

# Bargaining and News

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February 2017

# Motivation

A **central issue** in the bargaining literature

- ▶ Will trade be (inefficiently) delayed?

What is usually ignored

- ▶ If trade is in fact delayed, **new information** may come to light...

This paper = Bargaining + News

## A canonical setting

- ▶ An indivisible asset (e.g., firm, project, security)
    - Type of asset is either low or high
  - ▶ One informed seller and one uninformed buyer
    - Buyer makes price offers
    - Common knowledge of gains from trade
    - Efficient outcome: trade immediately
  - ▶ Infinite horizon; discounting; frequent offers; no commitment
- + **News**: information about the asset is gradually revealed

## Application 1: Catered Innovation

Consider a **startup** (the informed seller) that has “catered” its innovation to a large firm, say, **Google** (the uninformed buyer)

- ▶ This strategy has become increasingly common (Wang, 2015)
- ▶ The longer the startup operates independently, the more Google will learn about the value of the innovation
- ▶ But delaying the acquisition is inefficient because Google can leverage economies of scale and has a portfolio of complimentary businesses

**Questions:** How does Google's ability to learn about the startup affect

- the bargaining dynamics? their relative bargaining power?
- total surplus realized?

## Application 2: Due Diligence

“Large” transactions typically involve a due diligence period:

- ▶ Corporate acquisitions
- ▶ Commercial real estate transactions

This information gathering stage is inherently dynamic.

**Questions:** How does the acquirer’s ability to conduct **due diligence** and **renegotiate** the initial terms of sale influence

- Initial terms of sale? Eventual terms of sale?
- Likelihood of deal completion?
- Profitability of acquisition?

## Preview of Results

- ▶ The buyer's ability to leverage the information to extract more surplus is remarkably limited.
  - A negotiation takes place and yet the buyer **gains nothing** from the ability to negotiate a better price.
  - Coasian force overwhelms access to information.
- ▶ Buyer engages in a form of **costly experimentation**
  - Makes offers that are sure to lose money if accepted, but generate information if rejected
  - Seller benefits from buyer's ability to renegotiate terms
  - Seller may also benefit from buyer's ability to learn
- ▶ Introducing competition among buyers can lead to **worse outcomes**.
  - Under certain conditions, seller's payoff is higher and/or the outcome is more efficient with a single buyer than with competing ones.

# Literature

## Bargaining with independent values

- ▶ Coase (1972), GSW (1985), FLT (1987), Ausubel and Deneckere (1989, 1992), Ortner (2014)

## Bargaining with interdependent values

- ▶ Admati and Perry (1987), Evans (1989), Vincent (1989), Deneckere and Liang (2006), Fuchs and Skrzypacz (2010, 2012)

## News in competitive markets with adverse selection

- ▶ Daley and Green (2012, 2015), Asriyan, Fuchs and Green (2016)

## Setup: Players and Values

Players: seller and buyer

- ▶ Seller owns asset of type  $\theta \in \{L, H\}$
- ▶  $\theta$  is the seller's private information

Values:

- ▶ Seller's reservation value is  $K_\theta$ , where  $K_H > K_L = 0$
- ▶ Buyer's value is  $V_\theta$ , where  $V_H \geq V_L$ 
  - Independent values:  $V_H = V_L$
- ▶ Common knowledge of gains from trade:  $V_\theta > K_\theta$
- ▶ "Lemons" condition:  $K_H > V_L$

## Setup: Timing and Payoffs

The model is formulated in continuous time.

- ▶ At every  $t$  buyer makes offer,  $w$ , to seller
- ▶ If  $w$  accepted at time  $t$ , the payoff to the seller is

$$e^{-rt}(w - K_\theta)$$

and the buyer's payoff is

$$e^{-rt}(V_\theta - w)$$

- ▶ Both players are risk neutral

## Complete Information Outcome

Suppose  $\theta$  is public information.

- ▶ The buyer has all the bargaining power.
- ▶ The buyer extracts all the surplus.
- ▶ Offers  $K_\theta$  at  $t = 0$  and the seller accepts
- ▶ Payoffs:

$$\text{Buyer payoff} = V_\theta - K_\theta$$

$$\text{Seller payoff} = 0$$

Clearly, **knowing**  $\theta$  is beneficial to the buyer.

- ▶ What happens if the buyer does not know  $\theta$  but can **learn about**  $\theta$  gradually?

## Setup: News

- ▶ Represented by a **publicly observable** process:

$$X_t(\omega) = \mu_\theta t + \sigma B_t(\omega)$$

where  $B$  is standard B.M. and without loss  $\mu_H > \mu_L$

- ▶ The **quality of the news** is captured by the signal-to-noise ratio:

$$\phi \equiv \frac{\mu_H - \mu_L}{\sigma}$$

## Equilibrium objects

1. Offer process,  $W = \{W_t : 0 \leq t \leq \infty\}$
2. Seller stopping times:  $\tau^\theta$ 
  - Access to private randomization for mixing
  - Endows CDF over acceptance times:  $\{S_t^\theta : 0 \leq t < \infty\}$
3. Buyer's belief process,  $Z = \{Z_t : 0 \leq t \leq \infty\}$

We look for equilibria that are **stationary** in the buyer's beliefs:

- ▶  $Z$  is a time-homogenous Markov process
- ▶ Offer is a function that depends only on the state,  $W_t = w(Z_t)$

## Buyer's beliefs

Buyer starts with a prior  $P_0 = \Pr(\theta = H)$

- ▶ At time  $t$ , buyer conditions on
  - (i) the path of the news,
  - (ii) seller rejected all past offers
- ▶ Using Bayes Rule, the buyer's belief at time  $t$  is

$$P_t = \frac{P_0 f_t^H(X_t)(1 - S_{t^-}^H)}{P_0 f_t^H(X_t)(1 - S_{t^-}^H) + (1 - P_0) f_t^L(X_t)(1 - S_{t^-}^L)}$$

- ▶ Define  $Z \equiv \ln\left(\frac{P_t}{1 - P_t}\right)$ , we get that

$$Z_t = \underbrace{\ln\left(\frac{P_0}{1 - P_0}\right) + \ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right)}_{\hat{Z}_t} + \underbrace{\ln\left(\frac{1 - S_{t^-}^H}{1 - S_{t^-}^L}\right)}_{Q_t}$$

## Seller's problem

Given  $(w, Z)$ , the seller faces a stopping problem

### Seller's Problem

For all  $z$ , the seller's strategy solves

$$\sup_{\tau} E_z^{\theta} [e^{-r\tau} (w(Z_{\tau}) - K_{\theta})]$$

Let  $F_{\theta}(z)$  denote the solution.

## Buyer's problem

In any state  $z$ , the buyer essentially has three options:

1. **Wait:** Make a non-serious offer that is rejected w.p.1.
2. **Screen:** Make an offer  $w < K_H$  that only the low type accepts with positive probability
3. **Buy/Stop:** Offer  $w = K_H$  and buy the regardless of  $\theta$

Let  $F_B(z)$  denote the buyer's value function.

► Details

# Buyer's problem

## Lemma

For all  $z$ ,  $F_B(z)$  satisfies:

*Option to wait:* 
$$rF_B(z) \geq \frac{\phi^2}{2} (2p(z) - 1) F'_B(z) + \frac{\phi^2}{2} F''_B(z)$$

*Optimal screening:* 
$$F_B(z) \geq \sup_{z' > z} \left\{ \left( 1 - \frac{p(z)}{p(z')} \right) (V_L - F_L(z')) + \frac{p(z)}{p(z')} F_B(z') \right\}$$

*Option to buy:* 
$$F_B(z) \geq E_z[V_\theta] - K_H$$

where at least one of the inequalities must hold with equality.

