Signal or noise? Uncertainty and learning about whether other traders are informed

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Learning about other traders

Investors face uncertainty about:

- fundamentals
- characteristics / motives of other investors

Asset pricing models typically ignore the latter

- Characteristics / motives of others are common knowledge
- Even with asymmetric info, agents know whether others are informed
- Perhaps unrealistic: Uninformed investors know a lot!

Our framework: Uninformed investors are uncertain about other traders, and must learn about them
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Overview of Results

- Rational investors are **uncertain** about whether others are trading on informative signals or noise
- Over time, they **learn** using dividends and prices
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Generates a rich set of return dynamics:

- Price reacts asymmetrically to good news vs. bad news
- Stochastic, predictable expected returns and volatility
- Volatility clustering and the “leverage” effect
- Disagreement-return relation is non-monotonic and time-varying
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Underlying Mechanism:

(i) Uncertainty about others leads to non-linearity in prices,
(ii) Learning about others generates persistence.
Related Literature

- Uncertainty about others

- Stochastic volatility and expected returns through learning

- Non-linearity in prices

- Investors “agree to disagree” but update beliefs using prices
  - Banerjee, Kaniel and Kremer (2009)
Benchmark Model: Payoffs and Preferences

Two date, two securities

- Risk-free asset with return normalized to $R = 1 + r$
- Risky asset has price $P$ and pays dividends

$$D = \mu + d,$$
where $d \sim \mathcal{N}(0, \sigma^2)$
Benchmark Model: Payoffs and Preferences

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Mean-variance preferences over terminal wealth

$$x_i = \frac{\mathbb{E}_i[D] - RP}{\alpha \text{var}_i[D]}$$
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Aggregate supply of the risky asset is \textit{constant}:

$$\sum_{i} x_i = Z$$
Benchmark Model: Information and Beliefs

Two groups of investors, competitive, identical within group:

1) **Uninformed** \( (U) \): (e.g., arbitrageurs, liquidity providers) No private information, but can learn from prices and residual demand.

\[ S_I = d + \varepsilon_I, \quad \varepsilon_I \sim N(0, \sigma_e^2) \]

Noise / Sentiment \( (\theta = N) \): (e.g., retail) Receive uninformative signals

\[ S_N = u + \varepsilon_N, \quad \varepsilon_N \sim N(0, \sigma_e^2), \quad u \sim N(0, \sigma_u^2) \]

- Empirically relevant: over-confidence or differences of opinion
- Unconditional distribution of \( S_N \) and \( S_I \) are identical
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   Informed \((\theta = I)\): (e.g., institutions) Receive informative signals

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S_I = d + \varepsilon_I, \quad \varepsilon_I \sim \mathcal{N}(0, \sigma^2_e)
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   **Noise / Sentiment** ($\theta = N$): (e.g., retail) Receive uninformative signals
   
   \[ S_N = u + \varepsilon_N, \quad \varepsilon_N \sim \mathcal{N}(0, \sigma^2_e), \quad u \sim \mathcal{N}(0, \sigma^2) \]

   which they **incorrectly** believe to be informative about dividends.
   - Empirically relevant: over-confidence or differences of opinion
   - Unconditional distribution of $S_N$ and $S_I$ are identical
Uncertainty about other investors

**Key Feature:** U investors are uncertain about who they face

- At any date $t$, either $N$ or $I$ investors are present (but not both).
- Denote the type of trader at date $t$ by $\theta \in \{I, N\}$.
- Denote the likelihood of others being informed by $\pi = \Pr(\theta = I|\mathcal{I}_U)$.
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Model nests rational expectations (RE) and differences of opinions (DO)

- When $\pi = 1$, $U$ and $\theta$ investors have common priors (RE)
- When $\pi = 0$, $U$ and $\theta$ investors agree to disagree (DO)
Characterizing Equilibria

Following Kreps (1977), we assume investors can observe residual supply

- Non-existence when $U$ investors only observe price

Since aggregate supply $Z$ is fixed, $P$ and $Z - x_U$ can perfectly reveal $S_\theta$
Characterizing Equilibria

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**Definition**: An equilibrium is signal-revealing if uninformed investors can infer the signal $S_\theta$ from the price and the residual supply

- Unique equilibria in static model & dynamic benchmarks
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**Definition:** An equilibrium is *signal-revealing* if uninformed investors can infer the signal $S_\theta$ from the price and the residual supply

- Unique equilibria in static model & dynamic benchmarks

**Important:** Signal-revealing $\neq$ fully informative

- $U$ investors are uncertain about fundamentals since they don’t know whether $\theta$ is informed!
Learning about dividends

