An Extension to Statistical Discrimination in a Competitive Labor Market: The Case When Employees can Apply for Multiple Jobs Simultaneously.

Jonathan B. Berk

Haas School of Business
University of California, Berkeley

and

NBER

Email: berk@haas.berkeley.edu

December 15, 1998

In the original paper, our analysis abstracted away from the problem of employee search by simply assuming that an employee always applies for the job with the highest expected wage. Such a strategy is optimal in our (single-period) setting because agents are restricted to apply for only one job. Thus the opportunity cost of applying for the wrong job is high. In reality, such costs can be mitigated if the employee can apply for more than one job. Thus, one might conjecture that if the employees are allowed to apply for multiple jobs simultaneously, the self-selection bias will be less important. As the number of jobs an employee can simultaneously apply to gets very large, it might appear that the selection bias would disappear completely. In fact, this is not the case. In this section we demonstrate that the main result derived in this paper, that the self-selection bias is important in determining the nature of the resulting discrimination, remains true for the case in which employees can apply to more than one job simultaneously.

At first glance the claim that the main result in this paper is not sensitive to the number of jobs an employee can simultaneously apply for might appear surprising because it seemingly violates the following intuition. If there is no bound on the number of jobs employees can apply for, then all employees will apply for all jobs and no self selection will occur. Consequently, as the number of jobs employees can apply for gets large one might expect the equilibrium to approach this case. The reason it does not is that this line of reasoning ignores the optimal response of employers. If employers did not adjust their standards, then since the probability of getting a job increases in the number of jobs employees can apply for, employees would simply apply for better jobs. The fraction of lower quality applications would therefore rise so Equation 5 in the original paper will no longer be satisfied. To ensure that this equation is satisfied, employers must raise their standards. Consequently, the probability of getting a particular type of job is not necessarily increased when an employee can for multiple jobs simultaneously.

To model the effect of allowing employees to apply for multiple jobs simultaneously, we alter the model outlined in the original paper to allow employees to apply for M jobs simultaneously. Let $m_i^t(q)$ be the number of times an employee of quality q and type t applies for a job in the i^{th} sector. Since an employee of type t will still maximize her expected wage, she will solve the following problem:

$$\max_{m_1^t(q),\ldots,m_{I+1}^t(q)} E[w|q,t]$$

s.t.:

$$\begin{aligned} m_j^t(q) &\geq 0 \quad j=1,\ldots,I+1 \\ \sum_{i=1}^{I+1} m_i^t(q) &= M \end{aligned}$$

where

$$
\begin{aligned}
E[w|q,t] &= w_1 \left(1-(1-P_t(q,1))^{m_1^t(q)}\right) \\
&\quad + w_2 \left(1-(1-P_t(q,2))^{m_2^t(q)}\right)(1-P_t(q,1))^{m_1^t(q)} + \cdots \\
&\quad + w_{I+1} \prod_{j=1}^{I} (1-P_t(q,j))^{m_j^t(q)} \\
&= \sum_{i=1}^{I+1} w_i \left(1-(1-P_t(q,i))^{m_i^t(q)}\right) \prod_{j=1}^{i-1} (1-P_t(q,j))^{m_j^t(q)}, \qquad (1)
\end{aligned}
$$

and $P_t(q, I+1) \equiv 1$.

As before we assume that every job in the economy has at least some applicants. Providing a closed form solution for this problem is challenging because employees cannot apply to a fraction of job, so $m_i^t(q)$ is constrained to be an integer. For simplicity, we will proceed by ignoring this constraint and allowing $m_i^t(q)$ to be rational. The integer solution can be deduced from this solution by iterating through the set of integers immediately below and above the rational solution (simply picking the closest integer will not always provide the correct solution). Of course, as M gets large the distinction between the two solutions becomes less important. Under this assumption the following proposition provides the solution to this problem.

