

**Financial Integration:  
A New Methodology and an Illustration  
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*Comments Welcome*

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**Abstract**

This paper develops a simple methodology to test for asset integration, and applies it within and between American stock markets. Our technique relies on estimating and comparing expected risk-free rates across assets. Expected risk-free rates are allowed to vary freely over time, constrained only by the fact that they must be equal across (risk-adjusted) assets in well integrated markets. Assets are allowed to have standard risk characteristics, and are constrained by a factor model of covariances over short time periods. We find that implied expected risk-free rates vary dramatically over time, unlike short interest rates. Further, internal integration in the S&P 500 market is never rejected and is generally not rejected in the NASDAQ. Integration between the NASDAQ and the S&P, however, is always rejected dramatically.

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## 1: Defining the Problem

The objective of this paper is to propose and implement an intuitive and simple-to-use measure of asset-market integration. What does *asset-market integration* mean? We adopt the view that financial markets are integrated when assets are priced by the same stochastic discount rate. More precisely, we define security markets to be integrated if all assets priced on those markets satisfy the pricing condition:

$$p_t^j = E_t(m_{t+1}x_{t+1}^j) \quad (1)$$

where:  $p_t^j$  is the price at time  $t$  of asset  $j$ ,  $E_t(\cdot)$  is the expectations operator conditional on information available at  $t$ ,  $m_{t+1}$  is the intertemporal marginal rate of substitution (MRS), for income accruing in period  $t+1$  (also interchangeably known as the discount rate, stochastic discount factor, marginal utility growth, pricing kernel, and zero-beta return), and  $x_{t+1}^j$  is income received at  $t+1$  by owners of asset  $j$  at time  $t$  (the future value of the asset plus any dividends or coupons).

Our object of interest in this study is  $E_t m_{t+1}$  the time  $t$  expectation of the marginal rate of substitution. Agents behaving according to equation (1) use the entire perceived distribution of  $m_{t+1}$  to price assets at  $t$ . Nevertheless, we concentrate on its first moment for two reasons. First,  $E_t m_{t+1}$  is simple to measure. Second, cross-market differences in estimated values of  $E_t m_{t+1}$  turn out in practice to be highly illuminating. In particular, they allow us to use standard risk pricing models to discriminate for differences in market integration.

We emphasize at the outset that our test investigates a *necessary* but not *sufficient* condition for market integration. In other words if two portfolios are well integrated they will pass our test, but passing the test does not imply the portfolios to be well integrated. On the other hand if two portfolios fail the test, the portfolios are not well integrated.

## 2: Methodology

We use a standard decomposition of equation (1):

$$p_t^j = E_t(m_{t+1}x_{t+1}^j) = COV_t(m_{t+1}, x_{t+1}^j) + E_t(m_{t+1})E_t(x_{t+1}^j). \quad (2)$$

where  $COV_t(\cdot)$  denotes the conditional covariance operator. It is useful to rewrite this as

$$\begin{aligned} x_{t+1}^j &= -[1/E_t(m_{t+1})]COV_t(m_{t+1}, x_{t+1}^j) + [1/E_t(m_{t+1})]p_t^j + \mathbf{e}_{t+1}^j, & \text{or} \\ x_{t+1}^j &= \mathbf{d}_t(p_t^j - COV_t(m_{t+1}, x_{t+1}^j)) + \mathbf{e}_{t+1}^j \end{aligned} \quad (3)$$

where  $\mathbf{e}_{t+1}^j \equiv x_{t+1}^j - E_t(x_{t+1}^j)$ , a prediction error, and  $\mathbf{d}_t \equiv 1/E_t(m_{t+1})$ . The latter is the parameter of interest to us. In an integrated market, it is identical for all assets. Our work below is essentially concerned with exploiting and testing this restriction.

### 2a: The Traditional (Asset Return) Approach

It is typical in Finance to make equation (3) stationary by dividing the equation by  $p_{j,t}$ , resulting in:

$$x_{t+1}^j / p_t^j = \mathbf{d}_t (1 - COV_t(m_{t+1}, x_{t+1}^j / p_t^j)) + \mathbf{e}_{t+1}^j, \quad (4)$$

where  $\mathbf{e}_{t+1}^j$  is redefined appropriately. Dividing through by  $p_{j,t}$  also converts equation (3) into an asset-pricing equation – an equation relating one-period asset returns,  $x_{t+1}^j / p_t^j$ , to the market  $\mathbf{d}_t \equiv 1 / E_t(m_{t+1})$ , and to the asset-specific period risk premium.

Equation (4) is then given economic content by adding two assumptions:

- 1) *Rational Expectations*:  $\mathbf{e}_{t+1}^j$  is assumed to be uncorrelated with information available at time t, and
- 2) *Covariance Model*:  $COV_t(m_{t+1}, x_{t+1}^j / p_t^j) = \mathbf{b}_0^j + \sum_i \mathbf{b}_i^j f_{i,t}$ , for the relevant sample,

where:  $\mathbf{b}_0^j$  is an asset-specific intercept,  $\mathbf{b}_i^j$  is a set of I asset-specific factor coefficients and  $f_{i,t}$  a vector of time-varying factors.

Both assumptions are common in the literature; Campbell, Lo and MacKinlay (1997) and Cochrane (2001) provide excellent discussions.

With these two assumptions, equation (4) becomes a panel estimating equation. We use *time-series* variation to estimate the asset-specific factor loadings  $\{\mathbf{b}\}$ , coefficients that are constant across time. We exploit *cross-sectional* variation to estimate  $\{\mathbf{d}\}$ , the coefficients of interest that represent the risk-free return and are time varying but common to all assets.

