

Rational Valuation of an Internet Company:
The Case of Amazon.com¹

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¹This case was prepared by Professor Rossen Valkanov, The Anderson School, UCLA. We thank Eduardo Schwartz for allowing us to use his materials and programs. All remaining inconsistencies are our own.

Rational Pricing of Internet Companies—Schwartz and Moon (2000)

- In what follows, we will give a simplified exposition of the Schwartz and Moon (2000) approach. Their original model is written in continuous time and its implementation is done by simulations¹. However, the basic premises of their model can be understood in a simplified, discretized model.
- The main premise of the model is that the present value of internet companies must take into account the extreme uncertainty surrounding all variables that determine future cash flows. Moreover, the growth rates and the volatility of those variables, although currently very high, will undoubtedly decrease with time until they reach levels comparable to those of “mature” companies. For example, analysts project that Amazon’s revenues will grow by 55% over the next year. For a comparison, the entire US economy (GDP) grows at approximately 2% annually, and this growth has been more or less steady since the early 1910’s. No one really expects Amazon to keep growing 25 times faster than the entire US economy. If it did, in the not so distant future, the value of Amazon products and services will be greater than the products and services in the rest of the economy. In fact, it is more realistic to expect that, if Amazon is still around in the next 10-15 years, its growth will be in the neighborhood of 4-5% annually.
- When we write a model, there is always a trade-off between obfuscating the main points by adding too many details and oversimplifying the equations and rendering the model useless for practical applications. SM try to keep the modeling complexity to a minimum, while still giving enough richness to their story.

Model

SM assume that there are three basic sources of uncertainty in the valuation of Amazon. First, revenues are uncertain. Second, the growth of revenues is uncertain. Third, variable costs are uncertain. But we can’t just say that those three quantities are uncertain, without specifying how they change over time. In the following sections, we specify how those three variables evolve over time.

Revenues

SM specify the following processes for revenues (R_t) :

$$\frac{\Delta R_t}{R_t} = \mu_t + \sigma_t \varepsilon_{1,t} \quad (1)$$

where $\varepsilon_{1,t}$ has a standard normal distribution ($N(0,1)$) and is independent over time. As in class, we use the notation $\Delta R_t = R_{t+1} - R_t$. Recall that, if we write $\frac{\Delta R_t}{R_t} = \mu$, it means $\frac{R_{t+1} - R_t}{R_t} = \mu$ and μ is the growth rate of the variable R_t . To see why, write the last expression as: $R_{t+1} = (1 + \mu) R_t$.

¹Interested students can go to the UCLA Finance homepage and download the working paper, #26-00.

²A more accurate discretization specifies $\frac{\Delta R_t}{R_t} = \ln\left(\frac{R_{t+1}}{R_t}\right) = \mu$, so that $R_{t+1} = R_t e^\mu$.

In this example, the growth rate μ is deterministic and constant over time. However, this is not a realistic assumption for the revenues of internet companies. It is safe to say that the growth of those companies, although uncertain, will decrease in volatility over time, as will the revenues themselves.

First, we will assume that the volatility of revenues is decreasing deterministically over time, or

$$\sigma_{t+1} = (1 - \phi)\bar{\sigma} + \phi\sigma_t, \quad 0 < \phi < 1 \quad (2)$$

In the above expression, σ_{t+1} will converge to a steady state value of $\bar{\sigma}$ as time t progresses, or $t \rightarrow \infty$. The rate of convergence will depend on the parameter ϕ ($0 < \phi < 1$). Therefore, the above equation captures the fact that *volatility of revenues* will decrease with time until they reach a value of $\bar{\sigma}$.

