollees, we will also explore how these different policies influence college graduation rates and post-college income. Finally, we will assess how students from each demographic group alter their HC investment behavior in response to the different AA systems.

Since race-based AA has become legally questionable in the United States, we also consider the extent to which socioeconomic class can serve as a proxy for race from the perspective of a policy-maker who wishes to achieve more racially diverse college student bodies. Within our theoretical framework there is nothing fundamental about race *per se* for defining a target demographic group to benefit from preferential admissions practices. Therefore, we can also experiment with other observable characteristics to define target beneficiary demographics. In another counterfactual experiment we classify the students in our data set as economically advantaged or disadvantaged using their expected family contribution (EFC), a measure based on wealth and income that the Department of Education uses to determine access to means-tested government aid. We then compute equilibria of the model in which admissions quotas are implemented in favor of poorer students, for various cutoffs to define poor versus non-poor.

A final empirical question we address with our college admissions model focuses on the cost of competition. A novel aspect of our model is that we can separate the HC investment incentives into the productive channel, from holding extra HC, and the competitive channel, from the prospect of an improved school assignment in the admissions contest. We close our analysis by assessing the relative strength of these two channels.

6.1. Effects of AA on Minority Enrollment. Table 6 depicts the magnitudes of counterfactual enrollment shifts using the fraction of each demographic group enrolled in each college quality
Table 6. Enrollment by Group and School Quality Quintile

<table>
<thead>
<tr>
<th>College Quality Tier</th>
<th>MINORITIES</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Status Quo</td>
<td>Color-Blind</td>
<td>Proportional Quota</td>
<td></td>
</tr>
<tr>
<td>Top College Quintile</td>
<td>0.1295</td>
<td>0.1010</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Second College Quintile</td>
<td>0.1660</td>
<td>0.1339</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Third College Quintile</td>
<td>0.1486</td>
<td>0.1555</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Fourth College Quintile</td>
<td>0.2806</td>
<td>0.1907</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Bottom College Quintile</td>
<td>0.2753</td>
<td>0.4189</td>
<td>0.2000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>College Quality Tier</th>
<th>NON-MINORITIES</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Status Quo</td>
<td>Color-Blind</td>
<td>Proportional Quota</td>
<td></td>
</tr>
<tr>
<td>Top College Quintile</td>
<td>0.2133</td>
<td>0.2185</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Second College Quintile</td>
<td>0.2064</td>
<td>0.2123</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Third College Quintile</td>
<td>0.2096</td>
<td>0.2085</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Fourth College Quintile</td>
<td>0.1847</td>
<td>0.2019</td>
<td>0.2000</td>
<td></td>
</tr>
<tr>
<td>Bottom College Quintile</td>
<td>0.1859</td>
<td>0.1589</td>
<td>0.2000</td>
<td></td>
</tr>
</tbody>
</table>

The benchmark of full representation of each group in each quintile, 0.2, is mechanically achieved by a quota. Numbers below this imply under-representation and vice versa. For example, 13% of minority students enroll in colleges in the top quintile of the quality distribution under the status-quo AA scheme, but only 10% of them enroll in top colleges under a color-blind scheme. In other words, while the status-quo AA is substantially less generous to minorities at top colleges than a proportional quota would be, a ban of race-based AA would reduce minority enrollment in the top tier by nearly one quarter.

The status quo AA scheme has the intended result in that there is a first-order stochastic dominance (FOSD) shift in the distribution of the quality of colleges in which minority students enroll, relative to the color-blind case. Interestingly, the largest shift is from the lowest quintile of college quality into the second lowest quintile: in a color-blind world, minority enrollment in the bottom quintile would increase by almost 50%. A proportional quota yields an even stronger FOSD shift toward minority enrollment in better colleges, relative to the status quo. For completeness, Table 6 also provides the effects of AA on nonminorities. As expected, changing the AA scheme has the mirror opposite effect on the nonminority students, but the magnitudes of the shifts are small since the non-minority mass is over five times larger.

6.2. Effects of AA on HC Investment. Our second target of analysis is HC investment. It is theoretically ambiguous whether the minority HC choices will increase or decrease under a particular AA scheme. Figure 7 presents the change in minority HC investment under our two counterfactual admissions schemes, so positive (negative) values indicate increases (decreases) relative to the status quo. For ease of presentation, we describe the student’s type in terms of
quantiles of the minority cost distribution, and changes in HC are displayed as a fraction of a standard deviation in the status quo world.

Under a counterfactual color-blind scheme, minority students with learning costs above the median reduce HC output. The intuition for this change has to do with discouragement effects, a common phenomenon in rank-order contests influencing behavior of agents at a relative disadvantage.\footnote{The established contest literature which has explored discouragement effects has typically focused situations with a single prize under competition. One important difference in the current setting is the multi-object aspect of the competition, where everyone wins some prize, but prizes differ substantially by quality level.} Holding one’s own type fixed, if there is a shift in the distribution of competitors so that one falls far enough behind, effort will eventually decrease. Since shifting from color-blind to AA partially shields minority student quantiles from competition by the corresponding (and better-resourced) non-minority quantiles, it is as if the policy engineers such a shift in the type distribution. Therefore, the status quo AA scheme mitigates discouragement effects for high-cost minorities. In contrast, minority students with costs far enough below the median react to heightened competition for top seats under a color-blind scheme by investing more aggressively, since adjusting output upward is less costly for them. For nonminority students, the effects of shifting from status quo to color-blind are the opposite, but of much smaller magnitude since their mean college placement changes by a smaller margin.

The changes in HC investment from our second counterfactual of a proportional quota are of roughly the same magnitude and reversed in sign. We find that the minority students with learning costs above the median increase achievement, while the other minority students reduce it. The reasoning is essentially the same as before: a proportional quota splits the market into two separate competitions within demographic groups, where each one has the same distribution of seats “up for bids”. Since the quantiles of the minority cost distribution are all higher, discouragement effects are now further diminished for high-cost minority students. The lowest-cost minority students rationally reduce investment since they were placing at top schools already under the status quo, and now there is less competitive pressure from students placing at lower schools.
Table 7. Counterfactual Minority Graduation Probability...

<table>
<thead>
<tr>
<th>Learning-Cost Type Tier</th>
<th>Status Quo</th>
<th>Color-Blind</th>
<th>Proportional Quota</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Achiever Quintile</td>
<td>0.562</td>
<td>0.580</td>
<td>0.562</td>
</tr>
<tr>
<td>Second Achiever Quintile</td>
<td>0.389</td>
<td>0.399</td>
<td>0.402</td>
</tr>
<tr>
<td>Third Achiever Quintile</td>
<td>0.299</td>
<td>0.302</td>
<td>0.311</td>
</tr>
<tr>
<td>Fourth Achiever Quintile</td>
<td>0.236</td>
<td>0.229</td>
<td>0.249</td>
</tr>
<tr>
<td>Bottom Achiever Quintile</td>
<td>0.159</td>
<td>0.151</td>
<td>0.165</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>College Quality Tier</th>
<th>Status Quo</th>
<th>Color-Blind</th>
<th>Proportional Quota</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top College Quintile</td>
<td>0.612</td>
<td>0.661</td>
<td>0.560</td>
</tr>
<tr>
<td>Second College Quintile</td>
<td>0.439</td>
<td>0.481</td>
<td>0.401</td>
</tr>
<tr>
<td>Third College Quintile</td>
<td>0.353</td>
<td>0.391</td>
<td>0.310</td>
</tr>
<tr>
<td>Fourth College Quintile</td>
<td>0.270</td>
<td>0.307</td>
<td>0.251</td>
</tr>
<tr>
<td>Bottom College Quintile</td>
<td>0.178</td>
<td>0.195</td>
<td>0.166</td>
</tr>
</tbody>
</table>

In summary, the effect of the AA scheme on HC investment by minority students depends on the learning cost type of the particular student. In addition, the peak magnitude of these changes are significant and amount to roughly 25% of a standard deviation. The effect on non-minority students is much more limited since the distribution of college seats and the types of their competitors closely resembles the corresponding distributions under the status quo. The limited effect on nonminority students ought to be unsurprising since they represent 84% of the student population and their costs are on average lower.

