Financial Integration:
A New Methodology and
An Illustration

Robert P. Flood and Andrew K. Rose
Two Objectives:

1. Derive new methodology to assess integration of assets across instruments/borders/markets, etc.

2. Use methodology to illustrate technique empirically
   - Find remarkably little evidence of asset integration between S&P and NASDAQ
Definition of Asset Integration

• Assets are *integrated* if satisfy asset-pricing condition:

\[ p_t^j = E_t(d_{t+1}x_{t+1}^j) \]  (1)

• Completely standard general framework
Paper Focus: $E_t(d_{t+1})$

- Marginal Rate of Substitution/Discount Factor ties together all intertemporal decisions

- Subject of much research (Hansen-Jagannathan, etc.)

- Prices all assets

- Unobservable, even *ex post* (but estimable)

- Should be identical for all assets *in an integrated market*
Empirical Strategy

Definition of Covariance:

\[ p_t^j = E_t(d_{t+1}x_{t+1}^j) = COV_t(d_{t+1}, x_{t+1}^j) + E_t(d_{t+1})E_t(x_{t+1}^j). \]  

(2)

Rearrange and substitute actual for expected (WLOG):

\[ x_{t+1}^j = \frac{1}{E_t(d_{t+1})}COV_t(d_{t+1}, x_{t+1}^j) + \frac{1}{E_t(d_{t+1})}p_t^j + \varepsilon_{t+1}^j, \]

\[ x_{t+1}^j = \delta_t (p_t^j - COV_t(d_{t+1}, x_{t+1}^j)) + \varepsilon_{t+1}^j \]  

(3)

where \( \delta_t = \frac{1}{E_t(d_{t+1})} \)
Impose Two (Reasonable?) Assumptions for Estimation:

1) *Rational Expectations*: $\varepsilon_{t+1}^j$ is assumed to be white noise, uncorrelated with information available at time t, and

2) *Factor Model*:

$$COV_t(d_{t+1}, x_{t+1}^j) = \beta_j^0 + \sum_i \beta_j^i f_i^t$$, for the relevant sample.
Now we have an estimable Panel Equation:

\[ x_{t+1}^j = \delta_t (p_t^j - COV_t (d_{t+1}, x_{t+1}^j)) + \epsilon_{t+1}^j \]  

(3)

• Use *Cross-sectional* variation to estimate the coefficients of interest \{\delta\} – the shadow discount rates

• Use *Time-series* variation to estimate nuisance coefficients \{\beta\}

• Can estimate \{\delta\} for two sets of assets and compare them

  o Should be equal if assets are integrated – priced with same shadow discount rate
Why this Strategy?

• Natural to look at first moment (of MRS) first

• Easy to estimate

• Insensitive in practice

• Confirm priors, previous research, but discriminating
Are Assumptions Reasonable?

Easier

- Rational expectations in financial markets at relatively high frequencies
Harder

- Portfolio-specific covariances (payoffs with discount rates) are either constant or have constant relations with small number of factors, *for short samples*

- Standard assumption to make in literature

- Use standard factor model (Fama-French)
  - FF: 30 years; here for 2 months

- Sensitivity Analysis for robustness
Strengths of Methodology

1. Tightly based on general theory
2. Do not need particular asset pricing model held with confidence for long period of time
3. Do not model discount rate directly
4. Relatively loose assumptions required
5. Requires accessible, reliable data
6. Can be used at many frequencies

7. Can be used for many asset classes (stocks, bonds, foreign)

8. Requires no special/obscure software (E-Views/RATS/TSP/STATA all work – just NLLS)

9. Focused on intrinsically interesting object
Differences with Literature

• We focus on first-moment of $\delta$ (estimated discount rate/MRS)
  
  • Standard: $\beta$ (factor loadings), or second moment of $\delta$

• Our set-up is intrinsically non-linear

• We don’t fixate on asset-pricing model (though need it)
Most Importantly, don’t impose bond market integration

• Consider risk-free gov’t T-bill with price of $1, interest $i_t$:

\[ 1 = E_t(d_{t+1}(1+i_t)) \Rightarrow \frac{1}{1+i_t} = E_t(d_{t+1}) \]

• We do not use the T-bill rate since the T-bill market may not be integrated with the stock market!

