Mood, Memory, and the Evaluation of Asset Prices

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Abstract

We model the effect of imperfect memory on asset prices. Our model uses market affective state as a cue for information congruent with the affective state, a phenomenon called mood congruent memory. Our model also incorporates rehearsal, which implies information recalled in the recent past is more easily recalled in the present. When combined with a model of the dynamics of affect, our theory provides novel explanations for short-run continued overreaction to news and long-run correction of the short-run effects. In addition our model predicts asset price momentum, the unconditional autocorrelation of asset prices. We also predict that excess volatility is highest during market downturns, price biases are increasing in fundamental volatility, knowledge may exacerbate these biases, and asset prices exhibit excess comovement.[1]

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1 Introduction

Studies in behavioral finance have found a variety of regularities such as short-run asset price overreaction (and continued overreaction) to news and long-run correction of this overreaction that suggest that available information has significant predictive power for future price changes and returns even after controlling for current price. Asset prices changes also exhibit unconditional autocorrelation, which is referred to as price momentum.[2] Prices have also been shown to significantly respond to non-fundamental events such as weather and daylight length (Saunders [?], Hirshleifer and Shumway [?], Kamstra, Kramer and Levi [?]). Asset prices also tend to exhibit greater volatility than the volatility of the asset fundamentals would predict (Shiller [?] and [?] provide seminal discussions of this topic). Traditional asset pricing models have struggled to explain these phenomena, leading to a number of explanations founded on the psychology of biases in judgement.

We provide a model of long-term memory and mood that provides a unified explanation for all of the phenomena mentioned above. In addition our model provides a number of novel predictions that remain to be tested. First, our model suggests that excess volatility is greatest during market downturns when mood is depressed. Second our model implies that the magnitude of the bias is increasing in the underlying volatility of asset dividends. Third, our model predicts that investors with a great deal of knowledge (e.g., hedge fund managers, equity analysts) may be more biased by these effects than less knowledgeable investors. Fourth, our model implies that market prices for securities exhibit more comovement than predicted by the underlying fundamentals.

Long-term memory stores an enormous amount of information, most of which is irrelevant for any given decision. Cues present in our environment or state of mind prompt

[2] These findings are surveyed in Barberis, Shleifer and Vishny [?] and Daniel, Hirshleifer and Subrahmanyam [?].

the recollection of information salient for the decision at hand. While these cues provide an efficient process for focusing on important data, the cues can cause beliefs (and the resulting decisions) to be based on a systematically biased, incomplete set of information. Mood regulated memory is a psychological phenomenon wherein agent affective states (e.g., mood) serve as cues for information stored in long-term memory (Isen [?]). The valence of an affective state refers to the subjectively experienced goodness of the state. For example joy is a positively valanced affect, while anger and sadness are negatively valanced affective states. We focus on the valence of the affective state as a cue for information of the same valence.[3]

In addition to associative effects, our model includes rehearsal. Rehearsal is a phenomenon whereby information that was recalled in the recent past is more easily recalled in the present. The net effect in our model is that biases in beliefs are autocorrelated and decay with time, which forms the basis of our short-run continued overreaction and long-run correction predictions.

Introspection and everyday experience suggest that recall is imperfect and that cues can help recall associated data. However, we suspect that few individuals are aware of the magnitude of the effects of these cues or are fully cognizant of the cues present in their environment such as their own mood. Our model studies the impact of these effects in a financial market where agents must recall information from memory (e.g. information from industry insiders, interviews with corporate officers), and the mnemonic biases lead the agents to have biased beliefs about the economy that cause the mispricing of risky assets.

[3]Appraisal theories of emotion predict that the valence of the emotion elicited by and associated with a piece of information, situation, or a cognitive process is determined by the implications for the individual's well-being (Smith et al. [?]). This suggests that positive affective states of an agent are correlated with the satiation or expected satiation of an agent's desires and goals, which makes affective valence a natural point of connection between the psychology of emotion and financial theory.

The final component of our model is immune neglect, which is an important component of the dynamics of mood (Gilbert et al. [?]). Immune neglect refers to the fact that affect is less influenced by negative information than by positive information. In effect, our affective regulatory systems exhibit a kind of an "immune system" that helps individuals cope with emotional trauma. The neglect component, which we do not exploit in our model, refers to the fact that individuals tend to underestimate the power of this immune system and overestimate the length of negative moods.

Association and rehearsal represent the essence of long-term memory, which leads us to conclude that including these effects is required for any realistic model of long-term memory. Since our model focuses on the intersection of memory and affect, it is also important to leverage what the psychology literature can tell us about the dynamics of affect. Our assumptions that affect is influenced by news and that immune neglect predicts and asymmetry in the response to positive and negative news is at the core of the dynamics of affect. Again, any serious model of affective dynamics has to incorporate both of these effects to make realistic predictions.

We use a representative agent approach in our theory, and we interpret the biases of our representative agent as capturing the "average" mnemonic bias of the market participants. We first use our model to study short-run overreaction to news and long-run correction of the initial overreaction. Overreaction refers to positive autocorrelation of price changes over horizons of less than one year (Cutler, Poterba and Summers [?], Jegadeesh and Titman [?], Rouwenhorst [?]), which resembles repeated reaction to the same fundamental news in successive periods. Long-run correction denotes the negative correlation of asset price changes over time scales longer than a year (De Bondt and Thaler [?], Lakonishok, Shleifer and Vishny [?]), which suggests that asset prices react too much to news in the short-run. These patterns imply that current information is predictive of future price changes in a highly structured fashion.

Our model generates short-run continued overreaction through the autocorrelation of the asset price biases. Conditional on good news in the current period, the data recalled from memory through association are biased towards positive information about asset value and cause the price bias to increase. Although the affective state begins to return towards its ergodic distribution in the next period, if the rehearsal effect is sufficiently strong the asset price changes in future periods are expected to be positive. However, since agent affect has an ergodic distribution the recall biases decay with time, which generates the long-run correction effect.

Continued overreaction and the associated long-run correction reflect autocorrelation of price changes condition on news releases. Our model also predicts that asset price changes are autocorrelated without conditioning on the release of positive or negative news, which we term asset price momentum. Since most papers on asset price momentum and momentum strategies (e.g., Jegadeesh and Titman [?] and [?]) do not condition on whether the recents news regarding the asset has been positive or negative, the assesment of the unconditional autocorrelation is required to tie our predictions to the existing empirical literature.

We find that the dynamics of the mnemonic cues can cause excess volatility. When information about the fundamental value of the asset shift the affective state of the representative agent, changes in the representative agent's affect serve as a multiplier of the volatility effect of the information. Immune neglect (in the context of our model) suggests that affect is less volatile when it is positive - there is less room to increase affect, and negative information causes small decreases in affect. This asymmetry between the evolution of positive and negative affect leads our model to predict that excess volatility is higher during market downturns.

Another novel prediction of our model is that assets with greater fundamental volatility exhibit greater short-run continued overreaction, long-run correction, and excess volatility. Put simply, any asset with a high level of volatility realizes more extreme dividend innovations than a less volatile asset. Extreme positive (negative) dividend innovations dominate recall when the representative agent's affective state is positive (negative), which amplifies the biases of the high volatility asset's price relative to the price biases of a low volatility asset. This suggests that firms with unstable earnings (e.g., new firms, high growth firms) are more likely to exhibit the biases we predict. Kumar [?] and Zhang [?] use cash flow volatility and earnings volatility as a metric of valuation uncertainty under the assumption that this uncertainty exacerbates price biases. Our model provides a novel justification for these measures as metrics for an asset's susceptibility to price biases.

A related prediction of our model is that agents with more knowledge (e.g., equity analysts) may be more subject to mnemonic biases than less informed agents. Since knowledgeable agents are more likely to have extreme dividend growth information stored in memory, biases in the recall of that information can have more effect on these agents than on market participants with little prior knowledge. If the set of arbitrageurs that enforce market efficiency are also highly knowledgeable, then this effect may blunt the power of arbitrage to correct mispricings.

We also argue that mnemonic cues can induce comovement of asset prices in excess of the correlation of the fundamentals. Since affective cues have a general effect on cognitive processing, news or cues that change the representative agent's affect cause the bias in the price of all assets to shift correspondingly. In effect, the representative agent's affect becomes a source of aggregate, non-fundamental price shocks. Given the transience of the representative agent's affective state, these aggregate shocks can be identified by their temporary nature.

Relatedly when non-fundamental events serve as mnemonic cues, prices may respond to these events in a correlated fashion. For example, Saunders [?] and Hirshleifer and Shumway [?] provide evidence that market prices respond to meteorological events that convey (presumably) little information regarding future prices. Our model describes a clear link between the psychological foundations (mood congruent memory), beliefs about net present value, and market price dynamics.

1.1 Prior Finance and Economics Literature

Several prior works have provided behavioral models to explain some of the asset pricing puzzles that we study. Barberis, Shleifer and Vishny [?] explains short-run continued overreaction and long-run correction of asset prices to news with a model of investor sentiment.[4] Daniel, Hirshleifer and Subrahmanyam [?] develops a model of investor sentiment driven by overconfidence and positively biased self-attribution in order to explain these same anomalies. Hong and Stein [?] provides a model wherein traders are boundedly rational and limited to using simple forecasting models. Hong, Stein and Yu [?] provide a model of investors vacillating between different simple models and the resultant effect on asset prices. DeLong et al. [?] assumes that noise traders are subject to exogenous sentiment shocks and study how these shocks create risk that prevents arbitrageurs from eliminating mispricings and allow the noise traders to earn supernormal profits.

While there is overlap between the asset pricing predictions of these prior works and ours, four predictions distinguish our model. First, our model suggests that volatility will be highest during market downturns.[5] Second our model implies that the biases

[4]We have adapted the terminology of Barberis, Shleifer and Vishny [?] to the nomenclature used in this study. Several prior works refer to short-run continued overreaction and "underreaction" and long-run correction as "overreaction."

[5]Prior work (e.g. Officer [?], Schwert [?]) focuses on the time-series behavior of volatility (rather than excess volatility). These studies find that volatility is higher during market downturns and is only partially explained by factors such as leverage and macroeconomic volatility (Schwert [?]).

are increasing in an asset's fundamental volatility. Third, we predict that increases in agent knowledge may exacerbate the effects we find. In the case of models relying on misspecification (e.g., Hong et al. [?]), one might conjecture that the opposite occurs - knowledgeable, sophisticated investors are more likely to have the correct asset pricing model and serve as arbitrageurs. Fourth, our model suggests that price responses will be more correlated across assets than market fundamentals suggest.

Of particular interest for this paper is the branch of the behavioral finance literature that searches for correlations between asset prices and exogenous mood shifters such as weather, length of the day, and sporting event outcomes. In an early contribution, Saunders [?] finds that the cloud cover in Manhattan has a negative effect on the prices of stocks traded on the New York Stock Exchange. Hirshleifer and Shumway [?] extends this analysis to cover markets across the globe and finds the same negative correlation between cloud-cover and asset prices. Kamstra, Kramer and Levi [?] provides an analysis of seasonal effects around the world under the assumption that the depressed affect caused by short days during the winter in turn depresses stock prices. The Kamstra, Kramer and Levi [?] results are particularly striking since changing day length is both uninformative and predictable months in advance. Edmans, Garci and Norli [?] shows stock market valuations decline following a loss in important sports matches and that the magnitude of the loss is greater for smaller stocks and more important events. Yuan, Zheng and Zhu [?] and Keef and Khaled [?] argue that the lunar cycle influences asset prices through mood. Krivelyova and Robotti [?] finds evidence that solar events influence asset prices through the geomagnetic storm's effect on mood. Our model supplies foundations for these asset price anomalies in a Bayesian framework with the optimism caused by the biased recollection of information salient for estimating asset values.

While we interpret the effect of mood as a bias in beliefs, there is growing evidence that affect may influence preferences with a particular focus on the effect of mood on risk preferences. Kamstra et al. [?] models these effects under the assumption that seasonal mood swings cause predictable changes in risk aversion. Although most of the predictions are non-overlapping with ours, Kamstra et al. [?] predicts heightened volatility during seasons with low affective states.

Ljungqvist, Nanda and Singh [?] and Derrien [?] provide models wherein IPOs are timed to take advantage of the optimism of traders to explain why IPOs appear underpriced in the short-run and overpriced in the long-run. Our model provides microfoundations for the belief formation process that leads the initial beliefs of investors to be either bearish or bullish and the factors that cause these beliefs to change over time. Our predictions are supported by Purnanandam and Swaminathan [?] and Cook, Kieschnick and Van Ness [?].[6]

Tetlock [?] studies the influence of the Wall Street Journal's (WSJ) "Abreast of the Market" section on market affect. Included in this WSJ section are summaries of market events, explanations of the market behavior from third parties, and predictions about future market behavior. Tetlock conducts time-series analysis of the relationship between the number of negatively valenced words in the WSJ section and the performance of the Dow Jones Industrial Average (DJIA) as well as a variety of other financial market statistics. Tetlock [?] finds that negatively valenced words in the WSJ predict depressed performance of the DJIA in the following week. Tetlock [?] argues that the WSJ column does not solely have an informational effect since the price changes he studies partially revert with time, which contradicts the random walk pattern predicted by a purely informational model.

[6]We do not claim that these effects would not be realized if other models of asset price overreaction and correction were applied in this context. To isolate our model we would need to understand whether the IPO promotion efforts involved heightening positive affect or cueing positive information from memory (as our story suggests) or if the provision of new information played a crucial role in pre-IPO promotion (which might imply other models are salient).

