Dynamics, Asymmetric Information and News

Brett Green

Designed for PhD course on Daley and Green (2012)
Overview

The literature studies a plethora of static environments where agents have private information.

- Important to distinguish between mechanism design/contract theory (optimal incentive schemes) and competitive market equilibria. We’ll focus on the latter today.


- A natural question to ask is: How robust are the predictions of our static models to dynamic environments? This is what we will explore today.

- Then we will investigate the impact that gradual information revelation has on trade dynamics.
The Basic Setting

Single seller with asset of type $\theta \in \{L, H\}$

- Asset is $H$ (w.p. $\pi$), $L$ (w.p. $1 - \pi$)
- Seller privately knows $\theta$, buyers do not
- Seller has flow value $k_\theta$, buyers derive flow value $v_\theta$.
- Common knowledge of gains from trade: $k_\theta < v_\theta$
- All agents are risk neutral and have common discount rate $r$
Preliminaries

- We use this setup to analyze both Akerlof and Spence. Then investigate their dynamic counterpart.

- Seller’s outside option (i.e., if she never trades):

  \[ K_\theta = \int_0^\infty e^{-rs}k_\theta \, ds = \frac{k_\theta}{r} \]

- Next, consider how much a buyer is willing to pay assuming both types sell:

  \[ E[V_\theta] = \pi V_H + (1 - \pi)V_L \]

  where \( V_\theta = \frac{v_\theta}{r} \).
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Akerlof’s Market for Lemons

The extensive form:

- There is a single date at which trade can occur \((t = 0)\).
- Multiple \((> 2)\) buyers arrive and make-take-it-or-leave-it offers. Note: Buyers compete ala Bertrand so make zero-profit.
- If seller rejects, her payoff is \(K_\theta\).
- If seller accepts an offer of \(w\), her payoff is just \(w\).
- Winner buyer’s payoff is \(w - V_\theta\). Losing buyers make zero.

What is the equilibrium outcome?
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Two cases:

1. $K_H < E[V_\theta] \implies$ Market for Everything
   - Equilibrium price is $w = E[V_\theta]$
   - Both types trade w.p.1. $\implies$ Outcome is efficient.

2. $K_H > E[V_\theta] \implies$ Market for Lemons
   - Hit won’t sell for less than $K_H$.
   - Any $w \geq K_H$ will lose money on average.
   - Hence $w < K_H$. Zero profit implies $w = V_L$.
   - Hits don’t trade. Letdowns sell for $V_L$ w.p.1. $\implies$ Outcome is inefficient.
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Is the prediction from the Market for Lemons ($K_H > E[V_\theta]$) robust to a dynamic setting?

▶ What happens the next day ($t = 1$)?
  ◆ There will be more buyers
  ◆ Only H is left—price should increase to $V_H$
  ◆ Letdown’s regret their decision to sell at $t = 0$
  ◆ The equilibrium unravels

Note: Similar criticism applies to Myers and Majluf. If only certain types of firms issue equity and invest at $t = 0$, the decision not to issue/invest reveals information about the firms type. Beliefs/price tomorrow will be different.
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The extensive form:

- Seller can *commit* to any amount of costly delay: strategy is a mapping \( \sigma : \Theta \rightarrow \Delta(\mathbb{R}_+) \).

- Buyers observe seller’s action \((t \in \mathbb{R}_+)\) and update their beliefs from \( \pi \) to \( \mu(t) \).

- Buyers simultaneously and make offers at date \( t \). Buyer \( i \)'s strategy is a mapping \( w_i : \mathbb{R}_+ \rightarrow \Delta(\mathbb{R}) \).

- Seller decides which offer to accept (if any).
Spence’s Market Signaling

Payoffs:

- To seller from trading at time $t$ for price $w$:
  
  $$ u_\theta(t, w) = \int_0^t e^{-rs} k_\theta ds + e^{-rt} w = (1 - e^{-rt}) K_\theta + e^{-rt} w $$

- To buyers:
  
  $$ V_\theta - w \quad \text{if trades with type } \theta \text{ at price } w 
  
  0 \quad \text{if does not trade} $$

- Again, seller will be paid his expected value based on Buyers’ beliefs (i.e., Bertrand competition drives profits to zero).
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Spence’s Market Signaling

Typical parametric restriction in “signaling” environments:

- \( K_H < V_L \): No adverse selection problem
- \( K_L < K_H \): Single-crossing condition