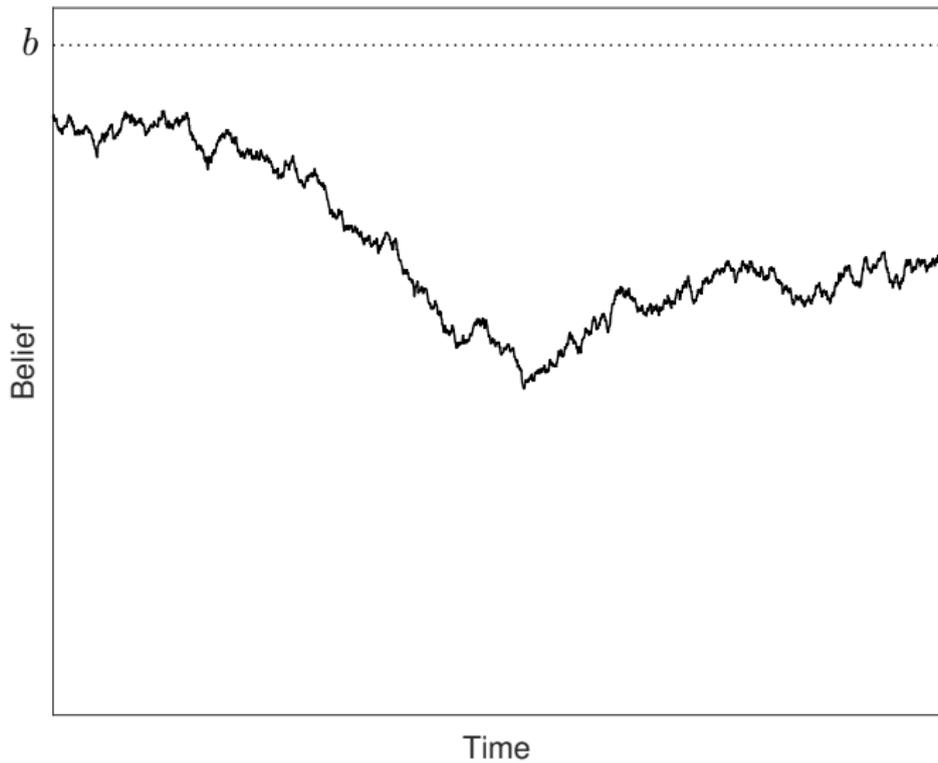
# Equilibrium

## Theorem

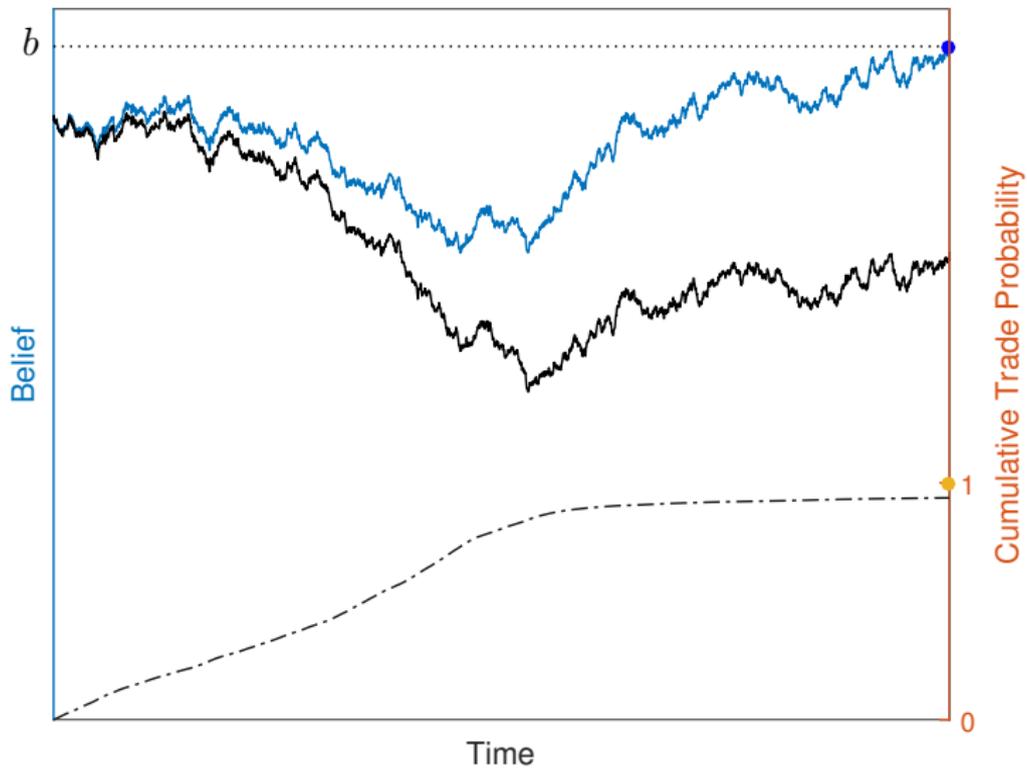
*There exists a unique equilibrium. In it,*

- ▶ *For  $P_t \geq b$ , trade happens immediately: buyer offers  $K_H$  and both type sellers accept*
- ▶ *For  $P_t < b$ , trade happens “smoothly”: only the low-type seller trades and with probability that is proportional to  $dt$ .*

## Equilibrium: sample path



## Equilibrium: sample path



## Equilibrium construction: sketch

1. Buyer's problem is linear in the rate of trade:  $\dot{q}$ 
  - Derive  $F_B$  (independent of  $F_L$ )
2. Given  $F_B$ , what must be true about  $F_L$  for smooth trade to be optimal?
  - Derive  $F_L$ , which implies  $w$
3. Low type must be indifferent between waiting and accepting
  - Indifference condition implies  $\dot{q}$  and therefore low-type acceptance rate.

**Summary:** Smooth  $\implies F_B \implies F_L \implies \dot{q}$

▶ Details

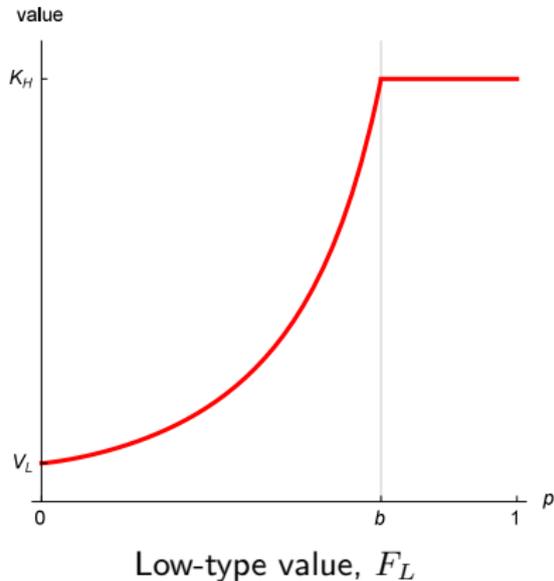
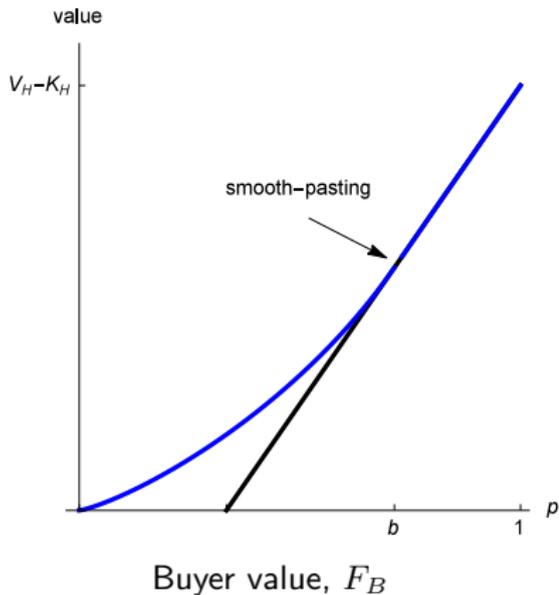
## A bit more about Step 1

For  $z < \beta$ ,

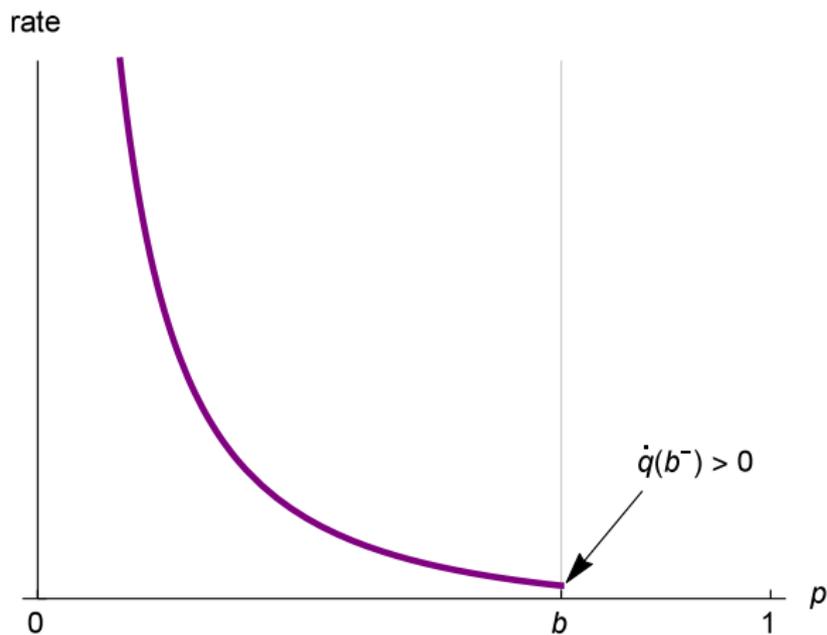
$$rF_B(z) = \underbrace{\frac{\phi^2}{2} (2p(z) - 1) F'_B(z) + \frac{\phi^2}{2} F''_B(z)}_{\text{Evolution due to news}} + \underbrace{\dot{q}(z) \left( (1 - p(z)) (V_L - F_L(z) - F_B(z)) + F'_B(z) \right)}_{\Gamma(z) = \text{net-benefit of screening at } z}$$

- ▶ Buyer's value is linear in  $\dot{q}$
- ▶ For “smooth” trade to be optimal, it must be that  $\Gamma(z) = 0$ 
  - $F_B$  does not depend on  $\dot{q}$  (and has simple closed-form solution)
- ▶ Therefore, buyer does not benefit from screening!
  - Otherwise, she would want to trade “faster”
  - Pins down exactly how expensive it must be to buy  $L$ , i.e.,  $F_L(z)$

# Equilibrium payoffs



# Equilibrium rate of trade



## Interesting Predictions?

1. Buyer does **not benefit** from the ability to negotiate the price.
  - Though she *must* negotiate in equilibrium.
2. The buyer is **guaranteed to lose money** on any offer below  $K_H$  that is accepted.
  - A form of costly experimentation.
3. The low-type seller may actually benefit from buyer's ability to learn his type.

## Who Benefits from the Negotiation?

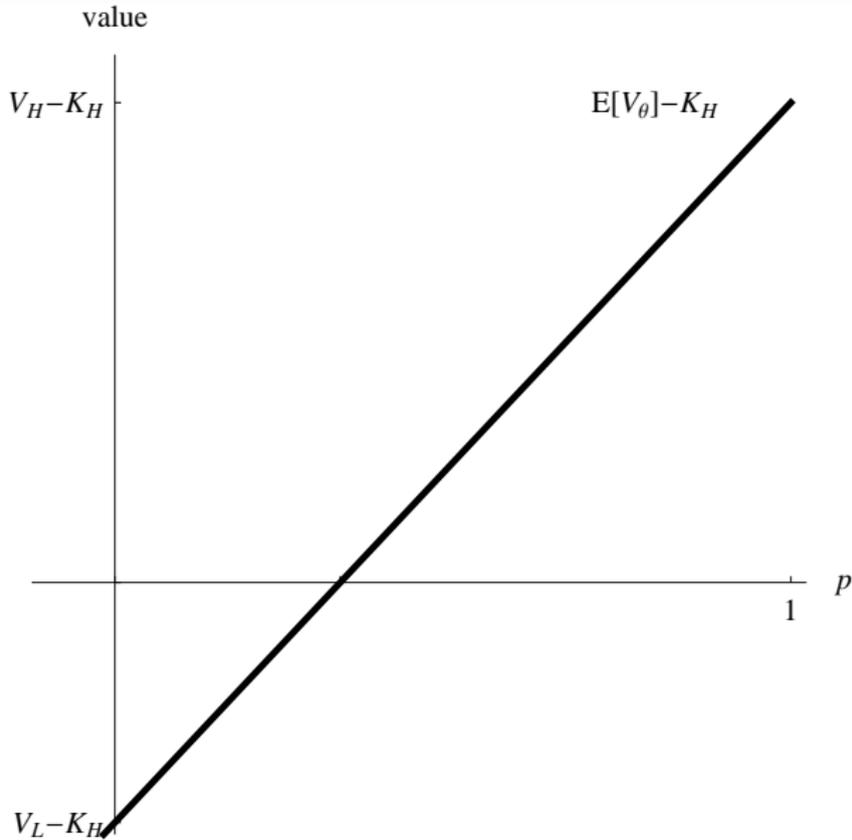
Suppose the price is **exogenously fixed** at the lowest price that the seller will accept:  $K_H$  (e.g., initial terms of sale).

