PRE-COLLEGE HUMAN CAPITAL INVESTMENT AND AFFIRMATIVE ACTION: A STRUCTURAL POLICY ANALYSIS OF US COLLEGE ADMISSIONS*

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ABSTRACT. We study a structural model of college admissions framed as a contest between a continuum of students for enrollment in a continuum of colleges where the contest outcome is decided by the students' choice of human capital (HC). Students have private information about their learning costs, and colleges have heterogeneous, observable qualities. Our econometric model is inspired by methods from the empirical auctions literature and allow us to separately identify the roles of school quality, HC, and unobserved learning costs on post-college household income. We use our estimates to conduct counterfactual experiments comparing different college admissions rules including color-blind admissions, a proportional quota for minority students, and means-tested affirmative action (AA). An AA ban would result in a large migration of minority students out of the best schools and into the lowest quality schools with a corresponding reduction in household income and mean graduation rates. However, the signs and magnitudes of changes to HC investment and individual graduation rate depend on the demographics and learning cost type of the particular student in question. We also argue that a means-tested AA plan does not significantly increase racial diversity. Finally, our estimates imply that the competitive incentive to accrue HC is stronger than the productive incentive for all but the top 4% achieving students.

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1. INTRODUCTION

There are many economically salient features of the competition between high school students for seats at colleges and universities in the United States. An ideal model of this market would include heterogeneous college qualities, heterogeneous unobserved student abilities, and endogenous human capital choices.[1] This already complex picture is complicated further by the roles of asymmetric distributions of student characteristics by race and affirmative action (AA) admissions policies that take into account student demographics. While prior studies often include some of these features, we believe that ours is the first econometric analysis to account for all of them in a single structural model.

Affirmative action has a history in the United States that stretches back to the Kennedy Administration in the 1960s, and is now a pervasive, though controversial, fixture of American higher education.[2] The US Supreme Court has ruled on legal challenges to racial considerations in college admissions in four separate cases, *Regents of the University of California v. Bakke* (1978)[1], *Gratz v. Bollinger* (2003)[2], *Grutter v. Bollinger* (2003)[3], and *Fisher v. Texas* (2016)[4]. Affirmative action is in general motivated by racial disparities in college placement, and a desire to achieve greater diversity within student bodies at top universities. For example, in 1996 18% of all new college freshmen were under-represented minorities (Black, Hispanic, or Native American) who accounted for only 11% of new enrollees within the top fifth of US colleges, despite substantial considerations for race in the admissions process. In a world of competitive admissions, this disparity is in turn driven by gaps in pre-college academic achievement: in that same year the median minority SAT score was at the 19$^{\text{th}}$ percentile for Whites and Asians. It is widely believed that racial achievement gaps are rooted in various stark racial disparities of access to resources that play a critical role in childhood development. Among these are income, healthcare, and high-quality schools, though the precise mechanisms through which these factors act are not yet fully understood.

Building on the theoretical work of Bodoh-Creed and Hickman [14], we estimate a model wherein a continuum of college applicants with differing unobserved learning costs compete for admission to colleges of varying quality through the accrual of productive human capital (HC), and colleges use distinct enrollment criteria for different demographic groups. One important concern that the previous literature has ignored is the extent to which AA alters students' incentives to invest in pre-college HC. Within our model, these incentives are two-fold. First, HC is a productive asset because it prepares students to fully benefit from higher education. We refer to this as the *productive channel* of incentives, and our model controls for the fact that if HC and school quality are complementary inputs in the match utility function, then altering college

[1] In the United States, the terms *college* and *university* are used synonymously to refer to bachelor's degree granting institutions. We apologize if this differs from the reader's familiar usage, but we maintain the American convention of using the two terms interchangeably for our empirical application.

[2] Throughout this paper, when we use the term AA we refer to the practice of granting preferential treatment in college admissions on the basis of race, unless otherwise stated.

assignments will alter its magnitude. This channel is present in the complete-information assortative matching model of Becker [11]. Second, HC bears an indirect return of determining access to high-quality colleges. In particular, colleges' preferences over applicants are largely based on observable measures of HC investment like exam scores and high school GPAs. These allow for efficient assortative matching of better students with more thorough preparation to better schools with higher per-student spending and more accomplished faculty. These indirect returns create a *competitive channel* of incentives that drives students to use achievement as a means of establishing their position above less-accomplished competitors, and it resembles the signaling phenomenon analyzed in Spence's [50] seminal model. The competitive channel creates a strategic interaction between one's own actions and the actions of others: if rivals never study mathematics and science, one might study less, consume more leisure time, and still place into a top college, and vice versa. In other words, a full understanding of a student's HC choice cannot focus only on her individual production technology, but rather, it must also take into account strategic pressures she faces from the rest of the market.

The first goal of our analysis is to tease apart the effects of college quality, human capital, and unobservable characteristics on post-college income. Since heterogeneous learning costs are private information to each student, a structural approach is crucial for identifying them and appropriately accounting for their influence. Our model allows us to apply techniques developed by Guerre, Perrigne, and Vuong [32] for first-price auctions in a novel way to identify student learning costs from the choice each student made, given the strategic incentives she faced. Intuitively, our theory model provides an inverse mapping from students' observable academic achievement into the underlying cost types which rationalize their choices as best responses to prevailing competition. This inverse mapping is embedded into a wage regression as a non-linear control function to separate the impacts of invested HC from one's permanent type in determining post-college income. In addition, the structural approach is crucial for any counterfactual analysis of different AA schemes, since changes to admissions rules have complex effects on both the productive and competitive channels of incentives, and in turn, on choices. For example, would a stronger AA scheme increase minority HC levels due to complementarity between school quality and HC? Or would the prospect of easier admissions reduce the incentive to accrue HC? And, are these effects different for students with distinct underlying types?

Our analysis uses individual-level data from the Baccalaureate and Beyond (B&B) survey conducted by the U.S. Department of Education, on student demography, academic achievement, and post-college household income. There are two main sources of variation in the attributes of students on a given college campus that feed into our identification strategy. First, although the market is highly assortative in that better colleges tend to attract more accomplished students, it is not perfectly so, and each university's student body exhibits a non-degenerate distribution of human capital, within race groups. This feature of the data implies that college quality and achievement are imperfectly correlated. The second source of variation is AA practices, which create different investment incentives for students in different demographic groups. This means that the mapping from a student's private information into her choice of HC accumulation will (indirectly) depend on her demographic group affiliation. Since colleges enroll students from all demographic groups, this implies variation in HC choices for students with the same private information on each campus, and therefore achievement and unobserved characteristics are imperfectly correlated as well. We combine this model structure with individual-level data on achievement, college of attendance, and post-graduation income to identify the wage regression function. In a preliminary step of estimation, we also use the B&B data to identify the joint distributions of school quality and achievement for new college enrollees by race, which requires a novel sample-selection correction procedure. To correct for sample selection—college dropouts do not appear in B&B—we also use college-level data on school quality and race-specific graduation rates, provided by U.S. News & World Report and the US Department of Education's Integrated Postsecondary Education Data System (IPEDS). Together, these data allow us to identify the form of status quo AA, the wage regression function, and unobserved student types.

We find a non-trivial role for AA to shape HC incentives and enrollment on all segments of the college quality spectrum above the 10% lowest-quality colleges. We find that college quality and pre-college HC both have a significant effect on the probability that a student graduates. On the other hand, HC has very little influence on household income 10 years following graduation, where college quality and unobserved student characteristics play dominant roles. We also find that the distribution of learning costs for minority students stochastically dominates the distribution of nonminority students. In other words, our empirical evidence suggests minority students in general have higher learning costs.[3] We discuss the connection between this result and a large body of empirical work documenting substantial racial disparity in childhood developmental and educational resources.

We then conduct a counterfactual analysis to study the effects of different AA schemes on enrollment of minority and nonminority students, graduation rates, and household income. Our first result is that admissions would be quite different in an alternative color-blind world. Under the status quo AA regime we estimate from the data, minorities are under-represented at the best schools and heavily over-represented at the worst ones, with 27.6% enrolling in the bottom quintile of the college quality distribution and another 28.1% in the second lowest quintile. However, a color-blind college admissions scheme would result in a large shift of minority students into lower ranked schools, with 42.0% in the bottom quintile and 18.7% in the second to last quintile. Under a proportional quota scheme, the opposite occurs: minorities enroll in higher quality programs with (mechanically) 20% of them in each college quality quintile.

In terms of the HC accumulation decisions of minority students, color-blind and proportional systems have roughly symmetric, but opposite, effects. In a color-blind scheme, minority students with learning costs above the median reduce their HC choices, relative to the status quo. The primary driver of the decrease in investment is that shifting to a color-blind system increases

[3]We do not believe that there is much if any reason to think that these learning costs are "innate" in a biological sense. For example, the higher socioeconomic strain that lower-income (on average) minority families tend to be under can make the actual and opportunity cost of early childhood monetary investments more difficult than for the typical nonminority family. This is compounded by institutional features such as the fact that minority students tend to be enrolled in primary and secondary schools with a high level of poverty and relatively low achievement.

the ratio of seats at low-quality colleges for high learning cost minority students, which makes it more difficult for them to obtain a seat at a better college by out-competing rivals. In contrast, minority students with learning costs below the median find themselves facing a restricted supply of high-quality seats, which motivates them to invest more aggressively in competition for them. For nonminority students, the effects of shifting from the status quo to color-blind are the opposite, but of smaller and generally inconsequential magnitude.

The changes in HC investment from a shift toward a proportional quota from the status quo are larger in magnitude and reversed in sign. We find that the 75% of minority students with the highest learning costs increase their HC invesments, while the 25% of minority students with the lowest learning costs reduce their HC investment. The reasoning is essentially symmetric to the color-blind case—high learning cost minority students now face a more favorable (*i.e.*, lower) ratio of college seats to competitors, while low learning cost minority students confront the opposite change.

We find that the effects of the AA programs on enrollment translate into intuitive effects on graduation rates and household incomes 10 years after graduation. More generous AA schemes generally lead to higher average graduation rates for minorities, with little effect on non-minorities. Since minority students are assigned to worse colleges under a color-blind admissions system, the average household income of minority students drops by $1,624, relative to the status quo, while nonminority students have $309 more annual income on average. Under a proportional system, minority students increase their household income 10 years after graduation by an average of $2,852 due to their better college placements, while nonminority student income drops by an average of $536.

Due to the legally contentious nature of AA programs, particularly at public universities, proposals have emerged for means-tested forms of AA that might have the benefit of encouraging racial diversity on college campuses in a less legally problematic way. To test this conjecture, we analyze several different proportional quotas that reserve a fixed fraction of the seats at each university for the most socioeconomically disadvantaged applicants, which we measure in terms of the *Expected Family Contribution* (EFC) to college expenses, a measure based on wealth and income used by the US Department of Education. We find that under an AA scheme based purely on socioeconomic status, the outcome is very close to that which obtains under a color-blind admissions system. As it turns out, socioeconomic status is a poor proxy for racial demographics when predicting learning costs within the population of college applicants.[4] Therefore, separating poor and non-poor students using EFC cannot induce sufficient *ex-ante* cost asymmetry for quotas to engineer substantial differences from color-blind admissions.

We close our empirical analysis by studying the relative magnitudes of the productive and competitive channels of incentives. To the best of our knowledge, ours is the first paper to separate the marginal benefit of pre-college HC accumulation into a competitive channel and a

[4]This result would seem to corroborate a relatively new body of evidence which suggests that educational differences by race in the US cannot be simply decomposed as differences by wealth and income, due in part to persistent racial segregation in housing and schooling (*e.g.*, see Reardon [44]).

productive channel. We find that the competitive channel is stronger than the productive channel for all but the highest-achieving 4% of students overall. The relative strength of the competitive channel increases for students with higher learning costs, being roughly three times as powerful as the productive channel for the middle 50% of the learning cost distribution.

The remainder of this paper has the following structure: we first briefly summarize the previous literature on AA and discuss its relation to the current model. In Section 2, we outline the theoretical model on which the econometric exercise is based. In Section 3.1, we describe the US college data that will be used for identification, and in Section 3.2 we formally outline our semi-parametric identification results based on methods pioneered by Guerre, *et. al.* [32] in the empirical auctions literature. In Section 3.3 we define a two-stage estimator for the structural model that falls within the class of Generalized Method of Moments estimators. In Section 4, we discuss the results of estimation and in 5 we present the counterfactual exercise. Section 6 concludes and briefly describes directions for future research.

1.1. **Related Literature.** Bowen and Bok [15] and Kane [37] were among the first to study how actual AA policies change college admissions. They found that it plays a substantial role in allocating high-quality college seats to minorities. Chung and Espenshade [20] and Chung, Espenshade, and Walling [21] undertake a similar study using matched student-college applications data from a small set of elite private universities to similar results. Several other papers have taken a structural approach to studying the impact of AA. Epple, Romano, and Sieg [25] model colleges setting tuition and admissions standards for different races. They calibrate the model to the US market and find that the data are broadly consistent with diversity preferences on the part of colleges. Two additional papers structurally estimate the college admissions process in order to control for changes in application behavior from counterfactual policy shifts. Arcidiacono [6] models applications, admissions, enrollment, major choice, and entrance into the labor market, with the aim of identifying counterfactuals when AA is eliminated from college admissions. Howell [35] performs a similar exercise with a more recent data set, but focusing solely on the admissions process. Both papers find that AA plays a significant role in shaping black educational outcomes, especially among the most selective institutions.

Another vein of empirical literature on AA focuses on *mismatch*, or the idea that AA may cause black students to be placed higher, but then graduate with lower probability, or to self-select into less lucrative majors. Some empirical studies have found evidence of mismatch (*e.g.*, Loury and Garman [40], Arcidiacono, Aucejo, Fang, and Spenner [7], and Arcidiacono, Aucejo, and Hotz [8]), while other empirical work has found evidence that the mismatch problem is small and likely outweighed by other benefits of higher-quality placement (*e.g.*, Long [39], Rothstein and Yoon [47], and Chambers, Clydesdale, Kidder, and Lempert [19]). Other evidence suggests, to the contrary, that all students generally benefit from attending higher-quality schools (*e.g.*, Dillon and Smith [24] and Badge, Epple, and Taylor [10]). Our work contributes to this debate in a unique way: by allowing for pre-college academic preparation to endogenously adjust to alternative admissions schemes, we allow for graduation rates to either rise or fall, counterfactually. We find that more generous AA schemes raise average graduation rates for minorities, but that the sign and magnitude of the change varies with student unobservable characteristics.

Along those lines, in most of the papers above, exam scores are used as a proxy for student ability, and assumed to be fixed with respect to variations in admissions criteria. However, if scores are jointly determined by both ability and market-based incentives for investment, then this is problematic. Our paper forms part of a new literature focusing on endogeneity of academic achievement with respect to admissions rules. Other such papers include Ferman and Assunção [27] who leverage a natural experiment in top Brazilian university admissions policies to uncover reduced-form evidence that pre-college academic achievement varies by admissions rules. Cotton, Hickman, and Price [22] seeks to replicate the Bodoh-Creed and Hickman [14] framework in a field experimental classroom setting in order to directly measure shifts of investment incentives. Cotton, *et. al.* set up a math competition among middle school students, with AA in place to assist students from lower grades who have less math preparation. These studies find evidence, in both observational data and in the field, that HC investment is influenced by AA, and therefore a model must take this into account when producing counterfactual estimates. Our paper takes a structural approach to control for behavioral shifts induced by changes to market allocation rules. This facilitates other market design questions such as, how do American allocation rules compare to other untested mechanisms?

Of course, this work is subject to its own limitations as well. We do not explicitly model "supply-side" concerns—*e.g.*, decisions on how many students to admit and how much to charge them—but instead we model college seats as fixed objects of known quality in order to concentrate just on student HC investment. To the extent that supply-side competition plays a role, this work can be seen as complementary to models such as Epple, *et. al.* [25], Chade, Lewis, and Smith [18], Fu [31], Azevedo and Leshno [9], and Fillmore [28] who treat these forces explicitly.

2. THEORETICAL MODEL

Following Bodoh-Creed and Hickman [14], we model college admissions as a Bayesian game where high-school students are characterized by a privately-known type that governs the costliness of HC production, as well as the payoff from enrolling in college. Students compete for enrollment at colleges of differing quality through their choice of HC level, which we view as a long-run plan for the accumulation of HC over several years leading up to the college admissions process. On the other side of the market are colleges that have preferences over students based on their HC and race. In general, colleges wish to attract the most academically advanced students, but they also prefer to have a demographically diverse student body as well, and their trade-off between race and pre-college academic achievement is reflected in their AA plan.

One empirical regularity of American college data is a nontrivial degree of academic heterogeneity among students on a given college campus (even within demographic groups). To capture this heterogeneity, our model includes market frictions in the form of a random matching shock to the colleges' perceptions of a particular student's HC choice. This shock is observable to colleges at the time the student applies, but not to the student while she is investing in her

HC. The unobservability of the shock to the student is consistent with an interpretation of HC choice as the gradual accumulation of learning over years, while the shock is the result of events that occur within a short time before her application to college, and that are by and large out of the student's control. A student's ex-post payoff is her match utility minus her HC investment cost. Her match utility is a function of the student's college assignment, her HC level, and her privately known learning cost type, while her cost of HC is a function of her type and the HC level she has chosen.

2.1. **Agents.** There are two demographic subgroups within the student population, minority students ($\mathcal{M}$) and nonminorities ($\mathcal{N}$), and the demographic class of each student is observable. There is a continuum of students of total mass 1 with mass μ of them in the minority group. We often refer to our model with a continuum of students as a *limit model* to emphasize the fact that it can be viewed as a limit of a model with a finite number of students, as their number approaches infinity. We elaborate on this point in Section 2.5. Each student has a privately-known learning cost type $\theta \in [\underline{\theta}, \overline{\theta}]$, and the distribution of θ in group $i \in \{\mathcal{M}, \mathcal{N}\}$ follows a cumulative distribution function (CDF) $F_i(\theta)$. For convenience, we denote the unconditional type distribution by $F_{\mathcal{K}}(\theta) \equiv \mu F_{\mathcal{M}}(\theta) + (1-\mu)F_{\mathcal{N}}(\theta)$. Each agent's strategy space, $\mathcal{S} = [\underline{s}, \infty)$, is the set of attainable HC levels. These are observable (*e.g.*, through standardized exam scores and high school GPAs) and $\underline{s}$ is the minimum required to attend some college. In order to produce s units of HC, a student incurs cost $C(s;\theta)$ which is increasing in both s and θ.

Investment costs can arise in various ways, such as from a consumption–investment tradeoff or psychic costs from difficult learning activities. Learning cost types θ may encapsulate both cognitive and non-cognitive characteristics, and they may be influenced by forces both internal to the individual (*e.g.*, innate ability or natural curiosity) and external (*e.g.*, home environment, primary/secondary school quality, or financial resources). We remain agnostic on the exact interpretation of θ, but we assume that it is fixed from the perspective of a student when she chooses her level of HC investment. Mathematically, the difference between types θ and investment s is that the former reflects the exogenous portion of HC costs and the latter arises from a costly decision under the control of the agent. For example, consider the case of a parent who may enrich his daughter's educational experience by spending time reading or doing schoolwork with her. In a low-income household where the parent must work two jobs, his time may be more constrained than in an affluent household where the parent has a single, high-paying job. In this example, the parent's and child's choice of how much time to spend on learning activities is encapsulated in s, whereas the pre-existing opportunity cost of time is reflected in θ.

A matching shock is applied to each student's choice of HC, s, to generate a noisy HC (NHC) value that is commonly observed by all of the colleges. We assume the noise enters additively, so if student i chooses s, the associated NHC is $t = s + \varepsilon$. We assume that ε is not observed by the student until after she has chosen s, so from her perspective it is a random variable with CDF F_ε. The shock allows the market to deviate from perfect assortativity so as to rationalize within-campus HC variation in the data. As the variance of ε gets very small it becomes perfectly assortative, and as the variance gets very large the market becomes a lottery.

2.2. **Payoffs.** On the other side of the market there is a continuum of colleges with total mass 1. Each college's quality is described by an index $p \in [\underline{p}, \overline{p}]$ that is distributed $P \sim F_{\mathcal{P}}(P)$. By assuming the measure of students and college seats are the same, we abstract from the extensive margin of college attendance, focusing only on the intensive margin of competition for admission to the best colleges, conditional on entering the market.Both college quality and HC are intrinsically valued: match utility $U(p, s, \theta)$ results from a student with type θ having HC s and enrolling in a college with quality p. The *ex post* payoff to agent i in group $j \in \{\mathcal{M}, \mathcal{N}\}$ is the match utility minus the cost of achievement, $U(p, s, \theta) - C(s, \theta)$.