Investor $\theta$’s beliefs about $d$ are:

$$E_\theta [d] = \lambda S_\theta \quad \text{and} \quad \text{var}_\theta [d] = \sigma^2 (1 - \lambda),$$

where $\lambda = \frac{\sigma^2}{\sigma^2 + \sigma^2_\varepsilon} \in [0, 1]$
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where \( \lambda = \frac{\sigma^2}{\sigma^2 + \sigma^2_\varepsilon} \in [0, 1] \)

Conditional on \( \pi = \Pr(\theta = I|\mathcal{I}_U) \), investor \( U \)'s beliefs are:

\[
\mathbb{E}_U [d] = \pi \lambda S_\theta + (1 - \pi) 0
\]

\[
\text{var}_U [d] = \pi \sigma^2 (1 - \lambda) + (1 - \pi) \sigma^2_\varepsilon + \pi (1 - \pi)(\lambda S_\theta)^2
\]

- expectation of cond. variance
- variance of cond. expectation
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Conditional on $\pi = \Pr(\theta = I | I_U)$, investor $U$’s beliefs are:

$$\mathbb{E}_U [d] = \pi \lambda S_\theta + (1 - \pi)0$$

$$\text{var}_U [d] = \pi \sigma^2 (1 - \lambda) + (1 - \pi)\sigma^2 + \pi (1 - \pi) (\lambda S_\theta)^2$$

**Note:** When $U$ is uncertain about $\theta$, the variance increases with $S_\theta^2$. 

*Banerjee & Green (2015) Signal or Noise?*
Benchmark Model: Equilibrium

- Static: Uncertainty about $\theta$, i.e., $\pi \in (0, 1)$, but no learning
  - Since unconditional distribution of $S_N$ and $S_I$ are same, cannot update $\pi$
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- Optimal $\theta$ demand is monotone in $S_\theta$:

$$x_\theta = \frac{\mathbb{E}_\theta[D] - RP}{\alpha \text{var}_\theta[D]} = \frac{\mu + \lambda S_\theta - RP}{\alpha \sigma^2 (1 - \lambda)}$$

$\Rightarrow$ Equilibrium is signal-revealing
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$\Rightarrow$ Equilibrium is signal-revealing

- Optimal $U$ demand depends on conditional beliefs:

$$x_U = \frac{\mathbb{E}_U[D] - RP}{\alpha \text{var}_U[D]} = \frac{1}{\alpha \pi(1 - \lambda)\sigma^2 + (1 - \pi)\sigma^2 + \pi(1 - \pi)(\lambda S_\theta)^2} \left( \mu + \pi \lambda S_\theta - RP \right)$$

- Solve for $P$ using market clearing: $x_U + x_\theta = Z$
Equilibrium price is non-linear in $S_\theta$

$$P = \frac{1}{R} \left( \kappa \mathbb{E}_\theta[D] + (1 - \kappa) \mathbb{E}_U[D] - \alpha \kappa \sigma^2 (1 - \lambda) Z \right)$$

where

$$\kappa = \frac{\text{var}_U [d|S_\theta]}{\text{var}_U [d|S_\theta] + \text{var}_\theta [d|S_\theta]}$$

is increasing in $S^2_\theta$ for $\pi \in (0, 1)$, and hump-shaped in $\pi$. 

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Both components are non-linear in $S_\theta$

- Uncertainty about $\theta \Rightarrow \text{var}_U[d|S_\theta]$ depends on $S_\theta$
- Price is linear in $S_\theta$ for standard RE / DO models ($\kappa$ constant)
Asymmetric price reactions to good vs. bad news

Prediction 1: *The price reacts more strongly to bad news*

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### Intuition:

- **Good news** (positive $S_\theta$) increases expectations about fundamentals, but also increases $U$'s uncertainty about fundamentals $\Rightarrow$ offsetting effects.
- **Bad news** (negative $S_\theta$) decreases expectations about fundamentals, and also increases $U$'s uncertainty about fundamentals $\Rightarrow$ reinforcing effects.

If risk concerns are large enough, price decreases with additional good news.

### Empirical evidence for asymmetric price reactions:

- **Aggregate level:** Campbell and Hentschel (1992)
- **Firm level:** Skinner (1994), Skinner and Sloan (2002)

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Banerjee & Green (2015)

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Dynamic Model: Setup

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- Competitive OLG, mean-variance investors

\[ x_{i,t} = \frac{\mathbb{E}_{i,t} \left[ P_{t+1} + D_{t+1} \right] - R P_t}{\alpha \text{var}_{i,t} \left[ P_{t+1} + D_{t+1} \right]} \]
Dynamic Model: Setup

