1 Why (18) is equivilent to (19)

1.1 Why (19) implies (18)

This direction is obvious.

1.2 Why (18) implies (19)

A little harder... First note that (19) implies that

$$E\left[\left\{U'(x) + E[U'(x)]\sigma_m^{-2}R_m(\bar{R}_m - r)]\right\}\sum_{j=1}^{J} b_{ji}\delta_j\right] = 0.$$

$$\Rightarrow \sum_{j=1}^{J} b_{ji}E\left[\left\{U'(x) + E[U'(x)]\sigma_m^{-2}R_m(\bar{R}_m - r)]\right\}\delta_j\right] = 0.$$

$$\Rightarrow \sum_{j=1}^{J} b_{ji}c_j = 0.$$

where

$$c_j = E\left[\left\{U'(x) + E[U'(x)]\sigma_m^{-2}R_m(\bar{R}_m - r)]\right\}\delta_j\right].$$

Thus (18) will hold so long as we can show that $c_i = 0 \forall i$. To show this assume the opposite, that some combination of the betas sum to 0 for every asset:

$$b_J i = \frac{\sum_{j=1}^{J} b_{ji}c_j}{c_J} \; \forall i$$

Now using this equation, equation (3) in the paper can be written as

$$R_i = \bar{R}_i + \sum_{j=1}^{J-1} b_{ji}\delta_j - \delta_J \sum_{j=1}^{J} \frac{b_{ji}c_j}{c_J} + \epsilon_i.$$

Simplifying gives,

$$R_i = \bar{R}_i + \sum_{j=1}^{J-1} b_{ji}\left(\delta_j - \frac{c_j}{c_J}\delta_J\right) + \epsilon_i$$

$$R_i = \bar{R}_i + \sum_{j=1}^{J-1} b_{ji}\theta_j + \epsilon_i$$

where,

$$\theta_j = \left(\delta_j - \frac{c_j}{c_J}\delta_J\right)$$

Since the θ's are also a factor structure,[1] this violates the assumption that the factor structure was minimal.

[1]The θ's do not satisfy the condition that they all have zero covariance with each other. However, this is no problem since they can be rotated to produce this result, i.e., the variance-covariance matrix is non singular so the new set of factors are $V^{-1}\theta$.