- ▶ The buyer conducts due diligence (observes  $\hat{Z}$ ) and decides when and whether to actually complete the deal.
- ▶ Buyer's strategy is simply a stopping rule, where the expected payoff upon stopping in state  $z$  is

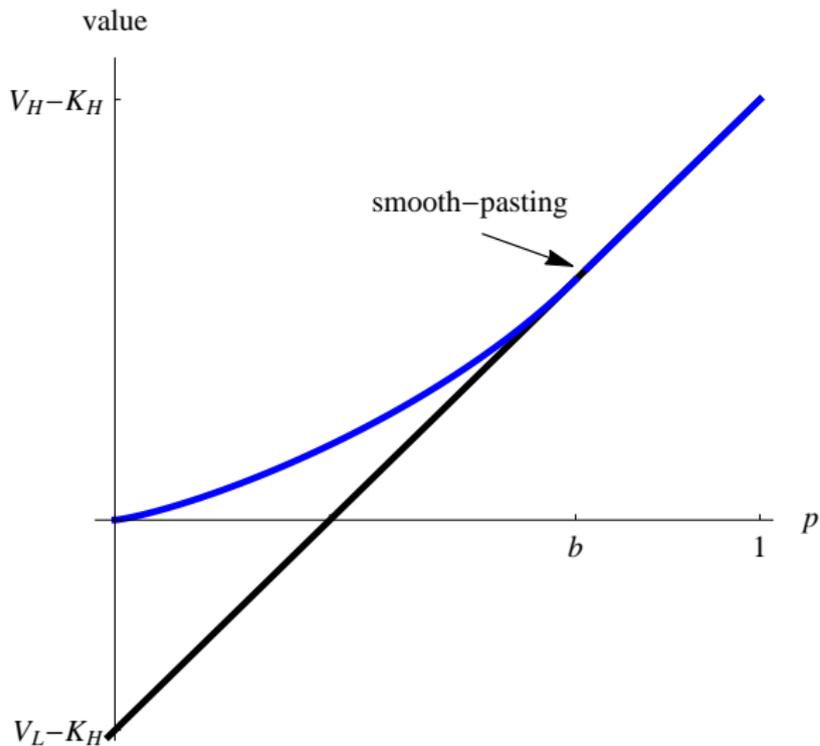
$$E_z[V_\theta] - K_H$$

- ▶ Call this the **due diligence game**.
  - NB: it is not hard to endogenize the initial terms.

# Due Diligence Game



# Due Diligence Game



# Who Benefits from the Negotiation?

## Result

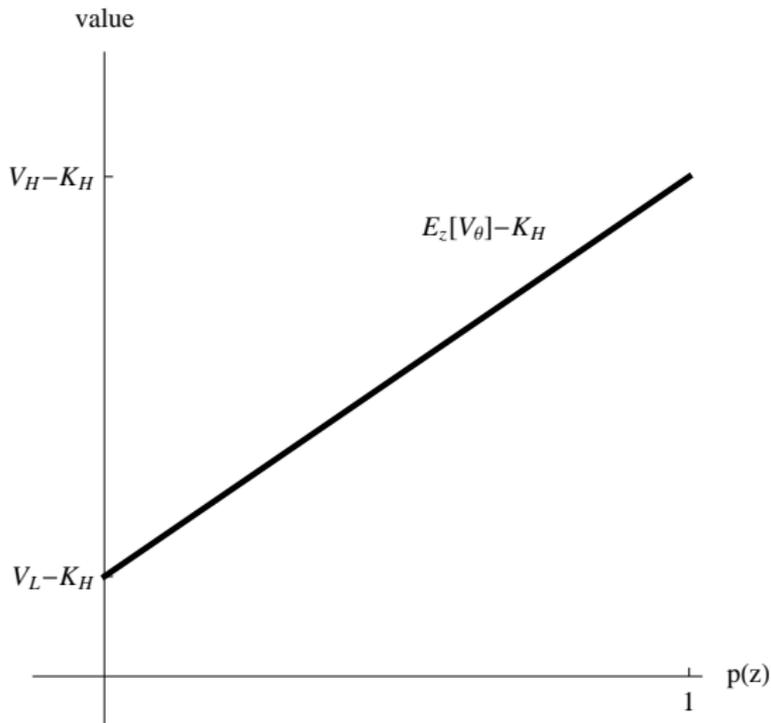
In the equilibrium of the bargaining game:

1. The buyer's payoff is **identical** to the due diligence game.
2. The (*L*-type) seller's payoff is **higher** than in the due diligence game.

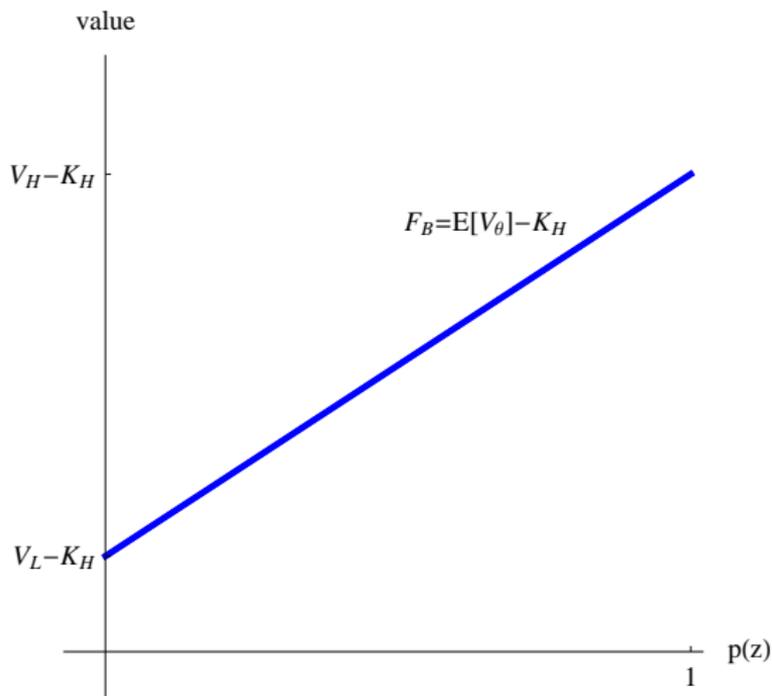
Total surplus higher with bargaining, but **fully captured** by seller.

- ▶ Despite the fact that the buyer makes all the offers.

# No Lemons $\implies$ No Learning



# No Lemons $\implies$ No Learning



## No Lemons $\implies$ No Learning

### Result

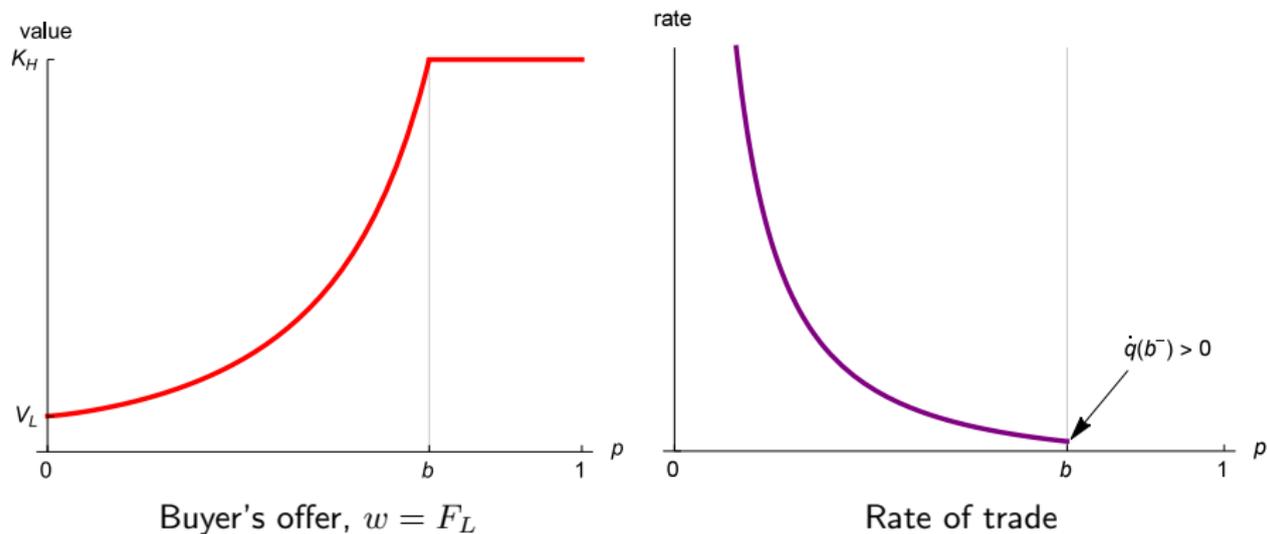
When  $V_L \geq K_H$ , unique equilibrium is immediate trade at price  $K_H$ .

- ▶ Absent a lemons condition, the Coasian force overwhelms the buyer's incentive to learn.

## Experimentation and regret

For all  $z$ , the buyer offers  $F_L(z)$ , which is strictly greater than  $V_L$ .

And for  $z < \beta$ , only the low type trades.



## Experimentation and regret

So below  $b$ , the buyer is making an offer that:

- (1) will ONLY be accepted by the low type
- (2) will make a loss whenever accepted

Why?