2.3. **Allocation Mechanisms.** In an admissions contest, students are allocated seats at colleges of varying quality, and the quality of the seat allocated to a student is a function of how her NHC realization compares to the distribution of NHC across the population of college applicants. Affirmative action schemes cause the NHC realizations of minority and nonminority students to be compared to the total distribution of NHC differently, which makes the contest asymmetric between the two demographic groups. We consider *color-blind* (cb), *proportional quota* (q), and *admissions preference* (ap) AA systems, which are described formally below. Despite the richness of the model, our contest mechanism provides a parsimonious characterization of endogenous HC investment within a complex market setting where HC plays dual roles of a productive asset and determining one's match prospects. One can also view the contest as a form of all-pay auction in which the admission scheme gives rise to an endogenous pricing rule that dictates the amount of HC an individual must "pay" to win a given seat.

The equilibrium allocation mechanism is an assignment mapping $P_j^r : \mathbb{R} \to [\underline{p}, \overline{p}]$, $j \in \{\mathcal{M}, \mathcal{N}\}$ and $r \in \{ap, cb, q\}$, that maps a student's NHC realization into an enrollment at some college. P_j^r is an endogenous object that depends on the form of AA at work in the market, the distribution of college qualities, and the equilibrium distribution of NHC in the population, which is in turn determined by the endogenous choices of HC by the students. To facilitate discussion we denote the equilibrium CDFs of HC and NHC as G_j^r and H_j^r, respectively, where $r \in \{ap, cb, pr\}$ indicates the allocation mechanism and $j \in \{\mathcal{M}, \mathcal{N}, \mathcal{K}\}$ indicates a demographic group or the unconditional population. Note that the distribution H_j^r is a convolution of G_j^r and F_ε with a density given by the familiar formula, $h_j^r(t) = \int_{-\infty}^{\infty} f_\varepsilon(\varepsilon) g_j^r(t - \varepsilon) d\varepsilon$.

The color-blind allocation mechanism is the simplest to define since it assigns students to schools assortatively with higher NHC realizations leading to assignment at higher quality colleges. Since demographics do not affect the assignment, $P_{\mathcal{M}}^{cb}(t) = P_{\mathcal{N}}^{cb}(t) = P^{cb}(t) = F_{\mathcal{P}}^{-1}\left(H_{\mathcal{K}}^{cb}(t)\right)$. In words, a student with NHC realization t at quantile rank $H_{\mathcal{K}}^{cb}(t)$ of the NHC distribution (for all students) is placed at a school with the same quantile rank in the college quality distribution, $F_{\mathcal{P}}^{-1}\left(H_{\mathcal{K}}^{cb}(t)\right)$. Since the assignment mapping is the same for both groups, marginal investment incentives, conditional on type θ, are also identical across groups.

The next simplest mechanism is a quota scheme, which is currently used in various parts of the world, although the US Supreme Court declared it unconstitutional in the college admissions context in 1978 [1]. Under an arbitrary quota scheme, students from group $j \in \{\mathcal{M}, \mathcal{N}\}$ are reserved sets of seats with qualities distributed $P \sim Q_j(P)$, where $\mu Q_{\mathcal{M}}(p) + (1-\mu)Q_{\mathcal{N}}(p) = F_{\mathcal{P}}(p)$, $\forall p$ is required for feasibility. Under a quota mechanism, members of each demographic group compete for prizes only against the other members of their same demographic group, in disjoint contests. The resulting assignment map is $P_j^{pq}(t) = Q_j^{-1}\left(H_j^{pq}(t)\right)$, which means a student with NHC realization t at quantile rank $H_j^q(t)$ of the NHC distribution of her own demographic group is placed at a school with the same quantile rank in the distribution colleges allocated to her demographic group, $Q_j^{-1}\left(H_j^q(t)\right)$. The most familiar member of this class is a *proportional quota* where $Q_{\mathcal{M}} = Q_{\mathcal{N}} = F_{\mathcal{P}}$, meaning that fraction μ of seats at each point in the college quality spectrum are reserved for minorities.

Although proportional quotas are not directly applicable in the US as a policy instrument, they provide a useful benchmark relative to a color-blind mechanism. When racial asymmetries in learning costs exist—because race is correlated with childhood school quality, for example—proportional quotas are designed so that these differences are not reflected in the fraction of students from each demographic group present on each college campus, particularly the best ones that would otherwise be out of reach in equilibrium. A color-blind mechanism allows asymmetries in learning costs between the demographic groups to be maximally reflected in the fraction of minority students enrolling in high-quality colleges.

Finally, an *admission preference* system refers to one in which the NHC values of both groups are compared to the distribution of the total student population, but the demographic status of a given student determines how this comparison is made. To the extent that American colleges engage in race-based AA at present, the only legally permissible form is an admissions preference system where race is taken as a "plus factor" among other considerations like grades and test scores. Formally, an admission preference is defined by a markup function $\widetilde{T} : \mathcal{S} \to \mathbb{R}$ that transforms the NHC levels of the minority students. The resulting assignment mappings are

$$P_{\mathcal{M}}^{ap}(t) = F_{\mathcal{P}}^{-1}\left(\mu H_{\mathcal{M}}^{ap}(t) + (1-\mu) H_{\mathcal{N}}^{ap}\left[\widetilde{T}(t)\right]\right)$$
$$P_{\mathcal{N}}^{ap}(t) = F_{\mathcal{P}}^{-1}\left(\mu H_{\mathcal{M}}^{ap}\left[\widetilde{T}^{-1}(t)\right] + (1-\mu) H_{\mathcal{N}}^{ap}(t)\right)$$

In words, a minority student's NHC level t is compared to the raw NHC of other minority students and marked up when compared to other nonminority students. Conversely, NHC for a nonminority student is compared to the raw NHC levels of other nonminority students and it is effectively "de-subsidized" for comparisons to NHC levels of minority students.

It is obvious that the admission preference mechanism nests the color-blind as a special case when $\widetilde{T}(t) = t$. Bodoh-Creed and Hickman [14, Theorem 4] proves that that all of the AA systems described above are equivalent under the conditions of our model in the following sense:

Theorem 2.1. *[Theorem 4 of Bodoh-Creed and Hickman (2017)]* $P_j^q(t) : \mathbb{R} \to \mathcal{P}$, $j \in \{\mathcal{M},\mathcal{N}\}$, *is the result of an equilibrium of some quota system with* $(Q_{\mathcal{M}}, Q_{\mathcal{N}})$ *if and only if there exists an admissions preference system with some* $\widetilde{T}$ *that has the same equilibrium assignment mappings and strategies.*

Because the allocation of students to schools is known *ex ante* under a quota scheme, our empirical approach will rely heavily on this useful equivalence result. We base our structural model on a (non-proportional) quota system that is outcome equivalent to the real-world admissions preference. This greatly simplifies identification and the construction of our estimator.

2.4. **Model Assumptions.** We do not get existence of an equilibrium without some assumptions. Although these assumptions are not used directly in our estimation, we reproduce them from Bodoh-Creed and Hickman [14] for completeness. Assumptions 2.2 - 2.4 require that the type, college quality, and matching shock distributions admit differentiable probability density functions (PDFs) with a connected support.

Assumption 2.2. $F_j(\theta) \in \mathcal{C}^2$, $j \in \{\mathcal{M},\mathcal{N}\}$ *and densities* $f_{\mathcal{M}}(\theta)$ *and* $f_{\mathcal{N}}(\theta)$ *are strictly positive on a common compact support* $[\underline{\theta}, \overline{\theta}]$ *with non-empty interior.*

Assumption 2.3. $F_P(p) \in \mathcal{C}^2$ *and the prize density* $f_P(p)$ *is strictly positive on a compact support* $[\underline{p}, \overline{p}]$ *with non-empty interior.*

Assumption 2.4. *The distribution of matching shocks is absolutely continuous with full support:* $\varepsilon \sim F_\varepsilon(\varepsilon)$, $F_\varepsilon \in \mathcal{C}^2$, *and* $f_\varepsilon(\varepsilon) > 0$, $\forall \varepsilon \in (\underline{\varepsilon}, \overline{\varepsilon}) \subseteq \overline{\mathbb{R}}$.

Assumption 2.5 imposes regularity conditions on the cost function. We associate high values of one's permanent type θ with high HC production costs, and low values of θ with low costs. Assumption 2.6 imposes regularity conditions on the match utility function $U(p,s,\theta)$. First we require that students benefit from enrolling in a high quality college (U is increasing in p), high levels of HC (U increasing in s), and we allow for permanent types to play a role as well (U decreasing in θ). Moreover, we require utility to be monotone in s, with convex costs and concave (in s) match utility.

Assumption 2.5. $C_s(s,\theta) > 0$, $C_{ss}(s,\theta) \geq 0$, *and* $C_\theta(s,\theta) > 0$.

Assumption 2.6. $U_p(p,s,\theta) > 0$, $U_s(p,s,\theta) \geq 0$, $U_\theta(p,s,\theta) \leq 0$ *and* $U_{ss}(p,s) \leq 0$.

Assumption 2.7 implies that the highest cost students find it optimal to choose the lowest level of HC that qualifies the student to attend college. From a formal perspective, this assumption provides a boundary condition for solution of the model equilibrium.

Assumption 2.7. $\sigma_{\mathcal{N}}^r(\overline{\theta}) = \arg\max_s E_\varepsilon \left[U\left(P_{\mathcal{N}}^r(s+\varepsilon), s, \overline{\theta}\right)\right] - C(s,\overline{\theta}) = \underline{s}$, $r \in \{cb, q, ap\}$

The following assumption ensures existence of a monotone equilibrium:

Assumption 2.8. $C(s,\theta)$ *is strictly supermodular in* (s,θ) *and* $U(p,s,\theta)$ *is supermodular in* $(p,s,-\theta)$.

Finally, we require that there is a highest possible HC that any student is willing to choose, which means that the effective action space is compact. Our assumption requires that this upper bound, denoted $\overline{s}$, will not be chosen by any type of student even if such a choice would result in enrollment into the best possible school.

Assumption 2.9. *There exists* $\overline{s}$ *such that for all* θ *we have:*

$$U(\overline{p}, \overline{s}, \theta) - C(\overline{s}, \theta) \leq U(\underline{p}, \underline{s}, \theta) - C(\underline{s}, \theta)$$

Finally, we require the following regularity condition on the markup function used in the admission preference system. This assumption is that $\widetilde{T}$ is strictly increasing (*i.e.*, the mechanism respects rank ordering within demographic groups) and that the markup function does not increase so steeply that students have an arbitrarily strong incentive to increase s.

Assumption 2.10. *There exists* $0 < \lambda_1 < \lambda_2 < \infty$ *such that for all* t *we have* $\lambda_1 < \widetilde{T}'(t) \leq \lambda_2$.

2.5. **Equilibrium.** The distribution of college seats, the form of the admission system, and the measures and distributions of student competitors are common knowledge prior to individual choices of HC investment. When combined with the equilibrium strategies of the minority and nonminority students, denoted $\sigma_{\mathcal{M}}(\theta)$ and $\sigma_{\mathcal{N}}(\theta)$ respectively, the students can forecast the form of the assignment mapping P_j^r arising in equilibrium. Each student solves the following optimization problem where $j \in \{\mathcal{M}, \mathcal{N}\}$ and $r \in \{cb, pq, ap\}$:

$$\sigma_j(\theta) = \arg\max_s \left\{ \mathrm{E}_\varepsilon \left[U\left(P_j^r(s+\varepsilon), s, \theta \right) \right] - C(s, \theta) \right\} \tag{1}$$

In equilibrium, students' beliefs about $\sigma_{\mathcal{M}}(\theta)$ and $\sigma_{\mathcal{N}}(\theta)$ must be consistent with the solution to Equation 1 so that the students' beliefs about $P_j^r(t)$ are correct. Given the assumptions described in Subsection 2.4, Bodoh-Creed and Hickman [14, Theorem 6] proves that such an equilibrium exists, and for completeness we reproduce the statement of that result here without proof.

Theorem 2.11. *[Theorem 6, Bodoh-Creed and Hickman* [14]*] There exists a monotone, pure strategy Nash equilibrium of our limit model in the color-blind, quota, or admissions preference systems.*

Actual college markets one might study empirically have only finitely many students and colleges, but on the other hand, the finite version of this model is computationally intractable. However, Bodoh-Creed and Hickman [14] establish empirical relevance of the continuum model by showing that it provides an accurate representation of a finite model with a large number of players. In this finite model, $K_{\mathcal{M}}$ minority students draw types from the distribution $F_{\mathcal{M}}(\theta)$, $K_{\mathcal{N}}$ nonminority students draw types from the distribution $F_{\mathcal{N}}$, and $K_{\mathcal{M}} + K_{\mathcal{N}}$ college seats draw their qualities from $F_{\mathcal{P}}$. In the limit as $K_{\mathcal{M}} + K_{\mathcal{N}} \to \infty$ and $K_{\mathcal{M}}/(K_{\mathcal{M}} + K_{\mathcal{N}}) \to \mu$, the primitives of the finite games approach those of the continuum model. Bodoh-Creed and Hickman [14] show that the continuum model approximates the finite game in the following sense:

Definition 2.12. *Given $\epsilon > 0$, an ϵ-approximate equilibrium of the K-agent game is a K-tuple of strategies $\boldsymbol{\sigma}^\epsilon = (\sigma_1^\epsilon, \ldots, \sigma_K^\epsilon)$ such that for all agents, almost all types θ, and all HC choices s' we have*

$$U\left(P_j^r(\sigma_i^\epsilon(\theta), \sigma_i^\epsilon(\theta)), \sigma_i^\epsilon(\theta), \theta_i\right) - C(\sigma_i^\epsilon(\theta), \theta_i) + \epsilon \geq U\left(P_j^r(s', \sigma_i^\epsilon(\theta))), s', \theta_i\right) - C(s', \theta_i)$$

Definition 2.12 describes an approximate equilibrium in terms of incentives: agents that follow an $\epsilon-$approximate equilibrium can gain at most ϵ by deviating. Intuitively, students lose little utility if they base their actions on the easy-to-compute limit game equilibrium. Our final theorem shows that we can choose $\epsilon > 0$ to be arbitrarily small as the size of the market increases.

Theorem 2.13. *[Theorem 7, Bodoh-Creed and Hickman* [14]*] Let σ_j^r, $j \in \{\mathcal{M}, \mathcal{N}\}$ and $r \in \{cb, q, ap\}$, denote an equilibrium of the game with a continuum of agents. We can choose K^* such that σ_j^r is a ϵ-approximate equilibrium of the K-agent game for any $K > K^*$.*

3. EMPIRICAL MODEL: IDENTIFICATION AND ESTIMATION

Our theoretical model produces a rich set of implications concerning investment behavior and equilibrium outcomes under different AA mechanisms and market conditions. We now use it to construct an empirical model to address our main research questions. These include 1) the effect of college quality, pre-college investment, and permanent type on the returns to a college education; 2) the link between AA and racial inequality of both achievement and college placement outcomes; and 3) the role of competition in pre-college investment.

The structural approach is helpful in overcoming several challenges faced by previous empirical work on AA. First, there is the need to control for endogenous investment responses to counterfactual changes in admissions criteria. Second, pinning down counterfactuals requires the researcher to separately identify the roles of both observable factors—investable HC and school quality—and unobserved factors—idiosyncratic learning costs—in determining economic outcomes. In this section we show how the structural primitives of our model can be identified from existing observational data. We begin with a brief discussion of the observables.

3.1. Observables.

We use US college data for academic year 1992-1993 for two main reasons. First, one can reasonably assume AA policies were stable and understood by decision-makers at that time. The only successful legal challenge prior to 1996 was in 1978, when the Supreme Court declared quotas unconstitutional in *University of California v. Bakke* [1]. The second reason for studying AY1992-1993 is that individual-level data are available from the Baccalaureate and Beyond (B&B) database, linking college quality and HC choices to the household income of college graduates from that year. Thus, our empirical application can be interpreted as a case study of how AA shaped the college landscape for the parents of today's high-school students.

3.1.1. *Colleges.* For a sample $\mathcal{L} = \{1, 2, \ldots, L\}$ of 4-year colleges we have a vector $\mathbf{Y}_l$ of school characteristics. The first is a quality measure derived from data and methodology by US News & World Report (USNWR) for their annual *America's Best Colleges* rankings (see Morse [41]).[5]

[5]USNWR computes its quality score as a weighted arithmetic mean of a school's quantile rank in 15 quality indicators, falling into 6 different categories: *Reputation Rank* (based on survey data from college presidents and deans;

We adopt this measure as the college quality index p_l, and we argue that interpreting this index as a reflection of meaningful quality rankings is sensible for three reasons. First, USNWR solves market information frictions by providing a wealth of data on many schools, along with advice on how to interpret the data. Consumers' response to this service has been large enough that rankings are now the primary focus of USNWR's business model. Second, the validity of USNWR rankings is undoubtedly reinforced in students' minds by the enthusiasm with which universities advertise their status in *America's Best Colleges*. Third, many previous studies have depicted college quality either with coarse, discrete measures, or with relatively simplistic ones such as mean student-body exam score alone. Our measure provides for a continuous transition from low quality to high, and it encompasses a host of factors influencing the college experience such as selectivity, per-student spending, and faculty quality.

The US postsecondary education industry is vast and diverse, with thousands of institutions offering students at least one type of 4-year degree, but many of these specialize in vocational training. Thus, we adopt the USNWR universe of schools as our definition of "the college market." This leaves us with 1,245 non-profit colleges and universities specializing primarily in liberal arts education leading up to a bachelor's degree. This set of schools accounts for the majority of 4-year degree production in the United States. Between the late 1980's and early 1990's the total size of the incoming freshman class for these schools was roughly 1.64 million students.

The other college-level are provided by the National Center for Education Statistics (NCES) through their Integrated Postsecondary Education Data System (IPEDS), which includes school-level enrollment for all first-time freshmen (including full-time and part-time) by race for Whites, Blacks, Hispanics, Asians or Pacific Islanders, American Indians or Alaskan Natives, non-resident aliens, and race unknown. For the 1988 incoming class, we have freshman headcount, denoted M_l, for underrepresented minorities—Blacks, Hispanics, and Native Americans—and a headcount, denoted N_l, for all others—Whites, Asians, race unknown, and non-residents. Aggregating this information across schools allows us to compute $\mu = \sum_{l=1}^{L} M_l / \sum_{l=1}^{L}(M_l + N_l)$. IPEDS also allows the researcher to compute race-specific, 6-year graduation rates for each college campus, which we denote by Γ_{jl}, $j \in \{\mathcal{M}, \mathcal{N}\}$. In total then, the data representing schools $l = 1, \ldots, L$ are denoted $\mathbf{Y}_l = \{p_l, M_l, N_l, \Gamma_{\mathcal{M}l}, \Gamma_{\mathcal{N}l}\}_{l=1}^{L}$.

Colleges are separated into five tiers with Tier I representing the top quintile of college quality. Entries in Table 1 represent the fraction of all students within a tier that are of a given demographic group. In a hypothetical world with no under-representation, each cell would be the

25% weight), *Student Selectivity* (comprising acceptance rate for 1992 freshman class, yield rate for 1992 freshman class, % of enrollees in top 25% of high school class, and mean SAT/ACT score among enrollees; 25% weight), *Faculty Resources* (comprising student-faculty ratios, % of full-time faculty with terminal degrees, % of faculty on part-time status, average salary and benefits for full-time faculty, and proportion of classes with fewer than 20 students; 20% weight), *Financial Resources* (comprising per-student education expenditures and per-student other expenditures; 15% weight), *Graduation Rate* (% of students in 1983-1986 freshman classes who graduated within 6 years; 10% weight), and *Alumni Satisfaction* (% of living alumni who donated to AY1991-1992 fund drives; 5% weight). USNWR computes the quality metric separately within 4 different groups of schools (National universities, national liberal arts colleges, regional universities, and regional liberal arts colleges, see Morse [41]), but we modify their method slightly and define quality indicator quantile ranks across the entire universe of schools. This produces a single index ranking that applies to all colleges in the sample. We used the 1994 edition because several of its quality indicators are on a 2-year lag, and we will be combining these data with student-level observations from the graduating class of AY1992-1993.

TABLE 1. Racial Representation Within Academic Quality Quintiles

Group	Total Share	Tier I Share	Tier II Share	Tier III Share	Tier IV Share	Tier V Share
Minority:	**15.81%**	10.23%	13.09%	11.75%	22.16%	21.81%
Non-Minority:	**84.19%**	89.77%	86.91%	88.25%	77.84%	78.19%

same as the overall share of each race group. "Minorities" as defined above are under-represented in the top three tiers, and over-represented in the bottom two. Similar patterns hold as well for race sub-categories: Blacks, Hispanics, and Native Americans are individually under-represented in top tiers and the reverse is true for Whites and Asians. Groups $\mathcal{M}$ and $\mathcal{N}$ are defined as they are because AA in college admissions specifically targets *under-represented* minority groups.