Under these conditions, the model admits two types of equilibria:

1. Full Pooling: \( \sigma_L = \sigma_H = t_p \) for any \( t_p \) such that

\[
u_L(t_p, E[V_\theta]) \geq V_L \iff t_p \leq \frac{1}{r} \ln \left( \frac{V_L - K_L}{E[V_\theta] - K_L} \right)
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2. Separating: \( \sigma_L = 0, \sigma_H = t_H \), where \( t_H \) is such that

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Equilibrium Selection in Signaling Models

- To make sharp predictions, we are left with the (difficult) task of selecting which equilibrium is “most reasonable.”
- Standard equilibrium refinements (Intuitive Criterion, D1, Universal Divinity), which are based on refining the set of off-equilibrium path beliefs, all arrive at the same conclusion.

**Proposition**

*The unique equilibrium outcome satisfying standard refinements is the least-cost-separating equilibrium.* That is, \( \sigma_L = 0 \) and \( \sigma_H = t^* \) where \( t^* \) is such that \( u_L(0, V_L) = u_L(t^*, V_H) \).

Intuition: From indifference curves (optional)
Separating Equilibria in Dynamic Settings

Question: Is this prediction robust to a dynamic setting?

- If only $H$ waits to sell, type is revealed by not selling at $t = 0$...
  - Buyers should snatch up any seller that remains for $V_H - \epsilon$
  - $L$ regrets decision to sell at $t = 0$
  - Equilibrium again unravels in a dynamic setting

Note: A similar criticism applies to Leland and Pyle.

- If a firm reveals its type through how much equity it retains at $t = 0$, then it should be able to sell the rest of the equity at fair market value at $t = 1$.
- But then low type firm who sold all their equity at $t = 0$ will regret their decision and the equilibrium unravels.
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Akerlof vs Spence

These static models are quite different:

- Akerlof - seller chooses whether to trade now or never
  - Somewhat stark as mentioned previously.
- Spence - seller can commit to any amount of costly delay
  - Is this commitment power reasonable? What stops buyers from making offers sooner?

Observation

*In a dynamic market (without commitment), these two strategic settings are virtually identical!*
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A Dynamic Market with Asymmetric Information

Reformulated version:

- Seller is interested in selling her asset but she is not forced to do so on any particular day
- If she does not sell today, she derives the flow value from the asset
- And can entertain more offers tomorrow
The Model: Daley and Green (2012)

Players:
- Initial owner, \( A_0 \)
- Mass of potential buyers (the market)

Preferences:
- All agents are risk-neutral
- Buyers have discount rate \( r \)
- \( A_0 \) discounts at \( \bar{r} > r \)
The Model

The Asset:

- Single asset of type $\theta \in \{L, H\}$
- Nature chooses $\theta$: $P(\theta = H) = P_0$
- $A_0$ knows $\theta$ and accrues (stochastic) flow payoff with mean $v_\theta$
- High-value asset pays more: $v_H > v_L$
- Let $V_\theta \equiv \int_0^\infty v_\theta e^{-rt} dt$ and $K_\theta \equiv \int_0^\infty v_\theta e^{-\bar{r}t} dt$
- Assume that $K_H > V_L$ (SLC)
News Arrival

- Brownian motion drives the arrival of news. Both type assets start with the same initial score $X_0$.
- Type $\theta$ asset has a publicly observable score $(X^\theta_t)$ which evolves according to:

$$dX^\theta_t = \mu_\theta \, dt + \sigma \, dB_t$$

where $B$ is standard B.M. and WLOG, $\mu_H \geq \mu_L$.
- The quality (or speed) of the news is measured by the signal-to-noise ratio: $\phi \equiv \frac{\mu_H - \mu_L}{\sigma}$.
- You can think about $\phi$ as either a quality: how much can be learned in a certain amount of time or as a rate: how fast can something be learned. It is a sufficient statistic for the drift terms and the volatility.
- One interpretation:
  - News=cashflows or information about them: $\mu_\theta = \nu_\theta$.
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Timing

- Infinite-horizon, continuous-time setting

- At every $t$:
  - Buyers arrive and make offers.
  - Owner decides which offer to accept (if any).
  - News is revealed about the asset.

