# A Theory of Decisive Leadership

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**Abstract**

We present a theory that rationalizes voters' preference for decisive leaders. Common definitions of decisiveness reference the speed of decision making. Although greater speed can be good or bad, agency problems between voters and politicians create a natural preference among voters for leaders who have a greater aversion to delay or more self-knowledge than the voters, and hence who make decisions more rapidly. In electoral contests, candidates with reputations for greater decisiveness prevail, and therefore earn larger rents from office holding. Officials who aspire to higher office therefore attempt to signal decisiveness by accelerating observable decisions. Perceived decisiveness also enhances a leader's ability to impose his or her own agenda on voters, and political polarization magnifies this effect.

"I'm the decider, and I decide what is best." - George W. Bush, April 18, 2006

## 1 Introduction

During elections, voters choose to support candidates partly based on stated positions concerning well-established issues. However, the voters also know new issues will arise after the winner of the election takes office, some of which will be entirely unanticipated. Consequently, voter support depends in part on perceptions of characteristics that bear on a candidate's ability to handle emerging situations and crises.

In this paper, we are concerned with one such characteristic, *decisiveness*. The literature on leadership defines decisiveness as the ability to come to a timely decision despite uncertainty (Simpson, French, and Harvey [31], Simon [30], Williams et al. [32]).[1] The

[1] Potworowski [26] lists definitions of indecisiveness distilled from psychological literature, including "[p]rolonged decision latency (in deciding or implementing decisions)," "[p]utting off decisions (e.g., decisional procrastination and strategic waiting)," and various other related concepts.

dominant perspective expressed in popular media holds that decisive leadership is advantageous because it promotes timely and consistent action (Williams et al. [32]).[2] This view portrays indecisiveness as especially costly during crises because "[s]peed matters, and time is a leader's enemy..." (Garcia [14]; see also Yukl [33], Holsti [15], and Pillai and Meindl [23]). A failure to act quickly and appropriately within the "Golden Hour of crisis response," such as the Bush Administration's delay in deploying resources to assist the victims of flooding in New Orleans after Hurricane Katrina, elicits widespread condemnation (Garcia [14]). Thus, labeling a leader "indecisive" is widely construed as an indictment of his or her suitability for office.[3] Indeed, Simon [30] goes so far to claim that, in contexts where actions must be taken quickly, "[b]ehavioral ambivalence is unacceptable... Leaders who vacillate will not retain their positions." Opinion surveys show that voters place substantial weight on perceived decisiveness,[4] and statistical analysis confirms that this perception predicts voting behavior (Williams et al. [32]).[5]

As a general matter, greater speed in the face of uncertainty can be either good or bad. In principle, effective leadership should require "negative capability," defined as "the capacity to sustain reflective inaction," as well as "positive capability," the capacity for action (Simpson et al. [31]). Fast decision makers may "know their own minds," or they may be hasty and impulsive. Slow decision makers may equivocate, or are they may be deliberate and thoughtful.[6] What then accounts for the widespread notion that effective leaders exhibit a "bias for action" rather than "paralysis by analysis" (Kelman et al. [18])? Why do "...dominant societal conditions militate against restraint and inaction on the

[2]Indeed, some studies have concluded that decisive leaders are more effective; see Deluga [12] and Kelman et al. [18].

[3]For example, Lindsey Graham criticized Barack Obama by calling him "a weak and indecisive president" (Washington Post, May 5, 2014).

[4]In one Gallup poll, respondents were asked to rate the importance sixteen characteristics when evaluating presidential candidates. Being "a strong and decisive leader" was considered "essential" by 77% of repsondents, highest of any characteristic, and "not that important" by 1% of respondents, lowest of any characteristic. (Jones [17]).

[5]Voter perception of politicians' personal traits and their implications for electoral results has not attracted much attention within the political economy literature. Until recently, political scientists typically viewed voting based on a candidate's personal characteristics as "irrational" (Shabad and Anderson [28]). However, recent studies have found that these characteristics significantly influence voters' beliefs and behavior (Maurer et al. [21], Pillai and Williams [24], Pillai et al. [25], Shamir [29]). Most of the literature studies broadly defined leadership traits such as "charisma." Rapoport, Metcalf, and Hartman [27] argue that voters make inferences about politicians' policy preferences from their perceived traits, and vice versa.

[6]For example, Angela Merkel justified delay in resolving the Greek financial crisis by saying that Germany would not make "hasty decisions" (The London Times, August 25, 2012).

part of leaders, however reflective their intent" (Simpson et al. [31])?[7]

This paper makes four main contributions. First, we formalize the notion of decisiveness and develop a theoretical framework suitable for studying it. Second, we show that a general electoral preference for decisive leaders, defined as a leader that is more willing than the typical citizen to reach decisions despite uncertainty, emerges naturally from the nature of the agency relationship between voters and the politician. Third, we point out that this preference creates an incentive for politicians to signal decisiveness by making highly visible decisions more rapidly, and we explore various implications of the signaling, summarized below. Fourth, we demonstrate that perceived decisiveness enhances a leader's ability to impose his or her own agenda on voters. Greater political polarization magnifies this benefit, and therefore amplifies the incentive to appear decisive by rushing through deliberations.

With respect to our first contribution, a central feature of our framework is the assumption that policy preferences consist of common and idiosyncratic components. When an issue arises, the office holder takes time to learn about these components, and selects a policy when the costs of further delay match the expected benefits of incremental information. As an example, suppose the office holder must formulate a policy response to a financial crisis that could potentially precipitate a severe recession. Taking the time to diagnose the underlying causes and explore suitable solutions may improve the response, but also allows the crisis to deepen. Additionally, investigation informs the office holder about both the aggregate and distributional implications of the various alternatives. While everyone shares common objectives with respect to the aggregate component, their interests plainly conflict with respect to distribution.

Within our framework, decision speed is interpretable as an index of decisiveness because it captures the individual's willingness to take action at any given point in time conditional on a fixed rate of information acquisition. It varies from one person to another for two reasons. First, some people see the costs of deferring a decision as higher than others (aversion to delay). Second, some start out with a more precise understanding of their idiosyncratic preferences than others (self-knowledge).[8] Thus, someone with high delay aversion and precise self-knowledge will display a "bias for action," while someone

[7] These observations encompass business enterprises, the public sector, and even voluntary organizations: "Although the language used in the different sectors varies, the imperative is similar" (Simpson et al., 2002).

[8] Decision speed could also vary if some people gather information more efficiently than others. That variation would not, however, reflect differences in decisiveness as the term is commonly understood. A more decisive individual decides more rapidly conditional on a fixed rate of information acquisition.

with low delay aversion and poor self-knowledge will exhibit "paralysis by analysis."

With respect to our second contribution, we show that, for each voter, the preferred politician perceives greater costs of delay than the voter and has greater self-knowledge, and as a result the preferred politician makes a given decision more rapidly than the voter. Because this statement holds for all voters, decisive candidates (defined as those who would make a given decision more rapidly than the median voter) have an electoral advantage over indecisive ones. Indeed, there exist Condorcet-winning candidates, and they are all decisive decision makers.

Intuitively, the preference for decisive leaders is a natural outgrowth of the agency problem between voters and politicians. A voter does not benefit from any information the office holder learns about the idiosyncratic component of the politician's policy preference. On the contrary, the acquisition of such information creates risk from the voter's perspective since the politician tunes the policy to the information he acquires about his idiosyncratic preference. Consequently, the voter would ideally like to elect a politician with perfect self-knowledge who shares the voter's views on the costs of delay. Unfortunately, no politician has perfect self-knowledge. To reduce the risk arising from the acquisition of information concerning the politician's idiosyncratic preferences, the voter would like any office holder with imperfect self-knowledge to make decisions more rapidly than the voter's ideal politician, who in turn decides more rapidly than the voter. Since all voters favor politicians who make decisions faster than themselves, a majority of the population would like to elect faster-than-average leaders who "know their own minds" and perceive greater-than-average costs of delay.

The preceding observation leads naturally to our third main point: politicians holding lower offices who aspire to higher office will attempt to signal decisiveness by accelerating their observable decisions. We formalize this observation in a setting where heterogeneity is limited to delay aversion, focusing on the case in which the median voter thinks all politicians are naturally inclined to make decisions too slowly.[9] We then consider implications for voter welfare, candidate self-selection, and transparency.

Because signaling increases decision speed, one might conjecture that it would benefit most voters. As we show, signaling is a rather poor solution for politicians' tendency to delay because its effect is smallest where the need for a corrective influence is greatest (i.e., when politicians are indecisive) and greatest where that need is smallest (i.e., when

[9]We examine the case in which politicians signal self-knowledge rather than an aversion to delay in an online appendix. Most of the results are similar, but there are some significant differences.

politicians are decisive). In the latter case, it can cause the politician to make decisions that are excessively hasty from every voter's perspective. We also demonstrate that, ironically, the signaling equilibrium provides the greatest rents to the median voter's least favorite politicians. This observation has potentially significant implications for the distribution of politicians' characteristics in settings with endogenous candidacy.

We define transparency as the ability of voters to observe the decision process underlying multiple choices made by the politician in a lower office prior to the current election. We show that greater transparency of decision making leads politicians to signal through a pattern of consistent decisiveness rather than occasionally displaying extreme decisiveness. As a result, greater transparency reduces the signaling distortion and favorably resolves the welfare ambiguity associated with signaling. However, too much transparency is welfare-reducing. From the perspective of institutional design, natural objectives therefore come into conflict: on the one hand, transparency promotes accountability; on the other hand, it may cause politicians to make decisions too slowly from the voters' perspective.

We make our fourth main point by extending the model to encompass the possibility that politicians with ex ante heterogeneous policy preferences can make ideological compromises by, for example, granting interest groups "seats at the table." In a Downsian-style electoral competition between two candidates whose policy inclinations are equally distant from those of the median voter, the more decisive candidate will prevail despite striking the more modest compromise. A greater ability to resist compromise translates into larger rents for the office holder. In effect, more decisive candidates impose their personal agendas on the electorate.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 sets forth the basic model, and section 4 uses it to rationalize voters' preference for decisiveness. Section 5 investigates the properties of equilibria wherein politicians signal decisiveness while in lower office. Section 6 extends our model to cover settings with ex ante heterogeneity among politicians' policy preferences, as well as mechanisms for ideological compromise. Section 7 provides some brief concluding remarks. All proofs appear in the appendix.

## 2 Related Literature

Our model is situated within the political agency literature. Most of this literature focuses on the potential misalignment between the policies chosen by politicians and the preferences of the electorate. In contrast, we focus on misalignments between politicians and voters involving decision-making strategies.

Our paper touches on the growing decision-theoretic literature concerning indecision. Starting with Aumann [1], most authors have treated indecisiveness as incompleteness of the preference relation. From this perspective, a person is deemed indecisive between two options if he or she has no preference ranking between them. Recent papers in this literature elaborate on this basic premise. For example, Ok, Ortoleva, and Riella [22] explore the difference between indecisive tastes and indecisive beliefs. More recently some papers have characterized indecision as a preference for choice deferral (e.g., Kopylov [19]), but these models also involve incomplete preferences. Like Kopylov [19], we take the view that decisiveness and indecisiveness are inherently dynamic phenomena. In contrast to the decision-theoretic literature, we characterize people as less decisive if they either have greater uncertainty about their own (complete) preferences or perceive smaller costs of delay. In our model, those individuals take longer to make up their minds, even though everyone acquires and accurately processes information at the same rate.

Our analysis also relates to a branch of the literature originating with Conlisk [10] that explores settings in which agents must take time or pay costs to learn about their own preferences; see also Ergin and Sarver [11] for a more recent decision-theoretic treatment. Our contribution is embedding a related model of choice into a model of political agency and exploring the implications.

Finally, our focus on signaling touches on the large agency literature studying settings where an agent signals through an action that he or she is disinclined towards rent seeking. If one interprets the time a politician takes to make a decision as rent-seeking, it may seem strange that signaling can both reduce the rent-seeking and reduce voter welfare. In our setting, the politicians choice of the time to make a decision is only partially rent-seeking. The voters would like the politician to take some time, but not too much, to make a decision. While the politician is naturally inclined to take too long to make a choice from the voters' perspective, it is relatively easy for signaling to induce hasty decision-making (from a voter's perspective).

# 3 The model

## 3.1 Agents and preferences

We study environments with two types of agents, voters and politicians. Voters select among politicians in elections. Ultimately one politician takes office and selects a policy $p \in \mathbb{R}$. Agents' preferences over policies depend on the state of the world, which is unknown at the outset. The politician spends time $\tau \geq 0$ gathering information and thinking about options before making a choice. Delay is costly, so agents also care about $\tau$.

We can decompose the fully informed ideal policy for each voter into two components. The first, $\theta$, is common across all agents. It represents the aspects of the policy over which there is broad agreement – for example, increasing defense spending during wartime. All agents have a common prior belief that $\theta$ is distributed normally with mean 0. The second component, $x_i$ for agent $i$, is idiosyncratic and distributed independently across agents. It represents aspects of the policy options for which there is no broad agreement – for example, $i$'s assessment of the moral and practical benefits and costs of nation-building missions. We assume $x_i$ is distributed normally with mean $\mu_i$, and we index voters so that $j > i$ if $\mu_i > \mu_j$. We use $f_\mu$ to denote the population distribution of $\mu_i$; we assume it has full support over $[\underline{\mu}, \overline{\mu}]$ and is symmetric around 0. The ideal point for agent $i$ is $\theta + x_i$.[10] The agent initially knows $\mu_i$, but not $x_i$ or $\theta$.

After taking office, politician $P$ invests time $\tau$ gathering signals about $\theta$ and $x_P$, incurring perceived costs of $c_P\tau$, $c_P > 0$, to the politician and $c_i\tau$, $c_i > 0$, to agent $i$. At the outset, we assume for the sake of simplicity that all politicians share the same *ex ante* idiosyncratic preferences, $\mu_P$. We introduce heterogeneity among politicians along this dimension in section 6. We allow for the possibility that agents disagree about the costs of delay. The values of $c_i$ are drawn independently from a distribution $f_c$; we assume it has full support over the interval $[\underline{c}, \overline{c}] \subset \mathbb{R}_{++}$, is symmetric around 0, and is independent of $\mu$.[11] The realized cost for any particular agent is private information.

We assume that the variances of agent $i$'s initial forecast errors, $E_0[\theta^2]$ and $E_0[x_i - \mu_i]^2$, are $\alpha/\phi$ and $\beta_i/\phi$, respectively. As mentioned in the introduction, we allow for the possibility that politicians differ with respect to the parameter $\beta_P$.[12] Accordingly,

[10] Bernhardt et al. [2] use a similar structure to model uncertainty about voters' preferences, but their focus differs from ours.

[11] Our distributional assumptions for $\mu$ and $c$ are inessential but simplify some of the analytics.

[12] Whether or not we allow for heterogeneity of $\beta_i$ for other agents is inconsequential.

some politicians initially understand their own idiosyncratic preferences ("know their own minds") better than others, which we model as a low value of $\beta_P$. We will use $f_\beta$ to denote the distribution of $\beta$ among the population of politicians, and we assume it has full support over $[\underline{\beta}, \overline{\beta}] \subset \mathbb{R}_{++}$ where $f_\beta(\beta) \geq \underline{f}_\beta > 0$ and $f_\beta(\beta) \leq \overline{f}_\beta < \infty$.[13]

Information gathering (which may include introspection about the decision maker's own preferences) and deliberation improve the accuracy with which the politician $P$ assesses $\theta$ and $x_P$. After investing time $\tau$, the posterior distributions $P$ holds about $E_\tau[\theta]$ and $E_\tau[x_P]$ are random variables that depend on $P$'s particular intervening observations. We assume that at any point in time $\tau$, $P$ can collapse all information received about $\theta$ and $x_P$, respectively, into two normally distributed sufficient statistics, $s^\theta$ and $s^x_P$, the first with mean $\theta$ and variance $\alpha/\tau$, the second with mean $x_P$ and variance $\beta_P/\tau$.[14] In that case, $P$'s forecasts as of time $\tau$ are

$$E_\tau[\theta] = \frac{\tau}{\tau+\phi} s^\theta$$
$$E_\tau[x_P] = \frac{\phi}{\tau+\phi}\mu_P + \frac{\tau}{\tau+\phi} s^x_P.$$

It is straightforward to verify that the time $\tau$ forecast error for $\theta$ (that is, $\theta - E_\tau[\theta]$) is distributed normally with mean zero and variance $\frac{\alpha}{\phi+\tau}$, while the time $\tau$ forecast error for $x_P$ (that is, $x_P - E_\tau[x_P]$) is distributed normally with mean zero and variance $\frac{\beta_P}{\phi+\tau}$. Thus, the variances of the time $\tau$ forecast errors asymptote to 0 as $\tau$ rises. As these formulas suggest, one can think of $\phi$ as parameterizing the amount of information contained in previously observed signals.

