Product Assortment and Pricing in the Presence of Retail Competition and Store Brands

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Abstract

Annual sales of store brands and retailer introductions of new store brands have grown quickly in recent years. We address retailers’ product assortment decisions (in terms of store and national brands) and related pricing decisions at two competing retailers, along with pricing decisions of a leading national brand manufacturer. Each retailer can offer the national-brand product and a competing store-brand product in the same category. We model the dynamics via a manufacturer-Stackelberg game. The national brand manufacturer first sets a wholesale price (the same for both retailers) and then the retailers engage in a Nash pricing game for the product(s) they choose to carry. Finally, customers, who are heterogeneous with respect to both their loyalty to the retailers and their willingness to pay for quality, decide whether and what to purchase.

Among other things, we derive conditions in which the retailers offer only one of the products or both. We also characterize the national brand manufacturer’s wholesale pricing strategy, and in particular, show how it depends upon the quality levels of the store-brand products. We explore the effects of customer loyalty and differential unit variable costs (as a function of quality) for the two store brands on equilibrium characteristics.

Keywords: store brands; national brands; product assortment; pricing; retail competition
1 Introduction

Sales of store brands have grown rapidly over the past decade, with an increase of 21% between 2007 and 2011 to over $90 billion annually. By comparison, sales of national brands have grown by only three percent during the same period. A few years ago, the growth of store brands could have been attributed to the economic recession (see, e.g., PLMA 2009). But as the economy has gradually recovered, store brand sales have continued to show signs of growth (see, e.g., Cardona 2009, Lariviere 2010, Store Brand Decisions 2012c and Store Brand Decisions 2012a).

What are the reasons underlying this trend? The general consensus from news articles and research reports is that there is a positive reinforcement cycle of store brand growth leading to retailers’ investments in store brands, which then leads to further growth. Specifically, during the economic recession, many consumers dropped national brands for store brands as they tightened their budgets. But once they switched, they tended not to switch back as they were “happy with” the new choices. Indeed, 46% of consumers think “it’s foolish to spend more if comparable quality is available from a store brand” (Consumer Edge Insight 2011). And 97% of consumers favorably compared store brand products to their previous national brand choices (PLMA 2010).

Increased customer acceptance has benefited retailers, as their stores have become better differentiated because of store brands. Stronger store brands have helped them increase store traffic and build store loyalty (see, e.g., Corstjens and Lal 2000; Ailawadi et al. 2008). Recognizing the benefits, retailers continue to invest in quality, merchandising and space allocation for store brands. For example, Supervalu plans to increase its Essential Everyday store brand offerings to 2700 products by 2013 (Store Brand Decisions 2012b). At Safeway, store brands can be found alongside competing national brands in categories “from dry cereal and frozen foods to paper towels and laundry detergent” (www.safeway.com). On its website, Safeway claims that the store brands are “the same as national brands but at a much lower price” (www.safeway.com).

An interesting case study concerning store brand assortment is the practice at Trader Joe’s. As much as 80% of the products carried there in 2010 were store brands (Kowitt 2010) and this strategy has been successful. Significantly, 92.6% of shoppers there reported they were “not bothered” by the lack of national brands and many of them even objected to the addition of national brands there. On a news webpage about Trader Joe’s product offerings, one customer commented, “Adding national brand products would be a huge mistake. Why offer a commodity you could purchase at any other retailer and therefore be at risk for ‘price comparison?’” (Thayer 2009).

This picture of competition between store brands and national brands is, however, far from complete. A key missing element is retail competition. When retailers try to win over customers from their competitors by developing their store brands, their competitors are likely to do the same. For example, in June 2012, a
few days after Kroger announced its plan to launch its store brand coffee pods for Keurig machines, Safeway launched Safeway brand Keurig coffee pods. (Kroger own the Ralph’s supermarket chain that competes with Safeway in several major metropolitan areas.) The store brand launches at each of the retail chains could then affect the other chain’s traffic. To maximize profit, both retailers need to respond to their competitor’s strategies (Geller 2012). Indeed, retail competition has become even more intense in recent years due to the expansion of non-traditional advertising channels such as social media, which allow very rapid dissemination of information on prices, perceptions of quality, etc.

Retailers have begun to realize the need to consider retail competition when devising store brand strategies. Notably, 12% of retailers reported that the primary reason they accelerated store brand development was the need to respond to expanded efforts by their competitors (Canning and Chanil 2011). Retailers are expanding their store brands into new categories. Categories such as refrigerated and frozen foods were considered “unbrandable” by retailers years ago, but now they are among the fastest-growing store brand categories (PLMA 2012). Retailers are also eliminating national brands from some product categories: 60% of retailers reported that they either already or were planning to eliminate some national brands to make room for their store brands (Canning and Chanil 2011). But questions remain. How exactly should a retailer adjust her store brand strategies, or when should she drop a national brand, in response to related strategies at her competitors? Moreover, how does retail competition affect the pricing strategy and the profit of an upstream national brand manufacturer?

We consider a setting with a manufacturer of a leading national-brand product and two retailers. Each retailer can offer the national-brand product and a competing store-brand product in the same product category. Each store brand is produced either in-house by the retailer or by a third-party manufacturer which is a non-strategic player in the game. We model the dynamics via a national brand manufacturer-Stackelberg game. First, the national brand manufacturer sets a single wholesale price offered to both retailers, taking retailers’ reactions into consideration. We assume the producer(s) of the store brand(s) is (are) non-strategic players. Major grocery chains including Safeway and Kroger own manufacturing facilities that produce some of their store-brand products. Also, there are regional brand manufacturers that produce store brands for specific markets (PLMA 2013) and do not have enough power to noticeably affect the outcome, except for a small markup over their variable costs. We leave scenarios with strategic store brand producers or production of store brands by the national brand manufacturer for future research. Given the wholesale price, the two retailers engage in a Nash pricing game for the products they choose to offer. If a retailer sets a sufficiently high price, this has the same effect as not carrying the product at all. In this way, retailers’ pricing decisions endogenize their product assortment decisions. Finally, customers decide whether and what to purchase.
Customers are heterogeneous in two dimensions: location and willingness to pay per unit of quality. In the first dimension, customers are distributed uniformly on a Hotelling line between two retailers and they incur a transportation cost for visiting each retailer, which allows us to capture the degree of customer loyalty to one retailer or the other. In the second dimension, customers’ willingness to pay is uniformly distributed within an interval. Each customer chooses a retailer to visit (if any) on the basis of which one offers the product offering the highest surplus (willingness to pay for the product less transportation cost less price). Upon arriving at the retailer, the transportation cost is sunk, so the customer then purchases the product that provides the higher difference between the willingness to pay for the product and its price, as long as it is nonnegative. Groznik and Heese (2010) use a similar model of demand; we explain the differences between their model and ours in more detail later.

From our analysis of the model, we offer insights into the following aspects of the equilibrium:

**Product assortment at retailers**
We study the product assortment and pricing problem facing competing retailers. We also compare how the retailer’s decision differs under retail monopoly and duopoly settings. Not surprisingly, we find that, ignoring the fixed cost of store brand introduction, a retailer should always introduce her store brand unless the national brand manufacturer intentionally underprices the national brand for reasons of market share while ignoring profit considerations. There is also a threshold wholesale price for the national brand product above which the retailer does not offer the national brand product. Surprisingly, although these threshold wholesale prices differ for the two retailers, under mild conditions on the customers’ transportation cost, for each retailer, the threshold is the same whether she is a monopolist or whether she faces competition from another retailer. If the national brand manufacturer offers a low wholesale price (below the smaller of the thresholds for two retailers), both retailers will offer the national brand. If the national brand manufacturer offers a high wholesale price (above the larger of the thresholds for the two retailers), neither retailer will offer the national brand. Between the two thresholds, only one retailer—the one with the store brand of lower quality—offers the national brand. This implies that, as the wholesale price of the national brand increases, the retailer with the higher-quality store brand will stop offering the national brand earlier. This partly explains the product assortment practice at Trader Joe’s: because the quality of its own-label products is high, it does not carry national brands in as many categories as its competitors do.

**Price gap between store and national brands**
The price gap between the national and store brands conveys the “value for the money” of the store brand, but it simultaneously sends a signal of the quality gap to consumers. Because of this, practitioners have been very interested in determining a good “price gap” between the national and store brands.
Hoch and Lodish (1998) conducted a study of consumer’s attitude toward prices of products in the analgesics category. Their results show that the price gap does not affect the consumers’ choice of stores. Instead, only the price level of the national and the store brand at a retailer (compared to the prices at the retailer’s competitors) matters. Moreover, the national brand price matters more in customers’ overall store choice probability. The authors thus recommended that retailers figure out whether their current prices are above or below the theoretical optimal values.

Our research can be used to aid in determining the optimal prices of the national and store brands for a retailer under competition. From our equilibrium results, we also find that retailers should respond to competition by lowering the prices of both the store and national brands, and should lower the price of the national brand to a greater extent. In other words, in the face of competition, retailers experience greater pressure on their price for the national brand than on their price for store brands. This is parallel to the empirical findings of Hoch and Lodish.

**National brand manufacturer’s product distribution**

The launch of the store brand coffee pods for Keurig machines described earlier caused the stock price of Green Mountain, a national brand, to plummet (Geller 2012). Thus, it is very important for the national brand manufacturer to develop counter-strategies. A key question is: when both retailers offer store brands, should a national brand manufacturer distribute through only one or both retailers?

We find that the answer to this question depends on the quality disparity between the two store brands. When the quality disparity is low, the national brand manufacturer prices in such a way that both retailers continue to sell the national brand. But when the quality disparity is greater than a threshold, the national brand manufacturer prices so that only the retailer with the lower store brand quality continues to offer the national brand product.

Sethuraman (2009) points out that few analytical papers derive results regarding national brand counterstrategies when facing competition from store brands; one notable exception of Mills (1999). Our work contributes to our understanding along these lines by characterizing, as part of our analysis of a three-party game, the national brands optimal strategies and how they change as costs, product quality, and market parameters change.

**Effect of customer loyalty**

Many large retail chains invest heavily in customer loyalty programs; virtually every major grocery and drug store chain has one. What is the effect of greater customer loyalty in our environment? We find that when customers exhibit no loyalty, retailers end up in a prisoner’s dilemma: both of them prefer a situation in which neither of them carries the national brand product. But at the equilibrium, both of them carry it, yet
the perfectly competitive environment leads both of the retailers to price at cost, so all of their profit comes from their store brands.

The remainder of this paper is organized as follows. We review the related literature in Section 2. Our model is presented in Section 3. In Section 4, we present each retailer’s problem of choosing her product assortment and prices, and properties of the equilibrium between the retailers for a given wholesale price of the national brand product. In Section 5, we analyze and discuss the national brand manufacturer’s optimal pricing policy, which determines which retailer(s) choose to offer the national brand. A discussion of the special cases and extensions appears in Section 6. We conclude the paper in Section 7.

2 Literature Review

A major stream within the literature on store brands investigates the strategic benefits that retailers can gain from them. First, a store brand serves as a strategic weapon for the retailer by increasing the retailer’s bargaining strength, thereby eliciting wholesale price reductions and non-price concessions (Du et al. 2005, Mills 1995, Narasimhan and Wilcox 1998, Mills 1999, Pauwels and Srinivasan 2004, Steiner 2004, Tarján 2004, Gabrielsen and Sørgard 2007). Second, store brands can be an instrument for retailers to enhance store differentiation and store loyalty (Corstjens and Lal 2000, Sudhir and Talukdar 2004, Avenel and Caprice 2006, Geylani et al. 2009). Third, store brands can help retailers to better discriminate among consumers by serving as one additional product version in its category (Wolinsky 1987, Soberman and Parker 2004, Soberman and Parker 2006). Finally, retailers may carry a store brand because they have more control over its positioning and production (Bergès-Sennou and Rey 2008, Morton and Zettelmeyer 2004). We are most interested in the role of store brands as a retailer’s strategic weapon in the vertical interaction with national brand manufacturers, particularly under retail competition.

