# Equity volatility and credit yield spreads*

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**Abstract**

We show that a simple structural model of credit risk is able to generate credit yield spreads for the low-rated bonds close to the historical spreads once the recent trends in the stock volatility are taken into account. We study the idiosyncratic and market volatility of stock returns in the cross-section of credit ratings. We find that the increase in the level of the firm-specific volatility, demonstrated recently by Campbell et al. (2001), refers only to the low-rated stocks. A time-varying deterministic volatility process is used to imply the asset volatilities, the asset risk premia and the default boundaries from the historical default rates. Stock volatility is modeled as an autoregressive process. Physical default probability of an investment-grade bond is primarily linked to the drift of the firm value process and default probability of a low-rated bond to the total asset volatility. We confirm this by finding that an increase in the firm-specific volatility affects credit spreads of the low-rated bonds and does not have an observable impact on the investment-grade bonds.

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# Contents

# 1 Introduction

We show that a simple structural model of credit risk is able to generate credit yield spreads for the low-rated bonds close to the historical spreads once the recent trends in the stock volatility are taken into account.

The motivation for this paper is straightforward. Idiosyncratic equity volatility has been increasing since the 1970s until recently, when the trend was stopped. We make an attempt to study the effect of that increase on the level of the credit yield spreads in the cross-section of credit ratings.

We present the implied asset volatilities, the implied asset risk premia and credit yield spreads for different maturities and credit ratings in the benchmark case, when the default boundary is constant. It is a straightforward application of the Longstaff and Schwartz (1995) model with constant interest rates. Estimated credit spreads are lower than those observed in the data.

A simple, time-varying volatility option pricing framework is applied to find the implied asset volatilities and credit spreads. We use a first-passage model with the exogenous default boundary. We observe that the stock price distribution, conditional on the mean variance over a specified period of time, depends only on that mean, the risk free rate and the time horizon.

Stock return volatility is estimated from the data in the cross-section of credit ratings. We use stock returns from major exchanges from 1970-2004. A time-varying deterministic process of the stock return volatility is assumed to hold. The AR(12) model is formulated. We follow closely the procedure for estimating the stock return volatility described in Schwert (1989). We estimate a cross-section of market volatility process by credit rating, using a seemingly unrelated regressions framework. Aggregate market return volatility is estimated for comparison purposes as only a limited number of traded stocks have a letter rating. We find that the predicted market volatility differs significantly in the cross-section of credit ratings. This is not surprising and consistent with the leverage effect, as increased debt to equity ratio brings about more volatility. Using the same estimation procedure and the methodology as in Campbell et al. (2001), we estimate the processes of idiosyncratic volatility in the cross-section of credit ratings and for the aggregate market. As before, credit rating is a significant covariate affecting volatility levels. Again, a seemingly unrelated regressions framework is applied due to the high cross-sectional correlation of error terms. Total equity volatility is then computed and transformed into asset volatility.

Estimates of the monthly asset volatility are applied using the option pricing framework of Hull and White (1987) to find the implied asset volatilities, asset risk premia and credit spreads for different maturities.

Main finding of the paper is that a simple structural model of credit risk is able to generate credit yield spreads for the low-rated bonds close to the historical spreads once the recent trends in the stock volatility are taken into account. The default probabilities and credit spreads predicted from the time-varying volatility model depend primarily on the market volatility for the investment-grade bonds and on the total volatility for the low-rated bonds. Market volatility enters into the drift term of the investment-grade bond. Asset volatility of the high quality firm is low and hence we expect that the drift of the firm value process is the main driver of the default process. Analogously, we anticipate that for the low-rated bonds the total volatility affects the implied default boundaries and credit spreads much more than the market volatility. This observation explains why a simple time-varying volatility model generates credit spreads for the low-rated bonds higher than the benchmark model.

Another finding is that the increase in the aggregate idiosyncratic volatility is primarily pronounced for the low-rated stocks. Firm-specific volatility levels for the investment-grade stocks remain much more stable and do not show a time trend.

The remainder of the paper is organized as follows. Section two presents an overview of the literature on the structural models of credit risk and option pricing in the stochastic volatility framework. Section three summarizes the theoretical framework used in the estimation of credit spreads and the results of the calibration of the benchmark model. Section four presents data summary, methodology and results of the estimation of the volatility process. We also show the implied asset volatilities, the asset risk premia and credit spreads predicted by the time-varying volatility model. Section five contains a summary and conclusions.

# 2 An overview of the literature

Structural models of credit risk attempt to find the sources of credit risk in the processes occurring within firms. A firm's value is modeled as a stochastic process. Contrary to the reduced-form approach, here default occurs once the firm's assets hit a specified default boundary. A default boundary can be constant or time-varying. Similarly, the risk free rate process can be driven by a Brownian motion or can be constant.

Structural modeling of credit risk begins with Black and Scholes (1973) and then Merton (1974) (BSM), who show that the firm's equity can be represented as a European call option with the strike price equal to the firm's face value of debt. Several important drawbacks of the BSM model were raised in the literature. The BSM model implies a hump-shaped or downward sloping term structure of credit spreads for the low-rated firms which is not found in the data. This was demonstrated by e.g. Sarig and Warga (1989).

Arguably, the most important shortcoming of the structural models is the credit spread puzzle. Empirical evidence that structural models of credit risk predict counterfactually low credit spreads for the investment-grade corporate bonds starts with Jones, Mason and Rosenfeld (1984). It was recently strengthened by Huang and Huang (2003), who demonstrate that structural models generate credit spreads falling well below observed levels of credit spreads. Structural models of credit risk suffered a lot of criticism for their counterfactually low predictions of credit spreads. Several authors have demonstrated that defaultable bond prices produced by structural models of credit risk are much too high to match the data, see e.g. Jones, Mason and Rosenfeld (1984), Sarig and Warga (1989), Huang and Huang (2003).

As a result a new line of research was initiated to demonstrate that credit risk accounts for only a fraction of observed credit spreads, see e.g. Collin-Dufresne, Goldstein and Martin (2001), Elton, Gruber, Agrawal and Mann (2001), Huang and Huang (2003). Conversely, other authors have shown that incorporating more realistic economic assumptions into structural models yields credit spreads much closer to the data. Anderson and Sundaresan (1996), Anderson, Sundaresan and Tychon (1996) as well as Mella–Barral and Perraudin (1997) argue that the possibility of strategic default in structural models increases their predictive power substantially. Other papers demonstrating that structural models of credit risk are in fact capable of generating credit spreads close to the data include Collin-Dufresne and Goldstein (2001), Longstaff and Schwartz (1995), Leland (1994, 1998) and Leland and Toft (1996).

Another critique of the structural modeling concerns its counterfactual prediction that for maturities close to zero, credit spreads are close to zero as well. Whenever a firm's assets process is modeled as a diffusion, its value is perfectly predictable over the short horizons and this implies zero credit spreads for short maturities.

Subsequently, *first passage* models were introduced with the work of Black and Cox (1976). Here the default boundary did not necessarily need to be equal to a firm's liabilities. Importantly, default could occur at any time now, not only at the debt's maturity as in the BSM model. Longstaff and Schwartz (1995) is another example of the first passage models. These two models have an exogenously fixed default boundary. Leland (1994) and Leland and Toft (1996) use an endogenous default boundary, which is specified by the shareholders' actions to maximize the firm's value. There were several other authors investigating the first passage models, e.g. Geske (1977), Kim, Ramaswamy and Sundaresan (1993), Nielsen et al. (1993), Briys and de Varenne (1997) and Hsu, Sa-Requejo and Santa-Clara (2004). These papers include structural models with stochastic or non-stochastic default boundaries and interest rates.

The first passage models overcome one of the drawbacks of the BSM model by allowing the default to occur at any time during the duration of the debt. However, certain issues remain unsolved, one of them being low short-term spreads. This was later somewhat remedied by an introduction of the jump-diffusion models, e.g. Zhou (2001).

The literature on forecasting the volatility is very rich as well. Our overview is limited to the most influential papers. We begin with the paper by Engle (1982), in which an ARCH-type of model is introduced. Here, variance of the stock return, conditional on the past information, evolves as a function of past errors. This paper, earlier work by Rosenberg (1972), and finally Bollerslev (1986) lead to the formulation of the GARCH-type of framework, in which volatility is allowed to follow a process enriched with a parsimonious parametrization of the lag structure. Proposed models have frameworks that accommodate for the stylized facts known about the volatility. Namely, they admit a varying degree of the persistence and mean-reversion exhibited by the stock return volatility. These features are documented by the early works of Mandelbrot (1963) and Fama (1965).

GARCH models assume that the innovations in the stock return process have a symmetrical effect on the volatility as the squared lagged innovation term enters the equation. Nelson (1991) and Glosten, Jagannathan, and Runkle (1993) provide a response to that problem, proposing an EGARCH-type of the volatility modeling.

Some papers suggest that an autoregressive process should be used for the volatility modeling. French, Schwert and Stambaugh (1987), and Schwert (1989) model volatility as monthly or daily AR processes with dummy variables capturing days of the week or monthly seasonal effects. Most recent literature on estimating the idiosyncratic stock volatility includes Campbell et al. (2001).

Other models assume the existence of an unobservable latent factor affecting the volatility. The Stochastic Volatility class of models was initiated by Taylor (1980).

Another strand of literature attempts to construct models of option prices using different measures of the time-varying volatility, deterministic or stochastic. Examples here are Hull and White (1987), Stein and Stein (1991) and Heston (1993). These papers rely heavily on different forecasts of the volatility of returns in the equity market. Instantaneous variance is modeled as a diffusion, namely a geometric Brownian motion or a square-root process. These processes are continuous-time limits of their discrete counterparts, presented in a form which is easy to estimate. Hull and White (1987) find a closed form solution for a European call option with an instantaneous variance following a diffusion process. Volatility and stock price processes are assumed to be independent. The distribution of the log of terminal stock price conditional on the mean variance is normal and depends solely on the mean of the variance process over a specified time interval. Stein and Stein (1991) show the option price when the volatility follows an arithmetic Ornstein-Uhlenbeck process (AR(1)). Heston (1993) finds the option price under the same assumption about the stock volatility process, but allows for an arbitrary level of correlation between the stock price and volatility.

# 3 Models

## 3.1 Benchmark model

A firm's equity value can be expressed as the call option with the strike price equal to the face value of debt. The Black-Scholes formula can be then applied to value the equity. The option is exercised when the assets' value falls below the default boundary. Assume that a firm's risk-neutral assets dynamics are given as

$$\frac{dV(t)}{V(t)} = (r - \delta)dt + \sigma dZ^Q(t), \tag{3.1}$$

where $\sigma$ is the total asset volatility. The risk free rate, dividend payout rate $\delta$ and the asset volatility are constant. Default occurs when the assets' value falls below the default boundary. It is defined as the level of a firm's assets value at which it enters into default. If $\tau$ is the default time, then we have the following relationship between the default time and the default boundary

$$\tau \equiv \min\{t : V(t) \leq K\},$$

where $K$ is the default boundary. We assume, following Huang and Huang (2003) (HH), an exogenous constant default boundary equal to 60% of the debt face value. It is a well known result that the risk-neutral survival probability until maturity $T$ is given by [see e.g. Black and Cox (1976)]

$$P^Q(\tau > T) = \Phi(d_1) - \left(\frac{V}{K}\right)^{1-2\frac{r-d}{\sigma^2}} \Phi(d_2), \tag{3.2}$$

where

$$\begin{aligned} d_1 &= \left(\frac{\ln V - \ln K + (r - d - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right), \\ d_2 &= \left(\frac{\ln K - \ln V + (r - d - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right), \end{aligned}$$

for a constant default boundary $K$ and dividend rate $d$. $\Phi$ denotes a standard normal cumulative distribution function. Equation (3.1) can be transformed under the physical measure,

$$\frac{dV^*(t)}{V^*(t)} = (r - \delta + \pi)dt + \sigma dZ(t), \tag{3.3}$$

where $\pi$ is the asset risk premium.