- Infinite horizon
- Competitive OLG, mean-variance investors
  \[ x_{i,t} = \frac{\mathbb{E}_{i,t}[P_{t+1} + D_{t+1}] - RP_t}{\alpha \text{var}_{i,t}[P_{t+1} + D_{t+1}]} \]
- Dividends are persistent
  \[ D_{t+1} = \rho D_t + (1 - \rho)\mu + d_{t+1} \]
  where \( d_{t+1} \sim \mathcal{N}(0, \sigma^2) \) and \( \rho < 1 \)
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where \(d_{t+1} \sim \mathcal{N}(0, \sigma^2)\) and \(\rho < 1\)

- We assume \(\theta_t\) follows a symmetric Markov switching process with

\[
\Pr(\theta_{t+1} = i | \theta_t = i) = q
\]

(Also look at i.i.d. case in paper)
Dynamic Model: Learning about $\theta$

Since $\theta$ investors are symmetric, $U$ cannot update $\pi_t$ using $x_{\theta,t}$ and $P_t$
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But, $U$ can update $\pi_t$ by comparing realized dividends to $S_{\theta,t}$, i.e.,

$$
\pi_{t+1} = \frac{\pi_t \Pr (S_{\theta,t} | \theta = \text{I}, d_{t+1})}{\pi_t \Pr (S_{\theta,t} | \theta = \text{I}, d_{t+1}) + (1 - \pi_t) \Pr (S_{\theta,t} | \theta = \text{N}, d_{t+1})}
$$

Intuitively:
- When the dividend is in line with the signal → increase $\pi_t$
- When the dividend is a surprise given the signal → decrease $\pi_t$
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Intuitively:

- When the dividend is in line with the signal $\rightarrow$ increase $\pi_t$
- When the dividend is a surprise given the signal $\rightarrow$ decrease $\pi_t$
Figure: Updated beliefs after observing $P_t, d_{t+1}$ starting from $\pi_t = 0.75$.

When $\theta$ is serially correlated, $\pi_t$ is stochastic and persistent
Proposition: In any signal-revealing equilibrium, the price is

\[
P_t = \frac{1}{R} \left( \begin{array}{c}
\bar{E}_t \left[ P_{t+1} + D_{t+1} \right] - \alpha \kappa_t \text{var}_{\theta,t} \left[ P_{t+1} + D_{t+1} \right] Z \\
\text{expectations}
\end{array} \right)
\]

\[
\begin{array}{c}
\text{risk-premium}
\end{array}
\]

where

\[
\bar{E}_t [\cdot] = \kappa_t E_{\theta,t} [\cdot] + (1 - \kappa_t) E_{U,t} [\cdot]
\]

is the weighted average expectation of future payoffs, and

\[
\kappa_t = \frac{\text{var}_{U,t} [P_{t+1} + D_{t+1}]}{\text{var}_{U,t} [P_{t+1} + D_{t+1}] + \text{var}_{\theta,t} [P_{t+1} + D_{t+1}]} \in (0, 1)
\]

measures the relative precision of the \( U \) and \( \theta \) investors.
Price components in the dynamic model

Similar comparative statics in the dynamic equilibrium

(a) Expectations component

(b) Risk premium component

Implications?
Predictability in expected returns and volatility

**Prediction 2:** *Learning about other traders leads to stochastic but predictable expected returns and volatility*
Predictability in expected returns and volatility

**Prediction 2:** Learning about other traders leads to stochastic but predictable expected returns and volatility

**Intuition:** Prices and, therefore, return moments depend on $\pi_t$

- Updates to $\pi_t$ depend on $S_{\theta,t}$ and $d_t \Rightarrow$ stochastic moments
- Persistence in $\pi_t \Rightarrow$ predictability of return moments
Volatility Clustering

Prediction 3: If $\pi_t$ is sufficiently large, a return surprise (in either direction) predicts higher volatility and expected returns in the future.
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Intuition: $\pi_{t+1}$ is decreasing in surprises in $d_{t+1}$

- Recall that, conditional on $\theta = I$, $E[S_{\theta,t}|d_{t+1}] = d_{t+1}$
- A large surprise in $d_{t+1}$, relative to $S_{\theta,t}$, decreases the likelihood that $\theta = I$, i.e., decreases $\pi$
- From high $\pi$, this implies $U$ faces more uncertainty about $\theta$
- Higher uncertainty $\Rightarrow$ higher expected return and higher volatility
Disagreement and Returns

**Prediction 4:** Relation between disagreement and expected returns is non-monotonic and varies over time (with $\pi_t$).

Unlike RE / DO models, disagreement depends on $\pi_t$:

$$E (|E_{U,t} [D_{t+1}] - E_{\theta,t} [D_{t+1}]|) \propto (1 - \pi_t) \lambda \sigma$$

When $\pi_t$ drives disagreement:

- **High disagreement (low $\pi_t$):** Returns decrease with disagreement  
  Disagreement $\uparrow \Rightarrow \pi_t \downarrow \Rightarrow$ lower uncertainty about others

- **Low disagreement (high $\pi_t$):** Returns increase with disagreement  
  Disagreement $\uparrow \Rightarrow \pi_t \downarrow \Rightarrow$ higher uncertainty about others
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Consistent with Banerjee (2011): high $\pi_t \approx$ RE and low $\pi_t \approx$ DO
Linear return-disagreement relation may be mis-specified!