Diagram:

- Buyers arrive and make private offers
- Seller accepts (and the game ends) or rejects
- News is revealed about the seller
- New buyers arrive observing everything except previous offers

$dt$
Payoffs

To the initial owner who accepts an offer of \( w \) at time \( t \) is:

\[
\int_0^t e^{-\bar{r}s} \nu_\theta ds + e^{-\bar{r}t}w = (1 - e^{-\bar{r}t})K_\theta + e^{-\bar{r}t}w
\]

To a buyer who purchases the asset for \( w \) at time \( t \):

\[
V_\theta - w
\]
Market Beliefs and Owner’s Status

 Buyers begin with common prior: \( \pi = \mathbb{P}_{t=0}(\theta = H) \)

- At time \( t \), buyers know:
  (i) The path of news arrival and shocks up to time \( t \)
  (ii) That trade has not occurred prior to \( t \)

Equilibrium beliefs will conditioned on all of the above.

- Let \( P \) denote the equilibrium belief process
- Define \( Z = \ln \left( \frac{P}{1-P} \right) \): “beliefs” in z-space
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We will construct a stationary equilibrium of the game.

- **The state** is the market belief $z$. Any history such that:
  - Market beliefs are $z$: $Z_t(\omega) = z$

- Note that the state evolves endogenously over time—since beliefs must be consistent with strategies.

- Buyers’ strategy is summarized by the function $w$
  - $w(z)$ denotes the (maximal) offer made in state $z$

- The owner’s strategy is a stopping rule $\tau$. Roughly, a mapping from $(\theta, z, w)$ into a decision of whether to “stop” (i.e., accept).
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Strategies and Equilibria

Given $w$ and $Z$, the owner’s problem is to choose a stopping rule to maximize her expected payoff given any state.

$$\sup_{\tau} E_t^\theta \left[ \int_0^\tau v_\theta e^{-\bar{\eta}s} \, ds + e^{-\bar{\eta}\tau} w(Z_\tau) \bigg| Z_t \right]$$  \hspace{1cm} (SP_\theta)

Definition

An equilibrium is a triple $(\tau, w, Z)$:

- Given $w$ and $Z$, the owner’s strategy solves $SP_\theta$.
- Given $\tau$ and $Z$, $w$ is consistent with buyers playing best responses.
- Market beliefs, $Z$, are consistent with Bayes rule whenever possible.
Evolution of Beliefs based only on News

A useful element in the equilibrium construction is the belief process that updates only based on news:

- Starting from the initial prior $P_0$
- Let $\hat{P}$ denote the belief process resulting from Bayesian updating based only on news:

$$\hat{P}_t \equiv \frac{\pi f^H_t(X_t)}{\pi f^H_t(X_t) + (1 - \pi)f^L_t(X_t)}$$

- Where $f^\theta_t$ is the normal pdf with mean $\mu_\theta t$ and variance $\sigma^2 t$
- The evolution of $\hat{P}$ is type dependent

Why is this useful? If strategies call for trade with probability zero over any interval of time, then the equilibrium (consistent) beliefs must evolve according to $\hat{P}$ over that interval.
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$$\hat{Z}_t = \ln\left(\frac{\hat{P}_t}{1 - \hat{P}_t}\right) = \ln\left(\frac{\hat{P}_0 f_t^H(X_t)}{(1 - \hat{P}_0) f_t^L(X_t)}\right)$$

$$= \ln\left(\frac{\hat{P}_0}{1 - \hat{P}_0}\right) + \ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right)$$

$$= \hat{Z}_0 + \ln\left(f_t^H(X_t) / f_t^L(X_t)\right) + \frac{\phi}{\sigma}(X_t - \frac{(\mu_H + \mu_L)t}{2})$$

Using Ito’s lemma and the law of motion of $dX^\theta_t$ gives:

$$d\hat{Z}_t^H = \frac{\phi^2}{2} dt + \phi dB_t$$

$$d\hat{Z}_t^L = -\frac{\phi^2}{2} dt + \phi dB_t$$
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$\hat{Z}_0$ $\hat{Z}_0 + \phi \frac{1}{\sigma} (X_t - (\mu_H + \mu_L)t)$

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Equilibrium Beliefs

In equilibrium, the market beliefs evolve based on news as well as:

- The owner’s equilibrium strategy and the fact that trade has not yet occurred.

That is,

\[ dZ_t = d\hat{Z}_t + dQ_t \]

Where \( dQ_t \) is the information contained in the fact that trade occurred or did not occur at time \( t \).

