Finance 590

Homework 2

This homework is due Tuesday, October 15.

Question 1

Reformulate the general equilibrium problem presented in class assuming that the endowments are in the asset span (i.e., explicitly incorporate this restriction into the general equilibrium formulation).

Question 2

Why, in the same general equilibrium problem, did we need to assume that $X_{0s} \geq 0$ for all s (and is strict for at least one s) to normalize $p_0 = 1$? Using this normalization, show how one market clearing constraint can be derived from the agents' budget constraits.

Question 3

Define,

$$\frac{\partial C_s^k}{\partial C_t^k} \equiv \text{ marginal rate of substitution.}$$

That is, *on the margin*, how much does k trade off consumption in state s for consumption in state t. Show that in a complete market the marginal rate of substitution between any two states is the same for all agents.

Question 4

Consider the following three agent, one period economy. Consumption takes place *at the beginning* and at the end of the period. The economy has only one (perishable) consumption good. Since there is no way to store the good, beginning of period consumption good can not be transferred into end of the period consumption good. Thus, in aggregate, all beginning of period consumption good must be consumed at the beginning of the period and all end of period consumption good must be consumed at the end of the period. At the beginning of the period agents know that the economy will be in one of four **equally likely** states at the end of the period. An asset market exists and the following three assets are traded on the market:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

Agents are each endowed 0.8 consumption good at the beginning of the period. Their end of period endowments are as follows:

$$e^1 = \begin{bmatrix} 1 \\ 2 \\ \frac{1}{2} \\ 1 \end{bmatrix} \quad e^2 = \begin{bmatrix} \frac{1}{2} \\ 2 \\ \frac{3}{2} \\ 1 \end{bmatrix} \quad e^3 = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

Each agent has the following utility function:

$$U(x) = u(x) + E[u(\tilde{y})],$$

where

$$u(z) = z - \frac{z^2}{100},$$

and x and $\tilde{y}$ are consumption at the beginning and at the end of the period respectively. Thus at the beginning of the period agents trade assets and beginning of period consumption in the asset market. They then consume, the state is revealed and they consume again.

1. Assume that the price of the consumption good is always 1. What are the equilibrium asset prices at the beginning of the period?

2. What do the agents consume, in equilibrium.

3. What are their net trades?

4. Explain (but do no calculations) what the implications would be of introducing the following security:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Hint: I would use Mathematica (or some other symbolic mathematics program) to solve this problem. Mathematica is available for your use under a university site license.