6.3. Effects of AA on Household Income and Graduation Probability. In order to give the reader a full sense for how AA shapes minority outcomes, we present graduation and income changes in two different ways. The first separates students by *achiever quintiles*, which represent the same set of individuals under each counterfactual scenario. We also present these outcomes by students enrolled in *college quintiles* in order to depict how outcomes change among students who enroll at different points in the college quality spectrum. It should be noted, however, that these do not represent the same sets of individuals across different counterfactual scenarios, as can be seen in Table 6. We do not display the quintile-specific effects for nonminority students as they are much smaller, but they tend to have the opposite signs. The top panel of Table 7 displays graduation probability changes by achievement quintile. Two main forces govern the results here. First, any change in the AA system alters investment incentives, which we discussed in Section 6.2. Since the graduation probability is affected by a student’s HC choice, this has a large effect. The second force is the counterfactual change in college assignments. Recall from the previous section that this force plays a secondary role to pre-college achievement and is most significant for students placing in higher-quality colleges.
The effects we see in the upper panel largely mirror what we found in Figure 7. High-achieving minority students have a stronger (weaker) incentive to make HC investments under a color-blind (proportional quota) system, and the extra investment is largely reflected in graduation rates, though mitigated somewhat by countervailing college placement shifts for these best students. The opposite incentives and outcome effects occur for the lowest achieving minority students, but their graduation rate shifts reflect pure investment changes to a larger extent.

The bottom panel of Table 7 breaks out the graduation rate by quintile of the college quality distribution. This perspective is useful because it reveals the total impact of the two forces on graduation probability. We see an increase for minority students who enroll at top colleges under a color-blind scheme, due mostly to composition effects: a smaller number of minority students with lower cost types enroll in the best colleges, relative to the status quo. In addition, these students accrue more HC as per Figure 7. Symmetrically, under a proportional scheme minority students enrolling at the top quintile of schools have a lower average rate of graduation because on average they have higher learning costs and their investment incentives (for top minority achievers) are weakened.

Elsewhere we find the same pattern continues to hold. Minority students within a college quintile graduate at a higher rate under a color-blind system than under a proportional quota, but the reasons are more subtle at the low end of the market. First, the large differences between successive college quintiles allows for this possibility, given counterfactual shifts of students across them. Second, note that minority students who enroll at the worst colleges have a weaker (stronger) incentive to invest in HC under a color-blind (proportional) system, which would seem to suggest lower (higher) graduation rates. However, because more minority students are shifted from better schools into the worst quintile under a color-blind system, their average learning costs are lower. This second effect, the lower average learning costs, is coupled with higher HC attainment by minority students enrolled at the worst schools in a color-blind system, relative to a proportional quota. It is also stronger than the first effect, which pushes the average minority graduation rate at the worst colleges above the minority graduation rates observed at the same colleges under a proportional quota.

The top and bottom panes of Table 7 are consistent with one another, but it is difficult to make comparisons because of high counterfactual flows of minority students between college quality quintiles. For example, a naive reading of the bottom pane of Table 7 would seem to suggest that the mean minority graduation rate is highest under a color-blind system. However, this reasoning ignores compositional shifts in where they enroll. Table 6 shows that 42% of minorities enroll in the bottom quintile of the college quality distribution in the color-blind counterfactual, which is almost the same fraction of the minority population that graduates in the bottom two quintiles of college quality in the proportional quota counterfactual. The average graduation rate across the bottom two achiever quintiles of the proportional quota counterfactual is 0.22, which is (slightly) higher than the average graduation rate of 0.19 in the bottom college quintile of the color-blind distribution.
Table 8. Aggregate Effect on Graduation Rates

<table>
<thead>
<tr>
<th></th>
<th>Average Graduation Rate of Minority Students</th>
<th>Average Graduation Rate of Nonminority Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status Quo</td>
<td>0.3291</td>
<td>0.4213</td>
</tr>
<tr>
<td>Color-Blind</td>
<td>0.3323</td>
<td>0.4230</td>
</tr>
<tr>
<td>Proportional</td>
<td>0.3378</td>
<td>0.4152</td>
</tr>
</tbody>
</table>

Table 9. Counterfactual Minority Household Income by Achievement Quintile

<table>
<thead>
<tr>
<th>Learning Cost Type Tier</th>
<th>Status Quo</th>
<th>Color-Blind</th>
<th>Proportional Quota</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Achiever Quintile</td>
<td>$80,873</td>
<td>$80,336</td>
<td>$81,560</td>
</tr>
<tr>
<td>Second Achiever Quintile</td>
<td>$75,709</td>
<td>$74,714</td>
<td>$77,223</td>
</tr>
<tr>
<td>Third Achiever Quintile</td>
<td>$71,308</td>
<td>$69,856</td>
<td>$73,350</td>
</tr>
<tr>
<td>Fourth Achiever Quintile</td>
<td>$67,360</td>
<td>$65,264</td>
<td>$69,143</td>
</tr>
<tr>
<td>Bottom Achiever Quintile</td>
<td>$61,606</td>
<td>$60,010</td>
<td>$62,673</td>
</tr>
</tbody>
</table>

This point highlights the importance of taking a market-wide perspective when investigating the impact of AA on the rates at which minorities graduate college: flows of heterogeneous students to alternative segments of the market can create a misleading picture if one focuses only on minorities who enroll within a narrow band of the quality spectrum under alternative admissions systems. Given the multiple forces at work determining graduation rates under different AA schemes, we also present the average effects across all students in Table 8. The net effect of moving to a proportional quota increases the graduation of minority students, while switching to a color-blind scheme has the opposite effect. In summary, admissions schemes that place minority students in better schools generally increase their graduation rates.

Since HC investment has only negligible influence on household income, conditional on graduation, the associated counterfactual impact of AA is mediated entirely by college assignment shifts. Table 9 depicts the effect of AA on minority household income by achievement quintiles. It shows that an AA ban would reduce household income across all five quintiles of the type distribution, whereas a proportional quota would increase it across all quintiles. These effects are weakest for the highest achievers, where the difference between a color-blind system and a quota is around $1,200/year. The strengths of the effects increase until the fourth quintile, where the difference between the two peaks at almost $3,900/year of household income. These changes for nonminorities tend to have the opposite sign but are small and relatively inconsequential, so we do not report them at the quintiles. Table 10 provides the average effect across the population for both groups. The effects on the minority students are on the order of a few thousand dollars, while the effect on nonminority students on average is relatively small. The effect of any change in AA policy averaged across the total population is on the order of ±$20, meaning AA entails a very small loss of total income production.
Table 10. Household Income (HHI) 10 Years After Graduation

<table>
<thead>
<tr>
<th></th>
<th>Average HHI of Minority Students</th>
<th>... Relative to Status Quo</th>
<th>Average HHI of Nonminority Student</th>
<th>... Relative to Status Quo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status Quo</td>
<td>$70,854</td>
<td>—</td>
<td>$74,130</td>
<td>—</td>
</tr>
<tr>
<td>Color-Blind</td>
<td>$70,036</td>
<td>$-818</td>
<td>$74,288</td>
<td>$158</td>
</tr>
<tr>
<td>Proportional</td>
<td>$72,790</td>
<td>$1,936</td>
<td>$73,768</td>
<td>$-363</td>
</tr>
</tbody>
</table>

In terms of the inequality in household income between the groups, a proportional quota results in the smallest gap and and a color-blind system causes the largest gap. In a crude sense, the former represents the most \textit{ex post} fair outcome, while the later represents the most \textit{ex ante} fair outcome. Of course, the \textit{ex ante} perspective is complicated by potential sources of the differences in the type distributions of the two demographic groups. As we discussed above, the sources of the difference in type distributions could encompass a huge array of factors within the control of society at large, such as racial differences in wealth/income and primary/secondary school quality that make it harder for some families to make early-life investments in their children, \textit{etc.} If one takes these factors into account, and if they drive the large \textit{ex ante} cost asymmetry, then it becomes harder to argue that the color-blind outcome is fair in the \textit{ex ante} sense.