For example, suppose trade does not occur at time \( t \):

- If strategies call for trade with probability zero: \( dQ_t = 0 \)
- If strategies call for a low type to trade with positive probability and a high type to trade with probability zero: \( dQ_t > 0 \)

Note: We have just linearized Bayesian updating (very convenient).
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In equilibrium, the market beliefs evolves based on news as well as:

- The owner’s equilibrium strategy and the fact that trade has not yet occurred.

That is,

\[ dZ_t = d\hat{Z}_t + dQ_t \]

Where \( dQ_t \) is the information contained in the fact that trade occurred or did not occur at time \( t \).

For example, suppose trade does not occur at time \( t \):

- If strategies call for trade with probability zero: \( dQ_t = 0 \)
- If strategies call for a low type to trade with positive probability and a high type to trade with probability zero: \( dQ_t > 0 \)

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Finding the Equilibrium
Some preliminaries

Proposition

Let $F_\theta(z)$ denote the seller’s value function. Properties that must be true of any equilibrium:

1. **Buyers make zero expected profit**
2. $F_L(z) \geq V_L$ and $F_H(z) \geq E[V_\theta|z]$ for all $z$
3. $F_L(z) \leq E[V_\theta|z]$ and $F_H(z) \leq V_H$ for all $z$
4. **The only prices at which trades occurs are $V_L$ and $E[V_\theta|z]$**
5. **If $E[V_\theta|z]$ is offered, it is accepted with probability 1**
Equilibrium for $\phi > 0$

**Main Result:**
There exists an equilibrium of the game which is characterized by a pair of beliefs $(\alpha, \beta)$ and the function s.t.

- If $z \geq \beta$: $w = B(z)$ and both type sellers accept with probability one.

- If $z \leq \alpha$: $w = V_L$, a high type seller rejects with probability one and a low type seller accepts with probability $\rho_L = 1 - e^{z-\alpha}$.

- If $z \in (\alpha, \beta)$: no trade occurs. Buyers make offers that are rejected by both type sellers with probability one.
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- If $z \in (\alpha, \beta)$: no trade occurs. Buyers make offers that are rejected by both type sellers with probability one.
Sample Path of Equilibrium Play

Low type mixes over accept/reject at \( p = a \)

Beliefs jump to zero if a low type accepts \( w = V_L \)

High type eventually reaches \( b \) and trades at \( E[V_0|b] \)

High type rejects w.p.1. at \( p = a \)

Figure: A low type may eventually sell for \( V_L \) at \( p = a \) (left), a high type never does (right). Notice that the low type accepts in such a way that the equilibrium beliefs reflect of the lower boundary. That is \( Z \) has a lower reflecting barrier at \( \alpha \).
Proof By Construction

Take \( w \) as given and assume that \( Z \) evolves as specified for some unknown \( \alpha \).

Let \( F_\theta(z) \) denote the seller’s value for the asset. The Bellman equation for the seller’s problem is:

\[
F_\theta(z) = \max \left\{ \text{payoff from accepting} \left( w(z) \right), \text{payoff from rejecting} \right\} \\
= \max \left\{ \int v_\theta \, dt + E^\theta \left[ e^{-\bar{r}dt} F_\theta(z + dZ) \right], \right\}
\]

In the no-trade region:

\[
F_\theta(z) = v_\theta \, dt + E^\theta \left[ e^{-\bar{r}dt} F_\theta(z + dZ) \right]
\]

And \( Z \) evolves only based on news, implying that \( \forall z \in (\alpha, \beta), F_\theta \) satisfies a second-order ODE. The two ODEs have simple closed-form solutions e.g.,

\[
F_L(z) = c_1 e^{u_1 z} + c_2 e^{u_2 z} + K_L
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$$F_L(z) = c_1 e^{u_1z} + c_2 e^{u_2z} + K_L$$
Six unknowns remain:

- Each ODE has two unknown constants (4 unknowns)
- $\alpha$ and $\beta$ also need to be determined (2 unknowns)

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There are six necessary boundary conditions:

- Market Belief ($z$)
- Asset Value
- $\alpha$, $\beta$
- $F_L(\alpha) = V_L$
- $F_L'(\alpha) = 0$
- $F_L(\beta) = B(\beta)$
Six unknowns remain:

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There are six necessary boundary conditions:

- **refl:** $F'_H(\alpha) = 0$
- **vm:** $F_H(\beta) = B(\beta)$
- **sp:** $F'_H(\beta) = B'(\beta)$
Six unknowns remain:

- Each ODE has two unknown constants (4 unknowns)
- $\alpha$ and $\beta$ also need to be determined (2 unknowns)

There are six necessary boundary conditions:

\[
\begin{align*}
\text{vm: } F_H(\beta) &= B(\beta) \\
\text{sp: } F_H'(\beta) &= B'(\beta) \\
\text{vm: } F_L(\beta) &= B(\beta) \\
\text{vm: } F_L(\alpha) &= V_L \\
\text{sp: } F_L'(\alpha) &= 0
\end{align*}
\]
Solution to the Seller’s Problem

Lemma

There exists a unique \((\alpha^*, \beta^*)\) that simultaneously solves the high and low-type sellers’ problem optimal stopping problem.

