An Extension to Statistical Discrimination in a Competitive Labor Market: The Case When the Signal Noise is Normally Distributed.

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December 15, 1998

The assumption that the signal noise is uniformally distributed is motivated by the fact that is allows the derivation of the relatively simple equilibrium conditions in used in the special cases studied in the original paper. In light of this, it is natural to be concerned about the role the uniform distribution plays in driving these results. In particular, the fact that this distribution has bounded support seems to be important. In this section we demonstrate, by example, the qualitative effect of relaxing this assumption.

Table 1: **The Equilibrium in the Economy with Normal Agents with Identical Normally Distributed Signal Noise**

Quality (marginal product) is distributed Normal$[9, 2]$ for agents of type l, Normal$[11, 2]$ for agents of type h and in both cases δ is distributed Normal$[0, 1]$. The offered wage or minimal expected marginal product (w), the equilibrium critical signal (s^*) and upper bound ($\bar{q}$) for each job available is listed below. The final column contains these values for the reservation job in the economy.

	Job			
	1	2	3	res
w	12.0	10.0	8.0	6.0
s_h^*	10.56	8.580	6.562	$-\infty$
s_l^*	10.64	8.677	6.704	$-\infty$
$\bar{q}^h$	∞	11.53	9.415	7.236
$\bar{q}^l$	∞	11.60	9.511	7.378

Consider, first, an economy in which underlying quality is distributed N[9,2] and N[11,2] for types l and h respectively. Assume that the signal noise for both types is distributed identically Normal[0,1]. Since solving for the resulting equilibrium is a computationally intensive process, we will restrict attention to an economy with four jobs, the reservation job, with wage 6, and three other jobs with wages 8, 10 and 12 respectively. As before, we will assume that $b_i = w_i$ for all i.

The resulting equilibrium is displayed in Table 1. At least in the context of this example, replacing the uniform signal noise distribution with a normal distribution makes little difference to the qualitative conclusions in the paper. The amount of legal discrimination is still small (never exceeding 0.15). The resulting economic discrimination is plotted in Figure 1. The difference in expected wages is also small — in no case does the expected difference in wages exceed 6%. In most cases this difference is less than 2%.

To investigate the impact of having normally distributed signal noise on an economy with identically distributed quality but differing signal noise, assume, in the above economy that the quality of both types is distributed N[11,2]. Let type h employees have the same signal noise as before (i.e., $G^h = \mathrm{N}[0, 1]$) and assume that type l employees have lower signal noise: $G^l = \mathrm{N}[0, \frac{1}{2}]$. The resulting equilibrium is displayed in Table 2. The implications of reduced signal noise are almost the same as in the uniform case — the magnitude of the

Figure 1: **Percentage Difference in Expected Wages in the Economy with Normal Agents with Identical Normally Distributed Signal Noise**

The percentage difference between the expected wage of two agents of the same quality is defined to be the expected wage of the agent of type h minus the expected wage of the agent of type l divided by the expected wage of the agent of type l (i.e., $\frac{E[w|h,q]-E[w|l,q]}{E[w|l,q]}$) expressed in percentage terms.