Following HH, we calibrate the model to the initial leverage ratios, the equity premium, the default boundary and the cumulative default probabilities. They are presented in table (1). The parameters to imply are the asset risk premium and the asset volatility. The remaining parameters: the interest rate, the payout rate and the default boundary, are chosen exogenously to match the historical data. We choose $r = 0.08$, $\delta = 0.06$. In particular, the interest rate of 8% is close to the historical Treasury rate from the period 1973–1998. The dividend rate of 6% comes from the average dividend yield of 4%, according to Ibbotson Associates (2002), and the average historical coupon rate of 9% in the period 1973–1998. Weights on the coupons and dividend payments come from the average leverage ratio of 35% in that period. Again, following HH, we assume the same payout rate for firms of all credit ratings, arguing that firms with a worse credit rating, and hence higher coupon payments, pay out less in dividends. The difference with the HH's benchmark model is that we work with zero coupon bonds for analytical tractability. Several authors argue that adding coupon payments has a small impact on credit spreads, especially for short maturities.

Given constant asset risk premium, $\pi$, the cumulative physical survival probability over period $T$ expressed in risk-neutral terms in equation (3.2) can be written as

$$P(\tau > T) = \Phi(d_1) + e^{-\frac{2\zeta b}{\sigma^2}} \Phi(d_2), \tag{3.4}$$

where

$$\begin{aligned} d_1 &= -\frac{b + \zeta T}{\sigma\sqrt{T}}, \\ d_2 &= \frac{-b + \zeta T}{\sigma\sqrt{T}}, \end{aligned}$$

and $b = \ln\left(\frac{V}{K}\right)$ and $\zeta = \pi + r - \delta - \frac{\sigma^2}{2}$.

Table (1) presents results of the calibration of the benchmark model to the historical cumulative default rates. We choose to show the results for the four- and ten-year horizon only. First, the asset volatility and the asset risk premium are implied from the historical default rates, the equity premia and the formula for the physical default probability given in equation (3.4). Next, credit spreads are computed from the risk-neutral pricing formula based on equation (3.2). Calculated credit spreads are lower than those observed in the data. Credit risk accounts for 2% to 92% of the credit yield spread for 4-year AAA bonds and 4-year B bonds respectively. The results are very similar to the base model in HH. The only difference comes from the fact that we use zero-coupon bonds. Calculated spreads differ by less than 10bp. The benchmark model explains more of credit spreads as the debt quality deteriorates. The predictive power of structural models is lower for the investment-grade firms.

## 3.2 Time-varying volatility model

Throughout this section of the paper we assume that the stock return volatility is time-varying but follows a deterministic process. We derive a simple structural model of credit risk with the time-varying deterministic volatility of the stock price. Then we compare the resulting credit spreads for various maturities to the benchmark model. The model used is a slight modification of the Hull and White (1987) model. We have two state variables: the asset volatility and the stock price. Volatility is not traded and is uncorrelated with the stock price. We also assume a constant correlation of the volatility with the aggregate consumption, which yields constant asset risk premia.

Risk-neutral dynamics of the firm value are given by,

$$\frac{dV(t)}{V(t)} = (r - \delta)dt + \sigma(t)\left[\gamma(t)dZ_m^Q(t) + \sqrt{1-\gamma(t)^2}dZ_i^Q(t)\right], \tag{3.5}$$

where $\sigma(t)$ is total asset volatility. In the appendix A we show the proof that $\gamma(t)$ can be treated as the share of market volatility in the total volatility. The process is driven by two sources of risk, namely the market and the idiosyncratic risk. As the volatility is a deterministic process, we drop the stochastic parameter in the usual formulation of the volatility. In the next sections we assume that the true volatility process is stochastic but can be approximated using a deterministic process. In particular, we assume that the volatility follows an AR(12) process. Hull and White (1987) demonstrate that the distribution of the $\log[S(T)/S(0)]$ conditional on $\bar{V}$ is normal with mean $rT - \bar{V}T/2$ and variance $\bar{V}T$, where

$$\bar{V} = \frac{1}{T}\int_0^T \hat{\sigma}^2(t)dt, \tag{3.6}$$

where $\hat{\sigma}$ is the asset volatility estimate. The original Hull and White (1987) argument uses a geometric Brownian motion for the volatility process. However, the distribution of $[S(T)/S(0)]|\bar{V}$ is log-normal and therefore depends only on the risk-free rate, the initial stock price, the time elapsed, and the mean variance over the period. Hence, in the case of no correlation between the stock price and the volatility process, a constant asset risk premium, the distribution of the terminal stock price depends on the variance process only through its mean over a specified period of time. This means that this framework can accommodate other models of the volatility, deterministic in particular. Hull and White (1987) show that in this setting the formula for the value of the call option is similar to the Black-Scholes formula and equals

$$C_t(\bar{V}) = S(t)\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2), \tag{3.7}$$

where

$$d_1 = \frac{\log[S(t)/K] + (r - \delta + \bar{V}/2)(T-t)}{\sqrt{\bar{V}(T-t)}} \tag{3.8}$$

$$d_2 = d_1 - \sqrt{\bar{V}(T-t)}. \tag{3.9}$$

The formula is analogous to the formula with constant volatility. Appendix B contains detailed description of the derivation of this formula and the application for the purpose of a deterministic volatility process. Using equations (3.2), (3.4) and (3.6), we can easily derive the risk-neutral and physical default probabilities.

It is important to note the difference in the calibration method as compared to the benchmark case. We calibrate the model to the target equity premium, the stock volatility and the cumulative default probability. The parameters to compute are the asset volatility, the asset risk premium and the default boundary.

In the sections below we estimate the volatility using the AR(12) process, following Schwert (1989). We use these estimates to predict a deterministic volatility process, after dropping the error terms from the stochastic model. In particular, we first estimate the market volatility process in the cross-section of credit ratings and then the idiosyncratic volatility process. Cross-sectional regressions are estimated using the seemingly unrelated regressions framework, following Zellner (1962). We show the correlation of error terms in the cross-section of credit ratings.

The CAPM implies that the stock return variance can be decomposed into two components, namely market and idiosyncratic variance. We argue, following Campbell et al. (2001), that stock and industry betas can be dropped in that decomposition and covariance terms average out. Total stock return volatility is then expressed as

$$\hat{\sigma} = \sqrt{\hat{\sigma}_m^2 + \hat{\sigma}_{id}^2}.$$

Next, we have to recognize that the inputs in equations (3.2) and (3.4) are asset volatilities. These are unobserved directly, however can be estimated from the equity volatilities. We use the following relationships for estimating the asset volatility and the asset risk premium in the cross-section of credit ratings,

$$\hat{\sigma}_A = (1-\zeta)\frac{\hat{\sigma}_E}{\Phi(d_1)}, \tag{3.10}$$

$$\hat{\pi}_A = (1-\zeta)\frac{\hat{\pi}_E}{\Phi(d_1)}, \tag{3.11}$$

where $\zeta$ denotes the implied default boundary, $\hat{\sigma}_E$ is the total stock volatility, $\hat{\pi}_E$ is the target equity premium, $\hat{\sigma}_A$ is the implied total asset volatility and $\hat{\pi}_A$ is the implied asset risk premium. We make a simplifying assumption that the equity premia are constant in the time-series dimension.

A system of nonlinear equations with two constraints on the asset volatility, the asset risk premium and default boundary, given by equations (3.10) and (3.11), is solved. The asset volatility, the asset risk premium and the default boundary meeting the constraints are next used to find the credit yield spreads in a similar fashion as in the benchmark case.

# 4 Empirical results

## 4.1 Data

We use the Compustat data. For all the stocks traded at Amex, New York Stock Exchange or Nasdaq in the period 1970-2004, we first limit our attention to those with an assigned Standard&Poors Long Term Domestic Issuer Credit Rating. The problem with restricting the sample only to rated companies is that only around 10% of firms listed on Amex/NYSE/Nasdaq in the period 1970-2004 have ever been assigned a rating. Stocks are divided according to their letter rating, into seven groups, AAA through CCC. For each day in the analysis period, an equal-weighted return of stocks in a particular group is computed. Panel one of table (2) demonstrates statistics of daily returns in the cross-section of ratings. The mean return remains close to zero and to the mean daily market return for all ratings. The highest volatility can be attributed to the lowest quality stocks in the sample. Panel two of table (2) presents monthly returns in the same period. Again, the mean monthly returns remain similar across the ratings, but lower than the mean market return. Here, the effect of the letter rating on the volatility of returns is much more pronounced than in the case of daily data. There is an almost monotone increase in monthly volatility as the credit quality deteriorates. Monthly returns show a negative skewness for all ratings and the market as a whole. Not surprisingly, the monthly kurtosis is much lower than the daily kurtosis. The monthly market standard deviation in the analyzed period was calculated at 0.044. Almost all rated stocks indicate volatility higher than this amount. The reason can be the relatively low fraction of rated stocks in all stocks. Also, apparently rated stocks may have more volatile returns on average than non-rated stocks.

The sample was also divided in the time-series dimension. Panels three and four demonstrate monthly return statistics in 1970-1990 and 1990-2004 respectively. Standard deviation of monthly returns is lower in the later sample, also kurtosis is substantially lower than in the earlier period. The first subperiod can be characterized as the one with much more turmoil in the financial markets due to oil shocks in the 1970s and the crash of 1987 and so it is natural to expect higher volatility in the market. The considerable decline of kurtosis of the monthly returns is worth noting. Kurtosis measures how fat-tailed the return distributions are. Its high values imply a higher probability of extremely low or high monthly returns. In the cross-sectional dimension, kurtosis is the lowest for CCC-rated stocks. This is probably because in the case of the low-rated stocks, it is hard to define what an extreme change in value means since such substantial drops or rises in the return occur quite frequently.

For comparison purposes, we use the S&P500 composite index return for estimating the aggregate market volatility, as well as the idiosyncratic volatility.