It is evident that this approach does not allow us to identify both the  $\mathbf{b}_0^j$  and the  $\mathbf{d}(t)$  - that would be an excessive number of constants and perfect multicollinearity. Empirical asset pricing models solve the problem either by setting  $\mathbf{d}(t) = 1 + i(t)$ , where  $i(t)$  is an appropriate short-term riskless interest rate, or by setting  $\mathbf{b}_0^j = 0$  for all  $j$ .

In the empirical work reported below, we follow this standard approach and estimate the  $\mathbf{d}(t)$  by setting  $\mathbf{b}_0^j = 0$  for all  $j$ . Anticipating the results, the factor loadings are estimated sharply; the  $\mathbf{d}(t)$  however, which should be just greater than unity are estimated implausibly and imprecisely. Our primary approach to estimating  $\mathbf{d}(t)$  thus takes a different tack.

## 2b: A Different Approach

When we normalize equation (3) by  $p_t^j$  we detach it from  $\mathbf{d}(t)$  and lose an information-laden regressor that helps to identify and estimate  $\mathbf{d}(t)$ .

Since our primary interest is  $\mathbf{d}(t)$ , we choose to normalize equation (3) by dividing the data by something *other than*  $p_t^j$ . In this paper we use  $p_{t-1}^j$ , but other normalizations, e.g., a market-wide price index, would do just as well. The equations we estimate in our new approach stem from:

$$x_{t+1}^j / p_{t-1}^j = \mathbf{d}_t((p_t^j / p_{t-1}^j) - COV_t(m_{t+1}, x_{t+1}^j / p_{t-1}^j)) + \mathbf{e}_{t+1}^j \quad (5)$$

The important point is to choose something that stabilizes the data (i.e., induces stationarity) and preserves the information in  $p_t^j$ , while still delivering moments in the  $COV_t(\ )$  term that can be

modeled as stable functions of a few aggregate sources of risk. This critical point is discussed further below.

Equation (5) is then given empirical content by adding the same two assumptions of the first approach, namely: 1) rational expectations, and 2) a relevant linear covariance model. To be clear, what we mean by the latter is:

2') *Covariance Model*:  $COV_t(m_{t+1}, x_{t+1}^j / p_{t-1}^j) = \mathbf{b}_0^j + \sum_i \mathbf{b}_i^j f_{i,t}$ , for the relevant sample.

We emphasize that equation (5), along with its empirical assumptions, is not a traditional asset-pricing regression of the type widely investigated in Finance. It is not designed to give information about period-to-period asset returns. Instead, it is designed to deliver information about  $\mathbf{d}(t)$ , and it does so primarily through the volatile regressor  $p_t^j / p_{t-1}^j$ .

Our test for integration is simple. Estimating, say, (5) for a set of assets  $j=1, \dots, J_0$  and then repeating the analysis for the same period of time with a different set of assets  $j=1, \dots, J_1$  gives us two sets of estimates of  $\{\mathbf{d}\}$ , a time-series sequence of estimated discount rates. These can be compared directly, using conventional statistical techniques, either one by one, or jointly. Under the null hypothesis of market integration, the two sets of  $\{\mathbf{d}\}$  coefficients are equal. If the two diverge, the hypothesis of market integration between the assets is rejected (jointly with the other two assumptions, of course). One can estimate  $\{\mathbf{d}\}$  via either equation (4), the traditional returns approach, or from our equation (5), our new approach. In our empirical analysis below we use both approaches, and they deliver the same message.

## 2c: Discussion

Choosing between the two approaches outlined above involves tradeoffs. The returns approach has been used widely in Finance to deliver empirical asset-pricing equations; see, e.g., Cochrane (2001). This approach was not, however, designed to estimate sharp and sensible estimates of  $\mathbf{d}(t)$ . Our new approach seems to deliver good  $\mathbf{d}(t)$  estimates, but there is clearly much less knowledge about the model's ability to produce stable covariance estimates through a factor model.

The factor model is potentially important in our new approach because getting it wrong might lead to inconsistent  $\mathbf{d}(t)$  estimates. Recall that  $m_{t+1}$  is asserted to depend only on market-wide aggregates, which implies that both  $COV_t(m_{t+1}, x_{t+1}^j / p_t^j)$  (used in the traditional returns approach) and  $COV_t(m_{t+1}, x_{t+1}^j / p_{t-1}^j)$  (used in our new approach) also depend only on aggregates. If the covariance factor model is mis-specified, the modeling error spills into the estimation error term, and could be correlated with  $p_t^j / p_{t-1}^j$ , thereby producing inconsistent  $\mathbf{d}(t)$  estimates.

Accordingly, we take precautions. First, in implementing our new approach, we use the same well-known aggregate factors used by Fama and French (1996) to model returns in the traditional approach, and we do robustness checks. Second, we require our factor model to hold with constant coefficients only over relatively short periods – generally two months of daily data. Third, we check our results against those generated in the same data set by the well-known Fama-French returns model, i.e., the analogues implied by the first approach.<sup>1</sup>

The measurements we produce are discriminating for market integration, yet they are robust, confirm our prior beliefs and previous research (e.g., Chen and Knez, 1995). In the examples below, our measure never rejects internal market integration for portfolios of S&P

stocks priced in the NYSE and seldom rejects for portfolios priced on the NASDAQ, but rejects integration strongly – by an order of magnitude – between NYSE and NASDAQ portfolios.