Few prior studies of memory exist in the behavioral finance and economics literature. Mullainathan [?] provides a model of long-term associative memory to explain deviations from the consumption paths predicted by the permanent income hypothesis. Sarafidis [?] applies Mullainathan [?] in a setting where politicians take advantage of the polity's associative memory by strategically releasing information. Hirshleifer and Welch [?] show decision-making can exhibit inertia or impulsiveness when beliefs are driven by perfectly recalled actions combined with imperfectly recollected signals.

Several models of memory of limited volume have appeared in the literature. Wilson [?] and Hellman and Cover [?] derive optimal memory processes and decision strategies in the context a of decision problem with an infinite repetition of informative signals, but where memory is of a fixed, finite length. Dow [?] discusses optimal memory schemes in the short-run context of a two period decision problem where the agent stores information from period to period in an optimal, but limited, fashion. Benabou and Tirole ([?], [?], [?], [?]) develop models of malleable, imperfect memory in the context of agents with self-control problems in order to explain the use of intrapersonal rules for self-regulation. Gottlieb [?] uses a model of agents with malleable memory and preferences over their own attributes to explain anomalies in the literature on choice under risk.

1.2 Psychology of Emotion and Belief

Affective state has a clear and significant effect on histories recollected by agents in experimental settings (see Isen [?] and Clore, Schwarz and Conway [?] for surveys).[7] Isen et al. [?] is the seminal study of mood congruent recall. The authors employed a field study to demonstrate a large effect on consumer product evaluations through the use of low cost

[7] Hirshleifer and Shumway [?] provides an excellent summary of the research on mood and individual judgement. Elster [?], Loewenstein [?], and Loewenstein and Lerner [?] provide overviews of the role of emotion in decision making.

affect manipulations and a supplementary laboratory study to isolate the cognitive processes underlying the phenomenon. In the field study shoppers in a suburban mall were assigned randomly to treatment and control groups, and subjects in the treatment group received free samples of products valued at $0.29 in 1977 dollars. The participants in the treatment group registered significantly higher product satisfaction ratings than those in the control group.

The laboratory experiment involved inducing a random affective state in the participants, requiring the subjects to memorize a list of words, inducing a (possibly different) random affective state in the subjects, and then determining the number and affective valence of the words that could be recollected in the new affective state. The affective state was created by having the subject play a computer game developed by the experimenters. Winning and losing were randomly determined by the experimenter for each repetition of the game. After playing the computer game once, the participants were provided a tape recording containing 36 words. 18 of the words were traits with 6 words each of positive, negative, and neutral valence, while 18 additional non-trait words were added as a control. After playing the computer game a second time, the participants were given 5 minutes to recall as many words as possible. Consistent with mood congruent recall, recollection was improved most when the valence of the material memorized matched the valence of the affective state at recollection.

Kida, Smith and Maletta [?] provides a series of experiments that reveal that affective reactions to information are related to the valence of the information in a straightforward fashion, encoded into memory, and have significant impact on the recall of the information at later times. The Kida, Smith and Maletta [?] experiments show that when the subjects, experienced managers, were provided with a natural numerical benchmark from which to establish the valence of experimental data (e.g., industry benchmarks), the managers were better able to recall their affective reactions to the data than the actual comparison to the benchmark. Further, the evidence suggests that the managers had difficulty recollecting data about a firm when the valence of the data did not conform to their overall impression of the firm. For example, if a firm's accounting data was typically below market benchmarks, subjects had difficulty recollecting the firm had a high cash flow relative to industry standards. This suggests that trained managers, who are exposed to large volumes of numerical data on a regular basis, are subject to affective influences on their recollection processes.

2 Model

In this section we present the asset pricing problem and develop a model of memory that describes the effects of mnemonic cues (such as agent affective state) on the belief updating process. We assume throughout that the beliefs under study are those of a risk neutral representative agent, which implies that asset prices equal the net present value of expected future dividends with a discount factor $r > 0$.[8] Prior to the time at which beliefs about asset valuations are formed, the agent is assumed to have observed a history of dividend innovations and stored this history in long-term memory. For the representative agent to compute the net present value of the asset, and hence determine the asset's price, the representative agent needs to form an expectation regarding future dividend growth. When forming a belief about dividend growth, the agent recollects a sample of data from memory in order to update his prior belief and form a posterior about the mean of the dividend innovation in each period. The representative agent in our model fully and correctly incorporates recollected information into beliefs about asset valuation, but the

[8]We thank an anonymous referee for suggesting the cleaner formulation using a representative agent structure. Prior versions employed a heterogeneous agent model with a partial equilibrium price setting assumption. Details are available upon request from the author.

agent neither corrects for his faulty memory nor does he foresee how shifts in his affective state will influence future beliefs (and hence prices).

Agent affective state is influenced by news in the present period with positive (negative) dividend innovations encouraging positive (negative) affective states. Mood-congruent memory causes positive (negative) affective states of a decision maker to cue the recollection of positive (negative) information from memory, and the beliefs founded on the recalled data will be optimistically (pessimistically) biased. We also assume that rehearsal causes recall of some data recalled by association in prior periods, which we prove leads to the autocorrelation of price biases. Our predictions are rooted in our model of the time-series relationship between changes in dividends, mnemonic biases, and price changes.

There are significant reasons to question whether biases, such as imperfect recall, that are plausible on the individual level aggregate to the level of a representative agent and have market-wide impact. We now address two objections to our approach.

First, since our model is fundamentally one of biased beliefs, one might conjecture that arbitrageurs would have significant power to mitigate the impact of biased beliefs on prices. However, there is a rich and growing literature on the limits of arbitrageurs to correct mispricings within the market (Shleifer [**?**] provides an early survey). Factors that limit arbitrage include risk aversion on the part of arbitrageurs and constraints on and costs of short sales. These limits are exacerbated by the potentially long time it can take for asset prices to correct when the ability to arbitrage is bounded.

Second, one might object to our model by arguing that although imperfect recall has plausible and significant effects on the individual level, market participants who are significantly affected by the resulting biased beliefs will be forced out of the market or choose not to participate. Chapman and Polkovnichenko [**?**] points out that in an economy with heterogeneous agents, potential participants with non-standard preferences who are aware of their preferences may choose to exit the market. The net effect of this selection is that the agents with non-standard preferences have a very limited effect on equilibrium prices. Most market participants recognize their own imperfect recall, but few of these agents are aware of the biases caused by associative memory. Therefore imperfect recall provides little motivation to exit the market.

Selection effects can also occur as agents are forced out of the market over time due to their poor decisions. Blume and Easley [?] and Easley and Yang [?] model this selection and prove that, when the selection occurs, the price impact of agents with incorrect beliefs vanishes. However, even if agents with severe biases due to imperfect recall are forced out of the market, new agents with these biases who enter over time would continue to have an impact on market prices.[9] The representative agent formulation captures the steady-state level of bias in the market.

2.1 Associative Recall and Rehearsal

We assume that dividends at time $t \in \{.., -1, 0, 1, 2, ...\}$, denoted d_t, are described by a stochastic process of the form $d_{t+1} = d_t + \varepsilon_{t+1}$ where ε_t are independently and identically distributed normal random variables with a known variance of 1 and an unknown mean of θ. The representative agent bases her beliefs about θ on recollected realizations of past dividend innovations (ε_t) and a prior belief with full support over the real numbers. For notational ease we assume the true value of θ is 0.

Asset prices are based on a representative agent's expectation regarding the net present value of the future dividends of an asset. Stated informally the agent's expectations are based on a posterior belief formed by updating a prior belief according to Bayes' rule using

[9] Both Blume and Easley [?] and Easley and Yang [?] assume that no new agents enter the economy. In addition, the strong selection results of these papers assume the agents are homogenous along all other preference dimensions. These works point out that if heterogeneity of intertemporal preferences exists, then the selection results can be reversed.

information recalled from memory. The information recalled from memory consists of a history of realizations of past dividend innovations. One could also interpret the recalled dividend innovation as other forms of information regarding the asset's fundamental value such as the opinions of equity analysts, information gleaned from contacting industry insiders, and interviews with corporate officers. The recall and posterior-formation process constitutes the only learning conducted by the agent, and the agent's recall and belief formation process is repeated in each period of our model.

In each period the representative agent recollects a subset of previously observed values of dividend innovations, ε_t, that are stored in long-term memory. A *history*, $H_t = \{..., \varepsilon_1, \varepsilon_2, ..., \varepsilon_t\}$, is a set of data observed by the agent prior to period t and stored in long term memory. A *recollected history* is a random set that consists of those events that are recalled by the agent in a particular period and used to form a posterior. Let a typical realization of the recollected history be denoted $H_t^R \subseteq H_t$. We assume naïveté on the part of the agent in the sense that the agent does not use knowledge of the affective state to correct for the biases in the recollection.

We incorporate two channel of recalls from memory into our model. First we assume that $l(N)$ data are recalled in each period from a complete history of data H_t of length N through associative recall.[10] The probability that each datum is recalled through associative memory in a given period depends on the affective state in that period. The second component of memory we model is rehearsal, the recall of information that was recalled in the prior period. As we show below, the data recalled through rehearsal depends on the past affective states that determined the data recalled via association in prior periods. The rehearsal processes generates autocorrelations in the bias in belief that are responsible for (most of) our time-series predictions.

[10] Our analysis focuses on the case where $N \to \infty$.

First we discuss our model of association. We let the variable $\varphi_t \in [-1, 1]$ denote the valence of the representative agent's affect in period t. In other words, higher values of φ_t denote more positive moods in period t. We describe the full history of affective valences using a vector indexed by time, $\overrightarrow{\varphi}_t = (..., \varphi_{-1}, \varphi_0, \varphi_1, ..., \varphi_{t-1}, \varphi_t)$. We discuss our model of the evolution of φ_t in section **??**.

As noted above, $l(N)$ denotes the number of data recalled through association from a length N complete history of data stored in long term memory, H_t. The contents of the recollected history are generated by a process of independent sampling without replacement from the complete history. The relative sampling probability of recalling ε_i given current affective state φ_t and a complete history H_t is $\tau(\varepsilon|\varphi_t, H_t) = \exp(m\varepsilon\varphi_t)$, $m > 0$, if $\varepsilon \in H_t$. For coherence we require $\tau(\varepsilon|\varphi_t, H_t) = 0$ if $\varepsilon \notin H_t$ to insure that only events that actually occurred can be recalled. The variable m parameterizes the intensity of the association effect. If m is large, then mood strongly drives the recall of information of a similar valence, whereas if $m = 0$ then each datum is equally likely to be recalled regardless of the agent's mood.

To interpret the relative sampling probability, consider two dividend innovations ε and ε' stored in long-term memory. If

$$\frac{\tau(\varepsilon|\varphi_t, H_t)}{\tau(\varepsilon'|\varphi_t, H_t)} = \beta$$

then ε is β times more likely to be recalled than ε' given affective state φ_t. We emphasize that $\tau(\cdot|\varphi_t, H_t)$ does not denote a probability distribution. Association is captured by the complementarities between recalling large (small) values of ε with large (small) values of φ_t. Written formally, for any $\varepsilon > \varepsilon'$ we have

$$\frac{\tau(\varepsilon|\varphi_t, H_t)}{\tau(\varepsilon'|\varphi_t, H_t)} = e^{m\varphi(\varepsilon-\varepsilon')}$$

is increasing in φ_t - in other words, the relative sampling probability of high (low) values of ε is increasing (decreasing) in the affective state φ_t. Although we have chosen a particular functional form, our results would hold for any choice of τ that is log-supermodular[11] in (ε, φ).

In figure 1 the solid line represents the probability density function (PDF) of an unbiased recollection (via association) of a large number of realizations of a normal distribution. The dashed line represents the distribution of points recollect through association by the representative agent when $\varphi > 0$.

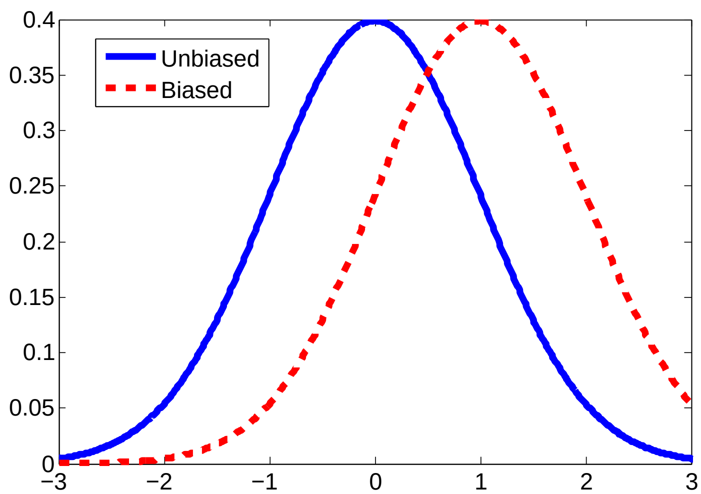

Figure 1: Biases in Recollection

The second component of recall we model is rehearsal, a phenomenon whereby information that was recalled in the recent past is more easily recalled in the present. To capture rehearsal we assume that the agent recollects each piece of information recalled in the prior period, H^R_{t-1}, with probability $\rho \in \mathbb{R}_+$. Note that the data recalled from period $t-1$ includes data retrieved through rehearsal from period $t-2$. Therefore data from period $t-2$

[11] See Appendix **??** for the formal definition of *log-supermodular.*

may be recalled through rehearsal, albeit indirectly, in period t. Since the data recalled by association in periods $t-1$ and $t-2$ was influenced by the respective affective states, φ_{t-1} and φ_{t-2}, the rehearsal causes the biases in past periods to influence the data used for inference in period t. It is recall through rehearsal that generates the autocorrelation of the biases in belief that are crucial for our prediction of short-run continued overreaction.