Proposition 1 *The solution to the above program is:*

$$
m_i^t(q) = \begin{cases}
0 & \textit{if } (i > \bar{\iota}) \textit{ or} \\
 & \textit{if } (i \le \underline{\iota}) \textit{ or} \\
 & \textit{if } \Big(i = \bar{\iota} \textit{ and } P_t(q, \bar{\iota}-1) \ge \frac{w_{\bar{\iota}}}{w_{\bar{\iota}-1}}\Big) \\
1 & \textit{if } \Big(i = \bar{\iota} \textit{ and } P_t(q, \bar{\iota}-1) < \frac{w_{\bar{\iota}}}{w_{\bar{\iota}-1}}\Big) \\
\max\left[M - \sum_{j=\underline{\iota}+2}^{\bar{\iota}} m_j^t(q)\ ,\ 0\right] & \textit{if } i = \underline{\iota}+1 \\
\max\left[\min\left(M - \sum_{j=i+1}^{\bar{\iota}} m_j^t(q)\, ,\ \frac{\ln\left(\frac{w_{i-1}-w_i}{\left(w_i - B(i+1)\right)\left(\frac{\ln(1-P_t(q,i))}{\ln(1-P_t(q,i-1))}-1\right)}\right)}{\ln\left(1-P_t(q,i)\right)}\right)\ ,\ 0\right] & \textit{o.w.}
\end{cases} \qquad (2)
$$

where

$$\begin{aligned}\bar{\imath} &= \min\{i|P_t(q,i)=1\}\\ \underline{\imath} &= \max\{0,\{i|P_t(q,i)=0\}\}\\ B(i) &= \sum_{k=i}^{\bar{\imath}} w_k\left(1-(1-P_t(q,k))^{m_k^t(q)}\right)\prod_{j=i}^{k-1}(1-P_t(q,j))^{m_j^t(q)},\end{aligned}$$

and

$$w_i - B(i+1) < w_{\bar{\imath}-1},\ \ i=\underline{\imath}+1,\ldots,\bar{\imath}-2. \quad (3)$$

Proof: First note that since no employee will apply for a job she has no chance of getting, $m_i^t(q)=0$, for $i\le\underline{\imath}$. Furthermore, of the jobs that an employee can get for sure, she will only apply to the best one so, $m_i^t(q)=0$, for $i>\bar{\imath}$. Thus, with loss of generality, these jobs can be eliminated from the problem.

We first solve the problem without job $\bar{\imath}$. Setting up the Lagrangian with the remaining jobs, taking partial derivatives with respect to $m_i^t(q)$ for each i provides the following set of equations:

$$\begin{aligned}-w_i(1-P_t(q,i))^{m_i^t(q)} &+ \sum_{k=i+1}^{\bar{\imath}-1} w_k\left(1-(1-P_t(q,k))^{m_k^t(q)}\right)\prod_{j=i}^{k-1}(1-P_t(q,j))^{m_j^t(q)}\\ &= \frac{\mu+\lambda_i}{\ln(1-P_t(q,i))\prod_{j=\underline{\imath}+1}^{i-1}(1-P_t(q,j))^{m_j^t(q)}} \qquad i=\underline{\imath}+1,\ldots,\bar{\imath}-1, \quad (4)\end{aligned}$$

where μ is the Lagrange multiplier for the constraint that $\sum_{i=\underline{\imath}+1}^{\bar{\imath}-1} m_i^t(q)=M$ and each λ_i is the Lagrange multiplier for the constraint that $m_i^t(q)\ge 0$. We will proceed in two steps. First we will describe a algorithm for finding the solution. Then we will show that this solution satisfies the above equations.

Consider first solving the problem without the constraints that $m_i^t(q)\ge 0$ for $i=\underline{\imath}+1,\ldots,\bar{\imath}-1$ (i.e., assume that $\lambda_i=0,\ \forall i$). Using the remaining constraint the above equations can be used to solve for $m_i^t(q)$ in terms of $m_{i+1}^t(q)\ldots m_{\bar{\imath}-1}^t(q)$:

$$m_i^t(q)=\begin{cases}\max\left[\dfrac{\ln\left(\dfrac{w_{i-1}-w_i}{(w_i-B(i+1))\left(\frac{\ln(1-P_t(q,i))}{\ln(1-P_t(q,i-1))}-1\right)}\right)}{\ln\left(1-P_t(q,i)\right)},0\right] & i=\underline{\imath}+2,\ldots,\bar{\imath}-1\\ M-\sum_{i=\underline{\imath}+2}^{\bar{\imath}-1} m_i^t(q) & i=\underline{\imath}+1\end{cases}. \quad (5)$$