3.1.2. *Students.* Our individual-level data on the student population comes from the 1993 Baccalaureate and Beyond Survey (B&B), which randomly samples colleges and then samples students graduating in AY1992-1993 within each college. The data contain several variables pertaining to pre-college investment for student $i \in \{1, 2, \ldots, I\}$. The two outcome variables which researchers and college admissions officers focus on most for assessing academic achievement are exam scores, denoted e_i, and academic record as measured by grade point average (GPA), denoted a_i. In the B&B data, e_i takes the form of either the ACT or the SAT, both of which are standardized college entrance exams. The companies which develop these exams also produce concordance tables which allow one to relate ACT scores into SAT scale and vice versa.

Unfortunately, B&B does not contain high-school GPA directly, but it has other information including combined GPA from declared major courses, combined GPA from declared minor courses, and combined GPA from all other courses. We adopt college non-major/non-minor GPA (NMGPA) as a proxy for high school GPA, and we argue this is a plausible substitution for several reasons. First, NMGPA primarily consists of coursework during the first one or two years of college immediately following high school. Second, non-major college coursework is qualitatively similar to high-school coursework in several ways. The American post-secondary education system follows a liberal arts model where students must progress through a structured, standardized learning regimen largely out of their control, unlike major and minor courses. This regimen spans many academic disciplines with a primary focus on introductory instruction. For these reasons we assume that exam scores plus high-school GPA together contain the same information about a student's pre-college academic preparation as exam scores plus NMGPA together. These two variables will be used to construct a single index representing the HC measure s_i within the model.

The other variable we draw from the B&B survey is annual household income after 10 years in the workforce, denoted w_i. Because our empirical model will focus on the returns to an undergraduate education, we drop from the sample all individuals who later enrolled in post-graduate studies or who were not in the labor force in 2003. Unlike the other variables, household

TABLE 2. Descriptive Statistics

Variable	Obs	Mean	Median	Std. Dev.
IPEDS/USNWR: (school-level data)				
6-Yr Graduation Rate: $\mathcal{M}$	961	0.3427	0.2812	0.2159
6-Yr Graduation Rate: $\mathcal{N}$	961	0.4831	0.4568	0.2295
Freshman Cohort Size: $\mathcal{M}$	1,245	145.37	46	254.56
Freshman Cohort Size: $\mathcal{N}$	1,245	660.86	378	788.68
College Quality Index	1,245	0.4842	0.4598	0.2132
BACCALAUREATE AND BEYOND*: (individual level data, college graduates only)				
SAT/SAT equivalent scores: $\mathcal{M}$	500	820	820	220
SAT/SAT equivalent scores: $\mathcal{N}$	4,980	990	980	190
Academic Record (GPA): $\mathcal{M}$	500	2.7	2.7	0.6
Academic Record (GPA): $\mathcal{N}$	4,980	3.0	3.0	0.6
College Quality: $\mathcal{M}$	500	0.5174	0.5236	0.2140
College Quality: $\mathcal{N}$	4,980	0.5846	0.6051	0.2024
10-Year Household Income: $\mathcal{M}$	260	$100,700	$89,500	$52,500
10-Year Household Income: $\mathcal{N}$	2,800	$108,300	$92,700	$77,300

*As per USDOE data security requirements, in order to protect anonymity of B&B respondents, sample sizes have been rounded to the nearest 10, dollar figures have been rounded to the nearest $100, SAT scores have been rounded to the nearest 10, and GPAs have been rounded to the nearest 0.1.

income is available for a random subsample of the B&B data.[6] By convention we denote $w_i = \emptyset$ whenever it is missing, and we use an indicator variable $D_{wi} = \mathbb{1}(w_i \neq \emptyset)$ for notational convenience when we define our estimator. Information on labor market payoffs is used to quantify match utility within the model. The top panel of Table 2 provides summary statistics for college-level variables and the bottom panel summarizes student-level data.

3.2. **Model Identification.** We now outline an empirical model of college admissions fashioned after the theoretical framework in Section 2. Our basic identification challenge is to disentangle the influence of college quality, HC investment, and privately known type on the returns to attending college. If the students who tend to enroll at more selective colleges have more advantageous characteristics to begin with, then raw correlations between college quality and earning power cannot be viewed as causal. Previous empirical work has attempted to address the issue essentially by instrumenting for the influence of college quality while subsuming unobserved student characteristics into the unexplained error term in the model (see Brewer, Eide,

[6]To verify that data loss on the income measure is truly random, we ran two-sample Kolmogorov-Smirnov tests on the other three variables in the data, P, E, and A, to see whether their distributions were different across the subsample which included household income versus the one that did not. All three tests fail to reject the null hypothesis of distributional equality at the 5% level. P-values for KS tests on the variables P, E, and A were 0.21, 0.48, and 0.23, respectively. The size of the subsample which included w_i was (rounding to the nearest 10) 3,060, and the size of the subsample which did not include w_i was 2,430.

and Ehrenberg [16]; Dale and Krueger [23]; Black and Smith [12]; and Long [39]). We take a novel approach to this problem by explicitly modeling the separate influences of school quality, pre-college HC investment, and unobserved student characteristics in determining post-college economic outcomes. A structural approach is necessary to identify the students' private information about their unobserved characteristics.

The empirical auctions literature has developed a set of tools specifically designed to identify private information in game-theoretic models. This literature was pioneered by Paarsch [43] and then revolutionized by Guerre, Perrigne, and Vuong [32, GPV] who proposed a computationally simple, non-parametric estimator for mapping observed bids into underlying private valuations in first-price auctions. We combine the approach of GPV with common techniques from labor econometrics to parse between the influences of student and school characteristics in producing post-college income. We outline an identification argument below wherein the equilibrium HC investment strategies serve as control functions within a wage equation. Intuitively, because AA changes the marginal investment incentives across race groups during high school, one can surmise that two students having the same achievement level but different race must have distinct underlying types. This and the matching shock provide a full-rank condition for the three explanatory factors in post-college income.

We begin this section by imposing some simplifying assumptions on the model for tractability, including functional forms for utility and and the distribution of matching shocks. Thus, our identification strategy is semi-parametric, although we do not require direct restrictions on the type distributions or the equilibrium college assignment mappings. Assumption 3.1 imposes a quadratic form on the single index equation for HC, with regularity conditions and a scale normalization to fix the units of S.

Assumption 3.1. ***(Single Index)*** *Human capital S is a single index function of exam scores E and academic record A,*

$$S_i = S(E_i, A_i) = \beta_1^s E_i + \beta_2^s E_i^2 + \beta_3^s A_i + \beta_4^s A_i^2 + \beta_5^s E_i A_i, \tag{2}$$

with $S_e(E_i, A_i) > 0,\ S_a(E_i, A_i) > 0\ \forall (E_i, A_i)$, and $\max_{(E,A)\in\mathbb{R}^2}\{S(E,A)\} = 1$.[7]

Assumption 3.2 imposes separability and a functional form on the cost function. Implicit here is also an assumption that individuals/households optimize their portfolio of investment activities (e_i, a_i) to generate the highest composite output s_i at the lowest possible cost. Recall that a student's type θ represents aspects of cost and match utility that are exogenous, whereas composite achievement S represents the component of costs and match utility under her control.

Assumption 3.2. ***(Separable Exponential Costs)*** *$C(s;\theta) = \theta c(s)$ with $c(s) = \exp(s)$.*[8]

[7] For the single index function, we also experimented with a more flexible cubic complete polynomial form $S(E_i, A_i) = \beta_0^s + \beta_1^s E_i + \beta_2^s E_i^2 + \beta_3^s E_i^3 + \beta_4^s A_i + \beta_5^s A_i^2 + \beta_6^s A_i^3 + \beta_7^s E_i A_i + \beta_8^s E_i^2 A_i + \beta_9^s E_i A_i^2$. This merely increased computational cost and sampling variability without producing a statistically or economically meaningful change in our estimates, relative to the quadratic form above.

[8] It is possible to add an additional parameter to the cost model, $c(s) = \exp(\nu(s - \underline{s}))$, for added flexibility. Identification would then require additional conditions, for example, individual rationality and incentive compatibility

Assumption 3.3 is an exclusion restriction that implies minority status plays no direct role in match utility. Although this assumption rules out some forms of racial discrimination on the labor market (*e.g.*, taste-based racial animus), our model allows for the existence of statistical discrimination. In the data, statistical discrimination would appear as minority and non-minority students with the same realizations of (s, p) receiving different wages because of differing values of θ that employers infer from the joint distributions of (p, s, θ) conditional on race.[9]

Assumption 3.3. ***(Exclusion Restriction)*** *Race does not directly affect match utility conditional on type θ, investment s, and college assignment p: if $D_{\mathcal{M}i} \equiv \mathbb{1}(i \in \mathcal{M})$ is an indicator for minority status then $U(p, s, \theta, D_{\mathcal{M}i}) = U(p, s, \theta)$.*

Assumption 3.2.4 adopts a form for the match utility function. We assume a Cobb-Douglas income production from a match. We further assume that a college dropout's income is $\kappa < 1$ times what it would have been had she had graduated, and we adopt a flexible, cubic complete polynomial form for the graduation probability function. This formulation allows the wage of college dropouts to vary by the student's underlying ability, HC, and college placement. We adopt a log utility form for the student's preferences over wage income, as it is a benchmark choice for lifetime consumption models.

Assumption 3.4. ***(Expected Log-Utility w/Cobb-Douglas Income Production)***

$$\begin{aligned} U(P_i, S_i, \theta_i) &= \rho(P_i, S_i) \log\left[u(P_i, S_i, \theta_i)\right] + \left[1 - \rho(P_i, S_i)\right] \log\left[\kappa u(P_i, S_i, \theta_i)\right], \textit{with } \kappa \in (0, 1) \\ \rho(P_i, S_i) &= \Pr\left[i \textit{ graduates college} \middle| P_i, S_i\right] \\ &= \beta_0^\rho + \beta_1^\rho P_i + \beta_2^\rho P_i^2 + \beta_3^\rho P_i^3 + \beta_4^\rho S_i + \beta_5^\rho S_i^2 + \beta_6^\rho S_i^3 + \beta_7^\rho P_i S_i + \beta_8^\rho P_i S_i^2 + \beta_9^\rho P_i^2 S_i \\ u(P_i, S_i, \theta_i) &= \mathrm{E}\left[W_i \middle| P_i, S_i, \theta_i\right] = \alpha_0 P_i^{\alpha_p} S_i^{\alpha_s} \theta_i^{-\alpha_\theta} \textit{ where } 0 < \alpha_0 \textit{ and } \alpha_p, \alpha_s, -\alpha_\theta \in (0, 1). \end{aligned}$$

Assumption 3.5 ensures that the agent's decision problem (Equation 1) has a unique solution, a needed property for mapping s into a corresponding θ. Assumptions 2.5 and 2.6 imply that $U(p, s, \theta) - \theta c(s)$ is concave in s, but without making assumptions on $P(s + \varepsilon)$ we cannot ensure the decision problem is concave. In lieu of this, we simply assume that Equation 1 has a unique solution. Note that this assumption is testable: given the supermodularity of the problem (Assumption 2.8), there would be jumps in the student's HC accumulation strategy if Assumption 3.5 failed to hold. Finally, Assumption 3.6 assumes normal matching shocks with zero mean.

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assumptions for marginal market participants who are nearly indifferent to college attendance versus entering the workforce instead. We experimented with this approach in our empirical implementation, and in practice the extra parameter adds little explanatory power within the exponential family of cost functions. It affects the behavior of the model most significantly for very low achieving college students where data are sparse. Elsewhere, changes in the curvature parameter ν are largely compensated by corresponding changes to the scale of θ, with little change to the economic implications of the point estimates. Thus, we focus on a simplified version of the cost model where $\nu = 1$. As a robustness check in our empirical application, we also estimated the model with an alternative power law cost function of the form $C(s, \theta) = \theta(s + 1)^2$ and found that model estimates were qualitatively very similar, suggesting that our functional form assumption for costs is not unduly driving the results.

[9]Beginning with Neal and Johnson [42], a body of empirical literature over the past two decades has emphasized non-animus factors for explaining the majority of current Black-White wage differentials. More recent work by Fryer, Pager, and Spenkuch [30] has found that roughly 89% of the Black-White wage gap is attributable to differentials in observable controls other than race, and has argued in favor of a statistical discrimination model to explain the remaininggap.

Assumption 3.5. ***(Unique Investment)*** *$U(P_j(s), s, \theta) - \theta c(s)$ is strictly concave in s.*

Assumption 3.6. ***(Normal Shocks)*** *Matching shocks $\varepsilon \sim N(0, \sigma_\varepsilon)$ are normally distributed with zero mean and variance σ_ε^2 and are independent of HC s and demographic status $j = \in \{\mathcal{M}, \mathcal{N}\}$.*

Ultimately, the structural objects to identify are the type distributions, $F_j(\theta)$, the matching shock variance, σ_ε^2, the assignment functions, P_j^{ap}, and the match utility parameters, $(\boldsymbol{\beta}^\rho, \boldsymbol{\alpha})$. The single index parameters $\boldsymbol{\beta}^s$ and the joint distributions of (P, S) across race groups are intermediate model components to be identified along the way.

3.2.1. *Identification: Single Index Parameters and Graduation Probabilities.* For simplicity of discussion, assume at first that the single index parameters $\boldsymbol{\beta}^s$ are known. Then the first hurdle to overcome is a problem of sample selection: because the B&B Survey only contains information for college graduates, we do not observe (P, S) pairs for anyone who failed to graduate. Thus, at first we can only treat the conditional school quality and HC distributions $f_{\mathcal{P}S}(p, s|\mathcal{M}, \text{grad})$ and $f_{\mathcal{P}S}(p, s|\mathcal{N}, \text{grad})$ as observables, and the sample selection problem stands in the way of correctly identifying the graduation probability parameters. However, note that $f_{\mathcal{P}S}(p, s|\mathcal{M}, \text{grad})$ and $f_{\mathcal{P}S}(p, s|\mathcal{N}, \text{grad})$ relate to the unconditional densities through the identity,

$$f_{\mathcal{P}S}(p, s|j) = \frac{f_{\mathcal{P}S}(p, s|j, \text{grad})\Gamma_j}{\rho(p, s)}, \quad j \in \{\mathcal{M}, \mathcal{N}\}, \tag{3}$$

where Γ_j is a constant that equals the total fraction of enrollees from group j who graduate college, and normalizes the joint density to integrate to one. The graduation parameters in turn are pinned down by

$$\begin{aligned} \Gamma_{jl} &= \beta_0^\rho + \beta_1^\rho p_l + \beta_2^\rho p_l^2 + \beta_3^\rho p_l^3 \\ &\qquad + \beta_4^\rho \overline{S}_{jl} + \beta_5^\rho \overline{S}_{jl}^2 + \beta_6^\rho \overline{S}_{jl}^3 + \beta_7^\rho p_l \overline{S}_{jl} + \beta_8^\rho p_l \overline{S}_{jl}^2 + \beta_9^\rho p_l^2 \overline{S}_{jl} + \epsilon_{jl} \\ &= \mathbf{Z}_{jl} \boldsymbol{\beta}^\rho + \epsilon_{jl}, \end{aligned} \tag{4}$$

where $\mathbf{Z}_{jl} = [1, p_l, p_l^2, p_l^3, \overline{S}_{jl}, \overline{S}_{jl}^2, \overline{S}_{jl}^3, p_l \overline{S}_{jl}, p_l \overline{S}_{jl}^2, p_l^2 \overline{S}_{jl}]$ contains the regressors for group j at school l, ϵ_{jl} is random and arises from finite sampling within campus l,

$$\overline{S}_{jl}^k = \int_{\underline{s}}^{\overline{s}} s^k f_{S|\mathcal{P}}(s|j, p_l) ds = \int_{\underline{s}}^{\overline{s}} s^k \frac{f_{\mathcal{P}S}(p_l, s|j)}{f_{\mathcal{P}_j}(p_l)} ds \tag{5}$$

is the conditional expectation, across both graduates and non-graduates, of the k^{th} power of s given p_l, and

$$f_{\mathcal{P}_j}(p_l) = \int_{\underline{s}}^{\overline{s}} f_{\mathcal{P}S}(p_l, s|j) ds, \; j \in \{\mathcal{M}, \mathcal{N}\} \tag{6}$$

is the unconditional marginal distribution of P for group j. Since Γ_{jl} is an aggregate, college-level variable, to compute its model-generated analog, $\mathbf{Z}_{jl}\boldsymbol{\beta}^\rho$, we compute the average of the individual-level graduation probabilities for group j on campus l. This is why equation (5) averages over individual-level powers of S. Equations (3) – (6) provide a sample selection correction to identify graduation probability parameters $\boldsymbol{\beta}^{\rho}$ as long as the single index parameters $\boldsymbol{\beta}^{s}$ are known. One appealing characteristic of this sample selection proposal is that it does not impose direct parametric restrictions on the form of the unconditional joint distributions of (P, S).

Now we require a further condition to pin the single index parameters down. Recall that one of the roles of S is to determine a student's access to a high-quality match partner. In other words, the HC index S represents all observable information about the student prior to the application process that predicts where he/she will place. Therefore, we adopt as our final condition the convention that the parameters $\boldsymbol{\beta}^{s}$ are such that college placement predictive power is maximized. Since the mapping between P and S arises from a rank-order contest, we adopt the *Kendall's* τ measure of rank correlation to formalize our notion of predictive power.

Briefly, Kendall's τ is defined in terms of concordance of random variables; we say that two ordered pairs (p_1, s_1) and (p_2, s_2) are *concordant* if the ordering of the first coordinate agrees with the ordering of the second, or $p_1 < p_2$ if and only if $s_1 < s_2$. Likewise, we say the two pairs are *discordant* when this condition is violated. For a joint distribution of (P, S), Kendall's τ is defined as the probability of concordance minus the probability of discordance for two iid realizations (P_1, S_1), (P_2, S_2): $\tau_{PS} \equiv \Pr\big[(P_1 - P_2)(S_1 - S_2) > 0\big] - \Pr\big[(P_1 - P_2)(S_1 - S_2) < 0\big]$. It is easy to see why this measure is directly relevant to our model with its perturbed rank-order contest structure. Within our context, Kendall's τ is directly interpretable as the probability that the ordering of two students' college assignments (within the same demographic group) respects the ordering of their pre-college achievement, minus the probability that it does not.

With the joint distribution (P, E, A) known, we formalize our assumption on $\boldsymbol{\beta}^{S}$ as:

$$\begin{aligned} \boldsymbol{\beta}^{s} = \arg\max \Bigg\{ &\mu \Big(\Pr\big[(P_1 - P_2)(S_1 - S_2) > 0 | \mathcal{M}\big] - \Pr\big[(P_1 - P_2)(S_1 - S_2) < 0 | \mathcal{M}\big] \Big) \\ &+ (1-\mu) \Big(\Pr\big[(P_1 - P_2)(S_1 - S_2) > 0 | \mathcal{N}\big] - \Pr\big[(P_1 - P_2)(S_1 - S_2) < 0 | \mathcal{N}\big] \Big) \Bigg\}. \end{aligned} \tag{7}$$

From the above arguments, the first part of our identification result follows:

Proposition 3.7. *There is a unique configuration of the single index and graduation probability parameters* $(\boldsymbol{\beta}^{s}, \boldsymbol{\beta}^{\rho})$ *that is consistent with the joint distributions of the observables* $\{ \{\mathbf{Y}_l\}_{l=1}^{L}, \{p_i, e_i, a_i\}_{i=1}^{I} \}$ *and equations (3), (4) and (7).*

3.2.2. *Identification: Matching Shock Variance.* At this point several important equilibrium objects can be treated as known, including the unconditional joint distribution of HC and school assignments. From this starting point, identifying the matching shock variance parameter σ_ε is simple since it uniquely determines the degree to which the joint distribution of P and S deviates from full rank correlation within each race group. Intuitively, the larger is the variance of the matching shock, the more latitude there is for students with lower HC levels to place above students with more HC. Thus, it is easy to see that Kendall's τ within each race group is decreasing in σ_ε. We use Theorem 2.1 to treat the data-generating process as equivalent to a quota mechanism that reserves a distribution of college seats for each group equal to $Q_j(p) = F_{\mathcal{P}_j}(p)$, $j \in \{\mathcal{M}, \mathcal{N}\}$, which are the marginal distributions of the selection-corrected $F_{\mathcal{PS}}(p, s|j)$'s from the previous section.

