# Correction to Ambiguous Beliefs and Mechanism Design

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**Abstract**

In this note I provide corrections to Theorems 1 and 2 of Bodoh-Creed [1], results that provide necessary and sufficient conditions for the implementability of allocation mechanisms in economies where agents are ambiguity averse. Neither issue affects the analysis of the auction or bargaining problems contained in Bodoh-Creed [1], but the revised results may be useful in the analysis of other mechanisms.

## 1 Introductory Comments

Bodoh-Creed [1] studies mechanism design problems when the participants have preferences of the minimum expected utility (MEU) form axiomatized by Gilboa and Schmeidler [2]. When the mechanisms are written in revelation form, the agent's have the following preferences over a type declaration, $\widehat{\theta}$, given the agent's true type, $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

$$\min_{\pi \in \Delta(\theta)} E^{\pi}_{\theta_{-i}}[\theta_i x_i(\widehat{\theta}_i, \theta_{-i}) - p_i(\widehat{\theta}_i, \theta_{-i})]$$

$x_i(\widehat{\theta}_i, \theta_{-i}) \in [0,1]$ represents the probability the agent is allocated the good, $p_i(\widehat{\theta}_i, \theta_{-i})$ is an expected transfer, and $\Delta(\theta)$ is a set of "plausible" beliefs parameterized by the agent's type. We refer to $\theta_i x_i(\widehat{\theta}_i, \theta_{-i}) - p_i(\widehat{\theta}_i, \theta_{-i})$ as the agent's ex post utility. The problem facing an agent is to choose the declaration that maximizes his utility. The mechanism designer chooses $(x, p)$ so that truthful declaration is optimal

$$\theta_i \in \arg\max_{\widehat{\theta}_i \in \Theta_i} \left\{ \min_{\pi \in \Delta} E^{\pi}_{\theta_{-i}}[\theta_i x_i(\widehat{\theta}_i, \theta_{-i}) - p_i(\widehat{\theta}_i, \theta_{-i})] \right\} \tag{1.1}$$

Equation 1.1 can be interpreted as a game between the agent choosing $\widehat{\theta}_i$ and Nature responding by "choosing" the measure that minimizes the expected utility for the agent from

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that action. I refer to the measure that minimizes the expected utility as the *minimal measure.*

Theorems 1 and 2 of Bodoh-Creed [1] provide necessary and sufficient conditions for truthfulness that completely characterize implementable mechanisms in this setting. The purpose of this note is to correct errors in the proofs of these theorems. Theorem 1 implicitly assumed saddle point properties apply to the agent's problem that may in fact not hold, so we provide a proof that requires an additional technical assumption regarding continuity of a derivative in lieu of saddle point properties. Theorem 2 requires a different (and arguably more natural) monotonicity assumption than the one used in Bodoh-Creed [1]. Interestingly, the new assumption is neither stronger nor weaker than the previous assumption.

Finally, we emphasize that neither of these corrections alter the analysis of the applications studied in Bodoh-Creed [1]. Hopefully the new results will prove useful in future work. For notational ease, throughout this work I will use the utility function

$$v(x, \theta) = \theta x$$

We highlight the assumptions required on more general forms of $v(x, \theta)$ in footnotes.

## 2 Saddle Points

An astute reader[1] raised the question of whether the agent's problem (P2) is a saddle point.

$$U(\theta_i) = \max_{\widehat{\theta}_i \in \Theta_i} \min_{\pi \in \Delta} E^{\pi}_{\theta_{-i}}[\theta_i x_i(\widehat{\theta}_i, \theta_{-i}) - p_i(\widehat{\theta}_i, \theta_{-i})] \tag{P2}$$

