Adverse Selection, Slow Moving Capital and Misallocation

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Fall 2013
Motivation

- Economies respond sluggishly to aggregate shocks

- and capital misallocation matters.
  → e.g., Syverson (2004); Foster, Haltiwanger, and Syverson (2008)

- ...especially in developing countries.
  → e.g., Hsieh and Klenow (2009)
Motivation (cont’d)

- Adjustment costs often used to explain these patterns:
  - ‘k-dot’ adjustment cost generate slow changes in the capital stock
  - ‘i-dot’ adjustment costs to generate slow changes in investment
    * Christiano, Eichenbaum and Evans, 2005)
  - Counter-cyclical adjustment costs generate pro-cyclical reallocation
    * (Eisfeldt and Rampini, 2006)

- But what do these costs represent? Physical costs vs market frictions

- Financial frictions can also generate persistent misallocation
  - However, most distortions on the extensive rather than intensive margin
This paper

1. A micro-foundation for capital adjustment costs based on adverse selection

   → **Main idea:** capital mobility requires transactions among asymmetrically informed agents. Separating equilibrium involves delay in transactions and leads to slow moving capital

   → Flexible model generates rich reallocation dynamics

   → Degree of misallocation increases with

      * productivity dispersion (degree of adverse selection)
      * frequency of productivity shifts
      * lower interest rate

2. Applications

   → Physical (or Human) Capital Reallocation

   → Technological Innovation and New Investment
Related Literature

• Convex Adjustment Cost and Time to Build Models:

• Search and Capital Mobility:

• Financial Constraints

• Adverse Selection and Delay:
**Reallocation environment**

- Different **locations** $\ell \in \{A, B\}$. Could represent
  - Sectors, industries, or physical locations

- Mass $M > 1$ of **firms** in each location
  - Firms can operate a unit of capital only in their own location

- Unit mass of **capital** of varying quality: $\theta \sim F$ on $[\theta, \theta]$, with $dF(\theta) > 0$.
  - Quality is privately observed by owner of capital

- **Output** depends on capital quality $\theta$ and location

  \[ dy_\ell(\theta) = \pi_\ell(\theta) dt, \quad \text{where } \pi'_\ell > 0 \]

- Suppose $\pi_B > \pi_A$ but all capital is initially **allocated** to sector $A$. 
Benchmarks

- Full information benchmark
  - Capital is reallocated immediately from $A$ to $B$ with no loss in efficiency

- Convex adjustment cost models
  - Capital is homogeneous ($\theta = \bar{\theta}$).
  - Specify costs in terms of measure of capital $k$ reallocated to sector $B$

\[
c(k, \dot{k}, \ddot{k}) = \begin{cases} 
  c \left( \frac{\dot{k}}{1-k} \right)^2 & \text{('kdot')} \\
  c \left( \frac{\dot{k}}{1-k} \right)^2 (1 - k) & \text{('ik')} \\
  c \left( \ddot{k} \right)^2 & \text{('idot')} 
\end{cases}
\]

  - Focus on the central planner’s problem.

\[
\max \int_0^\infty e^{-rt} \left( (1 - k_t)\pi_A + k_t\pi_B - c(k_t) \right) dt \tag{CPP}
\]
Solution to CPP: Reallocation Dynamics

- Dynamics implied by ‘kdot’ (red), ‘ik’ (black) and ‘idot’ (blue) models.
- Shape of reallocation dynamics depends on adjustment cost formulation.
To reallocate capital, trade must occur.

Firms can trade capital in a spot market starting at $t = 0$

Market is open continuously.

- No search, transactions, or adjustment costs.