## 4.2 Market volatility

This section presents results of the estimation of the market volatilities, both in the cross-section of credit ratings and for the market as a whole, including both rated and not-rated stocks. We conduct a preliminary test of the equality of unconditional means of monthly volatilities by the letter rating. A high F-statistic enables us to conclude that the letter rating is a significant factor when estimating volatilities. Next, we estimate the conditional volatility of the stock market following closely the procedure described in Schwert (1989), also e.g. French, Schwert and Stambaugh (1987), except for the fact that we estimate the conditional volatility process across rating classes. We restrict our attention to all companies listed on Amex, New York Stock Exchange or Nasdaq, having Standard&Poors Long Term Domestic Issuer Credit Rating (SPCR), to test whether volatility differs across credit ratings. All stocks with SPCR are grouped by the letter rating and the AR(12) processes are estimated jointly for all groups, based on the seemingly unrelated regressions methodology. For each trading day, an equal-weighted mean return of the credit rating group is calculated and used in further calculations. We eliminate the autocorrelation of residuals in the time-series dimension and estimate seemingly unrelated regressions assuming cross-sectional correlation of error terms.

Daily return data are used to estimate the monthly standard deviation of the return within each month in the sample,

$$\hat{\sigma}_m = \sqrt{\sum_{i=1}^{N_t} (R_{i,m} - \bar{R}_m)^2}, \tag{4.1}$$

where $R_{i,m}$ denotes the daily return and $\bar{R}_m$ denotes the average return in a given month for a particular rating return. French, Schwert and Stambaugh (1987) make no adjustment for the mean return, however their procedure does not guarantee the positive estimator of the volatility. We use nonoverlapping samples of the daily data to estimate the monthly volatility. Schwert (1989) shows that this errors-in-variables problem makes the estimated volatility time-series autocorrelation lower than the true volatility autocorrelation, however the rate of decay remains the same. This reduces the time-series autocorrelation of error terms, however does not impact the cross-sectional autocorrelation.

The monthly volatility is estimated rather than the daily volatility for practical reasons. As pointed out by Schwert (1989), it is difficult to present so many estimates of the daily volatility and hard to determine the persistence of volatility using high-order autoregressions.

Using the daily value of the return to estimate the monthly standard deviation is a generalization of the 12-month rolling standard deviation estimator used in e.g. Officer (1973). This allows for the conditional mean return to vary over time. However, this approach has a few advantages. By sampling the returns more often, we increase the accuracy of the volatility estimate for any interval. We obtain a much more precise estimate for any month using only returns in that month. Finally, the 12-month rolling standard deviation estimator uses overlapping samples of returns and generates correlated error terms.

Stock return volatility demonstrates a substantial amount of the time-series autocorrelation of residuals. The introduction of a lagged dependent variable into the set of covariates significantly reduces the time-series autocorrelation problem, however some time-series autocorrelation remains. This refers to primarily lower credit rating stocks. We perform the Breusch-Godfrey test for all the ratings. The null hypothesis is that the error terms are uncorrelated and the alternative that they follow the AR(12) process.

For the rating classes which demonstrate the time-series autocorrelation, we correct for it using the following procedure. First, ordinary least squares are applied to each equation in the system separately. Residuals from these regressions are then used to test for the autocorrelation. We have the null and alternative hypotheses,

$$\begin{aligned} \mathrm{H}_0 : \rho_1 &= \rho_2 = ... = \rho_{12} = 0 \\ \mathrm{H}_1 : \varepsilon_t &= \rho_1\varepsilon_{t-1} + \rho_2\varepsilon_{t-2} + ... + \rho_{t-12}\varepsilon_{t-12} + u_t, \end{aligned}$$

We use the estimated autoregression coefficients of error terms to transform the original dataset. We have for each rating class,

$$y_t^* = y_t - \sum_{i=1}^{12} \rho_i y_{t-i}$$

$$
\begin{aligned}
&\equiv \left[x_t - \sum_{i=1}^{12} \rho_i x_{t-i}\right]' \beta + u_t \\
&= x_t^{*\prime} \beta + u_t.
\end{aligned}
$$

This eliminates the time-series autocorrelation problem. This procedure is only used for the cross-sectional regressions. In case of the aggregate volatility, we use the Newey and West (1987) standard errors to correct for the autocorrelation.

Next, using the transformed dataset, we jointly estimate the seven autoregressive equations,

$$
\hat{\sigma}_m = \sum_{i=1}^{12} \alpha_i D_{im} + \sum_{j=1}^{12} \phi_j \hat{\sigma}_{m-j} + \varepsilon_m, \tag{4.2}
$$

assuming only the cross-sectional correlation of residuals. We introduce a dummy variable $D_{im}$ to allow for different mean monthly returns. The dummy variable captures the monthly seasonal effect in the volatility. We use the estimates of the AR(12) process to define the deterministic process driving the volatility,

$$
\hat{\sigma}_m^p = \sum_{i=1}^{12} \hat{\alpha}_i D_{im} + \sum_{j=1}^{12} \hat{\phi}_j \hat{\sigma}_{m-j}^p, \tag{4.3}
$$

where $\hat{\sigma}_m^p$ is the predicted value of the monthly volatility in the $m$th month. It is a good approximation of reality, assuming the true volatility process follows (4.2). Also, it captures basic properties of the volatility, like persistence and mean-reversion.

### 4.2.1 Cross-section of the market volatility

Letter rating has a significant influence on the stock market volatility. We perform a simple F-test of mean monthly volatilities. Means are ranging from 2.89% for BBB to 6.43% for CCC. The F-statistic of 165.29 provides strong evidence that enables us to reject the null of equal mean monthly volatilities with p-value of zero. Next, the AR(12) with twelve monthly dummy variables and the control covariate for the rating is estimated for all the rated stocks in the period 1970-2004. The rating coefficient has a robust t-statistic at 3.48. The first three lags and the sixth lag of the volatility are significant. The F-test of equal coefficients for monthly dummy variables equals 5.77 and the null is rejected with p-value zero.

We test for the presence of the time-series autocorrelation of residuals, using the Breusch-Godfrey test, described above. The p-values for ratings AAA through B are above 0.5. The null about lack of correlation is rejected for CCC rating. The data for that rating is transformed to eliminate the autocorrelation.

This leads us to the estimation of seven AR(12) processes in the cross-section of the sample by ratings. We use the estimated monthly standard deviations of stock returns from 1970-2004 for regressions. All equations are estimated jointly, following Zellner (1962). The upper panel of table (7) shows cross-sectional correlation of residuals for the market volatility. We observe very high correlation coefficients among the investment-grade bonds.

Tables (3) and (4) present results of the seemingly unrelated regressions estimation when the firms in the sample are divided by their letter rating. For all tested ratings, the first three lags and the fifth lag of the monthly volatility are significant with the p-vales below 5%. Grades AAA through BB also have the sixth lag which is significant. The persistence of the volatility measured as the sum of the autoregressive coefficients is within 0.73-0.80. The coefficients of determination range from 0.84 to 0.90. Table (4) presents estimates of the month dummy coefficients. Most coefficients are highly significant. We present results of the F-test of equal monthly dummy coefficients. The p-values are close to zero for all grades.

The upper panel of figure (2) shows the predicted volatilities for ratings A and CCC, for the market volatility case. Lower credit quality substantially increases the volatility predictions. This observation is consistent with the leverage effect, implying a positive relationship between the firm's equity volatility and its leverage.

### 4.2.2 Aggregate market volatility

The first two columns of table (8) present results of the estimation for the S&P500 composite index. First four lags and the sixth lag of the dependent variable are significant. The sum of the autoregressive coefficients equals 0.767 and indicates some level of persistence of the volatility. The reported t-statistics are adjusted for correlations of error terms across observations, following Newey and West (1987). The coefficient of determination equals 41.31%. We apply the F-test to verify the null hypothesis that the intercept in the conditional volatility autoregression is equal across the months of the year. The F-statistic was computed at 2.27. The null is rejected at 5% significance level, the p-value equals .0108.

Figure (1) presents the predicted values of the conditional monthly volatility in the period 1970-2004 versus their realized values. The maximum monthly volatility, approximately 27%, was realized during the 1987 crisis. There are several other spikes, including the 1970s oil shock crisis.

## 4.3 Idiosyncratic volatility

Our estimation of the idiosyncratic volatility is based on the methodology presented in Campbell et al. (2001) and Brandt, Brav, and Graham (2006). In particular, we use an updated classification of the SIC codes into 49 industry groups, as presented at Ken French's website[1]. Within each industry group, the equally-weighted industry-level mean return in computed for each month in the sample. Next, the sum of deviations of firm-specific daily returns from the monthly mean return is calculated for each firm. The industry's idiosyncratic volatility is then estimated as an equally-weighted mean volatility of all the firms in a given industry. For each month in the sample we have

$$\hat{\sigma}_{id,i} = \sqrt{\frac{\sum_{i=1}^{N_i} \hat{\sigma}_{id,i,j}^2}{N_i}}, \tag{4.4}$$

where $i = 1, ..., 49$, $N_i$ denotes the number of firms in industry $i$ and

$$\hat{\sigma}_{id,i,j} = \sum_{n=1}^{m} (r_{n,i,j} - \bar{r}_i)^2. \tag{4.5}$$

$\hat{\sigma}_{id,i,j}$ is the estimated idiosyncratic volatility of firm $j$ within industry $i$, $r_{n,i,j}$ denotes the firm-specific return in the $n$th day of the month, $\bar{r}_i$ is industry's mean monthly return and $m$ is the number of trading days in that month. Aggregate idiosyncratic volatility in a given month is reported as an equally-weighted mean of idiosyncratic volatilities of all industries. The results are very similar when value-weighted means are used instead.

The procedure described above is applied to both the aggregate market data as well as the cross-section of credit ratings.

### 4.3.1 Cross-section of the idiosyncratic volatility

We estimate the idiosyncratic volatility at a firm-specific level. Similarly to the market volatility, we first conduct a group mean F-test, indicating whether mean volatility over the analyzed period differed by credit rating. The mean monthly realized volatilities range from 7.56% for AAA to 14.02% for CCC. The calculated F-statistic with the null of equal means is 278.81, giving strong evidence against the null.

For ratings A, BB, B and CCC we need to make an adjustment for the time-series autocorrelation of residuals. The p-values from the Breusch-Godfrey test are below

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ ken.french/

5% for these grades. We proceed with the seemingly unrelated regressions estimation as in the case of market volatility.

We present estimates of the AR(12) process with the monthly dummy variables, in tables (5) and (6). For most ratings, only the first three lags are significant. The persistence measured by the sum of autoregressive coefficients is higher than for the market volatility.

The lower panel of figure (2) presents predicted idiosyncratic volatilities for ratings A and CCC. As for the market volatility case, lower credit quality substantially increases the overall anticipated volatility. One interesting feature of the predicted idiosyncratic volatility process is worth noting. Several authors claimed that the idiosyncratic volatility has been increasing over the last few decades. Presented results suggest that this claim concerns primarily lower quality firms. Realized monthly idiosyncratic volatilities in the cross-section of credit ratings are shown in figure (3). We choose to show ratings AAA, A, B and CCC. The figure leads to quite an interesting observation that the investment-grade firms' idiosyncratic volatility has not shown the same increasing trend, as the low-rated firms' idiosyncratic volatility.

### 4.3.2 Aggregate idiosyncratic volatility

We present the predicted aggregate idiosyncratic volatility in the lower panel of figure (1). The results are very similar to those reported in Campbell and Taksler (2003) and Brandt, Brav, and Graham (2006). However, we use a longer data sample as well as an updated version of industry division by the SIC codes.