We first discuss analytical models of the effect of store-brand introduction on national brand wholesale prices. These models are based on a manufacturer-Stackelberg game between a national brand manufacturer and one retailer whose store-brand product is available at a constant marginal cost. Under the assumption that the national brand manufacturer and the store brand producer share the same constant marginal cost, Mills (1995, 1999) finds that the wholesale price offered by the national brand manufacturer decreases as the quality of the store brand increases. When the quality of the store brand is not too low, the option of carrying a store brand imposes a threat, so the national brand manufacturer offers a lower wholesale price than when the retailer does not have a store-brand option and thereby successfully forecloses the store brand. If the quality of the store brand is very high, the store brand is sold, and as the quality of the store brand rise, the national brand manufacturer decreases its wholesale price to provide an incentive for the retailer to
sell a fair amount of the national brand instead of the store brand alternative.

Unlike Mills, Bontems et al. (1999) find that the wholesale price is not necessarily monotonically decreasing in the quality of the store brand when the unit production cost is convex and increasing with its quality. This is because the store brand suffers from a cost disadvantage when its quality exceeds a threshold, so it no longer imposes a threat. Both Narasimhan and Wilcox (1998) and Gabrielsen and Sørgard (2007) consider a model with two customer segments: one is loyal to the national brand and the other (so-called “switchers”) may choose the store brand if the price is attractive. Narasimhan and Wilcox (1998) assume that both segments have the same reservation price for the store and national brands, and obtain results that indicate the introduction of a store brand can only lead to a decrease in the national-brand wholesale price. Gabrielsen and Sørgard (2007) assume the loyal customers have a higher willingness to pay for quality, and their analysis indicates that the introduction of the store brand can lead to either an increase or a decrease in the wholesale price of the national brand, depending on the fraction of loyal customers. Soberman and Parker (2004, 2006) treat the level of advertising for the national brand as a decision variable and show that the direction of change in the average price for the product category after the store brand is introduced depends on whether advertising is expensive (with respect to its ability to increase the utility of the national brand among brand seekers).

Characteristics of vertical channel structures in the presence of store brands are also pertinent to our research. There are many analytical models of traditional vertical interactions between manufacturers and retailers (McGuire and Staelin 1983, Shugan 1985, Choi 1991, Lee and Staelin 1997), but we are not aware of any that addresses the specific characteristics of competition between store and national brands. There is, however, substantial empirical research on the nature of competition between store and national brands, which suggests that the type of interaction and the degree of competition between national and store brands is idiosyncratic across categories: it depends on whether or not the national brand is a leading product as well as the quality of the store brand. See Putsis and Dhar (1998), Cotterill and Putsis (2000), Sayman et al. (2002), Meza and Sudhir (2004) and the references therein.

How do consumers choose among different stores? How do they typically perceive store brands? A rich stream of empirical research addresses these questions. Bell et al. (1998) develop and test an empirical model to investigate the store choice behavior of households visiting a set of stores over a certain time horizon, based on the assumption that each shopper is most likely to visit the store with the lowest total shopping cost. They find that, in order to provide a comprehensive theory of store choice, both fixed and variable costs are necessary. The fixed cost is the cost independent of the shopping list whereas the variable cost depends on the shopping list. The fixed cost depends upon the travel distance, the shopper’s inherent preference for the store, and historic store loyalty, whereas the variable cost is the total expected cost of the items on the
shopping list if purchased at the store. Richardson et al. (1996) find that customers’ reliance on extrinsic cues (such as price, packaging, and brand) adversely affects customers’ propensity to purchase store brands. Baltas and Argouslidis (2007) find that quality plays a major role in the evaluation process from consumers develop their store brand preferences. Sayman et al. (2002) find that store brands are viewed as slightly more similar to secondary national brands than to the leading national brand. de Wulf et al. (2005) find that national brands enjoy brand equity while store brands do not.

Next, we review analytical models of demand when store and national brand(s) compete. In the literature, there are roughly five groups of demand models developed for such a context. Models in the first group derive demand from one representative consumer, or equivalently, by assuming consumers are homogenous (cf. Choi and Coughlan 2006 and Bergès-Sennou and Rey 2008). The second group of models uses an aggregate demand function which is linear with respect to price and is parameterized by differentiation and substitution factors (cf. McGuire and Staelin 1983, Choi 1991, Raju et al. 1995, Cotterill and Putsis 2000 and Sayman et al. 2002). Linear demand functions allow researchers to conduct sensitivity analysis on the cross-price sensitivity parameters and examine how they affect the equilibrium channel structure. However, linear demand functions are limited in their ability to accommodate the combination of complex customer choice behavior and interactions among multiple parties in a vertical channel.

The third group of models derives demand from individual utility functions of customers, which are assumed to be increasing with product quality and the customer’s willingness to pay for quality, and decreasing with the product price (cf. Mills (1995 and 1999), Bontems et al. 1999, Tarajian 2004 and Avenel and Caprice 2006). When such a utility function is used, the store brand and the national brand are assumed to differ in their quality levels, and the consumers’ willingness to pay for quality varies among consumers (a uniform distribution is usually assumed). The fourth group of models segments customers into those who are loyal to the national brand and those who are more willing to switch (cf. Narasimhan and Wilcox 1998, Gabrielsen and Sørgard 2007, Soberman and Parker 2004, Soberman and Parker 2006 and Corstjens and Lal 2000). Articles in this group examine the role of store brand introduction and study how the equilibrium would change in response to a change in segment sizes. Finally, the remaining models do not fall into any of the first four categories and are more context-specific (e.g., Morton and Zettelmeier 2004, Du et al. 2005 and Bergès-Sennou 2002).

There has been little analytical research that incorporates retail competition when both national- and store-brand products are offered. Indeed, in a recent survey paper by Sethuraman (2009) on models of national- and store-brand competition, only one article with retail competition is mentioned (Corstjens and Lal (2000)) and that article focuses on a situation with symmetric retailers. We offer a few comments on relevant articles here. Lal and Narasimhan (1996) study retail competition under the assumption that
retailers can affect consumers’ store choice by using advertising to inform them about the retail price of the national brand. They assume that each consumer demands one unit of the composite good and in addition, buys up to two units of the national brand depending on its price. They show that, because the retailers derive monopoly rents on the composite good, they compete vigorously for store traffic by advertising the price of the national brand. This intense competition for store traffic reduces retail margins on the national brand. Corstjens and Lal (2000) model a setting with retail competition and store-brand products, assuming customers are quality-sensitive and exhibit brand inertia. They show that brand inertia plays a part in customers’ store choice decisions because the store brands introduced in the first period play a role in store-differentiation in the second period of their model. Moreover, retail competition in the first period is intensified because retailers can later extract profits from customers who tried and liked store brands in the first period (due to their brand inertia).

Geylani et al. (2009) study store-brand introduction and pricing strategies for two competing retailers in the presence of “one-stop shopping” customers who visit only one retailer and view the national brand product and the store-brand product as exactly the same (except for price). The authors show that store brands enable a retailer to segment the market and thereby extract a higher price from the national-brand loyal customers because the store brand can be sold to price-sensitive one-stop shoppers. They assume the national brand manufacturer may offer different wholesale prices to the two retailers, and do not focus on how the equilibrium is shaped by retail competition.

Groznik and Heese (2010) focus on the impact of retail competition on the retailers’ decisions regarding store brand introduction. They show that under legislation associated with the Robinson Patman Act (i.e., non-discriminatory pricing), store brand introductions (or the potential for them) increase the retailers’ bargaining power vis-à-vis the national brand manufacturer, consistent with the result derived without retail competition. However, there are settings in which the retailers play a game of “chicken”. Both prefer that the competitor be the one to introduce a store brand product that enables both of them to secure a lower wholesale price, but each of them would prefer to avoid introducing a store brand themselves.

We address the retailers’ product assortment decisions along with their pricing decisions. To the best of our knowledge, the only article to explicitly address the assortment decision (including the option not to offer the national brand product) is by Fang et al. (2012), but this article treats a single-retailer setting. The authors derive conditions in which the retailer carries only the national brand product, only the store brand product, or both. They also propose a contract that coordinates the supply chain (i.e., achieves the first-best solution) when both products are offered.
3 The Model

We consider a scenario with a manufacturer of a leading national-brand product and two retailers, Retailer 1 (R1) and Retailer 2 (R2). Each retailer can offer the national-brand product and a competing store-brand product in the same product category. Each store brand is produced either in-house by the retailer or by a third-party manufacturer which is a non-strategic player in the game. We derive the equilibrium assuming that each retailer already has a store brand in place or that it is ready to be introduced. (If a retailer still needs to develop a store brand, then the firm can consider the results of the equilibrium analysis along with the fixed cost of store-brand introduction before making a decision.) Also, we assume that all parties have complete information.

The national brand manufacturer is the Stackelberg leader, and chooses the wholesale price, denoted by $w_n$, to offer to both retailers with the objective of maximizing his profit, taking into account both retailers’ reactions. We assume that the national brand manufacturer offers them same wholesale price. (In general, it is large retailers that are able to offer their own store brands. We assume the two retailers are similar in size and can therefore secure the same wholesale price. Various U.S. laws require the same pricing under the same terms of trade.) For any wholesale price offered by the national brand manufacturer, the retailers engage in a Nash game, choosing which products to offer and at what price(s), with the objective of profit maximization. The retail prices of the national-brand product and that of the store-brand product at retailer $i$ ($i = 1, 2$) are denoted by $p_n(i)$ and $p_s(i)$, respectively. In our model, choosing a very high price for either the store-brand or the national-brand product has the same effect as not offering the product at all. Therefore, when deriving the retail price equilibrium, we make the following assumption:

**Assumption.** Whenever a retailer finds it optimal not to offer some product, she sets the price at the lowest level that drives the customer demand for that product to zero.

In this way, given any wholesale price offered by the national brand manufacturer, we are implicitly modeling the product assortment decision via the Nash equilibrium in prices between retailers.

The quality level of the national-brand product is denoted by $q_n$, and those of the store-brand products at R1 and R2 are $q_{s1}$ and $q_{s2}$, respectively. Throughout our analysis, we assume these quality levels are exogenous, but we later explore how the quality levels and their differences affect the structure of the equilibrium. Without loss of generality, we assume $q_{s1} \leq q_{s2}$. We also assume that both $q_{s1}$ and $q_{s2}$ are less than $q_n$ to concentrate our attention on store brand products that have quality levels below that of similar national brand products. This applies to most store brands except so-called “premium store brands”. The marginal production cost of the national-brand product is denoted by $c_n$, and that of the store-brand product at retailer $i$ ($i = 1, 2$) is denoted by $c_{si}$. We assume that the production cost of each product is
proportional to its quality. That is, we assume \( c_{si} = k q_{si} \) for \( i = 1, 2 \) and that \( c_n = k q_n \) for some production parameter \( k > 0 \). In Section 6, we discuss results when this assumption is relaxed.

Customers are heterogeneous in two dimensions: location, which also can be interpreted as the degree of loyalty to one retailer or the other, and willingness to pay per unit of quality. We discuss each dimension in turn. For ease of exposition, we assume that in the first dimension, customers are distributed uniformly along a Hotelling line between the two retailers. Customer loyalty is captured via a transportation cost for visiting either of the retailers. Each customer’s transportation cost is the transportation cost per unit distance, \( t \), multiplied by his distance from the respective retailer. (Heterogeneity in per-unit-distance transportation costs and a more general distribution of customers vis-a-vis loyalty to the two retailers can be captured by an appropriate adjustment of the customer’s location. We discuss this further in Section 6. We assume \( t > 0 \) throughout the paper except in Section 6, where we consider the special case of no customer loyalty.) Mathematically, a customer’s location on a Hotelling line between the two retailers is denoted by \( x_1 (x_1 \in [0, 1]) \), with R1 located at \( x_1 = 0 \) and R2 at \( x_1 = 1 \). We use \( x_2 = 1 - x_1 \) to denote the customer’s distance from R2.