Growth of Revenues

SM also assume that the growth rate of revenues, μ_t is uncertain and follows the process:

$$\mu_t = (1 - \phi)\bar{\mu} + \phi\mu_{t-1} + \eta_t\varepsilon_{2,t} \quad (3)$$

where $\varepsilon_{2,t}$ has a standard normal distribution ($N(0,1)$) and is independent over time. The *volatility of the growth rate of revenues* is also a deterministic function of time, and dwindles to zero, as $t \rightarrow \infty$, or:

$$\eta_t = \phi\eta_{t-1} \quad (4)$$

In other words, as time passes, the growth rate will be less and less variable until it reaches levels of stability observed in other “mature” companies. Therefore, over time, we should observe *the growth rate of revenues* μ_t to converge to $\bar{\mu}$.

Note that, from the above discussions, we have established that

$$\sigma_t \rightarrow \bar{\sigma}, \text{ as } t \rightarrow \infty \quad (5)$$

$$\mu_t \rightarrow \bar{\mu}, \text{ as } t \rightarrow \infty \quad (6)$$

Therefore, returning back to equation (1), we can say that:

$$\frac{\Delta R_t}{R_t} = \bar{\mu} + \bar{\sigma}\varepsilon_{1,t}, \text{ as } t \rightarrow \infty \quad (7)$$

or that in the long run, the revenues of Amazon will grow at a steady rate of $\bar{\mu}$ and will have a steady volatility $\bar{\sigma}$.

Variable Cost

Lastly, we assume that production cost changes over time. In general, production cost can be decomposed into variable cost and fixed cost. Moreover, variable cost is usually a fraction of revenues. Therefore, we may be tempted to write an equation for cost as

$$Cost_t = \gamma R_t + F \quad (8)$$

where γ is a fraction between 0 and 1 and F represent all fixed cost. However, such an equation makes an implicit assumption that you are asked to discuss in your Project. Instead, we model cost as:

$$Cost_t = \gamma_t R_t + F \quad (9)$$

where

$$\gamma_t = (1 - \phi) \bar{\gamma} + \phi \gamma_{t-1} + \varphi_t \varepsilon_{3,t} \quad (10)$$

and $\varepsilon_{3,t}$ has a standard normal distribution ($N(0,1)$) and is independent over time. Note that the process γ_t has the same “flavor” as *the growth rate of revenues*, μ_t , described above. Indeed, let the volatility φ_t of γ_t be $\varphi_t = (1 - \phi) \bar{\varphi} + \phi \varphi_{t-1}$. From the above discussion, you should know that, as $t \rightarrow \infty$, we have $\varphi_t \rightarrow \bar{\varphi}$.

Rates of Convergence

So far, we have defined the three random processes, R_t, μ_t, γ_t . Note that they all depend on a very crucial parameter, ϕ , that determines how fast those processes converge to their long-horizon values. SM propose a way of calibrating the parameter ϕ by asking the question: “How long will it take for the effect ϕ to decrease by half?”. This is called the “half-life” of a process. We find the half-life of a process by letting:

$$\phi^k = 1/2 \quad (11)$$

Taking natural log, we can say that $k \ln(\phi) = \ln(1/2)$, or

$$k = \frac{\ln(1/2)}{\ln(\phi)} \approx \frac{-0.69}{\ln(\phi)} \quad (12)$$

If we specify ϕ , we have in fact specified the behavior of all processes. For example, if $\phi = 0.5$, then $k = \frac{\ln(0.5)}{\ln(0.5)} = 1$, or it takes one year for all of the above processes to reach half of their long-term equilibrium. If $\phi = 0.9$, then $k = \frac{\ln(0.5)}{\ln(0.9)} = 6.58$, or it will take approximately 6.58 years for all processes to reach half of their long-term equilibrium. In other words, by specifying k —the half-life of the processes—we are incorporating into the model how fast R_t, μ_t, γ_t , and all the variances converge to their long-run values.

Rest of the Model

Thus far, we have defined the random variables of our model. Now, we will put the rest of the model together. Assume that the corporate tax rate is τ_c and that Amazon’s investment (hardware, software, building, etc.) depreciates at rate Dep . Then, the after tax net income of Amazon is:

$$Y_t = (R_t - Cost_t - Dep)(1 - \tau_c) \quad (13)$$

You probably remember from your accounting class that corporate taxes are paid only if there is no loss-carry-forward, i.e. if there are no losses in previous periods that offset current gains. SM

allow for such a possibility in their model, but we will abstract from it, in the interest of simplicity and brevity. However, the distributed program incorporates those details.