We assume agents experience quadratic losses when the adopted policy departs from their ideal, as well as linear losses from delay:

$$U_i(p,\tau) = -(p - x_i - \theta)^2 - c_i\tau$$

[13] In principle, one could likewise introduce heterogeneity with respect to $\alpha$. We investigated this possibility, but it did not contribute much additional insight. In any case, differences in $\alpha$ pertain more to technical expertise in collecting information than to characteristics naturally construed as decisiveness, which is our focus.

[14] One can view these formulas as pertaining to the limiting case (as $n$ goes to infinity) of a sequence of environments in which $P$ starts out with $\phi n$ iid normal signals and receives $n$ additional signals per unit time, each with variance proportional to $n$. Focusing on the limiting case is analytically advantageous because the formulas become continuous in $\tau$.

Utility for the governing politician is given by the same expression, with $P$ substituting for $i$.

### 3.2 Timing

Our basic model consists of the following three stages.

In stage 1, all politicians hold (different) offices. Their preferences are as described in the preceding subsection. Each politician chooses the amount of time to deliberate and chooses a policy. The decisions of the politician in this period are influenced by the strategic incentive to signal a type that is preferred by the voters.

In stage 2, two politicians compete in an election. Voters observe the choices the candidates made when they previously held office ($p$ and $\tau$). Voters cannot directly observe a candidate's self-knowledge ($\beta_P$) or perceived costs of delay ($c_P$). Instead, the voters make inferences about those characteristics based on the candidate's decisions in prior office (stage 1).

In stage 3, the politician who wins the stage 2 election assumes office, deliberates, and chooses a policy. Because there is no subsequent interaction, stage 3 decisions concerning $p$ and $\tau$ are non-strategic. The payoff to the losing politician from stage 2 is the same as for a voter with the same characteristics.

To calculate a politician's total payoff, we sum across stages and attach weights of $\lambda$ and unity to the stage 1 and stage 3 payoffs respectively.[15] A larger value of $\lambda$ could reflect time discounting, a lower probability of running for higher office, or longer time spent in lower office.

We solve the model through backwards induction. Section 4 focuses on the politicians' behavior in stage 3 if they win the election, and we focus on assessing the endogenous valence component of voters' preference for decisive politicians. Section 5 studies how the rewards received by the elected politician in stage 3 define the incentives in stage 1 to signal a type that is preferred by the voters and how the signaling behavior impacts the voters and the politicians.

[15] Recall that there is no immediate payoff in stage 2.

# 4 Voter preference for decisiveness

We begin by investigating voters' preferences over politicians' characteristics, assuming that politicians behave non-strategically when deliberating (as they will in stage 3). As a first step, we must determine how different types of politician would behave if elected. Given the quadratic loss function assumed in the previous section, $P$'s optimal policy choice after a deliberation period of length $\tau$ is

$$p^* = E_\tau[\theta] + E_\tau[x_P]$$

Within each stage, we measure time from the start of that stage. As of time 0, the politician's expected utility is

$$\begin{aligned} W_P(\tau, \delta) &= E_0\left[U_P(p, \tau)\right] \\ &= -E_0\left[(\theta - E_\tau[\theta]) + (x_P - E_\tau[x_P])\right]^2 - c_P\tau \\ &= -\frac{\alpha + \beta_P}{\tau + \phi} - c_P\tau \end{aligned} \tag{1}$$

The first term captures the costs of making a decision before learning the exact values of $\theta$ and $x_P$, and the second reflects the welfare loss from delay. Thus, the optimal length of the deliberation period is the solution to the following maximization problem

$$\max_{\tau \geq 0} - \frac{\alpha + \beta_P}{\tau + \phi} - c_P\tau \tag{2}$$

From the first-order condition, we infer that

$$\tau^*(\beta_P, c_P) = \sqrt{\frac{\alpha + \beta_P}{c_P}} - \phi \tag{3}$$

To ensure $\tau^* > 0$, we assume

$$\bar{c} < \frac{\alpha + \beta_P}{\phi^2}$$

Voters' preferences over the politician's speed of deliberation are more complex. We can write the expected utility of voter $i$ as of time 0 with politician $P$ in office and a deliberation period of length $\tau$ as follows

$$\begin{aligned} W_i^P(\tau,\delta) &= E_0\left[U_i(p,\tau) \mid \mu_P, \beta_P, c_P\right] \\ &= -E_0\left[(\theta - E_\tau[\theta]) + (x_i - E_\tau[x_P])\right]^2 - c_i\tau \\ &= -\frac{\alpha}{\tau+\phi} - E_0\left(x_i - E_\tau\left[x_P\right]\right)^2 - c_i\tau \end{aligned} \tag{4}$$

The first term represents the costs of making a decision before $P$ learns the exact value of $\theta$. The second terms captures the welfare loss due to the disparity between the voter's ideal and $P$'s expected idiosyncratic preferences as of time $\tau$. The final term is the cost of delay.

To simplify equation (4), note that we can rewrite $E_\tau[x_P]$ as

$$E_\tau[x_P] = \frac{\phi}{\tau+\phi}\mu_P + \frac{\tau}{\tau+\phi}x_P + \frac{\tau}{\tau+\phi}\varepsilon_P^x$$

where, by our assumptions, $\varepsilon_P^x \equiv s_P^x - x_P$ is normally distributed with mean 0 and variance $\beta_P/\tau$. After some algebra one finds that

$$\begin{aligned} E_0\left(x_i - E_\tau\left[x_P\right]\right)^2 &= E_0\left(x_i - \frac{\phi}{\tau+\phi}\mu_P - \frac{\tau}{\tau+\phi}x_P - \frac{\tau}{\tau+\phi}\varepsilon_P^x\right)^2 \\ &= \frac{\beta_i}{\phi} + \frac{\beta_P}{\phi}\frac{\tau}{\tau+\phi} + (\mu_i - \mu_P)^2 \end{aligned}$$

Putting all of these parts together yields:[16]

$$W_i^P(\tau,\delta) = \underbrace{-(\mu_i - \mu_P)^2}_{\text{Policy Preference}} - \underbrace{\frac{\alpha}{\tau+\phi} - \frac{\beta_P}{\phi} + \frac{\beta_P}{\tau+\phi} - \frac{\beta_i}{\phi}}_{\text{Risk Preference}} - \underbrace{c_i\tau}_{\text{Cost of Delay}} \tag{5}$$

The preceding expression decomposes the voter's preferences into a policy preference component, a risk preference component, and a component that reflects the cost of delay. The policy preference term captures the difference between the voter's and politician's preferred policy choices as of $\tau = 0$. The *risk preference* term reflects the implications of uncertainty concerning the voter's and politician's ideal policies. The *cost of delay* term captures the opportunity costs the politician imposes on the members of the public when

[16]We will occasionally use the alternative form $W_i(p^*;\tau) = -\left(\mu_i - \mu_P\right)^2 - \frac{\alpha_P - \beta_P}{\tau+\phi} - \frac{\beta_i}{\phi} - \frac{\beta_P}{\phi} - c_i\tau$.

deciding slowly.

We can further decompose the risk preference term into three components, as follows

$$\underbrace{-\frac{\alpha}{\tau+\phi}}_{\substack{\text{Risk from}\\ \text{Common Concerns}}} \qquad \underbrace{-\frac{\beta_P}{\phi}+\frac{\beta_P}{\tau+\phi}}_{\substack{\text{Risk from}\\ \text{Idiosyncratic Preference}\\ \text{of the Politician}}} \qquad \underbrace{-\frac{\beta_i}{\phi}}_{\substack{\text{Risk from}\\ \text{Idiosyncratic Preference}\\ \text{of the Voter}}} \tag{6}$$

The first term captures risk from residual uncertainty about the common component of policy preferences. The second term reflects risk from the politician's efforts to fine-tune the policy according to his idiosyncratic preference. When $\tau = 0$, this term is 0 because the politician has learned nothing about $x_P$ beyond its a priori mean $\mu_P$, and hence the voter bears no risk. As $\tau$ increases, the politician becomes more informed about his own idiosyncratic preferences, and hence the voter bears more risk. As $\tau \to \infty$, this term approaches $-\frac{\beta_P}{\phi}$, which is the risk penalty inflicted on the voter when the politician perfectly accounts for his actual idiosyncratic preferences. The final term captures voters' uncertainty regarding their own idiosyncratic policy preferences. Unlike the second term, it does not depend on $\tau$: though voters may learn about their idiosyncratic preferences as time passes, that information does not inform the politician's choice, and hence the welfare cost of this risk is the same regardless of when the politician chooses.

Our first proposition shows that a voter's favorite candidate perceives greater costs of delay than the voter ($c_P > c_i$) and has maximal self-knowledge (the smallest possible value of $\beta_P$). For any fixed $\tau$, a preference for smaller values of $\beta_P$ follows immediately from Equation 5, but the endogeneity of $\tau$ renders preferences over $\beta_P$ more complex, so there is something more to prove. Preferences over the politician's costs of delay are more subtle.

**Proposition 1.** *From the perspective of voter $i$, the ideal politician has minimal uncertainty about his own preferences ($\beta_P = \underline{\beta}$) and perceives the cost of delay per unit time to be*

$$c_P = \begin{cases} c_i \frac{\alpha+\underline{\beta}}{\alpha-\underline{\beta}} \text{ if } \alpha > \underline{\beta} \text{ and the result is less than } \overline{c} \\ \overline{c} \text{ otherwise} \end{cases}$$

The intuition for this result is straightforward. Voter $i$ does not benefit from any information $P$ learns about $x_P$. On the contrary, such information adds risk from $i$'s perspective. Consequently, $i$ would ideally like to elect a politician $P^*$ with $\beta_{P^*} = 0$ and $c_{P^*} = c_i$, who would spend time $\tau^*(0, c_i)$ making a decision.[17] Of course, with $\underline{\beta} > 0$, $i$'s

[17]Recall that if $\beta_{P^*} = 0$, then the politician knows his own mind perfectly. The voters suffer no risk

ideal politician doesn't exist. The best available choice for $i$ is to select a politician $P'$ with $\beta_{P'} = \underline{\beta}$. To reduce the risk associated with $P'$ learning about $x_{P'}$, $i$ would like $P'$ to make the decision even more rapidly than $P^*$ (so $\tau^*(\underline{\beta}, c_{P'}) < \tau^*(0, c_i)$). With $c_{P'} \leq c_i$, $P'$ would actually take *more* time than $P^*$ (because then we would have $\tau^*(\underline{\beta}, c_P) > \tau^*(0, c_i)$ by equation (3)). Consequently, it must be the case that $c_{P'} > c_i$, as the theorem implies.

Notice that if voter $i$ were to become a politician and make the decision, $i$ would take more time than $P^*$ in order to learn about $x_i$ (i.e., $\tau^*(\beta_i, c_i) > \tau^*(0, c_i)$). Hence $i$ would also take more time than $i$'s most preferred politician $P'$ (because $\tau^*(\underline{\beta}, c_{P'}) < \tau^*(0, c_i)$). Since all voters favor politicians who make decisions faster than themselves, a majority of the population would like to elect faster-than-average leaders who "know their own minds" ($\beta_P = \underline{\beta}$) and perceive greater-than-average costs of delay. Thus we begin to arrive at an explanation for the electoral success of decisive politicians.

As a general matter, existence of a Condorcet winner can be a problematic issue, particularly in settings with more than one dimension of heterogeneity as we have in this model (even given our provisional assumption that all politicians share the same ex ante idiosyncratic preferences, $\mu_P$). Fortunately, the model has three special properties that simplify matters. First, voter $i$'s utility depends on $c_P$ only through the decision time, $\tau$. We can therefore think of each politician as offering an alternative in $(\tau, \beta_P)$-space rather than in $(c_P, \beta_P)$-space. Second, as shown in the proof of Theorem 1, preferences are single-peaked in $\tau$ for any fixed $\beta_P$. Third, as is clear from equation (5), all voters prefer politicians with lower values of $\beta_P$. Existence of a Condorcet-winning candidate then follows immediately: pairing the median preferred value of $\tau$ with $\underline{\beta}$, we identify a candidate who majority defeats all other $(\tau, \underline{\beta})$ combinations. Moreover, every other alternative is unanimously viewed as inferior to at least one of the latter. From Theorem 1, it then follows that the value of $c_P$ for the Condorcet-winning candidate is greater than the population median. Hence the winning candidate will make decisions faster than the typical member of the community.

Formally, we define decisiveness in terms of the speed with which the politician makes decisions relative to $\tau_{med}$, the population median of $\tau^*(\beta_i, c_i)$:

**Definition 1.** *A politician of type* $(\beta_P, c_P)$ *is* ***decisive*** *if* $\tau^*(\beta_P, c_P) < \tau_{med}$.

The preceding discussion then yields the following corollary of Theorem 1:

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from the politician tuning his policy to what he learns about his idiosyncratic preference as there is nothing left to learn.

**Corollary 1.** *There exists a Condorcet winner with $\beta_P = \underline{\beta}$ and*

$$c_P = \begin{cases} c_{med}\frac{\alpha+\beta_P}{\alpha-\beta_P} \text{ if } \alpha\beta_P \text{ and the result is less than } \overline{c} \\ \overline{c} \text{ otherwise} \end{cases}$$

*where $c_{med}$ is the median of the distribution of $c$. Moreover, this candidate is decisive.*

Instead of evaluating a politician's decision speed relative to that of the typical citizen, one could compare it to the speed with which any given voter $i$ would like the politician to make a decision, $\tau^v(\beta_P; c_i)$. In the proof of Proposition 1, we showed that

$$\tau^v(\beta_P; c_i) = \sqrt{\frac{\alpha - \beta_P}{c_i}} - \phi \tag{7}$$

Notice that $\tau^v$ depends on $\beta_P$: from the voter's perspective, $P$'s deliberation introduces more risk when $\beta_P$ is larger.

**Definition 2.** *Voter $i$ considers politician $P$* ***hesitant*** *if $P$'s decision time is greater than $\tau^v(\beta_P; c_i)$ and* ***hasty*** *if it is less than $\tau^v(\beta_P; c_i)$.*

It is trivial to verify that exactly half the population considers the Condorcet-winning candidate hesitant, while the other half considers that candidate hasty. In other words, a majority of voters would oppose either an increase or a decrease in the candidate's decision speed.

# 5 Signaling decisiveness

In this section we explore the idea that politicians who expect to seek higher office (stage 2 of our model) may attempt to cultivate reputations for decisiveness by accelerating decisions made in lower offices (stage 1 of our model). The implications of our analysis differ according to whether politicians wish to signal an aversion to delay (high $c$) or self-knowledge (low $\beta$). In the main text we focus on signaling an aversion to delay as this proves to be the more interesting case – for example, it implies that the median voter's favorite politician type earns the lowest utility in equilibrium. We analyze signaling self-knowledge in the online appendix. Throughout we focus on fully separating equilibria in which the voters infer (correctly in equilibrium) the types of the politicians from the politician's choices in period 1.