In the second dimension, customers have a willingness to pay per unit of quality, \( \theta \), which is uniformly distributed within an interval \([0, \theta]\). We assume \( \theta > k \) so that it is possible for the supply chain to profitably offer each product (in the absence of competition) to at least some customers. Mathematically, a customer with a willingness to pay per unit of quality \( \theta \) derives utility (willingness to pay for the product) \( \theta q_n \) from a unit of the national brand, and utility \( \theta q_{si} \) from a unit of the store brand at retailer \( i \) (\( i = 1, 2 \)). This representation of the second dimension of customer heterogeneity is a standard modeling approach and has been used in Moorthy 1988 and many papers investigating store brand strategies. Let \( \bar{v}_n \equiv \bar{\theta} q_n \) and \( \bar{v}_{si} \equiv \bar{\theta} q_{si} \), i.e., \( \bar{v}_n \) and \( \bar{v}_{si} \) denote the highest utility derived from the national-brand and store-brand products at R1 and R2, respectively.

The total number of potential customers is normalized to 1. Each customer visits only one retailer and purchases at most one unit of the product, either the national-brand or the store-brand product. When deciding which retailer to visit, each customer evaluates the maximum surplus he can derive from going to each of the retailers. At this stage, the customer calculates his willing to pay for the product under consideration as his willingness to pay per unit of quality multiplied by the product’s quality level. The customer then subtracts the sum of the transportation cost for visiting the relevant retailer and the price of the product to determine his surplus from this product. Finally, he adds the expected surplus he can derive from purchasing products in other product categories, \( M_i \), to determine the total surplus from going to each of the retailers. We assume each customer derives the same expected surplus from purchases in other product categories at either of the retailers, and that it is large enough that each customer visits one retailer or the
other. Expressed mathematically, a customer’s total surplus he can derive from going to retailer \( i \) \((i = 1, 2)\) is \( \max\{\theta q_n - tx_i - p_{ni} + M, \theta q_{si} - tx_i - p_{si} + M\} \). If \( \max\{\theta q_n - tx_1 - p_{n1} + M, \theta q_{s1} - tx_1 - p_{s1} + M\} \geq \max\{\theta q_n - tx_2 - p_{n2} + M, \theta q_{s2} - tx_2 - p_{s2} + M\} \), a customer located at \( x_1 \) chooses to visit R1. Otherwise, he/she visits R2.

After a customer located at a distance \( x_i \) from retailer \( i \) \((i = 1, 2)\) and a willingness to pay per unit of quality \( \theta \) arrives at his “preferred” retailer \( i \), the transportation cost is now sunk, so he buys the offered product that gives him the larger difference between his willingness to pay for the product and its price, if it is non-negative. That is, he buys the national-brand product if \( q_n \theta - p_{ni} \geq q_{si} \theta - p_{si} \) and \( q_n \theta - p_{ni} \geq 0 \), or he buys the store-brand product if \( q_n \theta - p_{ni} < q_{si} \theta - p_{si} \) and \( q_{si} \theta - p_{si} \geq 0 \). Otherwise, he does not buy a product in this category.

We note that Groznik and Heese (2010) use a model of demand that is identical in characterizing the heterogeneity of customers, but they assume that customers will not purchase unless the single product under purchase consideration provides the customer a surplus large enough to compensate for the transportation cost. We assume, instead, that the product in question is only one product in a market basket (as would be the case for a typical grocery shopper) and that the surplus from the market basket will outweigh the transportation cost, so each customer will visit one retailer or the other. But the customer will buy one of the products only if her utility minus the price is non-negative. Our representation allows us to consider high transportation costs, representing strong customer loyalty to the two retailers, without simultaneously driving demands down to negligible quantities. As such, our representation provides us more flexibility in exploring the effects of customer loyalty.

### 3.1 Customer Demand

Now we are ready to derive the customers’ demands given \((p_{ni}, p_{si})\) at retailer \( i = 1, 2 \). In the remainder of the paper, we use \( ni \) and \( si \) \((i = 1, 2)\) to denote the national-brand and the store-brand product at retailer \( i \) respectively. Define \( \theta_i \equiv \frac{p_{ni} - p_{si}}{q_n - q_{si}} \) and \( \hat{\theta}_i \equiv \frac{p_{si}}{q_{si}} \) for \( i = 1, 2 \). Then \( \theta_i \) represents the threshold willingness to pay per unit of quality at which customers are indifferent between purchasing the national-brand and store-brand products at retailer \( i \), and \( \hat{\theta}_i \) represents the threshold willingness to pay per unit of quality at which customers are indifferent between purchasing the store-brand product and purchasing nothing at retailer \( i \). We then have:

**Lemma 1.** For any positive wholesale price \( w_{ni} \), in any price equilibrium between the retailers, we have (i) \( \theta_1 \geq \hat{\theta}_1 \) for retailers \( i = 1, 2 \) and (ii) \( \theta_1, \theta_2, \hat{\theta}_1, \hat{\theta}_2 \in [0, \bar{\theta}] \).

All proofs can be found in the Appendix. In words, Lemma 1 says that each retailer sets prices in such a way that customers with high willingness to pay per unit of quality purchase the national brand, those with
low willingness to pay per unit of quality purchase nothing, and those in between purchase the store brand.

Define \( \tilde{x}_i \equiv t(x_i - x_j) \) for \( i = 1, 2, j = 3 - i \), i.e., the difference between the travel cost a customer incurs from going to retailer \( i \) versus retailer \( j \). Because customers’ locations are distributed uniformly on the Hotelling line between the retailers, \( \tilde{x}_i \) is uniformly distributed on \([-t, t]\). Also define \( b_{sn}^i(\theta) \equiv \theta(q_{si} - q_n) - (p_{si} - p_{nj}) \) for \( i = 1, 2, j = 3 - i \). Then, for a customer with a willingness to pay per unit of quality \( \theta \) who is located at \( \tilde{x}_i \), \( b_{sn}^i(\theta) - \tilde{x}_i \) is the difference in the customer’s surplus from purchasing a unit of the store-brand at retailer \( i \), and purchasing a unit of the national-brand product at retailer \( j \). Clearly, \( b_{sn}^i(\theta) - \tilde{x}_i = 0 \) defines the customers who are indifferent between purchasing products \( si \) and \( nj \). Define \( b_{sn}^j(\theta) \equiv \theta(q_{si} - q_{sj}) - (p_{si} - p_{nj}) \). Then, analogously, \( b_{sn}^j(\theta) - \tilde{x}_i \) is the difference in the customer’s surplus between purchasing a unit of the store-brand product from retailer \( i \) versus retailer \( j \), and customers who are indifferent between \( si \) and \( sj \) are defined by \( (\theta, \tilde{x}_i) \) that satisfy \( b_{sn}^j(\theta) - \tilde{x}_i = 0 \).

Figure 1 shows the partitioning of customer demand graphically on the \( \theta - \tilde{x}_i \) plane. Here, without loss of generality, we assume \( \theta_i > \theta_j \) (\( i = 1 \) or \( 2 \) and \( j = 3 - i \)). Under this assumption, we need to divide our analysis into two cases: \( \tilde{\theta}_i \leq \theta_j \) (shown in Figure 1(a)) and \( \tilde{\theta}_i > \theta_j \) (shown in Figure 1(b)). Note that in Figure 1, although it is not explicitly stated, \( \theta_i, \theta_j, \tilde{\theta}_i \) and \( \tilde{\theta}_j \) are not parameters, but depend upon the retailers’ pricing decisions. Expressions for the functions \( b_{sn}^i(\cdot) \) and \( b_{sn}^j(\cdot) \) also depend upon the pricing decisions. Also, for the diagrams in Figure 1, we are implicitly assuming that \( t \geq \max\{|p_{n2} - p_{n1}|, |p_{s2} - p_{s1}|, |b_{sn}^i(\theta_j)|\} \), i.e., the degree of customer loyalty is not too low, which guarantees that none of the demand regions represented in the diagrams vanish. For now, we proceed with our analysis under this assumption on \( t \), but address other cases later in this section.

With Figure 1 at hand, we can easily write the expressions for the demands for the two products at

![Figure 1: Graphical Representation of Customer Demands on the \( \theta - \tilde{x}_i \) Plane](image-url)
each retailer given a price vector \( \mathbf{p} = (p_{n1}, p_{s1}, p_{n2}, p_{s2}) \):

\[
D_{ni}(\mathbf{p}, \bar{\theta}) = \begin{cases} 
D_{ni}^H, & \text{if } \theta_i \geq \theta_j \\
D_{ni}^L, & \text{if } \theta_i < \theta_j 
\end{cases}
\]

\[
D_{si}(\mathbf{p}, \bar{\theta}) = \begin{cases} 
D_{si}^H, & \text{if } \theta_i \geq \theta_j \\
D_{si}^L, & \text{if } \theta_i < \theta_j 
\end{cases}
\]

where

\[
D_{ni}^H(\mathbf{p}, \bar{\theta}) = \frac{1}{2\theta} (p_{nj} - p_{ni} + t)(\bar{\theta} - \theta_i) \\
D_{ni}^H(\mathbf{p}, \bar{\theta}) = \frac{1}{2\theta} \left\{ \frac{1}{2} \left[ (p_{nj} - p_{ni} + t) + (b_{si}^t(\theta_j) + t) \right] (\theta_j - \theta_i) \right\} \\
D_{ni}^L(\mathbf{p}, \bar{\theta}) = \frac{1}{2\theta} \left\{ \frac{1}{2} \left[ (p_{nj} - p_{ni} + t) + (b_{si}^t(\theta_i) + t) \right] (\theta_i - \bar{\theta}) \right\} \\
D_{si}^L(\mathbf{p}, \bar{\theta}) = \frac{1}{2\theta} \left\{ \frac{1}{2} \left[ (t - b_{si}^t(\theta_i)) + (t - b_{si}^t(\theta_j)) \right] (\theta_j - \theta_i) \right\}
\]

and where, for example, \( D_{ni}^H \) is the demand for product \( ni \) if \( \theta_i \geq \theta_j \). It should be understood that \( \theta_i, \theta_j \) and \( \bar{\theta} \) are functions of \( \mathbf{p} \) as defined earlier. Similarly, \( b_{si}^t(\theta) \) and \( b_{sn}^t(\theta) \) are functions of both \( \mathbf{p} \) and \( \theta \).

Although the full expressions for these variables are \( \theta_i(\mathbf{p}), \theta_j(\mathbf{p}), \bar{\theta}_i(\mathbf{p}), b_{si}^t(\theta), \) and \( b_{sn}^t(\theta), \) in both (2) and in many formulas in the remainder of the paper, we omit the variable \( \mathbf{p} \) for brevity. It can be seen that we have four possible equilibrium scenarios, and from one scenario to another, the demand function \( \mathbf{p} \mapsto \mathbf{D} \equiv (D_{n1}, D_{s1}, D_{n2}, D_{s2}) \) is not continuous. Case 1 corresponds to \( \theta_1 \geq \theta_2 \) and \( \bar{\theta}_1 \geq \theta_2 \), Case 2 corresponds to \( \theta_1 \geq \theta_2 \) and \( \bar{\theta}_1 < \theta_2 \), Case 3 corresponds to \( \theta_1 < \theta_2 \) and \( \bar{\theta}_2 \geq \theta_1 \), and Case 4 corresponds to \( \theta_1 < \theta_2 \) and \( \bar{\theta}_2 < \theta_1 \).