6.4. Effects of AA on Net Utility. Section 6.3 broke out the effects of AA on household income and graduation probability, which are two of the primary components of student utility. In this section, we provide the most holistic look possible by examining the effect of AA on students’ net utility; that is, expected payoffs from investment, minus monetized utility costs. We present our results in terms of \textit{equivalent variation}, which we define as a proportional increase in status quo household income that would make the student just as well off under a shift toward another counterfactual admissions system. For example, from Figure 8, if we multiply the median minority student’s baseline, status quo, annual household income by a factor of roughly 1.027 then his utility would rise to the same level as in equilibrium under a proportional quota regime. Values above 1 indicate utility increases, and the effect will be different for different types of students.

Figure 8 shows that minority equivalent variation for a proportional quota ranges from 0% to 2.8% of annual household income, and the benefits are reaped by students across the type distribution. The equivalent variation for a color-blind system is around $-2.5\%$ for the middle 70% of the learning cost distribution, and the effect remains significant for all but the lowest cost minority students. Once again, the equivalent variation for nonminorities is much smaller, generally one fifth to one third as large.

6.5. Implementing AA Through Socioeconomic Quotas. In the wake of U.S. Supreme Court rulings that restrict forms that AA schemes can take, there have been attempts to devise alternative methods of encouraging minority enrollment in top schools. One alternative that has attracted attention is AA that reserves seats for economically deprived applicants. This approach will be effective if minority status and economic hardship are sufficiently correlated and equally
predictive of learning costs. In this section we test whether reserving seats for underprivileged applicants can can serve as an effective tool for achieving more racially diverse college campuses.

The B&B dataset includes information on the expected family contribution (EFC), a measure based on the wealth and income of each student’s pre-college household that is used by the US Department of Education for means-tested education aid. Students with a higher EFC grew up in more affluent homes, and therefore qualify for less aid since their families can be expected to contribute more resources to the cost of college. We adopt EFC as our measure of a student’s economic status, and we experiment with a new set of AA proportional quotas in favor of the $X\%$ poorest students, for various levels of $X \in [10, 50]$. We will refer to these as “$X\%$ EFC quotas”. The outcome measure we focus on in this section is the fraction of seats in each quintile of the college quality distribution that are occupied by (racial) minority students.
Table 11 provides the results of our analysis for three such EFC quotas. Somewhat surprisingly, socioeconomic quotas generally have almost no effect on the enrollment of minority students: regardless of the EFC cutoff, the outcome is very close to the admissions profile resulting from a purely color-blind admission rule. There are various reasons why. First, within the set of college enrollees, socioeconomic class is a poor proxy for the student’s cost type $\theta$, relative to racial demographics. Figure 9 plots the (weighted and selection-corrected) type distributions of the rich and poor students in both demographic groups under a 10% EFC quota. Affluence tends to predict lower cost types within each group, though its predictive power is weaker than race, as the figure shows. The cost distribution for affluent minorities is much more similar to that of poor minorities than it is to the cost distribution for affluent nonminority students.

Recent education research may provide an explanation for why this is so. Reardon, Townsend, and Fox [53] found that substantial racial segregation in housing persists to the present, and that Black and Hispanic primary/secondary students tend to live in substantially poorer neighborhoods, relative to their White and Asian counterparts of similar incomes. Reardon [51], and Reardon, Kalogrides, and Shores [52] also found that racial and socioeconomic segregation in schooling are among the strongest predictors of local racial achievement gaps in the data. Our results and theirs suggest that race in America is not yet reducible to socioeconomic differences alone. These results still leave open the possibility that other factors that are starkly stratified by race, like income (see Kena, et. al. [44]), healthcare (see Smith and Medalia [56]), and parental education (see Fox, Kewal, and Ramani [31]) play a role on the extensive margin of college attendance (outside our model), with primary/secondary school quality playing a predominant role in the intensive margin of college attendance (the main focus of our model).

Another recent paper which tests a socioeconomic-based form of AA, the Texas Top 10% program which guarantees admission at top Texas universities to any student in the top decile of her high school class, is Kapor [43]. He finds that the TTT program increases the admission of minority students by 10% relative to a color-blind benchmark. This is much higher than the effect we find by directly targeting EFC, though it is still weaker than the 25% increase in minority enrollment in the top college tier that we estimate would result from replacing color-blind admissions with the status-quo, race-based AA. Because of the racial segregation of Texas high schools, the high school one attends is likely a better predictor of racial background (and therefore, costs), which in turn means TTT increases on-campus diversity. In contrast, since socioeconomic background (EFC) is a relatively poor predictor of costs, our results show that an EFC quota does a poor job of increasing on-campus racial diversity.\(^{23}\)

To understand the theory behind why EFC quotas would have a limited effect on the competition for seats (relative to a racial color-blind scheme) in our application, Figure 10 plots the type distributions for economically disadvantaged minority and nonminority students under a 10% EFC quota and a 50% EFC quota. It demonstrates that the patterns in Figure 9 are quite robust

\(^{23}\)Another recent paper by Estevan, Gall, Legros and Newman [28] also found reduced-form evidence that a similar means-tested AA policy, the Texas Top 10% program, was ineffective at promoting minority enrollment in Texas flagship universities, but for different reasons: the policy as implemented in Texas was too gameable.
to substantially different cutoffs between rich and poor. The stability of the group-specific type distributions across EFC quotas and the relatively small changes in the fraction of poor students that are minorities means that the distributions of types of rich and poor students are almost identical across EFC quota systems. In absence of cost asymmetry, a proportional quota merely imposes ex ante allocations that would be achieved by a color-blind equilibrium ex post. Hence, our X% EFC quotas produce similar results as a color-blind contest where no attempt is made to racially diversify college campuses. We conclude from our analysis that implementing an AA program for minority students using economic status as a proxy for race is ineffective from the perspective of a policy-maker wishing to promote racial diversity on college campuses.

6.6. The Relative Force of the Competitive and Productive Channels of Investment. Before concluding we briefly discuss the underlying economic forces within our model that govern investment under any admissions scheme. In a world of complete information, the type of each student would be common knowledge, students would be assortatively assigned to colleges by their academic types before investment occurs, and students would choose the level of HC that is optimal given their college assignment. These “first-best” investments would then reflect only the direct benefits of HC that we have referred to as the productive channel of incentives. In the incomplete information world that students actually inhabit, society must rely on competitive investment to stratify students by unobserved characteristics, so that college assignment may still be assortative. Colleges infer rankings of a student’s type based on his or her HC output level, which in turn pushes students to accrue more HC than he or she would in a complete information world. The indirect incentive to accrue HC solely to obtain a seat at a better school reflects the competitive channel of incentives. Our goal in this section is to assess the relative strength of these two incentive channels in driving students’ observed behaviors.

To formalize this idea, we decompose the marginal benefit of HC investments into two components. Equation 13, a student’s first-order condition under incomplete information, places the
two marginal benefits of HC on the left-hand side, with the marginal costs on the right:

\[
E_{\epsilon} \left[ U_s \left( P_j(s + \epsilon), s, \theta; \alpha \right) \right] + E_{\epsilon} \left[ U_p \left( P_j(s + \epsilon), s, \theta; \alpha \right) P'_j(s + \epsilon) \right] = \theta c' \left( s \right)
\]

The productive channel can be thought of as the “Beckerian” incentives that represent direct marginal benefits of holding an additional unit of productive HC, which is the only one present in a world of complete information. The competitive channel is the indirect, “Spencerian” incentive, to invest in order to reveal one’s appropriate position ahead of higher-cost competitors in a separating equilibrium of a rank-order contest. To our knowledge, this is the first empirical framework capable of disentangling these two incentive channels.