- ▶ One interpretation: **costly experimentation**
- ▶ Buyer willing to lose money today (if offer accepted) in order to learn and reach  $\beta$  *faster* (if rejected)
- ▶ **News is critical** for this feature to arise

## Effect of news quality

### Proposition (The effect of news quality)

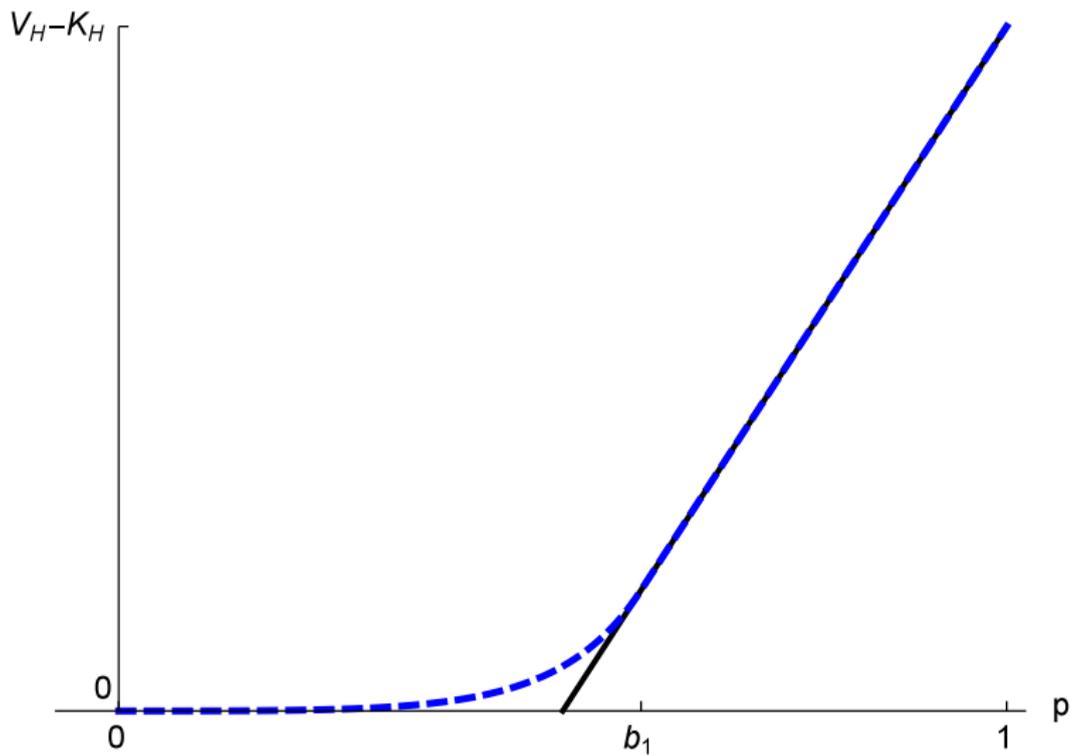
*As the quality of news increases:*

- 1. Both  $\beta$  and  $F_B$  increase*
- 2. The rate of trade,  $\dot{q}$ , decreases for low beliefs but increases for intermediate beliefs*
- 3. Total surplus and  $F_L$  increase for low beliefs, but decrease for intermediate beliefs*

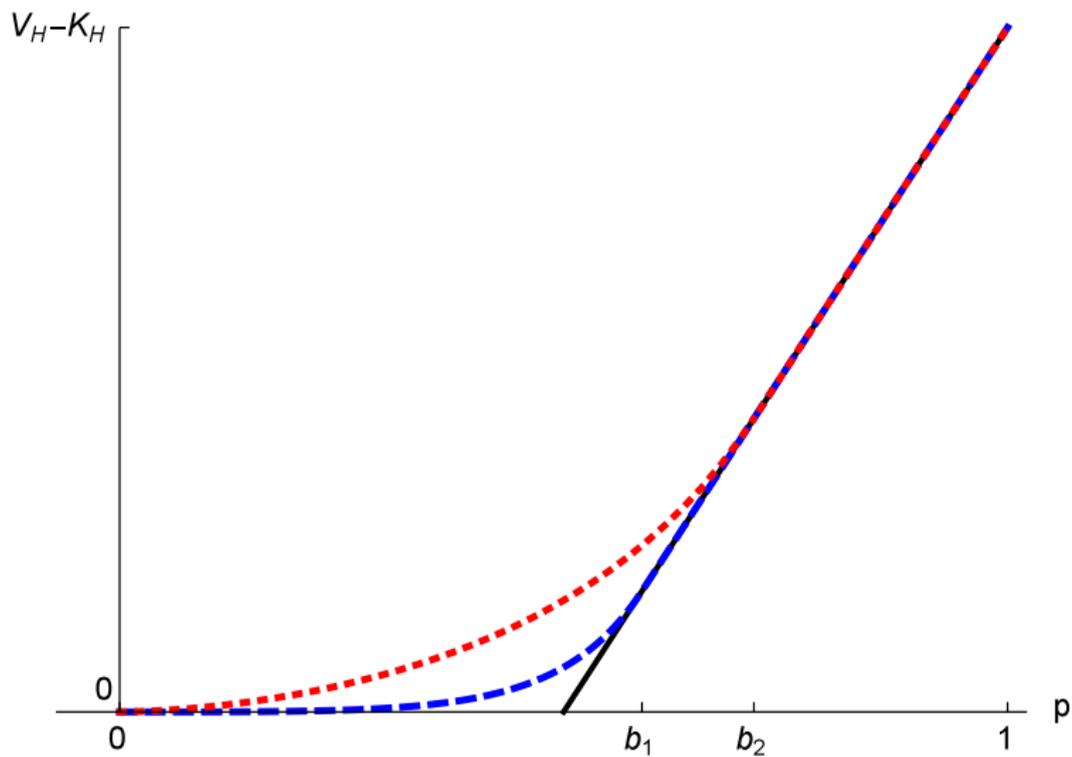
Two opposing forces driving 3.

- ▶ Higher  $\phi$  increases volatility of  $\hat{Z} \implies$  faster trade
- ▶ Higher  $\beta$  (and/or) lower  $\dot{q} \implies$  slower trade