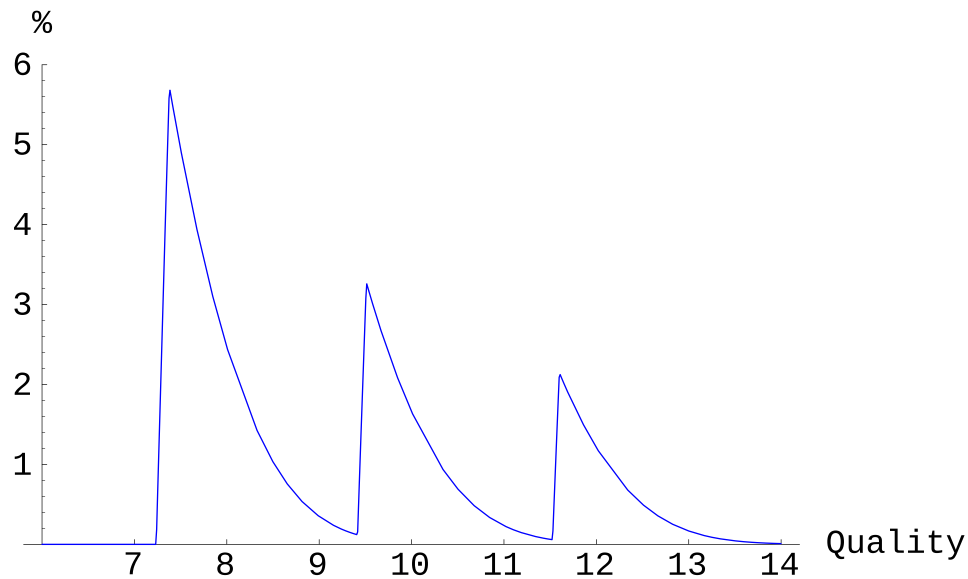
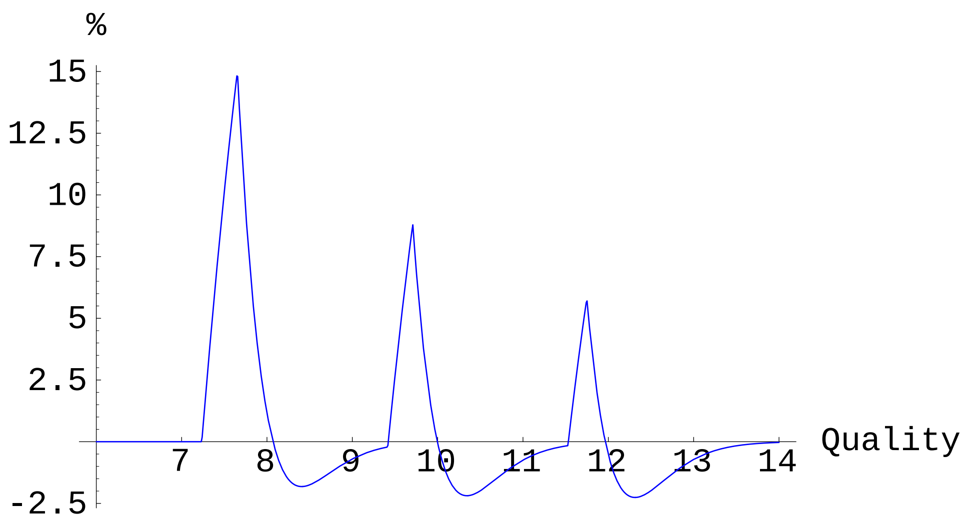


Table 2: **The Equilibrium in the Economy with Identical Normal Agents with Differing Normally Distributed Signal Noise**

Quality (marginal product) is distributed Normal$[11, 2]$ for agents of both types. Agent's of type l have smaller signal noise so $G^l = \text{Normal}[0, \frac{1}{2}]$ and $G^h = \text{Normal}[0, 1]$. The offered wage or minimal expected marginal product (w), the equilibrium critical signal (s^*) and upper bound ($\bar{q}$) for each job available is listed below. The final column contains these values for the reservation job in the economy.

	Job			
	1	2	3	res
w	12.0	10.0	8.0	6.0
s_h^*	10.56	8.580	6.562	$-\infty$
s_l^*	11.26	9.288	7.316	$-\infty$
$\bar{q}^h$	∞	11.53	9.415	7.236
$\bar{q}^l$	∞	11.05	9.001	6.899

Figure 2: **Percentage Difference in Expected Wages in the Economy with Identical Normal Agents with Differing Normally Distributed Signal Noise**
The percentage difference between the expected wage of two agents of the same quality is defined to be the expected wage of the agent of type h minus the expected wage of the agent of type l divided by the expected wage of the agent of type l (i.e., $\frac{E[w|h,q]-E[w|l,q]}{E[w|l,q]}$) expressed in percentage terms.



legal discrimination is increased and it is systematic. The group with the lower signal noise is always discriminated against. Figure 2, the plot of the percentage differences in expected wages shows that this also holds true in the case of economic discrimination. As in the uniform case, it is larger than the case with identical signal noise but not systematic.

Finally, consider the case in which group h has both higher quality and lower single noise variance, that is let,

$$F^h = N[11, 2] \qquad G^h = N[0, \frac{1}{2}]$$
$$F^l = N[9, 2] \qquad G^l = N[0, 1]$$

By comparing the third row of Table 1 to the third row of Table 2 it is clear that group h agents will be systematically legally discriminated against. Like the examples in the original paper, the pathological case in which the group with the better quality distribution and lower signal noise are systematically legally discriminated against exists in this case as well. Furthermore, although we do not explicitly derive it, it is easy to show that despite the systematic legal discrimination, systematic economic discrimination does not result.