Thus all the constraints that were ignored are satisfied except possibly for the last. If $m^t_{\underline{\iota}+1}(q) = M - \sum_{i=\underline{\iota}+2}^{\overline{\iota}-1} m^t_i(q) \geq 0$ then the solution to the unconstrained problem is also the solution to the constrained problem. If not, then the constraint that $m^t_{\underline{\iota}+1}(q) \geq 0$ is binding so set $m^t_{\underline{\iota}+1}(q) = 0$, and set $m^t_{\underline{\iota}+2}(q) = M - \sum_{i=\underline{\iota}+3}^{\overline{\iota}-1} m^t_i(q)$, leaving all the other $m^t_i(q)$ the same. Now if $m^t_{\underline{\iota}+2}(q) \geq 0$ then this is the solution. Otherwise, set $m^t_{\underline{\iota}+2}(q) = 0$ and repeat the process until an i is found such that $m^t_i(q) = M - \sum_{j=i+1}^{\overline{\iota}-1} m^t_j(q) \geq 0$. Note that this is the solution described in the statement of the proposition.

We next show is that this solution satisfies (4). To see that it does, let

$$\mu = -w_{\overline{\iota}-1} \ln(1 - P_t(q, \overline{\iota}-1)) \prod_{j=\underline{\iota}+1}^{\overline{\iota}-1} (1 - P_t(q, j))^{m^t_j(q)}$$

$$\lambda_i = \begin{cases} 0 & \text{if } m^t_i(q) > 0 \\ \mu \left(\frac{w_i - B(i+1)}{w_{\overline{\iota}-1}} - 1 \right) & \text{if } m^t_i(q) = 0 \end{cases}.$$

Note that since $\mu > 0$, by (3), $\lambda_i \leq 0$ for all i. Using the above definitions and (5) it is straightforward to verify, by substitution, that this solution satisfies (4).

The final step in the proof must show that the applicant applies for one job in Sector $\overline{\iota}$ if $P_t(\overline{\iota}-1, q) < \frac{w_{\overline{\iota}}}{w_{\overline{\iota}-1}}$, otherwise she does not apply in this sector. Before we can do this we need to take note of one fact. Let i^* be the best sector the applicant is applying in. By the properties of the optimum, her expected wage in the above solution is strictly greater than her expected wage if she applied for one less job in Sector i^* and one more job in Sector $\overline{\iota}-1$:

$$\begin{aligned} & w_{i^*} \left(1 - (1 - P_t(q, i^*))^{m^t_{i^*}(q)}\right) + B(i^*+1)(1 - P_t(q, i^*))^{m^t_{i^*}(q)} > \\ & \quad w_{i^*} \left(1 - (1 - P_t(q, i^*))^{m^t_{i^*}(q)-1}\right) + B(i^*+1)(1 - P_t(q, i^*))^{m^t_{i^*}(q)-1} \\ & \quad + w_{\overline{\iota}-1} \left(1 - (1 - P_t(q, \overline{\iota}-1))^{m^t_{\overline{\iota}-1}(q)+1}\right) (1 - P_t(q, i^*))^{-1} \prod_{j=i^*}^{\overline{\iota}-2} (1 - P_t(q, j))^{m^t_j(q)} \\ & \quad - w_{\overline{\iota}-1} \left(1 - (1 - P_t(q, \overline{\iota}-1))^{m^t_{\overline{\iota}-1}(q)}\right) (1 - P_t(q, i^*))^{-1} \prod_{j=i^*}^{\overline{\iota}-2} (1 - P_t(q, j))^{m^t_j(q)} \end{aligned}$$

where $B(\cdot)$ is calculated using the optimal solution. Simplifying this expression yields:

$$P_t(q, \overline{\iota}-1)\ w_{\overline{\iota}-1} \prod_{j=i^*+1}^{\overline{\iota}-1} (1 - P_t(q, j))^{m^t_j(q)} < (w_{i^*} - B(i^*+1))\ P_t(q, i^*). \qquad (6)$$

