Fin 590

Homework 5

Due: Monday, November 17.

Question 1

Roll wrote his critique well after many researchers had purportedly tested the CAPM. In light of the critique though, what were they testing?

Question 2

Go back to the general equilibrium problem you solved in mathematica (Q4 on HWK 4). Show that the CAPM holds in this economy with all four assets (i.e., the market is mean-variance efficient and two fund separation obtains when the asset in part 4 is added). Does the CAPM hold with only 3 assets?

Question 3

Show that in the class of utility functions that admit 2fs (regardless of the asset structure and endowments), only quadratic utility provides the mean-variance efficiency of the market portfolio. (In other words, show that if only restrictions on preferences are allowed, a necessary and sufficient condition for the CAPM is quadratic utility)

Question 4

If k-fund separation obtains, is each fund necessarily mean-variance efficient? What can you say about all mean-variance efficient portfolios?

Question 5

Assume that asset returns satisfy

$$\tilde{R}_i = \bar{R}_i + \sum_{j=1}^{J} b_{ji}\tilde{\delta}_j + \tilde{\epsilon}_i \tag{1}$$

with

$$E[\tilde{\epsilon}_i|\delta_1, \cdots, \delta_J] = 0 \quad \forall i \in \mathcal{I}$$

$$E[\tilde{\delta}_j] = 0 \quad j = 1, \ldots, J$$

$$E[\tilde{\delta}_i\tilde{\delta}_j] = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad i, j = 1, \ldots, J$$

in equilibrium. δ_j is referred to as the j^{th} *factor* and ϵ_i is the *residual* of the i^{th} security. Assume all agents have utility functions $U_\kappa(\cdot)$, always prefer more to less and are risk averse.

1. Show that, in equilibrium, if at least one individual in the economy has no idiosyncratic (i.e., $\tilde{\epsilon}$) risk in his consumption portfolio, then a portfolio that consists of only idiosyncratic risk has a price of zero.

   Hint: Find a positive pricing operator to price the idiosyncratic risk.

2. Assume that J portfolios can be formed with only factor risk and that the rank of the $J \times J$ matrix of factor b's is J. Show that, in equilibrium, either everybody holds some idiosyncratic risk in their portfolios or nobody holds idiosyncratic risk in their portfolios. (A result derive originally by Chen and Ingersoll)

   Hint: Use part 1.

3. We have seen in class that we can assume that the market portfolio has no residual risk, i.e., $\tilde{R}_m = E_m + \sum_{j=1}^{J} b_{jm}\tilde{\delta}_j$. If 2fs obtains in this economy show that

   $$E\left[\left\{U'_\kappa(\tilde{x}_\kappa) + E[U'_\kappa(\tilde{x}_\kappa)]\tilde{R}_m\frac{(E_m - r)}{\sigma_m^2}\right\}(\tilde{R}_i - E_i)\right] = 0 \quad \forall i$$

   if, and only if,

   $$E\left[\left\{U'_\kappa(\tilde{x}_\kappa) + E[U'_\kappa(\tilde{x}_\kappa)]\tilde{R}_m\frac{(E_m - r)}{\sigma_m^2}\right\}\tilde{\delta}_j\right] = 0 \quad j = 1, \ldots, J$$

   .