• Will test (and reject!) this assumption

• Do not violate replication/arbitrage since we are testing for integration across markets where replication is impossible
Implementation

Estimate:

\[ \frac{x_{t+1}^j}{p_{t-1}^j} = \delta_t \left( (p_t^j / p_{t-1}^j) + \beta_j^0 + \beta_j^1 f_t^1 + \beta_j^2 f_t^2 + \beta_j^3 f_t^3 \right) + \epsilon_{t+1}^j \]  \hspace{1cm} (4)

- Normalize to make Cov() more plausibly time-invariant (with factors)
- Use Fama-French (1996) 3 factors
- Estimate with NLLS, Newey-West covariances
  - Degree of non-linearity low
Notes

• Similar in nature to Roll and Ross (1980)

• Subsumes static CAPM through \( \{\beta^0\} \)

• Add three time-varying factors from Fama-French (their data!)
  
  o Market return less T-bill return
  
  o Small minus large return
  
  o High minus low book/market returns
• Use moderately high-frequency approach
  
  o Daily data for 2-month spans
First Example

• April-May 1999

• Use first 100 S&P 500 firms (by ticker symbol) that did not go ex-dividend (no obvious bias)

• Group randomly into 20 portfolios of 5 firms each (by ticker)

• Closing rates from “US Pricing” of Thomson Analytics

• 41 days, lose one each for lead/lag
Shadow Discount Rates

• Can easily estimate from sets of 10 S&P portfolios (along with confidence intervals):
• Two delta estimates look reasonably close, day by day

• Lots of time-series variation (Hansen-Jagannathan)

• Can reject hypothesis that $\delta =$ Treasury bill return (sluggish at 4.4% annual)
Likelihood-Ratio (Joint) Test for Asset Integration

• $2(2309-(1160+1166)) = 36$

• sits virtually at the median of $\chi^2(39)$

• Can’t reject null Ho of asset integration

• Bootstrapping (leptokurtosis!) implies p-value of .9
Broadening the Sample

- Five other samples (2 different sets of 2-month periods in 1999; same months in 2002) confirm integration

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<tr>
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<tbody>
<tr>
<td>First 10 portfolios</td>
<td>1160.</td>
<td>1302.</td>
<td>1157.</td>
</tr>
<tr>
<td>Second 10 portfolios</td>
<td>1166.</td>
<td>1299.</td>
<td>1172.</td>
</tr>
<tr>
<td>All 20 portfolios</td>
<td>2309.</td>
<td>2574.</td>
<td>2303.</td>
</tr>
<tr>
<td>Test (bootstrap P-value)</td>
<td>36 (.90)</td>
<td>54 (.37)</td>
<td>51 (.43)</td>
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Table 1: Integration inside the S&P 500, Fama-French-Factor Model
Add Different Asset Classes

• NASDAQ firms

• Same timing, samples
NASDAQ is usually (not always) integrated

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<tr>
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<tr>
<td><strong>First 10 portfolios</strong></td>
<td>881.</td>
<td>1066.</td>
<td>757.</td>
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<tr>
<td><strong>Second 10 portfolios</strong></td>
<td>816.</td>
<td>990.</td>
<td>945.</td>
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<tr>
<td><strong>All 20 portfolios</strong></td>
<td>1677.</td>
<td>2023.</td>
<td>1625.</td>
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<tr>
<td><strong>Test (bootstrap P-value)</strong></td>
<td>42 (.83)</td>
<td>65 (.20)</td>
<td>153** (.00)</td>
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<td><strong>First 10 portfolios</strong></td>
<td>1052.</td>
<td>1061.</td>
<td>991.</td>
</tr>
<tr>
<td><strong>Second 10 portfolios</strong></td>
<td>1174.</td>
<td>1003.</td>
<td>962.</td>
</tr>
<tr>
<td><strong>All 20 portfolios</strong></td>
<td>2185.</td>
<td>2035.</td>
<td>1919.</td>
</tr>
<tr>
<td><strong>Test (bootstrap P-value)</strong></td>
<td>82* (.03)</td>
<td>58 (.45)</td>
<td>69 (.08)</td>
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Table 2: Integration inside the NASDAQ, Fama-French-Factor Model
NASDAQ is *never* integrated with the S&P

- Test statistics for across-market integration are an order of magnitude higher than those for within-market integration