### **3: Relationship to the Literature**

Asset-market integration is a classic problem with a large associated literature, one which has grown along two branches. The first branch, based on parametric asset-pricing models, has been surveyed by Adams et. al. (2002), Cochrane (2001), and Campbell, Lo, and MacKinlay (1997). Karolyi and Stulz (2002) provide a survey of open-economy asset-market integration concepts and results. Along this branch, a parametric discount-rate model is used to price asset portfolios. Pricing errors are compared across portfolios. If the portfolios are integrated, the pricing errors should not be systematically identifiable with the portfolios in which they originate. Roll and Ross (1980) tested market integration this way using an (APT) arbitrage pricing theory model, and a large literature has followed, see e.g., Bekaert and Harvey (1995), hereafter “BH”.

The second branch of literature grows from the work of Hansen and Jagannathan (1991) and is represented by Chen and Knez (1995) and Chabot (2000). Along this branch, data from each supposed market is used to characterize the set of stochastic discount factors (SDF) that could have produced the observed data. Testing for cross market integration involves measuring the distance between admissible MRS sets, and asking if, and by how much, they overlap. If a common SDF exists the markets are integrated. If not, measures are available to judge the distance between the market-specific SDF sets.

Our work rests on the first branch, since we use parametric models to condition our estimation. It differs from previous work in three ways.

First, we do not measure integration by the full-blown cross-sectional pricing errors produced by a particular model. BH, working along the first branch used the definition “Markets are completely integrated if assets with the same risk have identical expected returns irrespective of the market.” Our market integration measure is based on a subset of the cross-market conditions demanded by BH. Instead of comparing all aspects of a fully parameterized SDF models, we measure integration by the implied first moment of the SDF. The condition we study, therefore, a necessary condition for integration. It is a subset of the conditions demanded by BH, and also Chen and Knez. Studying it will be valuable, therefore, only if it is simple to produce but still discriminating.

Second, parametric pricing models are often estimated with long data spans and are thus sensitive to parameter instability in time series long enough for precise estimation (e.g., Fama and French (1996); discussion is provided by Cochrane, 2001). We minimize (but do not avoid completely) the instability problem by concentrating attention on a parameter that is conditionally invariant to time-series instability. The measure we use is a free parameter, constant across assets but unconstrained across time. Our measure – borrowed from Roll and Ross - is therefore basically cross-sectional. We can estimate the measure precisely using a (very) short time-series dimension.

Finally, we do not assume that the bond market is integrated with other asset markets. When applied to a bond without nominal risk (e.g., a treasury bill), equation (1) implies

$$1 = E_t(m_{t+1}(1+i_t)) \quad \text{or} \quad \mathbf{d}_t \equiv 1 / E_t(m_{t+1}) = (1+i_t)$$

where:  $i_t$  is a risk-free nominal interest rate, and  $m_{t+1}$  is a nominal MRS. One tradition, common in Economics and Finance, is to assume that the SDF pricing bonds is the same for all bonds, and identical to that pricing all stocks (and other assets). We do not *impose* this assumption; instead we *test* it (and reject) it.

#### 4: Empirical Implementation

We begin by estimating our model (5') with asset-specific intercepts and the three time-varying factors used by Fama and French (1996). That is, we estimate:

$$x_{t+1}^j / p_{t-1}^j = \mathbf{d}_t ((p_t^j / p_{t-1}^j) + \mathbf{b}_0^j + \mathbf{b}_1^j f_{1,t} + \mathbf{b}_2^j f_{2,t} + \mathbf{b}_3^j f_{3,t}) + \mathbf{e}_{t+1}^j \quad (6)$$

for assets  $j=1, \dots, J$ , periods  $t=1, \dots, T$ . We allow  $\{\mathbf{d}_t\}$  to vary period by period, while we use a “three-factor” model and allow  $\{\mathbf{b}^j\}$  to vary asset by asset. The three Fama-French factors are: 1) the overall stock market return, less the treasury-bill rate, 2) the performance of small stocks relative to big stocks, and 3) the performance of “value” stocks relative to “growth” stocks. Further details and the data set itself are available at French’s website.<sup>2</sup> We also examine two other covariance models below.

Equation (6) can be estimated directly with non-linear least squares. The degree of non-linearity is not particularly high; conditional on  $\{\mathbf{d}_t\}$  the problem is linear in  $\{\mathbf{b}^j\}$  and vice versa. We employ robust (heteroskedasticity and autocorrelation consistent “Newey West”) covariance estimators.

We use a moderately high frequency approach. In particular, we use two-month spans of daily data. Using daily data allows us to estimate the coefficients of interest  $\{\mathbf{d}_t\}$  without assuming that firm-specific coefficients  $\{\mathbf{b}^j\}$  are constant for implausibly long periods of time.

Our empirical illustration examines the integration of American equity markets. Large American stocks are traded on liquid markets, which we consider *a priori* to be integrated. We begin by examining daily data over a quiet two-month period, April-May 1999 (about a year before the end of the Clinton bull market).<sup>3</sup> Two months gives us a span of over forty business day observations; this does not appear to stretch our reliance on a factor model of asset covariances excessively, while still allowing us to test financial market integration for an interesting span of data. We see no reason why higher- and/or lower-frequency data cannot be used.<sup>4</sup>

Our data set is drawn from the “US Pricing” database provided by Thomson Analytics. We collected closing rates for the first (in terms of ticker symbol) one hundred firms from the S&P 500 that did not go ex-dividend during the months in question. The absence of dividend payments allows us to set  $x_{t+1}^j = p_{t+1}^j$  (and does not bias our results in any other obvious way).