For each group $j \in \{\mathcal{M}, \mathcal{N}\}$, let $\tau_{\mathcal{PS}}(\sigma_\varepsilon|j)$ denote the rank correlation between HC and school assignment implied by shock variance parameter σ_ε holding $G_j(s)$ and $F_{\mathcal{P}_j}(p)$ fixed. Since school assignment within each group is determined by the rank ordering of perturbed HC levels and the perturbations are independent, the following must be true: (*i*) $\tau_{\mathcal{PS}}(0|\mathcal{M}) = \tau_{\mathcal{PS}}(0|\mathcal{N}) = 1$; (*ii*) $\tau'_{\mathcal{PS}}(\sigma_\varepsilon|j) < 0, j \in \{\mathcal{M}, \mathcal{N}\}$; and (*iii*) $\lim_{\sigma_\varepsilon \to \infty} \tau_{\mathcal{PS}}(\sigma_\varepsilon|\mathcal{M}) = \lim_{\sigma_\varepsilon \to \infty} \tau_{\mathcal{PS}}(\sigma_\varepsilon|\mathcal{N}) = 0$. The following result directly follows from these facts:

Proposition 3.8. *There is a unique value of σ_ε that is consistent with perturbed, rank-order allocations and the joint distributions $F_{\mathcal{PS}}(p, s|\mathcal{M})$ and $F_{\mathcal{PS}}(p, s|\mathcal{N})$.*

3.2.3. *Identification: Admission Preference Markups.* With the distribution of the matching shock known, we can now consider the equilibrium distributions of noisy HC as known objects since they are a convolution of HC and the shock, $H_j(t) = \left(G_j \circ F_\varepsilon\right)(t)$. These CDFs enter into the assignment mappings through Equations (2.3) to determine the allocation of college seats. One key observation about the admission preference mechanism relevant to identification is the following:

$$P_{\mathcal{M}}(t) = P_{\mathcal{N}}(\widetilde{T}(t)). \tag{8}$$

In other words, a minority student with perturbed investment $t = s + \varepsilon$ is matched to the same college as a non-minority student with perturbed investment $\widetilde{T}(s + \varepsilon)$. This observation allows us to recover $\widetilde{T}$ from the observables by determining what rule could have produced allocations $F_{\mathcal{P}_\mathcal{M}}$ and $F_{\mathcal{P}_\mathcal{N}}$ from the investment distributions $G_\mathcal{M}$ and $G_\mathcal{N}$.

Proposition 3.9. *Under assumptions 2.2 and 2.10 there exists a unique markup mapping $\widetilde{T}(\cdot)$ that is consistent with $(G_\mathcal{M}, G_\mathcal{N}, F_{\mathcal{P}_\mathcal{M}}, F_{\mathcal{P}_\mathcal{N}}, \sigma_\varepsilon)$*

Proof: For $\varphi \in (0, 1)$ define $t_\mathcal{N}(\varphi) \equiv H_\mathcal{N}^{-1}(\varphi)$ as the φ^{th} quantile in the non-minority noisy HC distribution. For minorities, let $\varphi_\mathcal{M}(\varphi) \equiv H_\mathcal{M}\left(\widetilde{T}^{-1}(t_\mathcal{N}(\varphi))\right)$ denote the quantile rank of the de-subsidized version of $t_\mathcal{N}(\varphi)$ within the minority noisy HC distribution. By Equation (8), it follows that $F^{-1}_{P_\mathcal{M}}(\varphi_\mathcal{M}[\varphi]) = F^{-1}_{P_\mathcal{N}}(\varphi)$, $\forall \varphi$. By substituting in $\varphi_\mathcal{M}$ and rearranging, we get $H^{-1}_\mathcal{N}(\varphi) = \widetilde{T}\left(H^{-1}_M\left[F_{P_\mathcal{M}}\left(F^{-1}_{P_\mathcal{N}}[\varphi]\right)\right]\right)$, from which it follows that

$$\widetilde{T}(t) = H^{-1}_\mathcal{N}\left[F_{P_\mathcal{N}}\left(F^{-1}_{P_\mathcal{M}}\left[H_\mathcal{M}(s)\right]\right)\right]. \ \blacksquare \tag{9}$$

The proof is constructive since the right-hand side of the above expression is a composition of distribution and quantile functions that can be estimated directly from data. Aside from basic regularity conditions, no a priori restrictions are imposed on the form of the markup function. In particular, one need not even assume that the markup aids minorities, since whether or not $\widetilde{T}(t) \geq t$ is left for the data to indicate.

3.2.4. *Identification: Utilty Parameters and Cost Types.* At this point, we can now treat the data-generating processes assignment mappings $P_{\mathcal{M}}^{ap}(t)$ and $P_{\mathcal{N}}^{ap}(t)$ as known. Since the remainder of our discussion on identification and estimation assumes the admission preference mechanism, we drop the superscript in order to simplify notation, unless it is needed for clarity.

A classic empirical approach for estimating strategic models with private information was proposed by Guerre, Perrigne, and Vuong [32] for first-price auctions. Their idea was simple but powerful: since the equilibrium distributions of bids is observable, one can reverse engineer a bidder's private valuation as that which rationalizes her bid as a best response to competitors. Our setting is similar to an auction in that each student's investment choice is a best response to the distribution of HC choices, given her type. The first-order conditions for the student's decision problem depicted in Equation (1) are:

$$\theta = \frac{\mathrm{E}_\varepsilon\left[U_p\left(P_j(s+\varepsilon), s, \theta; \boldsymbol{\alpha}\right) P_j'(s+\varepsilon)\right] + \mathrm{E}_\varepsilon\left[U_s\left(P_j(s+\varepsilon), s, \theta; \boldsymbol{\alpha}\right)\right]}{c'(s)}, \; j \in \{\mathcal{M}, \mathcal{N}\}, \tag{10}$$

where $\boldsymbol{\alpha} = \left[\log(\alpha_0), \alpha_p, \alpha_s, -\alpha_\theta\right]^\top$ is the vector of utility parameters governing wage production from a match of a student to a school, given her HC investment.

Equation (10) uniquely defines the inverse strategy mapping if the utility parameter vector $\boldsymbol{\alpha}$ is known. Let $\theta_j(s; \boldsymbol{\alpha})$ denote the implicit solution to the first-order condition given utility parameters $\boldsymbol{\alpha}$. Intuitively, Equation (10) indicates that with our knowledge of the distribution of college seats and the equilibrium HC distribution, we can reverse-engineer a cost type as a best response to the rest of the market once we can pin down the mapping between actions, college assignment, and income. To do this, we need an additional moment condition, which we get from the following wage regression model:

$$\log(w_i) = \log(\alpha_0) + \alpha_p \log(p_i) + \alpha_s \log(s_i) - \alpha_\theta \psi(S, D_{\mathcal{M}i}; \boldsymbol{\alpha}) + \varepsilon_{wi}, \tag{11}$$

where $\psi(S_i, D_{\mathcal{M}i}; \boldsymbol{\alpha}) \equiv \log\left[D_{\mathcal{M}i}\theta_{\mathcal{M}}(S_i; \boldsymbol{\alpha}) + (1 - D_{\mathcal{M}i})\theta_{\mathcal{N}}(S_i; \boldsymbol{\alpha})\right]$ is i's log-cost type and ε_{wi} is a transitory shock to 10-year household income. We assume transitory shocks are exogenous,

Assumption 3.10. $\mathrm{E}\left\{\left[\log(p_i), \log(s_i), \psi(S_i, D_{\mathcal{M}i}; \boldsymbol{\alpha})\right]^\top \varepsilon_{wi}\right\} = \mathbf{0}$.

Thus, our approach is to embed the inverse equilibrium equations into the wage regression as nonlinear control functions to account for the role of students' unobserved characteristics in the production of income. The intuition behind structural identification is as follows: in order for the parameters of the wage regression to be identified, we need an orthogonality condition and a full rank condition. Assumption 3.10 establishes orthogonality based on the idea that the same unobserved cognitive and non-cognitive characteristics that govern a student's pre-college achievement also govern the HC accumulation process during college as well.

For the full-rank condition, there must be something present in the data-generating process that prevents the regressors from being perfectly colinear. The matching shock breaks colinearity between college quality and HC. AA plus the exclusion restriction breaks the colinearity between

HC and learning cost types because AA implies a minority student and a non-minority student with the same observable HC output must have different underlying cost types.

More formally, note that $\sigma_\varepsilon > 0$ implies a non-degenerate distribution of HC types on each college campus under any college admissions rule, so that rank correlation between $\log(S)$ and $\log(P)$ must be less than one in absolute value. Second, denote the expected wage of minority (nonminority) students graduating from college p with HC level s as $U_{\mathcal{M}}(p,s)$ ($U_{\mathcal{N}}(p,s)$). Suppose we observe a positive measure of (p,s) such that $U_{\mathcal{M}}(p,s) = U(p,s,\theta_{\mathcal{M}}(s;\boldsymbol{\alpha})) \neq U_{\mathcal{N}}(p,s) = U(p,s,\theta_{\mathcal{N}}(s;\boldsymbol{\alpha}))$. Our exclusion restriction implies that it must be the case that $\theta_{\mathcal{M}}(s;\boldsymbol{\alpha}) \neq \theta_{\mathcal{N}}(s;\boldsymbol{\alpha})$, which insures that the rank correlation between s and θ is greater than -1. Thus, in expectation the matrix of regressors

$$\mathbf{X}(\boldsymbol{\alpha}) = \begin{bmatrix} 1 & \log(p_1) & \log(s_1) & \psi(s_1, D_{\mathcal{M}1};\boldsymbol{\alpha}) \\ 1 & \log(p_2) & \log(s_2) & \psi(s_2, D_{\mathcal{M}2};\boldsymbol{\alpha}) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \log(p_I) & \log(s_I) & \psi(s_I, D_{\mathcal{M}I};\boldsymbol{\alpha}) \end{bmatrix}$$

will have full rank for any configuration of the parameters $\boldsymbol{\alpha}$. Essentially, $\psi(S, D_{\mathcal{M}};\boldsymbol{\alpha})$, which is derived from economic theory of investment behavior, serves as a control function and allows the researcher to separate out the influence of unobserved student characteristics θ from achievement s and school quality p. Finally, recall that by supermodularity equations (10) produce unique, monotone solutions $\theta_j(s;\boldsymbol{\alpha})$, which implies that the type distributions

$$F_j(\theta) = G_j\left[\theta_j^{-1}(\theta;\boldsymbol{\alpha})\right], \quad j \in \{\mathcal{M}, \mathcal{N}\}$$

are known if $\boldsymbol{\alpha}$ is known. This logic yields the following, final result on structural identification:

Proposition 3.11. *Under Assumptions 2.2 – 3.10, wage parameters $\boldsymbol{\alpha}$ and cost type distributions $F_{\mathcal{M}}(\theta)$, $F_{\mathcal{N}}(\theta)$ are identified, provided that*

(i) $0 < \mu < 1$
(ii) $\sigma_\varepsilon > 0$,
(iii) $\exists (p,s)$ *such that* $U_{\mathcal{M}}(p,s) \neq U_{\mathcal{N}}(p,s)$
(iv) $\nabla^2_{\boldsymbol{\alpha}} \mathbf{Q}(\boldsymbol{\alpha})$ *is positive definite on* $(0,\infty) \times (0,1)^3$, *where*

$$\mathbf{Q}(\boldsymbol{\alpha}) = \left[\mathbf{D}_w\left(\mathbf{W} - \mathbf{X}(\boldsymbol{\alpha})\boldsymbol{\alpha}\right)\right]^\top \left[\mathbf{D}_w\left(\mathbf{W} - \mathbf{X}(\boldsymbol{\alpha})\boldsymbol{\alpha}\right)\right],$$

$$\mathbf{W} = [\log(w_1), \log(w_2), \ldots, \log(w_I)]^\top, \textit{ and } \mathbf{D}_w = [D_{w1}, D_{w2}, \ldots, D_{wI}].$$

3.3. **A Two-Stage, Semiparametric Estimator.** We now construct a two-stage GMM estimator to implement our identification strategy. First, we recover the preliminary model parameters that do not directly depend on our strategic investment model, $\boldsymbol{\beta}^s$, $\boldsymbol{\beta}^\rho$, $P_{\mathcal{M}}(s)$, $P_{\mathcal{N}}(s)$, and σ_ε. Then we use these estimated values and the first-order conditions to recover the utility parameters $\boldsymbol{\alpha}$ and the learning cost distributions $F_{\mathcal{M}}(\theta)$ and $F_{\mathcal{N}}(\theta)$.

3.3.1. *Stage I Estimation.* The first hurdle to overcome is to find a computationally tractable way of representing the selected joint distribution of (P, S) conditional on graduation. High-dimensional density estimation is a difficult problem both computationally and because of the rapid decay of optimal statistical convergence rates as the dimensionality of the underlying random variable grows.[10] Recent work in the auctions literature by Hubbard and Paarsch [36] has employed parametric copula functions to solve this problem. Sklar's Theorem states that any absolutely continuous joint distribution can be represented as a composition $F_{\mathcal{P}S}(p, s|j, \text{grad}) = \mathcal{C}_j\left[F_{\mathcal{P}_j}(p|\text{grad}), G_j(s|\text{grad})\middle|\text{grad}\right], j \in \{\mathcal{M}, \mathcal{N}\}$, where $\mathcal{C}_j(\cdot, \cdot|\text{grad})$ is a unique copula function. This implies that the rapidly increasing computational cost and data-hungriness of nonparametric estimators come from the complexity of the correlation structure $\mathcal{C}_j$ since the complexity of the marginal distributions does not increase with the dimension of the joint distribution. Hubbard and Paarsch [36] therefore propose a flexible approach to estimating the marginal distributions, while simplifying the copula with parametric assumptions for tractability. This allows the econometrician to maintain the familiar $\sqrt{I}$ convergence rate when estimating a multi-dimensional joint distribution. We follow this dimension reduction strategy by adopting the Gumbel-Hougaard copula, $\mathcal{C}(r, q; \nu) = \exp\left[-\left((-\log(r))^{\nu} + (-\log(q))^{\nu}\right)^{1/\nu}\right], \ \nu \geq 1$.[11] One advantage of the Gumbel-Hougaard copula is that it implies a closed-form expression for the Kendall's τ rank correlation index: $\tau^j_{\mathcal{P}S} = \frac{\nu_j - 1}{\nu_j}$, $j = \mathcal{M}, \mathcal{N}$.

For the selected marginal distributions we propose a flexible, semi-nonparametric approach based on B-splines. Like orthogonal polynomials, B-splines are defined as a linear combination of global basis functions, and B-splines can be made arbitrarily flexible while remaining much better behaved than global polynomials.[12] For the selected marginal distributions of school assignment, we begin by specifying knot vectors $\mathbf{k}^p_j = \left\{\underline{p} = k^p_{j1} < k^p_{j2} < \cdots < k^p_{j,K^p_j+1} = \overline{p}\right\}$ that uniquely define a set of $K^p_j + 3$ cubic B-spline basis functions $\mathcal{B}^p_{jk}(p) : [\underline{p}, \overline{p}] \to \mathbb{R}$, $k = 1, \ldots, K^p_j + 3$, which in turn define our parameterization of the CDFs:

$$F_{\mathcal{P}_j}\left(p|\text{grad}; \gamma^p_j\right) = \sum_{k=1}^{K^p_j+3} \gamma^p_{jk} \mathcal{B}^p_{jk}(p), \ j \in \{\mathcal{M}, \mathcal{N}\}.$$

For the marginal distribution of the HC index, S, we have an additional challenge: since its units (and therefore the relevant domain to span) are unknown ex ante, we instead parameterize

[10]See Silverman [48] and Campo, Perrigne, and Vuong [17] for a lengthy discussion on this concept.

[11]We also experimented with several other copula functions including the Frank copula, $\mathcal{C}(r, q; \nu) = -\frac{1}{\nu}\log\left[1 + \frac{(\exp(-\nu r)-1)(\exp(-\nu q)-1)}{(\exp(-\nu)-1)}\right]$, $\nu \in \mathbb{R}\backslash\{0\}$; the Clayton copula, $\mathcal{C}(r, q; \nu) = \left[\max\{r^{-\nu} + q^{-\nu} - 1; 0\}\right]^{-1/\nu}$, $\nu \in [-1, \infty)\backslash\{0\}$; and the Gaussian copula, $\mathcal{C}(r, q; \nu) = \Phi_R\left[\Phi^{-1}(r), \Phi^{-1}(q)\right]$ where Φ is a standard normal CDF and Φ_R is a bivariate normal CDF with correlation matrix $R = \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix}$, $\nu \in [-1, 1]$. All produced very similar results, which is consistent with the assumption that our parametric restriction of the copula function provides a robust approximation to the nonparametric correlation structure.

[12]For a brief primer on B-splines and their advantages in empirical auctions models, see Hickman, Hubbard, and Paarsch [34] and Bodoh-Creed, Boehnke, and Hickman [13].

the selected marginal quantile functions, whose domain is always $[0,1]$. Let the knot vectors and basis functions for selected HC quantile functions be $\mathbf{k}_j^q = \left\{0 = k_{j1}^q < k_{j2}^q < \cdots < k_{j,K_j^q+1}^q = 1\right\}$ and $\mathcal{B}_{jk}^q(r) : [0,1] \to \mathbb{R},\ k = 1, \ldots, K_j^q + 3$, respectively, with B-spline marginal quantile functions parameterized similarly as above by $Q_{S_j}\left(r|\text{grad}; \gamma_j^q\right) = \sum_{k=1}^{K_j^q+3} \gamma_{jk}^q \mathcal{B}_{jk}^q(r),\ j \in \{\mathcal{M}, \mathcal{N}\}$.

The parameterized, selected joint distributions are given by

$$F_{\mathcal{P}S}\left(p, s|j, \text{grad}; \gamma_j^p, \gamma_j^q, \nu_j\right) = \mathcal{C}\left[F_{\mathcal{P}_j}\left(p|\text{grad}; \gamma_j^p\right), Q_{S_j}^{-1}\left(s|\text{grad}; \gamma_j^q\right); \nu_j\right],\ j \in \{\mathcal{M}, \mathcal{N}\}.$$

Going forward, one important detail to note is that the parameters $\nu_{\mathcal{M}}$ and $\nu_{\mathcal{N}}$ reflect the empirical correlation structure for the selected joint distribution of P and S for college graduates only. Below we will define other notation for separate copula parameters that apply to the selection-corrected joint distribution for all college enrollees (*i.e.*, including dropouts).

In order to complete our GMM estimator, we also need to construct empirical analogs to the joint and marginal distributions of (p, s). In the case of CDFs, we use the standard Kaplan-Meier empirical distribution functions

$$\widehat{F}_{\mathcal{P}_j}(p|\text{grad}) = \frac{\sum_{i=1}^{I} \mathbb{1}(p_i \leq p)\mathbb{1}(i \in j)}{\sum_{i=1}^{I} \mathbb{1}(i \in j)}, \text{ and}$$

$$\widehat{F}_{\mathcal{P}S}(p, s|j, \text{grad}) = \frac{\sum_{i=1}^{I} \mathbb{1}(p_i \leq p)\mathbb{1}(s_i \leq s)\mathbb{1}(i \in j)}{\sum_{i=1}^{I} \mathbb{1}(i \in j)},\ j \in \{\mathcal{M}, \mathcal{N}\}.$$

For the empirical marginal quantiles of S, we use a new method developed by Hedblom, Hickman, and List [33] for smooth nonparametric quantile estimation. For a random sample $\mathbf{S}_j = \{S_{ji}\}_{i=1}^{I_j}$ of size $I_j \equiv \sum_{i=1}^{I} \mathbb{1}(i \in j)$, this estimator exists as a weighted average of the ordered data $\{S_j(1) \leq S_j(2) \leq \cdots \leq S_j(I_j)\}$. Specifically, for $k \in \{1, 2, \ldots, I_j\}$, the $(k/I_j)^{th}$ empirical quantile is estimated as

$$\widehat{Q}_{S_j}(k/I_j) = \sum_{i=1}^{I_j} \Pi_{ik}^{I_j} S_j(i),$$

where the weights $\Pi_{ik}^{I_j}$ are known and mimic the limiting behavior of a re-sampled quantile estimator as the number of simulated samples approaches infinity. Specifically, $\Pi_{ik}^{I_j}$ gives the probability that the i^{th} order statistic of $\mathbf{S}_j$ will occupy the k^{th} position in an ordered, randomly generated bootstrap sample from the raw data.[13] However, an important constraint for this estimator is that, for a fixed sample size I_j, it can only be evaluated at quantile ranks on the discrete grid $\left\{\frac{1}{I_j}, \ldots, \frac{I_j}{I_j}\right\}$. Therefore, assuming $I_j \geq 101,\ j \in \{\mathcal{M}, \mathcal{N}\}$, we also define $\mathbf{r}_j =$

[13] A benefit to this method is that it is differentiable and more efficient than the traditional empirical quantile estimator, $\inf\left\{s : \frac{1}{I_j}\sum_{i=1}^{I_j} \mathbb{1}(S_{ji} \leq s) \geq (k/I_j)\right\}$. Intuitively, the empirical quantile estimator, being the nearest-neighbor inverse of the Kaplan-Meier empirical CDF, incorporates cardinal information from only a single datum, with all other data providing only ordinal information. In contrast, $\widehat{Q}_{S_j}(k/I_j)$ as defined above uses both ordinal and cardinal information from the entire sample. See Hedblom, Hickman, and List [33] for additional details.