For most of this section we will assume the median voter's preferences are monotonic in $c$. Either $\alpha < \beta$ or $c_{med}\frac{\alpha+\beta}{\alpha-\beta} > \overline{c}$ suffices to guarantee this property; both conditions ensure that any increase in $c$ up to $\overline{c}$ benefits the median voter by mitigating the effects of the politician's imperfect self-knowledge. At the end of this section, we comment briefly on the case in which the median voter's ideal lies on the interior of $[\underline{c}, \overline{c}]$. We will also continue to assume for simplicity that candidates share the same *ex ante* policy preferences, $\mu_P$. At the cost of carrying around some extra terms, one can easily extend this analysis to subsume known differences in *ex ante* preferences, reflecting for example the differing ideologies of two political parties. One can also incorporate the types of ideological compromises we introduce in the next section without qualitatively altering our conclusions (as shown in an earlier version of this paper).

## 5.1 Separating equilibrium

Consider the stage 1 decision problem facing a politician with a perceived cost of delay $c$ who expects to play the role of a candidate in the stage 2 election. From the preceding discussion, the candidate knows he will win the election if voters believe he has a greater aversion to delay than his opponent. Because we are studying fully separating equilibria, the stage 1 outcome will fully reveal both candidates' types. Therefore, when choosing the image he wishes to project in stage 1, he knows he will win only if the voters believe his cost of delay is greater than his opponent's actual cost of delay.

In effect, a fully separating equilibrium presents each candidate with a menu of feasible action-perception pairs, where the action is stage 1 decision time and the perception pertains to the politician's aversion to delay. For our purposes it is convenient to represent this schedule as a function $\tau^S : [\underline{c}, \overline{c}] \rightarrow \mathbb{R}_+$. One can think of each candidate as choosing the desired perception $\widehat{c}$, and then making a stage 1 decision after deliberating for the amount of time required to create that perception, $\tau^S(\widehat{c})$. The electoral victor will be the candidate whom voters believe is most averse to delay (that is, for whom $\widehat{c}$ is greatest). As mentioned before, with full separation, each candidate knows he will win if his chosen $\widehat{c}$ is greater that his opponent's true cost-of-delay parameter. Accordingly, we can write his stage 3 payoff as follows

$$\Pi(c,\widehat{c}) = -\int_{\underline{c}}^{\widehat{c}} \left[\frac{\alpha+\beta}{\tau^*(c)+\phi} + c\tau^*(c)\right] f_c(s)ds \qquad (8)$$
$$- \int_{\widehat{c}}^{\overline{c}} \left[\frac{\alpha-\beta}{\tau^*(s)+\phi} + 2\frac{\beta}{\phi} + c\tau^*(s)\right] f_c(s)ds$$

The first integral reflects the politician's payoff when he wins the election,[18] and the second is the politician's payoff when he loses the election to an opponent with a higher perceived aversion to delay.

How do stage 3 payoffs vary with the inference, $\widehat{c}$? Taking the derivative, we have:

$$\left.\frac{\partial\Pi(c,\widehat{c})}{\partial\widehat{c}}\right|_{\widehat{c}=c} = \frac{2\beta}{\phi}\frac{\tau^*(c)}{(\tau^*(c)+\phi)} f_c(c) \qquad (9)$$

The right hand side of equation 9 represents the benefit of winning an election against an opponent with the same aversion to delay. We interpret this term as an endogenous rent from holding office. These rents include (1) the benefit of tuning the policy to the politician's ex post idiosyncratic preference and (2) the avoidance of the risk associated with another candidate doing the same.

Including stage 1, a politician's total payoff is

$$V(c,\widehat{c}) = \Pi(c,\widehat{c}) - \lambda\left[\frac{\alpha+\beta}{\tau^S(\widehat{c})+\phi} + c\tau^S(\widehat{c})\right]$$

To determine the candidate's optimal choice of $\widehat{c}$ given the signaling schedule $\tau^S$, we take the derivative of $V$ with respect to $\widehat{c}$ and set it equal to zero: $\frac{\partial}{\partial\widehat{c}}V(c,\widehat{c}) = 0$. In a fully separating equilibrium the solution is $\widehat{c} = c$, which implies $\frac{\partial}{\partial\widehat{c}}V(c,c) = 0$, or equivalently

$$\left.\frac{\partial\Pi(c,\widehat{c})}{\partial\widehat{c}}\right|_{\widehat{c}=c} = \lambda c\left[1 - \left(\frac{\tau^*(c)+\phi}{\tau^S(c)+\phi}\right)^2\right]\frac{\partial\tau^S}{\partial\hat{c}} \qquad (10)$$

Observe that equation 10 is a nonlinear first-order differential equation. As usual, the equilibrium leaves the choice of the "worst" type undistorted, so we also have a boundary

[18] Since we are integrating a constant function, we could simpy write this as the probability of winning times the payoff in the event he wins. We write the integral explicitly for easy comparison to the payoff in the event the politician loses and to facilitate a comparison with the extensions to electoral competition with policy committments in Section 6.

condition, $\tau^S(\underline{c}) = \tau^*(\underline{c})$. A somewhat unconventional feature of this differential equation is that the coefficient of $\frac{\partial \tau^S}{\partial c}$ is zero at the initial condition, which renders $\left.\frac{\partial \tau^S}{\partial c}\right|_{c=\underline{c}}$ undefined. We can finesse this difficulty by reversing the mathematical roles of $\tau^S$ and $c$, treating equation 10 as a differential equation for the function $c^S(\tau)$, with initial condition $c^S(\tau) = \underline{c}$ for $\tau = \tau^*(\underline{c})$. In that case, we have $\left.\frac{\partial c^S}{\partial \tau}\right|_{\tau=\tau^*(\underline{c})} = 0$. Our next result describes some properties of the solution.

**Proposition 2.** *$c^S(\tau)$ is strictly decreasing in $\tau$ with $\tau < \tau^*(c^S(\tau))$ for $\tau < \tau^*(\underline{c})$ and strictly increasing in $\tau$ with $\tau > \tau^*(c^S(\tau))$ for $\tau > \tau^*(\underline{c})$.*

This proposition offers us two candidates for the separating function $\tau^S$: we can invert the solution $c^S$ restricting attention either (1) to $\tau < \tau^*(\underline{c})$, which yields a downward-sloping function $\tau^1(c)$ satisfying $\tau^1(c) < \tau^*(c)$, implying that greater delay aversion translates into greater speed and signaling causes leaders to accelerate decisions or (2) to $\tau > \tau^*(\underline{c})$, which yields an upward-sloping function $\tau^2(c)$ satisfying $\tau^1(c) > \tau^*(c)$, implying the opposite. Given our assumptions, there is no guarantee that either solution is globally incentive-compatible. Our next result tells us that only $\tau^1$ can serve as the separating function $\tau^S$. We show that a sufficiently large value of $\lambda$ – which one can interpret (for example) as a limit on the likelihood with which lower officeholders expects to run for higher office – guarantees global incentive compatibility of $\tau^1$.[19] In contrast, $\tau^2$ is never globally incentive compatible. Accordingly, we will henceforth associate $\tau^S$ with $\tau^1$.

**Proposition 3.** *There exists $\lambda^*$ such that for $\lambda > \lambda^*$, $\tau^1$ is globally incentive-compatible. $\tau^2$ is never globally incentive-compatible.*

Together, Propositions 2 and 3 imply that signaling an aversion to delay accelerates decision making ($\tau^S(c) < \tau^*(c)$) – politicians act more decisive than they actually are. In addition, we learn that $\tau^S$ must be strictly decreasing in $c$, which raises the possibility that the non-negativity constraint ($\tau > 0$) may bind. We will eliminate this possibility by assuming, where necessary, that $\phi$ is sufficiently small.

Figure 1, which is based on a numerical simulation, depicts a typical equilibrium.[20]

[19] One might have hoped that, as in many signaling models, a supermodularity (or single-crossing) property would hold for $V(c, \widehat{c})$, as standard techniques for proving incentive compatibility (for example) would then apply. Unfortunately, that is not the case.

[20] The figure was generated using $\phi = 0.0001$, $\alpha = 1$, $\beta = 0.1$, $\underline{c}$= 100, and $\overline{c} = 500$. $c$ is distributed as per a truncated normal distribution with a mean of 0 and a standard deviation of 20. The example was checked for global incentive compatability numerically.

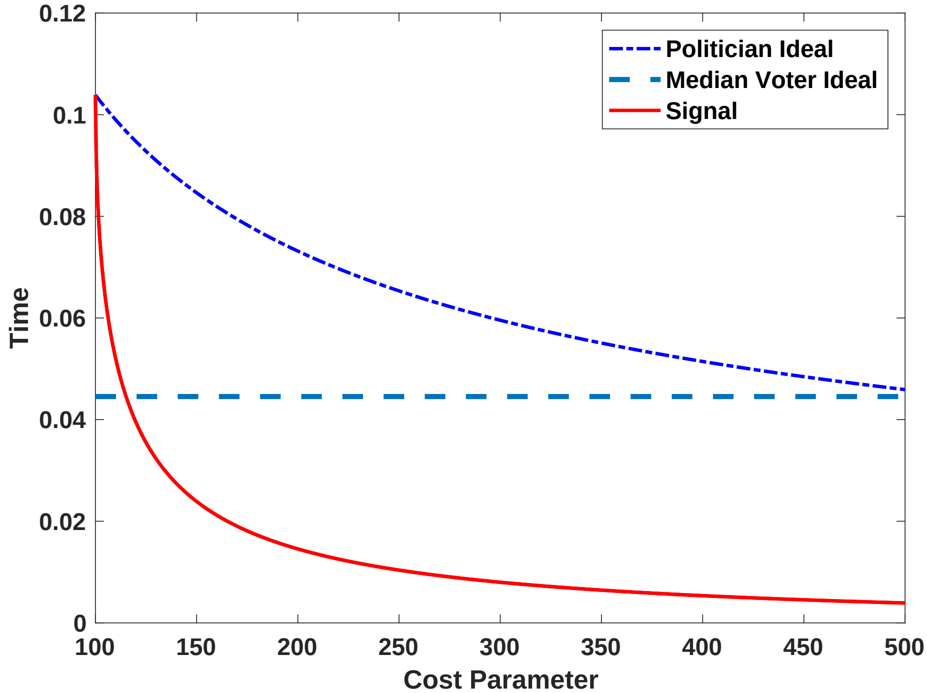


Figure 1: Equilibrium Versus Voter and Politician Optima With Signaling of Cost

The horizontal axis describes the politician's type, and the vertical axis describes the time taken to make a decision. The figure shows three functions relating the decision time, $\tau$, to the politician's delay aversion, $c$: the ideal from the politician's perspective ($\tau^*$), the median voter's ideal decision time ($\tau^v$), and the signaling equilibrium ($\tau^S$). Because $\alpha$ and $\beta$ are common across all agents, each voter has a single decision time they would prefer the politician to choose regardless of the politician's cost (see equation 7).[21]

## 5.2 Welfare

We have seen that voters regard non-strategic politicians as hesitant and that signaling increases decision speed. One might therefore think that signaling would benefit voters. In fact, that is not necessarily the case. Signaling is a rather poor solution for politicians' tendency to delay because the signaling incentive is smallest where the need for a corrective

[21] Recall that the politician's cost is irrelevant for determining the length of time a voter would like the politician to take to make a decision.

influence is greatest and greatest where that need is smallest. That pattern is evident from Figure 1. Signaling has no effect on politicians with the lowest values of $c$, for whom the gap between the voter and politician ideal is greatest. Moreover, its cumulative effect on politicians with the highest values of $c$, for whom the gap between $\tau^*$ and $\tau^v$ is smallest, can be enormous, causing them to spend little or no time pondering the common good even when they should. In the figure, the signaling curve is so steeply sloped that it crosses the voter ideal curve. Politicians to the left of the crossing remain hesitant from the voter's perspective, but those to the right become hasty. Thus the overall impact on voter welfare can be positive or negative, depending on the size of the signaling distortion and the distribution of politician types – specifically, whether it is skewed toward those with relatively high aversions to delay who overcorrect, or those with relatively low aversions to delay who undercorrect. That said, a "little bit" of signaling (sufficiently large $\lambda$) unambiguously improves welfare: when the signaling curve is sufficiently close to the politician ideal curve, it lies between the politician ideal and voter ideal curves.

## 5.3 Extensions

### 5.3.1 Endogenous candidate selection

As we saw in the previous section, the welfare effects of signaling can depend critically on the distribution of candidate types. While we have not endogenized that distribution, our model allows us to provide some useful insights. Our next result establishes that *lower* quality politicians receive higher expected payoffs in the separating equilibrium.

**Proposition 4.** *$V(c, c)$ is decreasing in $c$.*

Now imagine endogenizing the distribution of candidate types by appending the familiar citizen-candidate apparatus to our model (Besley and Coate [6]). Proposition 4 suggests that the distribution of politicians would be skewed toward types with low values of $c$. As a result, the effect of signaling would tend to be relatively small but beneficial, and voters would be more likely to complain that politicians are too hesitant.

### 5.3.2 The effect of transparency

Politicians generally make many decision while in office. We can classify a political institution as more or less transparent according to whether voters obtain information about the deliberations associated with a large or small fraction of these decisions. In this section, we examine the effect of changes in transparency on political outcomes. Intuitively, transparency leads politicians to “spread” their signals across many decisions. As a result, each choice is subject to less distortion, and deliberation times are closer to the politicians’ ideals. However, because politicians and voters have different ideals, transparency can hurt voters’ interests.

Formally, suppose the politician makes $N$ stage 1 decisions rather than one. Each has the structure described in section 3, and the $N$ stochastic realizations are independent. In $M \leq N$ instances, the electorate observes the deliberation process. For the purpose of studying transparency, we are interested in the effects of changing $M$ while holding $N$ fixed.

We denote the stage 1 actions as $\boldsymbol{\tau}^S = (\tau_1^S, ..., \tau_N^S)$. There are, of course, many ways to signal a single characteristic through multiple actions, and we consider fully separating equilibria with continuous action mappings $\tau_m^S : [\underline{c}, \overline{c}] \to \mathbb{R}_+$ for $m = 1, ..., N$, where voters observe $\tau_m$ for $m \leq M$. Note that in any such equilibrium, unobserved choices are undistorted: a politician with a perceived cost of delay $c$ sets $\tau_m = \tau^*(c)$ for $m = M+1, ..., N$.

The simplest equilibrium within this broad class treats all observable tasks symmetrically: $\tau_m^S = \underline{\tau}^0$ for $m = 1, ..., M$. With this restriction, the fully separating action function is the solution to the following differential equation

$$\left.\frac{\partial \Pi(c, \widehat{c})}{\partial \widehat{c}}\right|_{\widehat{c}=c} = M\lambda c \left[1 - \left(\frac{\tau^*(c) + \phi}{\tau^S(c) + \phi}\right)^2\right] \frac{\partial \underline{\tau}^0}{\partial c} \tag{11}$$

with boundary condition $\underline{\tau}^0(\underline{c}) = \tau^*(\underline{c})$. This is, of course, a slightly modified version of Equation 10.

How will politicians choose to signal their decisiveness? Will they do so through patterns of consistently decisive behavior, or will they occasionally display extreme decisiveness? We take the view that they will seek, and over time discover, the most efficient ways to signal.[22] Our next result establishes that this process drives them toward the symmetric equilibrium in which they are consistently decisive.

**Proposition 5.** *The symmetric equilibrium maximizes the payoff for every type of politi-*

[22] Plausible restrictions on out-of-equilibrium beliefs often point to the efficient separating equilibrium; see, for example, Cho and Kreps [8] or, for a problem with a more similar structure, Bernheim [3].

*cian within the set of fully separating equilibria.*

As the proof demonstrates, Proposition 5 holds because symmetry magnifies the rate at which a greater aversion to delay reduces the welfare loss from making decisions too quickly. It is unrelated to concavity of the representative politician's utility function.

Our final result shows that decision times converge to the politicians' ideals when politicians make many decisions that are observable to voters.

**Proposition 6.** *$\underline{\tau}^0(c)$ converges to $\tau^*(c)$ uniformly as $M \to \infty$.*

This result suggests that the desire to cultivate a reputation for decisiveness will distort any given deliberation to a smaller degree when the politician occupies a position that provides many opportunities to make visible decisions. Compare, for example, the signaling opportunities available to governors and legislators. As an executive, a governor has many opportunities to demonstrate decisiveness, but as a member of a larger deliberative body, a legislator likely has few. According to our theory, the legislator may feel compelled to act with extreme haste when opportunities for independent action arise. For example, she might craft and sponsor a bill in response to some emergent issue without adequate consideration or vetting.[23] In contrast, the governor might be in a position to act less precipitously without compromising his reputation.