Proof:

- For any \(\alpha\) (i.e., lower barrier on \(Z\)), there exists a unique \(\beta\) such that the stopping rule \(\tau_H = \inf\{t : Z_t \geq \beta\}\) solves \(SP_H\). Call this mapping \(\beta_H(\alpha)\).

- Similarly, for each \(\beta\), there exists a unique \(\alpha\) such that both \(\tau_H\) and \(\tau_L = \inf\{t : Z_t \notin (\alpha, \beta)\}\) solve \(SP_L\) (i.e., the low type is indifferent between playing \(\tau_H\) and \(\tau_L\)). Call this mapping \(\beta_L(\alpha)\).

- \(\beta_H\) and \(\beta_L\) intersect exactly once (requires \(K_H > V_L\)). The intersection is \((\alpha^*, \beta^*)\).
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Intersection of $\beta_L$ and $\beta_H$

Each curve represents a solution to a class of stopping problems:

- Lower Boundary ($\alpha$)
- Upper Boundary ($\beta$)
- $(\alpha^*, \beta^*)$
- $\beta_H(\alpha)$
- $\beta_L(\alpha)$
- 45-degree line
Completing the Seller Value Functions

Outside the no-trade region, the seller’s value function is as follows:

\[
F_L(z) = F_L(\alpha^+), \quad F_H(z) = F_H(\alpha^+) \quad \forall z \leq \alpha
\]

\[
F_L(z) = B(z), \quad F_H(z) = B(z) \quad \forall z \geq \beta
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\[ F_L(z) = B(z) \]
\[ F_H(z) = B(z) \quad \forall z \geq \beta \]
Why is there no trade for \( z \in (\alpha, \beta) \)?

Intuition for the no-trade region?

- \( H \) can always get \( B \) if she wants it. This endows \( H \) with an option. For \( z \in (\alpha, \beta) \), \( H \) does better by not exercising the option.

- \( L \) can always get \( V_L \) if he wants it. But for \( z \in (\alpha, \beta) \), \( L \) does better to mimic \( H \).
Equilibrium Beliefs at $z = \alpha$

- $F_L(\alpha) = V_L$, $F'_L(\alpha) = 0 \implies$ low-type seller is just indifferent

- Seller cannot accept with an atom at $z = \alpha$

- On the other hand, $Z$ cannot drift below $\alpha$

Proposition

The low-type sells at a flow rate $s/\sigma$ proportional to $dX_t$ at $p = a$ such that:

- $Z$ reflects off $z = \alpha$ if trade does NOT occur

- $Z$ is absorbed (drops to zero) at $z = a$ if trade does occur
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Reflection in Discrete Time
Reflection in Discrete Time

$p = a^+$

$p = a$

$p = a^-$

$t = \Delta$

$t = 2\Delta$

Low type accepts with some probability
Reflection in Discrete Time

\[ p = a^+ \]

\[ p = a \]

\[ p = a^- \]

\[ t = \Delta \]

\[ t = 2\Delta \]
Reflection in Discrete Time

\[ p = a^+ \]

\[ p = a \]

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\[ t = \Delta \]

\[ t = 2\Delta \]
Uniqueness Argument
Uniqueness Argument

High type never trades when $p < p$
Uniqueness Argument

\[ V_H \]

\[ K_H \]

\[ V_L \]

\[ K_L \]

Low type cannot be waiting for all \( p < p \)
Uniqueness Argument

There exists a $p_1$ where only the low type trades (with positive probability)
Uniqueness Argument

Beliefs then jump to $a$ following a rejection.
Uniqueness Argument

Only the low type is trading. The price must be $V_L$:

\[ F_L(p_1) = F_L(a) = V_L \]
Uniqueness Argument

Constrain $F_L$ to be non-decreasing
Uniqueness Argument

Low type is indifferent at \( a \)
Cannot get \( E[V|p] \) immediately
Waiting must occur