Adverse selection problem

- Capital is heterogeneous in quality: $\theta < \bar{\theta}$
- Quality is privately observed by owner.
- Lemons condition

$$\pi_A(\bar{\theta}) > \int \pi_B(\theta) dF(\theta)$$
• Firms in A choose **when** to sell capital. Their tradeoff
  → Sell now: Capture productivity gains in new sector
  → Sell later: Signal capital is of higher quality, get (potentially) better price

• Firms in B are competitive.
  → Have value $V(\theta) = \pi_B(\theta)/r$ for $\theta$-unit.
  → Prices determined by their break even condition (given seller’s strategy) and market clearing.
Equilibrium

**Seller's Problem**: Given $P_t$, sellers face a stopping problem:

$$\sup_{\tau} \int_0^\tau e^{-rt} \pi_A(\theta) dt + e^{-r\tau} P_t$$  \hspace{1cm} (SP)$$

Let $\chi_t$ denote the lowest quality asset that has not traded by time $t$:

$$\chi_t = \inf\{\theta_i : \tau_i \geq t\}$$

**Definition (Competitive Decentralized Equilibrium)**

(i) *Seller Optimality*. Given prices, $\tau(\theta)$ solves SP for all $\theta$

(ii) *Zero Profit*. Let $\Theta_t$ denote the set of types that trade at $t$. If $\Theta_t$ is not empty, then

$$P_t = \mathbb{E}[V(\theta)|\theta \in \Theta_t]$$

(iii) *Market Clearing*. The market price satisfies $P_t \geq V(\chi_t)$. 
Alternative interpretation: related to the allocation of human capital.

- Relabel ‘capital’ as ‘workers’, ‘quality’ as ‘ability’ and ‘prices’ as ‘wages’.
- Workers are privately informed of their ability $\theta$.
- Rather than the firms decision of when to sell its capital, it becomes the worker’s decision of when to migrate to sector $B$.
- Firms from sector $B$ do not observe workers ability, but compete for workers from sector $A$ through the timing and the wage they offer.

In the context of technological progress, the elasticity of substitution between worker quality and productivity $(1 - \alpha)^{-1}$ can be interpreted as the technology being skill biased ($\alpha < 1$) or ‘unskill’-biased ($\alpha > 1$).
Skimming

- Equilibria must satisfy the **skimming property**:  
  \[ \text{If it is optimal for type } \theta \text{ to trade at time } t, \text{ then strictly optimal for all } \theta' < \theta \text{ to trade at (or before) time } t. \]

- Therefore, \( \chi_t \) must be (weakly) increasing over time.
  
  \[ \rightarrow \text{Conjecture } \chi_t \text{ is strictly and continuously increasing. Fully separating equilibrium} \]
  
  \[ \rightarrow \text{Delaying trade is a signal of high quality.} \]

- Break even condition requires that
  
  \[ P_t = V(\chi_t) = \frac{\pi_B(\chi_t)}{r} \]
Equilibrium Characterization

To satisfy *Seller Optimality*, the cutoff type must be locally indifferent.

\[
\begin{align*}
\left( rP_t - \pi_A(\chi_t) \right) &= \frac{d}{dt} P_t \\
\text{Cost of Delay} & \quad \text{Benefit}
\end{align*}
\]

Combined with *Zero Profit*, we get

\[
rV(\chi_t) - \pi_A(\chi_t) = V'(\chi_t) \dot{\chi}_t
\]

where

\[
\dot{\chi}_t = \frac{d\chi_t}{dt}
\]

represents the rate at which capital ‘types’ transition across sectors.
Equilibrium Characterization (cont’d)

• From *Seller Optimality* and *Zero Profit*, we are left to solve a single differential equation

\[
\dot{\chi}_t = \frac{rV(\chi_t) - \pi_A(\chi_t)}{V'(\chi_t)}
\]

• The lowest type trades immediately. Thus, the boundary condition is given by

\[
\chi_0 = \theta
\]

• Like all separating equilibria, \( \dot{\chi}_t \) depends on distribution \( F \) only through support \( [\theta, \bar{\theta}] \).