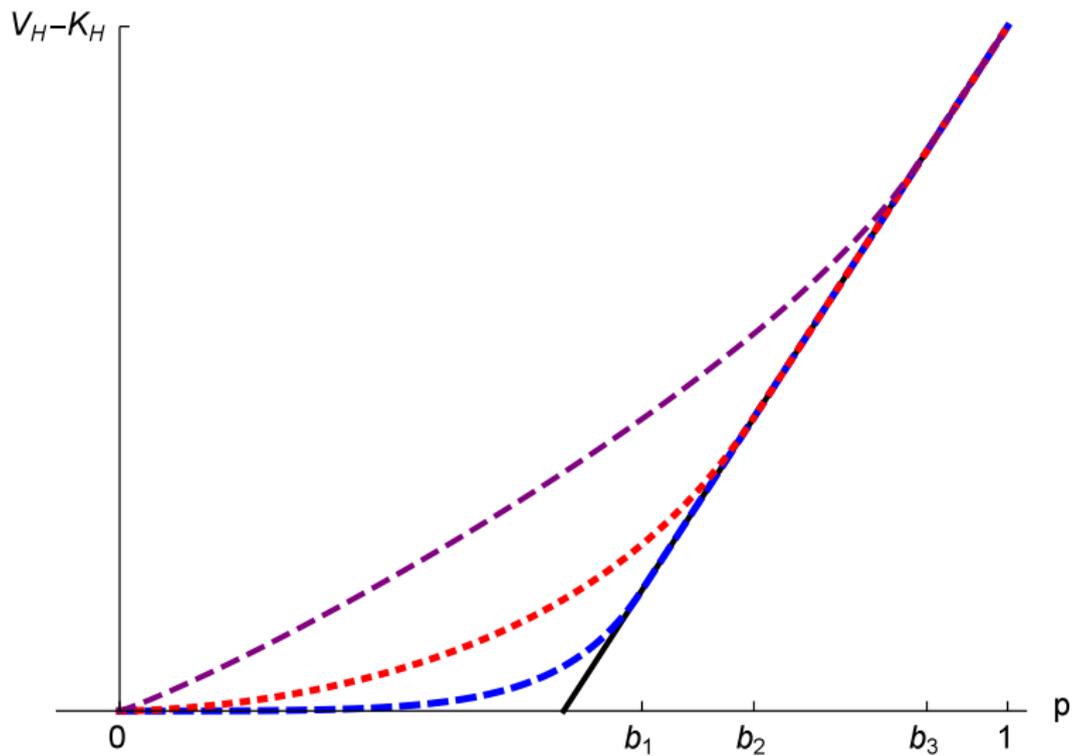
## Effect of news on buyer payoff



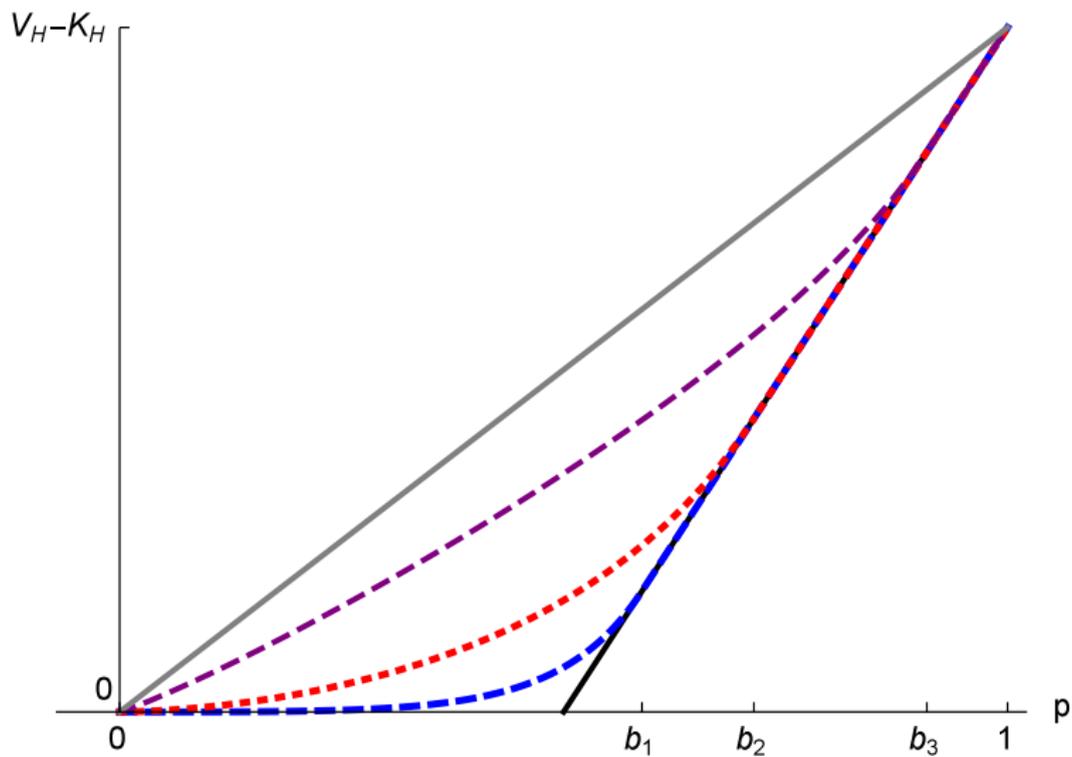
## Effect of news on buyer payoff



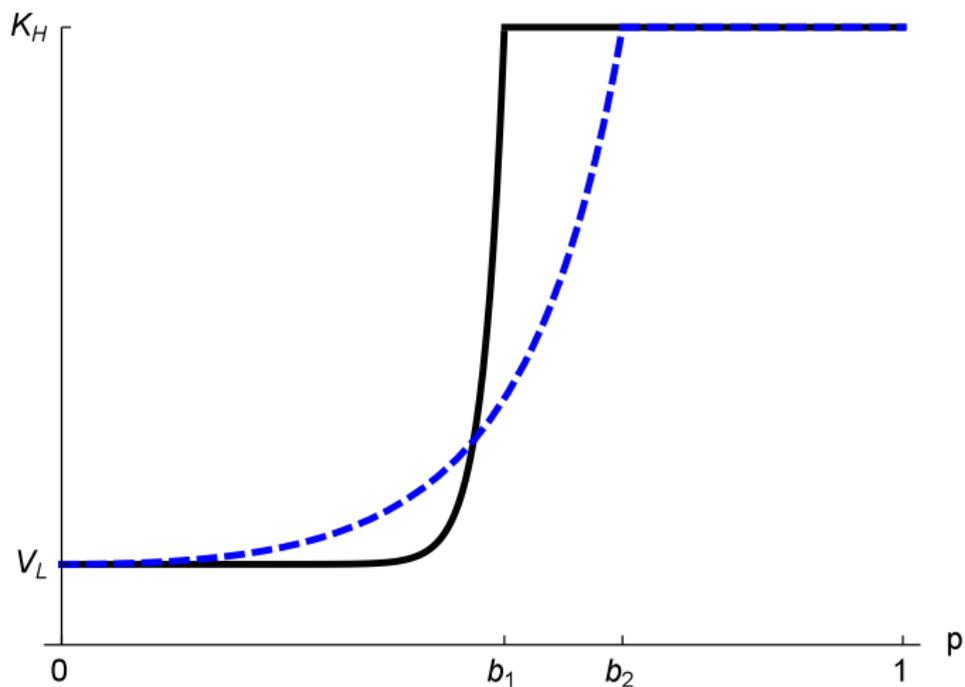
# Effect of news on buyer payoff



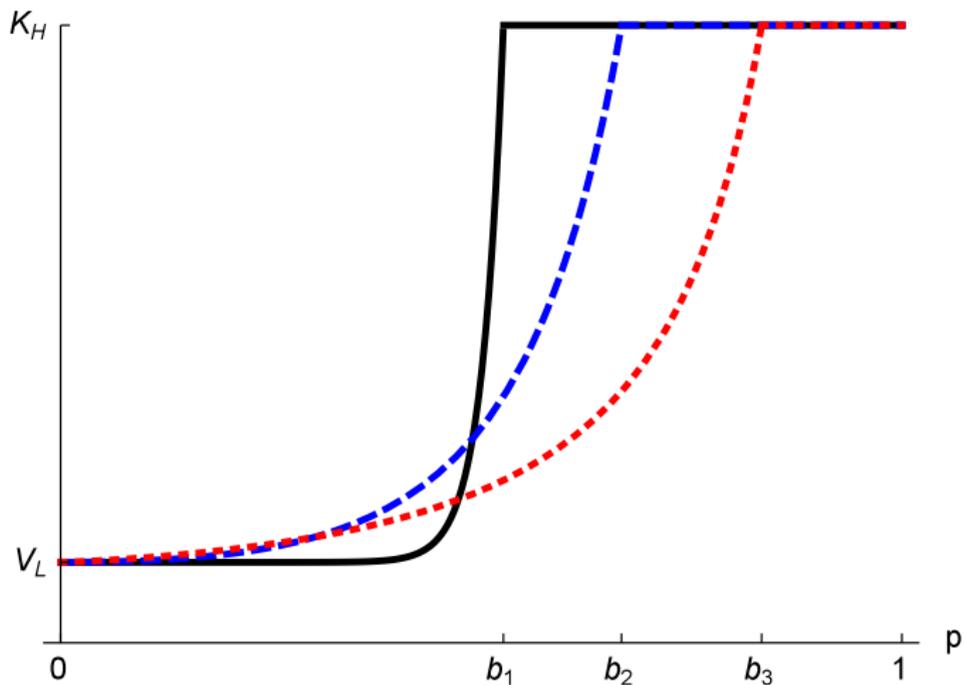
# Effect of news on buyer payoff



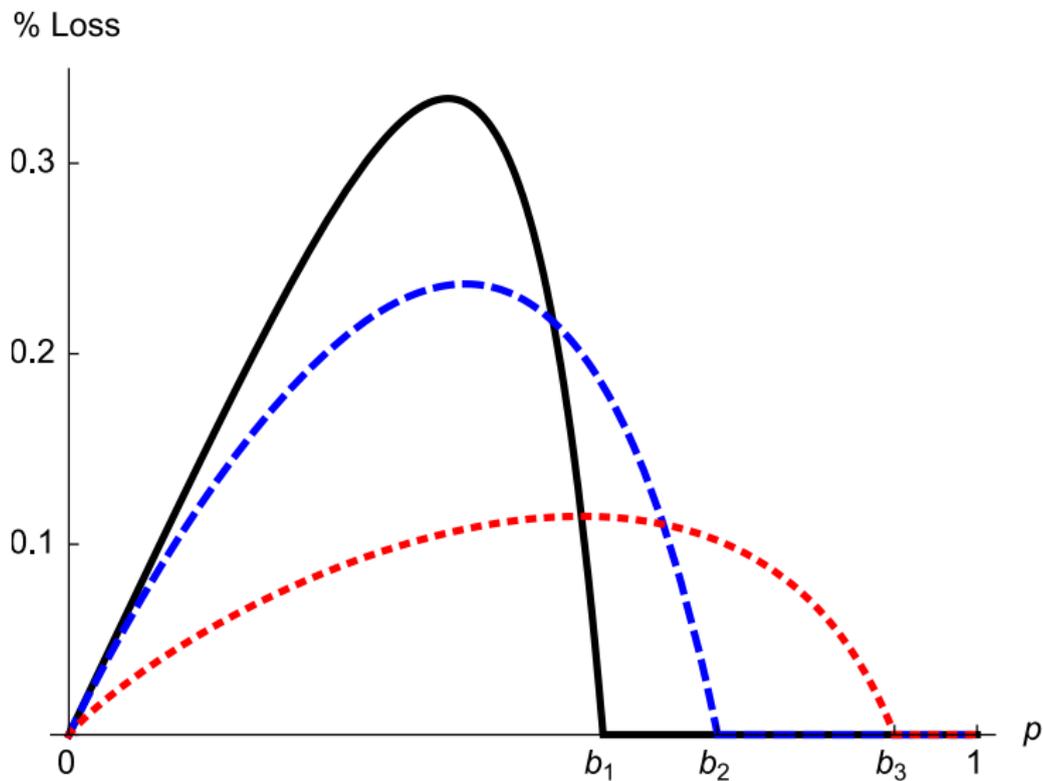
## Effect of news on low-type payoff



## Effect of news on low-type payoff



# (In)efficiency



## Competition and the Coase Conjecture

The buyer's desire to capture any future profits from trade leads to a form of intertemporal competition.

- ▶ Seller knows buyer will be tempted to increase price tomorrow
- ▶ Which increases the price seller is willing to accept today
- ▶ Buyer “competes” against future self

**Coase Conjecture:** Absent some form of commitment (delay, price, etc.), the outcome with a monopolistic buyer will resemble the outcome with competitive buyers.

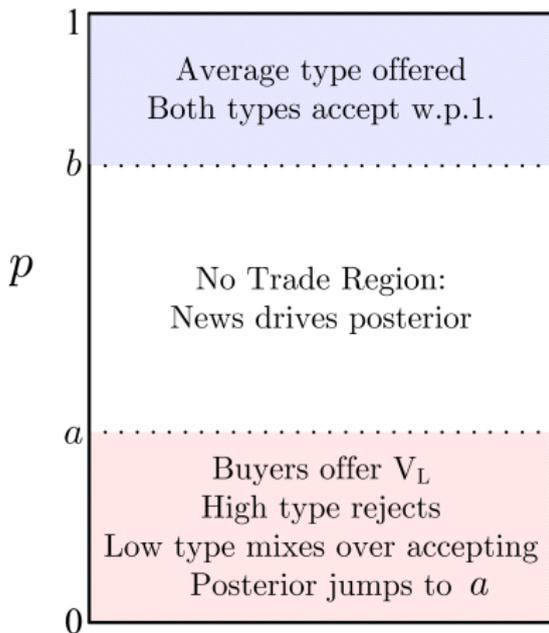
**Question:** How does **news** affect Coase's conjecture?