2.2 Asset Pricing

A note of interpretation is required to distinguish the two roles that the dividend innovation ε_t plays in our model. As in a classical model of asset pricing, ε_t provides information about future dividend payouts. In each period a history of dividend innovations is recollected, and none of the realized innovations (even the realization from the current period) need to be recollected or privileged over any other realization. The second role of the dividend innovations is to determine the evolution of the representative agent's affect, which is described below. In this second role ε_t provides a context for the formation of beliefs through the biased recall caused by the affective state.

A risk-neutral representative agent forms beliefs about the net present value of a security (and hence the market price) equal to

$$P_t(\overrightarrow{\varphi}_t) = E_t\left[\sum_{\tau=1}^{\infty} \frac{d_{t+\tau}}{(1+r)^{\tau}} | \overrightarrow{\varphi}_t\right] \tag{2.1}$$

where $E_t[\circ|\overrightarrow{\varphi}_t]$ represents the time t expectation of the agent given a history of affective valences $\overrightarrow{\varphi}_t$. The net present value equation can be written

$$P_t(\overrightarrow{\varphi}_t) = \frac{d_t}{r} + \frac{1+r}{r^2} E_t[\theta|\overrightarrow{\varphi}_t] \tag{2.2}$$

The predictions of our model turn on how changes in $\overrightarrow{\varphi}_t$ impact expectations regarding θ.

Since the true mean of ε_{t+1} is 0, we can describe the bias relative to a perfectly informed observer as

$$\delta(\overrightarrow{\varphi}_t) = \frac{1+r}{r^2} E_t[\theta|\overrightarrow{\varphi}_t] \tag{2.3}$$

We simplify our framework by focusing on situations where the representative agent recollects a large sample of data from memory, but this sample is incomplete and biased. In effect the representative agent is confident in his beliefs regarding θ, but these beliefs exhibit a bias that depends on $\overrightarrow{\varphi}_t$. An unbiased observer (such as an econometrician) would under these conditions know the true value of θ is 0 and update his or her expectations of future dividends based solely on the realized values of ε_t. Most of our predictions are stated in terms of how the mnemonic biases influence the confident, but incorrect, expectations of the representative agent relative to the confident, correct expectations of the hypothetical unbiased observer.

The following proposition implies that we can make use of a simple autocorrelation formula for the price bias given the affective state in the current and all prior periods.

Proposition 1. *Suppose that as $N \to \infty$ we have $l(N) \stackrel{a.s.}{\to} \infty$ and $l(N) = O(N^{1/4})$. Then for large N*

$$\delta(\overrightarrow{\varphi}_t) = \frac{1+r}{r^2} m(1-\rho) \sum_{\tau=0}^{\infty} \rho^{\tau} \varphi_{t-\tau} \tag{2.4}$$

Our two asymptotic assumptions capture the fact that associative memory provides us with a rich, but incomplete, set of facts on which to base our judgments. Taking asymptotic limits (as in proposition **??**) reflects markets where prices are based on a large ($l(N) \stackrel{a.s.}{\to} \infty$), but incomplete ($\frac{l(N)}{N} \to 0$) and biased, set of information. The assumption that $l(N) = O(N^{1/4})$ is largely technical, and we have not reason to believe we have found a tight bound on $l(N)$. In order to generate our closed form we need to account for the fact that data could be recalled through association in multiple periods in a row. We use the bound on the rate of growth of $l(N)$ relative to N to insure that this occurs with vanishingly small probability as $N \to \infty$, which implies that we can ignore complicated (but purely technical) issues with our asymptotic convergence arguments stemming from the same piece of information being recalled through association and rehearsal.[12]

For expositional purposes, we now unpack equation **??** to identify the sources of each term. In period t data is recalled through association, and this data is biased by the current period's affective state, φ_t. The data recalled through association have an average equal to $m\varphi_t$ and a volume of $l(N)$. Rehearsal recalls data used for inference in period $t-1$. The data recalled through rehearsal consists of $\rho l(N)$ data recalled through association in period $t-1$, $\rho^2 l(N)$ such data points from $t-2$, $\rho^3 l(N)$ such data points from $t-3$, etc. Note that the data recalled through association in period $t-\tau$ have an expectation equal to $m\varphi_{t-\tau}$. Averaging over all of these data (as Bayesian inference requires) yields

$$E_t[\theta|\overrightarrow{\varphi}_t] = m(1-\rho)\sum_{\tau=0}^{\infty}\rho^\tau \varphi_{t-\tau}$$

Inserting this into equation **??** provides our result.

Many of our predictions follow from proposition **??**, which proves that the bias in the beliefs of the representative agent are increasing in the valence of the agent's affect. A more direct model of mood's influence on asset pricing would be to assume the conclusion of proposition **??** and discard the model of memory entirely. However some of our most striking predictions, such as the interaction of the degree of bias and the volatility of the underlying asset (section **??**) and the interaction of mood and the level of agent knowledge (section **??**), would not have followed from directly assuming proposition **??**. These additional predictions stem from the fact our bias is informational rather than a direct shift of

[12] If $l(N)$ grows more rapidly, complex terms based on the relative values of affective state in all prior periods would have to be added to equation **??**.

expectations.

2.3 Dynamics of Affect

In order to generate predictions regarding the time-series behavior of asset prices, we must describe how the valence of the sentiment of the representative agent evolves over time. We assume that the valence of the representative agent's affect obeys:

$$\varphi_{t+1} = \varphi_t + \sigma_{t+1}$$

where σ_{t+1} is an innovation to the representative agent's affect. We assume that σ_{t+1} is determined by news in the current period, ε_{t+1}, and the current affective state, φ_t. The choice of the determinants of σ_{t+1} reflects an assumption as to what kinds of events are salient with respect to changes in affective state. Since there is little research in psychology that shines light on our financial model, it remains an empirical matter to use predictions based on different assumptions regarding the saliency of different market variables to choose the theory that makes superior empirical predictions.[13]

We employ the following formula for innovations to affect

$$\sigma_{t+1} = g(\varphi_t, \varepsilon_{t+1}) = \begin{cases} (1-\varphi_t)(1-e^{-\varepsilon_{t+1}}) \text{ if } \varepsilon_{t+1} \geq 0 \\ (1+\varphi_t)(e^{\alpha\varepsilon_{t+1}}-1),\, \alpha \in (0,1) \text{ if } \varepsilon_{t+1} < 0 \end{cases} \tag{2.5}$$

We view dividend innovations as a proxy for news regarding the fundamentals of the asset (e.g., earnings surprises, announcements regarding product lines).[14] Our theory in

[13] A leading alternative as a driver for changes in the representative agent's affect is total returns, $\sigma_{t+1} = P_t + d_t - (1+r)P_{t-1}$. Under this alternate theory we do not find short-run continued overreaction, but some of our other findings hold qualitatively. We provide a brief analysis of a model where returns drive mood changes in appendix **??**.

[14] In effect we assume that all changes in firm profitability are returned immediately to investors as dividends instead of being retained

effect assumes that these fundamental news events are salient and drive changes in the representative agent's mood. This innovation process implies that good news, the event $\{\varepsilon_t > 0\}$, increases the representative agent's affect, while bad news, the event $\{\varepsilon_t < 0\}$, has the opposite effect.

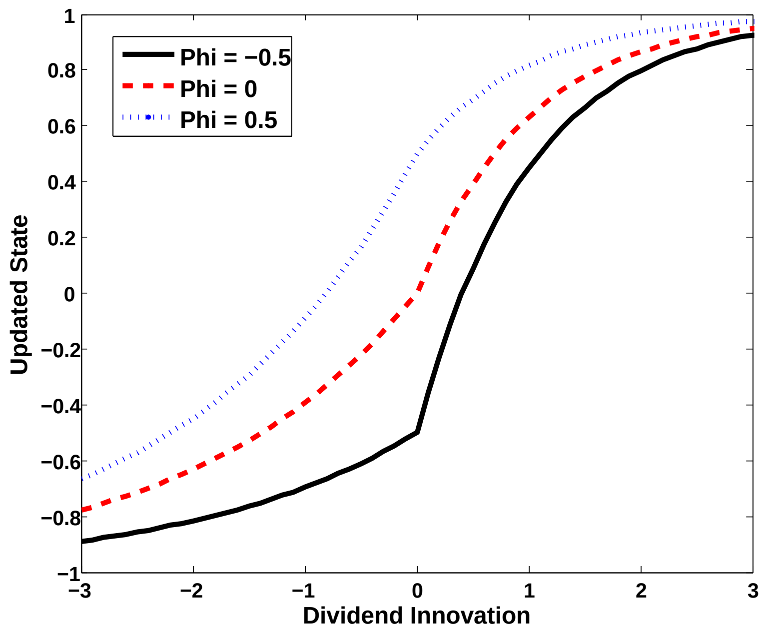

Figure 2: Evolution of Affective State

Two additional aspects of affect are implicit in our characterization. First, the affective state is bounded within the interval $[-1, 1]$, which captures the idea that there exists a natural upper and lower bound on the intensity of affective states.[15] The second stylized aspect of affect we capture is *immune neglect*, which denotes the empirical regularity that bad news has relatively little effect on affect (Gilbert et al. [**?**]). When $\alpha < 1$ the innovations described by g have the property that bad news ($\varepsilon_{t+1} < 0$) causes smaller changes in φ_{t+1} than an equivalent[16] good news event. In figure 2 we plot g for 3 different

[15]See Rayo and Becker [**?**] for arguments supporting this assumption.

[16]Given a state φ_t and innovation $\varepsilon_{t+1} \geq 0$, an equivalent bad news event is state $1-\ \varphi_t$ and innovation

affective states φ_t to illustrate the effects of bounded affect and immune neglect. The kink at $\varepsilon = 0$ is a result of the asymmetric treatment of positive and negative news on affect caused by immune neglect.

An important effect of the boundedness of the affective state and the unbounded support of the affect innovations is that the affective process of the representative agent is ergodic. Intuitively this implies that even the most extreme affective states are transient, which allows us to make predictions regarding the distribution of future affective states from the ergodic distribution. The transient nature of the affective impact of the information contained in dividend innovations differentiates the transient psychological component of our model from the permanent, informational component of dividend innovations.

Lemma 1. *φ_t is ergodic.*

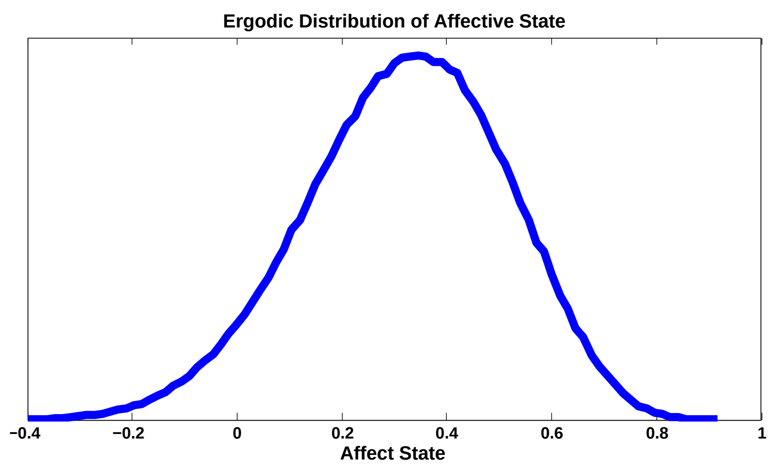

Figure 3: Long-Run Distribution of Affect

Although the ergodic distribution of affective state is intractable to describe analytically, we can easily compute the ergodic distribution (shown in figure 3) using numerical simulations. Note that since negative dividend innovations have less effect on the affect of

$-\varepsilon_{t+1}$.

the representative agent than positive events, the mood of the market is more likely to be positive than negative and as a result market prices are typically above the value indicated by the fundamentals alone. In this example we assume $\alpha = 0.5$, but the general pattern is robust to all other choices of α simulated.

3 Market Effects - Time Series

In this section we trace out linkages between affective state, beliefs, and market prices for financial assets. We show that the effects of dividend announcements on mood can explain short-run continued overreaction and long-run correction of securities prices to the release of news and predicts excess volatility of asset prices (especially during market downturns).

3.1 Dynamics of Overreaction and Correction

Let E_t^* refer to the expectation with respect to the true dividend process. We study short-run continued overreaction and long-run correction by examining the drift of prices following valenced news events observed by an unbiased econometrician. We assume that the information set of the outside observer does not include the affective valence. Since the hypothetical outside observer is not subject to our biases, our results on the predictability of future price changes based on dividend innovations in the current period generates identical predictions regarding the predictability of future returns.

Since shocks to fundamental value, ε_t, influence both the fundamental component of prices, d_t, and the evolution of the affective state, φ_t, the response to such a shock will evolve over several periods. To illustrate these effects we plot below the following based on Monte Carlo simulation in figure 4

$$E_t^* \left[\delta_{t+\tau}(\overrightarrow{\varphi}_{t+\tau})|\varepsilon_t > 0\right] - E_t^* \left[\delta_{t+\tau}(\overrightarrow{\varphi}_{t+\tau}))|\varepsilon_t < 0\right]$$

Note that, as expected, the impulse response to dividend innovations increases over the short-run due to the autocorrelation in the evolution of $\varphi_{t+\tau}$ in response to ε_t, but these effects fade as the affective state returns to its long-run ergodic distribution.