Rate of transition is proportional to the gains from doing so.
Example

- Suppose that $\pi_B(\theta) = c\theta + d$, $\pi_A(\theta) = \theta$

- The differential equation for the cutoff type is linear in $\chi$

  $$\dot{\chi}_t = r \cdot \frac{(c - 1)\chi_t + d}{c},$$

- Therefore both $\chi_t$ and $\dot{\chi}_t$ are proportional to $e^{(\frac{c-1}{c})rt}$
  
  - Case 1. $c = 1 \rightarrow \dot{\chi}_t$ constant over time as in to 'kdot' model
  
  - Case 2. $c > 1 \rightarrow \dot{\chi}_t$ increasing over time as in 'idot' model
  
  - Case 3. $c < 1 \rightarrow \dot{\chi}_t$ decreasing over time as in 'ik' model
Reallocation dynamics in our model

- Dynamics implied by $c = 1$ (black), $c < 1$ (red), $c > 1$ (blue).
- Shape of reallocation dynamics depends on how the gain from reallocation depends on quality.
Some Comparative Statics

- **As** $c \to \infty$:
  - $\chi_t \to \theta e^{rt}$
  - Rate of trade increases, but delay persists.

- **As** $d \to \infty$:
  - $\dot{\chi}_0 \to \infty$
  - All trade occurs immediately

Though lemons condition will eventually fail in both cases.
Key takeaway and next steps

- So far,
  - Adverse selection generates slow movements in capital reallocation
  - Generates flexible reallocation dynamics...consistent with several adjustment cost models

- Next steps, a stationary (symmetric) model with transitory shocks
  - $V(\theta)$ is endogenous...depends on capital ‘liquidity’
  - Dynamics of aggregate output and productivity
  - Response to shifts in structural parameters
  - Application to technology adoption and new investment
A symmetric model with transitory shifts

Locations are symmetric:

- $\phi_t$ is a Markov process with transition probability $\lambda$
- Output per $\theta$-unit of capital in location $\ell$ is given by $\pi_\ell(\theta, \phi_t)$.

<table>
<thead>
<tr>
<th>Location</th>
<th>State</th>
<th>$\pi_A$</th>
<th>$\pi_B$</th>
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<tbody>
<tr>
<td></td>
<td>$\phi_A$</td>
<td>$\pi_1(\theta)$</td>
<td>$\pi_0(\theta)$</td>
</tr>
<tr>
<td></td>
<td>$\phi_B$</td>
<td>$\pi_0(\theta)$</td>
<td>$\pi_1(\theta)$</td>
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where $\pi_1(\theta) > \pi_0(\theta)$

- Existing capital depreciates and new capital flows in at $\delta$.
  - New capital flows into most profitable sector
  - Efficient sector maintains full support over $[\underline{\theta}, \bar{\theta}]$.

- Firms maximize expected profits, discount at $r$, let $\rho \equiv r + \delta$.
- Firms have “deep pockets” (no shortage of financial capital)
First-Best Reallocation

Absent the information friction:

- Prices can depend on quality
- Inefficiently allocated capital trades immediately after shock
  \(\rightarrow\) No reason for firms with higher quality capital to delay
- Therefore, reallocation is costless:

\[
P^{FB}(\theta) = \frac{\pi_1(\theta)}{\rho}
\]

First-Best Dynamics:

- **Reallocation**: burst upon arrival of transition, then zero
- **Output and Productivity**: constant over time, equal to

\[
dY_t^{FB} = \int \pi_1(\theta) dF(\theta)
\]
Market for Capital

- As before:
  - Firms can trade capital across locations, no institutional frictions
  - Firms can observe the location but not the quality of capital they do not own.
  - $P_t^\ell$ denotes the market price of a unit of capital located in $\ell$ at time $t$.

- Note that prices are not immediately pinned down
  - Capital is illiquid: high quality capital
    - trades less frequently → more frequently misallocated
    - trades at a higher discount relative to $\pi_1(\chi_t)/r$
  - Need to solve for prices and $\chi_t$ jointly
Stationary Equilibria

• Conjecture a stationary (symmetric) separating equilibrium in which capital moves slowly across locations in response to a shock.

• Such an equilibrium can be characterized by a pair \((\tau, V)\).

• Let \(m_t \equiv t - \sup\{s \leq t : \phi_s^+ \neq \phi_s^-\}\) denote the amount of time that has elapsed since the last shift in the state variable occurred.