In Bodoh-Creed [1] I extend this problem by allowing the agent to randomize his declaration

$$\max_{\sigma \in \Sigma} \min_{\pi \in \Delta(\theta_i)} \int E^{\pi}_{\theta_{-i}}[\theta_i x_i(\widehat{\theta}_i, \theta_{-i}) - p_i(\widehat{\theta}_i, \theta_{-i})] * \sigma(d\widehat{\theta}_i) \tag{*}$$

where $\Sigma$ is the set of Borel measures over $\Theta$. Problem (*) is linear in both player's actions, which implies the problem faced by each agent is quasiconcave. Given this it is easy to show that problem (*) is a saddle point problem, and the application of Theorem 3 Milgrom and Segal [3] yields (ICFOC) as a *necessary* condition for truthfulness to be incentive compatible in (*). This is the basis for theorem 1 of Bodoh-Creed [1], which is restated below with some purely technical assumptions omitted.

**Theorem 1.** *If (P2) is truthful, then the utility function, $U(\theta_i)$, is differentiable almost*

[1] I would like to thank Alex Wolitzky for pointing out this potential problem.

*everywhere with a derivative equal to*

$$\frac{\partial}{\partial \theta_i} U(\theta_i) = U'(\theta_i) = E_{\theta_{-i}}^{\pi(\theta_i)} x_i(\theta_i, \theta_{-i})$$

The differential equation can be restated in integral form as

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} E_{\theta_{-i}}^{\pi(s)}[x_i(s, \theta_{-i})]ds \tag{ICFOC}$$

The theorem, as stated, is incorrect since it implicitly assumes that (P2) is a saddle point problem, which may not be true. The theorem would be correct if we substitute problem (*) for problem (P2) in the statement of the claim.

If we are designing a mechanism and realized (P2) is not a saddle point problem, then one might worry that we can achieve lower interim payoffs in (P2) than those described by the (ICFOC) and still preserve incentive compatibility. In other words, we could be "leaving money on the table." In the context of Bodoh-Creed [1], this would imply that the optimal auction as given is a lower bound .

In most applications of ambiguity aversion currently in the literature we can insure that (P2) is a saddle point. Theorem 11 of Bodoh-Creed [1] provides such a condition. Informally stated, Theorem 11 requires that (1) the ex post utility is decreasing in $\theta_{-i}$ and (2) there is a distribution in $\Delta$ that first order stochastically dominates all other distributions in $\Delta$.

We nor argue that under a weakening of a continuity assumption made in Bodoh-Creed [1] (assumption A8 specifically) we can show the (ICFOC) holds in (P2) if truthful behavior is optimal. First we need to define the following notation to describe the minimal measure when an agent deviates from truthful behavior

$$\pi\left(\theta, \widehat{\theta}\right) \in \arg\min_{\pi \in \Delta(\theta_i)} E_{\theta_{-i}}^{\pi}\left[\theta x_i\left(\widehat{\theta}, \theta_{-i}\right) - p_i\left(\widehat{\theta}, \theta_{-i}\right)\right]$$

To prove that (ICFOC) is necessary at any truthful equilibrium of (P2), we require the following continuity assumption. This assumption is subtly different than assumption (A8) in Bodoh-Creed [1] since it $\pi(\widetilde{\theta}, \theta)$ may be discontinuous in $\widetilde{\theta}$, which might entail discontinuities in $E_{\theta_{-i}}^{\pi(\widetilde{\theta},\theta)} x_i\left(\theta, \theta_{-i}\right)$ even in cases where assumption (A8) holds.

**Assumption.** *(AC)* $E_{\theta_{-i}}^{\pi(\widetilde{\theta},\theta)} x_i\left(\theta, \theta_{-i}\right)$ *is continuous in* $\widetilde{\theta}$ *in some open set around* $\theta = \widetilde{\theta}$ *for almost all* $\theta$.[2]

[2] For more general $v(x, \theta)$ we require that $E^{\pi(\widetilde{\theta},\theta)} v_\theta(x, \theta)$ be continuous in $\widetilde{\theta}$ in some open set around $\theta = \widetilde{\theta}$ for almost all $\theta$, which is a weakening of (A8) in Bodoh-Creed [1].