In our model, the effect of greater transparency on voter welfare is, as a general matter, ambiguous. In some cases it is beneficial because it reigns in hasty decision making. In others it is harmful because it slows down hesitant decisions. With sufficiently high levels of transparency, we have $\underline{\tau}^0(c) > \tau^v(c)$ and signaling necessarily becomes welfare-improving because it no longer causes politicians to overshoot the voter ideal. However, incremental transparency beyond that point is harmful because it causes deliberation times to approach $\tau^*(c)$, which all voters regard as hesitant. From the perspective of institutional design, natural objectives therefore come into conflict: on the one hand, transparency promotes accountability; on the other hand, for the reasons we have discussed, it may exacerbate other agency problems between voters and office holders.

#### 5.3.3 Non-monotonic voter preferences

The case in which the delay aversion of the median voter's ideal politician lies on the interior of $[\underline{c}, \overline{c}]$ is considerably more complex. Politicians with low $c$ will wish to project

[23] We acknowledge that legislators may act hastily for other reasons, for example to set the legislative agenda.

greater delay aversion than they feel, and those with high $c$ will do the opposite. One can show that these opposing inclinations rule out the existence of fully separating equilibria. Instead, as in Bernheim [3] and Bernheim and Severinov [4], they can generate equilibria with a single pool at a point in the interior of $[\underline{c}, \overline{c}]$, generally near the median voter's ideal point. Moreover, depending on the model's parameters, the fraction of politicians who join this pool may be arbitrarily close to unity. In such cases, voters would observe uniformly high quality decision making by politicians in lower office, only to observe that quality deteriorates systematically, but to a highly varying degree, when politicians reach higher office.

# 6 Heterogeneous policy preferences and ideological compromises

In this section, we extend the model to encompass the possibilities that politicians have ex ante heterogeneous policy preferences and they can commit themselves in advance to ideological compromises. Because perceived decisiveness confers an electoral advantage, it enhances a leader's ability to impose his or her own agenda on voters. Greater political polarization magnifies this benefit, and therefore amplifies the incentive to appear decisive by rushing through deliberations.

## 6.1 The extended model

We assume the stage 2 election involves two candidates whose ex ante policy ideals lie equally far from the population median, but in opposite directions: $\mu_1 = -\mu_2 \equiv \mu = E[x_P]$, where subscripts indicate the candidate. Voters observe these ideological orientations. As in the previous section, we will also assume that politicians share a common level of self-knowledge, $\beta$. One can relax these assumptions at the cost of extra notational complexity.

Because the common interest will depend on as-yet-unrealized states of nature, politicians do not try to commit themselves to particular policies. Instead, prior to the election, each politician forms political alliances that are intended to offset or reinforce their ideological biases, if elected. One can think of these alliances as involving choices of key advisors – for instance, a hawkish foreign policy expert – and/or commitments to provide particular interest groups with "seats at the table."

To keep matters relatively simple, we employ a reduced-form representation of this process. Specifically, in stage 2, politician $P$ commits to an ideological adjustment, $\delta$, which shifts the policy chosen after a deliberation period of length $\tau$ from $p^* = E_\tau[\theta] + E_\tau[x_P]$ to $p^* = E_\tau[\theta] + E_\tau[x_P] + \delta$. Setting $\delta \neq 0$ is costly to the politician because it shifts the ultimate policy (if he is elected) away from his ideal point. However, because $\delta$ is observable, this strategy may increase his chance of electoral victory.

Due to technical issues bearing on the existence of best responses, we assume that, in the event of an electoral tie, the winner is the candidate who appears more attractive to the majority of voters based on decision speed rather than policy predisposition. If a majority favors neither candidate based on this characteristic, each wins with probability 1/2.

### 6.2 Electoral equilibrium

For the purpose of this section, we focus on stages 2 and 3 of the model, and assume voters believe the candidates' characteristics are $(\mu_1, \widehat{c}_1)$ and $(\mu_2, \widehat{c}_2)$ (recall that we are assuming away heterogeneity in $\beta$). Our objective is to determine the equilibrium values of the ideological adjustments, $\delta_1$ and $\delta_2$, as well as the expected rewards to each politician. As we will see, the winning candidate is the one who appears more attractive to the majority of voters based on decisiveness. That advantage enables the candidate to prevail despite making a smaller ideological concession than the rival. Because the office holder can implement a policy closer to his idiosyncratic ideal, winning benefits the winner above and beyond the endogenous rents from holding office that are present in our original model.

An ideological compromise of $\delta$ reduces the politician's expected stage 3 payoff conditional on a deliberation period of length $\tau$ by the fixed amount $\delta^2$. It follows that ideological compromises do not alter $\tau^*(\beta_P, \widehat{c}_P)$, the length of the politician's optimal stage 3 deliberations. As a result, a voter's expected payoff from electing a politician with characteristics $(\mu_P, \widehat{c}_P)$ who commits to a compromise of $\delta$ is precisely the same as from electing a politician with characteristics $(\mu_P + \delta, \widehat{c}_P)$ who makes no commitment. With this adjustment, we can continue to apply the formulas for voter preferences derived in Section 4.

The incremental benefit voter $i$ receives from electing candidate 1 rather than candidate

2 is

$$\begin{aligned}\Delta_i &= W_1^P - W_2^P \\ &= 2\mu_i\left(2\mu + \delta_1 - \delta_2\right) - \left(\tau_1 - \tau_2\right)c_i + \\ &\qquad \left[\left(\delta_2 - \mu\right)^2 - \left(\delta_1 + \mu\right)^2 + \left(\frac{\alpha-\beta}{\tau_2+\phi} - \frac{\alpha-\beta}{\tau_1+\phi}\right)\right]\end{aligned}$$

where $\tau_j$ refers to the anticipated speed of candidate $j$. We will use $M$ to denote the voter whose characteristics $\mu_M = 0$ and $c_M$ are each population medians (henceforth, the "median voter"). Consider the set of voters who agree with $M$ about the net value of electing candidate 1: $\Delta_i = \Delta_M$. If $-\delta_1 = \delta_2 = \mu$ and $\tau_2 = \tau_1$, this set includes all voters. Otherwise, $\Delta_i = \Delta_M$ is a linear relation dividing the space of voters into two half spaces according to whether $\Delta_i \lesseqgtr \Delta_M$. Under our assumptions (independence and symmetry of the distributions of $\mu_i$ and $c_i$), those half spaces have equal mass. It follows that a (weak) majority of voters always prefers the same candidate as the median voter. Thus we have:

**Lemma 1.** *If a voter with characteristics $(\mu_M, c_M)$ strictly prefers candidate $i$ to candidate $j$, then so does a majority of voters.*

Lemma 1 establishes that the outcome of the election depends on the preferences of the median voter, $M$. A little algebra reveals that candidate 1 wins with certainty if

$$(\mu + \delta_1)^2 - (\mu - \delta_2)^2 < \Phi_M(\widehat{c}_1, \widehat{c}_2) \tag{12}$$

where

$$\Phi_M(\widehat{c}_1, \widehat{c}_2) = \left(\frac{\alpha-\beta}{\tau(\widehat{c}_2)+\phi} + c_M\tau(\widehat{c}_2)\right) - \left(\frac{\alpha-\beta}{\tau(\widehat{c}_1)+\phi} + c_M\tau(\widehat{c}_1)\right)$$

Notice that $\Phi_M$ captures the component of the median voter's preference attributable to the candidates' decisiveness. If $\Phi_M > 0$, candidate 1 can win the election with an anticipated policy choice that is further from the median voter's ideal than candidate 2's anticipated choice. The following lemma tells us that a better reputation for decisiveness (defined in terms of proximity to the median voter's ideal) puts a candidate in a better position to win the election.

**Lemma 2.** $\frac{\partial}{\partial \widehat{c}_1}\Phi_M\left(\widehat{c}_1, \widehat{c}_2\right)$ *has the same sign as* $c_M\frac{\alpha+\beta}{\alpha-\beta} - \widehat{c}_1$, *and* $\frac{\partial}{\partial \widehat{c}_2}\Phi_M\left(\widehat{c}_1, \widehat{c}_2\right)$ *has the same sign as* $\widehat{c}_2 - c_M\frac{\alpha+\beta}{\alpha-\beta}$.

The preceding analysis implies that the likelihood of winning shifts discontinuously from 0 to 1 as candidate $i$ varies $\delta_i$ through the point at which $(\mu+\delta_1)^2-(\mu-\delta_2)^2=\Phi_M$. This property generates Downsian convergence with respect to the ideological adjustments $\delta_1$ and $\delta_2$. In equilibrium, the candidate who is less attractive to the median voter in terms of decisiveness (that is, the one with the larger value of $\frac{\alpha-\beta}{\tau_i+\phi}+c_M\tau_i$) perfectly aligns with the median voter through an ideological adjustment, but a majority votes for the other candidate, whose ideological adjustment is just sufficient to make the median voter indifferent. Accordingly, we have:

**Proposition 7.** *The Nash equilibria depend on the model's parameters as follows:*

1. *If $\mu^2 \geq \Phi_M > 0$, the unique pure strategy Nash equilibrium involves $\delta_1 = \sqrt{\Phi_M}-\mu$, $\delta_2=\mu$ , and the election of candidate 1.*

2. *If $\Phi_M > \mu^2$, all pure strategy Nash equilibria involve $\delta_1 = 0$ and the election of candidate 1.*

3. *If $\mu^2 \geq -\Phi_M > 0$, the unique pure strategy Nash equilibrium involves $\delta_2 = \mu - \sqrt{-\Phi_M}$, $\delta_1 = -\mu$, and the election of candidate 2.*

4. *If $-\Phi_M > \mu^2$, all pure strategy Nash equilibria involve $\delta_2 = 0$ and the election of candidate 2.*

5. *If $\Phi_M = 0$, then $\delta_1 = \delta_2 = -\mu$ and each candidate is elected with probability $\frac{1}{2}$.*

Notice how $\delta_i$ varies with candidate $i$'s relative decisiveness. When $i$'s decisiveness advantage is sufficiently large, $i$ wins the election without compromising (cases 2 and 4). As the advantage declines, $i$ continues to win, but makes larger and larger concessions in the direction of the median voter (cases 1 and 3). Once the advantage shifts to the competitor, $i$ loses despite compromising fully with the median voter. Thus, more (appropriately) decisive candidates successfully impose their own agendas on the electorate.

The ability of (appropriately) decisive candidates to resist compromise and thereby earn greater rents from office-holding provides additional incentives for candidates to seek reputations for decisiveness. In the next section, we investigate the manner in which signaling is affected by the parameter $\mu$, which measures the degree of partisan disagreement.

### 6.3 Signaling and the degree of partisanship

Conventional wisdom holds that the U.S. electorate has become increasingly polarized.[24] The degree of partisanship also differs sharply across different types of elections. For instance, it is less pronounced when the main competition for an office occurs between members of the same party (as is the case for many seats in the U.S. House of Representatives) than when it involves candidates from opposing parties (as is the case for U.S. Presidential elections). In this section, we show that politicians signal decisiveness more aggressively in regimes with greater partisanship. The intuition is straightforward: greater partisanship increases the net gains from winning an election and therefore increases the incentive to cultivate a reputation for decisiveness.

As in section 5, we assume that the median voter's preferences are monotonic in the politician's perceived costs of delay. For the reasons discussed in section 5.3.3, this simplification keeps the analysis of signaling tractable.

With ex ante heterogeneous policy preferences, the expected utility of a type $c$ politician who is viewed as type $\widehat{c}$ becomes

$$\begin{aligned}\Pi\left(c,\widehat{c}\right) \;=\; & -\int_{\underline{c}}^{\widehat{c}}\left[\left(\mu-\sqrt{\Phi_M(\widehat{c},s)}\right)^2+\frac{\alpha+\beta}{\tau^*(c)+\phi}+c\tau^*(c)\right]f_c(s)ds \\ & -\int_{\widehat{c}}^{\overline{c}}\left[\left(\mu+\sqrt{-\Phi_M(\widehat{c},s)}\right)^2+\frac{\alpha-\beta}{\tau^*(s)+\phi}+2\frac{\beta}{\phi}+c\tau^*(s)\right]f_c(s)ds\end{aligned} \tag{13}$$

Provided one redefines $\Pi\left(c,\widehat{c}\right)$ in this way, equation 10 still describes the fully-separating equilibrium, and Propositions 2, 3, and 4 continue to hold with relatively minor changes to the proofs.

Although $\mu$ has no effect on a politician's marginal costs of signaling higher delay aversion (stage 1 payoffs), it changes the marginal benefits as follows:

$$\begin{aligned}\frac{\partial^2\Pi\left(c,\widehat{c}\right)}{\partial\mu\partial\widehat{c}} \;=\; & \int_{\underline{c}}^{\widehat{c}}\left[\frac{1}{\sqrt{\Phi_M(\widehat{c},s)}}\frac{\partial\Phi_M(\widehat{c},s)}{\partial\widehat{c}}\right]f_c(s)ds \\ & +\int_{\widehat{c}}^{\overline{c}}\left[\left(\frac{1}{\sqrt{-\Phi_M(\widehat{c},s)}}\frac{\partial\Phi_M(\widehat{c},s)}{\partial\widehat{c}}\right)^2\right]f_c(s)ds>0\end{aligned} \tag{14}$$

[24] The available evidence points to sharp polarization among the political elite. However, the evidence on mass polarization is less clear; see Fiorina and Abrams [13].

The inequality in Equation 14 implies that the marginal benefit of an increase in perceived aversion to delay (that is, an increase in $\widehat{c}$) is increasing in $\mu$, the degree of partisanship. Intuitively, partisan differences increase the stakes associated with winning and losing, and hence make politicians more willing to incur costs that will help them reach office.

**Proposition 8.** *In settings where politicians signal aversion to delay, $\tau^S(c)$ is decreasing in $\mu$.*

As a practical matter this result suggests, for example, that politicians will cultivate decisive images more aggressively while in lower office if they aspire to higher offices for which elections are more highly partisan (for example, U.S. President rather than a Representative from a district closely allied with one party).

# 7 Conclusion

We have presented a theory that rationalizes voters' preferences for decisive leaders who reach decisions expeditiously despite possessing no special skill at information gathering or processing. In the setting we have studied, policy preferences have both common and idiosyncratic components. Office holders can take time to learn about both, but at a cost. We have demonstrated that agency problems between voters and politicians create natural preferences among voters for leaders who perceive higher costs of delay and have greater self-knowledge than the voters, and hence who make decisions more rapidly. We have also shown that, in electoral contests, candidates with reputations for greater decisiveness prevail and earn larger rents from office holding. These rents incentivize officials who aspire to higher office to signal decisiveness by accelerating observable decisions.

The desire to signal decisiveness induces politicians to make decisions more rapidly. However, as our numerical examples demonstrate, this motive can drive politicians to make decisions far more rapidly than voters would prefer. Indeed, the signaling incentive to make fast decisions is strongest where the problem of politician indecision is least severe, which suggests that the signaling motive is not particularly well-suited to solving the politician-voter agency problem.

In settings with heterogeneous delay aversion, signaling equilibria provide the greatest rents to the lowest quality politicians. Consequently, were we to extend the model by appending the familiar citizen-candidate apparatus to endogenize candidacy, the distribution of politicians would be skewed toward those with low perceived costs of delay. Those individuals are minimally affected by signaling incentives, and consequently are regarded by voters as too hesitant. In Appendix B, which studies a version of the model in which politicians signal self-knowledge, we find the opposite: the most decisive politicians receive the highest utility, and would therefore be over-represented in a model with endogenous candidacy. They are also the ones who are most affected by signaling, and consequently who are most likely to overshoot the median voter's ideal.