Definition

The strategies and prices that are \textbf{consistent} with \((\tau, V)\) are given by:

\[
T_t^i(\theta) = \inf\{s \geq t : m_s = \tau(\theta), \phi_s \neq \phi_i\}
\]

\[
P_t^i = \begin{cases} 
V(\chi_{m_t}) & \text{if } \phi_t \neq \phi_i \text{ and } m_t < \tau(\bar{\theta}) \\
V(\bar{\theta}) & \text{otherwise}
\end{cases}
\]
• Key state variable is $\chi_t$: lowest quality of capital inefficiently allocated

• $V_0(\theta, \chi) = \text{(unit) value of quality } \theta \text{ capital inefficiently allocated}$

• $V(\theta) = \text{(unit) value of quality } \theta \text{ capital efficiently allocated}$

• In a competitive market, market price of capital is $P(\chi_t) = V(\chi_t)$
Equilibrium Construction

Follow same steps as before:

1. The seller’s Bellman equation is

\[
(r + \delta) V_0(\theta, \chi) = \pi_0(\theta) + \lambda (V(\theta) - V_0(\theta, \chi)) + \frac{\partial}{\partial \chi} V_0(\theta, \chi) \dot{\chi}_t
\]

2. The cutoff type (i.e., \( q = \chi \)) must be locally indifferent

\[
P'(\chi) = \frac{\partial}{\partial \chi} V(\theta, \chi) \bigg|_{\theta = \chi}
\]

3. Combining together

\[
\dot{\chi}_t = \frac{\rho V(\chi_t) - \pi_0(\chi_t)}{V'(\chi_t)}, \quad \chi(0) = \theta
\]

We are left to determine \( V(\theta) \), which is determined by \( \tau(\theta) \)
To determine $V_1$ from $\chi_t$

- Note first that

$$V_1(\theta) = \frac{\rho}{\rho + \lambda} \pi_1(\theta) + \frac{\lambda}{\rho + \lambda} V_0(\theta, \theta)$$  \hspace{1cm} (1)

- Then, we have that

$$V_0(\theta, \theta) = f(\tau(\theta)) \cdot \frac{\pi_0(\theta)}{\rho} + (1 - f(\tau(\theta))) \cdot V_1(\theta)$$  \hspace{1cm} (2)

where $f(t)$ is expected discount factor given delay period of length $t$.

- Combining (2) into (1), we get that...
Equilibrium Capital Prices

\[ V(\theta) = g(\tau(\theta)) \frac{\pi_0(\theta)}{\rho} + (1 - g(\tau(\theta))) \frac{\pi_1(\theta)}{\rho} \]

and so

\[
\frac{d}{dt} V(\chi_t) = \frac{1}{\rho} \left( \left[ g(t)\pi_0'(\chi) + (1 - g(t)) \cdot \pi_1'(\chi) \right] \dot{\chi}_t - \left( \pi_1(\chi) - \pi_0(\chi) g'(t) \right) \right).
\]

The indifference condition can now be written as

\[
\dot{\chi}_t = r \left( 1 - g(t) + \frac{g'(t)}{r} \right) \left( \pi_1(\chi) - \pi_0(\chi) \right)
\]

\[
= \frac{r \left( 1 - g(t) + \frac{g'(t)}{r} \right) \left( \pi_1(\chi) - \pi_0(\chi) \right)}{g(t)\pi_0'(\chi) + (1 - g(t))\pi_1'(\chi)},
\]

which (under modest regularity conditions) has a unique solution.
Theorem

There exists a unique \((\tau^*, V^*)\) such that the strategies consistent with \((\tau^*, V^*)\) constitute a fully separating equilibrium.

Uniqueness?

- Can there be a mass of trade at \(m_t = 0\)?
  - No. Price would jump, types prefer to wait.