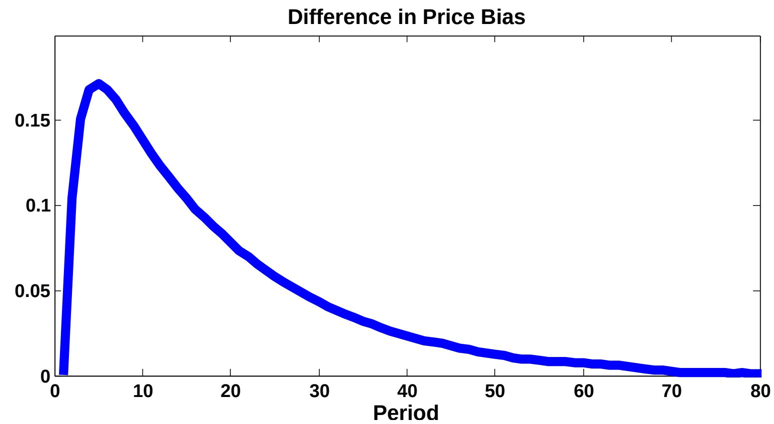

Figure 4: Price Impulse Response Difference

Recall that in a model with perfect recall the future price increments are unpredictable. Therefore under perfect recall we predict

$$E_t^*[P_{t+1} - P_t|\varepsilon_t \geq 0] = E_t^*[P_{t+1} - P_t|\varepsilon_t < 0]$$

since all of the information from ε_t is incorporated into P_t. Short-run continued overreaction to dividend innovations reflects a continued reaction to old information generated by the autocorrelation produced by rehearsal. To see this, consider

$$\begin{aligned} P_{t+1} - P_t &= \frac{\varepsilon_{t+1}}{r} + \frac{1+r}{r^2} m(1-\rho) \left[\sum_{\tau=0}^{\infty} \rho^\tau \varphi_{t+1-\tau} - \sum_{\tau=0}^{\infty} \rho^\tau \varphi_{t-\tau} \right] \\ &= \frac{\varepsilon_{t+1}}{r} + \frac{1+r}{r^2} m(1-\rho) \sum_{\tau=0}^{\infty} \rho^\tau \sigma_{t+1-\tau} \end{aligned}$$

where $E_t^*\left[\varepsilon_{t+1}\right] = 0$.

Two countervailing effects are at work. First, we have that $E_t^*[\sigma_t|\varepsilon_t \geq 0] > 0 > E_t^*[\sigma_t|\varepsilon_t < 0]$, which reflects the changes in the affective state driven by news in the current period. The autocorrelation of price biases implies that this effect is carried forward to period $t+1$ and discounted by the rehearsal probability ρ. Second, since the affective state has been moved away from the ergodic distribution and converges back to the ergodic distribution monotonically in expectation we have $E_t^*\left[\sigma_{t+1}|\varepsilon_t \geq 0\right] < E_t^*\left[\sigma_{t+1}|\varepsilon_t \geq 0\right]$. We predict short-run continued overreaction if ρ is sufficiently large that the first effect dominates the second effect in the short-run.

Proposition 2. *For $\rho \in (0,1)$ sufficiently large asset prices exhibit short-run continued overreaction*

$$E_t^*[P_{t+1} - P_t|\varepsilon_t \geq 0] > E_t^*[P_{t+1} - P_t|\varepsilon_t < 0] \tag{3.1}$$

Our result is not entirely satisfying since we must condition on ε_t, while most empirical studies of short-run continued overreaction use price changes as the conditioning variable. Due to the correlations between affective state and price changes, expectations conditional on $\{P_{t+1}-P_t \geq 0\}$ or $\{P_{t+1}-P_t \leq 0\}$ would employ different probability measures over $\overrightarrow{\varphi}_t$. We have been unable as of yet to produce an analogue of proposition **??** that conditions on price changes in period t. Therefore we supplement our analytical result with numerical simulations showing the results of proposition **??** hold when we condition on the sign of the price change in period t (figure 5 below).

Our second prediction on asset price time-series behavior concerns long-run correction of short-run continued overreaction. Our model predicts that an econometrician studying asset price time-series data finds that the effects of memory biases fade over time as the distribution of affective states asymptotically returns to the ergodic distribution. The ergodic convergence implies the affect component of any price change fades with time, so price changes become unpredictable in the long-run.

Proposition 3. *Our model predicts long-run correction of the price effect of changes in affect in the current period*

$$\underset{\tau\to\infty}{Lim}\ E_t^*[P_{t+\tau} - P_t|\varepsilon_t \geq 0] < \underset{\tau\to\infty}{Lim}\ E_t^*[P_t - P_{t-1}|\varepsilon_t < 0]$$

To show the results of propositions **??** and **??** extend to the case where we condition on price changes in period t we numerically compute

$$E_t^*[P_{t+\tau+1} - P_{t+\tau}|P_t - P_{t-1} \geq 0] - E_t^*[P_{t+\tau+1} - P_{t+\tau}|P_t - P_{t-1} < 0]$$

Importantly, our computations integrate over the distribution of $(\varphi_t, \varphi_{t-1}, \varphi_{t-2}, ...)$, which is correlated with $P_{t+\tau+1} - P_{t+\tau}$. As we see in figure 5, the difference in the response of

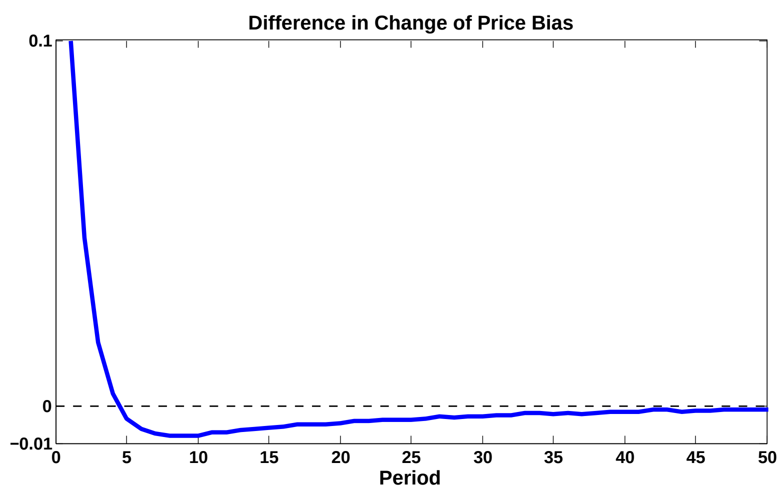

Figure 5: Autocorrelation of Bias

future price changes to the price change in the current period is positive in the short-run (similar to the predictions of proposition **??**) before reversing and becoming negative in the medium-run, which represents the correction of the short-run continued overreaction. At longer horizons the effect of mnemonic biases fades entirely as the affective state returns to the ergodic distribution (proposition **??**). The effects described in the figure match the findings of short-run continued overreaction and long-run correction found in the empirical literature.

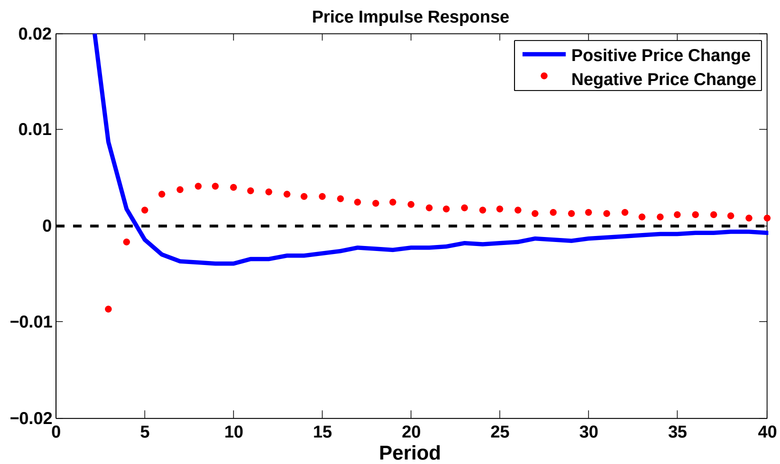

Figure 6: Autocorrelation of Bias, Deconstructed

In order to deconstruct the effects further, we separately plot $E_t^*[P_{t+\tau+1} - P_{t+\tau}|P_t - P_{t-1} \geq 0]$ and $E_t^*[P_{t+\tau+1} - P_{t+\tau}|P_t - P_{t-1} \leq 0]$ in figure 6. What we find is that $E_t^*[P_{t+\tau+1} - P_{t+\tau}|P_t - P_{t-1} \geq 0]$, denoted as "Positive Price Change," is in the short-run positive, which reflects short-run overreaction. In the medium–run the expected price changes become negative and return to 0 in the long-run. The plot of $E_t^*[P_{t+\tau+1} - P_{t+\tau}|P_t - P_{t-1} \geq 0]$, denoted as "Negative Price Change," displays symmetric features.

3.2 Momentum

Continued overreaction to news and long-run correction of the overeraction are statements regarding the autocorrelation of asset prices conditional on news received regarding the asset's underlying value. *Momentum* refers to autocorrelation of asset price changes that are not conditional on the news received by the agents in the current period. In this section we demonstrate through numerical simulations that our model predicts momentum in addition to overreaction.

In the context of our model the two phenomena are not equivalent since the asset price biases tend to exhibit drift back towards the long-run ergodic distribution on the asset price bias. Consider an asset that has received a long series of positive news announcements. The long series of positive news announcements has the effect of pushing market affect towards the upper bound of $\varphi_t = 1$. When affect approaches this bound, the unconditional expectation of future price changes is negative. To see this, first note that fundamental news has an expectation of 0, so the asymmetric effect must be rooted in the mnemonic bias. When φ_t is near 1, positive news ($\varepsilon_{t+1} > 0$) induces only a slight increase in φ_{t+1} (and hence a small increase in the asset price bias). On the other hand, a negative news ($\varepsilon_{t+1} < 0$) causes a significant decrease in φ_{t+1}. The net effect is an expected *negative* autocorrelation in asset price changes. However, for moderate level of φ_t it is reasonable to conjecture that short-run overreaction implies a *positive* autocorrelation in asset price changes.

Which effect dominates will depend on the ergodic distribution of affect. If affect is with high probability expected to take on extreme values, then negative autocorrelation results. If affect is more likely to be moderate in magnitude, then momentum will result. Due to the difficulty of analytically computing the ergodic distribution of affect, numerical simulations are required. A typical autocorrelation plot is displayed below, which shows that short-run momentum follows from our model.

Although the plot above represents a simulation using a particular choice of parameter values, the qualitative results demonstrated persist for a wide variety of choices of parameter values. Due to the robustness of our simulations, we conclude that momentum is a natural result of our model of mnemonic biases.

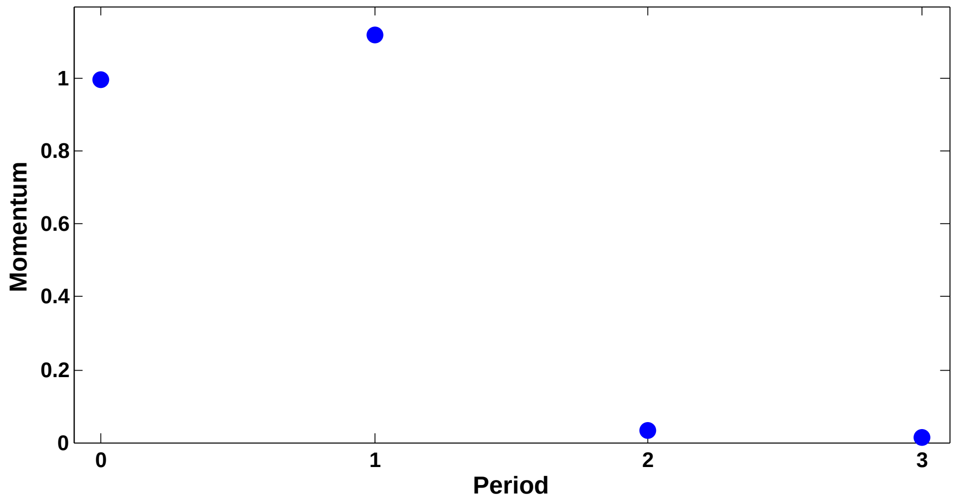

Figure 7: Biases in Recollection

3.3 Excess Volatility

In our model changes in asset prices are driven by two complementary effects. Information about future dividends is revealed by dividend innovations in the current period. This information is incorporated into the representative agent's beliefs about the net present value of the asset, and the movements of price caused by this updating are referred to as *fundamental volatility*. Empirical studies find that asset prices exhibit volatility greater than can be explained by the fundamental contribution alone (see, for example, Shiller [**?**], [**?**]), and our model of asset pricing suggests that variations in affect can (partially) explain the excess volatility. The movement of affective state will cause price volatility that we denote as *affective volatility*. Furthermore, information that drives fundamental changes in price also drives changes in affect, resulting in an *affective volatility of information*. We make two predictions regarding the effect of associative memory on excess volatility. First (and unsurprisingly), associative memory increases the volatility of asset prices. Second, we predict that excess volatility is highest when the representative agent's affect is depressed (e.g., during market downturns).

In our model the three forms of volatility can be separated allowing us to generate predictions about the relative magnitude of the affective and fundamental contributions to volatility. From period t to period $t+1$ the price increment is

$$\begin{aligned} P_{t+1} - P_t &= \frac{\varepsilon_{t+1}}{r} + \frac{1+r}{r^2} m(1-\rho) \sum_{\tau=0}^{\infty} \rho^{\tau} \left(\varphi_{t+1-\tau} - \varphi_{t-\tau} \right) \\ &= \frac{\varepsilon_{t+1}}{r} + \frac{1+r}{r^2} m(1-\rho) \sum_{\tau=0}^{\infty} \rho^{\tau} \sigma_{t+1-\tau} \end{aligned}$$

We interpret the variance of this price increment as price volatility. We identify the fundamental contribution to volatility as

$$\left[\frac{1}{r}\right]^2 Var\left(\varepsilon_{t+1}\right) > 0$$

As the variance of ε_{t+1} increases, the fundamental contribution to volatility increases.