$\{r_{j0}, r_{j1}, r_{j2}, \ldots, r_{j100}\}$ as a vector of quantile ranks at which the empirical quantile function is to be evaluated, where $r_{j0} = 1/I_j$ and $r_{jk} = \text{round}(kI_j/100)/I_j$, $1 \leq k$.

Finally, in order to estimate σ_ε we need to introduce four additional ancillary parameters. First, for notational ease let $\boldsymbol{\pi} = \left[\gamma^p_{\mathcal{M}}, \gamma^p_{\mathcal{N}}, \gamma^q_{\mathcal{M}}, \gamma^q_{\mathcal{N}}, \nu_{\mathcal{M}}, \nu_{\mathcal{N}}, \boldsymbol{\beta}^s, \boldsymbol{\beta}^\rho\right]^\top$ summarize all primary Stage I parameters except for the shock variance. Note that if $\boldsymbol{\pi}$ is known, Equation (3) defines the selection-corrected joint distribution of (P, S), $F_{\mathcal{P}S}(p, s|j; \boldsymbol{\pi})$, for group $j \in \{\mathcal{M}, \mathcal{N}\}$ as a function of the entire parameter vector $\boldsymbol{\pi}$ along with its two marginal distributions, $F_{\mathcal{P}_j}(p; \boldsymbol{\pi})$ and $G_j(s; \boldsymbol{\pi})$. Let ν^*_j denote the best-fit copula parameter of the selection-corrected joint distribution:

$$\nu^*_j \equiv \underset{n \in [1,\infty)}{\arg\min} \left\{ \int_S \int_P \left[\mathcal{C}\left(F_{\mathcal{P}_j}(p; \boldsymbol{\pi}), G_j(s; \boldsymbol{\pi}); n \right) - F_{\mathcal{P}S}(p, s|j; \boldsymbol{\pi}) \right]^2 dp\,ds \right\}.$$

Implicitly, ν^*_j is a function of $\boldsymbol{\beta}^s$ since these directly control how well the ranks of $S(e_i, a_i; \boldsymbol{\beta}^s)$ predict the ranks of i's placement in P space.

Now, let $F^\circ_{\mathcal{P}S}(p, s|j; \boldsymbol{\pi}, \sigma_\varepsilon)$ denote the joint distribution of (P, S) implied by the marginal distributions $F_{\mathcal{P}_j}(p; \boldsymbol{\pi})$ and $G_j(s; \boldsymbol{\pi})$, but assuming rank-order allocations with respect to NHC are generated by mean-zero, normal shocks with variance σ^2_ε. Let $\nu^\circ_j(\sigma_\varepsilon)$ denote the best-fit copula parameter for that joint distribution, in the following sense:

$$\nu^\circ_j(\sigma_\varepsilon) \equiv \underset{n \in [1,\infty)}{\arg\min} \left\{ \int_S \int_P \left[\mathcal{C}\left(F_{\mathcal{P}_j}(p; \boldsymbol{\pi}), G_j(s; \boldsymbol{\pi}); n \right) - F^\circ_{\mathcal{P}S}(p, s|j; \boldsymbol{\pi}, \sigma_\varepsilon) \right]^2 dp\,ds \right\}$$

Subject to :

$$\begin{aligned}
F^\circ_{\mathcal{P}S}(p, s|j; \boldsymbol{\pi}, \sigma_\varepsilon) &= \int_{\underline{s}}^{s} \int_{\underline{p}}^{p} f^\circ_{\mathcal{P}|S}(p|s, j; \boldsymbol{\pi}, \sigma_\varepsilon) g_j(s; \boldsymbol{\pi})\, dp\,ds \\
&= \int_{\underline{s}}^{s} \int_{\underline{p}}^{p} f_\varepsilon\left(P^{-1}_j(p; \boldsymbol{\pi}, \sigma_\varepsilon) - s; \sigma_\varepsilon \right) \frac{dP^{-1}_j(p; \boldsymbol{\pi}, \sigma_\varepsilon)}{dp} g_j(s; \boldsymbol{\pi})\, dp\,ds \\
P^{-1}_j(p; \boldsymbol{\pi}, \sigma_\varepsilon) &= H^{-1}_j\left[F_{\mathcal{P}_j}(p; \boldsymbol{\pi}); \boldsymbol{\pi}, \sigma_\varepsilon \right] \\
H_j(t; \boldsymbol{\pi}, \sigma_\varepsilon) &= \int_{-\infty}^{t} \int_{-\infty}^{\infty} f_\varepsilon(\varepsilon; \sigma_\varepsilon) g_j(x - \varepsilon; \boldsymbol{\pi}, \sigma_\varepsilon)\, d\varepsilon\, dx, \ j = \mathcal{M}, \mathcal{N}.
\end{aligned}$$

The ancillary parameters $\nu^*_{\mathcal{M}}$, $\nu^*_{\mathcal{N}}$, $\nu^\circ_{\mathcal{M}}(\sigma_\varepsilon)$, and $\nu^\circ_{\mathcal{N}}(\sigma_\varepsilon)$ are used below to define moment conditions for estimation of $\boldsymbol{\beta}^s$ and σ_ε. Intuitively, ν^*_j is the copula parameter that best reflects the correlation structure between P and S implied by the data (post-selection-correction), and $\nu^\circ_j(\sigma_\varepsilon)$ is the copula parameter that best reflects the correlation structure between P and S generated endogenously by our structural model of a noisy, rank-order college admissions contest given σ_ε and the empirical marginal distributions of P and S. With the above definitions, we can now formalize our Stage I GMM estimator:

$$
\begin{aligned}
\begin{bmatrix} \hat{\boldsymbol{\pi}} \\ \hat{\sigma}_\varepsilon \end{bmatrix} = \arg\min \Bigg\{ & \sum_{i=1}^{I} \Bigg(D_{\mathcal{M}i} \left[F_{\mathcal{P}_\mathcal{M}} \left(p_i | \text{grad}; \boldsymbol{\gamma}^p_\mathcal{M} \right) - \widehat{F}_{\mathcal{P}_\mathcal{M}}(p_i|\text{grad}) \right]^2 \\
& \qquad + (1 - D_{\mathcal{M}i}) \left[F_{\mathcal{P}_\mathcal{N}} \left(p_i | \text{grad}; \boldsymbol{\gamma}^p_\mathcal{N} \right) - \widehat{F}_{\mathcal{P}_\mathcal{N}}(p_i|\text{grad}) \right]^2 \Bigg) \\
& + \sum_{k=0}^{100} \Bigg(\left[Q_{S_\mathcal{M}} \left(r_{\mathcal{M}k} | \text{grad}; \boldsymbol{\gamma}^q_\mathcal{M} \right) - \widehat{Q}_{S_\mathcal{M}}(r_{\mathcal{M}k}) \right]^2 + \left[Q_{S_\mathcal{N}} \left(r_{\mathcal{N}k} | \text{grad}; \boldsymbol{\gamma}^q_\mathcal{N} \right) - \widehat{Q}_{S_\mathcal{N}}(r_{\mathcal{N}k}) \right]^2 \Bigg) \\
& + \sum_{i=1}^{I} \Bigg(D_{\mathcal{M}i} \left[F_{\mathcal{P}S} \left(p_i, s_i | \mathcal{M}, \text{grad}; \boldsymbol{\gamma}^p_\mathcal{M}, \boldsymbol{\gamma}^q_\mathcal{M}, \nu_\mathcal{M} \right) - \widehat{F}_{\mathcal{P}S}(p_i, s_i|\mathcal{M}, \text{grad}) \right]^2 \\
& \qquad + (1 - D_{\mathcal{M}i}) \left[F_{\mathcal{P}S} \left(p_i, s_i | \mathcal{N}, \text{grad}; \boldsymbol{\gamma}^p_\mathcal{N}, \boldsymbol{\gamma}^q_\mathcal{N}, \nu_\mathcal{N} \right) - \widehat{F}_{\mathcal{P}S}(p_i, s_i|\mathcal{N}, \text{grad}) \right]^2 \Bigg) \\
& + \sum_{l=1}^{L} \Bigg(\left[\Gamma_{\mathcal{M}l} - \mathbf{Z}_{\mathcal{M}l} \boldsymbol{\beta}^\rho \right]^2 + \left[\Gamma_{\mathcal{N}l} - \mathbf{Z}_{\mathcal{N}l} \boldsymbol{\beta}^\rho \right]^2 \Bigg) \\
& + \left(\frac{\nu^*_\mathcal{M} - 1}{\nu^*_\mathcal{M}} - 1 \right)^2 + \left(\frac{\nu^*_\mathcal{N} - 1}{\nu^*_\mathcal{N}} - 1 \right)^2 \\
& + \left(\frac{\nu^*_\mathcal{M} - 1}{\nu^*_\mathcal{M}} - \frac{\nu^\circ_\mathcal{M}(\sigma_\varepsilon) - 1}{\nu^\circ_\mathcal{M}(\sigma_\varepsilon)} \right)^2 + \left(\frac{\nu^*_\mathcal{N} - 1}{\nu^*_\mathcal{N}} - \frac{\nu^\circ_\mathcal{N}(\sigma_\varepsilon) - 1}{\nu^\circ_\mathcal{N}(\sigma_\varepsilon)} \right)^2 \Bigg\},
\end{aligned}
$$

Subject to :

$$s_i = S(e_i, a_i; \boldsymbol{\beta}^s),\ i = 1, \ldots, I$$

$$\frac{\partial S(e, a; \boldsymbol{\beta}^s)}{\partial e} > 0,\ \frac{\partial S(e, a; \boldsymbol{\beta}^s)}{\partial a} > 0\ \forall (e, a; \boldsymbol{\beta}^s),\ \ and \ \ \max_{(e,a)\in\mathbb{R}^2} \{S(e, a; \boldsymbol{\beta}^s)\} = 1$$

$$\gamma^v_{j,k-1} \leq \gamma^v_{jk},\ \ k = 2, \ldots, K^v_j + 3,\ \ v = p, s,\ \ j \in \{\mathcal{M}, \mathcal{N}\}$$

$$\min\{\Gamma_{\mathcal{M}1}, \ldots, \Gamma_{\mathcal{M}L}, \Gamma_{\mathcal{N}1}, \ldots, \Gamma_{\mathcal{N}L}, \} \leq \rho(p, s; \boldsymbol{\beta}^\rho) \leq 1\ \forall (p, s)$$

$$\frac{\partial \rho(p, s; \boldsymbol{\beta}^\rho)}{\partial p} > 0,\ \frac{\partial \rho(p, s; \boldsymbol{\beta}^\rho)}{\partial s} > 0,\ \frac{\partial^2 \rho(p, s; \boldsymbol{\beta}^\rho)}{\partial s^2} \leq 0,\ \forall (p, s)$$

$$\mathbf{Z}_{jl} = [1, p_l, p_l^2, p_l^3, \overline{S}_{jl}, \overline{S}^2_{jl}, \overline{S}^3_{jl}, p_l\overline{S}_{jl}, p_l\overline{S}^2_{jl}, p_l^2\overline{S}_{jl}],\ \ j \in \{\mathcal{M}, \mathcal{N}\},\ \ l = 1, \ldots, L$$

$\overline{S}^k_{jl}$ agrees with equations (3) (5) (6), $k = 1, 2, 3,\ \ j \in \{\mathcal{M}, \mathcal{N}\},\ \ l = 1, \ldots, L$

$\nu_\mathcal{M}, \nu_\mathcal{N}, \nu^*_\mathcal{M}, \nu^*_\mathcal{N}, \nu^\circ_\mathcal{M}(\sigma_\varepsilon), \nu^\circ_\mathcal{N}(\sigma_\varepsilon) \in [1, \infty)$; and $\sigma_\varepsilon > 0$

The first two summations in the objective function are the moment conditions for the selected marginal distributions of P and the selected marginal quantile functions of S. The third summation contains moment conditions for the copula of the selected joint distribution of (P, S). The fourth summation contains the selection-corrected regression equations for graduation probabilities. The second to last line of the objective function contains moment conditions for the single-index parameters: they minimize the distance between the selection-corrected rank correlations, $\tau^{\mathcal{M}*}_{\mathcal{P}S}$, $\tau^{\mathcal{N}*}_{\mathcal{P}S}$, and their theoretical maxima of one. Recall from the discussion in Section 3.2.1

that we define the parameters $\boldsymbol{\beta}^s$ within our model as those which maximize the Kendall's τ rank correlation between the implied single index S and P, which is why the estimate $\hat{\boldsymbol{\beta}}^s$ is chosen in this way. The final line of the objective function contains moment conditions for the matching shock variance: it is chosen to minimize the distance between the empirical rank correlations, $\tau_{\mathcal{PS}}^{\mathcal{M}*}$ and $\tau_{\mathcal{PS}}^{\mathcal{N}*}$, and their model-generated analogs , $\tau_{\mathcal{PS}}^{\mathcal{M}\circ}(\sigma_\varepsilon)$ and $\tau_{\mathcal{PS}}^{\mathcal{N}\circ}(\sigma_\varepsilon)$.

As for the constraints, the last line imposes natural bounds on the shock variance and copula parameters. The two lines above this establish the selection correction procedure for the graduation probability regressions, and the two lines above that impose regularity conditions on the graduation probability parameters. The third constraint from the top imposes monotonicity on the B-spline CDFs and quantile functions, and the first two lines define the s_i's as a single index in (e_i, a_i), and they impose monotonicity and a scale normalization to fix its units.

3.3.2. *Stage II Estimation.* Our Stage I estimator was based on a set of intuitive moment conditions that were notationally intense to formalize. Stage II estimation is the reverse: notationally compact and with considerable computational complexity under the surface. Amending notation somewhat, for computational convenience we first parameterize the assignment functions and inverse equilibrium strategies for group $j \in \{\mathcal{M}, \mathcal{N}\}$ as flexible B-splines. Similarly as above, these require knot vectors $\mathbf{k}_j^t = \{\underline{t} = k_{j1}^t < \cdots < k_{j,K_j^t+1}^t = \bar{t}\}$ for the assignment functins, and $\mathbf{k}_j^s = \{\underline{s} = k_{j1}^s < \cdots < k_{j,K_j^s+1}^s = \bar{s}\}$, for equilibrium strategies, which in turn uniquely define basis functions $\mathcal{B}_{jk}^t(t),\ k = 1, \ldots, K_j^t + 3$ and $\mathcal{B}_{jk}^s(s),\ k = 1, \ldots, K_j^s + 3$. When we combine these with weights $\boldsymbol{\lambda}_j^t \in \mathbb{R}^{K_j^t+3}$ and $\boldsymbol{\lambda}_j^s \in \mathbb{R}^{K_j^s+3}$ our parameterized B-spline functions have the form $P_j(t; \boldsymbol{\lambda}_j^t) = \sum_{k=1}^{K_j^t+3} \lambda_{jk}^t \mathcal{B}_{jk}^t(s)$, and $\theta_j(s; \boldsymbol{\lambda}_j^s) = \sum_{k=1}^{K_j^s+3} \lambda_{jk}^s \mathcal{B}_{jk}^s(s)$. Given a pre-specified grid of points $\{t_1, \ldots, t_{K^T}\}$ spanning $[\underline{t}, \bar{t}]$, the assignment function weights are chosen to satisfy

$$\hat{\boldsymbol{\lambda}}_j^t = \underset{\boldsymbol{\lambda} \in \mathbb{R}^{K_j^t}}{\arg\min} \left\{ \sum_{k=1}^{K^T} \Big(P_j(t_k; \boldsymbol{\lambda}) - F_{\mathcal{P}_j}^{-1} \left[H_j(t_k; \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon); \hat{\boldsymbol{\pi}} \right] \Big)^2 \right\}$$

$$\textbf{Subject to} : \lambda_k < \lambda_{k+1},\ k = 1, \ldots, K_j^t + 2.$$

The assignment mappings are a function of Stage I parameters, which are taken as fixed in Stage II, which is why we express the relevant B-spline weights using hat notation. On the other hand, the inverse equilibrium strategies solve

$$\boldsymbol{\lambda}_j^s(\boldsymbol{\alpha}) = \underset{\boldsymbol{\lambda} \in \mathbb{R}^{K_j^s}}{\arg\min} \sum_{i=1}^{I} D_{wi} \Big(\theta_j(s_i; \boldsymbol{\lambda}) - \ddot{\theta}_{ji} \Big)^2$$

$$\textbf{Subject to} :$$

$$\ddot{\theta}_{ji} c'(s_i) = \mathrm{E}_\varepsilon \Big[U_p \left(P_j(s_i + \varepsilon; \boldsymbol{\lambda}_j^t), s_i, \ddot{\theta}_{ji}; \boldsymbol{\alpha}, \hat{\boldsymbol{\pi}} \right) P_j'(s_i + \varepsilon; \boldsymbol{\lambda}_j^t) \Big] + \mathrm{E}_\varepsilon \Big[U_s \left(P_j(s_i + \varepsilon; \boldsymbol{\lambda}_j^t), s_i, \ddot{\theta}_{ji}; \boldsymbol{\alpha}, \hat{\boldsymbol{\pi}} \right) \Big]$$

$$\lambda_k > \lambda_{k+1},\ k = 1, \ldots, K_j^s + 2,\ j \in \{\mathcal{M}, \mathcal{N}\},$$

and are not fixed during Stage II. The B-spline strategies are a function of the Stage I parameters as well as the Stage II utility parameters $\boldsymbol{\alpha}$, and therefore must be adjusted each time these are updated during estimator runtime.

Essentially, the vectors $\hat{\boldsymbol{\lambda}}_j^t$ and $\boldsymbol{\lambda}_j^s(\boldsymbol{\alpha})$ are ancillary parameters that will be embedded inside the wage regression equation. The latter defines the control function for unobservable types and allows the econometrician to enforce monotonicity as required by our theory. With this in mind, we adopt a shorthand notation for our parameterized control function, $\psi\left(s, D_{\mathcal{M}i}; \boldsymbol{\alpha}, \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon, \hat{\boldsymbol{\lambda}}_{\mathcal{M}}^t, \hat{\boldsymbol{\lambda}}_{\mathcal{N}}^t\right) = \log\left[D_{\mathcal{M}i}\theta_{\mathcal{M}}\left(s; \boldsymbol{\lambda}_{\mathcal{M}}^s(\boldsymbol{\alpha})\right) + (1 - D_{\mathcal{M}i})\theta_{\mathcal{N}}\left(s; \boldsymbol{\lambda}_{\mathcal{N}}^s(\boldsymbol{\alpha})\right)\right]$, with the extra parameter arguments emphasizing its implicit dependence on Stage I objects. We also re-express the matrix of explanatory variables as

$$\mathbf{X}\left(\boldsymbol{\alpha}, \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon, \hat{\boldsymbol{\lambda}}_{\mathcal{M}}^t, \hat{\boldsymbol{\lambda}}_{\mathcal{N}}^t\right) = \begin{bmatrix} 1 & \log(p_1) & \log(s_1) & \psi\left(s_1, D_{\mathcal{M}1}; \boldsymbol{\alpha}, \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon, \hat{\boldsymbol{\lambda}}_{\mathcal{M}}^t, \hat{\boldsymbol{\lambda}}_{\mathcal{N}}^t\right) \\ 1 & \log(p_2) & \log(s_2) & \psi\left(s_2, D_{\mathcal{M}2}; \boldsymbol{\alpha}, \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon, \hat{\boldsymbol{\lambda}}_{\mathcal{M}}^t, \hat{\boldsymbol{\lambda}}_{\mathcal{N}}^t\right) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \log(p_I) & \log(s_I) & \psi\left(s_I, D_{\mathcal{M}I}; \boldsymbol{\alpha}, \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon, \hat{\boldsymbol{\lambda}}_{\mathcal{M}}^t, \hat{\boldsymbol{\lambda}}_{\mathcal{N}}^t\right) \end{bmatrix},$$

from which our utility parameter estimator is defined by

$$\hat{\boldsymbol{\alpha}} = \underset{\boldsymbol{\alpha} \in \mathbb{R}_+ \times [0,1]^3}{\arg\min} \left\{ \left[\mathbf{D}_w \left(\mathbf{W} - \mathbf{X}\left(\boldsymbol{\alpha}, \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon, \hat{\boldsymbol{\lambda}}_{\mathcal{M}}^t, \hat{\boldsymbol{\lambda}}_{\mathcal{N}}^t\right)\boldsymbol{\alpha}\right)\right]^\top \left[\mathbf{D}_w \left(\mathbf{W} - \mathbf{X}\left(\boldsymbol{\alpha}, \hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon, \hat{\boldsymbol{\lambda}}_{\mathcal{M}}^t, \hat{\boldsymbol{\lambda}}_{\mathcal{N}}^t\right)\boldsymbol{\alpha}\right)\right] \right\}.$$

In words, Stage II consists of estimating the wage regression, where the third explanatory variable is a nonlinear function of all model parameters, derived from the equilibrium conditions of our theory of HC investment.