- Can there be a mass of trade at some \(m_t > 0\)?
  - Possible that reallocation cycle concludes with a mass. Stronger “No Deals” condition (Daley and Green, 2012) may require it.
  - Must follow after an interval of time in which no reallocation occurs.
  - But, fully separating equilibrium is unique if there is no “gap” at the top

\[ \pi_0(\theta) = \pi_1(\theta). \]
Capital prices with transitory shocks

- With recurring shocks, prices must take into account the cost of reallocation.
- As a result, trades at a “discount” relative to first best (or if shock is permanent) due to future reallocation costs.

\[
\text{Discount}(\theta) = \frac{\pi_1(\theta)}{\rho} - V(\theta).
\]

- Discount depends on dispersion (i.e., support of \(\theta\)).
Reallocation with transitory shocks

How does $\lambda$ affect the rate of reallocation?

- **My initial intuition:**
  
  Higher $\lambda \implies$ less incentive to reallocate $\implies$ slower reallocation.

- **This intuition is not quite right.**
Reallocation with transitory shocks

How does $\lambda$ affect the rate of reallocation?

**Result**

*Consider any two symmetric economies $\Gamma_x$ and $\Gamma_y$, which are identical except that $\lambda_x < \lambda_y$. There exists a $\bar{t} > 0$ such that the rate of reallocation is strictly higher in $\Gamma_y$ than in $\Gamma_x$ prior to $\bar{t}$, i.e., $\chi'_y(t) > \chi'_x(t)$ for all $t \in [0, \bar{t}]$.***
Reallocation with transitory shocks

Correct Intuition:

• Fix equilibrium $\dot{\chi}_t$ in economy $\Gamma_x$ and consider an increase in $\lambda$
  
  Higher $\lambda$ $\implies$ delay incurred more frequently
  $\implies$ marginal cost of delay increases
  $\implies$ more incentive to mimic $\theta$

• Types arbitrarily close to $\theta$ were just indifferent in $\Gamma_x$, now strictly prefer to accept sooner in $\Gamma_y$.

• Reallocation must “speed up” at that bottom.

What about in the limit?
### Reallocation with transitory shocks

- Suppose firms were in autarky and never trade with one another

\[ V^{\text{autarky}}(\theta) = \frac{(\rho + \lambda)\pi_1(\theta) + \lambda\pi_0(\theta)}{\rho^2 + 2\rho\lambda} \]

- As \( \lambda \to \infty \)... 

\[ V^{\text{autarky}}(\theta) \to \frac{\pi_1(\theta) + \pi_0(\theta)}{2} \]

- Does autarky obtain in the limit?
  - Answer: No

### Result

*There exists a \( \theta^c \in [\underline{\theta}, \bar{\theta}] \) such that for all \( \theta \leq \theta^c \) and as \( \lambda \to \infty \),*

\[ V(\theta) \to \frac{\pi_1(\theta)}{\rho}, \]

*and for all \( \theta \geq \theta^c \),*

\[ V(\theta) \to V^{\text{autarky}}(\theta). \]
Capital values as $\lambda \to \infty$
Productivity Dispersion and Misallocation

Empirical evidence: *productivity dispersion correlated with misallocation*

- The model predicts a causal link
- Dispersion can be measured by:
  \[ \bar{\theta} - \theta \]
- Misallocation of quality \( \theta \) capital can be measured by:
  \[ m(\theta) = 1 - \frac{rV(\theta) - \pi_0(\theta)}{\pi_1(\theta) - \pi_0(\theta)} \]
- Aggregate misallocation by:
  \[ \int_{\theta}^{\bar{\theta}} m(\theta) dF(\theta) \]
- Aggregate rate of reallocation by:
  \[ \bar{R} = \frac{1}{\tau(\theta)} \int_{0}^{\tau(\bar{\theta})} R_t \, dt \]
Productivity Dispersion and Misallocation (cont’d)

$m(\theta)$

$\bar{m}(\theta)$

$\bar{R}$
Aggregate Output

Output of sector $i$ at time $t$:

$$Y^i_t = \int y^i_t(\theta) dF^i_t(\theta).$$

Total output is given by

$$Y_t = Y^A_t + Y^B_t$$

Figure: Response to a sectoral productivity shift, where at $t=0$, sector B becomes the more productive sector. The economy recovers slowly from a productivity shift even though aggregate potential output is unchanged.
Aggregate Productivity

The average productivity of capital in each sector as

\[ X^i_t = \frac{Y^i_t}{k^i_t}. \]

Since aggregate capital is constant, aggregate productivity is equal to total output, \( X_t = Y_t \).
Impulse Response to Structural Changes

• We consider the effect of an unanticipated change in the model’s structural parameters on aggregate quantities.
  → Link time variation in adjustment costs to economic environment.