Idiosyncratic volatility has been increasing quite consistently since the beginning of the 1970. Several papers [e.g. Campbell and Taksler (2003)] have argued that there is a time trend in the firm-specific return volatility. The trend was stopped in the early 2000s. Given that, Brandt, Brav, and Graham (2006), show that in fact the rise in idiosyncratic volatility occurs only during speculative episodes. We do not attempt to explain the sources of the firm-level volatility.

The same technique as in the market volatility estimation is used here. We estimate the AR(12) process with monthly dummy variables. The results of estimation are presented in the right panel of table (8). Only the first two autoregressive coefficients are significant, with the p-values below 5%. Only four of the monthly dummies are significant, quite the opposite in comparison to the market volatility. The sum of autoregressive coefficients equals 0.968 and indicates a very high level of persistence, much higher than for the market volatility. The coefficient of determination is 89.86%. One possible explanation for why we are able to fit the idiosyncratic volatility process to the data much better than the market volatility is that the latter is much more exposed to other factors, like macroeconomic shocks, and therefore a richer set of covariates would probably fit the data better.

## 4.4 Credit spreads

We calibrate the time-varying volatility framework to the estimates of the volatility process in order to compute the asset volatilities, the asset risk premia, the default boundaries and credit spreads. We calculate the implied default boundary by assuming that the physical default probability is given as in equation (3.4). The interest rate, the payout and recovery rate, as well as the average market price of risk are as in the benchmark case. We use equation (3.6) for the calculation of the mean predicted variance over one- or four-year rolling windows. Then, we calibrate the default boundary so that the physical default probability equals a historical default probability as reported by Moody's (2000).

Figure (4) presents the predicted credit spreads for the four-year and ten-year time horizons. Calculated spreads for the investment-grade bonds are pretty stable over the analyzed period. Predicted spreads for the low-rated bonds are fluctuating in response to the time-varying stock volatility. The upper panel of figure (4) shows spreads for the four-year maturity and the lower panel presents spreads for the ten-year maturity. In the case of the ten-year maturity credit spreads of the low-rated bonds show an increasing time trend, which is not observed in the investment-grade bonds.

Figure (5) shows the relative credit spreads between different rating classes, calculated from the time-varying volatility model. The upper panel shows results for the four-year maturity and the lower panel shows results for the ten-year maturity. It is clear from this figure that the credit spreads' reaction to the time-varying volatility is different for the investment-grade bonds than for the low-rated bonds. Only market volatility enters into the drift of the firm value process and this is the main driver of the default probability and credit spreads of the investment-grade bonds. Hence, as there is no clear trend in the market volatility in the analyzed period, credit spreads of the investment-grade bonds remain stable. Conversely, the default probability and credit spreads of the low-rated bonds are primarily linked to the total asset volatility and therefore are responsive to its fluctuations.

We compare calculated spreads to the historical average spreads in table (9). Spreads for the four-year maturity are from Duffee (1998) study of noncallable bonds. A 10bp is added to each spread to account for the fact that the presented ten-year spreads contain both callable and noncallable bonds, based on Crabbe(1991). The ten-year historical spreads are based on the Lehman bond index dataset. Historical spreads are from 1985-1995.

For the time-varying volatility model, we report the calculated credit spreads assuming constant interest and payout rate, constant recovery rate and default boundaries as implied by the time-varying volatility model. We compute spreads for each year between 1985 and 1995 and show the average spread.

The time-varying volatility model generates average credit spreads higher than the benchmark model. The only exception is the ten-year spread for the B-rated bonds, which is slightly lower than in the benchmark case.

# 5 Conclusions

We investigate the link between recent trends in the stock return volatility and the credit spreads in the simple structural models of credit risk.

We estimate autoregressive processes for the market and idiosyncratic volatility of the stock return in the cross-section of credit ratings. Both idiosyncratic and market volatility depend significantly on the credit rating and are higher for stocks of lower credit quality. This is consistent with the leverage effect, as stocks with lower credit rating carry more debt. Market stock volatility is relatively persistent, however idiosyncratic volatility is much more persistent with the sum of autoregressive coefficients close to one. We confirm that aggregate idiosyncratic volatility has increased from 1970s until the late 1990s, however in the cross-section of credit ratings this effect is only noticeable for the low-rated bonds.

The main finding of the paper comes from the estimation of credit spreads. An option pricing model with time-varying deterministic volatility is used. True asset volatility is modeled as the AR(12) process with monthly dummy variables. The predicted volatility after dropping error terms is used as an input to the model. We find that the time-varying volatility model generates higher credit spreads than the benchmark model after calibration to the historical probabilities of default.

We observe a clear relationship connecting the credit spreads to the share of the idiosyncratic volatility in the total volatility. The relationship is noticeable for both the four-year horizon and the ten-year horizon. Credit spreads of the investment-grade bonds are primarily linked to the level of market volatility through the drift of the physical firm's value process. Credit spreads of the low-rated bonds are related much more to the total volatility. Taking into account the recent increase of the idiosyncratic volatility of low credit quality firms' stock returns significantly improves predictive power of a simple structural model.

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# Appendix A

Assume we have two value processes,

$$\frac{dV(t)}{V(t)} = (\mu - \delta)dt + \sigma dZ_m(t), \tag{A.1}$$

$$\frac{dV^*(t)}{V^*(t)} = (\mu - \delta)dt + \sigma \left[\gamma dZ_m(t) + \sqrt{1-\gamma^2} dZ_i(t)\right]. \tag{A.2}$$

Process $V(t)$ is driven only by the market source of risk and process $V^*(t)$ is driven by both market and idiosyncratic sources of risk. $V(t)$ can be thought of as the market index process and $V^*(t)$ as the stock price process. The intuition is quite simple. Assuming that an average stock has $\beta = 1$, we can write

$$\frac{dV^*(t)}{V^*(t)} = \frac{dV(t)}{V(t)} + \sigma_i dZ_i(t), \tag{A.3}$$

where $\sigma_i$ denotes the idiosyncratic volatility. Assuming the market value process is given by (A.1), (A.2) follows.
We introduce the process

$$Z(t) = \gamma Z_m(t) + \sqrt{1-\gamma^2} Z_i(t), \tag{A.4}$$

which is a continuous martingale with $Z(0) = 0$ and

$$\begin{aligned} dZ(t)dZ(t) &= \gamma^2 dZ_m(t)dZ_m(t) + 2\gamma\sqrt{1-\gamma^2} dZ_m(t)dZ_i(t) + (1-\gamma^2)dZ_i(t)dZ_i(t) \\ &= \gamma^2 dt + (1-\gamma^2)dt = dt. \end{aligned}$$

By the one-dimensional Lévy Theorem, $Z(t)$ is a Brownian motion. Hence we can write

$$\frac{dV^*(t)}{V^*(t)} = \mu dt + \sigma dZ(t). \tag{A.5}$$

The Brownian motions $Z_m(t)$ and $Z(t)$ are correlated. We have

$$\begin{aligned} d(Z_m(t)Z(t)) &= Z_m(t)dZ(t) + Z(t)dZ_m(t) + dZ_m(t)dZ(t) \\ &= Z_m(t)dZ(t) + Z(t)dZ_m(t) + \gamma dt. \end{aligned}$$

After integration we obtain

$$Z_m(t)Z(t) = \int_0^t Z_m(s)dZ(s) + \int_0^t Z(s)dZ_m(s) + \gamma t.$$

Using properties of Itô integrals, we get that

$$\mathbb{E}[Z_m(t)Z(t)] = \gamma t. \tag{A.6}$$

This is the covariance between $Z_m(t)$ and $Z(t)$. We also know that both $Z_m(t)$ and $Z(t)$ have a standard deviation of $\sqrt{t}$, and so the correlation between $Z_m(t)$ and $Z(t)$ equals $\gamma$.

We are now going to show that the two sources of risk, $Z_m(t)$ and $Z_i(t)$ are indeed independent so that they can be treated as respectively systematic and idiosyncratic sources of risk. Using equation (A.6) we have that

$$dZ_m(t)dZ(t) = \gamma dt. \tag{A.7}$$

Hence

$$dZ_m(t)\left(\gamma dZ_m(t) + \sqrt{1-\gamma^2}dZ_i(t)\right) = \gamma dt \tag{A.8}$$

and given the properties of the Brownian motion we have that

$$dZ_m(t)dZ_i(t) = 0. \tag{A.9}$$

From equations (A.1) and (A.2) it can be inferred that the total volatility equals $\sigma$. The sum of the systematic and idiosyncratic volatilities yields,

$$\sqrt{\sigma^2\gamma^2 + \sigma^2(1-\gamma^2)} = \sigma. \tag{A.10}$$

This enables us to write the formulas for the market and idiosyncratic volatility as

$$\sigma_m = \sigma\gamma, \tag{A.11}$$
$$\sigma_i = \sigma\sqrt{1-\gamma^2}, \tag{A.12}$$

and interpret $\gamma$ as the share of market volatility in the total volatility. We have

$$\gamma \equiv \frac{\sigma_m}{\sqrt{\sigma_m^2 + \sigma_i^2}}. \tag{A.13}$$

# Appendix B

We follow the argument presented in Hull and White (1987) about the derivation of the option prices under time-varying volatilities. Option price in the time-varying volatility model is driven by the two state variables, the stock price and the asset volatility. Only the stock is a tradeable asset. The volatility process is not traded as there is no asset perfectly correlated with the stock return volatility. This poses a potential problem as it is not possible to create a hedge portfolio that would eliminate all the risk. Garman (1976) demonstrates that any security with the price $f$ driven by the state variables $\theta_i$ satisfies the following differential equation,

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sum_{i,j} \rho_{i,j}\sigma_i\sigma_j \frac{\partial^2 f}{\partial\theta_i\partial\theta_j} - rf = \sum_i \theta_i \frac{\partial f}{\partial\theta_i}[-\mu_i + \beta_i(\mu^* - r)], \tag{B.1}$$

where $\sigma_i$ is the instantaneous volatility of the state variable $\theta_i$, $\rho_{i,j}$ is the instantaneous correlation between the state variables $\theta_i$ and $\theta_j$, $\mu_i$ is the drift of $\theta_i$, $\beta_i$ is the vector of betas of the state variables on the market portfolio, $r$ is the risk free rate and $\mu^*$ is the vector of the instantaneous returns on the market and portfolios closely correlated to the state variables.

In our case the state variables are the stock price, $S$, and the volatility process, $\tilde{V}$. Assume the volatility follows the process,

$$\frac{d\tilde{V}(t)}{\tilde{V}(t)} = \mu dt + \xi dZ(t). \tag{B.2}$$

We have that $\rho_{S,\tilde{V}} = 0$ as the stock price and the volatility process are uncorrelated. The systematic risk of the volatility process is assumed to be constant. In other words the correlation of the volatility and the aggregate consumption is a constant. This yields a constant asset risk premium. We have $\beta_{\tilde{V}}(\mu^* - r) = \pi_{\tilde{V}}$. For analytical tractability, we proceed assuming $\pi_V = 0$. This does not change the results qualitatively. We can write the equation (B.1) as

$$\frac{\partial f}{\partial t} + \frac{1}{2}\left[\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + 2\rho\sigma^3\xi S \frac{\partial^2 f}{\partial S\partial\tilde{V}} + \xi^2\tilde{V}^2\frac{\partial^2 f}{\partial\tilde{V}^2}\right] - rf = -rS\frac{\partial f}{\partial S} - \mu\sigma^2\frac{\partial f}{\partial\tilde{V}}. \tag{B.3}$$

The above assumption about no correlation of the volatility process with the aggregate consumption is in practice equivalent to assuming the risk-neutrality. The option price can be therefore expressed as

$$f[S(t), \sigma^2(t), t] = e^{-r(T-t)} \int f[S(T), \sigma^2(T), T] p[S(T)|S(t),\, \sigma(t)^2] dS(T), \tag{B.4}$$

where $p[S(T)|S(t),\, \sigma(t)^2]$ is the conditional terminal stock price distribution.