The more mercurial affective state is, the greater the excess volatility will be, which we denote as affective volatility. We can write the affective volatility as proportional to

$$Var\left(\sum_{\tau=0}^{\infty} \rho^{\tau} \sigma_{t+1-\tau}\right) = \frac{Var(\sigma_{t+1})}{(1-\rho)^2} + \frac{2}{1-\rho} \sum_{\tau=1}^{\infty} \rho^{\tau} Cov(\sigma_{t+1}, \sigma_{t+1-\tau}) > 0$$

The final component of volatility, the affective volatility effect of information, is defined as follows

$$\frac{1+r}{r^3} m(1-\rho) Cov\left(\varepsilon_{t+1}, \sigma_{t+1}\right) > 0$$

The positive sign on the affective volatility of information is due to the fact that ε_{t+1} and σ_{t+1} by definition (i.e., equation **??**) have the same sign.

To summarize the predictions we use numerical simulations to compute the affective volatility as a function of φ_t in figure 7. Our plot reflects the force of two components of

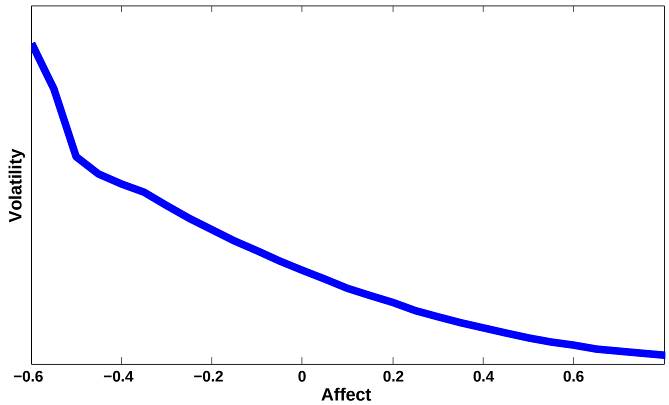

Figure 8: Volatility and Affect

our model of affect. First, the effect of positive (negative) shocks is highest when affective state is low (high). If this were the only effect in our model, one would predict that our volatility curve would be symmetric and single-peaked around $\varphi = 0$. The second effect we have included, immune neglect, dampens the effects of negative shocks at high affective states and generates asymmetric volatility for high and low affective states.

Our predictions regarding the asymmetry of excess volatility as a function of the market state mirror findings in the ARCH literature dating back to Black [**?**]. In particular, the earlier literature finds that return volatility rises following bad news and falls following good news. Our predictions are somewhat different since we focus on excess volatility generated by the behavioral bias we study rather than the total return volatility. One could test our predictions by analyzing the conditional volatility using a GARCH model with an affect measure (see section **??**) incorporated into the volatility equation alongside a traditional set of controls.

4 Market Effects - Cross-sectional

In this section we discuss three cross-sectional predictions of our model. First, we consider the case of two risky assets when the affective state is driven by a common, salient source such as an aggregate market index. Our model predicts that price changes of the two assets ought to be correlated through the common determinant of affective state even when their dividend processes are independent. We provide a variance decomposition that can be compared to the data to identify the magnitude of the asset price co-movement caused by mnemonic effects.

Second, we consider the case of two risky assets with different volatilities. A more volatile dividend innovation process generates a greater number of extreme news events. These extreme events are easily recollected when affect has a similar valence, which causes the high volatility asset to exhibit stronger biases in price relative to the low volatility asset. While we state this as a cross-sectional prediction, one could also interpret this as a time series prediction where we study a single risky asset that exhibits periods of low and high fundamental volatility.

Finally, we consider agents that have different levels of knowledge about an asset. For example, we could compare securities analysts at investment banks (high-knowledge agents) with casual investors (low-knowledge agents). A common intuition is that, ceteris paribus, knowledgeable investors are less biased than casual investors due to either experience or selection. Our theory suggests that the opposite may be true since the level of knowledge interacts with the mnemonic biases - knowledgeable investors are more likely to have stored extreme dividend realizations in memory, and these easily recollected extreme events amplify the bias suffered by knowledgeable agents. While both countervailing effects are likely at play in reality, this provides a sharp (if one-sided) prediction that delineates our theory from other psychological models.

Many of the predictions of this section turn on comparing various aspects of asset price performance with indices of market affect. Prior work has used measures as varied as weather over the relevant exchange (Saunders [?], Hirshleifer and Shumway [?]), seasonal changes in the length of the day (Kamstra, Kramer and Levi [?]), and sporting match outcomes (Edmans, Garci and Norli [?]). We close this subsection with suggestions for new measures of affect.

One novel mood measure is the Gallup Daily: U.S. Mood Poll.[17] In the context of our theory, the advantage of examining mood as a predictor is that it highlights the exact psychological channel we are studying whereas weather is an indirect measure of affect. Two issues arise with interpreting a naive regression using on the mood expressed in the Gallup poll. First, the U.S. Mood Poll is presumably correlated with diffuse information regarding future market performance (e.g., the poll is correlated with informative consumer confidence).[18] As suggested in Tetlock [?], if a correlation between the poll and asset prices is purely informational, one would expect a permanent shift in asset prices once the poll is released. On the contrary, if the poll's ability to predict asset price changes (at least partially) reflects the influence of mood, then our theory predicts partial reversion of these changes due to the transient nature of our affective biases. The second issue is that reverse causality could be present if positive stock market moves induce poll respondents to express a more positive mood. One could use lagged poll responses to limit effects of this nature.

Another possible index for market affect is the Michigan Consumer Sentiment Index (MCSI).[19] The questions on which the index are based query consumer expectations about current and future economic performance both at the personal and aggregate level. Also, the survey asks a question about the wisdom of a generic consumer's decision to purchase a

[17] Available at http://www.gallup.com/poll/106915/gallup-daily-us-mood.aspx

[18] I thank a referee for the excellent example of how the mood poll could reflect fundamental information.

[19] We thank an anonymous referee for suggesting this metric.

durable good. We believe that the MCSI holds promise as a measure of market affect, but the caveats regarding the potential information contained within the index noted above for the Gallup Poll also apply to the MCSI. Therefore, care must be taken to isolate the purely affective component of the index.

Baker and Wurgler [**?**] and [**?**] propose a new model of market sentiment based on the common component of variation amongst a number of previously used measure of biases in financial markets (e.g., the closed-end fund discount).[20] These papers use the term *market sentiment* to refer in the abstract to a variety of market biases. Our use of the term *affect* has been deliberately chosen to differentiate our concept from market sentiment, which is a likely confusion given the common English usage of the term *sentiment* to refer to an emotional state. We have no reason to suspect that our notion of affect is related to the sentiment index proposed except in that our theory may explain some of the underlying mispricings that are used as inputs to the index.

4.1 Effect of Volatility of Dividends

In this section we argue that the magnitude of the effect of mnemonic biases given a fixed affective valence is increasing in the level of fundamental volatility. We argue that an asset with a high level of fundamental volatility exhibits a greater magnitude of price biases than an asset with a lower level of fundamental volatility. As noted in the introduction, this provides an alternative explanation for why measures of fundamental volatility, as used in Kumar [**?**] and Zhang [**?**], predict the magnitude of price biases.[21]

Our comparative static result follows from the log-supermodularity of τ, which implies that the probability of recollecting an event of similar valence to the representative agent's affect is increasing in the extremity of the event. Therefore when the representative agent in

[20] We thank an anonymous referee for suggesting these references.

[21] We thank an anonymous referee for suggesting this interpretation.

affective state φ_t recollects data about the more volatile asset through associative memory, the data recalled is dominated by the extreme events (of the same sign as φ_t) generated by the highly volatile asset. The data recalled by the representative agent (in the same affective state) about the low volatility asset, while biased towards recollecting extreme events, is dominated by the more frequent moderate dividend innovations generated by the low volatility asset. Therefore the bias of the representative agent's beliefs about the high volatility asset in any given affective state is more extreme than the bias with regards to the price of the low volatility asset.

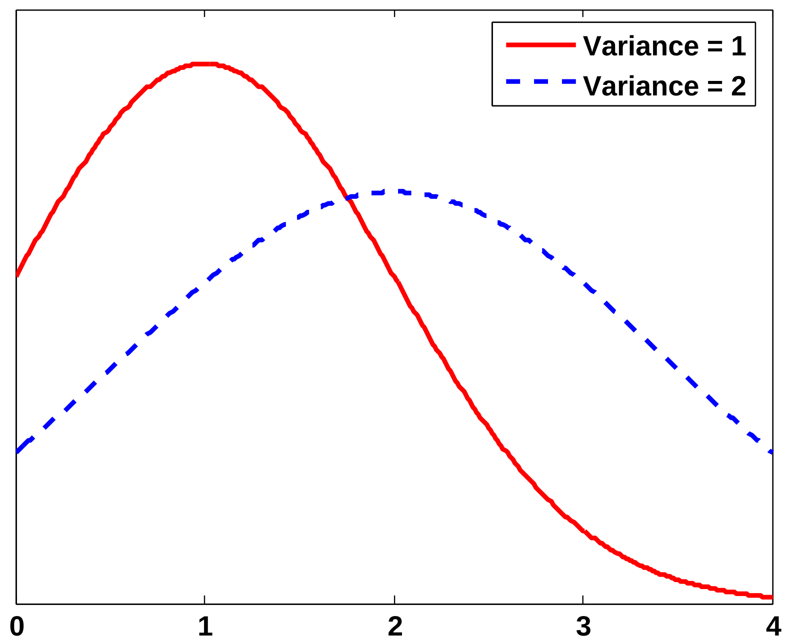

Figure 9: Volatility and Magnitude of Biases in Recollection

Our prediction is illustrated in figure 8. The dividend innovations are normally distributed with mean 0 and a variance equal to 1 for the low volatility asset and a variance of 2 for the higher volatility asset. We have plotted the as-if distribution of data recollected from the representative agent's memory through association. The mean of the recalled data for the high variance asset is approximately twice the mean of the recalled data for the lower variance asset, which implies that the bias in the representative agent's beliefs about the net present value of the high volatility asset is twice as large as the corresponding bias for the low volatility asset.

In order to test our prediction, one would need to first define a variable measuring either the representative agent's affective state or changes to the affective state. Candidates include measures such as the weather in or near the city where the exchange is located (Saunders [?], Hirshleifer and Shumway [?]) or one of the more direct measures of aggregate affective state suggested in section ??. The econometrician would then regress a cross-section of asset prices on a standard set of controls, the representative agent's affective state, and an interaction between fundamental volatility and the representative agent's affective state. The results of this section predict that the interaction term between volatility and the measure of affective state is positive, while the results of section ?? suggest the coefficient on the representative agent's affect is positive as well.

4.2 Asset Price Correlation

Our theory implies asset price comovement greater than the correlation of the fundamental values (i.e., dividend innovations) would suggest. Since the affect of the representative agent is a unitary quantity that affects the price of all assets, shifts in the affective state cause all prices to move in the same direction. This correlation acts as a nonfundamental aggregate shock to asset prices and, as such, is difficult to eliminate through diversification and could cause significant welfare losses. The analysis in terms of nonfundamental aggregate shocks provides an explanation for the results of prior works that focus on affect shifters that are clearly exogenous to economic outcomes (e.g. affect driven by weather patterns as in Saunders [?] and Hirshleifer and Shumway [?], sporting matches as in Edmans, Garci and Norli [**?**]).

In the previous section we assumed that dividend innovations for the risky asset are salient for changes in the affect of the representative agent. The question of what is salient for innovations to affect is more difficult in the presence of multiple risky assets. In choosing our candidates for drivers of changes in the representative agent's affect, we seek market variables that are relevant for a large number of firms or industries. For example, the S&P 500 market index is highly salient for all firms in the U.S. equity market. If we are interested in the correlation of price changes for small firms within the same industry, another candidate driver of market affect is the dividend innovation of a dominant firm within the industry. For example, the dividend innovations of large firms within the technology industry (e.g., Microsoft, Google) may drive the representative agent's affect when determining equity prices for smaller firms (e.g., eDiets.com).

For expositional purposes we consider the S&P 500 index and assume that the value of the index is a process of the form

$$\omega_{t+1} = \omega_t + \chi_t$$

where χ_t is an i.i.d. innovation. The innovation to market affect is then

$$\varphi_{t+1} = g(\varphi_t, \omega_{t+1})$$

In this subsection we assume that there are two risky assets with stochastically independent dividend innovations, which we denote asset i and asset j. For each asset we can use equations **??** and **??**, rewritten below, to describe the equilibrium price of the assets.

$$P_t^i(\overrightarrow{\varphi}_t) = \frac{d_t^i}{r} + \frac{1+r}{r^2} m(1-\rho) \sum_{\tau=0}^{\infty} \rho^\tau \varphi_{t-\tau}$$

We denote the price of asset i and j in period t as P_t^i and P_t^j with associated innovations ε_t^i and ε_t^j. We assume that ε_t^i, ε_t^j, and χ_t are independent.