The demands derived above enable us to formulate each retailer’s pricing problem. Given a wholesale price \( w_n > 0 \), and given the retail prices \( (p_{nj}, p_{sj}) \) set by the other retailer, retailer \( i \) \( (i = 1, 2 \text{ with } j = 3 - i) \) seeks a pair of prices \( (p_{ni}, p_{si}) \) that maximize her profit, \( \pi ri(w_n, \mathbf{p}, \bar{\theta}, c_{si}) \). Retailer \( i \)'s problem is:

\[
\max_{p_{ni} \geq w_n, p_{si} \geq c_{si}} \pi ri(w_n, \mathbf{p}, \bar{\theta}, c_{si}) \equiv (p_{ni} - w_n)D_{ni}(\mathbf{p}, \bar{\theta}) + (p_{si} - c_{si})D_{si}(\mathbf{p}, \bar{\theta})
\]

subject to

\[
\begin{align*}
\theta_i &\leq \bar{\theta} \\
\bar{\theta}_i &\leq \theta_i
\end{align*}
\]

Although there are four prices, retailer \( i \) chooses only \( p_{ni} \) and \( p_{si} \). Due to the form of the demand
functions shown in (1), retailer \( i \) needs to solve two subproblems, one for \( \theta_i \geq \theta_j \) and another for \( \theta_i < \theta_j \), and then choose the better solution. Moreover, due to the form of \( D^i_n(p, \bar{\theta}) \), the subproblem for \( \theta_i \geq \theta_j \) further divides into two subproblems, one for \( \bar{\theta}_i \geq \theta_j \) and one for \( \bar{\theta}_i < \theta_j \).

Let \( p^*(w_n) \) denote the retailers’ (joint) equilibrium reaction for any value of \( w_n \). Knowing \( p^*(w_n) \), the national brand manufacturer seeks to optimizes his own profit by solving the following problem:

\[
\max_{w_n \geq c_n} \pi_m(w_n, \bar{\theta}, c_n) = (w_n - c_n) \cdot \left( D_{n1}(p^*(w_n), \bar{\theta}) + D_{n2}(p^*(w_n), \bar{\theta}) \right)
\]

(4)

Before analyzing the equilibrium for the case of two retailers, we first derive properties of the equilibrium for a benchmark scenario in which there is no retail competition. The single retailer has the option of selling her store-brand as well as the national-brand product. Later, we will compare these results with those that we derive under retail competition.

### 3.2 Single Retailer Case

All customers will visit the monopolist retailer, but each customer will purchase a unit in the product category (either the store-brand or the national-brand product) only if (i) it provides a higher surplus than the other product and (ii) that surplus is non-negative. The following lemma characterizes the retailer’s optimal product assortment decision. Because there is only one retailer, we omit the subscript \( i \) for simplicity.

**Lemma 2.** [Assortment decision of a monopolist retailer] If \( w_n \leq c_n \), a monopolist retailer carries only the national brand. She sets \( p_n = \frac{1}{2}(\tilde{v}_n + w_n) \) and \( p_s = \frac{w_n}{\tilde{v}_s} q_s \) (at which price no customer would purchase the latter). If \( w_n - c_s \geq \tilde{v}_n - \tilde{v}_s \), she carries only the store brand and sets \( p_s = \frac{1}{2}(\tilde{v}_s + c_s) \) and \( p_n = p_s + (\tilde{v}_n - \tilde{v}_s) \) (at which price no customer would purchase the latter). If \( c_n < w_n < c_s + \tilde{v}_n - \tilde{v}_s \), the retailer carries both products and sets \( p_n = \frac{1}{2}(\tilde{v}_n + w_n) \) and \( p_s = \frac{1}{2}(\tilde{v}_s + c_s) \).

Lemma 2 states that a retailer’s product assortment decision depends on the value of the national brand’s wholesale price relative to two thresholds. This is consistent with the findings of Fang et al. (2012). More specifically, if \( w_n \leq c_n \), then \( \theta q_n - \theta q_s \geq w_n - c_n \) for all \( \theta \geq k \). This implies that the utility premium the national brand provides to all customers to whom the retailer can sell the store brand without incurring a loss (i.e., those with willingness to pay per unit of quality greater than \( k \)) is greater than or equal to the cost premium the retailer pays for the national brand over the store brand. Therefore the retailer does not carry the store brand because she can earn a higher profit margin on the national brand while not losing any market share. If \( \tilde{v}_n - \tilde{v}_s \leq w_n - c_s \), then we have \( w_n - c_s \geq \theta q_n - \theta q_s \) for all \( \theta \leq \bar{\theta} \). This implies that for every customer, the price premium he is willing to pay for the national brand over the store brand is less than the cost premium the retailer incurs by selling a unit of the national brand instead of the store
brand. Therefore the retailer completely forgoes the national-brand product.

If the wholesale price falls between $c_n$ and $c_s + \bar{v}_n - \bar{v}_s$, it is more profitable for the retailer to sell the national brand to the subset of customers who have a willingness to pay per unit of quality above a threshold and the store brand to the remaining customers, so the retailer carries both products. In this case, the retailer sets the retail prices in the same way as if each product were the sole product she carries. That is, she sets the price of the national (store) brand as half the sum of its procurement cost and the highest valuation for the national (store) brand among customers. As the wholesale price increases, $\hat{\theta} = p_s/q_s$ (set by the retailer implicitly via her choice of $p_s$) stays the same whereas $\theta = (p_n - p_s)/(q_n - q_s)$ increases, leading to more store brand sales and fewer national brand sales while the total sales stays the same.

We characterize the manufacturer’s equilibrium strategy and other equilibrium outcomes under this benchmark scenario in the following lemma.

**Lemma 3.** [Monopolist retailer with the option to offer a store brand] In a market with a monopolist retailer who has the option to offer a store brand, the equilibrium wholesale price is $w_n = \frac{1}{2}(\bar{v}_n + c_n) - \frac{1}{2}(\bar{v}_s - c_s)$. In response, the retailer carries both products and sets prices according to Lemma 2. The resulting customer demands are $D_n = \frac{\bar{v}}{q_n}$ for the national brand and $D_s = \frac{\bar{v}}{q_s}$ for the store brand, and the profits of the manufacturer and the retailer are, respectively, $\pi_m = \frac{\bar{v}}{q_n} \left[ (\bar{v}_n - c_n) - (\bar{v}_s - c_s) \right]$ and $\pi_r = \frac{\bar{v}}{q_n} \left[ (\bar{v}_n - c_n) + 5(\bar{v}_s - c_s) \right]$.

### 4 Assortment and Pricing under Retail Competition

The focus of this section is on the retailers’ optimal assortment and pricing strategies for a given national brand wholesale price. In Section 4.1, we derive the equilibrium and in Section 4.2, we study how the equilibrium prices and profits change with the level of customer loyalty.

#### 4.1 Retailers’ Product Assortment and Pricing

To simplify the analysis and for ease of exposition, we assume that for both retailers, their respective profit functions are quasi-concave in their prices, subject to certain conditions that we will explain later in this section. This is a common assumption and is supported by our observations from numerical examples. Given this, from results in game theory (see, for example, Osborne and Rubinstein (1994)) we know that a pure-strategy Nash equilibrium exists for the pricing game between the retailers. We now proceed to study its characteristics. The traditional approach to find a price equilibrium between two retailers is to (1) solve for the explicit expressions characterizing each retailer’s best responses and (2) solve for the crossing points of the retailers’ best responses and express them as explicit functions of $w_n$. However, in our model, the
demand functions are discontinuous and nonlinear in the price variables (see the discussion in Section 3.2). Indeed, it is impossible to obtain explicit expressions for retailers’ price responses and the equilibrium prices. We thus take an alternate approach: we derive and analyze the first-order-conditions for each retailer’s profit-maximization problem to characterize the retailers’ optimal strategies without explicitly solving the first-order-conditions. Details can be found in Appendix D. Here, we summarize the main results. We start by presenting each retailer’s optimal product assortment strategy given any pricing and assortment strategy taken by the other retailer. This result facilitates our proof (presented later) of the uniqueness of the equilibrium.

**Proposition 1.** Each retailer’s best response in terms of product assortment, given any strategy taken by the other retailer, is to carry only the national brand if \( w_n \leq c_n \), to carry only the store brand if \( w_n \geq c_{si} + (\bar{v}_n - \bar{v}_{si}) \), and to carry both products otherwise.

Proposition 1 implies that, under retail competition, for any given wholesale price, each retailer makes the product assortment decision in the same way as if she were a downstream monopolist. This result is certainly not intuitive. It implies that that a retailer’s optimal product assortment decision is not affected by the existence of the other retailer, nor is it affected by the assortment or pricing decisions of the other retailer. Intuitively, we would imagine in a Nash game between the two retailers, one retailer’s best response depends upon the strategy of the other retailer. But Proposition 1 says this is not so, which is quite surprising.

So what is the intuition behind Proposition 1? If it is less profitable for a retailer to sell one product (versus the other) to all customers, the retailer will not carry the less profitable product at all. For example, if \( w_n \geq c_{si} + (\bar{v}_n - \bar{v}_{si}) \), the per-unit cost premium retailer \( i \) incurs from selling the national brand (versus the store brand) is greater then the maximum per-unit price premium consumers are willing to pay. Thus, it is not worthwhile for the retailer to carry the national brand at all. This comparison between the two products remains the same whether a retailer is a monopolist or is facing competition. We will see later, however, that the national brand manufacturer takes into account the retailers’ reactions (including the assortment decisions) when choosing the wholesale price, so the introduction of retail competition may ultimately affect retailers’ assortment decisions. We study this issue in the next section.

Next, we establish the uniqueness of the equilibrium.

**Proposition 2.** There exists a \( t_0 \) with \( 0 < t_0 < +\infty \) such that, whenever \( t > t_0 \), the price equilibrium between retailer is unique for all \( w_n \).

In Appendix E, we prove the result for values of \( t \) for which the demand segmentation diagram has the form of Figure 1(a) or (b). As the value of \( t \) declines, the right and left boundaries in Figure 1(b) move inward, as shown in red in Figure 2(a). The resulting extracted diagram is shown in Figure 2(b). Then
finally, as $t$ becomes very small, the demand segmentation approaches the form shown in Figure 2(c): the diagonal lines flatten out and become horizontal at $t = 0$. When this happens, each retailer is able to seize a large share of the national brand demand from the other retailer by reducing the price of the national brand slightly. As a result, at equilibrium each retailer sets the national brand price at the wholesale price and gains profit exclusively from her store brand. The equilibrium is also unique for $0 \leq t \leq t_0$ (i.e., for the demand segmentation diagrams shown in Figures 2(b) and 2(c)) but we have omitted the proofs for the sake of brevity.

How do the prices a retailer sets when facing competition compare to those she would set as a monopolist, and how do the respective demands compare? We answer this question by first introducing the following proposition, in which $\theta_i^m$ and $\hat{\theta}_i^m$ denote the optimal values of $\theta$ and $\hat{\theta}$, respectively, that retailer $i$ would set when she is a downstream monopolist (cf. Lemma 3).

**Proposition 3.** If retailer $i$ decides to carry both the store and national brands, then she sets $\theta_i < \theta_i^m$ and $\hat{\theta}_i < \hat{\theta}_i^m$ for any pricing and assortment strategy of the other retailer ($i = 1, 2$).

Proposition 3 states that, under retail competition, each retailer will set prices in such a way that the willingness to pay per unit of quality of the customer who is indifferent between the store brand and the national brand, as well as the willingness to pay per unit of quality of the customer who is indifferent between the store brand and the no-purchase option, are smaller than when the retailer is a downstream monopolist. From this, we can immediately obtain Corollary 1.

**Corollary 1.** For any given value of $w_n$ that induces both retailers to carry both the store and national brands, we have the following results under retail competition:

(i) At each retailer, the retail price for each product as well as the price gap between the products are smaller than when the retailer is a downstream monopolist;
(ii) Total demand for the national brand and the aggregate demand for all products \((s1, s2 \text{ and the national brand})\) are greater than the respective demands when \(R2\) is a downstream monopolist.

(iii) The national brand manufacturer’s profit is greater under retail competition than when \(R2\) is a downstream monopolist. Moreover, there exists an \(\epsilon > 0\) such that when \(|q_{s2} - q_{s1}| < \epsilon\), the national brand manufacturer’s profit is greater than that when \(R1\) is a downstream monopolist.

Corollary 1 has several implications. First, although a retailer can ignore the other retailer without loss of optimality when deciding the assortment, it is suboptimal to do so when setting prices, as this would lead to prices that are too high and with too large a gap between them. Some reports in the business press have suggested that retailers should shrink this price gap to maximize profit (Nielsen 2011, SymphonyIRI 2011). Our results suggest one possible contributing reason for the larger-than-optimal price gaps is incomplete consideration of competition.