Although the continuity assumption is stated in terms of the endogenous variable $\pi\left(\theta,\widehat{\theta}\right)$, the assumption holds if $x_i\left(\theta,\theta_{-i}\right)$ is monotone in $\theta_{-i}$, $\Delta(\theta)$ is strictly convex (so $\pi\left(\theta,\widehat{\theta}\right)$ is continuous in $\theta$), and all $\pi\in\Delta(\theta)$ are nonatomic (so that small perturbations $\pi\left(\widetilde{\theta},\theta\right)$ from $\pi(\theta_i)$ in the weak-* topology do not upset the expectation of $x_i\left(\theta,\theta_{-i}\right)$).

We also require assumption (A7) from Bodoh-Creed [1], which we restate for completeness.

**Assumption.** *(A7) We can represent each agent's set of priors by $\Delta(\theta_i)=\{\pi: g(\pi,\theta_i)\geq 0\}$ for some $g:\Sigma\times\Theta\to\mathbb{R}$. We assume $g(\pi,\theta_i)$ is equidifferentiable in $\theta_i$, bounded and quasiconcave in $\pi$. We also assume $g_\theta(\pi,\circ)$ is continuous and integrable.*

Assumption (A7) puts enough structure on $\Delta(\theta)$ to allow us to write the (ICFOC) in cases where $\Delta(\theta)$ is nonconstant.

**Theorem 1'.** *Assume the mechanism $(x,p)$ is truthful and (AC) and (A7) hold.*[3] *Moreover, assume $g_\theta(\circ,\theta)$ is continuous for all $\theta$. $U(\theta)$ is absolutely continuous, and therefore almost everywhere differentiable, and*

$$U(\theta)=U(\underline{\theta})+\int_{\underline{\theta}}^{\theta}\left(E_{\theta_{-i}}^{\pi(s)}[x_i(s,\theta_{-i})]+\lambda(s)*g_\theta(\pi(s),s)\right)ds \qquad \text{(ICFOC)}$$

*for some multiplier $\lambda:\Theta_i\to\mathbb{R}_+$*

*Proof.* A double application of theorem 2 of Milgrom and Segal [3] proves that $U(\theta)$ is absolutely continuous (and so differentiable for almost all $\theta$). Our challenge is to provide upper and lower bounds for $U'(\theta)$ and use a sandwich theorem technique for finding $U'(\theta)$.

We repeatedly make use of the fact that we can write Nature's problem as

$$\begin{aligned}\pi\left(\theta,\widehat{\theta}\right) &\in \underset{\pi\in\Delta(\theta_i)}{\arg\min}\; E_{\theta_{-i}}^{\pi}\left[\theta x_i\left(\widehat{\theta},\theta_{-i}\right)-p_i\left(\widehat{\theta},\theta_{-i}\right)\right]\\ &= \underset{\pi\in\Sigma}{\arg\min}\; E_{\theta_{-i}}^{\pi}\left[\theta x_i\left(\widehat{\theta},\theta_{-i}\right)-p_i\left(\widehat{\theta},\theta_{-i}\right)\right]+\lambda\left(\theta,\widehat{\theta}\right)g(\pi,\theta)\end{aligned}$$

First we start with an upper bound. We denote the derivative of $U(\theta)$ from below as

[3] For more general $v(x,\theta)$ we require that $v$ be equidifferential in $\theta$ and that $v_\theta$ be continuous in $\theta$, which are encapsulated in assumption (A2) of Bodoh-Creed [1].