Helps reconcile the mixed empirical evidence on the return-disagreement relation:

- Positive relation: Qu Starks Yan (2004), Banerjee (2011)
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Existing specifications are univariate, linear, and constant over time

Need to control for $\pi_t$ (e.g., PIN (?), institutional ownership)
Parametrization

Show robustness of results

- Theory is in terms of dollar returns i.e., $Q_{t+1} = P_{t+1} + D_{t+1} - RP_t$
- Parametrize the model to generate moments of *rates of return* i.e., $r_{t+1} = Q_{t+1}/P_t$

Get a sense of economic magnitude
Parametrization

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Get a sense of economic magnitude

Set parameters such that for $\pi = 1$ and $\lambda = 0.75$, we have:

$$\mathbb{E}[r_{t+1} - r_f] = 7.5\% \text{ and } \sigma(r_{t+1}) = 22\%$$

- Dividend process: $\mu = 4\%, \sigma = 6\%, \rho = 0.95$,
- Other parameters: $r_f = 3\%, Z = 1, \alpha = 1$ and $q = 0.75$. 
Return Moments

(a) Expected Excess Rate of Return
(b) Volatility of Rate of Return

- For $\pi = 1$, change in $\lambda$ from 0.25 to 0.75
  $\Rightarrow$ exp returns: 9.2% to 7.5%; volatility: 25% to 22%

- For $\lambda = 0.75$, change in $\pi$ from 1 to 0.5
  $\Rightarrow$ exp returns: 7.5% to 10%; volatility: 22% to 30%
Volatility Clustering

- For $\pi = 1$ or $\pi = 0$, there is no response
- For $\pi = 0.95$, $\lambda = 0.75$,
  - one std. dev. surprise $\Rightarrow$ exp ret: 9% to 9.6%, vol: 25% to 27%
  - two std. dev. surprise $\Rightarrow$ exp ret: 9% to 10.2%, vol: 25% to 32%

(a) Future Excess Returns
(b) Future Volatility
Asymmetric Price Reaction $\rightarrow$ Leverage effect

(c) Excess return ($R_{e,t+1}$)

(d) Squared excess return ($R_{e,t+1}^2$)

Returns exhibit reversals: Signals are i.i.d. and short-lived

Asymmetric price reaction: Bigger reversals, higher volatility after negative returns
Robustness: Rational expectations with aggregate noise

Suppose investors have common prior beliefs, but aggregate supply is noisy

- The $\theta = N$ investor knows he does not have information

$$x_\theta = \frac{\mathbb{E}_\theta [d] - RP}{\alpha \text{var}_\theta [d]} = \begin{cases} \frac{\lambda S_\theta - RP}{\alpha \sigma (1 - \lambda)} & \text{if } \theta = I \\ \frac{0 - RP}{\alpha \sigma} & \text{if } \theta = N \end{cases}$$

- Market clearing condition is given by

$$x_\theta + x_U = Z + z, \quad z \sim N(0, \sigma_z^2)$$
Robustness: Rational expectations with aggregate noise

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- Market clearing condition is given by

$$x_\theta + x_U = Z + z, \quad z \sim N(0, \sigma_z^2)$$

- $U$ conditions on $P$ and residual supply $Z + z - x_\theta$ to construct:

$$y \equiv \alpha \sigma^2 (1 - \lambda) (x_\theta - z) + P$$

- Since $x_\theta$ is not symmetric, $y$ is informative about $\theta$
- Conditional on $\theta = I$, $y$ is informative about dividends
Price decomposition in the Noise Trader Version

Show existence of an equilibrium with price:

\[
P = \frac{1}{R} \left( (\kappa + (1 - \kappa) \pi \lambda_y) y - \kappa \alpha \sigma^2 (1 - \lambda) Z \right)
\]

(Expectations component)

Risk-premium component

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(a) Expectations Component

(b) Risk-premium Component
Summary

Key feature: Uncertainty and learning about whether others are informed

This uncertainty has rich implications for return dynamics

- Non-linear price that reacts asymmetrically to good news vs. bad news
- Stochastic, persistent return moments, even with i.i.d. shocks
- Volatility clustering and the “leverage” effect
- Disagreement-return relation is non-monotonic and time-varying

Model is stylized for tractability and to highlight intuition

- Intuition should be robust to alternative forms of uncertainty e.g., about proportion of informed trading
- Future work: Extension to study dynamic information acquisition