Next we make an observation that the conditional density function for any related three variables $x$, $y$ and $z$, can be presented as

$$p(x|y) = \int g(x|z)h(z|y)dz. \tag{B.5}$$

In particular, if the mean variance is given by equation (3.6), then the terminal stock price distribution can be expressed as

$$p[S(T)|\sigma^2(t)] = \int g[S(T)|\bar{V}]h[\bar{V}|\sigma^2(t)]d\bar{V}. \tag{B.6}$$

Using this equation in (B.4) enables us to write the security price as

$$f[S(t), \sigma^2(t), t] = \int \left[ e^{-r(T-t)} \int f[S(T)]g[S(T)|\bar{V}]dS(T) \right] h[\bar{V}|\sigma^2(t)]d\bar{V}. \tag{B.7}$$

With the assumption given above, namely that $\rho = 0$ and $\xi$ is independent of the stock price, the inner term in equation (B.7) is the Black-Scholes call option price.

For a deterministic volatility process, we set $\xi = 0$. All derived expressions hold. In that case the volatility can be modeled as an AR(1) process.

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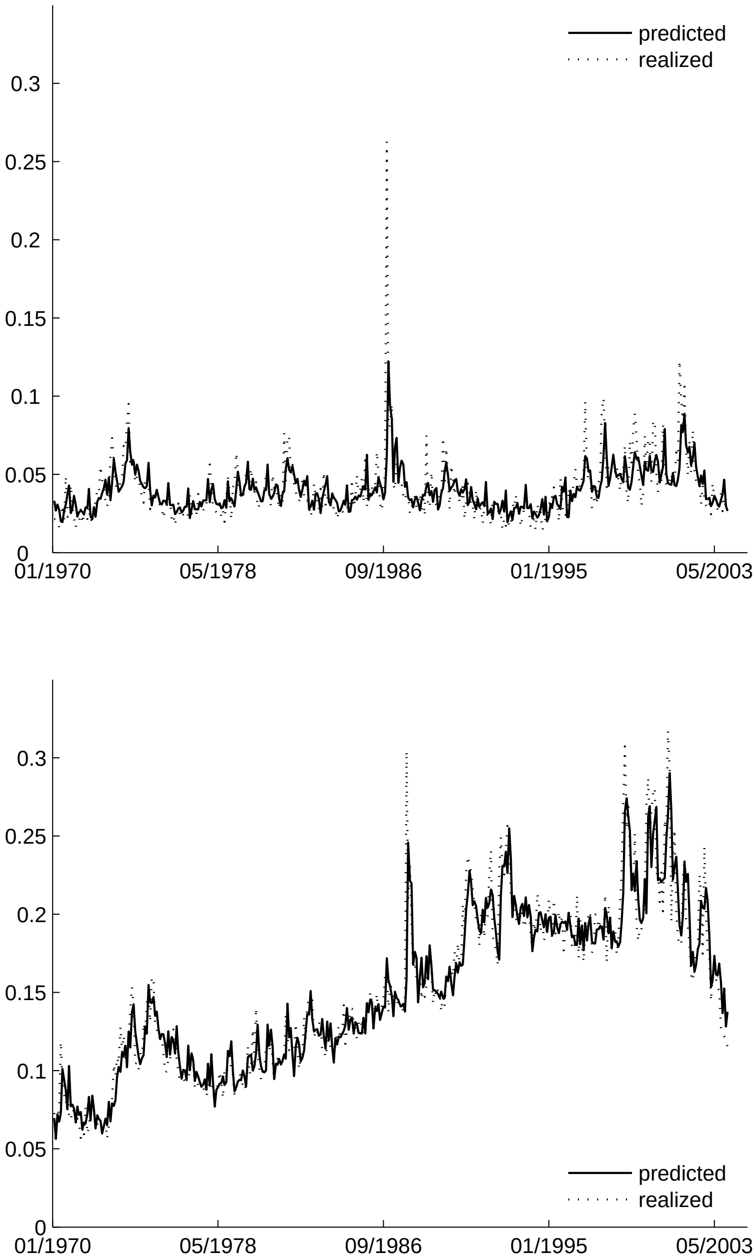


Figure 1: The upper panel shows predicted versus realized monthly conditional volatility from the AR (12) process for the aggregate market volatility. The lower panel shows predicted versus realized monthly conditional volatility from the AR (12) process for the aggregate idiosyncratic volatility. The market monthly predicted volatilities are obtained following closely the procedure described in Schwert (1989). Levels of realized idiosyncratic volatility are estimated following the procedure described in Campbell et al. (2001), however equal weights for firms within industry and for industries are assumed. Axis y is the conditional monthly standard deviation of S&P500 composite index return.

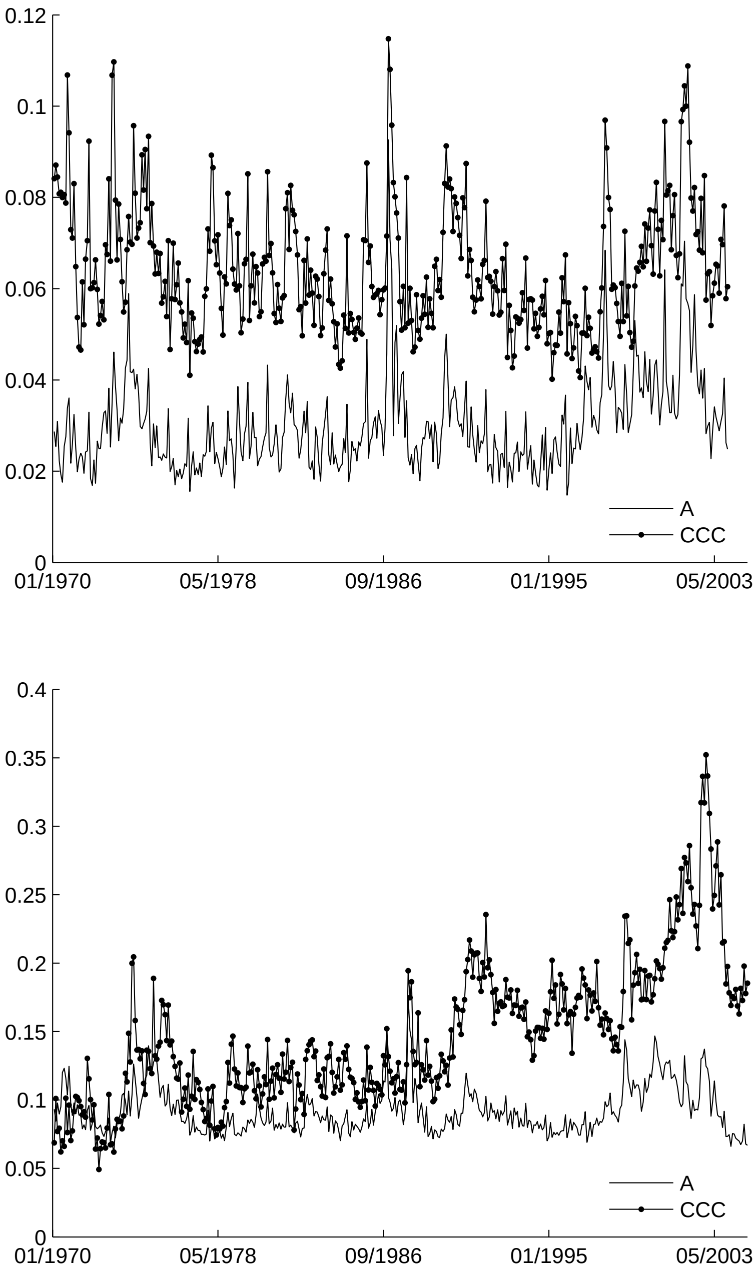


Figure 2: Figure presents the cross-section of predicted monthly volatility from AR (12) process for ratings A and CCC. The upper panel presents the cross-section of predicted monthly market volatility. The lower panel presents the cross-section of predicted monthly idiosycratic volatility. The monthly predicted volatilities are obtained following closely the procedure described in Schwert (1989) and Campbell et al. (2001). Axis y is the conditional monthly standard deviation of S&P500 composite index return.

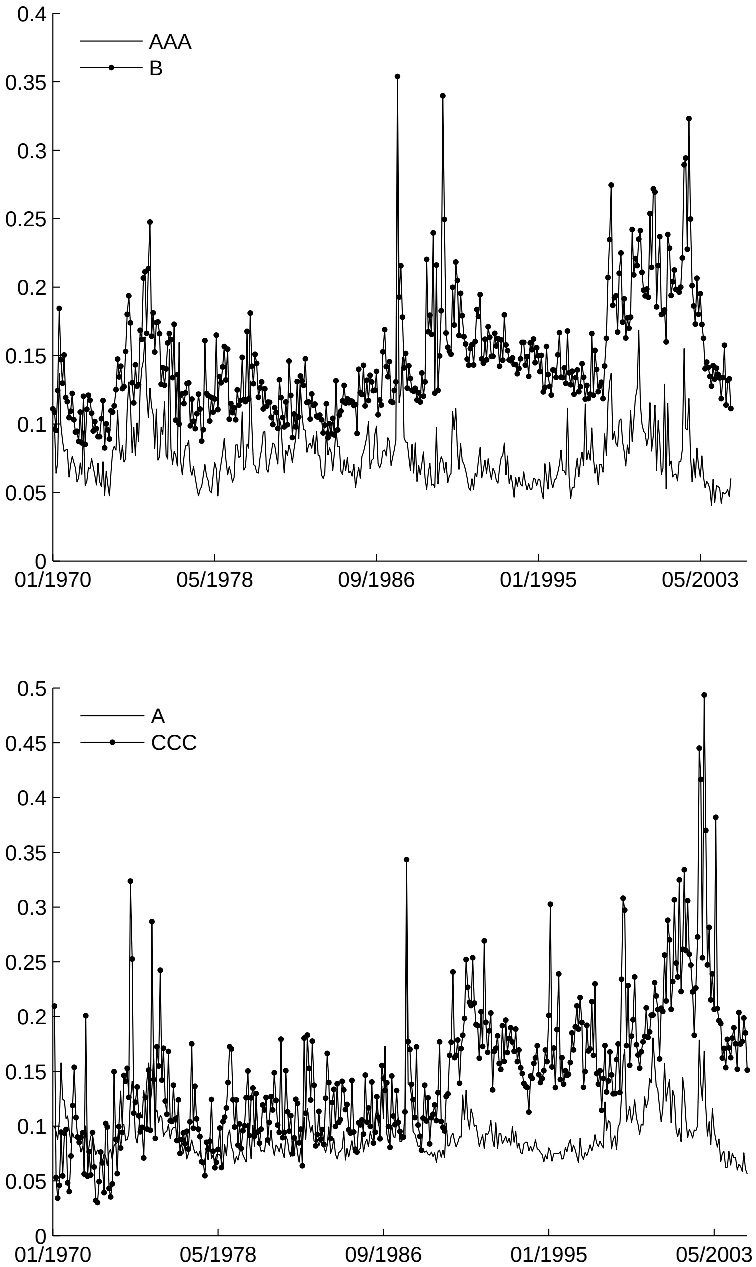


Figure 3: Figure presents the realized monthly idiosyncratic volatility. The upper panel shows ratings AAA and B, lower one A and CCC. Monthly realized idiosyncratic volatility is computed following the methodology described in Campbell et al. (2001).