We group our hundred firms into twenty portfolios of five firms each, arranged simply by ticker symbol. We use portfolios rather than individual stocks for the standard reasons of the Finance literature. In particular, as Cochrane (2001) points out, portfolios betas are measured with less error than individual betas because of lower residual variance. They also vary less over time (as size, leverage, and business risk change less for a portfolio of equities than any individual component). Portfolio variances are lower than those of individual securities, enabling more precise covariance relationships to be estimated. And of course portfolios are what investors tend to use (especially those informed by Finance theory!).

Our first sample period consists of 41 days. Since we lose the first and last observations because of lags ( $p_{t-1}^j$ ) and leads ( $x_{t+1}^j$ ), we are left with a total of 780 observations in our panel data set (20 portfolios x 39 days). Our data has been checked for transcription errors, both visually and with random crosschecking.

There is no reason that one cannot use more data (longer spans at different frequencies, for larger number of firms and/or portfolios grouped non-randomly). We choose this sample (only two months of daily price data for one hundred firms grouped randomly into twenty portfolios) deliberately to illustrate the power of our methodology and its undemanding data requirements. However, we also check for sensitivity with respect to the sample below.

## 5: Results

We start by splitting our 20 portfolios into two sets of 10 portfolios each (simply by ticker symbol) to estimate discount rates (i.e., estimates of  $\mathbf{d}_t \equiv [1/E_t(m_{t+1})]$ ). We provide time-series plots of the estimated deltas from the first 10 portfolios along with a plus/minus two standard error confidence interval in Figure 1. We also include the point estimates of delta from the second 10 portfolios, estimated in precisely the same way but using data from the last set of 10 portfolios.

There are two striking features of the graph. First, the time-series variation in delta is high, consistent with the spirit of Hansen and Jagannathan (1991). As shown in Table 1, the log likelihood of our equation estimated on the first 10 S&P portfolios is 1160. In April-May 1999, the US 3-month Treasury bill rate averaged 4.4%, a daily return of 1.00017 (with little time-series variation). The log likelihood for the default equation estimated with 1.00017 substituted in place of  $\{\mathbf{d}_t\}$  is only 1059. Under the null hypothesis of deltas that are constant and equal to

the T-bill interest rate,  $2*(1160-1059)$  is distributed as a chi-square with 39 degrees of freedom, grossly inconsistent with the null at any reasonable confidence level. (When we use all 20 portfolios, the analogue is  $2*(2309-2136)$ , again grossly inconsistent with the null.) That is, the hypothesis that the MRS is equal to the short t-bill rate is wildly inconsistent with the data. The MRS seems much more volatile than short-term interest rates.

Second, the estimates of delta from the two different sets of portfolios are similar; the deltas from the second set of portfolios almost always lie within the  $\pm 2$  standard error confidence interval of the first estimate of delta. That is, the two different sets of delta are usually statistically indistinguishable on any given day, consistent with the null hypothesis of integration within the S&P.

What about the two sets of delta examined jointly? The ocular evidence leads one to believe that the two sets of deltas are broadly equal. The statistical analogue is contained in the cells at the top left of Table 1. The log-likelihood of (5') estimated from the first set of 10 portfolios is 1160; that from the second set of 10 portfolios is 1166. When (6) is estimated from all 20 portfolios simultaneously so that only a single set of  $\{d_t\}$  is extracted, the log-likelihood is 2309. Under the hypothesis of integration (i.e., the same  $\{d_t\}$  for both sets of assets) and normally distributed errors, minus twice the difference in the log-likelihoods is distributed as a chi-square with 39 degrees of freedom; a likelihood ratio (LR) test. The test statistic is 36, consistent with the hypothesis of integration and normal residuals at the .61 confidence level.

It is well known that asset prices are not in fact normally distributed; Campbell, Lo, and MacKinlay (1997). Rather, there is strong evidence of fat tails or leptokurtosis, and this certainly characterizes our data.<sup>5</sup> Accordingly, we use a bootstrap procedure to estimate the probability

values for our likelihood ratio tests.<sup>6</sup> The bootstrapped p-value for the test of integration is even more consistent with the null hypothesis of integration at the .90 level.

To check for sample sensitivity, we also consider five other sample periods: July-August 1999, October-November 1999, and the same three two-month samples for the bear market of 2002. Results from these other sample periods are also included in Table 1 and are also consistent with the hypothesis of integration inside the S&P 500 at standard confidence levels.

What about the NASDAQ market for smaller stocks? We follow exactly the same procedures, but using data drawn from the NASDAQ market. We group (again on the basis of ticker symbol) data from 100 NASDAQ firms into 20 portfolios of 10 firms each, and test for equality of deltas (between the two different sets of deltas, estimated from the two sets of ten NASDAQ portfolios) using likelihood ratio tests with bootstrapped p-values. The results are presented in Table 2, and are generally consistent with the null hypothesis of integration inside the NASDAQ. However, one of our samples (April-May 2002) is inconsistent with integration at the .03 confidence level (this is marked with an asterisk), while integration is overwhelmingly rejected for Oct-Nov 1999 (two asterisks), shortly before the collapse of the NASDAQ. We think of these as intuitive, reasonable results, possibly consistent with the existence of “irrational exuberance” manifest in the NASDAQ just around the height of the internet bubble.