In a model without affective biases ($\varphi_t = 0$) we would predict that the price changes of the two assets have 0 covariance. To see this, note

$$Cov\left(P_{t+1}^i - P_t^i, P_{t+1}^j - P_t^j\right) = \frac{1}{r^2} Cov\left(\varepsilon_{t+1}^i, \varepsilon_{t+1}^j\right) = 0$$

When mnemonic biases are present we have

$$\begin{aligned} Cov\left(P_{t+1}^i - P_t^i, P_{t+1}^j - P_t^j\right) &= \frac{1}{r^2} Cov\left(\varepsilon_{t+1}^i, \varepsilon_{t+1}^j\right) + \left(\frac{1+r}{r^2} m(1-\rho)\right)^2 Var\left(\sum_{\tau=0}^{\infty} \rho^\tau \sigma_{t+1-\tau}\right) \\ &= \left(\frac{1+r}{r^2} m(1-\rho)\right)^2 Var\left(\sum_{\tau=0}^{\infty} \rho^\tau \sigma_{t+1-\tau}\right) > 0 \end{aligned}$$

Stated informally, our model predicts that the price biases of the assets are correlated through their common dependence on the salient driver of affect (the S&P 500), which generates correlation in the asset price changes.

In a more sophisticated analysis, we would need to decompose the covariance of asset price innovations to more carefully account for the correlations present in the data even without mnemonic biases. For example, the S&P 500 is driven by aggregate shocks that are correlated with the dividend innovations of all assets, which implies that $Cov(\varepsilon_t^i, \chi_t) \neq 0$. If we drop our assumption that ε_t^i, ε_t^j, and χ_t are independent we find the following formula

$$\begin{aligned} Cov\left(P_{t+1}^i - P_t^i, P_{t+1}^j - P_t^j\right) &= \frac{1}{r^2} Cov\left(\varepsilon_{t+1}^i, \varepsilon_{t+1}^j\right) + 2\left(\frac{1+r}{r^3} m(1-\rho)\right) Cov\left(\varepsilon_{t+1}^i, \sigma_{t+1}\right) \\ &\quad + \left(\frac{1+r}{r^2} m(1-\rho)\right)^2 Var\left(\sum_{\tau=0}^{\infty} \rho^\tau \sigma_{t+1-\tau}\right) \end{aligned}$$

First, we would need to determine the covariance of the the fundamentals of the assets, which can be determined from data on dividend innovations. Also, it is possible that ε_{t+1}^{i} is correlated with ω_{t+1} (and hence σ_{t+1}) if the two are driven by the same underlying economic forces. Again, this covariance term can be assessed using empirical data if we have a metric for φ_t.[22] Our theory would then attribute the residual of the variance decomposition to purely mnemonic effects, $\left(\frac{1+r}{r^2}m(1-\rho)\right)^2 Var\left(\sum_{\tau=0}^{\infty}\rho^{\tau}\sigma_{t+1-\tau}\right)$.

4.3 Levels of Knowledge

In this section we retreat from our use of a representative agent model and consider two classes of agents who possess different amounts of information stored in memory. One would expect mnemonic biases to remain for (or even be exacerbated in) individuals who possess a large store of knowledge since these knowledgeable agents could, potentially, recollect an extremely biased set of data. Therefore equity analysts and professional traders could be more subject to these biases than less knowledgeable agents.

We provide an example to illustrate this effect. There are two kinds of agents, inexperienced and experienced investors. Inexperienced investors have two realization of ε_t stored in memory, and experienced investors have $N \gg 2$ realizations in long-term memory. Agents recollect one piece of data prior to their decision (i.e., $l(1,N)=1$) and use the data to update a diffuse prior regarding the mean of θ.[23] Agents can be either in a neutral mood with unbiased recall ($\varphi=0$) or in a positive mood with optimistic recall ($\varphi=1$). Note that since the agents recall a single datum, their expectation is equal to the value recalled.

Under a neutral affect, the distribution of the estimates within both populations are standard normal distributions. Under optimistic recall, agents are biased towards recalling

[22]We cannot simply use the covariance of ε_t^i and χ_t since χ_t determines σ_t in a non-linear manner.

[23]These results are robust to having experienced agents recall more data from memory than less experienced agents so long as the experienced agents recollect a smaller fraction of the total information in memory.

(a single) higher signal stored in memory. For participants with a great deal of knowledge, the log-supermodularity of τ insures that a high realization of ε_t is more likely to be recalled (relative to the datum recalled by a less knowledgeable agent). Figure 9 depicts the distribution of beliefs of the agents with little knowledge (solid line) and those agents with a large store of knowledge in long-term memory (dashed line). Note that the population of less knowledgeable agents is biased by approximately $\frac{1}{3}$ of a standard deviation on average, while the knowledgeable agents are biased by more than 1 standard deviation on average.

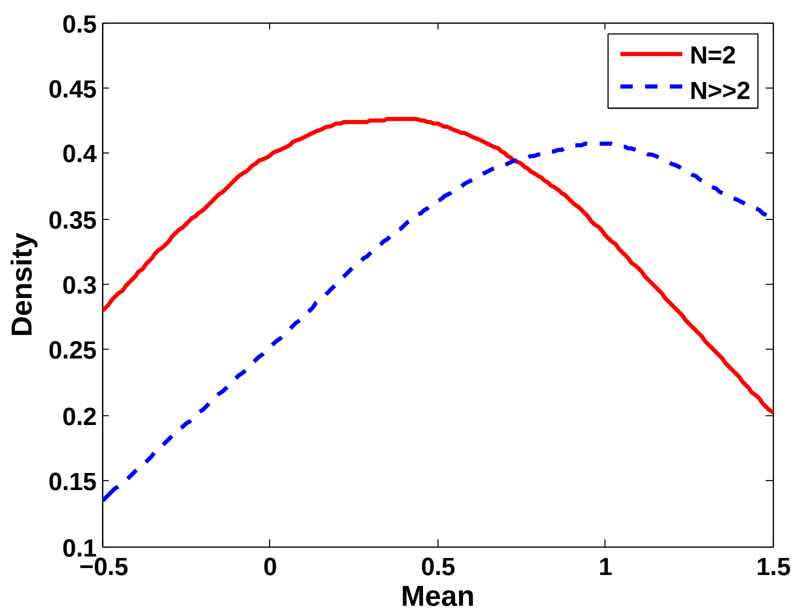

Figure 10: Biases in Recollection and Knowledge

Our predictions contrast with the usual conjecture that more informed and experienced agents ought to be *less* subject to biases in belief.[24] One justification for the usual conjecture is that knowledgeable agents are typically sophisticated, and the sophistication may render these agents less susceptible to biases. Implicitly this justification assumes

[24]Agents subject to different degrees of bias could be easily captured by attributing high values of m to very biased agents and values of m close to 0 to nearly unbiased agents.

that agents become aware of the bias and take measures to correct it. To the best of our knowledge, there is no evidence that suggests agents are aware of the bias in their recall (although most people appreciate that recall is only partial).

Another justification for the usual conjecture that experience ameliorates biases in judgement is that selection should drive traders with biased expectations from the market, although these results are sensitive to assumptions on preference parameters such as time discount rates and issues such as market completeness (Blume and Easley [?], Easley and Yang [?]). These effects are compatible with our model, but we would be required to condition our predictions on knowledge to capture our prediction and experience in the market to account for selection, which we discuss below.

Our goal in this section is not to claim that the usual conjecture must be wrong, but to point out that, unlike in many behavioral models, our theory suggests this intuition *may* be wrong. The predictions of this section provide a strong one-sided test for our theory.[25] Suggestive correlations are provided by examining the beliefs of agents with different levels of knowledge. La Porta [?] shows that forecasts by stock market analysts reflect the pattern of long-run correction, which suggests that the beliefs of experienced market participants may be subject to biases, but we do not claim that the result of La Porta [?] is more than suggestive. To test our prediction would require a differences-in-differences study of informed and uninformed investors influenced (or not influenced) by affective cues, which is substantially different than La Porta [?]. Controlling for selection would require variation in the experience of the agents or some other metric for the agents' exposure to selection pressures.

[25]In addition to testing against the usual conjecture that experience eliminates biases in judgement, our test also delineate our theory from a model that treats agent mood as an informative signal (Schwarz and Clore [?] and [?]). Under this theory market participants with large amounts of information at their disposal would be less influenced (relative to their less informed peers) by the effect of a single erroneous datum.

5 Conclusion

The principal goal of our paper is to model the effect of imperfect recall on asset prices. We are able to provide a unified explanation for short-run continued overreaction and long-run correction of prices to news, excess price volatility, and the response of prices to non-fundamental events. Our model provides the novel predictions that excess volatility will be highest in market downturns, the magnitude of the mispricings is increasing in the fundamental volatility of the assets, knowledgeable agents will be more prone to biases than less knowledgeable agents, and asset prices will be more correlated than fundamentals indicate.

Memory errors could also play a significant role in how shareholders evaluate executive competency and the timing of IPOs and other strategic market events. If firm performance has been exceptional, shareholders will be in a positive mood and optimistic about future performance. Shareholders might conclude that firm management is of higher quality than the evidence warrants and as a result fail to hold corporate officers accountable for relative performance. Market makers may time the sale of securities to take advantage of transient shifts in market affect, much as the bankers managing IPO publicity in Cook, Kieschnick, and Van Ness [?]. We hope to explore these applications in future work.

Facebook may be a recent example of the effects of increasing the affect of potential investors prior to an IPO. Although we do not wish to push our theory as the complete explanation for the precipitous price drop of Facebook's equity price following the IPO, Facebook is a firm where the investors have to estimate the (highly uncertain) growth rate of the firm's value in an affectively charged environment. Furthermore, the market price for Facebook stock in the days after the IPO (arguably) changed more than could be accounted for by the news releases; even though Facebook was previously a privately held company, a significant amount of information was available about recent performance and numerous studies of the Web 2.0 marketplace were available. Our theory suggests that a few negative announcements following the IPO could have had an outsized impact by dampening the previously buoyant market affect and changing the context in which the value of Facebook stock was estimated. Furthermore, if the performance of Facebook (and related firms) is perceived to be volatile, as is the case with many rapidly growing firms, then our theory predicts that swings in price could be very large (section **??**).

In addition to the predictions presented above, our theory presents an explanation for the effect of repeated exposure to the same information. While there are other explanations for some of our results (e.g. short-run continued overreaction), for repeated exposure to information to have an effect on market prices one of two explanations is most plausible. First, it could be that some market participants were not exposed to the first release of information due to the information's limited circulation. This is unlikely in the EntreMed case studied by Huberman and Regev [**?**] as both information releases were reported in at least one widely-read source, the *New York Times*. The second explanation is imperfect memory. The effects of repeated exposure in our model are mediated through two channels. First, a market participant may forget the data between exposures. Second, exposure to a datum a second time can cue the recall of related data, which can in turn bias beliefs. Our theory predicts that following the second exposure the news ought to have an outsized effect on the market's assessment of EntreMed's stock price due to associative recall and that these effects fade (in expectation) exponentially with the gradually decreasing effects of rehearsal.

We could further test our model using panel data of portfolio holdings in combination with data regarding cues the agents are facing. For example, we predict asset holders in cloudy cities divest from risky assets as their view of the net present value of future dividend flows decreases relative to the beliefs of asset holders in sunnier regions. If knowledgeable investors are more affected, then we would expect securities that are predominantly held by informed investors to exhibit these effects to a greater degree. Although it is well known that the equity of small firms is predominantly held by individual investors, it is not immediate that these investors are well-informed. This caveat aside, the results of Tetlock's analysis of the Fama-French small-minus-big factor provides tentative support for this prediction in the context of the WSJ "Abreast of the market" column (Tetlock [?]).

An additional empirical prediction is that generic mnemonic cues may affect how different components of a firm's earnings influence market price. For example consider General Electric, a firm that sells capital equipment important for green-tech projects in addition to other divisions operating in a variety of other industries. When cues for future profits in the green-tech industry are salient (e.g., the bankruptcy of Solyndra), information regarding green-tech related events is over-represented in the recalled data about GE and equity markets may overweight the contribution of green-tech to GE's future profits. If GE has done well in the green-tech market GE's equity will exhibit an upward mispricing, and if GE has done poorly in the green-tech market GE's equity will be downwardly mispriced.

Testing this prediction is complicated by the fact that these cues, such as the bankruptcy of Solyndra, may carry information about the fundamental profitability of GE. However, our theory predicts that the price effects of mnemonic biases will (partially) reverse as the cues become less salient with time. To test this prediction, the price of GE stock should be regressed on the valence of the cue (to control for short–run continued overreaction and long-run correction effects as documented in section **??**), the interaction of the presence of the cue (regardless of the cue valence) and the performance of GE in the relevant market, and a set of traditional control variables. Our prediction is that the interaction variable is positive in the short-run with negative values in the long-run.

In addition to the empirical tests proposed, we hope to test our theory using an experimental asset market. A controlled experiment would allow us to study the response of experimental subjects to manipulations of affective state and test for interactions between demographic traits and responses to mnemonic cues. In addition, we could monitor the data recalled by the subjects in each period of the experiment to insure that our affect manipulation has an effect on recall. Finally, we could elicit the subjects' beliefs to insure that our assumption of Bayesian updating provides a reasonable model of belief formation. Prior work has shown a significant effect of mood in the context of experimental asset markets, but these experiments have not isolated the source of the effect (Au et al. [**?**], Lahav and Meer [**?**]).

Although our model is founded on stylized facts of the psychology of memory, our theory could be reinterpreted to capture a number of other phenomena. For example, it could be that the agent has perfect recall, but the cues direct an agent's limited attention to particular pieces of information. If it is attention and not memory that is bounded, agents will require interventions that direct them to the most useful information rather than a memory aid that provides redundant information.

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A Definitions and Proofs

We now present the formal definition of log-suopermodularity, the crucial property we require of the relative sampling probability, $\rho(\varepsilon|\varphi)$. Milgrom [**?**] provides an analysis of the use of log-supermodularity as a regularity condition in economic models.