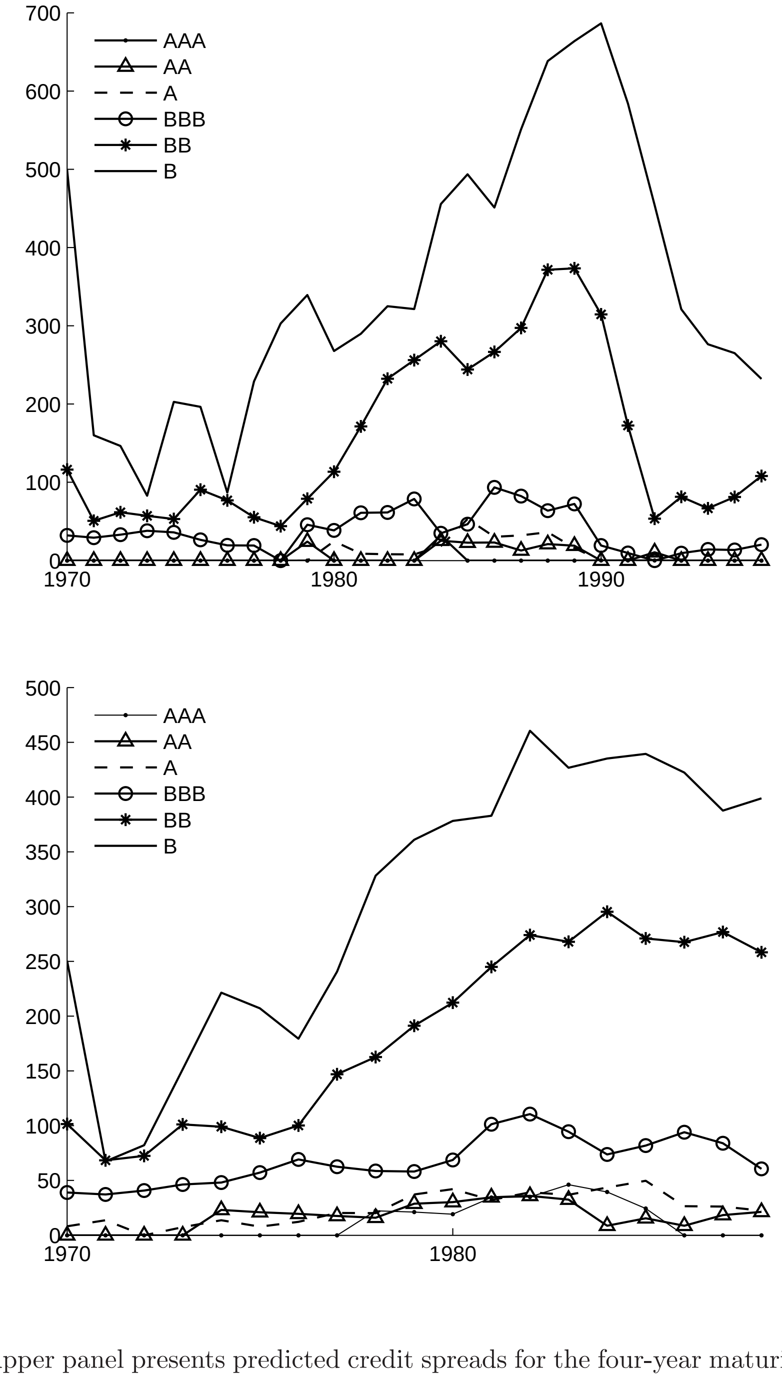


Figure 4: The upper panel presents predicted credit spreads for the four-year maturity and the lower panel presents predicted credit spreads for the ten-year maturity. Presented spreads are calculated using the time-varying volatility model. Axis y is the credit spread in basis points. Axis x is the beginning year of a four- or ten-year cohort.

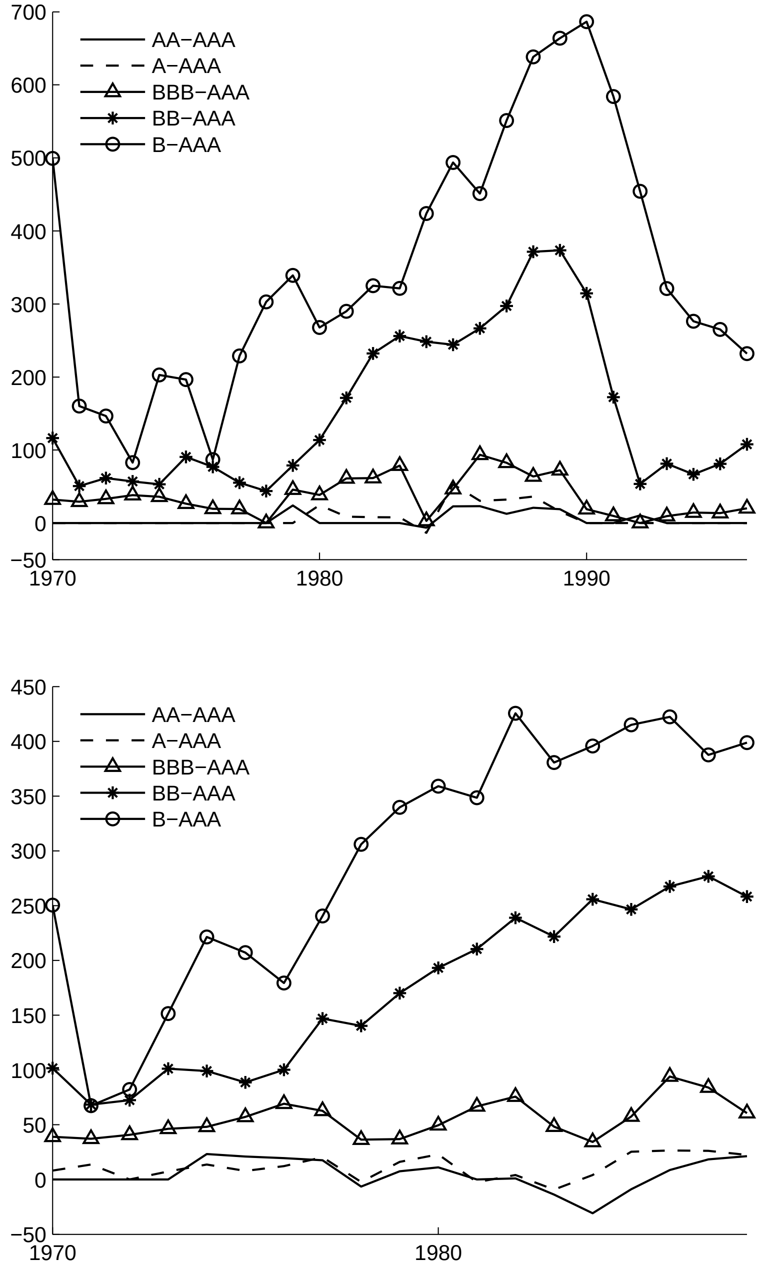


Figure 5: The upper panel presents predicted credit spreads between different rating classes for the four-year time horizon and the lower panel presents predicted credit spreads between different rating classes for the ten-year time horizon. Spreads are calculated relative to the predicted AAA spread, using the time-varying volatility model. Axis y is the credit spread in basis points. Axis x is the beginning year of a four- or ten-year cohort.

# List of Tables

| rating | target | | | implied | | credit spreads | |
|---|---|---|---|---|---|---|---|
| | leverage ratio | cum. def. prob. | equity premium | asset vol. | asset risk premium | historical | calculated |
| | % | | | | | basis points | |
| AAA | 13.08 | 0.04 | 5.38 | 36.18 | 4.96 | 55 | 1.25 |
| AA | 21.18 | 0.23 | 5.60 | 34.42 | 4.89 | 65 | 6.44 |
| A | 31.98 | 0.35 | 5.99 | 29.77 | 4.85 | 96 | 10.54 |
| BBB | 43.28 | 1.24 | 6.55 | 28.86 | 4.88 | 158 | 33.63 |
| BB | 53.53 | 8.51 | 7.30 | 34.21 | 5.17 | 320 | 174.39 |
| B | 65.70 | 23.32 | 8.76 | 39.35 | 6.00 | 470 | 432.68 |

| rating | target | | | implied | | credit spreads | |
|---|---|---|---|---|---|---|---|
| | leverage ratio | cum. def. prob. | equity premium | asset vol. | asset risk premium | historical | calculated |
| | % | | | | | basis points | |
| AAA | 13.08 | 0.77 | 5.38 | 32.12 | 4.98 | 63 | 12.55 |
| AA | 21.18 | 0.99 | 5.60 | 28.39 | 4.91 | 91 | 17.44 |
| A | 31.98 | 1.55 | 5.99 | 25.59 | 4.87 | 123 | 27.89 |
| BBB | 43.28 | 4.39 | 6.55 | 25.77 | 4.94 | 194 | 63.78 |
| BB | 53.53 | 20.63 | 7.30 | 32.32 | 5.46 | 320 | 197.45 |
| B | 65.70 | 43.91 | 8.76 | 39.86 | 6.73 | 470 | 360.60 |

Table 1: Table presents credit spreads and the implied asset volatilities and asset risk premia for a zero coupon bond, predicted by the benchmark model. The upper panel is for the four-year maturity and the lower panel is for the ten-year maturity. The historical default rates are from the Moodys report (2004). The risk free rate is 8%, the payout rate is 6%. The recovery rate is 51.31%. The default boundary is constant and equal to 60% of the debt face value. The asset volatility and asset risk premia are implied from the cumulative default probabilities, initial leverage ratios and equity premia given in the first three columns of the table. Average credit spreads of the ten-year investment-grade bonds are from the Lehman bond index data from 1973-1993. Historical credit spreads of the four-year investment-grade bonds are as in Duffee (1998), average spreads of the noncallable bonds from 1985-1995. For consistency with ten-year average spreads, which contain both callable and noncallable bonds, 10bp of call option spreads were added as in Crabbe (1991).