Finally, we have addressed the effects of the institutional context on the incentive to signal decisiveness. Increased transparency, defined according to the frequency with which the electorate can effectively gauge the speed of an office holder's responses to emerging issues, leads politicians to signal through consistent but less extreme decisiveness, rather than through intermittently extreme decisiveness. Once transparency reaches a threshold level, the effects of signaling are unambiguously beneficial. However, increases in transparency beyond that point reduce welfare by rendering politicians consistently too hesitant from the voters' perspective.

In settings with heterogeneous ex ante policy preferences, more decisive politicians can win elections while making smaller policy concessions, thereby imposing their own agendas on the electorate. Greater partisanship amplifies the tendency to act decisively because it makes the outcomes of elections more consequential to the candidates.

While our theory takes a specific stand on the meaning of decisiveness (one that is consistent with the literature cited in the introduction), we acknowledge that common usages of the term are somewhat vague and admit complementary interpretations. For example, decisiveness can also imply that a politician does not alter a decision once it has been made. A politician who is decisive in that sense would ignore useful information that might alter his decision and improve outcomes for voters, surely a negative feature of decision-making if the story ended there. One can, however, imagine benefits to inflexibility, for example if the leader must bargain on behalf of the voters with third parties. These additional facets of decisiveness are worth exploring in future work.

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# A Proofs

**Proposition 1.** *From the perspective of voter $i$, the ideal politician has minimal uncertainty about his own preferences ($\beta_P = \underline{\beta}$) and perceives the cost of delay per unit time to be*

$$c_P = \begin{cases} c_i \frac{\alpha+\beta}{\alpha-\underline{\beta}} \textit{ if } \alpha > \underline{\beta} \textit{ and the result is less than } \overline{c} \\ \overline{c} \textit{ otherwise} \end{cases}$$

*Proof.* Initially let us assume $\alpha > \beta_P$. The decision time a voter would like the politician to choose can be derived from the following first order condition, where $\tau^v$ denotes the voter's preferred time for the politician's decision

$$\begin{aligned}
\frac{\partial}{\partial \tau} W_i^P(p^*, \tau) &= -\frac{\partial}{\partial \tau}\left[\frac{\alpha}{\tau+\phi} + \frac{\beta_P}{\phi}\frac{\tau}{\tau+\phi} + c_i \tau\right] \\
&= \frac{\alpha}{(\tau^v+\phi)^2} + \frac{\beta_P}{\phi}\left(\frac{\tau^v}{(\tau^v+\phi)^2} - \frac{1}{\tau^v+\phi}\right) - c_i \\
&= \frac{\alpha-\beta_P}{(\tau^v+\phi)^2} - c_i = 0
\end{aligned}$$

Simplifying this we find

$$\tau^v = \sqrt{\frac{\alpha-\beta_P}{c_i}} - \phi \tag{15}$$

The second order condition is

$$\frac{\partial^2}{\partial \tau^2} W_i^P(p^*, \tau) = \frac{\partial}{\partial \tau}\left[\frac{\alpha-\beta_P}{(\tau+\phi)^2} - c_i\right] < 0$$

which implies the voter's utility is concave in $\tau$.

Comparing equation 15 with equation 3 we find

$$\tau^*\left(\beta_P, c_i\frac{\alpha+\beta_P}{\alpha-\beta_P}\right) = \sqrt{\frac{\alpha+\beta_P}{c_i}\left(\frac{\alpha-\beta_P}{\alpha+\beta_P}\right)} - \phi = \tau^v$$

In other words, this cost induces the politician to make decisions at the voter's preferred pace. If $c_i\frac{\alpha+\beta_P}{\alpha-\beta_P} > \bar{c}$, the voter's most preferred politician will have the highest cost possible, $\bar{c}$.

Now consider the case where $\alpha \leq \beta_P$, in which case $\frac{\partial}{\partial \tau} W_i^P(p^*, \tau) < 0$ for all $\tau$. Again, this implies that voter $i$ would like the politician to make decisions as quickly as possible, which implies that voter $i$ would like the politician's information gathering to be as high as possible (i.e., equal to $\bar{c}$) □

Our proofs for the slope and global incentive compatibility for equilibria of the game where politicians signal aversion to delay closely are based on the ODE that defines $c$ as a function of $\tau$

$$\frac{\partial c^S}{\partial \tau} = \lambda c \left[\left.\frac{\partial \Pi(c^S(\tau), \widehat{c})}{\partial \widehat{c}}\right|_{\widehat{c}=c^S(\tau)}\right]^{-1} \left[1 - \left(\frac{\tau^*(c^S(\tau)) + \phi}{\tau+\phi}\right)^2\right] \tag{16}$$

The following lemma insures that the right-hand side of equation 16 is well defined.

**Lemma 3.** *There exist finite* $d_L, d_U > 0$ *such that* $d_L < \frac{\partial \Pi(c,\widehat{c})}{\partial \widehat{c}}\Big|_{\widehat{c}=c} < d_U$ *for all* $c \in [\underline{c}, \overline{c}]$*.*

*Proof.* Taking the derivative, we have

$$\frac{\partial \Pi(c,\widehat{c})}{\partial \widehat{c}}\Bigg|_{\widehat{c}=c} = 2\beta \left[\frac{1}{\phi} - \frac{1}{\tau^*(c)+\phi}\right] f_c(c) \tag{17}$$

Since $\tau^*(c) \geq \tau^*(\overline{c}) > 0$, our claim must hold. □

**Proposition 2.** $c^S(\tau)$ *is strictly decreasing in* $\tau$ *with* $\tau < \tau^*(c^S(\tau))$ *for* $\tau < \tau^*(\underline{c})$ *and strictly increasing in* $\tau$ *with* $\tau > \tau^*(c^S(\tau))$ *for* $\tau > \tau^*(\underline{c})$*.*

*Proof.* Substituting the initial condition into equation 16, we obtain $\frac{\partial c^S}{\partial \tau}\Big|_{\tau=\tau^*(\overline{c})} = 0$. Using equation 16 along with lemma 3, we see that $\frac{\partial c^S}{\partial \tau} > 0$ when $\tau > \tau^*(c^S(\tau))$, and $\frac{\partial c^S}{\partial \tau} < 0$ when $\tau < \tau^*(c^S(\tau))$.

First consider $\tau > \tau^*(\underline{c})$. Starting at $\tau^*(\underline{c})$, a small increase in $\tau$ has a negligible effect on $c^S$, leaving us with $\tau > \tau^*(c^S(\tau))$, and hence $\frac{\partial c^S}{\partial \tau} > 0$. As $\tau$ increases, the slope remains strictly positive unless we reach a point at which $\tau = \tau^*(c^S(\tau))$. But that is impossible, because $\tau^*(c) < \tau^*(\underline{c})$ for all $c > \underline{c}$.

Next consider $\tau < \tau^*(\underline{c})$. Starting at $\tau^*(\underline{c})$, a small decrease in $\tau$ has a negligible effect on $c^S$, leaving us with $\tau < \tau^*(c^S(\tau))$, and hence $\frac{\partial c^S}{\partial \tau} < 0$. As $\tau$ decreases, the slope remains strictly negative unless we reach a value $\tau'$ at which $\tau' = \tau^*(c^S(\tau'))$. But that is impossible, because (i) $\frac{\partial \tau^*}{\partial c}$ is strictly negative and bounded away from 0 on $[\underline{c}, \overline{c}]$, and (ii) from equation 16 and lemma 3, $\frac{\partial c^S}{\partial \tau}$ would converge to zero as $\left(\tau, c^S(\tau)\right)$ converged to $\left(\tau', \tau^*(c^S(\tau'))\right)$; thus, further reductions in $\tau$ would widen the gap between $c^S(\tau)$ and $(\tau^*)^{-1}(\tau)$. □

**Proposition 3.** *There exists* $\lambda^*$ *such that for* $\lambda > \lambda^*$, $\tau^1$ *is globally incentive-compatible.* $\tau^2$ *is never globally incentive compatible.*

*Proof.* We begin by showing that $\tau^1$ converges to $\tau^*$ as $\lambda \to \infty$, and establishing a lower bound on $\frac{\partial \tau^1}{\partial c}$. To this end, we define $R(\lambda) = \max_{c\in[\underline{c},\overline{c}]} \frac{\tau^*(c)+\phi}{\tau^1(c)+\phi} > 1$. Continuity of $\tau^*$ and $\tau^1$ ensures existence of the maximum.

**Lemma 4.** *For any* $\lambda$ *and all* $c \in [\underline{c}, \overline{c}]$*, we have* $\frac{\partial \tau^1}{\partial c} < \frac{-d_L}{\lambda c(R(\lambda)^2-1)}$*. Furthermore,* $\lim_{\lambda\to\infty} R(\lambda) = 1$*.*

*Proof.* Fix a value of $\lambda$. From equation 10 and lemma 3, we have $d_L < \lambda c(1 - R(\lambda)^2)\frac{\partial \tau^1}{\partial c}$, or equivalently $\frac{\partial \tau^1}{\partial c} < \frac{-d_L}{\lambda c(R(\lambda)^2 - 1)}$, for all $c \in [\underline{c}, \overline{c}]$, which establishes the first part of the lemma.

For the second part of the lemma, we begin by observing that $\frac{\partial \tau^*}{\partial c} = \frac{-\sqrt{\alpha+\beta}}{2c^{3/2}}$, which is bounded above by $\frac{-g}{c}$ where $g \equiv \frac{1}{2}\sqrt{\frac{\alpha+\beta}{\overline{c}}}$. Now fix $r > 1$, and define $\lambda_r \equiv r\frac{d_H}{g(r^2-1)}$. From equation 10 and the definitions of $d_H$ and $g$, it follows that, if $\lambda > \lambda_r$ and $\frac{\tau^*(c)+\phi}{\tau^1(c)+\phi} \geq r$, we have $0 > r\frac{\partial \tau^1}{\partial c} > \frac{\partial \tau^*}{\partial c}$.

Next we show that, for all $\lambda > \lambda_r$, we have $R(\lambda) < r$. Suppose on the contrary that, for such $\lambda$, there is some $c$ for which $\frac{\tau^*(c)+\phi}{\tau^1(c)+\phi} \geq r$. Let $c'$ be the smallest value for which $\frac{\tau^*(c')+\phi}{\tau^1(c')+\phi} = r$. Differentiating, we obtain

$$\begin{aligned}
\frac{d}{dc}\left(\frac{\tau^*(c)+\phi}{\tau^1(c)+\phi}\right)\bigg|_{\beta=\beta'} &= \frac{\frac{d\tau^*}{dc}\left(\tau^1(c')+\phi\right) - \frac{d\tau^1}{dc}\left(\tau^*(c')+\phi\right)}{\left(\tau^1(c')+\phi\right)^2} \\
&= \frac{\frac{d\tau^*}{dc} - \frac{d\tau^1}{dc}r}{\tau^1(c')+\phi} \\
&< 0.
\end{aligned}$$

It follows that a small decrease in $c$ from $c'$ would result in $\frac{\tau^*(c)+\phi}{\tau^1(c)+\phi} > r$. But we know that $\frac{\tau^*(\underline{c})+\phi}{\tau^1(\underline{c})+\phi} = 1 < r$. Consequently, there would have to be some $c'' \in (\underline{c}, c')$ for which $\frac{\tau^*(c'')+\phi}{\tau^1(c'')+\phi} = r$. But that contradicts the definition of $c'$. We conclude that, for any $r > 1$, we have $R(\lambda) < r$ for all $\lambda > \lambda_r$. The lemma follows directly. □

**Lemma 5.** *There exists $\lambda^*$ such that for all $\lambda > \lambda^*$, we have $\frac{\partial^2 V(c,\widehat{c})}{\partial c \partial \widehat{c}} > 0$ for all $c \in [\underline{c}, \overline{c}]$.*

*Proof.* Using equation 3 to substitute for $\tau^*(c)$ in equation 8 and differentiating, we obtain

$$\begin{aligned}
\frac{\partial^2 V(c,\widehat{c})}{\partial \widehat{c} \partial c} &= \left(\tau^*(\widehat{c}) - \tau^*(c)\right)\, f_c(\widehat{c}) - \lambda \frac{\partial \tau^S(\widehat{c})}{\partial \widehat{c}} \\
&> \left(\tau^*(\overline{c}) - \tau^*(\underline{c})\right)\, \overline{f}_c + \frac{d_L}{c(R(\lambda)^2 - 1)}
\end{aligned} \tag{18}$$

where $\overline{f}_c$ is an upper bound on the density of $c$, and the inequality makes use of lemma 4. While the first term is negative, lemma 4 implies the second term becomes unboundedly large and positive as $\lambda \to \infty$. Accordingly, the entire expression is strictly positive for $\lambda$ sufficiently large. □

Now we prove the first part of the proposition. In light of the fact that $\frac{\partial^2 V(c,\widehat{c})}{\partial c \partial \widehat{c}} > 0$, the first-order condition for claiming to be of type $\widehat{c}$, $\frac{\partial V(c,\widehat{c})}{\partial \widehat{c}} = 0$, can be satisfied by at most one type, which is by construction $c = \widehat{c}$. Consequently the optimal value of $\widehat{c}$ for each $c$ must be $c, \underline{c}$, or $\overline{c}$. We can rule out $\underline{c}$ on the grounds that $\left.\frac{\partial V(c,\widehat{c})}{\partial \widehat{c}}\right|_{c,\widehat{c}=\underline{c}} = 0$ and $\frac{\partial^2 V(c,\widehat{c})}{\partial c \partial \widehat{c}} > 0$; similarly for $\overline{c}$. We conclude that $\tau^1$ is globally incentive-compatible.

Turning to the second part of the proposition, we know that $\lim_{c \to \underline{c}} \frac{\partial \tau^2}{\partial \widehat{c}} = +\infty$. Accordingly, the analog of equation 18 tells us that $\frac{\partial^2 V(c,\widehat{c})}{\partial c \partial \widehat{c}} < 0$ for $c$ and $\widehat{c}$ in a neighborhood of $\underline{c}$. But in that case, local incentive compatibility is violated: for $c_1$ and $c_2$ close to $\underline{c}$, the fact that $V(c_1, c_1) > V(c_1, c_2)$ would imply $V(c_2, c_1) > V(c_2, c_2)$. □

**Proposition 4.** *$V(c,c)$ is decreasing in $c$.*

*Proof.* Consider $c > c'$ and use the following decomposition of $V(c,c) - V(c',c')$ :

$$V(c,c) - V(c',c') = \left[V(c,c) - V(c',c)\right] + \left[V(c',c) - V(c',c')\right]$$

The second term is weakly negative since incentive compatibility requires $V(c',c') \geq V(c',c)$. Now we turn to the first term, which represent the utility difference between agents of type $c$ and $\widehat{c}$ when both claim to be of type $c$.

$$\begin{aligned} V(c,c) - V(c',c) \quad = \quad & -\int_{\underline{c}}^{c} \left[\frac{\alpha+\beta}{\tau^*(c)+\phi} + c\tau^*(c) - \frac{\alpha+\beta}{\tau^*(c')+\phi} - c'\tau^*(c')\right] f_c(s) ds \\ & - \int_{c}^{\overline{c}} (c - c')\tau^*(s) f_c(s) ds - \lambda \left(c - c'\right) \tau^S(c) \end{aligned} \quad (19)$$

While the second and third terms are clearly negative since $c > c'$, we need a bit more work to sign the first term. Note that

$$\frac{\alpha+\beta}{\tau^*(\widetilde{c})+\phi} + \widetilde{c}\tau^*(\widetilde{c}) = 2\sqrt{\widetilde{c}\,(\alpha+\beta)} - \widetilde{c}\phi \quad (20)$$

The first term in equation 19 is negative if the expression on the right-hand side of equation 20 is increasing in $\widetilde{c}$. Notice that

$$\frac{d}{dc}\left[2\sqrt{\alpha+\beta}\sqrt{c} - c\phi\right] = \sqrt{\frac{\alpha+\beta}{c}} - \phi = \tau^*(c) > 0$$

Therefore $V(c,c) < V(c',c)$ and, as a result, $V(c,c) < V(c',c')$. $\square$

**Proposition 5.** *The symmetric equilibrium maximizes the payoff for every type of politician within the set of fully separating equilibria.*