A priori it is not clear which channel is more important. HC does not play a significant role in household income conditional on graduating, but it does have a predominant role in determining whether or not a student graduates college to begin with. To get a sense for the relative magnitudes of these forces as a function of the student’s cost type, Figure 11 displays the ratio of the competitive channel incentives to the total marginal benefit, which is the sum of both the competitive and productive channels. The horizontal axis describes the quantile rank of the respective cost type in the group-specific type distribution.

Whenever the line is above a benchmark of 0.5 the competitive channel is dominant, and whenever it is below, the productive channel is most important. As one can see, the competitive channel is stronger than the productive channel for all but the lowest-cost agents. By extension, most students’ academic achievement levels would be significantly lower in a complete information world where the competitive channel is turned off. For roughly the top 14% of achieveers within the college universe, the productive channel is the dominant one. Of course, this discussion does not necessarily have bearing on optimal policy—for example, if HC spillovers are
important in the economy at large then the social planner may wish to use any means to max-
imize HC production—but it casts a new and interesting light on the motivations underlying
pre-college academic achievement.

7. CONCLUSION

This paper has developed identification and estimation results for a college assignment mar-
et based on contest models. By using individual-level data from the B&B survey, rather than
focusing only on elite private colleges, we can provide a market-wide analysis of how admis-
sions rules impact incentives and how changes to one’s college placement impact the returns
to a college education, conditional on individual characteristics. Our analysis adapts auction-
theoretic empirical techniques that allow us to identify the unobserved student characteristics
which influence pre-college investment and post-college outcomes. We find that while the qual-
ity of the college in which a student enrolls and the human capital the student accrued prior to
enrollment affect the probability that the student graduates, the wage conditional on graduation
is determined almost entirely by college quality and the student’s unobservable type.

AA is a prominent feature of the entire college market and plays a significant role in invest-
ment, redistribution, and welfare. A strong AA regime such as a proportional quota results
in minority students enrolling at better colleges, while a color-blind admissions rule results in
minority students predominantly enrolling in the bottom two quintiles of the college quality dis-
tribution. Interestingly, the effect on human capital investment incentives is more ambiguous. A
color-blind (quota) rule results in the best minority students increasing (reducing) their human
capital investments, while higher-cost minority students reduce (increase) them. The effect of
AA on graduation rates similarly varies by students’ types and the colleges in which they enroll.
Overall, however, stronger AA schemes increase the average graduation rate and wages condi-
tional on graduation for minority students. We also found that means-tested proportional quotas
serve as a poor substitute for race-based AA if the policy-maker’s primary objective is to increase
racial diversity in high-quality colleges.

Finally, we analyzed the strength of the incentive to accrue HC solely for its productive value
relative to the incentive to choose higher levels of HC to compete for access to a better college. We
find a surprisingly prominent role for the competitive channel of incentives, which is stronger
for all but the best students. Moreover, there is a stark contrast in the strength of these two
channels incentives for most students: the competitive channel is roughly twice as strong as the
productive channel for half of college-going students.

The original intention of AA schemes was that these programs could serve as a solution to
widespread demographic inequities tied to higher education. From a legal and political econ-
omy perspective, AA programs were thought to be long-term, but in the end, temporary. Al-
though stronger AA schemes have a positive effect on the average graduation probabilities and
post-graduation household income, the effect on HC accumulation is ambiguous. This makes it
difficult to form educated guesses about the long-term effects of these programs from our study.
There remain many unanswered questions that are the subject of ongoing research. For example, are the income effects of college quality, HC accumulation, and learning costs different for students in STEM fields relative to those in the humanities? Is it possible to design a better AA scheme than the prototypical examples we study? However, the biggest and most obvious question is whether one could use our model to say anything about the long-run impacts of different college admissions systems on the evolution of distributional inequalities over time. Our analysis, which was static by design, can only be the first step in such a research agenda. Providing a serious answer to this question will require considering the inter-generational effects of these programs. We leave these questions for future work which will build on the insights gained here.

References


A.1. Stage I GMM Estimator. The first hurdle to overcome is to find a computationally tractable way of representing the selected joint distribution of \((P, S)\) conditional on graduation. High-dimensional density estimation is a difficult problem both computationally and because of the rapid decay of optimal statistical convergence rates as the dimensionality of the underlying random variable grows.\(^{24}\) Recent work in the auctions literature by Hubbard, Li, and Paarsch [41] has employed parametric copula functions to solve this problem. Sklar’s Theorem states that any absolutely continuous joint distribution can be represented as a composition

\[
F_P(P|\text{grad}), G_j(S|\text{grad})|\text{grad}j \in \{M, N\}, \text{ where } C_j(\cdot, \cdot|\text{grad}) \text{ is a unique copula function.}
\]

This implies that the rapidly increasing computational cost and data-hungriness of nonparametric estimators come from the complexity of the correlation structure \(C\) since the complexity of the marginal distributions does not increase with the dimension of the joint distribution. Hubbard, Li, and Paarsch [41] therefore propose a flexible approach to estimating the marginal distributions, while simplifying the copula with parametric assumptions for tractability. This allows the econometrician to maintain the familiar \(\sqrt{T}\) convergence rate when estimating a multi-dimensional joint distribution. We follow this dimension reduction strategy by adopting the Gumbel-Hougaard copula, \(C(r, q; \nu) = \exp \left[\frac{1}{\nu} \log \left[ \frac{1}{1 + \frac{\exp(-r\nu)(1-\exp(-q\nu)(1-1))}{\exp(-r\nu)-1} \right] \right], \nu \in \mathbb{R}\setminus\{0\}\); the Clayton copula, \(C(r, q; \nu) = \max\left(\frac{\nu}{r} + \frac{\nu}{q} - 1; 0\right)^{-1/\nu}, \nu \in [-1, \infty)\setminus\{0\}\); and the Gaussian copula, \(C(r, q; \nu) = \Phi_{R}\left[\Phi^{-1}(r), \Phi^{-1}(q)\right]\) where \(\Phi\) is a standard normal CDF and \(\Phi_R\) is a bivariate normal CDF with correlation matrix \(R = \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix}, \nu \in [-1, 1]\). All produced very similar results, which is consistent with the assumption that our parametric restriction of the copula function provides a robust approximation to the nonparametric correlation structure.

\(^{24}\)See Silverman [55] and Campo, Perrigne, and Vuong [16] for a lengthy discussion on this concept.

\(^{25}\)We also experimented with several other copula functions including the Frank copula, \(C(r, q; \nu) = -\frac{1}{\nu} \log \left[ 1 + \frac{\exp(-r\nu)(1-\exp(-q\nu)(1-1))}{\exp(-r\nu)-1} \right], \nu \in \mathbb{R}\setminus\{0\}\); the Clayton copula, \(C(r, q; \nu) = \max\left(\frac{\nu}{r} + \frac{\nu}{q} - 1; 0\right)^{-1/\nu}, \nu \in [-1, \infty)\setminus\{0\}\); and the Gaussian copula, \(C(r, q; \nu) = \Phi_{R}\left[\Phi^{-1}(r), \Phi^{-1}(q)\right]\) where \(\Phi\) is a standard normal CDF and \(\Phi_R\) is a bivariate normal CDF with correlation matrix \(R = \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix}, \nu \in [-1, 1]\). All produced very similar results, which is consistent with the assumption that our parametric restriction of the copula function provides a robust approximation to the nonparametric correlation structure.