$U'(\theta-)$. Consider a sequence $\theta_k \to \theta$ where $\theta_k < \theta$

$$
\begin{aligned}
\frac{U(\theta) - U(\theta_k)}{\theta - \theta_k} &= \frac{E_{\theta_{-i}}^{\pi(\theta)}\left[\theta x_i\left(\theta, \theta_{-i}\right) - p_i\left(\theta, \theta_{-i}\right) + \lambda(\theta) g\left(\pi\left(\theta\right), \theta\right)\right]}{\theta - \theta_k} \\
&\quad - \frac{E_{\theta_{-i}}^{\pi(\theta_k)}\left[\theta_k x_i\left(\theta_k, \theta_{-i}\right) - p_i\left(\theta_k, \theta_{-i}\right) + \lambda(\theta_k) g\left(\pi\left(\theta_k\right), \theta_k\right)\right]}{\theta - \theta_k} \\
&\leq \frac{E_{\theta_{-i}}^{\pi(\theta)}\left[\theta x_i\left(\theta, \theta_{-i}\right) - p_i\left(\theta, \theta_{-i}\right) + \lambda(\theta) g\left(\pi\left(\theta\right), \theta\right)\right]}{\theta - \theta_k} \\
&\quad - \frac{E_{\theta_{-i}}^{\pi(\theta_k, \theta)}\left[\theta_k x_i\left(\theta, \theta_{-i}\right) - p_i\left(\theta, \theta_{-i}\right) + \lambda(\theta_k, \theta) g\left(\pi(\theta_k, \theta), \theta_k\right)\right]}{\theta - \theta_k} \\
&\leq \frac{E_{\theta_{-i}}^{\pi(\theta_k, \theta)}\left[\theta x_i\left(\theta, \theta_{-i}\right) - p_i\left(\theta, \theta_{-i}\right) + \lambda(\theta) g\left(\pi(\theta_k, \theta), \theta\right)\right]}{\theta - \theta_k} \\
&\quad - \frac{E_{\theta_{-i}}^{\pi(\theta_k, \theta)}\left[\theta_k x_i\left(\theta, \theta_{-i}\right) - p_i\left(\theta, \theta_{-i}\right) + \lambda(\theta_k, \theta) g\left(\pi(\theta_k, \theta), \theta_k\right)\right]}{\theta - \theta_k}
\end{aligned}
$$

The first inequality follows from the assumption of truthfulness (so type $\theta_k$ deviating to declaring $\theta$ reduces the agent's utility), and the second inequality comes from the fact that $\pi(\theta_k)$ minimizes $E_{\theta_{-i}}^{\pi}\left[\theta_k x_i\left(\theta_k, \theta_{-i}\right) - p_i\left(\theta_k, \theta_{-i}\right)\right]$. Note that $g\left(\pi(\theta_k, \theta), \theta_k\right) = 0$ since $\pi(\theta_k, \theta)$ is on the exterior or $\Delta(\theta_k)$. Collecting terms we have

$$
\frac{U(\theta) - U(\theta_k)}{\theta - \theta_k} \leq E_{\theta_{-i}}^{\pi(\theta_k, \theta)}\left[x_i\left(\theta, \theta_{-i}\right)\right] + \lambda(\theta) \frac{g\left(\pi(\theta_k, \theta), \theta\right) - g\left(\pi(\theta_k, \theta), \theta_k\right)}{\theta - \theta_k} \tag{2.1}
$$

This implies that when $U'(\theta-)$ exists that it must satisfy.

$$
U'(\theta-) \leq \underset{\theta_k \to \theta^-}{Lim} \left[E_{\theta_{-i}}^{\pi(\theta_k, \theta)}\left[x_i\left(\theta, \theta_{-i}\right)\right] + \frac{g\left(\pi(\theta_k, \theta), \theta\right) - g\left(\pi(\theta_k, \theta), \theta_k\right)}{\theta - \theta_k}\right]
$$

From assumption (AC) we have for almost all $\theta$

$$
\underset{\theta_k \to \theta^-}{Lim} E_{\theta_{-i}}^{\pi(\theta_k, \theta)}\left[x_i\left(\theta, \theta_{-i}\right)\right] = E_{\theta_{-i}}^{\pi(\theta)}\left[x_i\left(\theta, \theta_{-i}\right)\right]
$$