Still, the most interesting question to us is: Is the market for large (S&P 500) stocks integrated with the NASDAQ? It is easy to ask the question by comparing  $\{d_t\}$  estimates when (6) is estimated with: a) the twenty S&P portfolios; b) the twenty NASDAQ portfolios; and c) all forty portfolios pooled together (which is most efficient if the two markets are integrated). Our LR tests (with bootstrapped p-values) for this hypothesis are presented in Table 3 and are grossly inconsistent with the null hypothesis of market integration. The LR test statistics are an order of

magnitude bigger than those of Tables 1 and 2. That is, while the S&P always seems integrated and the NASDAQ is generally integrated, the S&P is never integrated with the NASDAQ. This result is similar to that of Chen and Knez (1995).

Time-series plots of  $\{d_t\}$  estimated from all (twenty) S&P and NASDAQ portfolios are provided in Figure 2 for all six sample periods, along with confidence intervals. Figure 3 provides scatterplots of S&P deltas against NASDAQ deltas. All these graphs indicate that there is no single obvious characteristic difference between the S&P and NASDAQ deltas.

## 6: Sensitivity Analysis

Thus far we have relied on the Fama-French model of asset covariances. That is, the covariance of each asset's return with the expectation of the MRS is characterized by four parameters: an intercept ( $\mathbf{b}_0^j$ ) and factor loadings on the market return minus the T-bill rate ( $\mathbf{b}_1^j$ ), the difference between small and large stock returns ( $\mathbf{b}_2^j$ ), and the difference between returns of stocks with high and low book to market ratios ( $\mathbf{b}_3^j$ ). Are our results sensitive to the number of factors used? It turns out that the answer is negative.

In Table 4 we provide test statistics (and bootstrapped p-values) to examine tests of integration within the S&P and NASDAQ and between the two markets, but using only the return on the market instead of the three Fama-French factors (while retaining the portfolio intercepts as well). The test statistics and conclusions are essentially unchanged.

Table 5 goes even further and drops the market factor from our covariance model, leaving only portfolios-specific intercepts ( $\mathbf{b}_0^j$ ). Thus the conditional covariance model is modeled as time-invariant for the months under consideration. Again, the results are essentially unchanged.

Finally we change the normalization of the data from lagged prices to contemporary prices. In so doing, we return to the traditional returns approach which has been widely investigated in Finance, stemming from equation (4). Thus the set of “nuisance” factor terms picks up the effects of  $COV_t(m_{t+1}, x_{t+1}^j / p_t^j)$  rather than  $COV_t(m_{t+1}, x_{t+1}^j / p_{t-1}^j)$ . This is *precisely* the covariance model that Fama and French had in mind when they developed their three-factor model, an advantage. On the other hand, we are forced to drop the factor-specific intercepts, i.e.,  $\{\mathbf{b}_0^j\}$ , since these are jointly perfectly collinear with the unity variable. That is, we estimate:

$$x_{t+1}^j / p_t^j = \mathbf{d}_t (1 + \mathbf{b}_1^j f_{1,t} + \mathbf{b}_2^j f_{2,t} + \mathbf{b}_3^j f_{3,t}) + \mathbf{e}_{t+1}^j \quad (6')$$

The integration test results are displayed in Table 6. Yet again, the results are similar to those which used different covariance models of Tables 3 through 5; integration within the S&P is never rejected, the NASDAQ has the same two episodes where integration can be marginally integrated, and the S&P is never close to being integrated with the NASDAQ.<sup>7</sup> This robustness is encouraging since it demonstrates the insensitivity of our methodology to reasonable perturbations in the exact factor model employed.

## 7: Summary and Conclusions

This paper developed a simple method to test for asset integration, and then applied it within and between American equity markets. It relies on estimating and comparing the expected risk-less returns implied by different sets of assets. Our technique has a number of advantages over those in the literature and relies on just two assumptions: 1) rational

expectations in financial markets; and 2) covariances between discount rates and normalized returns that can be modeled with a small number of factors for a short period of time.

We illustrated this technique with an application to stocks drawn from the NYSE and the NASDAQ, and found that a) the time-series variation in the expected Marginal Rate of Substitution is high; b) the NYSE always seems to be integrated; c) the NASDAQ is usually (but not always) integrated; and d) the NYSE and NASDAQ do not seem close to being integrated. Our results seem reasonably insensitive to the exact sample and conditioning model used.

If our finding of integration within but not across stock markets holds up to further scrutiny, the interesting question is not whether financial markets with few apparent frictions are poorly integrated but why? We leave that important question for future research.

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Log Likelihoods	<b>April-May 1999</b>	<b>July-Aug. 1999</b>	<b>Oct.-Nov. 1999</b>
<b>First 10 portfolios</b>	1160.	1302.	1157.
<b>Second 10 portfolios</b>	1166.	1299.	1172.
<b>All 20 portfolios</b>	2309.	2574.	2303.
<b>Test (bootstrap P-value)</b>	36 (.90)	54 (.37)	51 (.43)
	<b>April-May 2002</b>	<b>July-Aug. 2002</b>	<b>Oct.-Nov. 2002</b>
<b>First 10 portfolios</b>	1438.	1255.	1247.
<b>Second 10 portfolios</b>	1405.	1302.	1227.
<b>All 20 portfolios</b>	2805.	2525.	2456.
<b>Test (bootstrap P-value)</b>	75 (.06)	62 (.24)	37 (.90)

**Table 1: Integration inside the S&P 500, Fama-French-Factor Model**

Log Likelihoods	<b>April-May 1999</b>	<b>July-Aug. 1999</b>	<b>Oct.-Nov. 1999</b>
<b>First 10 portfolios</b>	881.	1066.	757.
<b>Second 10 portfolios</b>	816.	990.	945.
<b>All 20 portfolios</b>	1677.	2023.	1625.
<b>Test (bootstrap P-value)</b>	42 (.83)	65 (.20)	153** (.00)
	<b>April-May 2002</b>	<b>July-Aug. 2002</b>	<b>Oct.-Nov. 2002</b>
<b>First 10 portfolios</b>	1052.	1061.	991.
<b>Second 10 portfolios</b>	1174.	1003.	962.
<b>All 20 portfolios</b>	2185.	2035.	1919.
<b>Test (bootstrap P-value)</b>	82* (.03)	58 (.45)	69 (.08)