The final step of estimation is to recover the type distributions. One simple way to proceed would be to just compute the composition of the HC CDFs and the inverse investment strategies above, but this would make use of only the subsample where household income is available. Since during Stage II we recover functionals that map any HC level into its corresponding type, including s's for which income data were not available, we use an alternative approach that uses the full sample. We first parameterize the type distributions as flexible B-splines with knot vectors $\mathbf{k}_j^\theta = \{\underline{\theta} = k_{j1}^\theta < \cdots < k_{j,K_j^\theta+1}^\theta = \overline{\theta}\}$, basis functions $\mathcal{B}_{jk}^\theta(\theta)$, $k = 1, \ldots, K_j^\theta + 3$, and weights $\boldsymbol{\lambda}_j^\theta$ implying the familiar form $F_j(\theta; \boldsymbol{\lambda}_j^\theta) = \sum_{k=1}^{K_j^\theta+3} \lambda_{jk}^\theta \mathcal{B}_{jk}^\theta(\theta)$.

In order to fit these to the data we need to construct their empirical analogs. Let the inferred types for minority graduates in the full B&B sample be denoted $\hat{\theta}_{\mathcal{M}i} = \theta_{\mathcal{M}}(s_i; \boldsymbol{\lambda}_{\mathcal{M}}^s(\boldsymbol{\alpha}))$, and let those for non-minority graduates be $\hat{\theta}_{\mathcal{N}i} = \theta_{\mathcal{N}}(s_i; \boldsymbol{\lambda}_{\mathcal{N}}^s(\boldsymbol{\alpha}))$. We compute a selection-corrected empirical type distribution for all enrollees (including dropouts) as

$$\widehat{F}_j(\theta) = \frac{\sum_{i=1}^I \mathbb{1}(\hat{\theta}_{ji} \leq \theta)\mathbb{1}(i \in j)/\rho(p_i, s_i)}{\sum_{i=1}^I \mathbb{1}(i \in j)/\rho(p_i, s_i)}, \; j \in \{\mathcal{M}, \mathcal{N}\}, \tag{12}$$

so our B-spline estimator takes the form

$$\hat{\boldsymbol{\lambda}}_j^\theta = \underset{\boldsymbol{\lambda}\in\mathbb{R}^{K_j^\theta}}{\arg\min}\left\{ \sum_{i=1}^{I} \mathbb{1}(i\in j)\left(F_j(\theta_i;\boldsymbol{\lambda}_j^\theta) - \widehat{F}_j(\theta_i)\right)^2 \right\} \tag{13}$$

$$\textbf{Subject to}: \lambda_{j1}^\theta = 0,\ \lambda_{j,k-1}^\theta < \lambda_{jk}^\theta,\ k=2,\ldots,K_j^\theta,\ j=\mathcal{M},\mathcal{N}.$$

3.3.3. *Asymptotics and Standard Errors.* From the above definitions, it is evident that our estimator for model parameters $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\pi}}, \hat{\sigma_\varepsilon})$ falls within the class of Generalized Method of Moments. Therefore, standard asymptotic theory establishes consistency and asymptotic normality with convergence at the standard rate of $\sqrt{I}$, once tuning parameters (*e.g.*, knot vectors) have been specified. Other empirical objects, such as the type distributions, marginal distributions of P and S, and the college assignment mappings, are pointwise continuous functions of these parameters, and therefore inherit similar properties. However, due to the computational complexity involved in computing derivatives for our various moment conditions, we have elected to compute standard errors and confidence bounds on our estimates via the nonparametric bootstrap. Nevertheless, the established GMM asymptotic theory validates our use of the bootstrap as a well-founded method for inference.

Our block-bootstrap procedure involves re-sampling 1000 times from the race-specific B&B subsamples. Following the notation above, we separately re-sample $I_\mathcal{M}$ student observations (with replacement) from the minority subsample $\{w_i, p_i, e_i, a_i\}_{i=1}^{I_\mathcal{M}}$, and $I_\mathcal{N}$ student observations (with replacement) from the non-minority subsample $\{w_i, p_i, e_i, a_i\}_{i=1}^{I_\mathcal{N}}$. Because college-level variables represent the universe of four-year colleges and the full universe of students enrolled in these colleges, we hold these observables fixed during our bootstrap procedure. Moreover, we calibrate the minority mass μ from the IPEDS college-level data and hold it fixed as well. Thus, all variation in our standard errors comes from the finite sampling of the student-level B&B data.

3.3.4. *Practical Issues.* In the implementation of our GMM estimator of Stage I parameters, we adopt a simplification for computational convenience. First, during solver runtime we normalize $\beta_1^s = 1$ instead of constraining the objective function so that the maximal single index value is one. This simplifies the problem by reducing the number of parameters to choose and constraints to satisfy. After the estimator has run we re-scale the HC single index so that it's maximal possible value is one, and we accordingly make adjustments to the graduation probability parameters and HC distributions to reflect the re-scaling.

There are several model tuning parameters that we must specify, among which are knot vectors $\mathbf{k}_j^p$, $\mathbf{k}_j^q$, $\mathbf{k}_j^t$, and $\mathbf{k}_j^s$. We adopt the convention that knots are to be chosen uniformly in empirical quantile space, as this evenly spreads the statistical power of the data across all basis functions and simplifies the decision to a choice of the number of knots. Specifically, we chose $K_\mathcal{M}^p = K_\mathcal{N}^p = K_\mathcal{M}^t = K_\mathcal{N}^t = K_\mathcal{M}^s = K_\mathcal{N}^s = 5$ (*i.e.*, knots at the quintiles with 8 total B-spline basis functions) for the selected school quality distributions, assignment functions, and inverse strategies, respectively; and $K_\mathcal{M}^q = K_\mathcal{N}^q = 10$ (*i.e.*, knots at the deciles with 13 basis functions) for the selected HC quantile functions. Additional knots did not appreciably improve model fit. When approximating the assignment mappings, we imposed a truncation $\underline{t} = \min_i\{s_i\} - 5\hat{\sigma}_\varepsilon$, $\bar{t} = \max_i\{s_i\} + 5\hat{\sigma}_\varepsilon$ on the support of NHC, and we chose a set of evaluation points that included the modes of the B-spline basis functions and the midpoints between the modes.

We calibrated κ from the U.S. Census Bureau's Current Population Survey (CPS). Recall that the students in our sample were initially surveyed after graduation in 1993, and the household income data we use was collected in the 2003 follow-up survey. To get a benchmark for the fraction of college graduate incomes garnered by dropouts, we computed the ratio of the average household income of 33-year-olds in the 2003 CPS survey with some college to the average household income of 33-year-old college graduates (with no additional post-graduate education). The result is a value of $\kappa = 0.714$. As a robustness check we also repeated our estimates assuming κ values ranging from 0.5 to 0.9. Only two aspects of our analysis change. First, the choice of κ affects the estimated values of α_θ and the distribution of types, $F_j(\theta_i; \boldsymbol{\lambda}_j^\theta), j \in \{\mathcal{M}, \mathcal{N}\}$. However, the overall role of θ in the wealth equation, which we measure as α_θ times the standard deviation of $\log(\theta)$, is stable to changes in κ. The second thing that is affected is the productive channel of incentives, which we estimate to be weaker as κ rises. This is intuitive since, as we see in Section 4.5, s only affects the graduation probability, so anything that makes the utility gap between completing college and dropping out shrink will weaken the direct, productive benefit of HC.

Finally, two sources of sampling weights were used in our empirical implementation, but in order to avoid further complicating notation we left them out of the formal definition above. The first is cross-sectional sampling weights contained in the B&B data. In Stage I these were used for group $j \in \{\mathcal{M}, \mathcal{N}\}$ to calculate the empirical analogs of the joint distributions $\widehat{F}_{\mathcal{PS}}(p, s|j, \text{grad})$, the marginal quantile functions $\widehat{Q}_{S_j}(r)$, and the marginal distributions $\widehat{F}_{\mathcal{P}_j}(p|\text{grad})$. In simple terms, each of these functions at a point (p, s) is a sample mean of indicator functions evaluated at each datapoint, and we converted them into weighted sample means (in the usual way) using the B&B cross-sectional weights. In Stage II they were also used to weight each component of the wage regression (in the usual way that weighted regressions are constructed) and to calculate the empirical analog of the type distributions $\widehat{F}_j(\theta)$. The second source of sampling weights came from IPEDS. In Stage I, the graduation probability regression (the fourth summation in the definition of $[\hat{\boldsymbol{\pi}}, \hat{\sigma}_\varepsilon]^\top$ above) is converted into a weighted regression in the usual way by using the number of individuals in each school-race freshman cohort in 1988 as weights.

4. ESTIMATION RESULTS

4.1. **ESTIMATES: Single Index Function $S(e, a)$ and Matching Shock Variance σ_ε^2.**

Recall that the HC single index function was specified as a quadratic polynomial in SAT score e and GPA a. The values for these parameters along with their standard errors are contained in Table 3.[14] The HC index in convex in both e and a and admits significant complementarities between the

[14]Recall that the single index function is scale-free and normalized to attain a maximum value of one. Recall also that a more flexible cubic form did not improve our model fit (see footnote 7).

TABLE 3. ESTIMATES: Single Index Function and Matching Shock Variance

Parameter	Point Est.	Std. Err.	P-Value	95% Confidence Interval
β_1^s (e)	0.0507***	(9.63×10^{-4})	< 0.001	[0.0488,0.0527]
β_2^s (e^2)	0.1580***	(0.0095)	0.006	[0.1415,0.1695]
β_3^s (a)	0.3023***	(0.0085)	< 0.001	[0.2877, 0.3179]
β_4^s (a^2)	0.2906***	(0.0096)	< 0.001	[0.2754,0.380]
β_5^s $(e \cdot a)$	0.22291***	(0.0090)	< 0.001	[0.2105,0.2410]
σ_ε	0.0275	(7.25×10^{-4})	—	[0.026, 0.029]
$\tau_{\mathcal{PS}}(\sigma_\varepsilon \vert \mathcal{M})$	0.867	(0.01458)	—	[0.820, 0.885]
$\tau_{\mathcal{PS}}(\sigma_\varepsilon \vert \mathcal{N})$	0.893	(0.0021)	—	[0.889, 0.896]

*NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.*

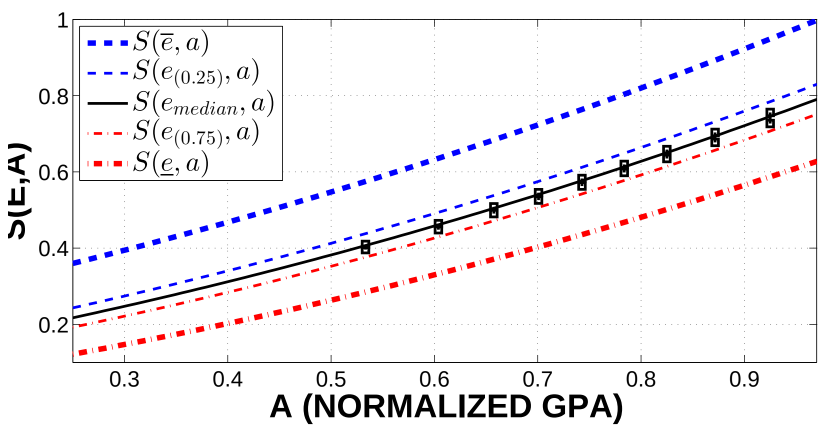


FIGURE 1. Single Index Function

arguments. To help the reader understand the marginal effects of e and a on the HC function, we present marginal effect statistics at the median values found in the B&B dataset. The marginal effect of a one standard deviation change of e is 0.411, while the marginal effect of a one standard deviation change in a is equal to 0.131. In other words, a is roughly 2.5 times as important as e for determining a median student's HC single index. Due to convexity of $s(e, a)$, these marginal effects are maximized for the highest values of (e, a) and minimized for the lowest values.

Figure 1 illustrates our single index equation. Each line depicts the effect of academic record a on the HC index while holding exam score e fixed at one of it's quartiles. The lesser importance of e relative to a is reflected in the fact that the difference between the 75^{th} and 25^{th} percentile lines is less than 0.1, while the difference between the 75^{th} and 25^{th} quintiles on the line describing $S(e_{median}, a)$ is close to 0.2. The upward curve of the lines is a result of the convexity of the single index with respect to the student's GPA. We include 95% confidence intervals on the line describing $S(e_{median}, a)$ at each decile of the distribution of a.

TABLE 4. Graduation Probability Estimates $\widehat{\rho}(p,s)$

Parameter	Point Est.	Std. Err.	P-Value	95% Confidence Interval
β_1^ρ (p)	0.0500	(0.0805)	0.415	[-0.087,0.2377]
β_2^ρ (p^2)	-0.2042^{*}	(0.1391)	0.097	[-0.4568, 0.0689]
β_3^ρ (p^3)	0.2190^{*}	(0.1305)	0.054	[-0.0028,0.4972]
β_4^ρ (s)	0.8445^{***}	(0.1658)	< 0.001	[0.6097,1.3085]
β_5^ρ (s^2)	-0.0938^{***}	(0.1421)	< 0.001	[-0.9295,-0.0136]
β_6^ρ (s^3)	0.0077	(0.0129)	0.146	[-0.0172,0.036]
β_7^ρ $(p \cdot s)$	0.0735	(0.3753)	0.936	[-0.4381,1.2508]
β_8^ρ $(p \cdot s^2)$	0.0732	(0.1443)	0.476	[-0.0795,0.1990]
β_9^ρ $(p^2 \cdot s)$	0.0760	(0.2739)	0.633	[-0.3497,0.8685]
β_0^ρ (const.)	-0.0497^{*}	(0.0236)	0.065	[-0.0814,0.0048]

*NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.*

Table 3 also presents our estimate of the matching shock standard deviation, σ_ε, and group-specific Kendall's τ in order to provide different perspectives on the matching frictions in the college admissions market. The matching shock variance provides insight into how informative the NHC realizations are for students' underlying HC choices. Our estimates imply that the noise-to-signal ratio of the matching shock (the ratio of σ_ε to a standard deviation of HC) is 18.9%. Kendall's τ provides an alternative way to interpret the economic meaning of the matching shock variance. Our rank-order contest structure implies Kendall's τ can never be negative, but a value near 0 would mean that the matching shock almost entirely determines the allocation of students to schools. On the other hand, a value close to 1 means the matching shock plays only a negligible role in assignment of students to colleges. Since Kendall's τ for minority students is estimated at 0.867, this means for two randomly selected minority students there is a 93.4% chance that the student with higher HC will enroll in a higher quality college. Likewise, this probability for the minority group is 94.6%. Thus, while matching shocks play a nontrivial role, our empirical model suggests a high degree of assortativity in the college market.

4.2. **ESTIMATES: Graduation Probability Function, $\rho(p,s)$.** Point estimates and standard errors for the graduation probability parameters are displayed in Table 4. We again compute the marginal effect of a one standard deviation change in p and s (from the median values) on $\rho(p,s)$ as our metric of the relative importance of these variables for determining the probability of graduation. In this case, we compute the median and standard deviation statistics using the selection-corrected distributions of the respective variables. The marginal effect of a one standard deviation change in p is 0.026, while the marginal effect of a one standard deviation change in s in 0.143. This means that s is roughly 5.5 times as important as p for determining college graduation probabilities.

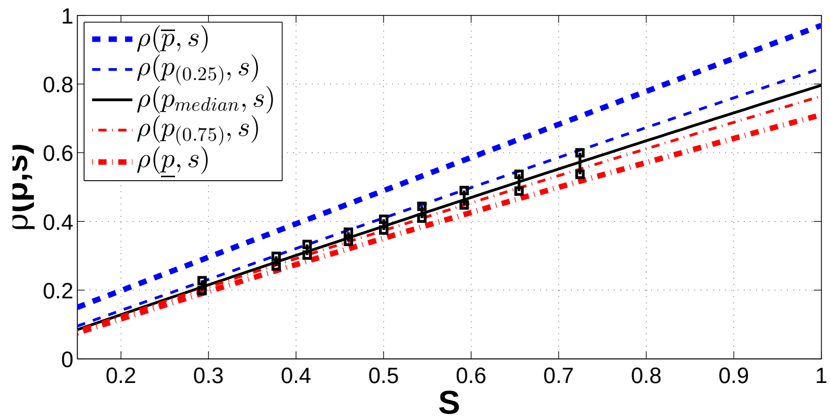


FIGURE 2. Graduation Probability Function

Figure 2 illustrates the relative impact of p and s on graduation probabilities. Each line depicts the effect of HC on graduation probability holding that college quality fixed at one of its quartiles. We have also plotted bootstrapped 95% confidence bounds on graduation rates at a college of median quality at the deciles of the distribution of HC. The convexity of $\rho(p, s)$ is evident from the increasing difference between the lines as the college quality improves, meaning that college quality matters most above the 75^{th} percentile of college quality. The complementarity between p and s results in an increasing spread between the lines as s increases, meaning that college quality is more important for higher achieving students.

4.3. **ESTIMATES: Selection-Corrected Joint Distributions $f_j(p, s)$.** Figure 3 displays the distributions of HC levels for each demographic group, including the selected sample of graduates from the raw data (dashed lines) and the selection-corrected distributions for all enrollees (solid lines). Figure 4 displays the distributions of college seats allocated to each group, including the selected raw samples (dashed lines) and the selection corrected distributions for all enrollees (solid lines). The CDF plots also include 95% confidence bounds at the deciles of the population-wide distributions. The first plot illustrates the achievement gap, a stochastic dominance relationship between minority HC and non-minority HC. The second plot illustrates the enrollment gap, a similar stochastic dominance relationship between minority and non-minority college quality.

Figure 4 does not have any large jumps in the CDFs, which is consistent with our model's assumption that there is a continuum of college seats. In 1988 (four years prior to the graduating class of AY1992-1993) there were a total of 1,644,340 freshman seats in the market with any single school having only a negligible market share. The largest college in 1988 (Ohio State) had a total market share of only 0.76% of new freshman seats, and the next ten largest schools (in descending order of size: UT-Austin, Michigan State, Akron, BYU, Purdue, Minnesota, Northeastern, Toledo, Texas A&M, and Pittsburgh) combined for only 5.4% of total market supply. The mean, median, and standard deviation of market shares for individual universities were 0.091%, 0.047%, and 0.102%, respectively. Thus, a large, atomless market approximation appears reasonable.

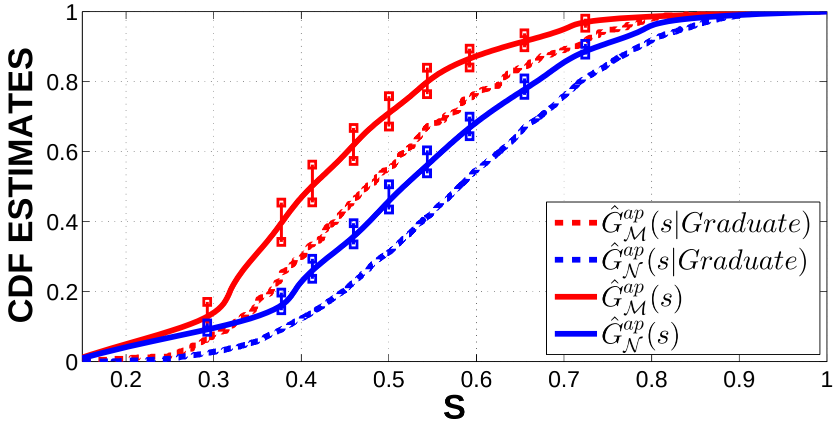


FIGURE 3. Human Capital Distribution

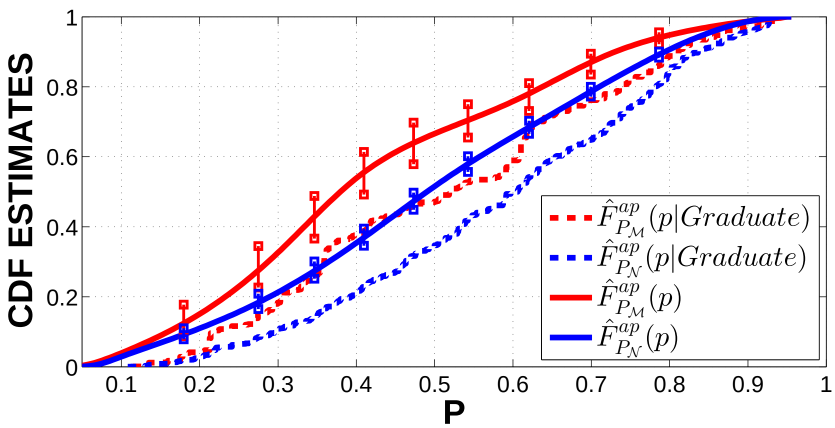


FIGURE 4. School Quality Distribution

4.4. **ESTIMATES: Minority Markup Function, $\widetilde{T}$.** Our estimate of the markup function is based on equation 9. Figure 5 describes the markup function in two separate ways. The horizontal axes of both panels display quantile ranks of NHC for nonminority students. The top panel describes the shape of of $\widetilde{T}$. Its vertical axis displays the quantile rank of subsidized NHC within the nonminority NHC distribution. If a minority student has an NHC at the quantile rank marked on the horizontal axis, the student gets the same college assignment as a nonminority student with an NHC at the quantile rank denoted on the vertical axis. For example, the plot shows that a minority student with an NHC equal to the median value of the nonminority population gets the same college assignment as a nonminority student at the 64^{th} percentile of the nonminority population. The dashed line denotes the 45° line for reference.