Second, when retail competition arises because the retailer with the lower store brand quality enters the market (i.e., \(R1\) in our model), demand for the national brand increases at any given wholesale price. The reason is that the lower-quality store brand competes less directly with the national brand, so the retailers set prices in a way that is less unfavorable to the national brand compared to the scenario in which \(R2\) is a downstream monopolist. Therefore, for any wholesale price, the demand for the national brand, and hence also the profit of the national brand manufacturer, are higher than in the scenario in which \(R2\) is a downstream monopolist.

On the other hand, when retail competition arises because the retailer with the higher store brand quality (i.e., \(R2\) in our model) enters the market, the national brand manufacturer’s profit increases only if the two store brands have very similar quality (i.e., when \(|q_{s2} - q_{s1}| < \epsilon\)). Otherwise his profit decreases with the entry of the retailer with the higher store-brand quality, as the level of competition between the national brand and the store brands as a whole becomes fiercer.

Next, we study the effect of customer loyalty (or transportation cost), \(t\), on the price equilibrium between the retailers. As before, we restrict our attention to positive \(t\), but we examine how the equilibrium prices, demands and profits change as \(t\) increases from zero.

### 4.2 Effect of \(t\) on Retailers’ Product Assortment and Pricing

To study how equilibrium retail prices change with \(t\), we resort to numerical approaches as it is impossible to derive closed form expressions for the equilibrium decisions. For any given input parameters \((\bar{\theta}, k, q_n, q_{s1}, q_{s2}, t, w_n)\), we solve for the equilibrium by starting with an arbitrary pair of initial prices for \(R1\) and alternately solving one retailer’s problem given the other retailer’s prices from the most recent iteration.
If \( t \) is sufficiently large, i.e., \( t \geq \max\{|p_{n2} - p_{n1}|, \ |p_{s2} - p_{s1}|, \ |b_{sn}(\theta)|\} \) for all prices selected during the iterations, we can use the demand system (2) and this process converges. But we cannot guarantee that \( t \) will always satisfy this condition for all iterations before the equilibrium is identified. To address this problem, we utilize a more general representation of demand which is valid even if \( t \) is very small. This system of demand functions is discontinuous at more points than is the demand system (2). (We will illustrate this below). We present these alternate expressions for demands in Appendix G. Just as in (2), the demand functions in the new system are discontinuous because \( \theta_2 \) may be either greater or less than \( \theta_1 \). However, if \( \theta_2 \geq \theta_1 \), we now may have either \( \tilde{\theta}_2 \geq \theta_1, p_{n2} - p_{n1} \in (-\infty, -t], (-t, t] \) or \( (t, +\infty) \), \( b_{sn}(\theta_1) \in (-\infty, -t], (-t, t] \) or \( (t, +\infty) \), \( b_{sn}^2(\tilde{\theta}_1) \in (-\infty, -t], (-t, t] \) or \( (t, +\infty) \). Altogether, there are \( 2 \times 3 \times 3 \times 3 = 162 \) different cases if \( \theta_2 \geq \theta_1 \). (In contrast, in the demand system (2), there are only two subcases if \( \theta_2 \geq \theta_1 \), i.e., \( \tilde{\theta}_2 \geq \theta_1 \).) Similarly, if \( \theta_1 > \theta_2 \), we may have either \( \tilde{\theta}_1 \geq \theta_2, p_{n2} - p_{n1} \in (-\infty, -t], (-t, t] \) or \( (t, +\infty) \), \( b_{sn}(\theta_2) \in (-\infty, -t], (-t, t] \) or \( (t, +\infty) \), \( b_{sn}^2(\tilde{\theta}_2) \in (-\infty, -t], (-t, t] \) or \( (t, +\infty) \), again yielding 162 cases. In total, there are \( 162 \times 2 = 324 \) different cases. Although it is possible to state the demands using common expressions for all cases (see Appendix G), it should be understood that the demand functions are discontinuous in the retail prices and therefore each retailer’s profit function is, as well.

To solve each retailer’s problem at a given iteration (given the other retailer’s prices from the previous iteration), we first solve the problem for each of the aforementioned 324 cases using Sequential Quadratic Programming (SQP). We then compare the best feasible solutions for each of the 324 cases and choose the best among them. We repeat this process until the changes in both retailers’ price vectors from one iteration to the next (measured as the Euclidean distance between them) is below a threshold. We use a threshold of \( 10^{-6} \).

We next present some numerical examples to illustrate the impact of \( t \) on equilibrium prices and profits. We use parameter values \( q_{s1} = 0.3, q_{s2} = 0.4, q_n = 0.6, \bar{\theta} = 0.5 \) and \( k = 0.1 \). In Figures 3 through 5, we plot equilibrium retail prices, retailers’ profit and customer demands, respectively, as a function of \( w_n \) for \( t = 0.1, 0.3, 0.5 \) and 0.9. (Other examples exhibit similar patterns.)

Figure 3 illustrates some characteristics of the price equilibrium. First, the retail price for the store brand is higher at R2 than at R1, but the retail prices of the national brand exhibit the opposite relationship. Also, at each retailer, the retail prices for both the store and national brands strictly increase with \( w_n \) until \( w_n \) reaches the threshold at which the retailer stops carrying the national brand. For \( w_n \) above that threshold, the retail prices remain unchanged. Moreover, as predicted by Proposition 1, the product assortment decision at each retailer is not affected by a change in \( t \). For any value of \( t \), both retailers choose to offer the store brand if \( w_n \) exceeds \( kq_n = 0.06 \), as can be seen from the kink on each curve in Figures 3(c) and 3(d) at

20
Figure 3: Equilibrium retail prices for different levels of $t$

$w_n = 0.06$. The analogous kinks can also be identified in Figures 5(c) and 5(d). R1 drops the national brand if $w_n$ exceeds $kq_{s1} + \theta(q_n - q_{s1}) = 0.18$, which is evident from the kink on each curve in Figures 3(a) and 5(a) at $w_n = 0.18$. R2 stops carrying the national brand if $w_n$ exceeds $kq_{s2} + \theta(q_n - q_{s2}) = 0.14$, as can be identified from the kinks on the curves in Figures 3(b) and 5(b) at $w_n = 0.14$. These thresholds do not vary with the value of $t$. However, all of the retail prices increase with $t$, as expected, due to reduced competition.

The retailers’ profits for different values of $t$ are shown in Figure 4. Comparing Figures 4(a) and 4(b), it can be seen that the profits of the two retailers are equal and both are decreasing with $w_n$ for $w_n \leq 0.06$ (that is, when both of them carry only the national brand). For $w_n > 0.06$, R1’s profit is smaller than that of R2. R1’s profit keeps decreasing with $w_n$ until the retailer stops carrying the national brand (at about $w_n = 0.18$). In comparison, R2’s profit first decreases and then increases with $w_n$ until both retailers stop carrying the national brand. The fact that R2’s profit increases with $w_n$ for a range of $w_n$ is quite counterintuitive, as one would conjecture that both retailers should become worse off as $w_n$ increases. What is happening here is the following: as $w_n$ increases in this range, R2 prices so as to sell greater quantities of
her store brand. This not only shields her from the declining margin on the national brand as $w_n$ increases, but also lessens the degree of competition she faces from R1. As a result, R2 may be better off as $w_n$ increases in this range. (In some other numerical examples, although not in all of them, we also observe that R2’s profit may increase as $w_n$ increases for a range of $w_n$ before both retailers drop the national brand.) Note that this would never happen if R2 were a downstream monopolist. Finally, as $t$ increases, both retailers’ profits increase for all values of $w_n$ as the level of retail competition declines.

Figure 5 shows the customer demands at the equilibrium prices for different values of $t$. The national brand demand at each retailer strictly decreases with $w_n$ before it reaches zero, as expected. The store brand demand at each retailer increases with $w_n$ for the interval of $w_n$ in which the retailer carries both products. Also, as $t$ increases, the demands for all products decline due to lessened competition which drives up retail prices, which in turn drive down customer demands.

5 Manufacturer’s Strategy under Retail Competition

In this section, we investigate the manufacturer’s strategy regarding the breadth of product distribution, i.e., whether to distribute through both retailers or only one. Due to the structure of each retailer’s product assortment strategy (see Proposition 1), the national brand manufacturer can either sell through both retailers by setting the wholesale price in the range $[c_n, \ c_{s2} + (\bar{\nu}_n - \bar{\nu}_{s2})]$, or sell it only through R1, whose store brand competes less directly with the national brand. The national brand manufacturer implements the latter decision by setting the wholesale price in the range $[c_{s2} + (\bar{\nu}_n - \bar{\nu}_{s2}), \ c_{s1} + (\bar{\nu}_n - \bar{\nu}_{s1})]$.

To understand when the manufacturer chooses each option, we start with the special case in which each customer is extremely loyalty to the closer retailer. We examine the general case subsequently.
**Proposition 4.** There exists a threshold \( H(q_{s2}) \) such that when \( t \to +\infty \), the manufacturer distributes through both retailers if \( q_{s2} - q_{s1} < H(q_{s2}) \) and distributes only through R1 if \( q_{s2} - q_{s1} > H(q_{s2}) \). He is indifferent between these two options if \( q_{s2} - q_{s1} = H(q_{s2}) \), where \( H(q_{s2}) = 2(q_{n} - q_{s2}) \).

Proposition 4 states that, when the two retailers serve separate markets, the national brand manufacturer will distribute his product only through the retailer that competes less directly with him if the quality gap between the two store brands is greater than a threshold, \( H(q_{s2}) \). If the quality disparity is above this threshold, if the manufacturer sells through both retailers, he needs to compromise to a large degree when setting the wholesale price. (Ideally, he would like to offer different prices to the two retailers but he is not allowed to do so.) In such a scenario, the manufacturer prefers to sell through only one retailer.

The value of \( H(q_{s2}) \) decreases with \( q_{s2} \) because, as \( q_{s2} \) increases, the store brand at R2 competes more directly with the national brand. This makes selling through R2 less attractive to the national brand manufacturer. He is then not willing to sell through both retailers unless the quality disparity between the two store brands is below an even smaller threshold. In particular, when the quality level of \( s2 \) is equal to that of the national brand, this threshold falls to zero, meaning that the manufacturer will sell through only
R1 for all levels of the quality disparity between $s_2$ and $s_1$.

Figure 6 shows the manufacturer’s distribution breadth for different pairs of $(q_{s_1}, q_{s_2})$ at different levels of $t$. (Although our discussion focuses on the half plane in which $q_{s_2} \geq q_{s_1}$, for completeness, we show the other half plane in Figure 6 as well.) We can see that the threshold $H(q_{s_2})$ increases as $t$ decreases, which implies that as the competition between retailers becomes fiercer, the manufacturer is willing to sell through both retailers when the quality levels of their store-brand products are even more disparate. Intuitively, when $t$ is small, the national brand manufacturer can effectively create strong competition between retailers by selling his product through both of them, and the national brand manufacturer benefits from this competition. On the other hand, when $t$ is relatively large, the level of competition between retailers remains low even if the national brand manufacturer sells through both of them. In this case the manufacturer is even more inclined to forgo the opportunity of distributing through R2.

Other than a change in $H(q_{s_2})$, the qualitative insights from Proposition 4 carry over to the case of finite $t$. First, the manufacturer sells through both retailers only if the quality disparity between the store brands is smaller than a threshold. Proposition 5 formally establishes this result for the case where customer loyalty is large enough that the manufacturer’s profit function is piecewise concave.

![Diagram](image)

Figure 6: Manufacturer’s distribution breadth for different $(q_{s_1}, q_{s_2})$ pairs for different levels of $t$. Other parameters are $(k, \theta) = (0.4, 2.0)$.

![Diagram](image)

Figure 7: $\pi_m(w_n)$ for different $q_{s_2} - q_{s_1}$. Other parameters are $(q_n, t, \theta, k) = (9.64, 9.09, 1.06, 0.00)$.