| | AAA | AA | A | BBB | BB | B | CCC | market |
|---|---|---|---|---|---|---|---|---|
| | | | | #1 | | | | |
| mean | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 |
| st. dev | 0.010 | 0.008 | 0.008 | 0.008 | 0.009 | 0.010 | 0.016 | 0.010 |
| skewness | -0.336 | -0.762 | -0.779 | -0.823 | -0.575 | -0.712 | 0.498 | -0.940 |
| kurtosis | 17.228 | 20.967 | 23.793 | 20.424 | 15.185 | 14.696 | 11.232 | 27.144 |
| | | | | #2 | | | | |
| mean | 0.015 | 0.015 | 0.015 | 0.015 | 0.016 | 0.017 | 0.018 | 0.007 |
| st. dev | 0.045 | 0.043 | 0.044 | 0.046 | 0.058 | 0.065 | 0.090 | 0.044 |
| skewness | -0.008 | -0.542 | -0.563 | -0.626 | -0.583 | -0.442 | -0.187 | -0.447 |
| kurtosis | 5.433 | 5.213 | 5.822 | 6.174 | 5.756 | 6.191 | 4.582 | 4.604 |
| | | | | #3 | | | | |
| mean | 0.015 | 0.015 | 0.015 | 0.014 | 0.015 | 0.016 | 0.019 | 0.007 |
| st. dev | 0.049 | 0.046 | 0.047 | 0.050 | 0.061 | 0.068 | 0.092 | 0.046 |
| skewness | 0.040 | -0.583 | -0.523 | -0.489 | -0.436 | -0.344 | -0.050 | -0.374 |
| kurtosis | 5.257 | 5.421 | 5.934 | 5.906 | 5.858 | 6.788 | 4.668 | 4.960 |
| | | | | #4 | | | | |
| mean | 0.014 | 0.014 | 0.016 | 0.016 | 0.018 | 0.018 | 0.017 | 0.008 |
| st. dev | 0.038 | 0.040 | 0.039 | 0.039 | 0.053 | 0.062 | 0.087 | 0.042 |
| skewness | -0.163 | -0.456 | -0.609 | -0.901 | -0.828 | -0.597 | -0.406 | -0.561 |
| kurtosis | 4.691 | 4.404 | 4.678 | 5.634 | 5.300 | 4.941 | 4.392 | 3.806 |

Table 2: Table presents means, standard deviation, skewness and kurtosis of the returns used in the volatility estimation. Two upper panels of the table present sample statistics for the daily and monthly data for the whole period 1970-2004. The lower panels present monthly data sample statistics in the subperiods. #1 denotes 1970-2004, daily data, #2 denotes 1970-2004, monthly data, #3 is 1970-1990 monthly data and #4 is 1990-2004 monthly data. The market return is defined as the S&P500 composite index return.

| # of lags | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| 1 | 0.244*** | 0.261*** | 0.263*** | 0.237*** | 0.219*** | 0.231*** | 0.235*** |
| | (8.410) | (10.090) | (11.638) | (10.775) | (8.966) | (9.332) | (6.210) |
| 2 | 0.128*** | 0.135*** | 0.126*** | 0.120*** | 0.123*** | 0.128*** | 0.117** |
| | (4.206) | (4.963) | (5.384) | (5.310) | (4.930) | (5.008) | (2.998) |
| 3 | 0.090** | 0.095*** | 0.079*** | 0.078*** | 0.089*** | 0.074** | 0.105** |
| | (2.942) | (3.445) | (3.361) | (3.424) | (3.538) | (2.894) | (2.660) |
| 4 | 0.049 | 0.032 | 0.046 | 0.048* | 0.041 | 0.048 | 0.093* |
| | (1.597) | (1.159) | (1.943) | (2.121) | (1.645) | (1.867) | (2.360) |
| 5 | 0.064* | 0.091*** | 0.114*** | 0.125*** | 0.116*** | 0.143*** | 0.116** |
| | (2.073) | (3.298) | (4.857) | (5.535) | (4.638) | (5.575) | (3.098) |
| 6 | 0.096** | 0.079** | 0.061* | 0.065** | 0.061* | 0.031 | 0.022 |
| | (3.116) | (2.887) | (2.571) | (2.860) | (2.433) | (1.195) | (0.596) |
| 7 | 0.031 | -0.013 | 0.009 | 0.006 | 0.010 | 0.013 | 0.018 |
| | (0.994) | (-0.483) | (0.384) | (0.263) | (0.397) | (0.510) | (0.493) |
| 8 | 0.015 | 0.065* | 0.042 | 0.030 | 0.039 | 0.044 | -0.012 |
| | (0.491) | (2.371) | (1.804) | (1.332) | (1.576) | (1.705) | (-0.328) |
| 9 | 0.022 | 0.015 | 0.013 | 0.015 | -0.001 | 0.017 | -0.064 |
| | (0.730) | (0.562) | (0.562) | (0.654) | (-0.040) | (0.673) | (-1.718) |
| 10 | 0.007 | 0.022 | 0.028 | 0.045* | 0.045 | 0.036 | 0.045 |
| | (0.231) | (0.799) | (1.202) | (1.987) | (1.802) | (1.399) | (1.214) |
| 11 | 0.051 | 0.012 | 0.012 | -0.002 | 0.009 | -0.004 | -0.032 |
| | (1.692) | (0.448) | (0.504) | (-0.071) | (0.347) | (-0.149) | (-0.870) |
| 12 | -0.019 | -0.009 | 0.010 | 0.011 | 0.022 | 0.017 | 0.091* |
| | (-0.661) | (-0.336) | (0.426) | (0.510) | (0.923) | (0.693) | (2.559) |
| sum of AR | 0.78 | 0.79 | 0.80 | 0.78 | 0.77 | 0.78 | 0.73 |
| $R^2$ | 0.88 | 0.87 | 0.85 | 0.85 | 0.84 | 0.86 | 0.90 |

Table 3: Table presents autoregression coefficients and robust t-statistics for the cross-section of the market volatility. Estimates of the AR(12) process with dummy variables for different monthly mean returns are obtained following closely the procedure described in Schwert (1989), however covariance matrix of error terms assumes cross-sectional correlation of residuals. Monthly standard deviations are calculated based on the daily data of S&P500 composite index return in the period 1970-2004. Sum of the autoregressive coefficients remains within 0.73-0.80 for all ratings. The coefficient of determination ranges from 0.84 to 0.90, stars denote significance level, $*\,p < .05$, $**\,p < .01$, $***\,p < .001$.

| # of month | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| 1 | 0.012*** | 0.009*** | 0.008** | 0.008*** | 0.009** | 0.010** | 0.015** |
| | (3.948) | (3.488) | (3.173) | (3.300) | (3.255) | (3.172) | (2.775) |
| 2 | 0.004 | 0.003 | 0.001 | 0.001 | 0.002 | 0.003 | 0.009 |
| | (1.442) | (1.017) | (0.364) | (0.586) | (0.807) | (0.949) | (1.656) |
| 3 | 0.011*** | 0.009*** | 0.006** | 0.006** | 0.007* | 0.007* | 0.011* |
| | (3.735) | (3.650) | (2.696) | (2.593) | (2.374) | (2.161) | (1.987) |
| 4 | 0.006* | 0.006* | 0.006* | 0.006* | 0.007* | 0.008** | 0.016** |
| | (2.073) | (2.396) | (2.318) | (2.429) | (2.411) | (2.753) | (2.897) |
| 5 | 0.006 | 0.003 | 0.004 | 0.004 | 0.004 | 0.005 | 0.013* |
| | (1.929) | (1.264) | (1.479) | (1.653) | (1.354) | (1.611) | (2.389) |
| 6 | 0.005 | 0.004 | 0.002 | 0.002 | 0.003 | 0.004 | 0.012* |
| | (1.542) | (1.408) | (0.735) | (0.852) | (0.929) | (1.147) | (2.237) |
| 7 | 0.008** | 0.007** | 0.006* | 0.007** | 0.008** | 0.009** | 0.015** |
| | (2.839) | (2.687) | (2.306) | (2.761) | (2.865) | (2.783) | (2.686) |
| 8 | 0.010*** | 0.009*** | 0.007** | 0.008** | 0.008** | 0.010** | 0.018*** |
| | (3.481) | (3.621) | (3.043) | (3.275) | (2.962) | (3.181) | (3.403) |
| 9 | 0.010** | 0.009*** | 0.008*** | 0.008*** | 0.009** | 0.011*** | 0.023*** |
| | (3.270) | (3.598) | (3.309) | (3.409) | (3.233) | (3.527) | (4.223) |
| 10 | 0.021*** | 0.019*** | 0.018*** | 0.018*** | 0.022*** | 0.023*** | 0.032*** |
| | (7.068) | (7.666) | (7.585) | (7.706) | (7.599) | (7.286) | (5.904) |
| 11 | 0.004 | 0.004 | 0.003 | 0.004 | 0.005 | 0.006* | 0.014** |
| | (1.518) | (1.658) | (1.074) | (1.869) | (1.832) | (1.977) | (2.623) |
| 12 | 0.005 | 0.004 | 0.004 | 0.005* | 0.007* | 0.007* | 0.021*** |
| | (1.593) | (1.438) | (1.582) | (2.145) | (2.352) | (2.326) | (3.780) |
| F-test | 3.55 | 4.04 | 4.00 | 3.80 | 3.74 | 3.34 | 2.61 |

Table 4: Table presents dummy coefficients from the AR(12) regression for the market volatility. The F-statistics from the Wald test indicate that the null of equal coefficients is rejected at 5% level for all ratings with p-values equal zero.

| # of lags | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| 1 | 0.200*** | 0.309*** | 0.313*** | 0.296*** | 0.355*** | 0.349*** | 0.275*** |
| | (6.059) | (10.348) | (11.461) | (11.201) | (11.770) | (9.786) | (6.503) |
| 2 | 0.153*** | 0.123*** | 0.146*** | 0.180*** | 0.174*** | 0.184*** | 0.156*** |
| | (4.470) | (3.810) | (5.007) | (6.331) | (5.352) | (4.768) | (3.475) |
| 3 | 0.137*** | 0.116*** | 0.106*** | 0.110*** | 0.099** | 0.036 | 0.167*** |
| | (3.932) | (3.558) | (3.608) | (3.808) | (2.970) | (0.908) | (3.723) |
| 4 | 0.016 | 0.036 | 0.046 | 0.034 | 0.074* | 0.081* | 0.070 |
| | (0.469) | (1.099) | (1.571) | (1.191) | (2.230) | (2.051) | (1.533) |
| 5 | 0.032 | 0.032 | 0.039 | 0.037 | 0.020 | -0.002 | 0.063 |
| | (0.906) | (0.980) | (1.330) | (1.298) | (0.610) | (-0.060) | (1.381) |
| 6 | 0.102** | 0.070* | 0.028 | 0.063* | 0.042 | 0.054 | 0.045 |
| | (2.899) | (2.132) | (0.933) | (2.177) | (1.257) | (1.367) | (0.986) |
| 7 | -0.016 | -0.005 | 0.008 | -0.033 | -0.024 | 0.075 | 0.090* |
| | (-0.451) | (-0.148) | (0.254) | (-1.157) | (-0.726) | (1.901) | (1.985) |
| 8 | 0.019 | 0.055 | 0.031 | 0.057* | 0.017 | 0.023 | -0.050 |
| | (0.538) | (1.679) | (1.064) | (2.003) | (0.507) | (0.582) | (-1.069) |
| 9 | 0.091** | 0.041 | 0.046 | 0.031 | 0.058 | 0.047 | 0.003 |
| | (2.623) | (1.256) | (1.572) | (1.107) | (1.783) | (1.215) | (0.062) |
| 10 | -0.017 | -0.007 | 0.005 | -0.013 | -0.001 | 0.030 | 0.103* |
| | (-0.492) | (-0.211) | (0.184) | (-0.471) | (-0.038) | (0.765) | (2.229) |
| 11 | 0.060 | -0.020 | 0.011 | 0.006 | -0.026 | 0.071 | -0.028 |
| | (1.756) | (-0.611) | (0.386) | (0.208) | (-0.823) | (1.848) | (-0.616) |
| 12 | -0.023 | 0.030 | 0.057* | 0.014 | 0.086** | -0.049 | 0.032 |
| | (-0.707) | (1.007) | (2.113) | (0.563) | (2.948) | (-1.377) | (0.745) |
| sum of AR | 0.75 | 0.78 | 0.84 | 0.78 | 0.87 | 0.90 | 0.92 |
| $R^2$ | 0.94 | 0.96 | 0.98 | 0.98 | 0.98 | 0.98 | 0.95 |