**Table 2: Integration inside the NASDAQ, Fama-French -Factor Model**

Log Likelihoods	<b>April-May 1999</b>	<b>July-Aug. 1999</b>	<b>Oct.-Nov. 1999</b>
<b>20 S&amp;P Portfolios</b>	2309.	2574.	2303.
<b>20 NASDAQ Portfolios</b>	1677.	2023.	1625.
<b>Combined</b>	3706.	4396.	3633.
<b>Test (bootstrap P-value)</b>	559** (.00)	403** (.00)	590** (.00)
	<b>April-May 2002</b>	<b>July-Aug. 2002</b>	<b>Oct.-Nov. 2002</b>
<b>20 S&amp;P Portfolios</b>	2805.	2525.	2456.
<b>20 NASDAQ Portfolios</b>	2185.	2035.	1919.
<b>Combined</b>	4735.	4352.	4170.
<b>Test (bootstrap P-value)</b>	511** (.00)	416** (.00)	410** (.00)

**Table 3: Integration between S&P 500 and NASDAQ, Fama-French -Factor Model**

Test Statistics (bootstrap P-value)	April-May 1999	July-Aug. 1999	Oct.-Nov. 1999
<b>Within S&amp;P</b>	36 (.93)	48 (.75)	30 (.99)
<b>Within NASDAQ</b>	47 (.79)	65 (.27)	127** (.00)
<b>S&amp;P vs. NASDAQ</b>	548** (.00)	388** (.00)	594** (.00)
	April-May 2002	July-Aug. 2002	Oct.-Nov. 2002
<b>Within S&amp;P</b>	44 (.88)	55 (.61)	35 (.98)
<b>Within NASDAQ</b>	80 (.09)	58 (.61)	72 (.13)
<b>S&amp;P vs. NASDAQ</b>	497** (.00)	432** (.00)	422** (.00)

**Table 4: Integration within and between S&P 500 and NASDAQ, One-Factor Model**

Test Statistics (bootstrap P-value)	April-May 1999	July-Aug. 1999	Oct.-Nov. 1999
<b>Within S&amp;P</b>	33 (.97)	46 (.71)	34 (.94)
<b>Within NASDAQ</b>	42 (.80)	62 (.28)	114** (.00)
<b>S&amp;P vs. NASDAQ</b>	534** (.00)	378** (.00)	591** (.00)
	April-May 2002	July-Aug. 2002	Oct.-Nov. 2002
<b>Within S&amp;P</b>	46 (.76)	47 (.77)	36 (.95)
<b>Within NASDAQ</b>	86* (.03)	52 (.63)	68 (.12)
<b>S&amp;P vs. NASDAQ</b>	506** (.00)	416** (.00)	419** (.00)

**Table 5: Integration within and between S&P 500 and NASDAQ, Only Asset Intercepts**

Test Statistics (bootstrap P-value)	April-May 1999	July-Aug. 1999	Oct.-Nov. 1999
<b>Within S&amp;P</b>	29 (.93)	72 (.13)	33 (.93)
<b>Within NASDAQ</b>	37 (.87)	65 (.31)	114** (.01)
<b>S&amp;P vs. NASDAQ</b>	520** (.00)	372** (.00)	550** (.00)
	April-May 2002	July-Aug. 2002	Oct.-Nov. 2002
<b>Within S&amp;P</b>	59 (.33)	40 (.70)	22 (.98)
<b>Within NASDAQ</b>	103* (.02)	51 (.60)	62 (.22)
<b>S&amp;P vs. NASDAQ</b>	540** (.00)	411** (.00)	501** (.00)