**Proposition 5.** For any $\epsilon > 0$, there exists $t_0 = t_0(\epsilon) < +\infty$ such that whenever $t > t_0$, the manufacturer distributes through both retailers whenever $q_{s_2} - q_{s_1} \leq 2(q_n - q_{s_2}) - \epsilon$ and distributes only through R1 whenever $q_{s_2} - q_{s_1} \geq 2(q_n - q_{s_2}) + \epsilon$.

Second, the threshold on $q_{s_2} - q_{s_1}$ at which the manufacturer changes his distribution breadth decreases
with the absolute quality level of the higher-quality store brand \( s_2 \). In particular, when the quality of \( s_2 \) is as high as that of the national brand, the threshold falls to zero. Formally, we have the following result.

**Proposition 6.** For all positive values of \( t \), if \( q_{s_2} = q_n \), the national brand manufacturer always distributes the national brand only through R1.

Figure 7 shows the manufacturer’s profit as a function of \( w_n \) for different levels of the quality disparity between the store brands while keeping the average store brand quality constant (at 4.6). We can see that the profit of the national brand manufacturer is piecewise concave with two modes, one for each distribution strategy. The national brand manufacturer chooses between the candidate wholesale prices corresponding to the two modes. For this example, he will choose the smaller candidate wholesale price and thus distribute the national brand through both retailers when \( q_{s_2} - q_{s_1} = 1, 2, 4, \) or 5. He will choose the higher candidate wholesale price, distributing through only R1 when \( q_{s_2} - q_{s_1} = 6 \) or 8. We note here that an interesting implication of this is that, keeping the other retailer’s store brand quality fixed, as the retailer continues to improve its store brand quality, the wholesale price she needs to pay for the national brand will first decreases and then, at some point, jumps up. Past research (Morton and Zettelmeyer 2004 and Mills 1995) suggests that a retailer is able to extract a lower wholesale price from a national brand manufacturer by introducing a store brand of similar quality. We find that this may not be the case when the national brand manufacturer has the option of selling solely through the competing retailer.

We close this section by noting that the manufacturer’s wholesale price at equilibrium under retail competition is between the wholesale prices he would set when each retailer is a downstream monopolist. To understand the underlying intuition, notice that the higher is the average quality of the store brands, the lower is the bargaining power of the manufacturer. If R2 is introduced into the market in which R1 is a monopolist, the overall competitiveness of the store brands increases, and therefore the manufacturer lowers the wholesale price. Similarly, he increases the wholesale price if R1 enters the market.

Because the equilibrium wholesale price differs under retail competition versus the two monopolist scenarios, the retailers’ equilibrium assortments may change even though, for any given wholesale price, each makes the decision in the same way as if she were a monopolist retailer.

## 6 Special Cases, Extensions and Discussions

In this section, we briefly describe several additional results. Details, including proofs, are omitted here but are available from the authors.
6.1 No Customer Loyalty

When the retailers do not enjoy any store loyalty, i.e., when $t = 0$, if customers purchase the national brand product, they will do so at the retailer with the lower price. This has the effect of driving the retail prices of the national brand at both retailers down to the wholesale price. As a result, neither retailer can secure any profit from the national brand. (This result was also obtained by Moorthy 1988 in the absence of store brands.) On the other hand, if either retailer drops the national brand, her competitor can earn a profit by offering it, so at equilibrium, both retailers offer it, although both would prefer a scenario in which neither carries it.

With the national brand providing no profit, the store brands are the sole source of profit and the sole means by which the retailers are able to differentiate themselves. In practice, major retailers almost always enjoy some level of customer loyalty. Therefore we rarely see retailers getting no margins from national brands. However, for retailers facing strong head-to-head competition, this result implies that the markup on national brands can be fairly low and that retailers are essentially deriving most of their profit from store brands. Indeed, there is empirical evidence showing that some major retailers set high markup ratios on store brands, while at the same time their markup ratios for national brands are close to 1 (see Barsky et al. 2003). Although there may be other explanations for this phenomenon (for example, differences in marginal costs for the products), it is consistent with our findings. Thus, retail competition could be one factor that leads to low markup ratios on national brands compared to those on store brands.

We conclude that when retailers compete head-to-head, the structure of retailers’ product assortment and pricing differs from when each enjoys some degree of customer loyalty. Therefore, for the sake of answering the questions raised in Section 1, it is important to include a positive parameter $t$ in our model to capture reality.

6.2 Production Cost

Thus far, we have assumed that both retailers and the manufacturer share the same production cost function, i.e., $c = kq$. If one retailer has a cost advantage over the other, we can let $k_1 \neq k_2$. Without loss of generality, we assume $k_i < k_j \ (i \in \{1, 2\}$ and $j = 3 - i$). In this scenario, the retailers’ product assortment strategy can be characterized as a generalization of Proposition 2:

**Corollary 2.** If $c_{ai} = k_iq_{ai}$ for $i = 1, 2$ and $k_1 \neq k_2$, each retailer’s best response in terms of product assortment is to carry only the national brand if $w_n \leq k_iq_{ai}$, to carry only the store brand if $w_n \geq c_{ai} + (\bar{v}_n - \bar{v}_{ai})$, and to carry both products otherwise.

Recall that when the retailers have the same production cost parameters, the lower wholesale price threshold (above which the retailers choose to carry a store brand) coincide with each other. This no longer
holds if their production cost parameters differ. Retailer $i$ (i.e., the retailer with the smaller cost parameter) starts to carry the store brand at a lower wholesale price than the other retailer. When $w_n \in (k_i q_n, k_j q_n]$, retailer $i$ carries both the national and the store brand and retailer $j$ carries the national brand only. This phenomenon has additional implications for the manufacturer’s strategy, as we delineate below.

**Case (1):** $k \geq \max\{k_1, k_2\}$. In this case, the national brand manufacturer never sets the wholesale price such that one or both retailer(s) only carry the national brand product. Therefore, just as in our basic model, he chooses between two candidate wholesale prices, which is equivalent to choosing between distributing the national-brand product through one or both retailers. At either candidate wholesale price, both retailers will carry their respective store brand.

**Case (2):** $k_i < k < k_j$. In this case, the national brand manufacturer chooses among three candidate wholesale prices, each corresponding to a different distribution strategy: selling through both retailers while foreclosing the store brand at retailer $j$, selling through both retailers but foreclosing neither of the store brands, and selling through only the retailer with the lower-quality store brand while not foreclosing her store brand.

**Case (3):** $k < k_i < k_j$. In this case, the national brand manufacturer chooses among four candidate wholesale prices, each corresponding to a different distribution strategy. One strategy is selling through both retailers while foreclosing the store brand at both of them. The other three are the same as those mentioned above in the case of $k_i < k < k_j$.

Mills (1995, 1999) finds that it is optimal for the national brand manufacturer to foreclose the store brand if its quality falls into a particular range. We note here that this result is contingent on his assumption that the national brand has an advantage in terms of cost-per-unit-of-quality (as in Case (2) or (3) above). If the national brand does not enjoy a cost advantage over either of the store brands (as in Case (1)), the national brand manufacturer would never find it optimal to foreclose a store brand because he would need to set the wholesale price below his production cost to accomplish this. This effect has been found by Fang et al. (2012) in the case of a single retailer. We have shown that it also applies when when retail competition is introduced.

### 6.3 Heterogeneity in Transportation Costs

In our basic model, we have assumed that customers are uniformly distributed on a Hotelling line between the retailers. Our model can be extended to capture heterogeneity in transportation costs by an appropriate placement of each customer on the Hotelling line between the two retailers and using a general distribution of customers along the Hotelling line. To implement this, one can use the demand expressions in Appendix G and then substitute a general distribution of customers’ locations into the demand expressions. The analysis
could then be conducted in a similar fashion. From numerical examples, we have found that retailers’ product
assortment strategy (i.e., Proposition 1) and the manufacturer’s strategy regarding distribution breadth (i.e.,
distributing through both retailers only if the quality disparity between the two store brands is low) remain
valid for common symmetric bell-shaped distributions of customers along the Hotelling line.

6.4 Market Share versus Profitability

In Section 5, we establish that it is sometimes profitable for the manufacturer to sell through only one of
retailers. In reality, however, national brand manufacturers may still sell through both retailers to maintain
market share even though they realize that doing so may have an adverse effect on profit. Indeed, manufac-
turers are facing a tradeoff between market share and profitability when making the decision of whether
to sell through a retailer with a very high quality store brand. For example, a senior manager of a large
national brand revealed to us that he chooses to continue to sell through all the major retailers although
profit may suffer in the short run. Maintaining a large market share has benefits if it reduces competition
in the long-run, but whether this actually occurs is an ongoing topic of discussion in the literature. Many
researchers have suggested that increasing market share at a cost may not prove to be profitable in the
that firms do well by attacking in the early stages of the product life cycle but are better off not overplaying
their cards in the stable periods of the life cycle.

7 Conclusion

In this paper, we study a retailer’s product assortment and pricing problem when she has the option to carry
a store brand, a national brand, or both. We first derive the structure of the retailer’s optimal assortment
and prices in two settings: (1) when she is a downstream monopolist and (2) when she faces competition
from another retailer who may also offer the same national brand and a competing store brand. Past research
has established that store brands help generate store traffic and help a store better differentiate itself. As
a result, one would expect that a retailer would be more likely to introduce a store brand as a competitive
strategy when she faces retail competition. In contrast, we find that in the presence of retail competition, for
any given wholesale price of the national brand product, a retailer makes the product assortment decision
in the same way as if she were a downstream monopolist. The underlying reason for this result is that
a retailer’s assortment decision is determined by a comparison between the profitability of her store and
national brands, which does not involve the other retailer. However, the presence of multiple retailers affects
the national brand manufacturer’s choice of a wholesale price, which then affects the ultimate assortments
and prices at the retail level.
We also characterize how each retailer’s optimal assortment decision depends on the national brand’s wholesale price. For wholesale prices below a lower threshold, the retailer carries only the national brand and for wholesale prices above an upper threshold, the retailer carries only the store brand. For wholesale prices between the lower and upper thresholds, the retailer carries both brands. The thresholds may, in general, differ by retailer, but if the production cost per unit of quality is the same for both retailers, then the lower thresholds are the same for both retailers. Although a retailer in a duopoly is able to make the optimal assortment decision in the same way as if she were a monopolist, she needs to take into account retail prices set by her competitor to set the optimal retail prices. Failing to do so may have a sizable effect on profits.

Not surprisingly, the equilibrium retail prices of all offered products decline with the introduction of retail competition, but interestingly, the optimal gap between the prices of the store and national brands also decline. Thus, the introduction of retail competition puts greater pressure on the retail price of the national brand than it does on the retail prices of the store brands. Several news reports and market research suggest that the retailers are essentially “leaving money on the table” by setting store-brand prices too low. (See, e.g., Kumar and Steenkamp 2007).

We also study the national brand manufacturer’s optimal pricing decision which affects the retailers’ product assortments. From our characterization of how each retailer’s assortment changes as a function of the wholesale price, we can infer that the national brand manufacturer needs to choose between two regimes: (1) selling through both retailers and optimizing the wholesale price within the interval in which both retailers choose to offer the national brand (along with their respective store brand), and (2) selling through only the retailer with the lower-quality store brand and optimizing the wholesale price within the relevant price interval.

Our results suggest that the manufacturer should distribute his national brand through only one retailer if the quality disparity between the two store brands is larger than a threshold. The rationale is that under this condition, the manufacturer would prefer to charge the retailers different wholesale prices if he were allowed to do so. But if the quality disparity is large, he needs to compromise a lot when setting a single wholesale price, in which case he may be better off distributing through one retailer.

We also study the effect of customer loyalty. The structure of the retailers’ optimal assortment decisions when there is no customer loyalty differs from that when the customer loyalty is strong. In the former case, both retailers stop carrying the national brand when the wholesale price exceeds the same threshold. Thus, the national brand manufacturer always distributes through both retailers. The retailers end up in a prisoner’s dilemma at the equilibrium: both of them could have earned a higher profit if neither of them had carried the national brand, but they both end up carrying it. As the degree of customer loyalty increases, the
national brand manufacturer may or may not distribute through both retailers depending upon the quality levels of the two store-brand products, as discussed above.

