Long-Run Growth

Solow’s “Neoclassical” Growth Model
Simple Growth Facts

• Growth in real GDP per capita is non-trivial, but only really since Industrial Revolution
• Dispersion in real GDP per capita across countries is sluggish; countries tend to “catch up” slowly
Production

- Output (Y) is produced by combining:
  - Capital (K)
  - Labor (L) in a
    - Production function $F(.)$ linking inputs to output

- So $Y = F(K, L)$
Constant Returns to Scale

- Assume constant returns to scale (CRS)
  - Doubling inputs doubles output
- \( Y = F(K, L) = L \cdot F(K/L, L/L=1) \rightarrow Y/L = F(K/L) \rightarrow y = F(k) \)
  - \( y \) is output *per capita*; \( k \) is capital *per capita*
- \( y = F(k) \) is positively sloped, concave (diminishing marginal returns)
Capital “Requirements”

How Much Capital “Needs” to be Accumulated?

– Have to accumulate some capital just to keep capital stock per capita constant.

1. *Depreciation* (to replace erosion, wear and tear) is a constant fraction of the capital stock, $D = dK$
Population/Labor Force Growth

2. Assume population is growing at constant rate

   – Algebraically $\%\Delta L = n$

• Reasonable? Probably not for two reasons.
Two Problems with Assuming Constant Growth in Labor/Population

1. Strong *negative* correlation between real GDP per capita and fertility. Why?
   - Parents invest in children for purely economic reasons
   - Rising opportunity cost – quality of children matters, not quantity
   - Falling cost of birth control, falling infant mortality

Andrew Rose, Global Macroeconomics 3
Also ...

2. The *population* is not the *labor force*

   – Reasons why Population Growth\(>\)LF growth

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“Supply” of New Capital

Where does the Capital come from?

• Assume savings a constant fraction of output,

\[ S = sY \]

— Consistent with consumption model
Equilibrium – Technical Stuff

• Define \textit{steady state} where \( k = k^* \), i.e., \textit{per capita} capital stock is fixed

• Savings equals “capital requirements”

• \textit{Net change in the capital stock} is \( \Delta K = sY - dK \)

• Population is growing, so in a steady state
  – \( \%\Delta K = n, \Delta K = nK \)

• \textit{In steady state}, \( nK + dK \[ = (n+d)K \] = sY \)

• That is per capita, \textit{capital requirements} \( (n+d)k = \text{savings sy} \)

• If \( \%\Delta K > (n+d)k \), then \( k \) grows, and conversely
\[ y, c, s \]

\[ y^e \]

\[ y^e \]

\[ s^e \]

\[ c^e \]

\[ (n+d)k \]

\[ y = f(k) \]

\[ s \cdot y \]
Notes

1. Countries *converge* to steady state (same $y^e$!)
   - Poor grow faster than rich as they “catch up” – for how long?
   - A *very* testable hypothesis (works well for OECD, but not all)

2. In steady state, output growth per capita is *nil* (!)

3. In steady state, GDP grows at rate $n$ (independent of savings rate)
   - Algebraically, in equilibrium, $\%\Delta Y = \%\Delta K = \%\Delta L = n$; $\%\Delta y = 0$
This Assumes Countries are Equally Productive!

• In practice productivity levels (for similar inputs) differ dramatically
  
  – Why? Hall and Jones (NBER WP 5812) empirically find that productivity depends on five factors
Hall & Jones Characteristics of Highly Productive Countries

1. Government Institutions that promote production not “diversion”
2. Openness to International Trade
3. Private Ownership
4. Speaking International Language
5. Being Far from Equator
Change Savings Rate (s)

• Many policies can change savings rate
Graphically
Change Savings Rate ($s$)

• If savings rate rises, steady state value of $k$ rises; 
  *per capita level of income is higher, but the growth rate of output will not be affected* (after initial burst)

• $s \uparrow$ implies $y \uparrow$, $k \uparrow$ *but not $\% \Delta y$*
  
  – Another testable hypothesis (works very well: Levine and Renelt)
So can Choose Optimal Savings

• Raising $s$ can raise *or lower* consumption $(c)$ per capita (smaller fraction, bigger pie)

• Phelps’ *Golden Rule* is the optimal choice of $s$, maximizes consumption
Change Population Growth Rate (n)

- Many policies can change fertility rate or labor force growth
Graphically

$y = f(k)$

$(n+d)*k$

$s*y$

$y^0$

$y^1$

$k_1$

$k_0$
Change Population Growth Rate (n)

• As (n+d)k line rotates up, y (GDP per capita) and k *fall* (individually, people are worse off)
  – Testable: population growth and y are negatively correlated
    • Since both s and n affect y, can test for “conditional” convergence taking into account variation in s and n across countries/time

• But *steady state growth rate of Y rises!* (country as a whole grows faster)

• Equilibrium: %ΔY=ΔK=ΔL=n’ (higher)
One-Shot Technological Progress (F)

• Many examples of large technological leaps
Graphically

\[ y = f(k) \]

\[ (n+d) \cdot k \]

\[ s \cdot y \]
One-Shot Technological Progress (F)

• Production function $F(.)$ shifts; same inputs yield more output

• Both $y$ and $k^*$ to rise; output per capita rises (one-time), but *growth rate* of output **not permanently affected**

• In steady state equilibrium: $\%\Delta Y = \%\Delta K = \%\Delta L = n$ (unchanged); $\%\Delta y = n$, higher $y$
Key Conclusion

- Only *continuing* technological progress can explain continuing growth in $y$ (GDP per capita) over a) long periods of time for b) rich countries

- The production function shifts out continuously because of technological progress
Growth Accounting

• \( Y = F(K,L) \cdot \text{Tech} \)
  – Then take logarithmic derivative:

• \( \% \Delta Y = \left( \frac{rK}{Y} \right) \% \Delta K + \left( \frac{wL}{Y} \right) \% \Delta L + \% \Delta \text{Tech} \)

• Growth can be decomposed into parts due to: a) capital accumulation; b) labor growth; c) productivity.
  – Since \( \left( \frac{rK}{Y} \right) - \alpha \) is small, capital doesn’t contribute much to growth.
Summary

• Why do *Levels* of GDP Differ?
  – Institutions
  – Savings rates (investment)

• Why do *Growth Rates* Differ?
  – Labor force growth rates
  – “Catch-Up” growth during convergence (initial conditions)
  – Continuing growth in TFP (innovation)
Key Takeaways

• Growth is driven by: a) technological innovation; b) labor growth; and only a little by c) capital accumulation

• Poor countries “converge” but usually slowly
  – Countries need not converge to the same levels of income (varying savings rates, institutions, etc.)