We now explicitly allow the applicant to apply for a job in Sector $\overline{\iota}$. Clearly since $P_t(\overline{\iota}, q) = 1$, she will apply for at most one job in this sector. She will do this if and only if her expected wages are higher by applying for one less job in Sector i^* and applying for this job, that is,

$$
\begin{aligned}
w_{i^*} &\left(1-(1-P_t(q,i^*))^{m^t_{i^*}(q)}\right) + B(i^*+1)(1-P_t(q,i^*))^{m^t_{i^*}(q)} < \\
&w_{i^*}\left(1-(1-P_t(q,i^*))^{m^t_{i^*}(q)-1}\right) + B(i^*+1)(1-P_t(q,i^*))^{m^t_{i^*}(q)-1} \\
&+w_{\bar{\iota}}(1-P_t(q,i^*))^{-1}\prod_{j=i^*}^{\bar{\iota}-1}(1-P_t(q,j))^{m^t_j(q)},
\end{aligned}
$$

where $B(\cdot)$ is calculated as above. Simplifying as before:

$$
(w_{i^*} - B(i^*+1))\ P_t(q,i^*) < w_{\bar{\iota}} \prod_{j=i^*+1}^{\bar{\iota}-1}(1-P_t(q,j))^{m^t_j(q)}, \tag{7}
$$

Equations (6) and (7) together imply that,

$$
P_t(q,\bar{\iota}-1)\ w_{\bar{\iota}-1} < w_{\bar{\iota}}
$$

which completes the proof. ■

Although the solution is complicated to write down formally, it is relatively easy to describe. The first condition derives from the fact that an employee obviously does not apply to any job she has zero probability of getting so she does not apply in any sector for which $i \leq \underline{\iota}$. She also will only consider applying to the best job in the set of jobs she can get for sure, thus she will not apply for any job in a sector strictly greater than the $\bar{\iota}^{th}$ sector. If she decides to apply in the $\bar{\iota}^{th}$ sector, since she gets jobs in this sector for sure, it only makes sense to apply once. To figure out how many jobs to apply for in the sectors were it makes sense to apply, she starts with the worst sector ($\bar{\iota}$) and computes the unconstrained optimum number of jobs to apply for. If this number is greater than M she applies for all M jobs in this sector. If it is less than M, she applies for the unconstrained optimal number of jobs in this sector and then computes what the unconstrained optimal number of jobs to apply for in the next best sector is. If the total number of jobs is larger than M then she applies for as many jobs as she can in this sector and does not apply in any other sector. If not, she applies for the unconstrained optimal number of jobs in this sector and computes the unconstrained optimal in the next best sector. She continues this process until the total number of jobs exceeds M or she runs out of sectors. An important implication of this solution is that if M is increased by one, the employee will always use the additional opportunity either to apply for a job in the best sector she is currently applying or she will apply in the next best sector. She will not change the number of applications in any other sector.

There are a number of characteristics of this solution that have intuitive appeal. Employees apply for multiple jobs in different sectors so the equilibrium displays heterogeneity of hiring standard across jobs that a given employee applies for. They apply for jobs in sectors in which their probability of being hired is high as well as for better jobs where their probability of being hired is much lower. What is particularly interesting is the dynamics of how they allocate the number of jobs to apply for in each sector. They begin with the sector in which they applied in the single job case. They continue to apply in this sector until some number of applications are submitted and then apply in the next best sector. Once the optimal number of jobs in this sector is reached they apply in the next best sector until they have submitted M applications. Thus their optimal strategy is to first ensure that their probability of getting a job in a particular sector is very high, before gambling and applying for a better job in a lower probability sector. Finally, the solution also has the intuitively appealing property that *if the response of employers is ignored*, then increasing the number of job applications that are simultaneously allowed increases the number of better jobs the employee applies for.

Of course, the optimal response of employers cannot be ignored. Given this set of optimal $\{m_i^t(q)\}$, employers solve the same problem as before. Since employers hire any applicant whose expected quality exceeds b_i, the critical signal, s_{ti}^*, solves:

$$E[q|s_{ti}^*, t] = b_i, \tag{8}$$

where

$$E[q|s,t] = \left(\frac{1}{\int_{-\infty}^{\infty} g^t(s-u) \frac{m_i^t(u)}{M} \; dF^t(u)} \right) \int_{-\infty}^{\infty} u \; \frac{m_i^t(u)}{M} \; g^t(s-u) \; dF^t(u). \tag{9}$$