We also discuss how our model can be extended to handle different production cost functions (as a function of quality) for the two store-brand products as well as heterogeneity in customer loyalty to the retailers.

In this paper, we assume that store-brand quality levels are exogenous and that the retailers already offer or are ready to offer their respective store brands. Further research is needed to determine how a retailer should choose the quality of a new store brand when facing competition from a retailer that has, or can offer, her own store brand, along with the national brand. We are pursuing research along these lines. Further research is also needed to study retail competition in settings in which the manufacturers of the store-brand products—both national brand manufacturers and third-party producers—are strategic players. For these settings, equilibria are much more difficult to derive because there will be four or more parties in the competitive game.

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31


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33


APPENDIX: Proofs of Lemmas and Propositions

Appendix A: Proof of Lemma 1

If any of the relations in (1) or (2) fails to hold for retailer \( i \) at the equilibrium, she could decrease one of the retail prices without affecting either her own profit or the pricing decisions of the other retailer. Then given Assumption 1, she will decrease the relevant price.

Appendix B: Transformed Problems

Before presenting proofs of the other lemmas and propositions, we first establish the equivalence of a market with parameters \( (\bar{\theta}, k) \) and a transformed market with parameters \( (\tilde{\theta}, 0) \). Define \( p' \equiv (p'_{a1}, p'_{a2}, p'_{s2}) \equiv p - c \) where \( c \equiv (c_n, c_s, c_{a1}, c_{a2}) \), \( w'_n \equiv w_n - c_n, \bar{\theta}' \equiv \bar{\theta} - k, \theta'_1 \equiv \theta_1 - k = \frac{p'_n - p'_s}{q_n - q_s} \) and \( \tilde{\theta}_i = \bar{\theta}_i - k = \frac{p'_n}{q_n} \) for \( i = 1, 2 \). Then we have the following lemma, which can be obtained from straightforward algebraic manipulations.

**Lemma 4.** \( D_{ns}(p, \tilde{\theta}) = \frac{\partial^2}{\partial \theta^2} D_{ns}(p', \theta') \). Moreover, \( \pi_r(w_n, \ p, \ \tilde{\theta}, c_s) = \frac{\theta'}{\bar{\theta}} \pi_r(w'_n, \ p', \ \theta', 0) \) and \( \pi_m(w_n, \ \tilde{\theta}, c_n) = \frac{\theta'}{\bar{\theta}} \pi_m(w'_n, \ \bar{\theta}', 0) \).

As such, if the equilibrium prices for the transformed market are \( (w'_n, p'(w'_n)) \), we can immediately write the equilibrium prices of the original market as \( (w_n + c_n, p'(w'_n) + c) \). As such, here and throughout the Appendix, we will only present proofs of the lemmas and propositions for a market with parameters \( (\bar{\theta}, 0) \) and omit the proofs for the original markets (i.e., the markets with parameters \( (\bar{\theta}, k) \)). Nevertheless, the results immediately generalize with appropriate transformations of the prices.

Appendix C: Proofs of Lemmas 2 and 3

The customer demands under this scenario are \( D_n(p, \bar{\theta}) = \frac{1}{\bar{\theta}} (\bar{\theta} - \frac{p_n - p_s}{q_n - q_s}) \) and \( D_s(p, \bar{\theta}) = \frac{1}{\bar{\theta}} (\bar{\theta} - \frac{p_n - p_n}{q_n - q_s}) \). It is easy to confirm that \( \pi_r(w_n, \ p, \ \bar{\theta}, 0) \) is concave in the retail prices. Setting the first derivatives of \( \pi_r(w_n, \ p, \ \bar{\theta}, 0) \) with respect to the prices equal to zero, we obtain \( p_n(w_n) = \frac{1}{2} (\bar{\theta} q_s + w_n) \) and \( p_s(w_n) = \frac{1}{2} \bar{\theta} q_s \). It remains to verify whether the solution is an interior point in the region of \( (p_n, p_s) \) satisfying

\[
\bar{\theta} > \frac{p_n - p_s}{q_n - q_s} \quad \text{[5]}
\]

and

\[
\frac{p_n}{q_n} > \frac{p_s}{q_s} \quad \text{[6]}
\]

Otherwise, we have a boundary solution which corresponds to a case in which either the store brand is not sold at equilibrium (which occurs if [6] is violated), or the national brand is not carried by the retailer (which occurs if [5] is violated). It is easy to confirm that [6] is satisfied if and if only \( w_n > 0 \), and [5] is satisfied if and only if \( w_n \leq (q_n - q_s) \bar{\theta} \). Therefore the retailer sells both products if \( 0 < w_n < (q_n - q_s) \bar{\theta} \) and sets \( p_n(w_n) = \frac{1}{2} (\bar{\theta} q_n + w_n) \) and \( p_s(w_n) = \frac{1}{2} \bar{\theta} q_s \). She sells only the national brand if \( w_n \leq 0 \) and sets \( p_n(w_n) = \frac{1}{2} (\bar{\theta} q_n + w_n) \) and \( p_s(w_n) = q_n \cdot \frac{p_n(w_n)}{q_n} \). She sells only the store brand if \( w_n \geq (q_n - q_s) \bar{\theta} \) and sets \( p_s(w_n) = \frac{1}{2} \bar{\theta} q_s \) and \( p_n(w_n) = \bar{\theta} (q_n - q_s) + p_s(w_n) \). Taking the reaction of the retailer into account, the objective of the national-brand manufacturer is \( \max_{(q_n, q_s, \bar{\theta}), \bar{\theta} > 0, (q_n, q_s) \geq 0} \pi_m(q_n, q_s) \equiv \frac{\alpha}{\bar{\theta}} (\bar{\theta} - \frac{1}{2} (\bar{\theta} q_n + w_n) - \frac{1}{2} \bar{\theta} q_s) \). Solving this optimization problem, we obtain the equilibrium wholesale price. Equilibrium demands and profits follow directly.
Appendix D: Proof of Proposition 1

Define the following change of variables: $b_i \equiv \frac{\partial x_i}{\partial y_i} \delta_i \equiv \frac{\partial x_i - \partial y_i}{\partial y_i} \left( = 1 - \beta_i \right)$, $\alpha \equiv \frac{1}{\beta_i} \delta_i \beta \equiv \frac{\alpha_i \delta_i}{\beta_i}$, $\gamma_i \equiv \frac{\delta_i}{\beta_i}$, $D_{ni} = 2D_{ni}$ and $\hat{D}_{ni} = 2D_{ni}$ for $i = 1, 2$. Also define $\hat{D}_{n+i} \equiv \hat{D}_{n+i} + \hat{D}_{ni}$ and $\hat{\pi}_{ri} = \frac{2}{\delta_i} \pi_{ri}$. Then the problem facing retailer $i \left( i = 1, 2 \right)$ can be written as

$$\max_{0 \leq \bar{\phi}_i \leq \phi_i \leq 1} \hat{\pi}_{ri}(\phi_i, \bar{\phi}_i) = d_i \hat{D}_{ni}(\phi_i, \bar{\phi}_i) \cdot (\phi_i - \gamma_i) + b_i \hat{D}_{n+i}(\phi_i, \bar{\phi}_i) \cdot (\bar{\phi}_i) \quad (7)$$

where $\hat{D}_{n+i}(\phi_i, \bar{\phi}_i)$ and $\hat{D}_{n+i}(\phi_i, \bar{\phi}_i)$ can be derived directly from [1] and [2]. Without loss of generality assume $l, h \in \{1, 2\}$ such that $\eta_l \leq \eta_h$, then

$$\hat{D}_{n+i} = (1 - \phi_i) + \frac{1}{\alpha} \left[ (1 - \phi_i)A_i + \frac{1}{2} d_h(1 - \phi_i)^2 - \frac{1}{2} d_h(1 - \phi_i)^2 \right]$$

$$\hat{D}_{n+i} = (1 - \phi_i) + \frac{1}{\alpha} \left[ (1 - \phi_i)A_i + \frac{1}{2} d_h(1 - \phi_i)^2 - \frac{1}{2} d_h(1 - \phi_i)^2 \right]$$

$$\hat{D}_{n+i} = (1 - \phi_i) + \frac{1}{\alpha} \left[ [1 - \phi_h] A_i \right]$$

$$\hat{D}_{n+i} = \left\{ \begin{align*}
(1 - \phi_i) + \frac{1}{\alpha} \left[ (1 - \phi_i) A_i + (\phi_i - \phi_h)(d_h - d_i) + b_i \phi_h(1 - \phi_i)^2 \right], \bar{\phi}_h \leq \phi_i \\
(1 - \phi_i) + \frac{1}{\alpha} \left[ (1 - \phi_i) A_i + \frac{1}{2} d_h(1 - \phi_i)^2 \right], \bar{\phi}_h > \phi_i
\end{align*} \right. \quad (8)$$

where $A_i \equiv - (b_i \phi_i - b_h \phi_h) + (b_i - b_h) \phi_i$ and $A_h \equiv (b_i \phi_i - b_h \phi_h) - (d_h \phi_h - d_i \phi_i)$.

To establish Proposition 1, we only need to show that the solution to (7) satisfies (i) $\phi_i = 1$ when $\gamma_i \geq 1$ and $\phi_i < 1$ when $\gamma_i < 1$; and (ii) $\phi_i = \phi_i$ when $\gamma_i \leq 0$ and $\phi_i < \phi_i$ when $\gamma_i > 0$.

We first prove (i). Notice that for any $\phi_i \in [0, \phi_i]$ we have

$$\frac{\partial \hat{\pi}_{ri}}{\partial \phi_i} \bigg|_{\phi_i \rightarrow 1^-} = d_i \left[ \hat{D}_{ni}(\phi_i, \bar{\phi}_i) \bigg|_{\phi_i \rightarrow 1^-} + \frac{\partial \hat{D}_{n+i}}{\partial \phi_i} \bigg|_{\phi_i \rightarrow 1^-} \times (\phi_i - \gamma_i) \bigg|_{\phi_i \rightarrow 1^-} \right] + b_i \left[ \frac{\partial \hat{D}_{n+i}}{\partial \phi_i} \bigg|_{\phi_i \rightarrow 1^-} \times \phi_i \right]$$

It is easy to confirm that $\hat{D}_{ni}(\phi_i, \bar{\phi}_i) \bigg|_{\phi_i \rightarrow 1^-} = 0$, $\frac{\partial \hat{D}_{n+i}}{\partial \phi_i} \bigg|_{\phi_i \rightarrow 1^-} = 0$, and $\frac{\partial \hat{D}_{n+i}}{\partial \phi_i} \bigg|_{\phi_i \rightarrow 1^-} = \frac{1}{2}(1 - \beta_i) \phi_i^2 < 0$. Therefore

$$\frac{\partial \hat{\pi}_{ri}}{\partial \phi_i} \bigg|_{\phi_i \rightarrow 1^-} \begin{cases} > 0 & \text{if } \gamma_i > 1 \text{ and as } \phi_i \rightarrow 1^-; \\ < 0 & \text{if } \gamma_i < 1 \text{ and } \phi_i > \gamma_i \text{ and as } \phi_i \rightarrow 1^-; \\ = 0 & \text{if } \gamma_i = 1 \text{ and as } \phi_i \rightarrow 1^-.
\end{cases}$$

Thus, when $\gamma_i > 1$, a retailer $i$ who chooses $\phi_i$ slightly less than 1 can locally improve her profit by increasing $\phi_i$ to 1. Along with the quasi-concavity of the profit function, this implies the solution to (7) satisfies $\phi_i = 1$. Similarly, when $\gamma_i < 1$, retailer $i$ can obtain a local improvement by slightly decreasing $\phi_i$. Therefore the solution to (7) satisfies $\phi_i < 1$. Finally, when $\gamma_i = 1$, we have $\frac{\partial \hat{\pi}_{ri}}{\partial \phi_i} \bigg|_{\phi_i \rightarrow 1^-} = 0$, therefore the solution to (7) satisfies $\phi_i = 1$.