\(^{26}\)For a brief primer on B-splines and their advantages in empirical auctions models, see Hickman, Hubbard, and Paarsch [38] and Bodoh-Creed, Boehnke, and Hickman [12].
define a set of \( K_j^p + 3 \) cubic B-spline basis functions \( B_{jk}^p(p) : [p, \bar{p}] \rightarrow \mathbb{R}, k = 1, \ldots, K_j^p + 3 \), which in turn define our parameterization of the CDFs:

\[
F_{P_j}(p|\text{grad}; \gamma_j^p) = \sum_{k=1}^{K_j^p+3} \gamma_{jk}^p B_{jk}^p(p), j \in \{\mathcal{M}, \mathcal{N}\}.
\]

For the marginal distribution of the HC index, \( S \), we have an additional challenge: since its units (and therefore the relevant domain to span) are unknown ex ante, we instead parameterize the selected marginal quantile functions, whose domain is always \([0, 1]\). Let the knot vectors and basis functions for selected HC quantile functions be \( k_j^q = \left\{ 0 = k_{j1}^q < k_{j2}^q < \cdots < k_{jk_j^q+1}^q = 1 \right\} \) and \( B_{jk}^q(r) : [0, 1] \rightarrow \mathbb{R}, k = 1, \ldots, K_j^q + 3 \), respectively, with B-spline marginal quantile functions parameterized similarly as above by \( Q_{S_j}(r|\text{grad}; \gamma_j^q) = \sum_{k=1}^{K_j^q+3} \gamma_{jk}^q B_{jk}^q(r), j \in \{\mathcal{M}, \mathcal{N}\} \).

Going forward, one important detail to note is that the parameters \( \nu_M \) and \( \nu_N \) reflect the empirical correlation structure for the selected joint distribution of \( P \) and \( S \) for college graduates only. Below we will define other notation for separate copula parameters that apply to the selection-corrected joint distribution for all college enrollees (i.e., including dropouts).

In order to complete our GMM estimator, we also need to construct empirical analogs to the joint and marginal distributions of \((p, s)\). In the case of CDFs, we use the standard Kaplan-Meier empirical distribution functions

\[
\hat{F}_{P_j}(p|\text{grad}) = \frac{\sum_{i=1}^{l_j} \mathbb{I}(p_i \leq p) \mathbb{I}(i \in j)}{\sum_{i=1}^{l_j} \mathbb{I}(i \in j)}, \quad \text{and}
\]

\[
\hat{F}_{PS}(p, s|j, \text{grad}) = \frac{\sum_{i=1}^{l_j} \mathbb{I}(p_i \leq p) \mathbb{I}(s_i \leq s) \mathbb{I}(i \in j)}{\sum_{i=1}^{l_j} \mathbb{I}(i \in j)}, j \in \{\mathcal{M}, \mathcal{N}\}.
\]

For the empirical marginal quantiles of \( S \), we use a new method developed by Hedblom, Hickman, and List [37] for smooth nonparametric quantile estimation. For a random sample \( S_j = \{S_{ji}\}_{i=1}^{l_j} \) of size \( l_j = \sum_{i=1}^{l_j} \mathbb{I}(i \in j) \), this estimator exists as a weighted average of the ordered data \( \{S_{j1} \leq S_{j2} \leq \cdots \leq S_{jl_j}\} \). Specifically, for \( k \in \{1, 2, \ldots, l_j\} \), the \((k/l_j)\)th empirical quantile is estimated as

\[
\hat{Q}_{S_j}(k/l_j) = \sum_{i=1}^{l_j} \Pi_{ik} S_j(i),
\]

where the weights \( \Pi_{ik} \) are known and mimic the limiting behavior of a re-sampled quantile estimator as the number of simulated samples approaches infinity. Specifically, \( \Pi_{ik} \) gives the probability that the \( i \)th order statistic of \( S_j \) will occupy the \( k \)th position in an ordered, randomly
generated bootstrap sample from the raw data.\footnote{A benefit to this method is that it is differentiable and more efficient than the traditional empirical quantile estimator, \( \text{inf}\{ s : \frac{1}{\sum_{j=1}^{n} 1(S_{ij} \leq s)} \geq (k/100) \} \). Intuitively, the empirical quantile estimator, being the nearest-neighbor inverse of the Kaplan-Meier empirical CDF, incorporates cardinal information from only a single datum, with all other data providing only ordinal information. In contrast, \( \tilde{Q}_{S}(k/l) \) as defined above uses both ordinal and cardinal information from the entire sample. See Hedblom, Hickman, and List [37] for additional details.} However, an important constraint for this estimator is that, for a fixed sample size \( l_i \), it can only be evaluated at quantile ranks on the discrete grid \( \{1/l_i, \ldots , l_i \} \). Therefore, assuming \( l_i \geq 101, j \in \{M,N\} \), we also define \( r_j = \{r_{j0}, r_{j1}, r_{j2}, \ldots , r_{j100} \} \) as a vector of quantile ranks at which the empirical quantile function is to be evaluated, where \( r_{j0} = 1/l_i \) and \( r_{jk} = \text{round}(k/l_i)/l_i, 1 \leq k \).

Finally, in order to estimate \( \sigma_e \) we need to introduce four additional ancillary parameters. First, for notational ease let \( \pi = [\gamma^p_M, \gamma^p_N, \gamma^q_M, \gamma^q_N, v_M, v_N, \beta^s, \beta^e]^{\top} \) summarize all primary Stage I parameters except for the shock variance. Note that if \( \pi \) is known, Equation (3) defines the selection-corrected joint distribution of \( (P, S) \), \( F_{PS}(p, s|j; \pi) \), for group \( j \in \{M,N\} \) as a function of the entire parameter vector \( \pi \) along with its two marginal distributions, \( F_{p_j}(p; \pi) \) and \( G_j(s; \pi) \). Let \( v^*_j \) denote the best-fit copula parameter of the selection-corrected joint distribution:

\[
v^*_j = \arg \min_{n \in [1, \infty)} \left\{ \int_S \int_P \left[ C \left( F_{p_j}(p; \pi), G_j(s; \pi); n \right) - F_{PS}(p, s|j; \pi) \right]^2 dp ds \right\}.
\]

Implicitly, \( v^*_j \) is a function of \( \beta^e \) since these directly control how well the ranks of \( S(e_i, a_i; \beta^e) \) predict the ranks of \( i \)’s placement in \( P \) space.

Now, let \( F_{PS}^0(p, s|j; \pi, \sigma_e) \) denote the joint distribution of \( (P, S) \) implied by the marginal distributions \( F_{p_j}(p; \pi) \) and \( G_j(s; \pi) \), but assuming rank-order allocations with respect to NHC are generated by mean-zero, normal shocks with variance \( \sigma_e^2 \). Let \( v^0_j(\sigma_e) \) denote the best-fit copula parameter for that joint distribution, in the following sense:

\[
v^0_j(\sigma_e) = \arg \min_{n \in [1, \infty)} \left\{ \int_S \int_P \left[ C \left( F_{p_j}(p; \pi), G_j(s; \pi); n \right) - F_{PS}^0(p, s|j; \pi, \sigma_e) \right]^2 dp ds \right\}
\]

Subject to:

\[
F_{PS}^0(p, s|j; \pi, \sigma_e) = \int_S \int_P f^0_{P|S}(p|s,j; \pi, \sigma_e) g_j(s; \pi) dp ds,
\]

\[
= \int_S \int_P f_\pi \left( P_j^{-1}(p; \pi, \sigma_e) - s; \sigma_e \right) \frac{dP_j^{-1}(p; \pi, \sigma_e)}{dp} g_j(s; \pi) dp ds,
\]

\[
P_j^{-1}(p; \pi, \sigma_e) = H_j^{-1} \left[ F_{p_j}(p; \pi); \pi, \sigma_e \right],
\]

\[
H_j(t; \pi, \sigma_e) = \int_{-\infty}^t \int_{-\infty}^\infty f_\pi(\epsilon; \sigma_e) g_j(x - \epsilon; \pi, \sigma_e) d\epsilon dx, j = M,N.
\]