The bottom pane of Figure 5 describes the effect of the admissions preference schemes in terms of school quality. Again, the horizontal axis denotes quantile ranks of the distribution of nonminority NHC levels. The vertical axis denotes the gap in the quantile rank of college quality between a minority student and a nonminority student at each NHC quantile. For example,

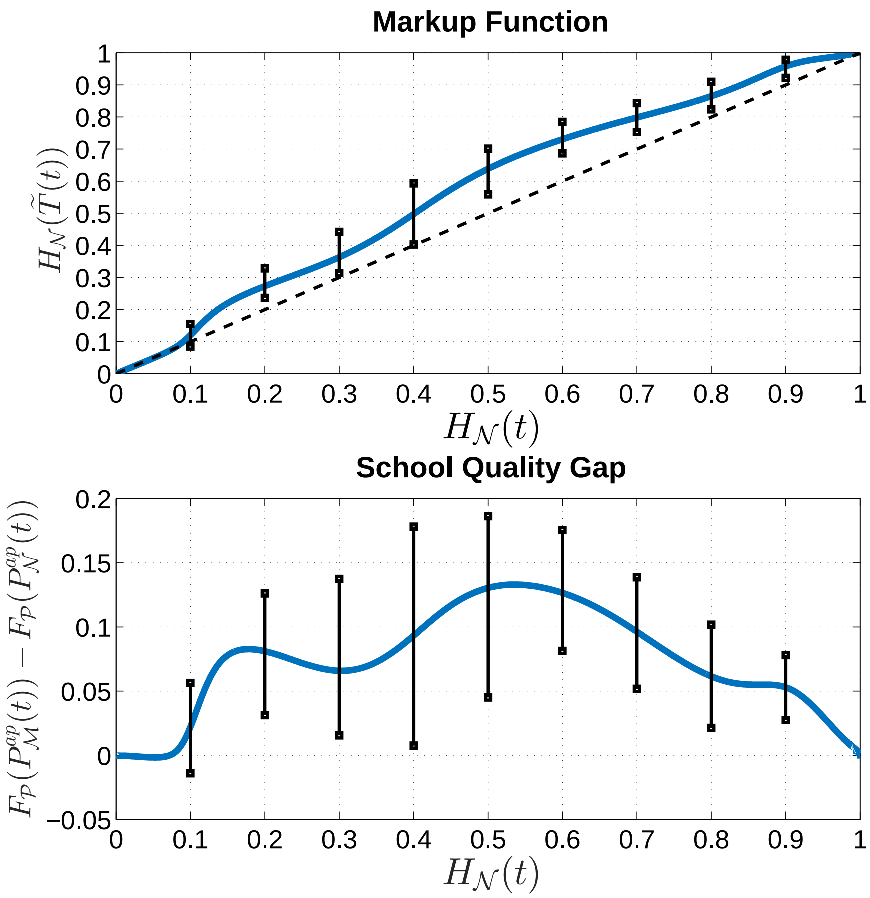


FIGURE 5. Noisy Human Capital Markup Function, $\widetilde{T}$

the plot shows that if two students from different groups both have an NHC value equal to the median of the nonminority population, then the minority student is assigned to a school whose quantile rank is 0.13 higher in the school quality distribution.

We display 95% confidence bounds at the deciles of NHC. What our plot reveals is that the effect of the status quo admissions preference scheme is insignficant at colleges in the bottom decile, but is statistically and economically significant across the rest of the college quality spectrum. This result improves upon previous empirical work in various ways. Several papers have estimated a substantial impact of AA, including Bowen and Bok [15]; Chung and Espenshade [20]; and Chung, Espenshade, and Walling [21], but these studies used data from elite colleges, whereas ours uncovers a market-wide picture. The most similar previous study is Kane [37], which also used a nationally representative sample (the High School and Beyond (HS&B) survey), but estimated a significant role for AA only in the top quintile of the market.

Several differences exist between Kane [37] and our study. First, we use measures of final market allocations (enrollment data), whereas Kane [37] uses applications data which may not fully reflect final enrollment decisions. Second, the HS&B data contain potentially important sources of sample selection that could affect probit regression results in unpredictable ways.

HS&B respondents were asked their two top choices (sample truncation) among the schools to which they applied (endogenous selection in student-school pairs), and whether they were accepted. Third, HS&B focuses on students who entered college in 1980 and 1982, while the B&B focuses largely on the freshman class of 1988.

Given the above factors, it is hard to pinpoint the source of the differences between our results and Kane [37]. At the end of the day, our estimates of the markup function are the most directly data-driven component of the empirical model: they do not depend on Bayes-Nash equilibrium theory, but instead hinge only on a Stage I reduced-form sample-selection correction to map our observed set of college graduates into the original set of college enrollees. The intuition behind our result is that we see marginal distributions of HC and college quality by race, and a color-blind world implies a very specific form to which the latter must conform, given the former. However, our Stage I reduced-form data products deviate from this specific form, and in such a way that more generous admissions practices toward minorities must exist on the majority of the market in order to rationalize observed allocations from observed achievement.

4.5. **ESTIMATES: Match Utility and Learning Cost Type Distributions.** The type distributions are presented in Figure 6. The top panel displays the type CDFs $F_{\mathcal{M}}(\theta)$ and $F_{\mathcal{N}}(\theta)$ with 95% confidence bounds represented by the shaded areas. The type distribution of the minority students stochastically dominates the the type distribution of nonminority students, which is consistent with minority students having higher learning costs than nonminority students. The bottom panel of Figure 6 plots the difference between the CDFs with 95% confidence bounds as well. Our estimates strongly suggest that the difference between the distribution of types between the two groups is significant at all cost quantiles.

This result would seem to conform with a large body of empirical evidence on stark racial differences in access to resources that affect childhood development in the US. For example, Black and Hispanic children are nearly three times as likely to live below the poverty line (see Kena, *et. al.* [38]). They are also much less likely to be covered by health insurance (see Smith and Medalia [49]) or to be raised by parents with bachelor degrees (see Fox, Kewal, and Ramani [29]). Moreover, holding income level fixed, Blacks and Hispanics are also much more likely to attend under-performing schools that serve poorer student bodies relative to their White and Asian counterparts of similar incomes (see Reardon, Townsend, and Fox [46], Reardon [44], and Reardon, Kalogrides, and Shores [45]). With these empirical facts in mind, the estimated stochastic dominance relationship in learning costs would seem a natural, though unfortunate, consequence of resource stratification by race.

Table 5 presents our estimates of the returns to various inputs of income production including a more selective college, more HC, and lower learning costs. The first takeaway is that pre-college HC choice, while important for determining the graduation probability, has very little effect on wages conditional on graduation. The second takeaway is that both school quality and the agent's unobservable type have economically and statistically significant effects on income production. To get a sense for the scale of the relative importance of these variables, we again compute the marginal effect of a one standard deviation change in the inputs to the income

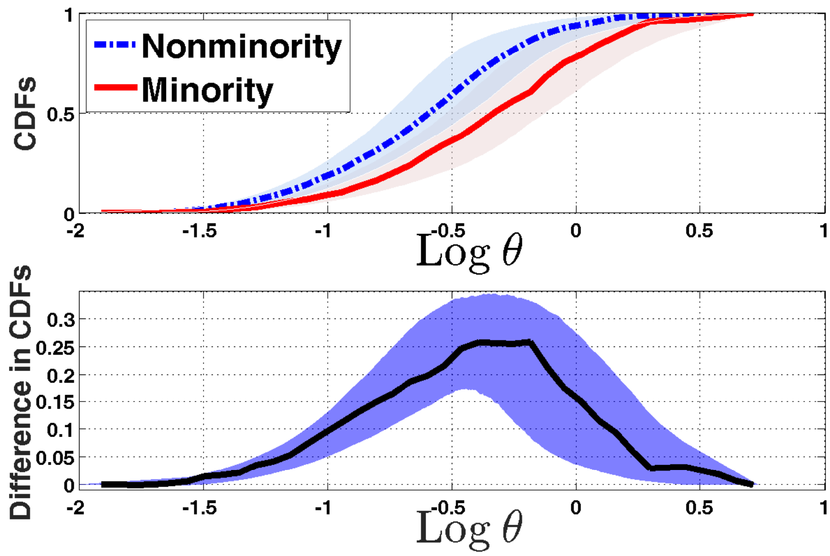


FIGURE 6. Agent Type Distribution

TABLE 5. Estimates of the Wealth Equation and Cost Curvature Parameters

Parameter	Point Est.	Std. Err.	95% Confidence Interval
α_p	0.1852	(0.0269)	[0.1414,0.2293]
α_s	1.63×10^{-6}	(0.0181)	[2.81×10^{-8}, 0.0452]
α_θ	0.0400	(0.0270)	[6.01×10^{-8},0.0858]
α_0	\$101,273	(\$3,558)	[\$95,765, \$107,634]

production function. The marginal effect of a one standard deviation increase in $log(p)$ is 6.25 times larger than the marginal effect of a one standard deviation reduction $log(\theta)$, which is in turn 3×10^4 times larger than the marginal effect of a one standard deviation increase in $log(s)$.

In summary, we estimate that household income 10 years following graduation is driven primarily by the quality of the college attended, although an agent's unobservable type also plays a significant role. There is a sizable literature on the returns to a higher quality college, including Brewer, Eide, and Ehrenberg [16], Dale and Krueger [23], Black and Smith [12], Long [39], and Andrews, Li, and Lovenheim [5]). In general, the evidence is clear that more selective colleges benefit poorer students significantly. Other than that, there is some disagreement as to the magnitude of the return for more affluent students. Our results support the view that a higher quality college is a resource from which all students benefit. Holding s and θ fixed at their relative medians, a move from being placed at the 25^{th} percentile college to the 75^{th} percentile college induces an estimated shift of roughly \$13,000 in year-10 income. We are also the first paper to directly quantify the role of unobserved heterogeneity in the returns to an education, which we find is significant though not dominant: a similar shift from the lower quartile of θ to the upper quartile, holding p fixed at the median, induces an increase of roughly $\$2,000$ in year-10 income. On the other hand, while a student's HC choices prior to college do not significantly affect the wage conditional on graduation, these investments have a strong influence on the probability of graduating college.

5. COUNTERFACTUAL POLICY EXPERIMENTS

The issue of demographic diversity on college campuses has been controversial, and the subject of repeated judicial scrutiny in the United States. The cornerstone of Supreme Court jurisprudence regarding AA in college admissions is the 1978 case *University of California Regents v. Bakke* [1]. Justice Powell's opinion established that the government has a compelling interest in encouraging diversity in university admissions founded on principles of academic freedom and a university's right to take what actions it feels necessary to provide a high quality education to its students, but using racial admission quotas is an illegal method to do so. With ongoing legal challenges to race-based AA in college admissions its future may be uncertain, and in recent years some state education systems have begun to experiment with alternative forms of AA that use proxies for race, such as socioeconomic status.

We turn our attention now to an exploration of the economic implications of various changes to the status quo AA we estimated from the data-generating process. We will compare how the 1988 AA admissions policies shaped college diversity relative to the benchmarks of a color-blind system (*i.e.*, a hypothetical AA ban) and a proportional quota system (an even more generous form of AA). For each of the three admissions schemes, we numerically solved for the equilibrium of the model holding our structural point estimates fixed (for technical details see online appendix). Given the importance of school quality in determining outcomes of college enrollees, we will also explore how these different policies influence college graduation rates and post-college income. Finally, we will assess how students from each demographic group alter their HC investment behavior in response to the different AA systems.

Since race-based AA has become legally questionable in the United States, we also consider the extent to which socioeconomic class can serve as a proxy for race from the perspective of a policy-maker who wishes to achieve more racially diverse college student bodies. Within our theoretical framework there is nothing fundamental about race *per se* for defining a target demographic group to benefit from preferential admissions practices. Therefore, we can also experiment with other observable characteristics to define target beneficiary demographics. In another counterfactual experiment we classify the students in our data set as economically advantaged or disadvantaged using their *expected family contribution* (EFC), a measure based on wealth and income that the Department of Education uses to determine access to means-tested government aid. We then compute equilibria of the model in which admissions quotas are implemented in favor of poorer students, for various cutoffs to define poor versus non-poor.

A final empirical question we address with our college admissions model focuses on the cost of competition. A novel aspect of our model is that we can separate the HC investment incentives into the productive channel, from holding extra HC, and the competitive channel, from the

TABLE 6. Enrollment by Group and School Quality Quintile

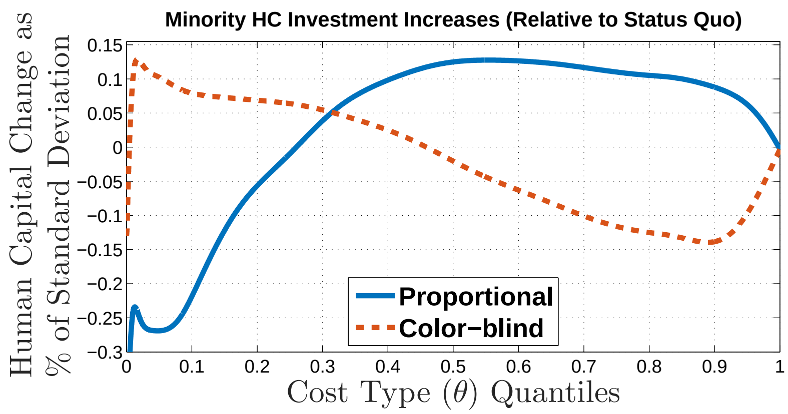
MINORITIES			
College Quality Tier	**Status Quo**	**Color-Blind**	**Proportional Quota**
Top College Quintile	0.1297	0.1034	0.2000
Second College Quintile	0.1656	0.1354	0.2000
Third College Quntile	0.1484	0.1544	0.2000
Fourth College Quintile	0.2806	0.1865	0.2000
Bottom College Quintile	0.2757	0.4203	0.2000
NON-MINORITIES			
College Quality Tier	**Status Quo**	**Color-Blind**	**Proportional Quota**
Top College Quintile	0.2131	0.2180	0.2000
Second College Quintile	0.2065	0.2123	0.2000
Third College Quntile	0.2097	0.2087	0.2000
Fourth College Quintile	0.1850	0.2025	0.2000
Bottom College Quintile	0.1856	0.1585	0.2000

prospect of an improved school assignment in the admissions contest. We close our analysis by assessing the relative strength of these two channels.

5.1. **Effects of AA on Minority Enrollment.** Table 6 depicts the magnitudes of counterfactual enrollment shifts using the fraction of each demographic group enrolled in each college quality quintile under each system. The benchmark of full representation of each group in each quintile, 0.2, is mechanically achieved by a quota. Numbers below this imply under-representation and vice versa. For example, 12.97% of minority students enroll in colleges in the top quintile of the quality distribution under the status-quo AA scheme, but only 10.34% of them enroll in top colleges under a color-blind scheme. In other words, while the status-quo AA is substantially less generous to minorities at top colleges than a proportional quota would be, a ban of race-based AA would reduce minority enrollment in the top tier by one fifth.

The status quo AA scheme has the intended result in that there is a first-order stochastic dominance (FOSD) shift in the distribution of the quality of colleges in which minority students enroll, relative to the color-blind case. Interestingly, the largest shift is from the lowest quintile of college quality into the second lowest quintile: in a color-blind world, minority enrollment in the bottom quintile would increase by almost 50%. A proportional quota yields an even stronger FOSD shift toward minority enrollment in better colleges, relative to the status quo. For completeness, Table 6 also provides the effects of AA on nonminorities. As expected, changing the AA scheme has the mirror opposite effect on the nonminority students, but the magnitudes of the shifts are small since the non-minority mass is over five times larger.

FIGURE 7. Minority Investment Strategy



5.2. **Effects of AA on HC Investment.** Our second target of analysis is HC investment. It is theoretically ambiguous whether the minority HC choices will increase or decrease under a particular AA scheme. Figure 7 presents the change in minority HC investment under our two counterfactual admissions schemes, so positive (negative) values indicate increases (decreases) relative to the status quo. For ease of presentation, we describe the student's type in terms of quantiles of the minority cost distribution, and changes in HC are displayed as a fraction of a standard deviation in the status quo world.

Under a counterfactual color-blind scheme, minority students with learning costs above the median reduce HC output. The intuition for this change has to do with *discouragement effects*, a common phenomenon in rank-order contests influencing behavior of agents at a relative disadvantage.[15] Holding one's own type fixed, if there is a shift in the distribution of competitors so that one falls far enough behind, effort will eventually decrease. Since shifting from color-blind to AA partially shields minority student quantiles from competition by the corresponding (and better-resourced) non-minority quantiles, it is as if the policy engineers such a shift in the type distribution. Therefore, the status quo AA scheme mitigates discouragement effects for high-cost minorities. In contrast, minority students with costs far enough below the median react to heightened competition for top seats under a color-blind scheme by investing more aggressively, since adjusting output upward is less costly for them. For nonminority students, the effects of shifting from status quo to color-blind are the opposite, but of much smaller magnitude since their mean college placement changes by a smaller margin.

The changes in HC investment from our second counterfactual of a proportional quota are larger in magnitude and reversed in sign. We find that the 75% of minority students with highest learning costs increase achievement, while the other 25% of minority students reduce it. The reasoning is essentially the same as before: a proportional quota splits the market into two separate

[15]The established contest literature which has explored discouragement effects has typically focused situations with a single prize under competition. One important difference in the current setting is the multi-object aspect of the competition, where everyone wins some prize, but prizes differ substantially by quality level.

TABLE 7. Counterfactual Minority Graduation Probability...

...BY ACHIEVEMENT QUINTILE			
Learning-Cost Type Tier	**Status Quo**	**Color-Blind**	**Proportional Quota**
Top Achiever Quintile	0.572	0.577	0.553
Second Achiever Quintile	0.395	0.396	0.410
Third Achiever Quntile	0.303	0.298	0.327
Fourth Achiever Quintile	0.241	0.225	0.259
Bottom Achiever Quintile	0.164	0.148	0.176
...BY ENROLLED COLLEGE QUALITY QUINTILE			
College Quality Tier	**Status Quo**	**Color-Blind**	**Proportional Quota**
Top College Quintile	0.623	0.657	0.552
Second College Quintile	0.444	0.475	0.409
Third College Quntile	0.357	0.386	0.327
Fourth College Quintile	0.275	0.304	0.261
Bottom College Quintile	0.182	0.191	0.176

competitions within demographic groups, where each one has the same distribution of seats "up for bids". Since the quantiles of the minority cost distribution are all higher, discouragement effects are now further diminished for high-cost minority students. The lowest-cost minority students rationally reduce investment since they were placing at top schools already under the status quo, and now there is less competitive pressure from students placing at lower schools.

In summary, the effect of the AA scheme on HC investment by minority students depends on the learning cost type of the particular student. In addition, the peak magnitude of these changes are significant and amount to roughly 25% of a standard deviation. The effect on non-minority students is much more limited since the distribution of college seats and the types of their competitors closely resembles the corresponding distributions under the status quo. The limited effect on nonminority students ought to be unsurprising since they represent 84% of the student population and their costs are on average much lower.

5.3. **Effects of AA on Household Income and Graduation Probability.** In order to give the reader a full sense for how AA shapes minority outcomes, we present graduation and income changes in two different ways. The first separates students by *achiever quintiles*, which represent the same set of individuals under each counterfactual scenario. We also present these outcomes by students enrolled in *college quintiles* in order to depict how outcomes change among students who enroll at different points in the college quality spectrum. It should be noted, however, that these do not represent the same sets of individuals across different counterfactual scenarios, as can be seen in Table 6. We do not display the quintile-specific effects for nonminority students as they are much smaller, but they tend to have the opposite signs.

The top panel of Table 7 displays graduation probability changes by achievement quintile. Two main forces govern the results here. First, any change in the AA system alters investment incentives, which we discussed in Section 5.2. Since the graduation probability is affected by a student's HC choice, this has a large effect. The second force is the counterfactual change in college assignments. Recall from the previous section that this force plays a secondary role to pre-college achievement and is most significant for students placing in higher-quality colleges. The effects we see in the upper panel largely mirror what we found in Figure 7. High-achieving minority students have a stronger (weaker) incentive to make HC investments under a color-blind (proportional quota) system, and the extra investment is largely reflected in graduation rates, though mitigated somewhat by countervailing college placement shifts for these best students. The opposite incentives and outcome effects occur for the lowest achieving minority students, but their graduation rate shifts reflect pure investment changes to a larger extent.