Denote by $Capx_t$ the capital expenditures that Amazon has in period t . Therefore, we can define the amount of cash available to Amazon in any given period by X_t , where

$$X_t = (1 + r) X_{t-1} + Y_{t-1} + Dep_t - Capx_t \quad (14)$$

SM include the untaxed interest earned on the cash available in the dynamics to make the valuation results insensitive to when the cash flows are distributed to the security-holders³.

SM assume that Amazon will go bankrupt when its available cash reaches a predetermined negative number, say X^* . The threshold X^* is allowed to be negative to accommodate the possibility of future financing.

The ultimate objective of this model is to determine the value of Amazon today. SM show that the payoff from Amazon from today to T periods ahead in the future, under the *risk-neutral valuation*⁴, is $X_T + M \times (R_T - Cost_T)$, where M is a multiple. Therefore, the value of the firm today until horizon T is:

$$V_0 = E \{ (X_T + M \times (R_T - Cost_T)) e^{-rT} \} \quad (15)$$

Note that the expectation is taken with respect to the risk-neutral probabilities⁵. Here the mathematics are a bit more complicated, but the idea is exactly the same.

We have found the rational market value of Amazon! In order to determine the price of a share of Amazon, we need to know the capital structure of the company in detail, i.e. how many shares are outstanding, how many shares are likely to be issued to stock option holders, and convertible bondholders. We also need to know how much of the cash flows will be available to the shareholders after coupon (interest) and principal payment to the bondholders. SM make some minor assumptions in order to tackle those complications, but none of them are essential enough to require our attention.

After writing a few more formulas, we can finally express the stock price of Amazon, S , as a function of the initial variables, $R_t, \mu_t, \gamma_t, X_t$, and t , or

$$S = Function (R_t, \mu_t, \gamma_t, X_t, t) \quad (16)$$

Characterizing the behavior of this function involves Ito's Lemma and is beyond the scope of this class. Moreover, the above problem is too complicated to be solvable by hand. SM use simulations in order to find a solution of the entire value of Amazon and its price per share.

Relating β to Market price of Risk, λ

In this model, SM are able to relate the market price of revenues risk to the beta of Amazon, β , where

$$\beta = \frac{cov(R_A, R_M)}{var(R_M)} \quad (17)$$

³In the risk-neutral valuation, r is the risk-free interest rate.

⁴Note that SM have already found the risk neutral probabilities in order to do those calculations.

⁵Hence the appropriate discount rate r is the risk-free rate.

R_A is the return from Amazon and R_M is the stock market return. This is exactly the definition of β from our CAPM lectures. Using a somewhat complicated formula, SM show that, given β and a few other parameters, we can find the market price of revenues risk. In summary, once we estimate β by running our usual regression of Amazon returns on market return, we can find the market price of revenues risk, called λ . The other way also works. If we specify a market price of risk λ , one can solve for the implied β of the model.

Let's Recapitulate

If you are overwhelmed from reading the above section, do not worry. This model is not easy. But it is not difficult, either. Here are the main components, without formulas.

- We specified three sources of uncertainty, or variation, in the model: Revenues(R_t), growth of revenues (μ_t) and variable cost (γ_t).
- The variances of all those processes decrease either to zero or to a fixed number, over time.
- We specified how fast all of the above processes converge toward their long-run states using the notion of a “half-life”, denoted by k .
- We added a few other necessary things, such as taxes, depreciation, etc, in order to find the cash flows of Amazon
- If cash flows become too negative, below X^* , the company goes into bankruptcy.
- We were able to find the value of the company using risk neutral valuation.
- We were able to find the stock price of Amazon.
- We were able to relate the β of Amazon to the market price of revenues risk.
- The model can also deliver the probability of default at any given date, provided that we specify a threshold of bankruptcy X^*

Program

Description

As mentioned above, the above model cannot be solved analytically (by formulas). SM have written a program that takes as inputs all the parameters of the model and outputs some key results, such as the value of the company (V), its standard deviation, the share price (S), its standard deviation, the probability of bankruptcy within a certain period of years, and the implied beta of the stock.