The ancillary parameters \( v^*, v^0_\pi, v^0_M(\sigma_e) \), and \( v^0_N(\sigma_e) \) are used below to define moment conditions for estimation of \( \beta^e \) and \( \sigma_e \). Intuitively, \( v^*_j \) is the copula parameter that best reflects the
correlation structure between $P$ and $S$ implied by the data (post-selection-correction), and $v_j^\sigma(\sigma_e)$ is the copula parameter that best reflects the correlation structure between $P$ and $S$ generated endogenously by our structural model of a noisy, rank-order college admissions contest given $\sigma_e$ and the empirical marginal distributions of $P$ and $S$. With the above definitions, we can now formalize our Stage I GMM estimator:

$$
\left[ \hat{\pi} \right] = \arg \min \left\{ \sum_{i=1}^I \left( D_{M_i} \left[ F_{P_M}(p_i|\text{grad}; \gamma^p_M) - \tilde{F}_{P_M}(p_i|\text{grad}) \right]^2 
+ (1 - D_{M_i}) \left[ F_{P_N}(p_i|\text{grad}; \gamma^p_N) - \tilde{F}_{P_N}(p_i|\text{grad}) \right]^2 \right) 
+ \sum_{k=0}^{100} \left( \left[ Q_{S_M}(r_{Mk}|\text{grad}; \gamma^q_M) - \tilde{Q}_{S_M}(r_{Mk}) \right]^2 + \left[ Q_{S_N}(r_{Nk}|\text{grad}; \gamma^q_N) - \tilde{Q}_{S_N}(r_{Nk}) \right]^2 \right) 
+ \sum_{i=1}^I \left( D_{M_i} \left[ F_{PS}(p_i, s_i|\mathcal{M}, \text{grad}; \gamma^p_M, \gamma^q_M, v_M) - \tilde{F}_{PS}(p_i, s_i|\mathcal{M}, \text{grad}) \right]^2 
+ (1 - D_{M_i}) \left[ F_{PS}(p_i, s_i|\mathcal{N}, \text{grad}; \gamma^p_N, \gamma^q_N, v_N) - \tilde{F}_{PS}(p_i, s_i|\mathcal{N}, \text{grad}) \right]^2 \right) 
+ \sum_{i=1}^I \left( [\Gamma_{M_i} - \mathbf{z}_{M_i} \mathbf{b}^\sigma]^2 + [\Gamma_{N_i} - \mathbf{z}_{N_i} \mathbf{b}^\sigma]^2 \right) 
+ \left( \frac{v^e_M - 1}{v^e_M} - 1 \right)^2 \left( \frac{v^e_N - 1}{v^e_N} - 1 \right)^2 
+ \left( \frac{v^e_M - 1}{v^e_M} - \frac{v^e_M(\sigma_e) - 1}{v^e_M(\sigma_e)} \right)^2 \left( \frac{v^e_N - 1}{v^e_N} - \frac{v^e_N(\sigma_e) - 1}{v^e_N(\sigma_e)} \right)^2 \right\},
$$

**Subject to:**

$$s_i = S(e_i, a_i; \mathbf{b}^\sigma), \quad i = 1, \ldots, I$$

$$\frac{\partial S(e, a; \mathbf{b}^\sigma)}{\partial e} > 0, \quad \frac{\partial S(e, a; \mathbf{b}^\sigma)}{\partial a} > 0 \forall (e, a; \mathbf{b}^\sigma), \quad \text{and} \quad \max_{(e, a) \in \mathbb{R}^2} \{S(e, a; \mathbf{b}^\sigma)\} = 1$$

$$\gamma^p_{j,k-1} \leq \gamma^p_{j,k}, \quad k = 2, \ldots, K^p + 3, \quad v = p, s, \quad j \in \{\mathcal{M}, \mathcal{N}\}$$

$$\min\{\Gamma_{M1}, \ldots, \Gamma_{ML}, \Gamma_{N1}, \ldots, \Gamma_{Nl}\} \leq \rho(p, s; \mathbf{b}^\sigma) \leq 1 \forall (p, s)$$

$$\frac{\partial \rho(p, s; \mathbf{b}^\sigma)}{\partial p} > 0, \quad \frac{\partial \rho(p, s; \mathbf{b}^\sigma)}{\partial s} > 0, \quad \frac{\partial^2 \rho(p, s; \mathbf{b}^\sigma)}{\partial s^2} \leq 0, \quad \forall (p, s)$$

$$\mathbf{z}_{M} = [1, p_1, p_1^2, p_1^3, s_{j1}, s_{j1}^2, s_{j1}^3, p_1 s_{j1}, p_1^2 s_{j1}], \quad j \in \{\mathcal{M}, \mathcal{N}\}, \quad l = 1, \ldots, L$$

$$\mathbf{z}_{N} = [1, p_1, p_1^2, p_1^3, s_{j1}, s_{j1}^2, s_{j1}^3, p_1 s_{j1}, p_1^2 s_{j1}], \quad j \in \{\mathcal{M}, \mathcal{N}\}, \quad l = 1, \ldots, L$$

$$v_{M}, v_{N}, v_{M}^e, v_{N}^e, v_{M}^\sigma(\sigma_e), v_{N}^\sigma(\sigma_e) \in [1, \infty); \quad \text{and} \quad \sigma_e > 0.$$
The first two summations in the objective function are the moment conditions for the selected marginal distributions of \( P \) and the selected marginal quantile functions of \( S \). The third summation contains moment conditions for the copula of the selected joint distribution of \((P, S)\). The fourth summation contains the selection-corrected regression equations for graduation probabilities. The second to last line of the objective function contains moment conditions for the single-index parameters: they minimize the distance between the selection-corrected rank correlations, \( \tau_{M,P} \) and \( \tau_{N,P} \), and their theoretical maxima of one. Recall from the discussion in Section 4.1.1 that we define the parameters \( \beta_s \) within our model as those which maximize the Kendall’s \( \tau \) rank correlation between the implied single index \( S \) and \( P \), which is why the estimate \( \hat{\beta}_s \) is chosen in this way. The final line of the objective function contains moment conditions for the matching shock variance: it is chosen to minimize the distance between the empirical rank correlations, \( \tau_{M,P} \) and \( \tau_{N,P} \), and their model-generated analogs , \( \tau_{M,P}(\sigma_{\varepsilon}) \) and \( \tau_{N,P}(\sigma_{\varepsilon}) \).

As for the constraints, the last line imposes natural bounds on the shock variance and copula parameters. The two lines above this establish the selection correction procedure for the graduation probability regressions, and the two lines above that impose regularity conditions on the graduation probability parameters. The third constraint from the top imposes monotonicity on the B-spline CDFs and quantile functions, and the first two lines define the \( s_i \)’s as a single index in \((e_i, a_i)\), and they impose monotonicity and a scale normalization to fix its units.

A.2. Stage II Type Distribution GMM Estimator. The final step of estimation is to recover the type distributions. One simple way to proceed would be to just compute the composition of the HC CDFs and the inverse investment strategies above, but this would make use of only the subsample where household income is available. Since during Stage II we recover functionals that map any HC level into its corresponding type, including \( s_i \)’s for which income data were not available, we use an alternative approach that uses the full sample. We first parameterize the type distributions as flexible B-splines with knot vectors \( k_{\theta}^j = \{\theta = k_{j1}^\theta < \cdots < k_{jK+1}^\theta = \bar{\theta}\} \), basis functions \( B_{jk}^\theta(\theta) \), \( k = 1, \ldots, K+3 \), and weights \( \lambda_j^\theta \) implying the familiar form \( F_j(\theta; \lambda_j^\theta) = \sum_{k=1}^{K+3} \lambda_j^\theta B_{jk}^\theta(\theta) \).