Definition 1. *The function $\rho(\varepsilon|\varphi)$ is log-supermodular if for any choice of $\varepsilon > \varepsilon'$ and $\varphi > \varphi'$ we have*

$$\ln \rho(\varepsilon|\varphi) - \ln \rho(\varepsilon'|\varphi) \geq \ln \rho(\varepsilon|\varphi') - \ln \rho(\varepsilon'|\varphi')$$

The definition of log-supermodularity is equivalent to the following equation that we use in the main body of our text

$$\frac{\rho(\varepsilon|\varphi)}{\rho(\varepsilon'|\varphi)} \geq \frac{\rho(\varepsilon|\varphi')}{\rho(\varepsilon'|\varphi')}$$

A.1 Proofs from Section Two

Lemma 1. *φ_t is ergodic.*

Proof. Since $g(\varphi_t, 0) = 0$ the process is strongly aperiodic. Since φ_{t+1} has full support over $(-1, 1)$, the process is Harris recurrent. Combined with strong aperiodicity this implies that the affect process has an invariant measure and is irreducible (Theorem 10.4.2 of Meyn and Tweedie [**?**]). Together the properties of irreducibility, existence of an invariant measure, and Harris recurrence imply the process is ergodic and converges to the invariant measure π in the total variation norm (Theorem 13.3.1 of Meyn and Tweedie [**?**]) □

Proposition 1. *Suppose that as $N \to \infty$ we have $l(N) \overset{a.s.}{\to} \infty$ and $l(N) = O(N^{1/4})$. Then for large N, we have*

$$\delta(\overrightarrow{\varphi}_t) = \frac{1+r}{r^2} m(1-\rho) \sum_{\tau=0}^{\infty} \rho^{\tau} \varphi_{t-\tau}$$

Proof. The representative agent uses a set of recalled data $\{\varepsilon_1, ..., \varepsilon_R\}$ to form beliefs regarding the true value of θ, where R denotes the length of the data set recalled. Standard results on Bayesian estimation (Chapter 10 of DeGroot [**?**]) imply that the agent's beliefs in the limit as $N \to \infty$ are concentrated on

$$\widehat{\theta}_{l(N)}(\overrightarrow{\varphi}_t) \in \arg\max_{\widehat{\theta}} \frac{h(\widehat{\theta})}{R} + \frac{1}{R} \sum_{i=1}^{R} \ln f(\varepsilon_i | \widehat{\theta}) \tag{A.1}$$

where $f(\circ|\widehat{\theta})$ denotes the probability density function of a normal variable with mean $\widehat{\theta}$

and variance 1, and $h(\widehat{\theta})$ represents the representative agent's prior beliefs. In the limit as $l(N) \to \infty$ (and hence $R \to \infty$) we can neglect the prior belief term since h is bounded. Since the representative agent is attempting to estimate the mean of a normal variable, we can replace equation **??** with the simpler formula

$$\widehat{\theta}_{l(N)}(\overrightarrow{\varphi}_t) = \frac{1}{R}\sum_{i=1}^{R} \varepsilon_i \tag{A.2}$$

The data set is comprised of two separate components: data recalled through association and data recalled through rehearsal.

We deal first with the $l(N)$ data recalled by association in the current period by proving a weak law of large numbers for this data set. Since our model is based on sampling without replacement from the data in memory, we need to account for the covariance between the data recalled. The ε_i recalled by association are (in effect) generated by stratified sampling where we need to account for the weights placed on the different strata.[26] Our assumption that $\frac{l(N)}{N} \to 0$ allows us to ignore second order corrections due to the sampling without replacement (see Hanif and Brewer [**?**] for an overview). To prove the weak law of large number, we use Chebyshev's inequality as follows

$$\Pr\left\{\frac{1}{l(N)}\sum_{i=1}^{l(N)} \varepsilon_i - E\left[\varepsilon_i|\theta\right] \geq k\sigma\right\} \leq \frac{1}{k^2}$$

[26] We could formally argue for this formula by taking successively finer finite approximations of ρ using traditional formulas for stratified samples where we stratify with respect to the values of ε_i. Since ρ is monotone, the limit would yield our result.

where

$$
\begin{aligned}
\sigma^2 &= Var\left(\frac{1}{l(N)}\sum_{i=1}^{l(N)}\varepsilon_i\right) \\
&= \frac{Var(\varepsilon_i)}{l(N)} + \frac{1}{l(N)^2}\sum_{i=1}^{l(N)}\sum_{j\neq i} Cov(\varepsilon_i, \varepsilon_j) \\
&= \frac{Var(\varepsilon_i)}{l(N)} + \frac{l(N)(l(N)-1)}{l(N)^2} Cov(\varepsilon_i, \varepsilon_j)
\end{aligned}
$$

Note that the first term goes to 0 since $l(N) \to \infty$ and since $\frac{l(N)}{N} \to 0$ the covariance term, $Cov(\varepsilon_i, \varepsilon_j)$, vanishes as $N \to \infty$. Therefore $\sigma^2 \to 0$ and $\frac{1}{l(N)}\sum_{i=1}^{l(N)} \varepsilon_i \to E[\varepsilon_i]$ in probability as $N \to \infty$ where the expectation is taken with respect to the biased sampling from memory. Proving convergence almost surely follows from standard techniques in probability theory and is omitted.

The remainder of the data recalled is generated by rehearsal. From the stationarity of the model, we know that R data were recalled in the prior period, a fraction ρ of which is recalled this period. In period t the data recalled from the prior period consists of $\rho l(N)$ data recalled by associative memory in period $t-1$, $\rho^2 l(N)$ data points recalled by association in period $t-2$, and so on. This implies that the total volume of data recalled is

$$
R = \frac{l(N)}{1-\rho}
$$

To generate the closed form above, we need to prove that the data recalled by association in any finite number of prior periods is independent of the data recalled in the current period in the limit as $N \to \infty$. It suffices to prove that data recalled by association in period $t-1$ are independent of the data recalled in period t in the limit as $N \to \infty$. Consider a particular datum recalled by association in period t. The probability that this datum was not recalled by association in period $t-1$ is bounded by

$$\left[1-\mu\frac{l(N)}{N}\right]^{l(N)} \tag{A.3}$$

where μ is defined as

$$\mu = \max_{\varepsilon,\varphi}\frac{f(\varepsilon|0)\rho(\varepsilon|\varphi)}{\int_{-\infty}^{+\infty} f(s|0)\rho(s|\varphi)ds} < \infty$$

If we demand equation **??** hold for all $l(N)$ data points recalled in period t we have that

$$\left[1-\mu\frac{l(N)}{N}\right]^{l(N)^2} \tag{A.4}$$

From our assumption that $l(N) = O(N^{1/4})$, standard results imply that equation **??** converges to 0 as $N \to \infty$. Therefore the data recalled by association and rehearsal can be treated independently in the limit as $N \to \infty$.

To find the closed form for $\delta(\varphi)$, first note that the mean of the data newly recalled in each period through association (as opposed to rehearsal) is sampled according to the recall probability function

$$\exp\left(m\varepsilon_i\varphi_t\right)$$

Therefore the data recalled through association in affective state φ_t will be distributed with a probability density function proportional to

$$\begin{aligned}\exp\left(-\frac{{\varepsilon_i}^2}{2}\right)\exp\left(m\varepsilon_i\varphi_t\right) &= \exp\left(-\frac{{\varepsilon_i}^2-2m\varepsilon_i\varphi_t+m^2\varphi_t^2}{2}\right)\exp\left(-\frac{m^2\varphi_t^2}{2}\right)\\ &= \exp\left(-\frac{(\varepsilon_i-m\varphi_t)^2}{2}\right)\exp\left(-\frac{m^2\varphi_t^2}{2}\right)\end{aligned}$$

Once this is normalized to a total probability of 1 we find the probability density function of a normal random variable with mean $m\varphi_t$. This logic for the associative recall applies in each period of the economy. When we average over the data recalled by association in period t and the data recalled through rehearsal (which is in effect a combination of data recalled by association in prior periods) we find the equation defined in our proposition. $\square$

A.1.1 Proofs of Price Time-Series Effects

Proposition 2. *For $\rho \in (0,1)$ sufficiently large asset prices exhibit overreaction in the short-run*

$$E_t^*[P_{t+1} - P_t | \varepsilon \geq 0] > E_t^*[P_{t+1} - P_t | \varepsilon_t < 0] \tag{A.5}$$

Proof. We first prove that our claim holds for any fixed choice of $\overrightarrow{\varphi}_t = (\varphi_t, \varphi_{t-1}, \varphi_{t-2}, ...)$ (i.e., we condition the expectations on $\overrightarrow{\varphi}_t$), although the value of ρ clearly depends continuously on $\overrightarrow{\varphi}_t$. Since $\overrightarrow{\varphi}_t$ resides in a compact space, we can choose ρ sufficiently for equation **??** to hold (i.e., without the conditioning on $\overrightarrow{\varphi}_t$).

Consider an arbitrary choice of $\overrightarrow{\varphi}_t$. First we write the price change formula as

$$\begin{aligned} P_{t+1} - P_t &= \frac{\varepsilon_{t+1}}{r} + \frac{1+r}{r^2} m(1-\rho) \left[\sum_{\tau=0}^{\infty} \rho^\tau \varphi_{t+1-\tau} - \sum_{\tau=0}^{\infty} \rho^\tau \varphi_{t-\tau} \right] \\ &= \frac{\varepsilon_{t+1}}{r} + \frac{1+r}{r^2} m(1-\rho) \sum_{\tau=0}^{\infty} \rho^\tau \sigma_{t+1-\tau} \end{aligned}$$

Note that $E_t^*[\varepsilon_{t+1}] = 0$ and given our selection of a sequence of affective states, the innovations to affect before period t are the same in the event $\{\varepsilon_t \geq 0\}$ and $\{\varepsilon_t \leq 0\}$. Using these two facts we can write

$$\begin{aligned}
E_t^*[P_{t+1}-P_t|\varepsilon_t &\geq 0,\overrightarrow{\varphi}_t]-E_t^*[P_{t+1}-P_t|\varepsilon_t<0,\overrightarrow{\varphi}_t] \\
&= \frac{1+r}{r^2}m(1-\rho)(E_t^*[\sigma_{t+1}+\rho\sigma_t|\varepsilon_t\geq 0]-E_t^*[\sigma_{t+1}+\rho\sigma_t|\varepsilon_t<0]) \\
&= \frac{1+r}{r^2}m(1-\rho)\left(E_t^*[\sigma_{t+1}|\varepsilon_t\geq 0,\overrightarrow{\varphi}_t]-E_t^*[\sigma_{t+1}|\varepsilon_t<0,\overrightarrow{\varphi}_t]\right)+ \\
&\qquad \frac{1+r}{r^2}m(1-\rho)\rho\left(E_t^*[\sigma_t|\varepsilon_t\geq 0,\overrightarrow{\varphi}_t]-E_t^*[\sigma_t|\varepsilon_t<0,\overrightarrow{\varphi}_t]\right)
\end{aligned}$$

To sign this equation, we must analyze the the expectations. Note that the events on which our two expectations are conditioned can be broken down into individual events where $\varepsilon_t>0$ and $\widetilde{\varepsilon}_t=-\varepsilon_t$. Under this notation, let σ_t , σ_{t+1} denote the innovation to affect given ε_t and $\widetilde{\sigma}_t$, $\widetilde{\sigma}_{t+1}$ denote the innovation to affect given $\widetilde{\varepsilon}_t$ Consider a particular realization $\varepsilon_t,\varepsilon_{t+1}>0$, which yields

$$\begin{aligned}
\sigma_t-\widetilde{\sigma}_t &= (1-\varphi_{t-1})(e^{-\alpha\varepsilon_t}-e^{-\varepsilon_t})>0 \\
\sigma_{t+1}-\widetilde{\sigma}_{t+1} &= -(1-\varphi_{t-1})(e^{-\alpha\varepsilon_t}-e^{-\varepsilon_t})(1-e^{-\varepsilon_{t+1}})
\end{aligned}$$

This then yields

$$(\sigma_{t+1}-\widetilde{\sigma}_{t+1})+\rho\left(\sigma_t-\widetilde{\sigma}_t\right)=(1-\varphi_{t-1})(e^{-\alpha\varepsilon_t}-e^{-\varepsilon_t})\left(\rho-(1-e^{-\varepsilon_{t+1}})\right)$$

which is positive for $\rho<1$ sufficiently large. If we instead consider realizations $\varepsilon_t>0,\varepsilon_{t+1}<0$ we find

$$\begin{aligned}
\sigma_t-\widetilde{\sigma}_t &= (1-\varphi_{t-1})(e^{-\alpha\varepsilon_t}-e^{-\varepsilon_t})>0 \\
\sigma_{t+1}-\widetilde{\sigma}_{t+1} &= -(1-\varphi_{t-1})(e^{-\alpha\varepsilon_t}-e^{-\varepsilon_t})(1-e^{\alpha\varepsilon_{t+1}})
\end{aligned}$$

which in turn yields

$$(\sigma_{t+1} - \widetilde{\sigma}_{t+1}) + \rho\,(\sigma_t - \widetilde{\sigma}_t) = (1 - \varphi_{t-1})(e^{-\alpha\varepsilon_t} - e^{-\varepsilon_t})\,(\rho - (1 - e^{\alpha\varepsilon_{t+1}}))$$

which is again positive for $\rho < 1$ sufficiently large. Since the required value of ρ depends (in a continuous fashion) on the realizations of $\varepsilon_t, \varepsilon_{t+1}$ considered, we can find a value of $\rho < 1$ sufficiently large that integrating over values of $\varepsilon_t, \varepsilon_{t+1}$ yields

$$E_t^*[P_{t+1} - P_t|\varepsilon_t \geq 0, \overrightarrow{\varphi}_t] > E_t^*[P_{t+1} - P_t|\varepsilon_t < 0, \overrightarrow{\varphi}_t]$$