No Trade Here
Uniqueness Argument

High type must accept $E[V | p]$ eventually

No Trade Here
Uniqueness Argument

High type must accept $E[V|p]$ eventually

No Trade Here
Uniqueness Argument

High type must be just indifferent at $p = b$

Smooth Pasting Condition: Slopes Match
Uniqueness Argument

Optimal for \( H \) to accept \( E[V|p] \) everywhere above \( b \)

High type prefers to accept here
Uniqueness Argument

Hence, $E[V \mid p]$ must be offered for all $p > b$
Uniqueness Argument

Low type waits in hopes of getting good news
Uniqueness Argument

$V_H$

- **High type**
- **Low type**

$K_H$

$V_L$

$p \quad a \quad b \quad 1$
When News is Completely Uninformative

Figure: Notice, the payoffs are the same as in the static Akerlof model.
As News Becomes More Informative
As News Becomes More Informative
Welfare Analysis

Starting from $p_0$, total welfare is:

$$p_0 F_H(p_0) + (1 - p_0) F_L(p_0)$$

Total potential welfare is the expected value of the asset to buyers:

$$p_0 V_H + (1 - p_0) V_L$$

All seller’s trade eventually so delay is the only source of inefficiency
Welfare with No News
News “Shifts” Inefficiency

\[ V_H \]

- High type
- Low type

\[ K_H \]

\[ V_L \]

Efficiency Increases
Efficiency Decreases
Dynamic Signaling in Non-Lemons Markets

What happens when the Static Lemons Condition does not hold?

- Up to this point, we have interpreted the flow payoff to the seller as a “benefit”

- In some cases, delay may impose a “cost” to the seller
  - e.g., signaling through education or the used car dealer

- In such cases, the Static Lemons Condition may not hold
  - *That is, the high type prefers to trade at* $V_L$ *rather than never trade*
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- In such cases, the Static Lemons Condition may not hold
  - That is, the high type prefers to trade at $V_L$ rather than never trade
Equilibrium when $K_H < V_L$

Theorem

When the Static Lemons Condition does not hold the equilibrium depends on the quality of the news:

1. When the news is sufficiently informative $(\phi > \bar{\phi})$, there exists an equilibrium of the same (three-region) form. Moreover it is the unique equilibrium in which the seller’s value is non-decreasing in $p$.

2. When news is sufficiently uninformative, the unique equilibrium involves immediate trade at $B(z)$ for all $z$ (similar to Swinkels, 1999).

3. For some parameter values, there exists another type of equilibrium...
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3. *For some parameter values*, there exists another type of equilibrium...
Another Type of Equilibrium

The other equilibrium looks like this:

- **Average type offered:** Both types accept w.p.1
  - **β** region

- **No Trade Region:** Non-serious offers made
  - Both types reject w.p.1
  - News drives posterior
  - **p** region

- **Average type offered:** Both types accept w.p.1
  - **α** region
Seller’s Value in Other Equilibrium

Intuition?

- $(\alpha, \beta)$ determined solely from high type
- Low-type plays no role in determining the structure
- As $s$ increases, $\alpha$ decreases
  - $\beta$ increases
  - $\alpha$ decreases

![Marginal Benefit of Waiting](Image)
Seller’s Value in Other Equilibrium

Intuition?

- \((\alpha, \beta)\) determined solely from high type
- Low-type plays no role in determining the structure
- As \(s\) increases \(\rightarrow\) more incentive to wait
  - \(\beta\) increases
  - \(\alpha\) decreases
Seller’s Value in Other Equilibrium

Intuition?

- $(\alpha, \beta)$ determined solely from high type
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- As $s$ increases $\Rightarrow$ more incentive to wait
  - $\beta$ increases
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Intuition?

- \((\alpha, \beta)\) determined solely from high type
- Low-type plays no role in determining the structure
- As \(s\) increases \(\Rightarrow\) more incentive to wait
  - \(\beta\) increases
  - \(\alpha\) decreases
Remarks

Summary thus far...

- Introduced gradual information revelation into a dynamic lemons market
- The equilibrium involves three distinct regions: capitulation, no trade and liquid markets
- News can have a dramatic effect on trade
  - More is not necessarily better
- Developed a framework to encompass both signaling lemons markets
  - The two have the same equilibrium when news is sufficiently informative