In order to fit these to the data we need to construct their empirical analogs. Let the inferred types for minority graduates in the full B&B sample be denoted \( \hat{\theta}_{Mi} = \theta_M(s_i; \lambda_{M}(\alpha)) \), and let those for non-minority graduates be \( \hat{\theta}_{Ni} = \theta_N(s_i; \lambda_{N}(\alpha)) \). We compute a selection-corrected empirical type distribution for all enrollees (including dropouts) as

$$
\hat{F}_j(\theta) = \frac{\sum_{i=1}^{I} \mathbbm{1}(\hat{\theta}_{ji} \leq \theta) \mathbbm{1}(i \in j)/\rho(p_i, s_i)}{\sum_{i=1}^{I} \mathbbm{1}(i \in j)/\rho(p_i, s_i)}, \quad j \in \{M, N\},
$$

where \( \mathbbm{1}(\cdot) \) is the indicator function. This estimator is efficient under the assumption of a correctly specified model and allows for the estimation of the joint distribution of the type and the observed variables, which is necessary for the estimation of the return on investment. The estimator is consistent and asymptotically normal, and the standard errors can be estimated using the outer product of gradients (OPG) method, which is a standard approach in econometrics.
so our B-spline estimator takes the form

\[
\hat{\lambda}_j^\theta = \arg\min_{\lambda \in \mathbb{R}^K} \left\{ \sum_{i=1}^I \mathbb{1}(i \in j) \left( F_j(\theta_i; \lambda_j^\theta) - \bar{F}_j(\theta_i) \right)^2 \right\}
\]

Subject to: \( \lambda_{j1}^\theta = 0, \lambda_{jk}^\theta < \lambda_{jk}^\theta, k = 2, \ldots, K_j, j = \mathcal{M}, \mathcal{N} \).

A.3. Practical Issues for Implementation of the Estimator. In the implementation of our GMM estimator of Stage I parameters, we adopt a simplification for computational convenience. First, during solver runtime we normalize \( \beta^s_1 = 1 \) instead of constraining the objective function so that the maximal single index value is one. This simplifies the problem by reducing the number of parameters to choose and constraints to satisfy. After the estimator has run we re-scale the HC single index so that it’s maximal possible value is one, and we accordingly make adjustments to the graduation probability parameters and HC distributions to reflect the re-scaling.

There are several model tuning parameters that we must specify, among which are knot vectors \( k^p_j, k^q_j, k^t_j, \) and \( k^s_j \). We adopt the convention that knots are to be chosen uniformly in empirical quantile space, as this evenly spreads the statistical power of the data across all basis functions and simplifies the decision to a choice of the number of knots. Specifically, we chose \( K^p_M = K^p_N = K^t_M = K^t_N = K^s_M = K^s_N = 5 \) (i.e., knots at the quintiles with 8 total B-spline basis functions) for the selected school quality distributions, assignment functions, and inverse strategies, respectively; and \( K^q_M = K^q_N = 10 \) (i.e., knots at the deciles with 13 basis functions) for the selected HC quantile functions. Additional knots did not appreciably improve model fit. When approximating the assignment mappings, we imposed a truncation \( t = \min_i \{s_i\} - 5 \hat{\sigma}_\epsilon, \bar{t} = \max_i \{s_i\} + 5 \hat{\sigma}_\epsilon \) on the support of NHC, and we chose a set of evaluation points that included the modes of the B-spline basis functions and the midpoints between the modes.

We calibrated \( \kappa \) from the U.S. Census Bureau’s Current Population Survey (CPS), which is consistent with the structural estimates of Heckman, Humphries, and Veramendi [36]. Recall that the students in our sample were initially surveyed after graduation in 1993, and the household income data we use was collected in the 2003 follow-up survey. To get a benchmark for the fraction of college graduate incomes garnered by dropouts, we computed the ratio of the average household income of 33-year-olds in the 2003 CPS survey with some college to the average household income of 33-year-old college graduates (with no additional post-graduate education). The result is a value of \( \kappa = 0.714 \). As a robustness check we also repeated our estimates assuming \( \kappa \) values ranging from 0.5 to 0.9. Only two aspects of our analysis change. First, the choice of \( \kappa \) affects the estimated values of \( \alpha_\theta \) and the distribution of types, \( F_j(\theta_i; \lambda_j^\theta), j \in \{\mathcal{M}, \mathcal{N}\} \). However, the overall role of \( \theta \) in the wealth equation, which we measure as \( \alpha_\theta \) times the standard deviation of \( \log(\theta) \), is stable to changes in \( \kappa \). The second thing that is affected is the productive channel of incentives, which we estimate to be weaker as \( \kappa \) rises. This is intuitive since, as we see in Section 5.5, \( s \) only affects the graduation probability, so anything that makes the utility gap between completing college and dropping out shrink will weaken the direct, productive benefit of HC.
Finally, two sources of sampling weights were used in our empirical implementation, but in order to avoid further complicating notation we left them out of the formal definition above. The first is cross-sectional sampling weights contained in the B&B data. In Stage I these were used for group $j \in \{M,N\}$ to calculate the empirical analogs of the joint distributions $\hat{F}_{ps}(p,s|j,\text{grad})$, the marginal quantile functions $\hat{Q}_{sj}(r)$, and the marginal distributions $\hat{F}_{pj}(p|\text{grad})$. In simple terms, each of these functions at a point $(p,s)$ is a sample mean of indicator functions evaluated at each datapoint, and we converted them into weighted sample means (in the usual way) using the B&B cross-sectional weights. In Stage II they were also used to weight each component of the wage regression (in the usual way that weighted regressions are constructed) and to calculate the empirical analog of the type distributions $\hat{F}_j(\theta)$. The second source of sampling weights came from IPEDS. In Stage I, the graduation probability regression (the fourth summation in the definition of $[\hat{\pi}, \hat{\sigma}_\epsilon]^\top$ above) is converted into a weighted regression in the usual way by using the number of individuals in each school-race freshman cohort in 1988 as weights.

Appendix B. Determinants of $\theta$

If our exclusion restriction is violated and race directly influences household income (i.e., racial animus has a significant effect), then our analysis will load the effects of this animus onto the estimates values of $\theta$ for the minority students. If this is the case, then it may be possible to predict $\theta$ using minority status in combination with other demographic traits, which we reference in Section 5.6 on robustness checks of our empirical method. Of course, understanding the determinants of $\theta$ is of independent interest, but a broad investigation of this question is outside of the scope of this paper. One method for assessing the drivers of $\theta$ is to regress $\theta$ on other plausibly exogenous demographic traits contained in the B&B data.

For the duration of this section we work with $\ln(\theta)$ as the outcome variable of interest. Given our interpretation of $\theta$ as student characteristics that are influenced by forces internal to the individual (e.g., innate ability) and external conditions (e.g., primary/secondary school quality, parental education and financial resources), the most plausibly exogenous explanatory variables describe properties of the students’ parents or family. The variables we consider are described in Table 12. The data set includes the state of residence of the student at enrollment, and this has been condensed into dummy variables for the northeast, southeast, midwest, southwest, mountain west, and west coast regions. We consider only students that have nonmissing values for all of these variables.

The variable risk index is the sum of seven binary risk factors for failure to complete college (e.g., GED recipient). Mother college graduate and father college graduate are dummies that indicate whether the respective parent graduated from college. Parents married is a dummy variable indicating whether both parents are married (but perhaps not to each other). Parental cash savings refers to the amount of cash and savings reported on financial aid forms (submitted at college enrollment).