From the equidifferentiability of $g$ with respect to $\theta$ and the continuity of $g_\theta$ we have

$$
\underset{\theta_k \to \theta^-}{Lim} \frac{g\left(\pi(\theta_k, \theta), \theta_k\right) - g\left(\pi(\theta_k, \theta), \theta\right)}{\theta_k - \theta} = g_\theta\left(\pi(\theta), \theta\right)
$$

Therefore for almost all $\theta$

$$
U'(\theta-) \leq E_{\theta_{-i}}^{\pi(\theta)}\left[x_i\left(\theta, \theta_{-i}\right)\right] + \lambda(\theta) g_\theta\left(\pi(\theta), \theta\right) \tag{2.2}
$$

If we consider a sequence $\theta_k \to \theta$ where $\theta_k > \theta$, similar arguments yield that the derivative from above, $U'(\theta+)$ must satisfy

$$U'(\theta+) \geq E_{\theta_{-i}}^{\pi(\theta)} x_i\left(\theta, \theta_{-i}\right) + \lambda(\theta) g_\theta\left(\pi(\theta), \theta\right) \tag{2.3}$$

When $U(\theta)$ is differentiable we have $U'(\theta+) = U'(\theta) = U'(\theta-)$, so combining this with equations 2.2 and 2.3 yields

$$E_{\theta_{-i}}^{\pi(\theta)} x_i\left(\theta, \theta_{-i}\right) + \lambda(\theta) g_\theta\left(\pi(\theta), \theta\right) \geq U'(\theta) \geq E_{\theta_{-i}}^{\pi(\theta)} x_i\left(\theta, \theta_{-i}\right) + \lambda(\theta) g_\theta\left(\pi(\theta), \theta\right)$$

Therefore we have that almost everywhere that $U(\theta)$ is differentiable, and the derivative must equal

$$U'(\theta) = E_{\theta_{-i}}^{\pi(\theta_i)}[x_i(\theta_i, \theta_{-i})] + \lambda(\theta) g_\theta\left(\pi(\theta), \theta\right)$$

From the absolute continuity of $U$ we have

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \left( E_{\theta_{-i}}^{\pi(s)}[x_i(s, \theta_{-i})] + \lambda(s) g_\theta\left(\pi(s), s\right) \right) ds$$

□

$E_{\theta_{-i}}^{\pi(\theta_k, \theta)}\left[x_i\left(\theta, \theta_{-i}\right)\right]$ may not be continuous in the limit as $\theta_k \to \theta$ (i.e., assumption AC may fail) due to failures of the continuity[4] of $\pi(\theta_k, \theta)$ as $\theta_k \to \theta$ even when $x(\theta_i, \theta_{-i})$ is continuous in $\theta_{-i}$. In particular, this may be the case if the set of maximizers of Nature's problem is merely upper hemicontinuous. In cases like this we can bound the directional derivatives of $U(\theta)$. Note that we do not prove the existence of these derivatives.

For the following result, we need to define a topology on the space of measures $\Delta$. For typical applications the weak-* topology suffices. For example, if $x_i\left(\widehat{\theta}, \theta_{-i}\right)$ and $p_i\left(\widehat{\theta}, \theta_{-i}\right)$ are monotone in $\theta_{-i}$ and all $\pi \in \Delta(\theta)$ are nonatomic then the weak-* topology is appropriate since $E_{\theta_{-i}}^{\pi}\left[\theta x_i\left(\widehat{\theta}, \theta_{-i}\right) - p_i\left(\widehat{\theta}, \theta_{-i}\right)\right]$ will be continuous in $\pi$. In the statement of the theorem, we use the notation

$$\Pi(\theta, \widehat{\theta}) = \underset{\pi \in \Delta(\theta)}{\arg\min} E_{\theta_{-i}}^{\pi}\left[\theta x_i\left(\widehat{\theta}, \theta_{-i}\right) - p_i\left(\widehat{\theta}, \theta_{-i}\right)\right]$$

to refer to Nature's choice of $\pi$ given the agent's true type $\theta$ and declared type $\widehat{\theta}$. When $\theta = \widehat{\theta}$ we simply write $\Pi(\theta)$. Our corollary places limits on how quickly $U$ can change with $\theta$, which indirectly bounds the directional derivatives (if the directional derivatives exist).