*Proof.* Suppose we have a separating equilibrium with action functions $\tau^S(c) = (\tau_1^S(c), ..., \tau_m^S(c))$. Defining

$$\Gamma(c,\tau) \equiv \sum_{m=1}^{M} \left( \frac{\alpha+\beta}{\tau_m+\phi} + c\tau_m \right), \tag{21}$$

we can rewrite the first-order condition for type $c$'s optimal choice as

$$\left.\frac{\partial \Pi(c,\widehat{c})}{\partial \widehat{c}}\right|_{\widehat{c}=c} = \frac{\partial \Gamma(c,\tau^S(c))}{\partial \tau} \frac{d\tau^S(c)}{dc}, \tag{22}$$

where $\frac{\partial \Gamma(c,\tau^S(c))}{\partial \tau}$ and $\frac{d\tau^S(c)}{dc}$ are $M$-dimensional vectors.[25] We are interested in determining type $c$'s total payoff in equilibrium. Definitionally,

$$V(c,c) = \Pi(c,c) - \Gamma(c,\tau^S(c)), \tag{23}$$

and using the Envelope Theorem we have

$$\frac{dV(c,c)}{dc} = \frac{d\Pi(c,c)}{dc} - \frac{\partial \Gamma(c,\tau^S(c))}{\partial c}$$

Notice that only the final term depends on the particular separating equilibrium. Let $\tau^0$ denote the symmetric separating equilibrium with payoffs $V^0$, and $\tau^A$ denote an asymmetric separating equilibrium with payoffs $V^A$. To demonstrate that payoffs in the symmetric separating equilibrium are strictly higher than in the asymmetric separating equilibrium, we will establish the following Property (capitalized for clarity of subsequent references): if it were the case for some $c$ that either (i) $V^0(c,c) = V^A(c,c)$ and $\tau^0(c) \neq \tau^A(c)$, or

[25] We assume for convenience that each $\tau_m^S$ is differentiable. However, the argument only requires differentiability of $\Gamma(c,\tau^S(c))$, which is a slightly weaker condition. As an example, consider $M = 1$ with an equilibrium where $\tau_1^S$ and $\tau_2^S$ are differentiable. We could define another equilibrium $\widetilde{\tau}_1^S$ and $\widetilde{\tau}_2^S$ where

$$\widetilde{\tau}_1^S(c) = \begin{cases} \tau_1^S(c) \text{ if } c \text{ is a rational number} \\ \tau_2^S(c) \text{ otherwise} \end{cases}$$

and $\widetilde{\tau}_2^S(c)$ is defined symmetrically. Obviously the equilibrium strategies would not be differentiable.

(ii) $V^0(c,c) < V^A(c,c)$, then we would have $\frac{dV^0(c,c)}{dc} > \frac{dV^A(c,c)}{dc}$.[26] To understand why this property delivers the desired conclusion, note that $V^A(c',c') - V^0(c',c')$ would then shrink with $c'$ over $[\underline{c}, c]$, violating the boundary conditions $V^0(\overline{c},\overline{c}) = V^A(\overline{c},\overline{c}) = \Pi(\overline{c},\overline{c}) - \Gamma(c,\tau^*(\overline{c}))$. In light of equation 23, we can rewrite the Property as follows: if it were the case for some $c$ that either (i)$'$ $\Gamma(c,\tau^0(c)) = \Gamma(c,\tau^A(c))$ and $\tau^0(c) \neq \tau^A(c)$, or (ii)$'$ $\Gamma(c,\tau^0(c)) > \Gamma(c,\tau^A(c))$, then we would have $\frac{\partial\Gamma(c,\tau^0(c))}{\partial c} < \frac{\partial\Gamma(c,\tau^A(c))}{\partial c}$.

We now establish the Property. Supposing condition (i)$'$ were satisfied for some $c > \underline{c}$, we would begin by defining[27]

$$\overline{\tau}_m = \begin{cases} \tau_m^A(c) \text{ if } \tau_m^A(c) \leq \tau^*(c) \\ \\ \tau \leq \tau^*(c) \text{ s.t. } \frac{\alpha+\beta}{\tau+\phi} + c\tau = \frac{\alpha+\beta}{\tau_m^A(c)+\phi} + c\tau_m^A(c) \text{ otherwise} \end{cases}$$

Let $Q \equiv \left\{m \mid \tau_m^A(c) > \tau^*(c)\right\}$. Then

$$\frac{\partial\Gamma(c,\overline{\tau})}{\partial c} - \frac{\partial\Gamma(c,\tau^A(c))}{\partial c} = \sum_{m\in Q} \left(\overline{\tau}_m - \tau^A(c)\right) \leq 0,$$

with strict inequality if $Q$ is non-empty.

If $\tau^0(c) = \overline{\tau}$, we are done. If not, then since $\Gamma(c,\tau^A(c)) = \Gamma(c,\overline{\tau})$ by construction, there must exist $i$ and $j$ such that $\overline{\tau}_i > \tau^0(c) > \overline{\tau}_j$.

Define the function $\widetilde{\tau}(\tau_i)$ as follows: $\widetilde{\tau}_i(\tau_i) = \overline{\tau}_i$, $\widetilde{\tau}_k(\tau_i) = \overline{\tau}_k$ for $k \neq i,j$, and $\Gamma(c,\widetilde{\tau}(\tau_i)) = \Gamma(c,\overline{\tau})$. In other words, $\widetilde{\tau}_j(\tau_i)$ indicates how $\tau_j$ must vary in response to changes in $\tau_i$ to keep the value of $\Gamma$ constant at its equilibrium value. Implicit differentiation reveals that for $\overline{\tau}_i > \overline{\tau}_j$

$$\left.\frac{d\widetilde{\tau}_j}{d\tau_i}\right|_{\tau_i=\overline{\tau}_i} = -\frac{\frac{\alpha+\beta}{(\overline{\tau}_i+\phi)^2} - c}{\frac{\alpha+\beta}{(\widetilde{\tau}_j(\tau_i)+\phi)^2} - c}$$

Plainly, there exists a unique value $\tau_i^e < \tau^*(c)$ such that $\widetilde{\tau}_j(\tau_i^e) = \tau_i^e$. For $\tau_i \in$

[26] Suppose our claim is true. Then if either condition (i) or (ii) holds for $c$, then condition (ii) must hold for all $c' \in (\underline{c}, c)$.

[27] Although we ruled out $\tau_m^A(c) > \tau^*(c)$ when $M = 1$, in principle this need be true when $M > 1$. $\overline{\tau}_m$ is defined so that an equivalent signaling cost is incurred, but all of the actions are in the intutive direction of $\overline{\tau}_m(c) \leq \tau^*(c)$.

$[\tau_i^e, \tau_i^A(c)]$, we have[28]

$$\begin{aligned}\frac{d}{d\tau_i}\left(\frac{\partial\Gamma(c,\widetilde{\tau}(\tau_i))}{\partial c}\right) &= \frac{d}{d\tau_i}\left(\tau_i + \widetilde{\tau}_j(\tau_i)\right) \\ &= 1 + \frac{d\widetilde{\tau}_j}{d\tau_i} > 0\end{aligned}$$

where the final inequality follows from the fact that $\widetilde{\tau}_j(\tau_i) < \tau_i$ (and hence $\frac{d\widetilde{\tau}_j}{d\tau_i} > -1$). If follows that $\frac{\partial\Gamma(c,\widetilde{\tau}(\tau_i^e))}{\partial c} < \frac{\partial\Gamma(c,\overline{\tau})}{\partial c}$ since $\overline{\tau}_i > \tau^0(c)$ is being reduced in this equalization step. Through repeated application of this equalization argument, we conclude that $\frac{\partial\Gamma(c,\tau^0(c))}{\partial c} < \frac{\partial\Gamma(c,\overline{\tau})}{\partial c} \leq \frac{\partial\Gamma(c,\tau^A(c))}{\partial c}$, as desired.

Next, supposing condition (ii)$'$ were satisfied for some $c > \underline{c}$, we would begin by defining $\tau'$ s.t. $\tau_1' = \tau_2' = ... = \tau_M' < \tau^*(c)$ and $\Gamma(c,\tau') = \Gamma(c,\tau^A(c))$. By the same argument as for condition (ii)$'$, we infer $\frac{\partial\Gamma(c,\tau')}{\partial c} \leq \frac{\partial\Gamma(c,\tau^A(c))}{\partial c}$. (The inequality is weak because we include the possibility that $\tau' = \tau^A(c)$). Because $\Gamma(c,\tau^0(c)) < \Gamma(c,\tau^A(c)) = \Gamma(c,\tau')$ by assumption, we have $\tau_m^0(c) < \tau_m'$. Accordingly,

$$\frac{\partial\Gamma(c,\tau^0(c))}{\partial c} - \frac{\partial\Gamma(c,\tau')}{\partial c} = \sum_{m=1}^{M}\left(\tau_m^0(c) - \tau_m'\right) < 0.$$

It follows that $\frac{\partial\Gamma(c,\tau^0(c))}{\partial c} < \frac{\partial\Gamma(c,\tau^A(c))}{\partial c}$, as desired.

Having established that the Property holds, the Proposition follows for the reasons given above. □

**Proposition 6.** *$\underline{\tau}^0(c)$ converges to $\tau^*(c)$ uniformly as $M \to \infty$.*

*Proof.* A comparison between Equations 11 and 10 reveals that the two are isomorphic, inasmuch as one can simply absorb $M$ into $\lambda$. The Proposition then follows directly from Lemma 4. □

**Lemma 1.** *If a voter with characteristics $(\mu_M, c_M)$ strictly prefers candidate $i$ to candidate $j$, then so does a majority of voters.*

*Proof.* The difference in utility that voter $(\mu_i, c_i)$ receives if candidate 1 is elected versus

[28]Since $\overline{\tau}_i > \overline{\tau}_j$, it must be that $\overline{\tau}_i > \tau_i^e > \overline{\tau}_j$

candidate 2 where candidate $j$ commits to a power sharing agreement $\delta_j$ is

$$\Delta_i = 2\mu_i\left(2\mu + \delta_1 - \delta_2\right) - \left(\tau_1 - \tau_2\right)c_i + \left[\left(\delta_2 - \mu\right)^2 - \left(\delta_1 + \mu\right)^2 + \left(\frac{\alpha - \beta_2}{\tau_2 + \phi} - \frac{\alpha - \beta_1}{\tau_1 + \phi}\right) + \frac{\beta_2 - \beta_1}{\phi}\right]$$

where the bracketed term is independent of the voter's type. Clearly the indifference curves implied by $\Delta_i$ are linear, which implies that the voters that prefer candidate 1 to candidate 2 can be described using a half-plane. The claim regarding the pivotality of $(\mu_M, c_M)$ follows from elementary geometric arguments invoking the symmetry of the distribution of types in each $\square$

**Lemma 2.** $\frac{\partial}{\partial \widehat{c}_1}\Phi_M\left(\widehat{c}_1, \widehat{c}_2\right)$ *has the same sign as* $c_M\frac{\alpha+\beta}{\alpha-\beta} - \widehat{c}_1$*, and* $\frac{\partial}{\partial \widehat{c}_2}\Phi_M\left(\widehat{c}_1, \widehat{c}_2\right)$ *has the same sign as* $\widehat{c}_2 - c_M\frac{\alpha+\beta}{\alpha-\beta}$.

*Proof.* Suppose the ideal politician type for the median voter has an aversion to delay larger than $\overline{c}$. Since the median agents prefers politicians that have higher aversion to delay, an increase in the perceived aversion of delay in candidate 1 increases the appeal of that candidate (i.e., $\Phi_M$ rises). An identical argument implies $\frac{\partial}{\partial \widehat{c}_2}\Phi_M\left(\widehat{c}_1, \widehat{c}_2\right) < 0$. If the median voter's ideal politician has a cost within $[\underline{c}, \overline{c}]$, then the sign convention in our lemma captures the single-peakedness of the median voter's ideal politician type. $\square$

**Proposition 7.** *The Nash equilibria depend on the model's parameters as follows:*

1. *If* $\mu^2 \geq \Phi_M > 0$*, the unique pure strategy Nash equilibrium involves* $\delta_1 = \sqrt{\Phi_M} - \mu$*,* $\delta_2 = \mu$ *, and the election of candidate 1.*

2. *If* $\Phi_M > \mu^2$*, all pure strategy Nash equilibria involve* $\delta_1 = 0$ *and the election of candidate 1.*

3. *If* $\mu^2 \geq -\Phi_M > 0$*, the unique pure strategy Nash equilibrium involves* $\delta_2 = \mu - \sqrt{-\Phi_M}$*,* $\delta_1 = -\mu$*, and the election of candidate 2.*

4. *If* $-\Phi_M > \mu^2$*, all pure strategy Nash equilibria involve* $\delta_2 = 0$ *and the election of candidate 2.*

5. *If* $\Phi_M = 0$*, then* $\delta_1 = \delta_2 = -\mu$ *and each candidate is elected with probability* $\frac{1}{2}$.

*Proof.* In each case, it is easily verified that the indicated actions constitute a Nash equilibrium. In cases (ii) and (iv), any values for $\delta_1$ and $\delta_2$, respectively, will suffice.

We establish uniqueness for case (i) by dividing the alternatives into the following categories.

- First suppose $\delta_1 = \sqrt{\Phi_M} - \mu$ and $\delta_2 \neq \mu$, in which case candidate 1 wins the election. Then there exists $\varepsilon > 0$ such that candidate 1 can deviate to $\widetilde{\delta}_1 = \sqrt{\Phi_M} - \mu + \varepsilon$, still win the election, and achieve a preferred outcome.
- Next suppose $\delta_1 < \sqrt{\Phi_M} - \mu$ and candidate 1 wins. Then candidate 1 could deviate to $\widetilde{\delta}_1 = \sqrt{\Phi_M} - \mu$, still win the election, and achieve a preferred outcome.
- Next suppose $\delta_1 < \sqrt{\Phi_M} - \mu$ and candidate 1 loses. Then candidate 1 could deviate to $\widetilde{\delta}_1 = \sqrt{\Phi_M} - \mu$, win the election, and achieve a preferred outcome.
- Next suppose $\delta_1 > \sqrt{\Phi_M} - \mu$ and candidate 1 wins. Then candidate 2 could deviate to $\widetilde{\delta}_2 = \mu$, win the election, and achieve a preferred outcome.
- Finally suppose $\delta_1 > \sqrt{\Phi_M} - \mu$ and candidate 1 loses. Then candidate 1 could deviate to $\widetilde{\delta}_1 = \sqrt{\Phi_M} - \mu$, win the election, and achieve a preferred outcome.

