

# Fin 590

## Homework 4

Due: Monday, November 3

### Question 1

In a one period model (i.e., agents trade assets at the beginning of the period on a spot market and the assets pay off in units of the consumption good at the end of the period), if  $p_i$  and  $\tilde{C}_i$  are the price and end of period cash flow of each asset  $i$ , a *pricing operator* is any r.v.  $\tilde{x}$  such that  $p_i = E[\tilde{x}\tilde{C}_i] \quad \forall i$ . A *positive pricing operator* is a pricing operator whose realizations are strictly positive, i.e.,  $\tilde{x} > 0$

1. The (one period) CAPM pricing relationship is usually expressed in terms of expected returns. What is the corresponding relationship of the price of any asset as a function of the covariance between the market *return* and the end of period *cashflow* of the asset, i.e.,  $\text{cov}(\tilde{R}_m, \tilde{C}_i)$ ?
2. Use your result to derive a pricing operator when the CAPM holds.
3. Show that in a complete market, if the market return is normally distributed and no investor is satiated, then the CAPM will *not* hold in equilibrium.

### Question 2

Consider an economy in which the CAPM holds but no riskless asset exists.

1. Draw the efficient frontier in mean - standard deviation space.
2. Mark the market portfolio.
3. Can you mark mark the set of zero beta portfolios? If you can, mark it and justify why you picked that location. If you can not, explain why.

4. Now consider adding a riskless asset in zero net supply to the economy. Can you mark the riskless rate on the above graph? If you can, mark it and justify why you picked that location. If you can not, explain why.

### Question 3

Assume that the CAPM holds exactly in an economy with a finite number of assets. Now consider an asset that only consists of idiosyncratic risk (i.e., has a beta of zero). Clearly it has an expected return equal to the riskless rate. Now the standard explanation of this result is that agents are not rewarded for holding idiosyncratic risk because it can be costlessly diversified away. However, in an economy with a finite number of assets, this argument is flawed because the number of assets in the economy has nothing to do with whether the CAPM holds. Thus, if the CAPM holds in an economy with very few assets, diversifying across these assets will not by itself remove all the idiosyncratic risk, yet the risk will not be priced in equilibrium. Why then does an asset that only consists of idiosyncratic risk earn the riskfree rate on average?

Hint: Is this asset in the market portfolio?

### Question 4

Show that if a riskless asset exists, the following relation always holds in the economy (regardless of the distribution of asset returns or preferences):

$$E[\tilde{r}_i] = r + \beta_i(E[\tilde{r}_m] - r)$$

where  $r_i$  is the return on the  $i^{\text{th}}$  stock;  $r_m$  is the return on a mean-variance efficient portfolio;  $r$  is the riskfree rate; and  $\beta_i = \frac{\text{cov}(\tilde{r}_i, \tilde{r}_m)}{\text{var}(\tilde{r}_m)}$ .