[4] We use continuity in the sense of the weak-* topology.

**Corollary 1.** *Assume the mechanism $(x,p)$ is truthful, $\Delta(\theta)=\Delta$, and assumption (A8) of Bodoh-Creed [1] holds. Then*[5]

$$\begin{aligned} Lim\sup_{\theta_k\to\theta}\frac{U(\theta_k)-U(\theta)}{\theta_k-\theta} &\leq \max_{\pi\in\Pi(\theta)} E^{\pi}_{\theta_{-i}}\left[x_i\left(\theta,\theta_{-i}\right)\right] \\ Li\min_{\theta_k\to\theta} f\frac{U(\theta_k)-U(\theta)}{\theta_k-\theta} &\geq \min_{\pi\in\Pi(\theta)} E^{\pi}_{\theta_{-i}}\left[x_i\left(\theta,\theta_{-i}\right)\right] \end{aligned}$$

*If $U$ is directionality differentiable, these bounds (obviously) apply to $U'(\theta-)$ and $U'(\theta+)$.*

*Proof.* Since the objective function is continuous in $\pi,\theta$ and $\widehat{\theta}$ and $\Delta(\theta)$ is compact and continuous, Berge's theorem of the maximum implies $\Pi(\theta,\widehat{\theta})$ is upper hemicontinuous in both arguments. From equation 2.1 we have

$$\frac{U(\theta)-U(\theta_k)}{\theta-\theta_k}\leq E^{\pi(\theta_k,\theta)}_{\theta_{-i}}\left[x_i\left(\theta,\theta_{-i}\right)\right]+\lambda(\theta)\frac{g\left(\pi(\theta_k,\theta),\theta\right)-g\left(\pi(\theta_k,\theta),\theta_k\right)}{\theta-\theta_k}$$

This inequality combined with the upper hemicontinuity of $\Pi(\theta,\widehat{\theta})$ and the equidifferentiability of $g$ implies our first claim

$$Lim\sup_{\theta_k\to\theta}\frac{U(\theta_k)-U(\theta)}{\theta_k-\theta}\leq\max_{\pi\in\Pi(\theta)} E^{\pi}_{\theta_{-i}}\left[x_i\left(\theta,\theta_{-i}\right)\right]+\lambda(\theta)g_{\theta}(\pi(\theta),\theta)$$

Since $\Delta(\theta)=\Delta$, we have $g_{\theta}(\pi,\theta)=0$ and our first claim is proven.

[5]The bound on the lim sup could be extended to include a term for the term associated with changes in $\Delta(\theta)$ easily since the mutliplier is fixed when taking limits of the upper bound. Unfortunately, the lower bound involves a limit of a multiplier, and I have not discovered an easy way of formulating a closed form lower bound on the limit of this multiplier. At the expense of a great deal of notational complexity we could state a bound, but it is unclear whether it would be of any technical use or yield any mathematical insight.