The bottom panel of Table 7 breaks out the graduation rate by quintile of the college quality distribution. This perspective is useful because it reveals the total impact of the two forces on graduation probability. We see an increase for minority students who enroll at top colleges under a color-blind scheme, due mostly to composition effects: a smaller number of minority students with lower cost types enroll in the best colleges, relative to the status quo. In addition, these students accrue more HC as per Figure 7. Symmetrically, under a proportional scheme minority students enrolling at the top quintile of schools have a lower average rate of graduation because on average they have higher learning costs and their investment incentives (for top minority achievers) are weakened.

Elsewhere we find the same pattern continues to hold. Minority students within a college quintile graduate at a higher rate under a color-blind system than under a proportional quota, but the reasons are more subtle at the low end of the market. First, the large differences between successive college quintiles allows for this possibility, given counterfactual shifts of students across them. Second, note that minority students who enroll at the worst colleges have a weaker (stronger) incentive to invest in HC under a color-blind (proportional) system, which would seem to suggest lower (higher) graduation rates. However, because more minority students are shifted from better schools into the worst quintile under a color-blind system, their average learning costs are lower. This second effect, the lower average learning costs, is coupled with higher HC attainment by minority students enrolled at the worst schools in a color-blind system, relative to a proportional quota. It is also stronger than the first effect, which pushes the average minority graduation rate at the worst colleges above the minority graduation rates observed at the same colleges under a proportional quota.

The top and bottom panes of Table 7 are consistent with one another, but it is difficult to make comparisons because of high counterfactual flows of minority students between college quality quintiles. For example, a naive reading of the bottom pane of of Table 7 would seem to suggest that the mean minority graduation rate is highest under a color-blind system. However, this reasoning ignores compositional shifts in *where* they enroll. Table 6 shows that 42% of minorities enroll in the bottom quintile of the college quality distribution in the color-blind counterfactual,

TABLE 8. Aggregate Effect on Graduation Rates

	Average Graduation Rate of Minority Students	**Average Graduation Rate of Nonminority Students**
Status Quo	0.3349	0.4203
Color-Blind	0.3289	0.4182
Proportional	0.3449	0.4216

TABLE 9. Counterfactual Minority Household Income by Achievement Quintile

Learning Cost Type Tier	**Status Quo**	**Color-Blind**	**Proportional Quota**
Top Achiever Quintile	$99,621	$98,790	$100,674
Second Achiever Quintile	$92,208	$90,656	$94,795
Third Achiever Quntile	$85,265	$82,934	$88,766
Fourth Achiever Quintile	$79,727	$76,301	$82,594
Bottom Achiever Quintile	$70,960	$68,417	$72,648

which is almost the same fraction of the minority population that graduates in the bottom two quintiles of college quality in the proportional quota counterfactual. The average graduation rate across the bottom two achiever quintiles of the proportional quota counterfactual is 0.218, which is higher than the average graduation rate of 0.191 in the bottom college quintile of the color-blind distribution.

This point highlights the importance of taking a market-wide perspective when investigating the impact of AA on the rates at which minorities graduate college: flows of heterogeneous students to alternative segments of the market can create a misleading picture if one focuses only on minorities who enroll within a narrow band of the quality spectrum under alternative admissions systems. Given the multiple forces at work determining graduation rates under different AA schemes, we also present the average effects across all students in Table 8. The net effect of moving to a proportional quota increases the graduation of minority students, while switching to a color-blind scheme has the opposite effect. In summary, admissions schemes that place minority students in better schools generally increase their graduation rates.

Since HC investment has only negligible influence on household income, conditional on graduation, the associated counterfactual impact of AA is mediated entirely by college assignment shifts. Table 9 depicts the effect of AA on minority household income by achievement quintiles. It shows that an AA ban would reduce household income across all five quintiles of the type distribution, whereas a proportional quota would increase it across all quintiles. These effects are weakest for the highest achievers, where the difference between a color-blind system and a quota is under $\$2,000/year$. The strengths of the effects increase until the fourth quintile, where the difference between the two peaks at $\$6,293/year$ of household income. These changes for nonminorities tend to have the opposite sign but are small and relatively inconsequential, so we

TABLE 10. Household Income (HHI) 10 Years After Graduation

	Average HHI of Minority Students	**... Relative to Status Quo**	**Average HHI of Nonminority Student**	**... Relative to Status Quo**
Status Quo	$85,044	—	$89,333	—
Color-Blind	$83,420	−$1,624	$89,642	$309
Proportional	$87,895	$2,852	$88,797	−$536

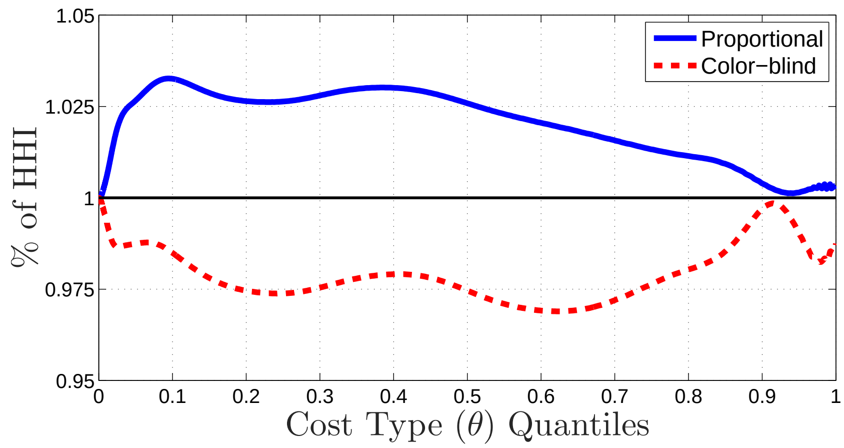
do not report them at the quintiles. Table 10 provides the average effect across the population for both groups. The effects on the minority students are on the order of a few thousand dollars, while the effect on nonminority students on average is relatively small. The effect of any change in AA policy averaged across the total population is on the order of $+/-$ \$50, meaning AA entails a very small loss of total income production.

In terms of the inequality in household income between the groups, a proportional quota results in the smallest gap and and a color-blind system causes the largest gap. In a crude sense, the former represents the most *ex post* fair outcome, while the later represents the most *ex ante* fair outcome. Of course, the *ex ante* perspective is complicated by potential sources of the differences in the type distributions of the two demographic groups. As we discussed above, the sources of the difference in type distributions could encompass a huge array of factors within the control of society at large, such as racial differences in wealth/income and primary/secondary school quality that make it harder for some families to make early-life investments in their children, *etc.* If one takes these factors into account, and if they drive the large *ex ante* cost asymmetry, then it becomes harder to argue that the color-blind outcome is fair in the *ex ante* sense.

5.4. **Effects of AA on Net Utility.** Section 5.3 broke out the effects of AA on household income and graduation probability, which are two of the primary components of student utility. In this section, we provide the most holistic look possible by examining the effect of AA on students' net utility; that is, expected payoffs from investment, minus monetized utility costs. We present our results in terms of *equivalent variation*, which we define as a proportional increase in status quo household income that would make the student just as well off under a shift toward another counterfactual admissions system. For example, from Figure 8, if we multiply the median minority student's baseline, status quo, annual household income by a factor of roughly 1.027 then his utility would rise to the same level as in equilibrium under a proportional quota regime. Values above 1 indicate utility increases, and the effect will be different for different types of students.

Figure 8 shows that minority equivalent variation for a proportional quota ranges from 0% to 3% of annual household income. The largest benefits are reaped by the lowest half of the type distribution (*i.e.*, the highest achieving half), while the benefits taper off when moving from the median cost student towards the highest-cost students. The equivalent variation for a color-blind system is around −2.5% for the middle 70% of the learning cost distribution, and the effect tapers to nothing for the very highest- and very lowest-cost students. Once again, the equivalent

FIGURE 8. Minority Equivalent Variations



variation for nonminorities is much smaller, generally one fifth to one third as large, so we leave the analogous plot for nonminorities to an online appendix.

5.5. **Implementing AA Through Socioeconomic Quotas.** In the wake of U.S. Supreme Court rulings that restrict forms that AA schemes can take, there have been attempts to devise alternative methods of encouraging minority enrollment in top schools. One alternative that has attracted attention is AA that reserves seats for economically deprived applicants. This approach will be effective if minority status and economic hardship are sufficiently correlated and equally predictive of learning costs. In this section we test whether reserving seats for underprivileged applicants can can serve as an effective tool for achieving more racially diverse college campuses.

The B&B dataset includes information on the *expected family contribution* (EFC), a measure based on the wealth and income of each student's pre-college household that is used by the US Department of Education for means-tested education aid. Students with a higher EFC grew up in more affluent homes, and therefore qualify for less aid since their families can be expected to contribute more resources to the cost of college. We adopt EFC as our measure of a student's economic status, and we experiment with a new set of AA proportional quotas in favor of the $X\%$ poorest students, for various levels of $X \in [10, 50]$. We will refer to these as "$X\%$ EFC quotas". The outcome measure we focus on in this section is the fraction of seats in each quintile of the college quality distribution that are occupied by (racial) minority students.

Table 11 provides the results of our analysis for three such EFC quotas. Somewhat surprisingly, socioeconomic quotas generally have almost no effect on the enrollment of minority students: regardless of the EFC cutoff, the outcome is very close to the admissions profile resulting from a purely color-blind admission rule. There are various reasons why. First, within the set of college enrollees, socioeconomic class is a poor proxy for the student's cost type θ, relative to racial demographics. Figure 9 plots the (weighted and selection-corrected) type distributions of the rich and poor students in both demographic groups under a 10% EFC quota. Affluence tends to predict lower cost types within each group, though its predictive power is weaker than race,

TABLE 11. Effect of EFC-based AA on Minority Enrollment by School Quality Quintile

Percent Reserved	**10%**	**25.5%**	**50%**
EFC Threshold	\$1,000	\$2,552	\$5,655
Fraction Minority	0.295	0.252	0.213
Top Quintile	0.100	0.099	0.103
Second Quintile	0.132	0.137	0.137
Third Quntile	0.158	0.160	0.156
Fourth Quintile	0.190	0.192	0.196
Bottom Quintile	0.421	0.413	0.407

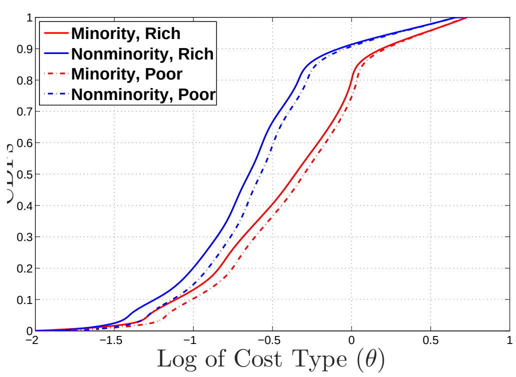


FIGURE 9.

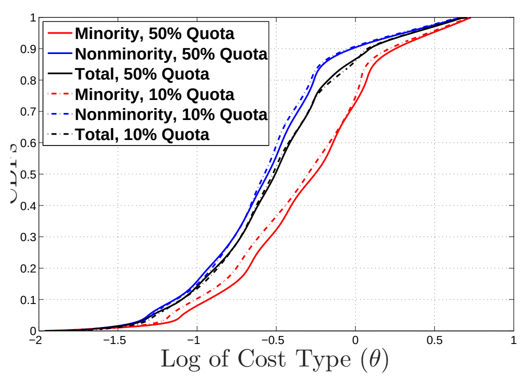


FIGURE 10.

as the figure shows. The cost distribution for affluent minorities is much more similar to that of poor minorities than it is to the cost distribution for affluent nonminority students.

Recent education research may provide an explanation for why this is so. Reardon, Townsend, and Fox [46] found that substantial racial segregation in housing persists to the present, and that Black and Hispanic primary/secondary students tend to live in substantially poorer neighborhoods, relative to their White and Asian counterparts of similar incomes. Reardon [44], and Reardon, Kalogrides, and Shores [45] also found that racial and socioeconomic segregation in schooling are among the strongest predictors of local racial achievement gaps in the data. Our results and theirs suggest that race in America is not yet reducible to socioeconomic differences alone. These results still leave open the possibility that other factors that are starkly stratified by race, like income (see Kena, *et. al.* [38]), healthcare (see Smith and Medalia [49]), and parental education (see Fox, Kewal, and Ramani [29]) play a role on the extensive margin of college attendance (outside our model), with primary/secondary school quality playing a predominant role in the intensive margin of college attendance (the main focus of our model).

To understand the theory behind why EFC quotas would have a limited effect on the competition for seats (relative to a racial color-blind scheme) in our application, Figure 10 plots the type distributions for economically disadvantaged minority and nonminority students under a 10%

EFC quota and a 50% EFC quota. It demonstrates that the patterns in Figure 9 are quite robust to substantially different cutoffs between rich and poor. The stability of the group-specific type distributions across EFC quotas and the relatively small changes in the fraction of poor students that are minorities means that the distributions of types of rich and poor students are almost identical across EFC quota systems. In absence of cost asymmetry, a proportional quota merely imposes *ex ante* allocations that would be achieved by a color-blind equilibrium *ex post*. Hence, our X% EFC quotas produce similar results as a color-blind contest where no attempt is made to racially diversify college campuses. We conclude from our analysis that implementing an AA program for minority students using economic status as a proxy for race is ineffective from the perspective of a policy-maker wishing to promote racial diversity on college campuses.[16]

5.6. **The Relative Force of the Competitive and Productive Channels of Investment.** Before concluding we briefly discuss the underlying economic forces within our model that govern investment under any admissions scheme. In a world of complete information, the type of each student would be common knowledge, students would be assortatively assigned to colleges by their academic types *before* investment occurs, and students would choose the level of HC that is optimal given their college assignment. These "first-best" investments would then reflect only the direct benefits of HC that we have referred to as the *productive channel* of incentives. In the incomplete information world that students actually inhabit, society must rely on competitive investment to stratify students by unobserved characteristics, so that college assignment may still be assortative. Colleges infer rankings of a student's type based on his or her HC output level, which in turn pushes students to accrue more HC than he or she would in a complete information world. The indirect incentive to accrue HC solely to obtain a seat at a better school reflects the *competitive channel* of incentives. Our goal in this section is to assess the relative strength of these two incentive channels in driving students' observed behaviors.

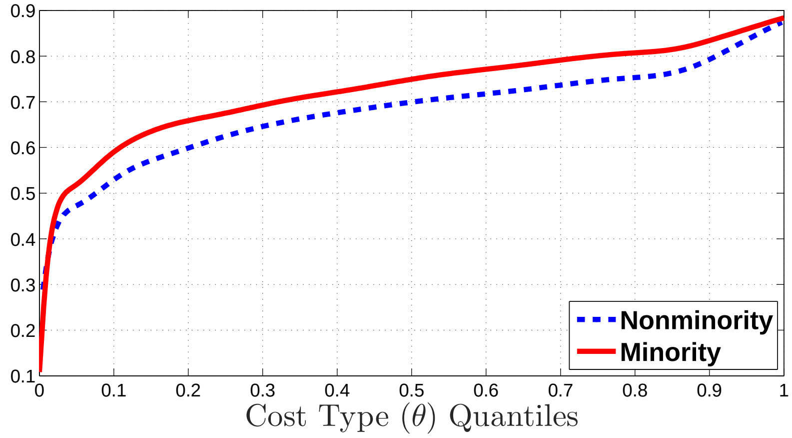
To formalize this idea, we decompose the marginal benefit of HC investments into two components. Equation 14, a student's first-order condition under incomplete information, places the two marginal benefits of HC on the left-hand side, with the marginal costs on the right:

$$\underbrace{\mathrm{E}_\varepsilon\left[U_s\left(P_j(s+\varepsilon),s,\theta;\boldsymbol{\alpha}\right)\right]}_{\text{Productive Channel}}+\underbrace{\mathrm{E}_\varepsilon\left[U_p\left(P_j(s+\varepsilon),s,\theta;\boldsymbol{\alpha}\right)P_j'(s+\varepsilon)\right]}_{\text{Competitive Channel}}=\underbrace{\theta c'(s)}_{\text{Marginal Cost}} \tag{14}$$

The productive channel can be thought of as the "Beckerian" incentives that represent direct marginal benefits of holding an additional unit of productive HC, which is the only one present in a world of complete information. The competitive channel is the indirect, "Spencerian" incentive, to invest in order to reveal one's appropriate position ahead of higher-cost competitors in a separating equilibrium of a rank-order contest. To our knowledge, this is the first empirical framework capable of disentangling these two incentive channels.

[16]Another recent paper by Estevan, Gall, Legros and Newman [26] also found reduced-form evidence that a similar means-tested AA policy, the Texas Top 10% program, was ineffective at promoting minority enrollment in Texas flagship universities, but for different reasons: the policy as implemented in Texas was too gameable.

FIGURE 11. Relative Force of Competitive and Productive Channels



A priori it is not clear which channel is more important. HC does not play a significant role in household income conditional on graduating, but it does have a predominant role in determining whether or not a student graduates college to begin with. To get a sense for the relative magnitudes of these forces as a function of the student's cost type, Figure 11 displays the ratio of the competitive channel incentives to the total marginal benefit, which is the sum of both the competitive and productive channels. The horizontal axis describes the quantile rank of the respective cost type in the group-specific type distribution.

Whenever the line is above a benchmark of 0.5 the competitive channel is dominant, and whenever it is below, the productive channel is most important. As one can see, the competitive channel is stronger than the productive channel for all but the lowest-cost agents. By extension, most students' academic achievement levels would be significantly lower in a complete information world where the competitive channel is turned off. For roughly the top 4% of achievers within the college universe, the productive channel is the dominant one. Of course, this discussion does not necessarily have bearing on optimal policy—for example, if HC spillovers are important in the economy at large then the social planner may wish to use any means to maximize HC production—but it casts a new and interesting light on the motivations underlying pre-college academic achievement.

6. CONCLUSION

This paper has developed identification and estimation results for competitive human capital within a college assignment market. By using individual-level data from the B&B survey, rather than focusing only on elite private colleges, we can provide a market-wide analysis of how admissions rules impact incentives, and how changes to one's college placement impact the returns to a college education, conditional on individual characteristics. Our analysis adapts auction-theoretic empirical techniques that allow us to identify the unobserved student characteristics which influence pre-college investment and post-college outcomes. We find that while the quality of the college in which a student enrolls and the human capital the student accrued prior to enrollment affect the probability that the student graduates, the wage condition on graduation is determined almost entirely by college quality and the student's unobservable type.

AA is a prominent feature of the entire college market, and plays a significant role in investment, redistribution, and welfare. A strong AA regime such as a proportional quota results in minority students enrolling at better colleges, while a color-blind admissions rule results in minority students predominantly enrolling in the bottom two quintiles of the college quality distribution. Interestingly, the effect on human capital investment incentives is more ambiguous. A color-blind (quota) rule results in the best minority students increasing (reducing) their human capital investments, while higher-cost minority students reduce (increase) them. The effect of AA on graduation rates similarly varies by students' types and the colleges in which they enroll. Overall, however, stronger AA schemes increase the average graduation rate and wages conditional on graduation for minority students. We also found that means-tested proportional quotas serve as a poor substitute for race-based AA if the policy-maker's primary objective is to increase racial diversity in high-quality colleges.

Finally, we analyzed the strength of the incentive to accrue HC solely for its productive value relative to the incentive to choose higher levels of HC to compete for access to a better college. We find a surprisingly dominant role for the competitive channel of incentives, which is stronger for all but the best students. Moreover, there is a stark contrast in the strength of these two channels incentives for most students: the competitive channel is at least three times stronger than the productive channel for half of college-going students.

The original intention of AA schemes was that these programs could serve as a solution to widespread and longstanding institutionalized inequities in the treatment of the members of different demographic groups. From a legal and political economy perspective, AA programs were thought to be long-term, but in the end, temporary. Although stronger AA schemes have a positive effect on the average graduation probabilities and post-graduation household income effects, the effect on HC accumulation is ambiguous. This makes it difficult to even form educated guesses about the long-term effects of these programs from our study.

There remain many unanswered questions that are the subject of ongoing research. For example, are the income effects of college quality, HC accumulation, and learning costs different for students in STEM fields relative to those in the humanities? Is it possible to design a better AA scheme than the prototypical examples we study? However, the biggest and most obvious question is whether one could use our model to say anything about the long-run impacts of different college admissions systems on the evolution of distributional inequalities over time. Our analysis, which was static by design, can only be the first step in such a research agenda. Providing a serious answer to this question will require considering the inter-generational effects of these programs. We leave these questions for future work which will build on the insights gained here.

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