**Table 6: Integration within and between S&P 500 and NASDAQ, Current Price Normalization (no Asset Intercepts)**

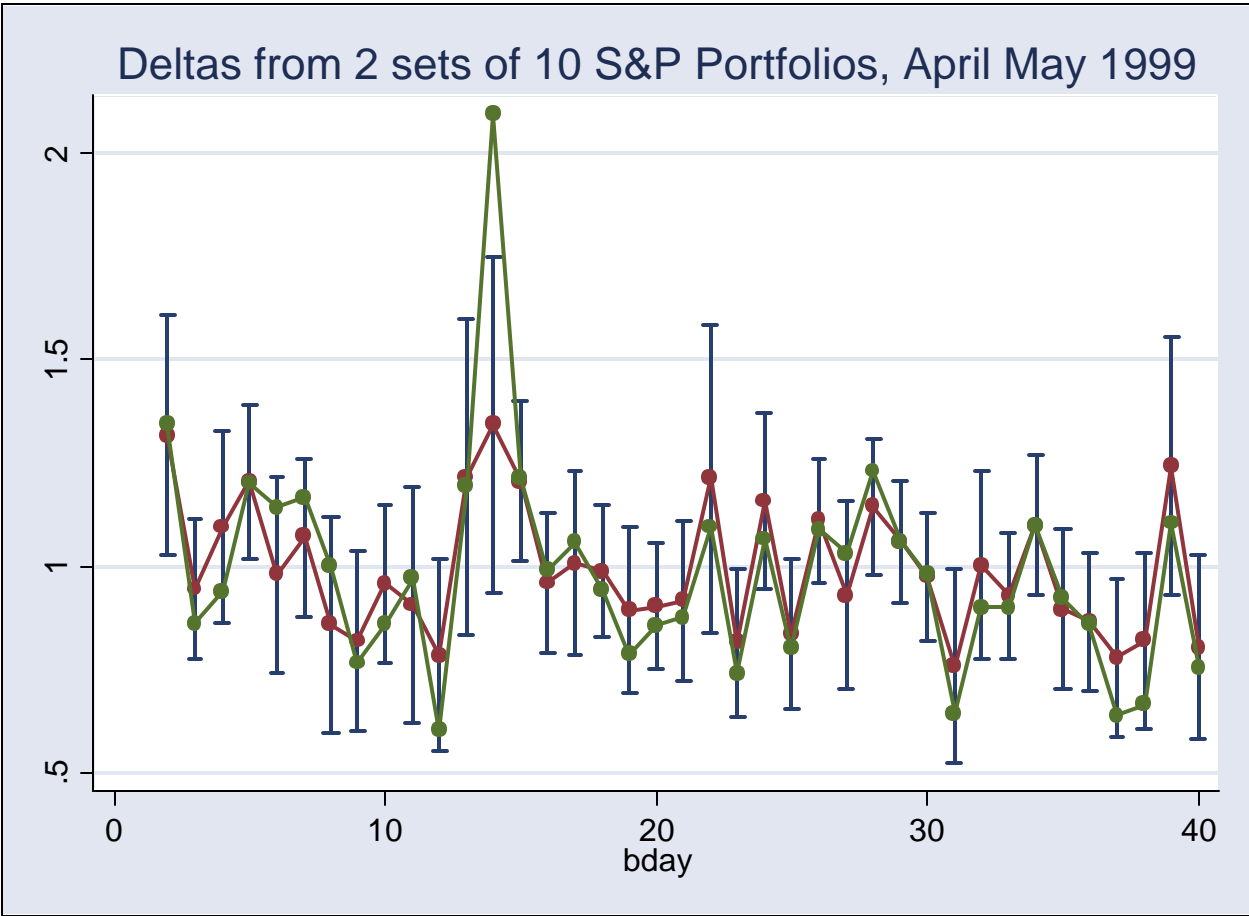
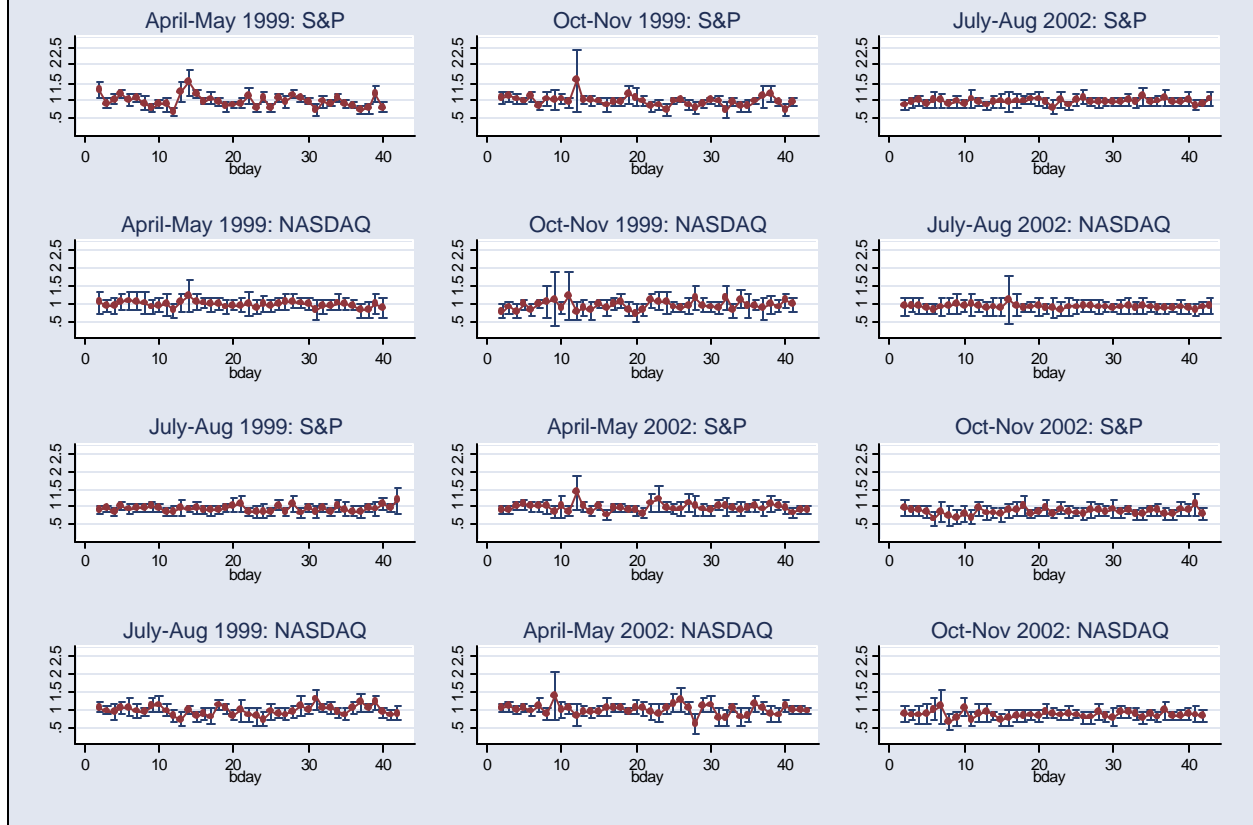
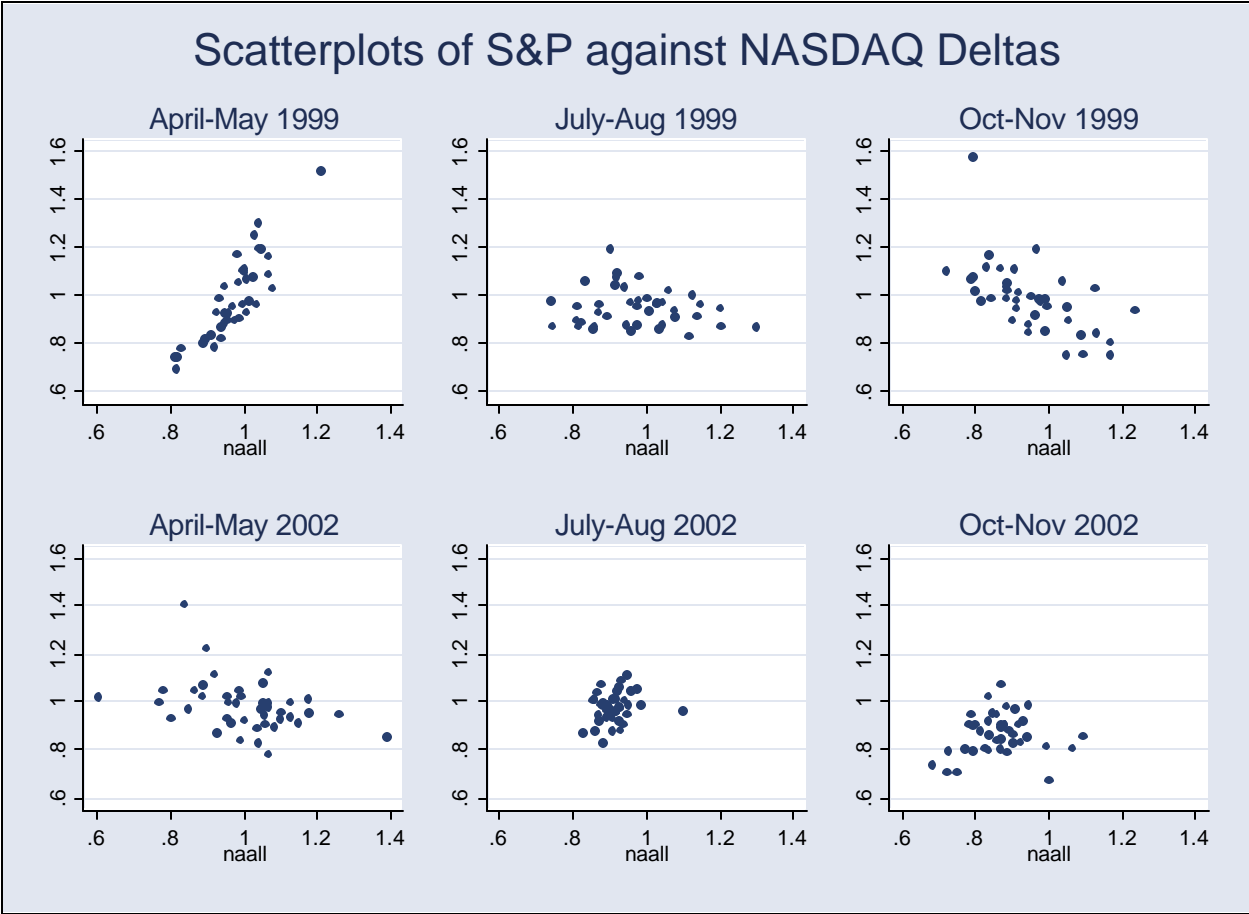