Using symmetric algebra we find

$$\begin{aligned}
\frac{U(\theta)-U(\theta_k)}{\theta-\theta_k} &= \frac{E^{\pi(\theta)}_{\theta_{-i}}\left[\theta x_i\left(\theta,\theta_{-i}\right)-p_i\left(\theta,\theta_{-i}\right)+\lambda(\theta)g\left(\pi\left(\theta\right),\theta\right)\right]}{\theta-\theta_k} \\
&\quad -\frac{E^{\pi(\theta_k)}_{\theta_{-i}}\left[\theta_k x_i\left(\theta_k,\theta_{-i}\right)-p_i\left(\theta_k,\theta_{-i}\right)+\lambda(\theta_k)g\left(\pi\left(\theta_k\right),\theta_k\right)\right]}{\theta-\theta_k} \\
&\geq \frac{E^{\pi(\theta,\theta_k)}_{\theta_{-i}}\left[\theta x_i\left(\theta_k,\theta_{-i}\right)-p_i\left(\theta_k,\theta_{-i}\right)+\lambda(\theta,\theta_k)g\left(\pi\left(\theta,\theta_k\right),\theta\right)\right]}{\theta-\theta_k} \\
&\quad -\frac{E^{\pi(\theta_k)}_{\theta_{-i}}\left[\theta_k x_i\left(\theta_k,\theta_{-i}\right)-p_i\left(\theta_k,\theta_{-i}\right)+\lambda(\theta_k)g\left(\pi\left(\theta_k\right),\theta_k\right)\right]}{\theta-\theta_k} \\
&\geq \frac{E^{\pi(\theta,\theta_k)}_{\theta_{-i}}\left[\theta x_i\left(\theta_k,\theta_{-i}\right)-p_i\left(\theta_k,\theta_{-i}\right)+\lambda(\theta,\theta_k)g\left(\pi\left(\theta,\theta_k\right),\theta\right)\right]}{\theta-\theta_k} \\
&\quad -\frac{E^{\pi(\theta,\theta_k)}_{\theta_{-i}}\left[\theta_k x_i\left(\theta_k,\theta_{-i}\right)-p_i\left(\theta_k,\theta_{-i}\right)+\lambda(\theta_k)g\left(\pi\left(\theta_k\right),\theta_k\right)\right]}{\theta-\theta_k}
\end{aligned}$$

Collecting terms we have

$$\frac{U(\theta)-U(\theta_k)}{\theta-\theta_k} \geq E^{\pi(\theta,\theta_k)}_{\theta_{-i}}\left[x_i\left(\theta_k,\theta_{-i}\right)\right]+\lambda(\theta,\theta_k)\frac{g\left(\pi\left(\theta,\theta_k\right),\theta\right)-g\left(\pi\left(\theta_k\right),\theta_k\right)}{\theta-\theta_k}$$

Again since $\Delta(\theta)=\Delta$, we have $g_\theta(\pi,\theta)=0$ and our second claim is proven.[6] □

# 3 Correction to Theorem 2

In Bodoh-Creed [1] I assumed the following monotonicity restriction on the mechanisms.

**Assumption.** *(M)* $x(\widehat{\theta},\theta_{-i})$ *is monotone increasing in* $\widehat{\theta}$.

Leaving aside some purely technical assumptions, Theorem 2 of Bodoh-Creed [1] claims

**Theorem 2.** *Given (ICFOC) and (M) hold, the mechanism is truthful and globally incentive compatible.*

In this section we amend an error in the proof of Theorem 2 of Bodoh-Creed [1]. The corrected theorem requires the following monotonicity assumption.[7]

[6] It is only at this point that we really need $\Delta(\theta)=\Delta$.

[7] For general $v(x,\theta)$ we require that the following satisfy the single crossing property in $(\theta,\widehat{\theta})$

$$E^{\pi(\widehat{\theta})}\left[v_\theta\left(x(\widehat{\theta},\theta_{-i}),\theta\right)\right]$$

**Assumption.** *(M') $E^{\pi(\theta)}\left[x(\theta,\theta_{-i})\right]$ is monotone increasing in $\theta$.*

(M') implies that when agents behave truthfully, agents with higher types "believe" that they are more likely to be allocated to the good. Bayesian implementation in SEU settings typically requires that $E^{\pi}\left[x(\theta,\theta_{-i})\right]$ be monotone given prior beliefs $\pi$, which suggests M' as the natural generalization to the MEU preference case. As a practical matter I believe it would be odd if a monotone mechanism (such as the ones studied in Bodoh-Creed [1]) resulted in a higher value agent being more pessimistic about being allocated the object than a lower value agent. Finally, all of the applications and examples in Bodoh-Creed [1] satisfy (M').