Substituting (9) into (8) and simplifying provides,

$$\int_{-\infty}^{\infty} (u - b_i) \; m_i^t(u, s_{ti-1}^*, \ldots, s_{tI}^*) \; g^t(s_{ti}^* - u) \; dF^t(u) = 0, \tag{10}$$

where, for clarity, we have made the dependency of m_i^t on $s_{ti-1}^*, \ldots, s_{tI}^*$ explicit. The equilibrium is obtained by solving (10) for the critical signal for each type in each sector, where $m_i^t(u, s_{ti-1}^*, \ldots, s_{tI}^*)$ is given by Proposition 1. Since we have seen that if an additional opportunity to apply for a job is provided, employees will use it to apply in the highest feasible sector, as M increases the critical signal increases as well — employers react to the increased ability of employees to apply for multiple jobs by increasing the required qualifications for the job.

We next illustrate some of the quantitative implications of allowing employees to apply to multiple jobs in the context of an example. To properly assess the robustness of the qualitative results obtained in that example, we will consider the case in which the number of jobs an employee can simultaneously apply is extremely large — in this case $M = 100$.

Assume that underlying quality is distributed N[9,2] and N[11,2] for types l and h respectively. We will limit attention to the case in which the signal noise is identical across the two types. Table 1 summarizes the resulting equilibrium. In every case the critical signal is now much higher than in the single job case, reflecting the fact that employers optimally increase their standards. Like the single job case, the difference in the critical signal between the types in sectors in which both types apply is still small (it does not exceed 0.3) in comparison to the overall quality differences. However, this difference is larger than when employees could apply for only one job. Furthermore, no lower quality agent applies for any job in Sector 10. The reason is that the next worst job is the reservation job. Since every applicant is guaranteed this job, it does not make sense to apply more than once for this job. Hence, in Sector 10 very few applications are discouraged by raising the critical signal, since applying instead to the reservation job does not affect the probability of getting the reservation job if the applicant is already applying for that job. On the other hand raising the critical signal also decreases the number of higher quality applications because some of these applicants responded by applying for the next better job. In this case the latter effect dominates so no critical signal exists that solves (10). Employers in Sector 10 optimally respond by refusing to hire any applicant of type l. Another consequence of this is that in the next highest sector the discrimination favors employees of type l — the critical signal is actually lower for type l applicants. Hence, unlike the single job case, systematic legal discrimination does not exist in this example.

Table 1: **The Equilibrium in the Economy with Normal Agents with Identical Signal Noise and the Ability to Apply for 100 Jobs Simultaneously**
Quality (marginal product) is distributed Normal$[9,2]$ for agents of type l, Normal$[11,2]$ for agents of type h and $\Delta_h = \Delta_l = 1.95$. The offered wage or minimal expected marginal product (w) and the equilibrium critical signal (s^*) for each job available is listed below. The final column contains these values for the reservation job in the economy.

	Job										
	1	2	3	4	5	6	7	8	9	10	res
w	20.0	18.0	16.0	14.0	12.0	10.0	8.0	6.0	4.0	2.0	1.0
s_h^*	21.52	19.44	17.32	15.15	12.93	10.69	8.511	6.200	4.251	1.683	$-\infty$
s_l^*	21.59	19.53	17.44	15.32	13.15	10.93	8.691	6.502	4.180	—	$-\infty$

The resulting economic discrimination is plotted in Figure 1, the percentage difference in expected wages (given by (1)) between applicants of the same quality but of differing types. Economic discrimination, like the legal discrimination, is not systematic. Some low quality type l agents have higher expected wages than their type h counterparts. Furthermore, the economic discrimination, *when it occurs*, is substantial. Even if Sector 10 is ignored, the resulting difference in wages can be as high as 50%, which is substantially greater than in the single job case. However, in another important dimension — the fraction of workers of the same quality whose expected wage differs — the amount of economic discrimination is substantially *lower* than the single job case. Thus, by increasing the number of jobs that can be simultaneously applied for, the number of workers who suffer from economic discrimination is substantially reduced.