Table 5: Table presents autoregression coefficients and robust t-statistics for the cross-section of the idiosyncratic volatility process. The estimates of the AR(12) process with dummy variables for different monthly mean returns are obtained following closely the procedure described in Schwert (1989), however covariance matrix of error terms assumes cross-sectional correlation of residuals. The sum of the autoregressive coefficients remains within 0.75-0.92 for all ratings. The coefficient of determination ranges from 0.94 to 0.98, stars denote significance level, $*\ p < .05$, $**\ p < .01$, $***\ p < .001$.

| # of month | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| 1 | 0.024*** | 0.021*** | 0.019*** | 0.027*** | 0.020*** | 0.027*** | 0.018* |
| | (4.726) | (5.329) | (4.975) | (6.652) | (4.171) | (4.779) | (2.199) |
| 2 | 0.015** | 0.013** | 0.008* | 0.013** | 0.004 | 0.007 | -0.000 |
| | (3.012) | (3.291) | (2.057) | (3.205) | (0.928) | (1.274) | (-0.024) |
| 3 | 0.021*** | 0.020*** | 0.016*** | 0.022*** | 0.017*** | 0.016** | -0.001 |
| | (4.121) | (5.086) | (4.360) | (5.582) | (3.600) | (2.803) | (-0.172) |
| 4 | 0.015** | 0.016*** | 0.014*** | 0.019*** | 0.013** | 0.015** | 0.010 |
| | (2.999) | (4.075) | (3.833) | (4.799) | (2.848) | (2.590) | (1.232) |
| 5 | 0.016** | 0.014*** | 0.011** | 0.016*** | 0.013** | 0.004 | 0.011 |
| | (3.145) | (3.410) | (2.976) | (4.005) | (2.677) | (0.646) | (1.282) |
| 6 | 0.016** | 0.013** | 0.011** | 0.017*** | 0.015** | 0.014* | 0.004 |
| | (3.073) | (3.189) | (2.853) | (4.265) | (3.145) | (2.457) | (0.451) |
| 7 | 0.020*** | 0.017*** | 0.015*** | 0.023*** | 0.016*** | 0.011 | 0.008 |
| | (3.900) | (4.151) | (3.911) | (5.774) | (3.368) | (1.934) | (0.974) |
| 8 | 0.021*** | 0.022*** | 0.014*** | 0.022*** | 0.018*** | 0.017** | 0.016 |
| | (4.200) | (5.356) | (3.824) | (5.666) | (3.844) | (2.911) | (1.899) |
| 9 | 0.019*** | 0.017*** | 0.016*** | 0.021*** | 0.015** | 0.008 | 0.017* |
| | (3.630) | (4.256) | (4.310) | (5.197) | (3.198) | (1.492) | (2.003) |
| 10 | 0.034*** | 0.035*** | 0.030*** | 0.040*** | 0.037*** | 0.036*** | 0.036*** |
| | (6.683) | (8.694) | (8.096) | (9.958) | (7.796) | (6.282) | (4.287) |
| 11 | 0.015** | 0.013** | 0.012** | 0.020*** | 0.015** | 0.013* | 0.009 |
| | (2.903) | (3.126) | (3.186) | (5.077) | (3.062) | (2.344) | (1.038) |
| 12 | 0.013** | 0.015*** | 0.013*** | 0.021*** | 0.018*** | 0.015** | 0.021* |
| | (2.610) | (3.667) | (3.598) | (5.276) | (3.748) | (2.670) | (2.476) |
| F-test | 2.62 | 4.54 | 4.21 | 5.94 | 5.36 | 4.00 | 2.05 |

Table 6: Table presents dummy coefficients from the AR(12) regression for the idiosyncratic volatility. F-statistics from the Wald test indicate that the null of equal coefficients is rejected at 5% level for investment for all ratings. The p-values are zero for ratings AAA through B and 0.02 for CCC.

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| AAA | 1.00 | | | | | | |
| AA | 0.88 | 1.00 | | | | | |
| A | 0.89 | 0.95 | 1.00 | | | | |
| BBB | 0.87 | 0.91 | 0.97 | 1.00 | | | |
| BB | 0.80 | 0.85 | 0.92 | 0.95 | 1.00 | | |
| B | 0.79 | 0.84 | 0.91 | 0.94 | 0.96 | 1.00 | |
| CCC | 0.56 | 0.61 | 0.64 | 0.67 | 0.67 | 0.69 | 1.00 |

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| AAA | 1.00 | | | | | | |
| AA | 0.77 | 1.00 | | | | | |
| A | 0.77 | 0.83 | 1.00 | | | | |
| BBB | 0.79 | 0.85 | 0.89 | 1.00 | | | |
| BB | 0.68 | 0.75 | 0.82 | 0.84 | 1.00 | | |
| B | 0.54 | 0.62 | 0.69 | 0.67 | 0.70 | 1.00 | |
| CCC | 0.42 | 0.45 | 0.49 | 0.51 | 0.49 | 0.51 | 1.00 |

Table 7: Table presents cross-sectional error terms correlation from the seemingly unrelated regressions. The upper panel presents results for the market volatility and the lower panel for the idiosyncratic volatility.

| # of lags/month | market volatility | | idiosyncratic volatility | |
|---|---|---|---|---|
| | AR coeff. | month coeff. | AR coeff. | month coeff. |
| 1 | 0.371*** | 0.011** | 0.621*** | 0.003 |
| | (5.169) | (3.025) | (9.428) | (0.935) |
| 2 | 0.200*** | 0.006* | 0.158* | -0.011*** |
| | (5.611) | (2.129) | (2.241) | (-3.509) |
| 3 | 0.104** | 0.010** | 0.106 | 0.006 |
| | (2.922) | (3.213) | (1.493) | (1.956) |
| 4 | -0.122*** | 0.008** | -0.096 | 0.003 |
| | (-3.522) | (2.768) | (-1.464) | (1.097) |
| 5 | 0.062 | 0.007** | 0.081 | 0.005 |
| | (1.222) | (2.591) | (1.489) | (1.090) |
| 6 | 0.146** | 0.005 | 0.032 | 0.001 |
| | (2.825) | (1.559) | (0.580) | (0.317) |
| 7 | -0.007 | 0.008** | -0.074 | 0.005 |
| | (-0.147) | (2.665) | (-1.118) | (1.439) |
| 8 | 0.006 | 0.013** | 0.010 | 0.008** |
| | (0.207) | (3.259) | (0.194) | (2.799) |
| 9 | 0.015 | 0.011** | 0.109 | 0.001 |
| | (0.400) | (2.942) | (1.672) | (0.303) |
| 10 | 0.048 | 0.023* | -0.028 | 0.021*** |
| | (1.000) | (2.514) | (-0.562) | (3.357) |
| 11 | -0.007 | 0.004 | 0.038 | 0.006 |
| | (-0.188) | (1.241) | (0.500) | (1.557) |
| 12 | -0.049 | 0.005 | 0.012 | 0.013*** |
| | (-0.867) | (1.639) | (0.227) | (3.485) |

Table 8: Table presents the results of the estimation of the aggregate volatility process. The estimates of the AR(12) process with dummy variables for different monthly mean returns are obtained following closely the procedure described in Schwert (1989). The left panel (first two columns) present results of regression for the aggregate market volatility and the right panel for aggregate idiosyncratic volatility. The first column of each panel presents autoregressive coefficients and second column monthly dummy variables coefficients. Monthly standard deviations of market returns are calculated based on the daily data of S&P500 composite index return in the period 1970-2004. Levels of realized idiosyncratic volatility are computed using the methodology presented in Campbell et al. (2001). Reported t-statistics, presented in parentheses, are adjusted for correlations of error terms across observations, following Newey and West (1987). The sum of the autoregressive coefficients equals 0.767 for the market and 0.968 for the idiosyncratic volatility. The coefficients of determination are 41.31% and 89.86% for the market and the idiosyncratic volatility respectively, stars denote significance level, $* \ p < .05$, $** \ p < .01$, $*** \ p < .001$.

| rating | implied | | credit spreads | |
|---|---|---|---|---|
| | asset vol. | asset risk premium | historical | calculated |
| | % | | basis points | |
| AAA | 27.29 | 4.93 | 55 | 2.89 |
| AA | 22.68 | 4.20 | 65 | 11.28 |
| A | 30.24 | 4.86 | 96 | 19.20 |
| BBB | 18.84 | 3.36 | 158 | 56.66 |
| BB | 30.35 | 4.33 | 320 | 270.91 |
| B | 40.82 | 5.48 | 470 | 496.37 |

| rating | implied | | credit spreads | |
|---|---|---|---|---|
| | asset vol. | asset risk premium | historical | calculated |
| | % | | basis points | |
| AAA | 27.49 | 4.84 | 63 | 22.03 |
| AA | 23.74 | 4.57 | 91 | 23.69 |
| A | 24.36 | 4.03 | 123 | 31.05 |
| BBB | 12.48 | 2.32 | 194 | 76.07 |
| BB | 42.87 | 6.40 | 320 | 205.00 |
| B | 67.14 | 10.09 | 470 | 349.03 |

Table 9: Table presents credit spreads, the implied asset volatilities and the asset risk premia for a zero coupon bond predicted by the time-varying volatility model. The upper panel presents the results for the four-year maturity and the lower panel presents the results for the ten-year maturity. The historical default rates are from the Moodys report (2004). The risk free rate is 8%, the payout rate is 6%. The recovery rate is 51.31%. The default boundary is calculated based on equation (3.4), assuming $K = e^{-b}$, after calibrating to the cumulative default probabilities and equity premia given in table (1). Average credit spreads of the ten-year investment-grade bonds are from the Lehman bond index data. Historical credit spreads of the four-year investment-grade bonds are average spreads of the noncallable bonds, as in Duffee (1998). For consistency with the ten-year average spreads, which contain both callable and noncallable bonds, 10bp of call option spreads were added as in Crabbe (1991). Average spreads for the low-rated bonds are as in Caouette, Altman and Narayanan (1998).