As noted at the outset of our proof, since $\overrightarrow{\varphi}_t$ resides in a compact space we can choose ρ sufficiently high that our result holds unconditionally of $\overrightarrow{\varphi}_t$,[27] which implies equation **??**. □

Proposition 3. *Our model predicts long-run correction of the price effect of changes in affect in the current period*

$$\underset{\tau\to\infty}{Lim}\; E_t^*[P_{t+\tau} - P_t|\varepsilon_t \geq 0] < \underset{\tau\to\infty}{Lim}\; E_t^*[P_{t+\tau} - P_t|\varepsilon_t < 0]$$

Proof. Note that we can write the price in each period as

$$\begin{aligned} P_t &= \frac{d_t}{r} + \frac{1+r}{r^2} m(1-\rho) \sum_{k=0}^{\infty} \rho^\tau \varphi_{t-k} \\ P_{t+\tau} &= \frac{d_{t+\tau}}{r} + \frac{1+r}{r^2} m(1-\rho) \sum_{k=0}^{\infty} \rho^\tau \varphi_{t+\tau-k} \end{aligned}$$

Let $\overline{\varphi}$ denote the mean of the ergodic distribution of φ_t. Since our expectation is not

[27] In effect we choose ρ sufficiently high that it holds for $\overrightarrow{\varphi}_t$ with sufficiently high probability that our result extends to expectations unconditional on $\overrightarrow{\varphi}_t$.

conditional on price changes, we know that in all periods priors to t the expected value of φ_t is equal to the mean of the ergodic distribution, which implies

$$(1-\rho)E_t^*\left[\sum_{k=0}^{\infty}\rho^{\tau}\varphi_{t-k}|\varepsilon_t \geq 0\right] = (1-\rho)E_t^*\left[\varphi_t|\varepsilon_t \geq 0\right] + \rho\overline{\varphi}$$

Similarly we have as $\tau \to \infty$

$$(1-\rho)E_t^*\left[\sum_{k=0}^{\infty}\rho^{\tau}\varphi_{t+\tau-k}|\varepsilon_t \geq 0\right] \to \overline{\varphi}$$

Since φ_{t-1} is drawn from the ergodic distribution, we have

$$E_t^*\left[\varphi_t|\varepsilon_t \geq 0\right] = \overline{\varphi} + E_t^*\left[\sigma_t|\varepsilon_t \geq 0\right] > \overline{\varphi}$$

since $E_t^*\left[\sigma_t|\varepsilon_t \geq 0\right] > 0$, which implies

$$\underset{\tau\to\infty}{Lim} E_t^*[P_{t+\tau} - P_t|\varepsilon_t \geq 0] < 0 \tag{A.6}$$

Symmetrically, since $E_t^*\left[\sigma_t|\varepsilon_t \leq 0\right] < 0$ we have

$$E_t^*\left[\varphi_t|\varepsilon_t \leq 0\right] < \overline{\varphi}$$

so we have

$$\underset{\tau\to\infty}{Lim} E_t^*[P_{t+\tau} - P_t|\varepsilon_t \leq 0] > 0 \tag{A.7}$$

Combining equations **??** and **??** yields our result. $\square$

B Total Returns as Determinant of Affect

In this appendix we provide a brief discussion of an alternative model that uses total returns to define the innovations to affect. We assume the following form for the innovations to affect

$$\begin{aligned} \sigma_{t+1} &= P_{t+1} + d_{t+1} - (1+r)P_t \qquad \text{(B.1)} \\ \varphi_{t+1} &= \varphi_t + \sigma_{t+1} \end{aligned}$$

This formulation does not incorporate immune neglect and does not presume an upper or lower bound to affect. We are required to abandon these aspects of our model due to the need to work with closed forms (the necessity of which is clear below).

Our pricing equations still apply - all that needs change is the process governing the evolution of the affect of the representative agent. Note that a positive return in period t is driven in part by the change in the agents affect in period t, which is in turn driven by the return in period t. The change in affect is, in this sense, endogenous. Since m is a free parameter, we normalize it so that we can write the pricing equation as

$$P_t(\varphi) = \frac{d_t}{r} + m \sum_{\tau=0}^{\infty} \rho^{\tau} \varphi_{t-\tau} \qquad \text{(B.2)}$$

One might worry that the endogeneity of innovations to affect might lead to indeterminacy of predictions. For example, intuition suggests that, especially for mild dividend innovations, changes to affective state may become a self-fulfilling prophecy in that downward changes in affect in period t reduce the price in period t, which in turn reduces the period t return, which (closing the loop) implies a downward change in affect. If this intuition were correct, then asset prices would, to the extent that they are determined by affect, be arbitrary.

However, we can show that this is impossible in our structure and that a single price prediction can be made. We use the linearity of equation **??** to express the evolution of affect (and hence market prices) through a closed form solution for our model. Assuming that innovations to affect are linear in returns is crucial for this part of our analysis. Without assuming linearity (or an equally convenient functional form) we would not have been able to "close the endogenous loop" caused by returns in the current period influencing affect in the current period, which in turn determine the returns in the current period.

To make our argument precise, we combine our affect innovation and price equations to find

$$
\begin{aligned}
\sigma_{t+1} &= P_{t+1} + d_{t+1} - (1+r)P_t \\
&= \frac{d_{t+1}}{r} + d_{t+1} - \frac{1+r}{r}d_t + m\sum_{\tau=0}^{\infty}\rho^{\tau}\left[\varphi_{t-\tau+1} - (1+r)\varphi_{t-\tau}\right] \\
&= \frac{1+r}{r}\left(d_t + \varepsilon_{t+1} - d_t\right) + m\sum_{\tau=0}^{\infty}\rho^{\tau}\left[\sigma_{t-\tau+1} - r\varphi_{t-\tau}\right] \\
&= \frac{1+r}{r}\varepsilon_{t+1} + m\sum_{\tau=0}^{\infty}\rho^{\tau}\left[\sigma_{t-\tau+1} - r\varphi_{t-\tau}\right]
\end{aligned}
$$

As a final step we bring together the σ_{t+1} terms to find the (rather complicated) autocorrelation process of innovations to affective state.

$$
\sigma_{t+1} = \frac{(1+r)}{r(1-m)}\varepsilon_{t+1} - \frac{mr}{1-m}\varphi_t + \frac{m}{1-m}\sum_{\tau=1}^{\infty}\rho^{\tau}\left[\sigma_{t-\tau+1} - r\varphi_{t-\tau}\right] \tag{B.3}
$$

Inserting equation **??** into our formula for affective valence, we find that the representative agent's affect is

$$\begin{aligned}\varphi_{t+1} &= \varphi_t + \sigma_{t+1} \\ &= \left(\frac{1-m-mr}{1-m}\right)\varphi_t + \frac{(1+r)}{r(1-m)}\varepsilon_{t+1} + \frac{m}{1-m}\sum_{\tau=1}^{\infty}\rho^{\tau}\left[\sigma_{t-\tau+1} - r\varphi_{t-\tau}\right]\end{aligned} \tag{B.4}$$

Our next step is to state the equation solely in terms of affective state. First we have

$$\sigma_{t-\tau+1} - r\varphi_{t-\tau} = \varphi_{t-\tau+1} - (1+r)\varphi_{t-\tau}$$

which implies we can write equation **??** as

$$\varphi_{t+1} = \left(\frac{1-m-mr}{1-m} + \rho\right)\varphi_t + \frac{(1+r)}{r(1-m)}\varepsilon_{t+1} - \frac{m}{1-m}\sum_{\tau=1}^{\infty}\rho^{\tau}\left((1+r)-\rho\right)\varphi_{t-\tau} \tag{B.5}$$

Equation **??** has a number of unappealing properties that lead us to reject it on both psychological and economic grounds. Psychology research on emotion (as well as common intuition) suggest that affective states are transient. However, the dynamics described by equation **??** are not obviously ergodic,[28] which implies that our model would not reflect the transient nature of emotion.

In addition, we find it counter-intuitive that affective state today should be negatively correlated with past affective states. Equation **??** implies that affect in period $t+1$ is negatively correlated with the affective state in period $t-1$ (and all earlier periods). Intuition suggests that, if anything, the valence of one's affect should be positively correlated with any given past affective state. In other words if a person has a more positive mood two days ago that person should be more likely (rather than less likely) to have a positive mood today, which is contradicted by equation **??**.

[28] If m is small and ρ is large, then φ_t resembles a process with a root exceeding 1. This would imply that affective state ought to grow unboundedly in either the positive or negative direction.

Finally, we note that numerical simulations of the alternative model using plausible values for m and ρ show that while the model evinces ergodic behavior (and so exhibits long-run correction), there is no short-run continued overreaction to information. In other words, the model where affective state is driven by total returns predicts that positive (negative) price changes in the present period predict negative (positive) price changes in all future periods. In addition, the novel prediction that excess volatility is highest during market downturns is a result of our assumption of immune neglect, which cannot be incorporated into a model where innovations to affect and total returns are determined jointly (and endogenously).

The predictions we made that are based solely on our asset pricing formula, equation **??**, continue to hold since the we use the same asset pricing equation in the analysis of the main text and this appendix. In fact, these predictions, which include all of the predictions regarding the cross-sectional features of asset pricing developed in section **??**, would hold regardless of the source of the changes in the representative agent's affective state.

In summary, of the predictions of our model, only those that are independent of the source of changes to the representative agent's affect continue to hold when we assume that affect is driven by total returns (as opposed to dividend innovations). To the extent that we judge a model (at least partially) by the ability to match regularities discovered in the data, our assumption that affect is driven by dividend innovations is superior to the assumption that affect is driven by total returns. Admittedly, we are required to use an unbounded, linear formula to map returns into changes in affect in addition to assuming that total returns drive affect. It is unclear how one could resolve the joint determination of affect and total returns without significantly modifying this component of our model from the form used in the main text.

C Returns Instead of Price Changes

The bulk of our time-series analysis, carried out in sections **??** and **??**, study price changes rather than returns. The reason for this is simple: it is exceedingly difficult to obtain closed form results regarding claims of, for example, return momentum. To see why, it suffices to examine the equation for asset returns

$$\begin{aligned} P_{t+1} + d_{t+1} - (1+r)P_t &= \frac{1+r}{r} d_{t+1} + \delta(\varphi_{t+1}) - (1+r)\left(\frac{d_t}{r} + \delta(\varphi_t)\right) \\ &= \frac{1+r}{r}\varepsilon_{t+1} + \big(\delta(\varphi_{t+1}) - \delta(\varphi_t)\big) - r\delta(\varphi_t) \end{aligned}$$

The first term has a mean of 0, while the second term reflects the time path of price biases that drove the results in the main body of the paper. The final term, $-r\delta(\varphi_t)$, reflects the reversion of returns. To put it another way, if we (for example) wanted to study short-run overreaction of returns, we would need to compute

$$E_t^*\left[P_{t+1} + d_{t+1} - (1+r)P_t|\varepsilon_t > 0\right] = E_t^*\big[\delta(\varphi_{t+1}) - \delta(\varphi_t)|\varepsilon_t > 0\big] - rE_t^*\left[\delta(\varphi_t)|\varepsilon_t > 0\right]$$

Our proof for the short-run overreaction of price changes was based on the fact that

$$E_t^*\big[\delta(\varphi_{t+1}) - \delta(\varphi_t)|\varepsilon_t > 0\big] > 0 > E_t^*\big[\delta(\varphi_{t+1}) - \delta(\varphi_t)|\varepsilon_t < 0\big]$$

To complete an analysis of the short-run overreaction of returns, we would need to compute

$$\begin{aligned} E_t^*\left[\delta(\varphi_t)|\varepsilon_t > 0\right] - E_t^*\left[\delta(\varphi_t))|\varepsilon_t < 0\right] &= m(1-\rho)\left(E_t^*\left[\varphi_t|\varepsilon_t > 0\right] - E_t^*\left[\varphi_t|\varepsilon_t < 0\right]\right) \\ &= m(1-\rho)\big(E_t^*\big[(1-e^{-\varepsilon_t})(1-\varphi_{t-1})|\varepsilon_t > 0\big] - E_t^*\big[(1+\varphi_{t-1})(e^{\alpha\varepsilon_t} - \end{aligned}$$

Note that since φ_{t-1} is unknown to the econometrician, this expectation involves an integration over the ergodic distribution of φ_{t-1} as well as the distribution of ε_t. To understand this difficulty, note that if the ergodic distribution was focused around $\varphi_{t-1} = 1$, then equation **??** would clearly be negative and short-run overreaction of returns would hold. On the other hand, if the ergodic distribution is more diffuse and α is not too large, then assessing whether short-run overreaction holds would depend on r and the magnitude of the short-run overreaction of prices to news.

Although we cannot provide analytical results on the impact of mnemonic biases on returns, we have conducted numerical simulations which show that for all of the parameters considered the results of sections **??** and **??** carry over to returns.[29] In other words, the numerical simulations imply that returns also exhibit short-run overreaction, long-run correction, and momentum.

[29] We suggested above that if α is small and the ergodic distribution is diffuse, then our results may not carry-over to returns. However, our simulations seem to imply that low values of α cause the ergodic distribution to place a large amount of probability mass near $\varphi_t = 1$, which implies that returns do exhibit the described time-series effects.