## Competitive buyers

### Theorem (Daley and Green, 2012)

*There is a unique equilibrium satisfying a mild refinement on off-path beliefs. In it,*

- ▶ *For  $P_t \geq b$ : trade happens immediately, buyers offer  $V(P_t)$  and both type sellers accept*
- ▶ *For  $P_t < a$ : buyers offer  $V_L$ , high types reject w.p.1. Low types mix such that the posterior jumps to  $a$*
- ▶ *For  $P_t \in (a, b)$ : there is no trade, buyers make non-serious offers which are rejected by both types.*



## Intuition for equilibrium play

1.  $H$ -seller can get  $V(p)$  whenever she wants it. For  $p < b$ , she does better by waiting for news.
2. For high enough  $p$ ,  $H$  has little to gain by waiting, so exercises the option to trade at  $V(p)$ . The low type (happily) pools.
3.  $L$  can always get  $V_L$ . But for  $p \in (a, b)$ , he does better to mimic  $H$ .
4.  $L$ 's prospects of reaching  $b$  decrease as  $p$  falls.
  - At  $p = a$ , she is indifferent  $\implies$  willing to mix.

Buyer competition eliminates incentive for experimentation.

# Effect of competition

## Bilateral

Trade is efficient for  $p \geq b_b$ .

For  $p < b_b$

- ▶ probability of trade is proportional to  $dt$ .
- ▶ rate is decreasing in  $p$ .

## Competitive

Trade is efficient for  $p \geq b_c$ .

For  $p < b_c$

- ▶  $p \in (a_c, b_c)$ , complete trade breakdown.
- ▶  $p < a_c$ , atom of trade

## Result

Efficient trade requires higher belief in the competitive market:  $b_b < b_c$

## Difference in the efficient-trade threshold

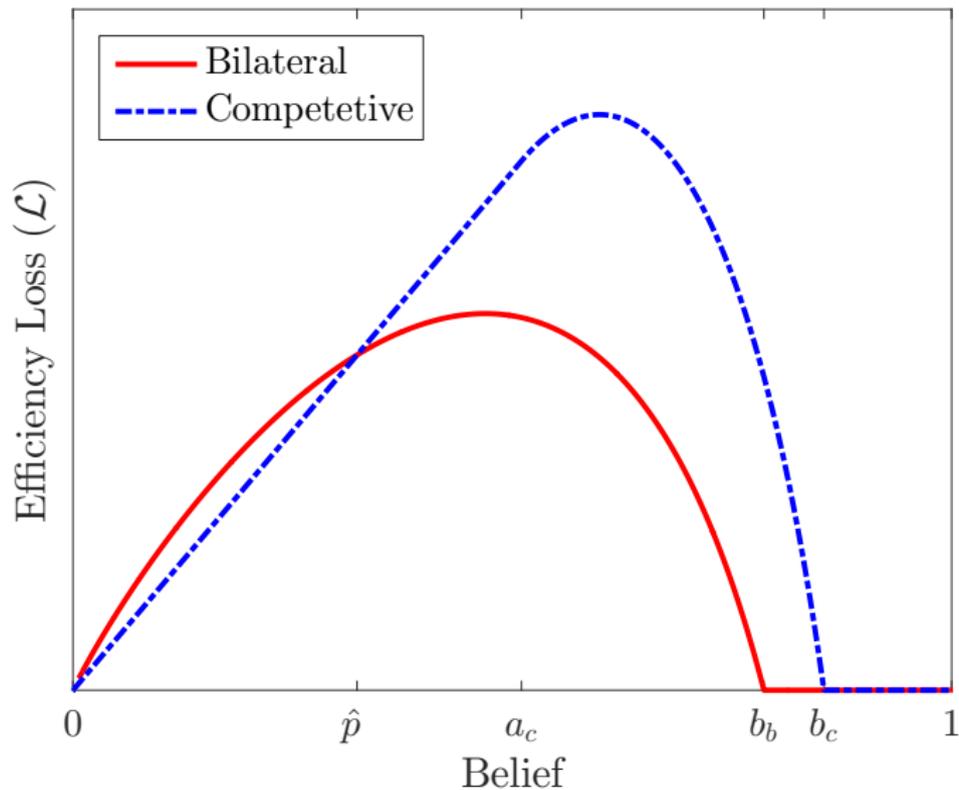
Intuition?

- ▶ Buyers and sellers differ in their expectations about the realization of future news.
- ▶ With competitive buyers, the high-type seller decides when to “stop” and net  $E_z[V_\theta] - K_H$ .
- ▶ With one buyer, the buyer decides when to “stop” and net  $E_z[V_\theta] - K_H$ .
- ▶ *But the high-type seller expects good news, while the buyer does not.*

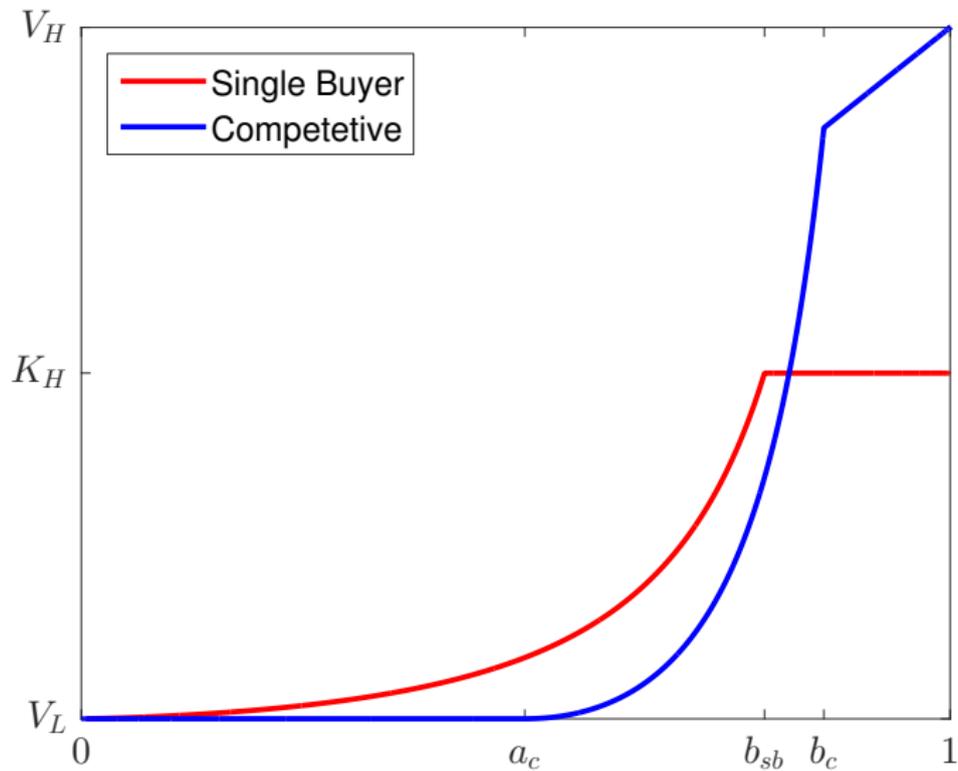
**More generally:** competition does not necessarily lead to more efficient outcomes in dynamic models with adverse selection

- ▶ Pushes prices up in later periods  $\implies$  more incentive to wait
- ▶ See also Asyrian et al (2016)

## Efficiency: bilateral vs competitive



## Low-type value: bilateral vs. competitive



# Implications

- ▶ Entrepreneurs who cater their innovation are more likely to have negative private information.
  - All else equal, catered innovations are less valuable innovations
- ▶ Acquisitions that take place at a price below the initial terms add less value for the acquirer.
  - In fact, they necessarily lose value for the acquirer.
  - A downward renegotiation of the acquisition price should negatively affect acquirer's share price.
  - E.g., when Verizon announced the Yahoo merger is going through but at a price \$300M below the original bid.

## Hot off the press

Suppose there is competition for the right to conduct due diligence.

- ▶ Multiple bidders submit bids in an auction at  $t = 0$
- ▶ The seller selects a winner
- ▶ The winner can conduct due diligence
- ▶ No renegotiation of price allowed

### Preliminary Result

A higher bid is not necessarily better for the seller because it induces *stricter due diligence*.

- ▶ The winning bid lies strictly between  $K_H$  and  $V_H$
- ▶ The winning bidder makes strictly positive profit

To do list:

- ▶ Incorporate/allow for renegotiation
- ▶ Enrich the space of contracts

# Summary

We explore the effect of news in a canonical bargaining environment

- ▶ Construct the equilibrium (in closed form).
- ▶ Show that uninformed player's ability to leverage news to extract surplus is remarkably limited.
  - Buyer negotiates based on new information in equilibrium, but gains nothing from doing so!
- ▶ More news does not necessarily lead to more efficient outcomes
  - Seller may actually benefit from buyer's ability to learn.
- ▶ Relation to the competitive outcome
  - Competition among buyers eliminates the Coasian force and may reduce both total surplus and seller payoff.

# Additional Results

- ▶ Uniqueness
- ▶ The no-news limit
- ▶ Extensions
  1. Costly acquisition
  2. Arrival of “perfect” news

## Other equilibria?

We focused on the (unique) smooth equilibrium. Can other stationary equilibria exist?

- ▶ No

By [Lesbegue's decomposition theorem](#) for monotonic functions

$$Q = Q_{abs} + Q_{jump} + Q_{sing}$$

To sketch the argument, we will illustrate how to rule out:

1. Atoms of trade with  $L$  (i.e., jumps)
2. Reflecting barriers (i.e., singular component)

# Uniqueness

Suppose there is some  $z_0$  such that:

- ▶ Buyer makes offer  $w_0$
- ▶ Low type accepts with atom

Let  $\alpha$  denote the buyer's belief conditional on a rejection. Then

- ▶  $F_L(z_0) = F_L(\alpha) = w_0$ , by  $L$ -seller optimality
- ▶  $F_L(z) = w_0$  for all  $z \in (z_0, \alpha)$ , by Buyer optimality

Therefore, starting from any  $z \in (z_0, \alpha)$ , the belief conditional on a rejection jumps to  $\alpha$ .

- ▶ If there is an atom, the behavior must resemble the competitive-buyer model...