• Consider 3 types of changes
  1. Increase in dispersion of capital quality $\bar{\theta} - \theta$
  2. Increase in frequency of productivity shifts: $\lambda$
  3. Reduction in the interest rate: $r$

• Methodology:
  → First simulate a sequence of shocks assuming no structural shifts.
  → Holding the sequence of shocks fixed, permute the model by introducing an unanticipated parameter change at time 0
  → Compute the deviation across the two paths. Repeat 1,000,000 times.
  → Report mean deviations over all simulations in the rate of reallocation $(\Delta R_t = R_t - R_t^{SS})$, the percentage of misallocation $(\Delta M_t)$, and total output $(\Delta \log(Y_t))$. 
Impulse Response: Capital Dispersion

Figure: Response to an increase in the dispersion of capital quality. Figures plot mean difference from steady state across simulations.
Impulse Response: Expansionary Monetary Policy

Figure: Reducing $r$ decreases the cost of delaying $\Rightarrow$ reallocation occurs more slowly.

- Standard model: lower $r$ increases investment.
- Our model: lower $r$ reduces reallocative efficiency.
New Investment vs Reallocation

- So far, we have focused on reallocation of existing capital.
- Most of the macroeconomic literature focused on ‘new’ investment.
- Goal: Develop extension to capture this...
Technological Innovation and New Investment

- Unit mass of entrepreneurs, their type $q \sim F(\theta)$ on $[\theta, \bar{\theta}]$.
  - They must sell old firms before they can start a new firm.

- Firms generate cash flows $\pi_i(q)$ depending on the vintage of innovation $i$ and the quality $\theta$ of the entrepreneur.
  - Competitive investors (or households) can buy existing firms from the entrepreneurs.
  - Investors can observe vintage $i$ but not the quality $\theta$.

- New technology or investment opportunity arises at rate $\lambda$.
  - For all $\theta$ and $i < j$:
    $$\pi_i(\theta) < \pi_j(\theta)$$

- Type is persistent:
  - When starting a new firm, with probability $\kappa \in [0, 1]$ the entrepreneur retains its previous type and with $(1 - \kappa)$ it draws a new type $\theta \sim F(\theta)$.
The graphs show the functions $Y_i(t)$, $K_i(t)$, $\bar{Y}(t)$, $\bar{K}(t)$, $Y_i(t)/K_i(t)$, and $\bar{Y}(t)/\bar{K}(t)$ for different technologies labeled as Tech 1, Tech 2, and Tech 3.
Jovanovic and Rousseau (2005) on comparing Electrification with IT

Productivity growth in the two General Purpose Technology (GPT) eras tended to be lower than it was in other periods, with productivity slowdowns taking place at the start of the two eras and the IT era slowdown stronger than that seen during Electrification. Both improved as they were adopted...

Proposition

Upon the arrival of an innovation, economy wide TFP is initially decreasing (and eventually increasing) over time if and only if persistence ($\kappa$) is sufficiently high.

Intuition: Less talented entrepreneurs migrate first to the new technology and type is persistent; low-skill entrepreneur’s are more likely to create low-type projects. Eventually, good types migrate, raising average productivity in new technology.
Conclusion

- Presented an adverse selection based mechanism for generating slow movements in capital
- A micro-foundation for convex adjustment cost models
- Capable of generating rich dynamics/predictions
  - Delayed response to shocks
  - Productivity dispersion amplifies misallocation
  - TFP slowdowns in response to innovation
- A number of possible applications
  - Physical capital reallocation
  - Human capital reallocation
  - Technological innovation and new investment
  - Slow moving financial capital