To run the program, you need three files, all saved in the same directory:

1. Amazon-AIF.exe
2. Amazon-Basic.dat
3. Amazon-aif.dat

The file *Amazon-AIF.exe* is the executable file that simulates the model. It needs the input from file *Amazon-aif.dat* in order to run. The inputs in *Amazon-aif.dat* can be modified by the user. The file *Amazon-Basic.dat* contains the entire list of parameters needed to simulate the model, including those in *Amazon-aif.dat*. The descriptions of the parameters are self-explanatory, once you have understood the model above. However, most of those parameters are hard-wired within the *Amazon-AIF.exe* file.

The only parameters you can change are those in *Amazon-aif.dat*. You can open this file in any text editor, such as Wordpad, and modify the parameters. The input parameters are explained below:

- HALF-LIFE OF DEVIATIONS (k)—in years
- INITIAL VOLATILITY OF GROWTH RATE IN REVENUES (η_0)—yearly percentage points
- LONG-TERM VARIABLE COSTS —fraction between 0 and 1.
- MARKET PRICE OF RISK (λ)—parameter
- MAXIMUM FUTURE FINANCING (BANKRUPTCY LEVEL) (X^*)—in millions of dollars

Those parameters have the most impact on the formulas of V and S .

Running the Program

You need to have all three files in the same directory, say C: \DIRECTORY. To run the program in Windows:

1. Press the “Start” button
2. Go to: “Run”
3. In the window, marked “Open”, write: C:\DIRECTORY\Amazon-AIF.exe

4. The program will run in a DOS window. It might take several seconds for the simulations to complete.
5. Once the program terminates, do back to C:\ DIRECTORY. You must have a new file, called AMAZON-AIF.OUT
6. If you open the text file AMAZON-AIF.OUT you will see the output of the Amazon-AIF.exe program. The first few lines show the variables used in the program. You will see a section, named “PROGRAM TO VALUE AMAZON.COM” which contains the input values that you can modify. Immediately after that, you will see the final output of the program, in a section called “ VALUATION OF AMAZON.COM”. The output must be self-explanatory.

Questions

1. Run the program with the following parameters, and find the value of the company, its share price, and the implied beta.

HALF-LIFE OF DEVIATIONS = 1.5

INITIAL VOLATILITY OF GROWTH RATE IN REVENUES= 0.180

LONG-TERM VARIABLE COSTS = 0.890

MARKET PRICE OF RISK = 0.140

MAXIMUM FUTURE FINANCING (BANKRUPTCY LEVEL) = -3000.
2. The above specification assumes a decay rate of only 1.5 years. In other words, the convergence of Amazon to a “mature” company is quite rapid. Lengthen the convergence period by setting the half-life to be 2.5 years. What is the effect on the output variables? Now shorten the half-life. Can you provide an intuition for the change of results?
3. Suppose that we were wrong in assuming that long-term variable costs were 0.89. How would the results change if those costs were changed to 0.85? How about if they were 0.92? Explain the intuition behind the results. (Use the half-life value you obtained in part (c).)
4. Suppose that, in a credit crunch, Amazon is not able to secure external financing and it turns out that the maximum level of future financing decreases from 3 billion to 2 billion dollars. How would such a change impact the probability of default? Explain whether this result is intuitive.
5. In all of the above exercises, what was the range of the implied β 's. How close is it to the current β of Amazon (you can find the current β of Amazon on any financial website)? Would the β (and the market price of revenues risk) change with time?

In preparing your written write up of the case, it is very important that you do not just present the sensitivity analysis. Rather, you need to demonstrate that you understand WHY the sensitivity is the way it is. That is, we are looking for an economic explanation that demonstrates that you will take some lessons away from doing this case.