# Uniqueness

To rule out the dynamics of the competitive-buyer model:

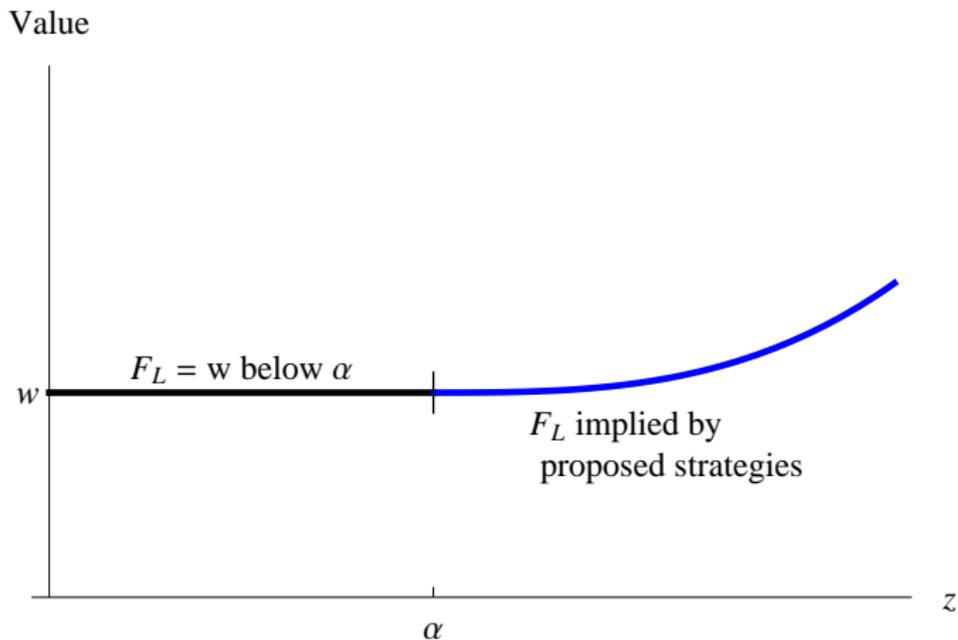
Suppose  $\alpha$  is a reflecting barrier, such that

- for  $z \leq \alpha$ , the offer is  $w$ , and rejection jumps the belief to  $\alpha$ ,
- there is no trade on an interval  $(\alpha, \bar{z})$ ,
- so,  $Z$  reflects upward at  $\alpha$  (conditional on rejection).

Hence, the low type is mixing at  $\alpha$ , implying the boundary condition:

$$F'_L(\alpha^+) = 0.$$

# Uniqueness



# Uniqueness

Consider now the buyer's incentives.

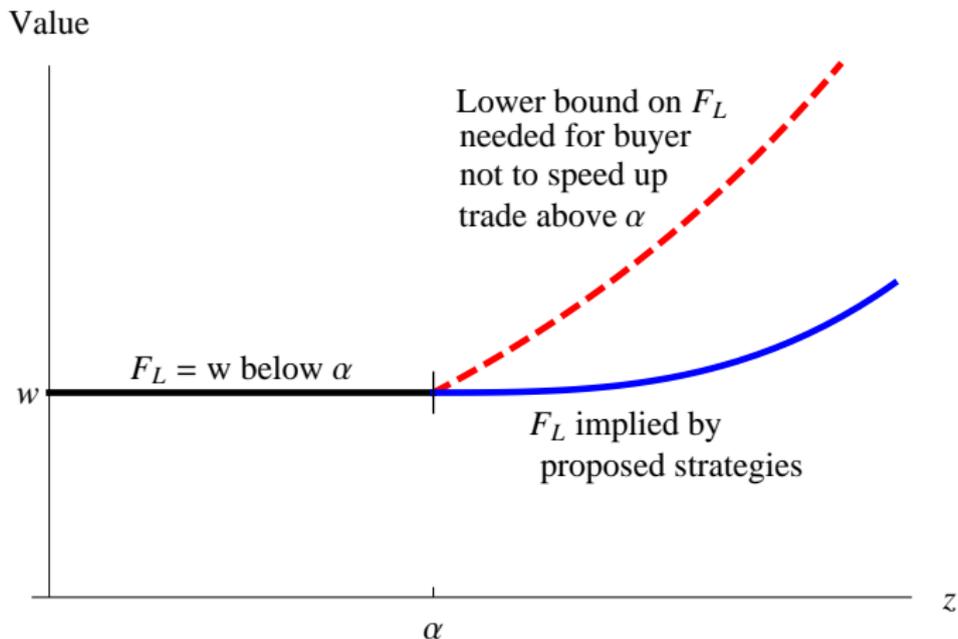
On  $(\alpha, \bar{z})$ , he is not screening, so:

- ▶  $F_B$  evolves according to the “waiting” ODE,
- ▶ It must be that  $\Gamma \leq 0$ . Hence,

$$F_L(z) \geq V_L - F_B(z) + \frac{1}{1-p(z)} F'_B(z)$$

which must hold with equality at  $z = \alpha$ .

# Uniqueness



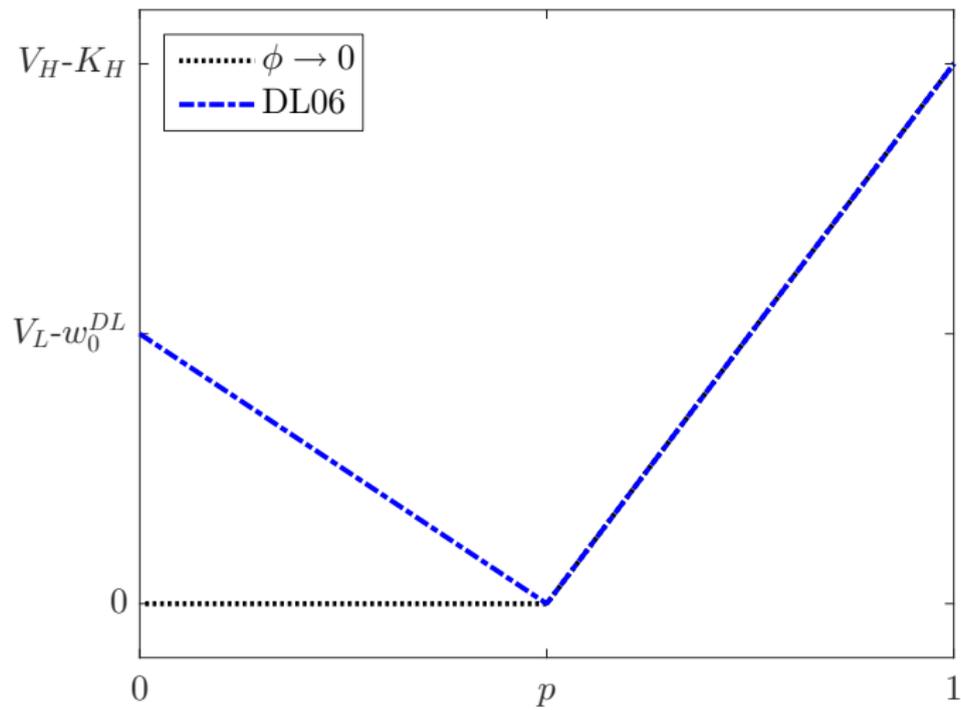
# Uniqueness

Intuitively,

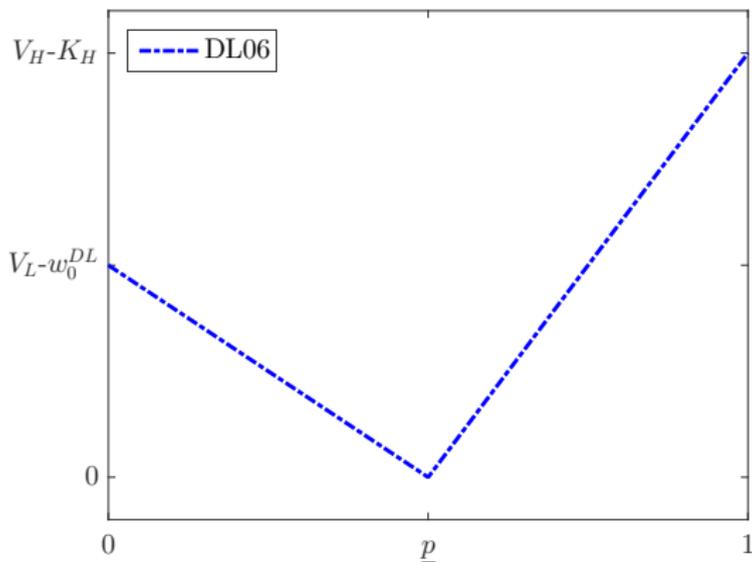
- ▶ The low type is no more expensive to trade with at  $z = \alpha + \epsilon$  than at  $z = \alpha$ .
- ▶ If the buyer wants to trade with the low type at price  $w$  at  $z = \alpha$ , he will want to extend this behavior  $z = \alpha + \epsilon$  as well.

## Effect of news

Our  $\phi \rightarrow 0$  limit differs from Deneckere and Liang (2006)



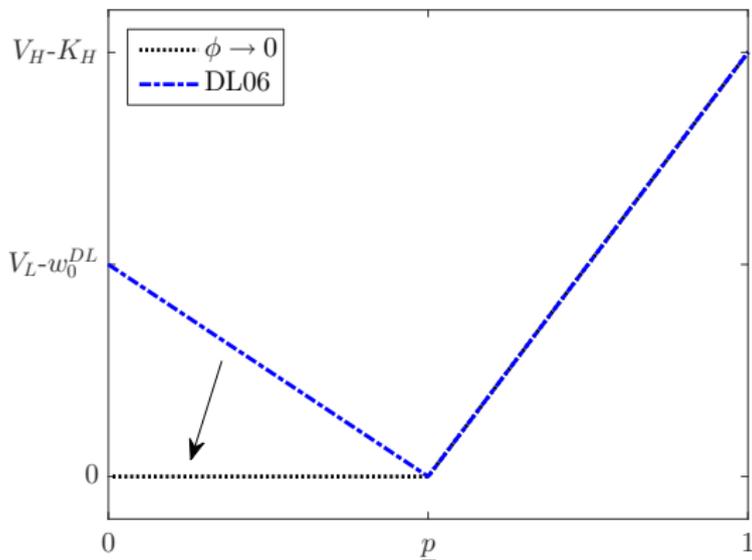
## Effect of news



Intuition for DL06:

- ▶ Coasian force disappears at precisely  $Z_t = \underline{z}$
- ▶ Buyer leverages this to extract concessions from low type at  $z < \underline{z}$

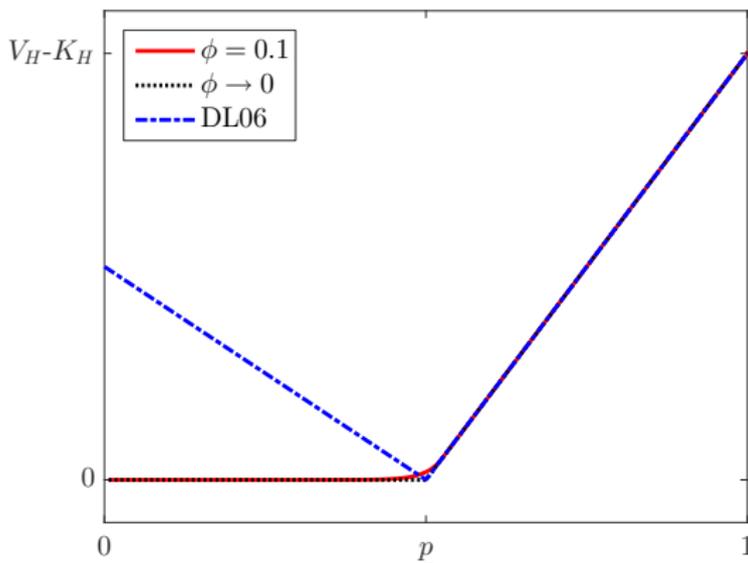
## Effect of news



With news, his belief cannot just “sit at  $\underline{z}$ ”, so this power evaporates.