Figure 1: Estimates of Marginal Rate of Substitution from two sets of (10) S&P portfolios

## Deltas from Different Markets and Samples



**Figure 2: Estimates of Marginal Rate of Substitution from sets of (20) portfolios**



**Figure 3: Estimates of Marginal Rate of Substitution from sets of (20) portfolios**

## Endnotes

<sup>1</sup> That said, we again stress that the factor model we use in our new approach has not been designed to model the covariance term of interest, nor has it been discussed by the literature as has the traditional returns approach.

<sup>2</sup> [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

<sup>3</sup> We choose these months to avoid January (and its effect), February (a short month), and March (a quarter-ending month), but test for sample sensitivity extensively below.

<sup>4</sup> For instance, we could use data at five-minute intervals for a day, making our assumption of constant asset-specific effects even more plausible; but the question of whether financial markets are integrated over hours (not weeks) is less interesting to us.

<sup>5</sup> Jarque-Bera tests are inconsistent with the null hypothesis for  $\{\varepsilon\}$  at all reasonable confidence levels.

<sup>6</sup> Our bootstrap procedure is as follows. We estimate the deltas from (say) all 20 portfolios under the null hypothesis of integration. This gives us an estimate of  $\{\varepsilon\}$ . We then draw with randomly with replacement from this vector to create an artificial vector of  $\{\varepsilon\}$  which we use to construct an artificial regressand variable  $\{x\}$ . Using this artificial data we then generate a likelihood ratio test by estimating the model from the first set of 10 portfolios, the second set of 10 portfolios, and the combined set of 20. We then repeat this procedure a large number of times to generate a distribution for the LR test statistic.

<sup>7</sup> The point estimates of the expected MRS are much more noisy and volatile than those estimated with the lagged price normalization.