**Theorem 2'.** *Assume $\Delta(\theta)=\Delta$. Given (ICFOC) and (M') hold, the mechanism is truthful and globally incentive compatible.*

*Proof.* First define the following value function for player $i$

$$V(\theta,\widehat{\theta})=\min_{\pi\in\Delta} E^{\pi}_{\theta_{-i}}\left[\theta x_i\left(\widehat{\theta},\theta_{-i}\right)-p_i\left(\widehat{\theta},\theta_{-i}\right)\right]$$

For global incentive compatibility to fail, it must be the case that

$$\max_{\widehat{\theta}}\max_{\theta} V(\theta,\widehat{\theta})-U(\theta)>0$$

We now prove that this cannot be case.

Consider some declaration $\widehat{\theta}$ and the associated continuum of incentive constraints

$$\max_{\theta} V(\theta,\widehat{\theta})-U(\theta)$$

Let us find the value of $\theta$ where this equation is maximized. Note that

$$V(\theta,\widehat{\theta})-U(\theta)=E^{\pi(\widehat{\theta})}_{\theta_{-i}}\left[\theta x_i\left(\widehat{\theta},\theta_{-i}\right)-p_i\left(\widehat{\theta},\theta_{-i}\right)\right]-U(\theta)$$

Since $U(\theta)$ is differentiable almost everywhere with respect to $\theta$, we can write

$$\frac{\partial}{\partial\theta}\left\{E^{\pi(\widehat{\theta})}_{\theta_{-i}}\left[\theta x_i\left(\widehat{\theta},\theta_{-i}\right)-p_i\left(\widehat{\theta},\theta_{-i}\right)\right]-U(\theta)\right\}=E^{\pi(\widehat{\theta})}_{\theta_{-i}}\left[x_i\left(\widehat{\theta},\theta_{-i}\right)\right]-E^{\pi(\theta)}_{\theta_{-i}}\left[x(\theta,\theta_{-i})\right]$$

Note that if $\widehat{\theta}>\theta$ we have $E^{\pi(\widehat{\theta})}_{\theta_{-i}}\left[x_i\left(\widehat{\theta},\theta_{-i}\right)\right]\geq E^{\pi(\theta)}_{\theta_{-i}}\left[x(\theta,\theta_{-i})\right]$, and if $\widehat{\theta}<\theta$ we have $E^{\pi(\widehat{\theta})}_{\theta_{-i}}\left[x_i\left(\widehat{\theta},\theta_{-i}\right)\right]\leq E^{\pi(\theta)}_{\theta_{-i}}\left[x(\theta,\theta_{-i})\right]$. This implies that the global optimum is $\widehat{\theta}=\theta$, so we have

$$\max_{\theta} V(\theta,\widehat{\theta})-U(\theta)=V(\widehat{\theta},\widehat{\theta})-U(\widehat{\theta})=0$$

Therefore $\max_{\widehat{\theta}} \max_{\theta} V(\theta, \widehat{\theta}) - U(\theta) \leq 0$ as required for global incentive compatibility. $\square$

# References


[1] Bodoh-Creed, A., 2012. Ambiguous Beliefs and Mechanism Design, Games and Economic Behavior, 75 (2), 518 - 537.

[2] Gilboa, I. and D. Schmeidler (1989) "Maxmin Expected Utility with Non-Unique Prior," *Journal of Mathematical Economics*, 18, pp. 141-153.

[3] Milgrom, P. and I. Segal, 2002. Envelope Theorems for Arbitrary Choice Sets, Econometrica, 70 (2), 583-601.