We include the variables in Table 12 as well as a dummy for minority status and a full set interactions with the minority status dummy in our regressions. Our regression results are
Table 12. Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BACCALAUREATE AND BEYOND</strong>:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minority Dummy:</td>
<td>0.118</td>
<td>0</td>
<td>0.322</td>
</tr>
<tr>
<td>Gender Dummy:</td>
<td>0.578</td>
<td>1</td>
<td>0.494</td>
</tr>
<tr>
<td>Risk Index (7 Point Scale):</td>
<td>0.3139</td>
<td>0</td>
<td>0.536</td>
</tr>
<tr>
<td>Parent Adj. Gross Income ($K) :</td>
<td>42.4</td>
<td>39.1</td>
<td>24.7</td>
</tr>
<tr>
<td>Mother College Grad.:</td>
<td>0.244</td>
<td>0</td>
<td>0.429</td>
</tr>
<tr>
<td>Father College Grad.:</td>
<td>0.399</td>
<td>0</td>
<td>0.490</td>
</tr>
<tr>
<td>Parents Married:</td>
<td>0.760</td>
<td>1</td>
<td>0.4273</td>
</tr>
<tr>
<td># Members of Family:</td>
<td>3.97</td>
<td>4</td>
<td>1.27</td>
</tr>
<tr>
<td>Parental Cash Savings ($K):</td>
<td>5.81</td>
<td>0.83</td>
<td>22.1</td>
</tr>
</tbody>
</table>

Table 13. Estimates of $\log(\theta)$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Est.</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minority Dummy:</td>
<td>0.0894</td>
<td>0.419</td>
</tr>
<tr>
<td>Gender Dummy:</td>
<td>-0.0855***</td>
<td>0.0240</td>
</tr>
<tr>
<td>Gender Dummy x Minority Dummy:</td>
<td>-0.0224</td>
<td>0.0787</td>
</tr>
<tr>
<td>Risk Index&gt;0:</td>
<td>0.113***</td>
<td>0.0273</td>
</tr>
<tr>
<td>Risk Index x Minority Dummy:</td>
<td>0.0792</td>
<td>0.829</td>
</tr>
<tr>
<td>Parent Adj. Gross Income :</td>
<td>$5.42 \times 10^{-4}$</td>
<td>$5.62 \times 10^{-4}$</td>
</tr>
<tr>
<td>Parent Adj. Gross Income x Minority Dummy :</td>
<td>$1.39 \times 10^{-4}$</td>
<td>$2.20 \times 10^{-3}$</td>
</tr>
<tr>
<td>Mother College Grad.:</td>
<td>$7.75 \times 10^{-3}$</td>
<td>0.0314</td>
</tr>
<tr>
<td>Mother College Grad. x Minority Dummy:</td>
<td>$-7.60 \times 10^{-3}$</td>
<td>0.102</td>
</tr>
<tr>
<td>Father College Grad.:</td>
<td>-0.152***</td>
<td>0.028</td>
</tr>
<tr>
<td>Father College Grad. x Minority Dummy:</td>
<td>-0.0407</td>
<td>0.0841</td>
</tr>
<tr>
<td>Parents Married:</td>
<td>-0.0803***</td>
<td>0.0341</td>
</tr>
<tr>
<td>Parents Married x Minority Dummy:</td>
<td>0.144</td>
<td>0.0932</td>
</tr>
<tr>
<td># Members of Family:</td>
<td>$2.65 \times 10^{3}$</td>
<td>0.0104</td>
</tr>
<tr>
<td># Members of Family x Minority Dummy:</td>
<td>-0.0258</td>
<td>0.0824</td>
</tr>
<tr>
<td>Parental Cash Savings&gt;0:</td>
<td>-0.0894***</td>
<td>0.0382</td>
</tr>
<tr>
<td>Parental Cash Savings&gt;0 x Minority Dummy:</td>
<td>-0.0144</td>
<td>0.0816</td>
</tr>
</tbody>
</table>

| Region Dummies                               | Included   |
| Region Dummies x Minority Dummies            | Included   |
| N:                                           | 1196       |
| $R^2$:                                        | 0.451      |

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

contained in Table 13. An F test reveals that the minority dummy and the minority interaction terms are not jointly significant at the 90% confidence level. In addition, the coefficient on the minority dummy is either of comparable magnitude or smaller than several other demographic variables such as the gender of the student, the parent’s marital status, whether the father has graduated from college, and whether the parents have liquid savings.
It is interesting to note that in Section 5.5 we estimate a substantial race gap in learning costs, whereas through the current exercise we can conclude that the race gap in costs disappears when home environment controls are used. Our analysis implies that the black-white household income gap in our data is due to the role of learning-costs in determining household income combined with a black-white gap in the distribution of learning-costs. If racial animus played a significant role in the household income gap (contrary to Assumption 4.3 and Fryer et al. [32]), then one might expect to find that the minority dummy is a significant predictor of learning-costs. However, our analysis suggests that inter-group differences in learning costs are not driven by minority status once one controls for other demographic traits. In short, we do not find evidence that our exclusion restriction is violated and causing the effect of racial animus to be loaded onto our estimates of $\theta$.

**Appendix C. Solving the Model Numerically**

Counterfactuals that compare different AA regimes necessarily require solving the model to determine the student behavior (i.e., human capital investments), which in turn determines college enrollments and final outcomes. There are two endogenous objects that need to be computed for each demographic group: the student strategies and the assignment functions that map NHC to college assignment.

We numerically approximate all four objects: two group-specific strategies and two group-specific inverse assignment functions. B-splines were used for the numerical approximations because these functions allow for accurate approximations of both a function and its derivatives using relatively few parameters. We chose to approximate the inverse assignment function because while the range of these functions (the colleges) are known ex ante, the domain (the noisy human capitals) is endogenous. We used six knots in our approximation, but found that using up to 20 knots had a negligible effect on our results. We insist that the strategies be consistent with the first order conditions for the student’s decision problem in the AA regime of interest (Equation 1). The assignment functions (i.e., the inverse of the inverse assignment function approximated by our B-splines) are required to be consistent with the human capital choices of the students.\(^{28}\)

We solved the model using an optimization algorithm that tries to minimize the inconsistencies between the approximated objects and the theoretical analogs described above. The optimization algorithm adjusts the variables describing all four of the numerical approximations simultaneously. Our first step within each iteration is to compute the induced inverse assignment function for each group, which is simply the inverse of the assignment function generated by the approximate student strategies and the contest structure generated by the form of AA we are studying. We then use an $L^2$ penalty function for the distance between the B-spline fit of the inverse assignment function and the induced inverse assignment function, which is computed at 50 points evenly...\(^{28}\)We solved for the status-quo system as a quality assurance check on our algorithms. Because quota and admissions preference schemes are outcome equivalent (Bodoh-Creed and Hickman [13]), we treated the status quo admissions preference scheme as quota wherein each group competed separately for the distribution of seats allocated to members of that group in the status quo.
spaced across the support of the distribution of college qualities. The measure applied in the $L_2$ norm is the estimated CDF of the distribution of college qualities available to that demographic group.\footnote{In the color-blind and proportional quota counterfactuals, this is the total measure of college qualities. In the status-quo model, the distribution of school available to students in a given demographic group is equal to the distribution of college qualities in which those students enroll.}

We now turn to our metric of the inconsistency between the approximate and exact equilibrium strategies. Our third step within each iteration is to compute the assignment function implied by our B-spline of the inverse assignment function. The fourth step of our algorithm is to calculate the first order condition given the assignment function for each group implied by the associated B-spline of the inverse assignment function and the B-spline of the strategy for that group. Our error function was an $L_2$ penalty function enforced at 50 evenly gridded values over the support of $\theta$. The measure used in the $L_2$-norm is the type distribution of the respective group.

The complete objective function for the optimization problem is the sum of the penalty functions for the inverse assignment mapping and the first order condition, which we chose for simplicity and the fact that the weighting did not seem to significantly affect the optimization results. If an optimal value of 0 is found, then the approximated inverse assignment functions and the approximate strategies are consistent with the first order conditions of the decision problem. In addition, an optimal value of 0 implies that the approximation strategies and inverse assignment functions are also consistent with each other as required by equilibrium.

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