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<tr>
<td><strong>20 S&amp;P Portfolios</strong></td>
<td>2309.</td>
<td>2574.</td>
<td>2303.</td>
</tr>
<tr>
<td><strong>Combined</strong></td>
<td>3706.</td>
<td>4396.</td>
<td>3633.</td>
</tr>
<tr>
<td><strong>Test (bootstrap P-value)</strong></td>
<td>559** (.00)</td>
<td>403** (.00)</td>
<td>590** (.00)</td>
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<td>2805.</td>
<td>2525.</td>
<td>2456.</td>
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<td><strong>20 NASDAQ Portfolios</strong></td>
<td>2185.</td>
<td>2035.</td>
<td>1919.</td>
</tr>
<tr>
<td><strong>Combined</strong></td>
<td>4735.</td>
<td>4352.</td>
<td>4170.</td>
</tr>
<tr>
<td><strong>Test (bootstrap P-value)</strong></td>
<td>511** (.00)</td>
<td>416** (.00)</td>
<td>410** (.00)</td>
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Table 3: Integration between S&P 500 and NASDAQ, Fama-French Model
Sensitivity Analysis

- Does exact factor model matter?

- Can drop 2 “extra” Fama-French factors; similar results

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<td><strong>Within S&amp;P</strong></td>
<td>36 (.93)</td>
<td>48 (.75)</td>
<td>30 (.99)</td>
</tr>
<tr>
<td><strong>Within NASDAQ</strong></td>
<td>47 (.79)</td>
<td>65 (.27)</td>
<td>127** (.00)</td>
</tr>
<tr>
<td><strong>S&amp;P vs. NASDAQ</strong></td>
<td>548** (.00)</td>
<td>388** (.00)</td>
<td>594** (.00)</td>
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<td><strong>Within S&amp;P</strong></td>
<td>44 (.88)</td>
<td>55 (.61)</td>
<td>35 (.98)</td>
</tr>
<tr>
<td><strong>Within NASDAQ</strong></td>
<td>80 (.09)</td>
<td>58 (.61)</td>
<td>72 (.13)</td>
</tr>
<tr>
<td><strong>S&amp;P vs. NASDAQ</strong></td>
<td>497** (.00)</td>
<td>432** (.00)</td>
<td>422** (.00)</td>
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Table 4: Integration between S&P 500 and NASDAQ, 1 factor (market) Model
In fact, Time-Varying Factors Make Little Difference!

- Can estimate with only firm-specific intercepts
- Very similar results and conclusions

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<td>33 (.97)</td>
<td>46 (.71)</td>
<td>34 (.94)</td>
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<tr>
<td>Within NASDAQ</td>
<td>42 (.80)</td>
<td>62 (.28)</td>
<td>114** (.00)</td>
</tr>
<tr>
<td>S&amp;P vs. NASDAQ</td>
<td>534** (.00)</td>
<td>378** (.00)</td>
<td>591** (.00)</td>
</tr>
<tr>
<td>April-May 2002</td>
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<tr>
<td>Within S&amp;P</td>
<td>46 (.76)</td>
<td>47 (.77)</td>
<td>36 (.95)</td>
</tr>
<tr>
<td>Within NASDAQ</td>
<td>86* (.03)</td>
<td>52 (.63)</td>
<td>68 (.12)</td>
</tr>
<tr>
<td>S&amp;P vs. NASDAQ</td>
<td>506** (.00)</td>
<td>416** (.00)</td>
<td>419** (.00)</td>
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Table 5: Integration between S&P 500 and NASDAQ, Only Firm Intercepts
Deltas from Different Markets and Samples

April-May 1999: S&P

Oct-Nov 1999: S&P

July-Aug 2002: S&P

April-May 1999: NASDAQ

Oct-Nov 1999: NASDAQ

July-Aug 2002: NASDAQ

July-Aug 1999: S&P

April-May 2002: S&P

Oct-Nov 2002: S&P

July-Aug 1999: NASDAQ

April-May 2002: NASDAQ

Oct-Nov 2002: NASDAQ
Scatterplots of S&P against NASDAQ Deltas

April-May 1999

July-Aug 1999

Oct-Nov 1999

April-May 2002

July-Aug 2002

Oct-Nov 2002
Future Work

• Monte Carlo work for small samples

• Examine before/after crises

• Lower frequencies (housing? more factors? trends?)

• Higher frequencies

• Is the finding of little integration general?

Most Importantly

• Causes of low integration?