

Fin. 590

Homework 3

Question 1

In an incomplete market the marginal rate of substitution of consumption among states do not match. Can you find something equivalent that does?

Question 2

Show that the equality of marginal rates of substitution for consumption across individuals is necessary and sufficient for Pareto optimality.

Question 3

Let the set of all possible state price vectors in a incomplete market equilibrium be denoted $Q = \{q^{ic}\}$. Now consider adding (zero net supply) assets that complete the market and denote an equilibrium state price vector q^c . Show that if the incomplete market equilibrium is Pareto optimal then there must exist a complete market equilibrium with $q^c \in Q$ and if $q^c \in Q$ then the incomplete market equilibrium is Pareto optimal.

Question 4

Investors must choose one of two lotteries: \tilde{x} which pays off a with probability p and b with probability $1 - p$ and \tilde{y} which pays off c with probability p and d with probability $1 - p$. Show by appropriately choosing the parameters that a mean-variance investor exists that strictly prefers \tilde{y} over \tilde{x} even though $\min\{a, b\} > \max\{c, d\}$.

Question 5

How is the paradox in Question 4 handled in the derivations of the CAPM?

Question 6

1. The global minimum variance portfolio is clearly mean-variance efficient. Does it price assets? (i.e., does it satisfy the beta relation?)

Question 8

1. Explain how you would go about identifying a portfolio that has zero covariance with the market portfolio.
2. Can you identify such a portfolio that is also on the mean-variance frontier (of risky assets)?

Question 9

Prove that the combination of any two mean-variance efficient portfolios is mean-variance efficient.