The argument for uniqueness is symmetric in case (iii), and proceeds similarly in case (v). □

**Proposition 8.** *In settings where they signal aversion to delay, $\tau^S(c)$ is decreasing in $\mu$.*

*Proof.* Consider $\mu > \widetilde{\mu}$, and suppose contrary to the proposition that there exists some $c' > \underline{c}$ such that $\tau^S(c'; \mu) \geq \tau^S(c'; \widetilde{\mu})$. Then there must be some $c \in [\underline{c}, c')$ with $\tau^S(c; \mu) \geq \tau^S(c; \widetilde{\mu})$ such that $\frac{d\tau^s(c;\mu)}{dc} \leq \frac{d\tau^s(c;\widetilde{\mu})}{dc}$ – otherwise we would have $\tau^S(\underline{c}; \mu) > \tau^S(\underline{c}; \widetilde{\mu})$, which violates the boundary condition.[29] However,

[29] If $\tau^S(c'; \mu) > \tau^S(c'; \widetilde{\mu})$, then the existence of a $c$ such that $\tau^S(c; \mu) > \tau^S(c; \widetilde{\mu})$ follows from the continuity of $\tau^S(\circ; \mu)$ and $\tau^S(\circ; \widetilde{\mu})$. If $\tau^S(c'; \mu) = \tau^S(c'; \widetilde{\mu})$, then we know there exists a $c$ such that $\tau^S(c; \mu) > \tau^S(c; \widetilde{\mu})$ because $\frac{\partial \Pi}{\partial c \partial \mu} > 0$, which implies

$$\left.\frac{\partial \tau^S(\circ; \mu)}{\partial c}\right|_{c=c'} > \left.\frac{\partial \tau^S(\circ; \widetilde{\mu})}{\partial c}\right|_{c=c'}$$

$\left.\frac{\partial \Pi(c,\widehat{c})}{\partial \widehat{c}}\right|_{\widehat{c}=c} = \lambda c \left[1 - \left(\frac{\tau^*(c)+\phi}{\tau^S(c)+\phi}\right)^2\right] \frac{\partial \tau^S}{\partial \hat{c}}$

$$\begin{aligned}
\frac{d\tau^s(c;\mu)}{dc} &= \frac{1}{\lambda c}\left[1 - \left(\frac{\tau^*(c)+\phi}{\tau^S(c;\mu)+\phi}\right)^2\right]^{-1} \left(\left.\frac{\partial \Pi(c,\widehat{c};\mu)}{\partial c}\right|_{\widehat{\beta}=\beta}\right) \\
&> \frac{1}{\lambda c}\left[1 - \left(\frac{\tau^*(c)+\phi}{\tau^S(c;\widetilde{\mu})+\phi}\right)^2\right]^{-1} \left(-\left.\frac{\partial \Pi(c,\widehat{c};\mu)}{\partial \beta c}\right|_{\widehat{\beta}=\beta}\right) \\
&> \frac{1}{\lambda c}\left[1 - \left(\frac{\tau^*(c)+\phi}{\tau^S(c;\widetilde{\mu})+\phi}\right)^2\right]^{-1} \left(-\left.\frac{\partial \Pi(c,\widehat{c};\widetilde{\mu})}{\partial \widehat{c}}\right|_{\widehat{\beta}=\beta}\right) \\
&= \frac{d\tau^s(c;\widetilde{\mu})}{dc},
\end{aligned}$$

where the first inequality follows from $\tau^S(c;\mu) \geq \tau^S(c;\widetilde{\mu})$, and the second from $\frac{\partial}{\partial \mu}\frac{\partial \Pi}{\partial \widehat{c}} > 0$ (which we demonstrated in the text). Thus we have a contradiction. □

# B Online Appendix: Signaling self-knowledge

For the purpose of this section, we assume all agents perceive the same costs of delay, $c$. Candidates therefore differ only in the precision of their self-knowledge, $\beta_P$. As in the case where politicians signal an aversion to delay, the victor in the stage 2 election will be the candidate voters believe has the greatest decisiveness – in other words, the one with the lowest value of $\widehat{\beta}$. Since low values of $\beta_P$ yield rapid decisions in stage 3, each candidate has an incentive to signal decisiveness in stage 1. Throughout this extension we include ideological adjustments as per section 6 to illustrate how these adjustments affect our proofs.

Now consider the stage 1 decision problem facing a politician with self-knowledge $\beta$ who expects to play the role of candidate 1 in the stage 2 election. (The analysis for a candidate who expects to play the role of candidate 2 is symmetric.) From the preceding discussion, the candidate knows he will win the election if voters believe he has greater self-knowledge than his opponent. Because we are studying fully separating equilibria, the stage 1 outcome will fully reveal his opponent's type. Therefore, when choosing the image he wishes to project in stage 1 ($\widehat{\beta}$), he knows he will win if it turns out that his chosen $\widehat{\beta}$ is less than the actual value of his opponent's self-knowledge parameter, and lose if it is greater. For the purpose of simplifying some of the analytic expressions, we assume that $\mu$ is large enough that $\delta$ is strictly less than $\mu$ on the entire support of $f_\beta$ (i.e., $\Phi_M(\underline{\beta}, \overline{\beta}) < \mu$). Using equations 1 and 5 along with proposition 7, we can write the stage 3 payoff of a politician with type $\beta$ who chooses a perception $\widehat{\beta}$ as follows:

$$\Pi\left(\beta, \widehat{\beta}\right) = -\int_{\widehat{\beta}}^{\overline{\beta}} \left[\left(\mu - \sqrt{\Phi_M(\widehat{\beta}, s)}\right)^2 + \frac{\alpha + \beta}{\tau^*(\beta) + \phi} + c\tau^*(\beta)\right] f_\beta(s) ds \tag{24}$$
$$- \int_{\underline{\beta}}^{\widehat{\beta}} \left[\left(\mu + \sqrt{-\Phi_M(\widehat{\beta}, s)}\right)^2 + \frac{\alpha - s}{\tau^*(s) + \phi} + \frac{\beta + s}{\phi} + c\tau^*(s)\right] f_\beta(s) ds$$

Thus his total payoff including stage 1 is:

$$V\left(\beta, \widehat{\beta}\right) = \Pi\left(\beta, \widehat{\beta}\right) - \lambda\left[\frac{\alpha + \beta}{\tau^S(\widehat{\beta}) + \phi} + c\tau^S(\widehat{\beta})\right]$$

where $\lambda$ reflects the relative importance of the payoffs in stages 1 and 3.

To determine the candidate's optimal choice of $\widehat{\beta}$ given the signaling schedule $\tau^S$, we take the derivative of $V$ with respect to $\widehat{\beta}$ and set it equal to zero: $\frac{\partial}{\partial \widehat{\beta}} V(\beta, \widehat{\beta}) = 0$. In a fully separating equilibrium the solution is $\widehat{\beta} = \beta$, which implies $\frac{\partial}{\partial \widehat{\beta}} V(\beta, \widehat{\beta})\Big|_{\widehat{\beta}=\beta} = 0$. Notice that we can rewrite this equilibrium condition as follows:

$$\left.\frac{\partial \Pi(\beta, \widehat{\beta})}{\partial \widehat{\beta}}\right|_{\widehat{\beta}=\beta} = \lambda c\left[1 - \left(\frac{\tau^*(\beta) + \phi}{\tau^S(\beta) + \phi}\right)^2\right] \frac{\partial \tau^S}{\partial \hat{\beta}} \tag{25}$$

Now observe that equation 25 is a nonlinear first-order differential equation. As usual, the equilibrium leaves the choice of the "worst" type undistorted, so we also have a boundary condition, $\tau^S(\overline{\beta}) = \tau^*(\overline{\beta})$.

A somewhat unconventional feature of this differential equation is that the coefficient of $\frac{\partial \tau^S}{\partial \beta}$ is zero at the initial condition, which renders $\frac{\partial \tau^S}{\partial \beta}\Big|_{\beta=\overline{\beta}}$ undefined. We can finesse this difficulty by reversing the mathematical roles of $\tau^S$ and $\beta$, treating equation 25 as a differential equation for the function $\beta^S(\tau)$, with initial condition $\beta^S(\tau) = \overline{\beta}$ for $\tau = \tau^*(\overline{\beta})$. In that case, we have $\frac{\partial \beta^S}{\partial \tau}\Big|_{\tau=\tau^*(\overline{\beta})} = 0$. Our next result describes some properties of the solution.

**Proposition 9.** *$\beta^S(\tau)$ is strictly increasing in $\tau$ with $\tau < \tau^*(\beta^S(\tau))$ for $\tau < \tau^*(\overline{\beta})$, and* *strictly decreasing in* $\tau$ *with* $\tau > \tau^*(\beta^S(\tau))$ *for* $\tau > \tau^*(\overline{\beta})$.

This proposition offers us two candidates for the separating function $\tau^S$: we can invert the solution $\beta^S$ restricting attention either to $\tau < \tau^*(\overline{\beta})$, which yields an upward-sloping function $\tau^1(\beta)$ satisfying $\tau^1(\beta) < \tau^*(\beta)$, or to $\tau > \tau^*(\overline{\beta})$, which yields a downward-sloping function $\tau^2(\beta)$. Given our assumptions, there is no guarantee that either solution is globally incentive-compatible. Our next result tells us that only $\tau^1$ can serve as the separating function $\tau^S$. We show that a sufficiently large value of $\lambda$ – which one can interpret (for example) as a limit on the likelihood with which lower officeholders expects to run for higher office – guarantees global incentive compatibility of $\tau^1$. The proof also supplies a relatively simple analytic condition that one can check after solving any parametrized version of the model numerically; see the Appendix. In contrast, $\tau^2$ is never globally incentive compatible. Accordingly, we will henceforth associate $\tau^S$ with $\tau^1$.

**Proposition 10.** *There exists* $\lambda^*$ *such that for* $\lambda > \lambda^*$, $\tau^1$ *is globally incentive-compatible.* $\tau^2$ *is never globally incentive compatible.*

Together, propositions 9 and 10 imply that $\tau^S(\beta) < \tau^*(\beta)$. Thus, politicians signal by acting more decisive than they actually are. In addition, we learn that $\tau^S$ must be strictly increasing in $\beta$, which raises the possibility that the non-negativity constraint ($\tau > 0$) may bind. We will eliminate this possibility by assuming, where necessary, that $\phi$ is sufficiently small.

Figure 2 depicts a typical equilibrium. We produced the figure by numerically solving a parametrized version of our model. The horizontal axis describes the politician's type, and the vertical axis describes the time take to make a decision. For this case, we have also verified global incentive-compatibility.[30] The figure shows three functions relating the decision time, $\tau$, to the politician's self-knowledge, $\beta$: the ideal from the politician's perspective ($\tau^*$), the ideal from a voter's perspective ($\tau^v$), and the signaling equilibrium ($\tau^S$). Because we have assumed the cost of delay, $c$, is the same for all voters, the ideal $\tau^v$ does not depend on the voter's identity.

For the moment, focus on the curves representing the voters' and politician's ideals. One important feature of the figure is that the former lies below the latter. In other words, absent heterogeneity in $c$, voters regard all non-strategic politicians as hesitant.

[30]The figure was generated using $\mu = 0$, $\phi = 0.001$, $\alpha = 1, c = 25$, $\underline{\beta} = 0$, and $\overline{\beta} = 0.2$. $\beta_P$ is distributed as per a truncated normal distribution with a mean of 0.2 and a standard deviation of 0.0667.

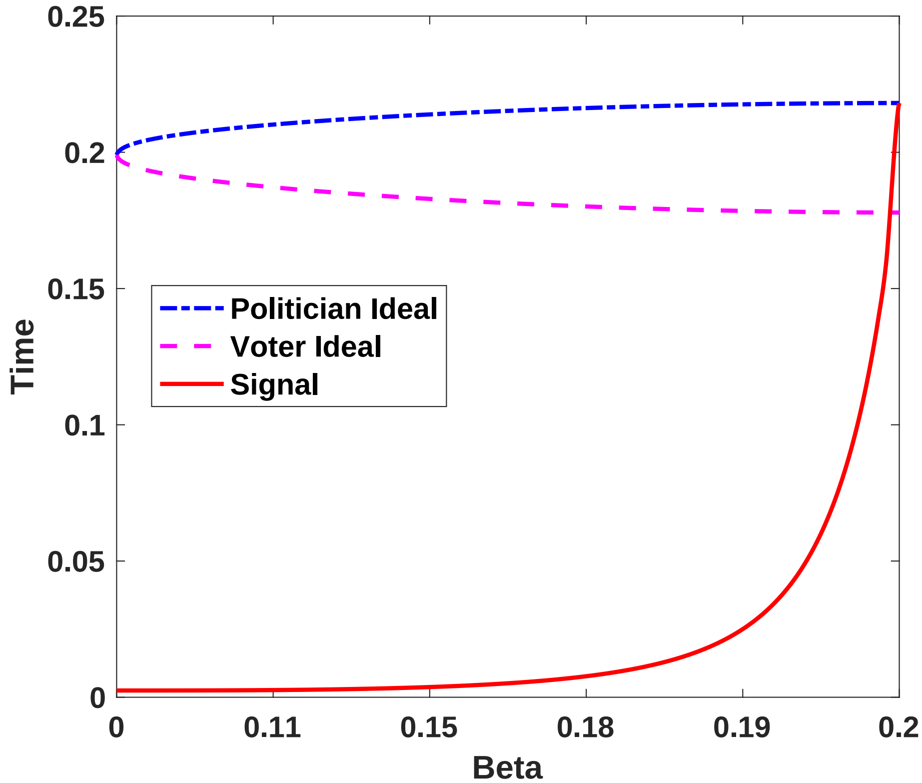


Figure 2: Equilibrium Versus Voter and Politician Optima With Signaling of Self-Knowledge

The intuition is simply that voters bear additional risk when politicians take extra time to better inform their own idiosyncratic preferences. Notice also that the curve representing the politician's ideal slopes upward, while the one representing the voter's ideal slopes downward. Intuitively, if the politician starts out with less precise knowledge of his own preferences, incremental time spent deliberating will lead to a greater reduction in the risk he bears, but a greater increase in the risk born by the voter; hence the ideal duration of the deliberation period increases with $\beta$ for the politician and decreases for the voter.

As in Section 5, signaling does not necessarily improve voter welfare. Signaling is a rather poor solution for politicians' tendency to delay because the signaling incentive is smallest where the need for a corrective influence is greatest, and greatest where that need is smallest. That pattern is evident from the figure. Signaling has no effect on politicians with the largest values of $\beta$, for whom the gap between the voter and politician ideal is greatest. Moreover, its cumulative effect on politicians with the lowest values of $\beta$, for whom that gap is smallest, can be enormous, causing them to spend little or no time pondering the common good even when they should. In the figure, the signaling curve is so steeply sloped that it crosses the voter ideal curve. Politicians to the right of the crossing remain hesitant from the voter's perspective, but those to the left become hasty, overshooting the voter's ideal. Thus the overall impact on voter welfare can be positive or negative, depending on the size of the signaling effect and the distribution of politician types – specifically, whether it is skewed toward those with relatively good self-knowledge who overcorrect, or those with relatively poor self-knowledge who undercorrect.

Because we have not endogenized the decision to become a politician, we cannot draw formal conclusions about the distribution of candidate types. However, we can still provide some insight concerning this issue. Our next result establishes that higher quality politicians receive higher expected payoffs in equilibrium.

**Proposition 11.** *$V(\beta, \beta)$ is decreasing in $\beta$.*

Were we to endogenize the distribution of candidate types, for example by appending the familiar citizen-candidate apparatus to our model, proposition 11 suggests that the distribution of politicians would likely be skewed toward types with low values of $\beta$ that the voters' prefer. However, coupled with the preceding analysis, this additional observation raises the possibility that the political system may encourage most politicians to act with excessive haste in order to project decisiveness.

# C Online Appendix: Proofs

Our proofs for the equilibrium for the game wherein agents signal self-knowledge are similar to the arguments made when analyzing the model wherein agents signal an aversion to delay. For our next result, we rewrite the first-order condition, equation (10), so that it describes a differential equation for $\beta$ in terms of $\tau$, rather than the other way around:

$$\frac{\partial \beta^S}{\partial \tau} = \lambda c \left[ \left. \frac{\partial \Pi(\beta^S(\tau), \widehat{\beta})}{\partial \widehat{\beta}} \right|_{\widehat{\beta} = \beta^S(\tau)} \right]^{-1} \left[ 1 - \left( \frac{\tau^*(\beta^S(\tau)) + \phi}{\tau + \phi} \right)^2 \right] \tag{26}$$

The following lemma ensures that the right hand side of equation 26 is well-defined:

**Lemma 6.** *There exist finite $d_L, d_U > 0$ such that $d_L < -\frac{\partial \Pi(\beta,\widehat{\beta})}{\partial \widehat{\beta}}\Big|_{\widehat{\beta}=\beta} < d_U$ for all $\beta \in [\underline{\beta}, \overline{\beta}]$.*

*Proof.* Taking the derivative, we have

$$-\frac{\partial \Pi(\beta,\widehat{\beta})}{\partial \widehat{\beta}}\Bigg|_{\widehat{\beta}=\beta} = 2\beta\left[\frac{1}{\phi} - \frac{1}{\tau^*(\beta)+\phi}\right] f_\beta(\beta) - \int_\beta^{\overline{\beta}} \left(\frac{\mu}{\sqrt{\Phi_M(\beta,s)}} - 1\right)\frac{\partial \Phi_M(\beta,s)}{\partial \widehat{\beta}_1} f_\beta(s)ds - \int_{\underline{\beta}}^{\beta} \left(\frac{\mu}{\sqrt{-\Phi_M(\beta,s)}} + 1\right)\frac{\partial \Phi_M(\beta,s)}{\partial \widehat{\beta}_1} f_\beta(s)ds \tag{27}$$

Let

$$d_L \equiv 2\underline{\beta}\left[\frac{1}{\phi} - \frac{1}{\tau^*(\underline{\beta})+\phi}\right]\underline{f}_\beta > 0,$$

where $\underline{f}_\beta$ is the lower bound on density. By construction, the first term in equation 27 exceeds $d_L$. The second and third terms are positive because (i) $\sqrt{|\Phi_M(\beta,s)|} < \mu$ by assumption, and (ii) $\Phi_M$ is decreasing in its first argument. Thus, the sum exceeds $d_L$.