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# Stochastic control problem

The buyer must decide:

- ▶ How quickly to trade with only the low type (i.e., choose  $Q$  given  $F_L$ )
- ▶ When to “buy the market” (i.e., choose  $T$  at which to offer  $K_H$ )

## Buyer's Problem

Choose  $(Q, T)$  to solve, for all  $z$ ,

$$\sup_{Q, T} \left\{ (1 - p(z)) E_z^L \left[ \int_0^T e^{-rt} (V_L - F_L(\hat{Z}_t + Q_t)) e^{-Q_t} dQ_t \right. \right. \\ \left. \left. + e^{-(rT+Q_T)} (V_L - K_H) \right] + p(z) E_z^H \left[ e^{-rT} (V_H - K_H) \right] \right\}$$

Let  $F_B(z)$  denote the solution.

# Buyer's problem

## Lemma

For all  $z$ ,  $F_B(z)$  satisfies:

*Option to wait:* 
$$rF_B(z) \geq \frac{\phi^2}{2} (2p(z) - 1) F'_B(z) + \frac{\phi^2}{2} F''_B(z)$$

*Optimal screening:* 
$$F_B(z) \geq \sup_{z' > z} \left\{ \left( 1 - \frac{p(z)}{p(z')} \right) (V_L - F_L(z')) + \frac{p(z)}{p(z')} F_B(z') \right\}$$

*Option to buy:* 
$$F_B(z) \geq E_z[V_\theta] - K_H$$

where at least one of the inequalities must hold with equality.

## Equilibrium construction

1. For  $z < \beta$ ,  $w(z) = F_L(z)$  and the buyer's value is

$$F_B(z) = (V_L - F_L(z)) (1 - p(z)) \dot{q}(z) dt + \left( 1 - \frac{\dot{q}(z)}{1 + e^z} dt \right) E_z [F_B(z + dZ_t)]$$

and  $dZ_t = d\hat{Z}_t + \dot{q}(Z_t) dt$ . So,

$$rF_B(z) = \underbrace{\frac{\phi^2}{2} (2p(z) - 1) F'_B(z) + \frac{\phi^2}{2} F''_B(z)}_{\text{Evolution due to news}} + \dot{q}(z) \underbrace{\left( (1 - p(z)) (V_L - F_L(z) - F_B(z)) + F'_B(z) \right)}_{\Gamma(z) = \text{net-benefit of screening at } z}$$

## Equilibrium construction

2. Observe that the buyer's problem is linear in  $\dot{q}$

$$rF_B(z) = \underbrace{\frac{\phi^2}{2} (2p - 1) F'_B + \frac{\phi^2}{2} F''_B}_{\text{Evolution due to news}} + \sup_{\dot{q} \geq 0} \underbrace{\dot{q} \left( (1-p)(V_L - F_L - F_B) + F'_B \right)}_{\Gamma(z) = \text{net-benefit of screening}}$$

Hence, in any state  $z < \beta$ , either

- (i) the buyer strictly prefers  $\dot{q} = 0$ , or
- (ii) the buyer is indifferent over all  $\dot{q} \in \mathbb{R}_+$

## Equilibrium construction

3. In either case

$$\dot{q}(z)\Gamma(z) = 0$$

4. This simplifies the ODE for  $F_B$  to just

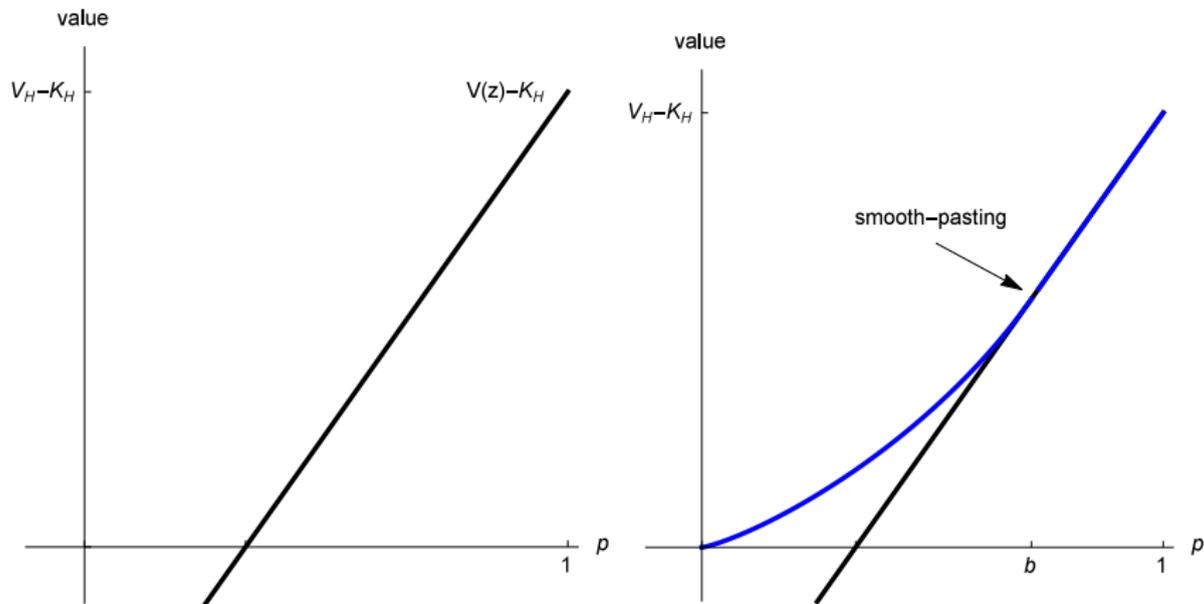
$$rF_B = \frac{\phi^2}{2} (2p - 1) F'_B + \frac{\phi^2}{2} F''_B$$

- $F_B$  does not depend on  $\dot{q}$
- Buyer gets same value he would get from  $\dot{q} = 0$
- Buyer gains nothing from the ability to screen using prices!

## Equilibrium construction

Using the appropriate boundary conditions, we find  $F_B(z) = C_1 \frac{e^{u_1 z}}{1+e^z}$ ,

- ▶ where  $u_1 = \frac{1}{2} \left( 1 + \sqrt{1 + 8r/\phi^2} \right)$  and  $C_1$  solves VM and SP at  $z = \beta$ .



## Equilibrium construction

Next, conjecture that  $\dot{q}(z) > 0$  for all  $z < \beta$ . Then, it must be that

$$\Gamma(z) = 0$$

Or equivalently

$$F_L(z) = (1 + e^z)F_B'(z) + V_L - F_B(z)$$

This pins down exactly how “expensive” the low type must be for the buyer to be indifferent to the speed of trade (i.e.,  $F_L$ ).

## Equilibrium construction

For  $z < \beta$ , the low-type must be indifferent between accepting  $w(z)$  and waiting.

The waiting payoff is

$$F_L(z) = \mathbb{E}_z^L \left[ e^{-rT(\beta)} K_H \right]$$

which evolves as

$$rF_L(z) = \left( \dot{q}(z) - \frac{\phi^2}{2} \right) F_L'(z) + \frac{\phi^2}{2} F_L''(z)$$

So,  $\dot{q}(z)$  must satisfy

$$\dot{q}(z) = \frac{rF_L(z) + \frac{\phi^2}{2} F_L'(z) - \frac{\phi^2}{2} F_L''(z)}{F_L'(z)}$$

# Equilibrium verification

## Seller optimality ✓

- ▶ By construction, low type is indifferent between accepting and rejecting at all  $z < \beta$ .

## Buyer optimality: Recall the **three necessary conditions** ✓

- ▶ By construction, **option to wait** holds with equality for all  $z < \beta$ .
- ▶ In addition, we verify directly that for all  $z' > z$ ,

$$\underbrace{F_B(z)}_{\text{Eq payoff}} > \underbrace{\left(1 - \frac{p(z)}{p(z')}\right) (V_L - F_L(z')) + \frac{p(z)}{p(z')} F_B(z')}_{\text{Payoff from deviating to } w=F_L(z')}$$

- ▶ So buyer cannot benefit from deviating to a higher (or lower) offer.

## No “no-trade” regions

We assumed  $\dot{q} > 0$ , could there exist an interval on which  $\dot{q} = 0$ ?

► No.

If there was, the low-type's waiting payoff would be strictly lower:

$$E_z^L \left[ e^{-rT(\beta)} K_H \right] = F_L(z) < (1 + e^z) F_B'(z) + V_L - F_B(z)$$

or, equivalently

$$\Gamma(z) > 0,$$

which means the buyer actually wants to speed up trade, a contradiction.

**Intuition:** If trade rate ever slows to zero, the low type becomes too cheap for the buyer not to want to trade (i.e.,  $\Gamma(z)$  turns positive).