Intuitively, the effect on economic discrimination of allowing employees to apply for multiple jobs is straightforward to understand. Because the total number of jobs an employee can apply for is so high, a very small probability of landing a job in a particular sector on one application can translate into almost certainty when the employee can apply up to 100 times.[1] For any reasonable probability of landing the job, given 100 chances the employee will get the job with certainty. When the probability of landing the job on any one application is large, small changes in this probability make very little difference in the employees probability of landing the job on 100 tries. Given the fact that the legal discrimination is in most cases small in this economy, this implies that most employees of the same quality but different types apply for most of their jobs in the same sector — the best one in which they both have a near certain probability of landing the job. In these cases, the difference in expected wages will be minimal and so they account for the fact that the vast majority of applicants suffer no economic discrimination.

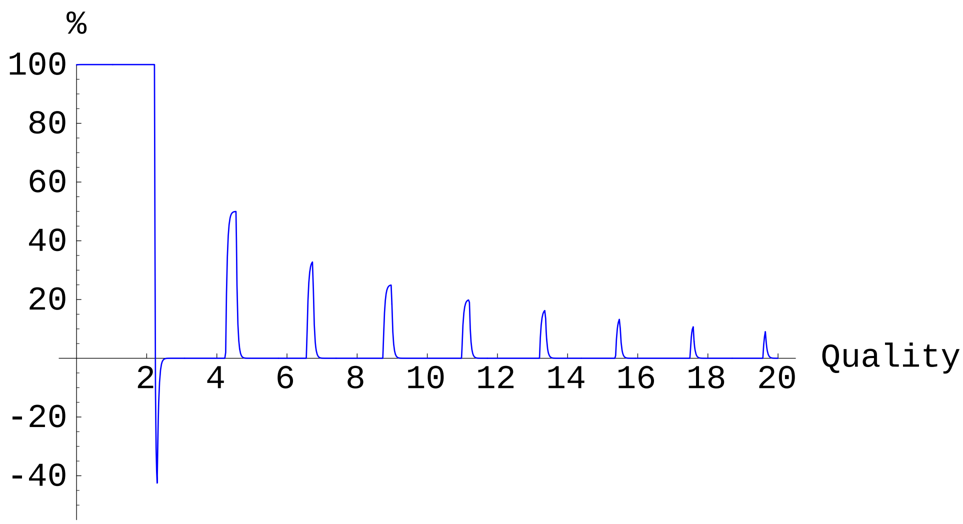
The problem comes in when the small difference in the probability of getting a job forces employees of the same quality but of different type to apply for a substantially different amount of jobs in each sector. This will occur, for instance, when one type has a zero probability of getting a job in the next best sector while the other type has a small, but non-zero, probability of getting a job in this sector. As before, in both cases both types will most likely be hired in the main sector in which they are applying, however, now the wage differences between the sectors will loom large. Thus the 100% difference in expected wages in Figure 1 reflects the fact that in this range type l applicants are forced to take the reservation wage, 1, with certainty, while the type h agents get a tenth sector job with wage 2 almost certainty. Similarly, the 50% difference for quality levels between 4 and 5 reflects the range where type l agents get a ninth sector job with wage 4 almost certainly, while type h agents get a eighth sector job with wage 6 almost certainly.

Although a complete investigation of the robustness of our results to the number of

[1] For example, to have a 99% probability of getting the job on 100 applications the employee needs only a $4\frac{1}{2}$% probability of getting the job on each application.

Figure 1: **Percentage Difference in Expected Wages in the Economy with the Ability to Apply for 100 Jobs Simultaneously**

The expected wage is defined by (1). The percentage difference between the expected wage of two agents of the same quality is defined to be the expected wage of the agent of type h minus the expected wage of the agent of type l divided by the expected wage of the agent of type l (i.e., $\frac{E[w|h,q]-E[w|l,q]}{E[w|l,q]}$) expressed in percentage terms.



jobs employees can apply for is beyond the scope of this paper, the above example does demonstrate that many of the qualitative results derived in the original paper could be derived when the employees are allowed to apply for multiple jobs. Most importantly, the self-selection bias materially affects the analysis even when the number of jobs an employee can apply for is unrealistically large. In the context of one example, we show that even with substantial differences in type distributions, systematic legal and economic discrimination does not exist. Furthermore, like the single job case, the amount of legal discrimination is still surprising small. Perhaps the most important difference between the single job and multiple job example is the nature of the resulting economic discrimination. In the context of this example, more employees suffer from economic discrimination when they can only apply for one job. Yet the magnitude of this discrimination is much smaller than when they can apply for multiple jobs.