Next, let

$$d_U^1 \equiv 2\overline{\beta}\left[\frac{1}{\phi} - \frac{1}{\tau^*(\overline{\beta})+\phi}\right]\overline{f}_\beta > 0$$

where $\overline{f}_\beta$ is the upper bound on density. By construction, $d_U^1$ exceeds the first term in equation 27. From the proof of Lemma 2, it is easily seen that there exists finite values $\underline{e}, \overline{e} > 0$ such that $\underline{e} < -\frac{\partial \Phi_M(\widehat{\beta}_1,\widehat{\beta}_2)}{\partial \widehat{\beta}_1}, \frac{\partial \Phi_M(\widehat{\beta}_1,\widehat{\beta}_2)}{\partial \widehat{\beta}_2} < \overline{e}$ for all $\widehat{\beta}_1, \widehat{\beta}_2 \in [\underline{\beta}, \overline{\beta}]$. Accordingly,

$$\begin{aligned}
-\int_\beta^{\overline{\beta}} \left(\frac{\mu}{\sqrt{\Phi_M(\beta,s)}} - 1\right)\frac{\partial \Phi_M(\beta,s)}{\partial \widehat{\beta}_1} f_\beta(s)ds &\leq -\frac{\overline{e}\overline{f}}{\underline{e}}\int_{\underline{\beta}}^{\overline{\beta}} \left(\frac{\mu}{\sqrt{\Phi_M(\beta,s)}} - 1\right)\frac{\partial \Phi_M(\beta,s)}{\partial s}ds \\
&= -\frac{\overline{e}\overline{f}}{\underline{e}}\left[2\mu\sqrt{\Phi_M(\beta,s)} - \Phi_M(\beta,s)\right]_{\underline{\beta}}^{\overline{\beta}} \\
&\leq \frac{\overline{e}\overline{f}}{\underline{e}}\left[2\mu\sqrt{\Phi_M(\underline{\beta},\overline{\beta})} + \Phi_M(\underline{\beta},\overline{\beta})\right] \\
&\equiv d_U^2
\end{aligned}$$

where we have used the bounds on the derivatives of $\Phi_M$ to switch the partial derivative in the first line. A completely analogous argument yields an upper bound, $d_H^3$, on the final term in equation 27. To complete the proof, we take $d_U = d_U^1 + d_U^2 + d_U^3$. $\square$

**Proposition 9.** *$\beta^S(\tau)$ is strictly increasing in $\tau$ with $\tau < \tau^*(\beta^S(\tau))$ for $\tau < \tau^*(\overline{\beta})$, and strictly decreasing in $\tau$ with $\tau > \tau^*(\beta^S(\tau))$ for $\tau > \tau^*(\overline{\beta})$.*

*Proof.* Substituting the initial condition into equation 26, we obtain $\frac{\partial \beta^S}{\partial \tau}\Big|_{\tau=\tau^*(\overline{\beta})} = 0$, as claimed in the text. Using equation 26 along with Lemma 6, we see that $\frac{\partial \beta^S}{\partial \tau} < 0$ when $\tau > \tau^*(\beta^S(\tau))$, and $\frac{\partial \beta^S}{\partial \tau} > 0$ when $\tau < \tau^*(\beta^S(\tau))$.

First consider $\tau > \tau^*(\overline{\beta})$. Starting at $\tau^*(\overline{\beta})$, a small increase in $\tau$ has a negligible effect on $\beta^S$, leaving us with $\tau > \tau^*(\beta^S(\tau))$, and hence $\frac{\partial \beta^S}{\partial \tau} < 0$. As $\tau$ increases, the slope remains strictly negative unless we reach a point at which $\tau = \tau^*(\beta^S(\tau))$. But that is impossible, because $\tau^*(\beta) < \tau^*(\overline{\beta})$ for all $\beta < \overline{\beta}$.

Next consider $\tau < \tau^*(\overline{\beta})$. Starting at $\tau^*(\overline{\beta})$, a small decrease in $\tau$ has a negligible effect on $\beta^S$, leaving us with $\tau < \tau^*(\beta^S(\tau))$, and hence $\frac{\partial \beta^S}{\partial \tau} > 0$. As $\tau$ increases, the slope remains strictly positive unless we reach a value $\tau'$ at which $\tau' = \tau^*(\beta^S(\tau'))$. But that is impossible, because (i) $\frac{\partial \tau^*}{\partial \beta}$ is strictly positive and bounded away from 0 on $[\underline{\beta}, \overline{\beta}]$, and (ii) from (26) and lemma 6, $\frac{\partial \beta^S}{\partial \tau}$ would converge to zero as $\big(\tau, \beta^S(\tau)\big)$ converged to $\big(\tau', \tau^*(\beta^S(\tau'))\big)$; thus, further reductions in $\tau$ would widen the gap between $\beta^S(\tau)$ and $(\tau^*)^{-1}(\tau)$. $\square$

**Proposition 10.** *There exists $\lambda^*$ such that for $\lambda > \lambda^*$, $\tau^1$ is globally incentive-compatible. $\tau^2$is never globally incentive compatible.*

*Proof.* We begin by showing that $\tau^1$ converges to $\tau^*$, and establishing a lower bound on $\frac{\partial \tau^1}{\partial \beta}$. To this end, we define $R(\lambda) = \max_{\beta \in [\underline{\beta}, \overline{\beta}]} \frac{\tau^*(\beta)+\phi}{\tau^1(\beta)+\phi} > 1$. Continuity of $\tau^*$ and $\tau^1$ ensures existence of the maximum.

**Lemma 7.** *For any $\lambda$ and all $\beta \in [\underline{\beta}, \overline{\beta}]$, we have $\frac{\partial \tau^1}{\partial \beta} > \frac{d_L}{\lambda c(R(\lambda)^2-1)}$. Furthermore, $\lim_{\lambda \to \infty} R(\lambda) = 1$.*

*Proof.* Fix a value of $\lambda$. From equation 25 and lemma 6, we have $-d_L \leq \lambda c(1 - R(\lambda)^2) \frac{\partial \tau^1}{\partial \beta}$, or equivalently $\frac{\partial \tau^1}{\partial \beta} > \frac{d_L}{\lambda c(R(\lambda)^2-1)}$, for all $\beta \in [\underline{\beta}, \overline{\beta}]$, which establishes the first part of the lemma.

For the second part of the lemma, we begin by observing that $\frac{\partial\tau^*}{\partial\beta} = \frac{1}{2\sqrt{c(\alpha+\beta)}}$, which is bounded below by $g \equiv \frac{1}{2\sqrt{c(\alpha+\overline{\beta})}}$.

Now fix $r > 1$, and define $\lambda_r \equiv r\frac{d_H}{g(r^2-1)}$. From equation 25 and the definitions of $d_H$ and $g$, it follows that, if $\lambda > \lambda_r$ and $\frac{\tau^*(\beta)+\phi}{\tau^1(\beta)+\phi} \geq r$, we would have $r\frac{\partial\tau^1}{\partial\beta} < \frac{\partial\tau^*}{\partial\beta}$.

Next we show that, for all $\lambda > \lambda_r$, we have $R(\lambda) < r$. Suppose on the contrary that, for such $\lambda$, there is some $\beta$ for which $\frac{\tau^*(\beta)+\phi}{\tau^1(\beta)+\phi} \geq r$. Let $\beta'$ be the largest value for which $\frac{\tau^*(\beta')+\phi}{\tau^1(\beta')+\phi} = r$. Differentiating, we obtain

$$\begin{aligned} \frac{d}{d\beta}\left(\frac{\tau^*(\beta)+\phi}{\tau^1(\beta)+\phi}\right)\bigg|_{\beta=\beta'} &= \frac{\frac{d\tau^*}{d\beta}\left(\tau^1(\beta')+\phi\right) - \frac{d\tau^1}{d\beta}\left(\tau^*(\beta')+\phi\right)}{\left(\tau^1(\beta')+\phi\right)^2} \\ &= \frac{\frac{d\tau^*}{d\beta} - \frac{d\tau^1}{d\beta}r}{\tau^1(\beta')+\phi} \\ &> 0. \end{aligned}$$

It follows that a small increase in $\beta$ from $\beta'$ would result in $\frac{\tau^*(\beta)+\phi}{\tau^1(\beta)+\phi} > r$. But we know that $\frac{\tau^*(\overline{\beta})+\phi}{\tau^1(\overline{\beta})+\phi} = 1 < r$. Consequently, there would have to be some $\beta'' \in (\beta', \overline{\beta})$ for which $\frac{\tau^*(\beta'')+\phi}{\tau^1(\beta'')+\phi} = r$. But that contradicts the definition of $\beta'$.

We conclude that, for any $r > 1$, we have $R(\lambda) < r$ for all $\lambda > \lambda_r$. The lemma follows directly. $\square$

**Lemma 8.** *There exists $\lambda^*$ such that for all $\lambda > \lambda^*$, we have $\frac{\partial^2 V(\beta,\widehat{\beta})}{\partial\beta\partial\widehat{\beta}} > 0$ for all $\beta \in [\underline{\beta}, \overline{\beta}]$.*

*Proof.* Using equation 3 to substitute for $\tau^*(\beta)$ in equation 24 and differentiating, we obtain

$$\begin{aligned} \frac{\partial^2 V(\beta,\widehat{\beta})}{\partial\widehat{\beta}\partial\beta} &= \lambda\left(\frac{1}{\tau^S(\widehat{\beta})+\phi}\right)^2 \frac{\partial\tau^1}{\partial\widehat{\beta}} - \left(\frac{1}{\phi} - \sqrt{\frac{c}{\alpha+\beta}}\right) f_\beta(\widehat{\beta}) \\ &> \left(\frac{1}{\tau^*(\overline{\beta})+\phi}\right)^2 \frac{d_L}{\lambda c(R(\lambda)^2-1)} - \left(\frac{1}{\phi} - \sqrt{\frac{c}{\alpha+\overline{\beta}}}\right)\overline{f}_\beta, \end{aligned} \tag{28}$$

where $\overline{f}_\beta$ is an upper bound on the density of $\beta$, and the inequality makes use of Lemma 4. Observe that (i) $\left(\frac{1}{\tau^*(\overline{\beta})+\phi}\right)^2$ is strictly positive and independent of $\lambda$, (ii) $\frac{d_L}{\lambda c(R(\lambda)^2-1)}$ is strictly positive and, by Lemma 4, converges to $+\infty$ as $\lambda \to +\infty$, and (iii) $\left(\frac{1}{\phi} - \sqrt{\frac{c}{\alpha+\overline{\beta}}}\right)\overline{f}_\beta$ is finite. Accordingly, the entire expression is strictly positive for $\lambda$ sufficiently large. $\square$

Now we prove the first part of the proposition. In light of the fact that $\frac{\partial^2 V(\beta,\widehat{\beta})}{\partial\beta\partial\widehat{\beta}} > 0$, the first-order condition for claiming to be of type $\widehat{\beta}$, $\frac{\partial V(\beta,\widehat{\beta})}{\partial\widehat{\beta}} = 0$, can be satisfied by at most one type, which is by construction $\beta = \widehat{\beta}$. Consequently the optimal value of $\widehat{\beta}$ for each $\beta$ must be $\beta, \underline{\beta}$, or $\overline{\beta}$. We can rule out $\underline{\beta}$ on the grounds that $\left.\frac{\partial V(\beta,\widehat{\beta})}{\partial\widehat{\beta}}\right|_{\beta,\widehat{\beta}=\underline{\beta}} = 0$ and $\frac{\partial^2 V(\beta,\widehat{\beta})}{\partial\widehat{\beta}\partial\beta} > 0$; similarly for $\overline{\beta}$. We conclude that $\tau^S$ is globally incentive-compatible.

Turning to the second part of the proposition, we know that $\lim_{\beta\to\overline{\beta}} \frac{\partial\tau^2}{\partial\widehat{\beta}} = -\infty$. Accordingly, the analog of equation 28 tells us that $\frac{\partial^2 V(\beta,\widehat{\beta})}{\partial\widehat{\beta}\partial\beta} < 0$ for $\beta$ and $\widehat{\beta}$ in a neighborhood of $\overline{\beta}$. But in that case, local incentive compatibility is violated: for $\beta_1$ and $\beta_2$ close to $\overline{\beta}$, the fact that $V(\beta_1,\beta_1) > V(\beta_1,\beta_2)$ would imply $V(\beta_2,\beta_1) > V(\beta_2,\beta_2)$. □

**Remark 1.** *The proof also supplies a sufficient condition for global compatibility that we can check after solving for $\tau^S$ numerically: for all $\beta,\widehat{\beta} \in [\underline{\beta},\overline{\beta}]$:*

$$\Omega(\beta,\widehat{\beta}) \equiv \frac{1}{\left(\tau^S(\widehat{\beta})+\phi\right)^2}\frac{d\tau^s(\widehat{\beta})}{d\widehat{\beta}} - \left(\frac{1}{\phi} - \frac{1}{\tau^*(\beta)+\phi}\right) f_\beta(\widehat{\beta}) > 0$$

**Proposition 11.** *$V(\beta,\beta)$ is decreasing in $\beta$.*

*Proof.* Consider $\beta < \beta'$ and the following decomposition of $V(\beta,\beta) - V(\beta',\beta')$

$$V(\beta,\beta) - V(\beta',\beta') = \left[V(\beta,\beta) - V(\beta,\beta')\right] + \left[V(\beta,\beta') - V(\beta',\beta')\right]$$

The first term is positive since incentive compatibility requires $V(\beta,\beta) \geq V(\beta,\beta')$. Using some algebra we find

$$\begin{aligned} V(\beta,\beta') - V(\beta',\beta') = &-\int_{\beta'}^{\overline{\beta}} \left[\frac{\alpha+\beta}{\tau^*(\beta)+\phi} - \frac{\alpha+\beta'}{\tau^*(\beta')+\phi} + c\left[\tau^*(\beta) - \tau^*(\beta')\right]\right] f_\beta(s)ds \\ &- \int_{\underline{\beta}}^{\beta'} \left[\frac{\beta-\beta'}{\phi}\right] f_\beta(s)ds - \lambda\frac{\beta-\beta'}{\tau^S(\beta')+\phi} \end{aligned} \tag{29}$$

Using the fact that

$$\frac{\alpha+\beta}{\tau^*(\beta)+\phi} = \sqrt{c(\alpha+\beta)} = c\sqrt{\frac{\alpha+\beta}{c}} = c(\tau^*(\beta)+\phi)$$

we obtain the following

$$
\begin{aligned}
V\left(\beta,\beta'\right)-V\left(\beta',\beta'\right) &= -2c\int_{\beta}^{\overline{\beta}}\left[\tau^{*}(\beta)-\tau^{*}(\beta')\right]f_{\beta}(s)ds \\
&\quad -\int_{\underline{\beta}}^{\beta}\left[\frac{\beta-\beta'}{\phi}\right]f_{\beta}(s)ds-\lambda\frac{\beta-\beta'}{\tau^{S}(\beta')+\phi}
\end{aligned}
\tag{30}
$$

Since $\beta<\beta'$ implies $\tau^{*}(\beta)<\tau^{*}(\beta')$, all of these terms are positive, so we have $V\left(\beta,\beta'\right)-V\left(\beta',\beta'\right)\geq 0$. Therefore $V\left(\beta,\beta\right)\geq V\left(\beta',\beta'\right)$. $\square$