We next show (ii). Define $x_i = d_i \phi_i + b_i \bar{\phi}_i$ and $y_i = d_i \phi_i - b_i \bar{\phi}_i$. Then problem (7) is equivalent to one of maximizing $\hat{\pi}_{ri}(x_i, \phi_i, \bar{\phi}_i, y_i(\phi_i, \bar{\phi}_i)) \equiv \hat{\pi}_{ri}(\phi_i, \bar{\phi}_i)$ with respect to $x_i$ and $y_i$ subject to $0 \leq x_i \leq b_i + d_i$ and $-(b_i - d_i) x_i \leq y_i \leq b_i$.
\[
\frac{\partial \hat{\pi}_{xi}}{\partial y_i}(x_i(\phi_i, \tilde{\phi}_i), y_i(\phi_i, \tilde{\phi}_i)) = 1 \left [ \frac{1}{d_i} \frac{\partial \hat{\pi}_{xi}}{\partial \phi_i} - \frac{1}{b_i} \frac{\partial \hat{\pi}_{xi}}{\partial \phi_i} \right ] \\
= \frac{1}{2} \left [ \hat{D}_{xi}(\phi_i, \tilde{\phi}_i) + \left ( \frac{\partial D_{xi}}{\partial \phi_i} \tilde{\phi}_i + \frac{\partial D_{xi}}{\partial \phi_i} \phi_i \right ) + \left ( \frac{\partial D_{xi}}{\partial \phi_i} - \frac{b_i}{d_i} \frac{\partial D_{xi}}{\partial \phi_i} \right ) \phi_i - \left ( \frac{\partial D_{xi}}{\partial \phi_i} - \frac{d_i}{b_i} \frac{\partial D_{xi}}{\partial \phi_i} \right ) \phi_i \right ]
\]

For any \( x_i \in [0, b_i + d_i] \), as \( y_i \rightarrow \left [ -(b_i - d_i)x_i \right ]^+ \) (equivalently, as \( \tilde{\phi}_i \rightarrow \phi_i^- \)), it is easy to confirm that we have \( \hat{D}_{xi}(\phi_i, \tilde{\phi}_i) \rightarrow 0 \), \( \frac{\partial D_{xi}}{\partial \phi_i} - \frac{b_i}{d_i} \frac{\partial D_{xi}}{\partial \phi_i} \rightarrow 0 \), \( \frac{\partial D_{xi}}{\partial \phi_i} - \frac{d_i}{b_i} \frac{\partial D_{xi}}{\partial \phi_i} > 0 \), and \( \frac{\partial D_{xi}}{\partial \phi_i} \phi_i + \frac{\partial D_{xi}}{\partial \phi_i} \phi_i \rightarrow 0 \). Therefore,

\[
\frac{\partial \hat{\pi}_{xi}}{\partial y_i} \begin{cases} 
> 0 & \text{if } \gamma_i < 0 \text{ and as } y_i \rightarrow \left [ -(b_i - d_i)x_i \right ]^+ \text{ (equivalently, as } \tilde{\phi}_i \rightarrow \phi_i^- \text{);} \\
< 0 & \text{if } \gamma_i > 0 \text{ and as } y_i \rightarrow \left [ -(b_i - d_i)x_i \right ]^+ \text{ (equivalently, as } \tilde{\phi}_i \rightarrow \phi_i^- \text{);} \\
= 0 & \text{if } \gamma_i = 0 \text{ and as } y_i \rightarrow \left [ -(b_i - d_i)x_i \right ]^+ \text{ (equivalently, as } \tilde{\phi}_i \rightarrow \phi_i^- \text{).}
\end{cases}
\]

In words, if \( \gamma_i < 0 \), a retailer \( i \) who sets her \( \tilde{\phi}_i \) slightly less than \( \phi_i \) can locally improve her profit by increasing \( \tilde{\phi}_i \). Along with the quasi-concavity of the profit function, this implies that if \( \gamma_i < 0 \), the solution to (7) satisfies \( \tilde{\phi}_i = \phi_i \). Similarly, if \( \gamma_i > 0 \), retailer \( i \) can strictly increase her profit by decreasing \( \tilde{\phi}_i \) from very close to \( \phi_i \). Therefore the solution to (7) satisfies \( \tilde{\phi}_i < \phi_i \). Finally, when \( \gamma_i = 0 \), the solution to (7) satisfies \( \tilde{\phi}_i = \phi_i \).

**Appendix E: Proof of Proposition 2**

We present a proof for the assortment scenario in which both retailers carry both the store and the national brands (i.e. when \( 0 \leq \gamma_1, \gamma_2 \leq 1 \)). The uniqueness of the price equilibrium under the other assortment scenarios can be shown in a similar way. For this case, the first order conditions of the two retailers can be written as \( \phi = T(\phi) \) where \( \phi \) represents a vector \( (\phi_1, \tilde{\phi}_1, \phi_2, \tilde{\phi}_2)^T \in [0, 1]^4 \). We can confirm that for all \( \alpha \rightarrow +\infty \), \( T(\phi) = \begin{bmatrix} 1 \frac{1}{2}(1 + \gamma_1) \frac{1}{2}(1 + \gamma_2) \frac{1}{2}(1 + \gamma_2) \frac{1}{2} \end{bmatrix}^T \in [0, 1]^4 \). Therefore, there exists \( \alpha_1 \) with \( 0 < \alpha_1 < +\infty \) such that for all \( \alpha > \alpha_1 \), \( T(\phi) \in [0, 1]^4 \). Therefore, for all \( \alpha \) satisfying \( \alpha > \alpha_1 \), \( T(\phi) \) is a mapping from \( [0, 1]^4 \) to \( [0, 1]^4 \). Define \( q(\alpha) \equiv \sup_{\phi_x, \phi_y \in [0, 1]^4} \Gamma(\phi_x, \phi_y) \) where \( \Gamma(\phi_x, \phi_y) \equiv \sqrt{\|T(\phi_x) - T(\phi_y)\|^2} / \|\phi_x - \phi_y\| \) if \( \phi_x \neq \phi_y \).

Then, because \( \lim_{\alpha \rightarrow +\infty} T(\phi) = \begin{bmatrix} 1 \frac{1}{2}(1 + \gamma_1) \frac{1}{2}(1 + \gamma_2) \frac{1}{2}(1 + \gamma_2) \frac{1}{2} \end{bmatrix}^T \) for any \( \phi \in [0, 1]^4 \), we have \( \lim_{\alpha \rightarrow +\infty} q(\alpha) = 0 \). Then we can immediately establish that there exists \( \alpha_2 \) with \( 0 < \alpha_2 < +\infty \) such that, for all \( \alpha > \alpha_2 \), there exists a \( q \equiv q(\alpha) \) with \( 0 < q < 1 \) such that \( \|T(\phi_x) - T(\phi_y)\| \leq q\|\phi_x - \phi_y\| \) for any \( \phi_x, \phi_y \in [0, 1]^4 \). Define \( \alpha_0 \equiv \max\{\alpha_1, \alpha_2\} \). Then for all \( \alpha > \alpha_0 \), \( T(\cdot) \) is a contraction mapping on \( [0, 1]^4 \). By the Banach Fixed Point Theorem, there exists a unique solution to the system of the retailers' first order conditions. This implies that for all \( t > t_0 \equiv a_0 q t_0 \), the price equilibrium between the retailers is unique.

**Appendix F: Proof of Proposition 3**

First, notice that \( \hat{D}_{ni} \) and \( \hat{D}_{nxi} \) are both strictly decreasing and convex in \( \phi_i \), and that given \( \phi_i, \hat{D}_{ni} \) and \( \hat{D}_{nxi} \) are both strictly decreasing and convex in \( \tilde{\phi}_i \) \((i = 1, 2) \). Next, from a Taylor series expansion of \( \hat{D}_{n1}(1, \tilde{\phi}_i) \), we have \( \hat{D}_{n1}(1, \tilde{\phi}_i) = \)
Appendix G: Another Way to Express Consumer Demand

Denote the p.d.f. and c.d.f. of $\tilde{x}$ as $f(\cdot)$ and $F(\cdot)$, respectively, and suppose $\theta_t \geq \theta_j$ for $i = 1, 2, 3$ for a moment. Then for all finite positive values of $t$, the demand functions are $D_{n1} = \int_0^\theta \frac{1}{\beta} \frac{1}{\theta} df_{\tilde{x}^{-\theta_1}} f(x) dx, D_{n3} = \int_{\theta_t}^{\theta_1} \frac{1}{\beta} \frac{1}{\theta} df_{\tilde{x}^{-\theta_1}} f(x) dx$ if $\theta_t < \theta_1, D_{n4} = \int_{\theta_t}^{\theta_1} \frac{1}{\beta} \frac{1}{\theta} df_{\tilde{x}^{-\theta_1}} f(x) dx + \int_{\theta_t}^{\theta} \frac{1}{\beta} \frac{1}{\theta} df_{\tilde{x}^{-\theta_1}} f(x) dx, D_{n5} = \int_{\theta_t}^{\theta} \frac{1}{\beta} \frac{1}{\theta} df_{\tilde{x}^{-\theta_1}} f(x) dx$. These expressions can be integrated explicitly as $\theta$ is uniformly distributed on $(0, \theta)$. We obtain the following demand functions: $D_{n1}^\beta = \frac{1}{\theta} (\theta - \theta_j) F(\tilde{x}_{n1}), D_{n3}^\beta = \frac{1}{\theta} (\theta - \theta_j) F(\tilde{x}_{n3}), D_{n4}^\beta = \frac{1}{\theta} (\theta - \theta_j) F(\tilde{x}_{n4}), D_{n5}^\beta = \frac{1}{\theta} (\theta - \theta_j) F(\tilde{x}_{n5})$, and $D_{n2}^\beta = \frac{1}{\theta} (\theta - \theta_j) F(\tilde{x}_{n2})$, where $(\theta_1, \theta_2)$ is defined in [1] and $X(\alpha, \beta) = \int_0^\theta f(x) dx$. The advantages of the above demand functions in comparison to those in [2] are that (i) the above demand functions hold for any distribution of $\tilde{x}$, whereas (ii) [2] holds only when $\tilde{x}$ has a uniform distribution; and (ii) (2) represents demands for the case where $t \geq \max\{|p_{n2} - p_{n1}|, |p_{n2} - p_{n1}|, |b_{n3}(\theta_1)|\}$, whereas the above demand functions are more general and are valid for all positive, finite values of $t$. We thus use the above demand functions when performing our numerical analysis. In our numerical analysis, given all the exogenous parameters and a set of retail prices, we substitute expressions for $F(\tilde{x}), f(\tilde{x})$ and $X(\tilde{x}_1, \tilde{x}_2)$ into the above demand functions to calculate the ultimate demand for each product. Specifically, for the case of uniform distributions, $F(\tilde{x}) = \frac{\tilde{x} + t}{m}, 1 \{t < \tilde{x} < t\}, f(\tilde{x}) = \frac{1}{m}, 1 \{t < \tilde{x} < t\},$ and $X(\tilde{x}_1, \tilde{x}_2) = X(\tilde{x}_1, \tilde{x}_2) = 1 \{\tilde{x}_1 \leq t\} + \frac{\tilde{x} - t}{m}, 1 \{t < \tilde{x}_1 < \tilde{x}_2 < t\} + X(\tilde{x}_1, t) \{\tilde{x}_2 \geq t\}$.

Appendix H: Proof of Proposition 4

When $t \rightarrow +\infty$, we have $D_{n1} = 1 - \phi_t, D_{n3} = 1 - \phi_t, \pi_{n4} = d_1(1 - \phi_t) \phi_t + b_1(1 - \phi_t) \phi_t$ for $i = 1, 2$ and $\pi_m = \mathbb{P}_{\pi_m} = [(1 - \phi_t) + (1 - \phi_t) \beta]/(1 + \beta)$. Given $\beta$, retailer i solves $\max \pi_i$. The solution is $(\phi_t, \phi_t) = (1, \frac{1}{2})$ if $\gamma_t \geq 1, (\phi_t, \phi_t) = (1, \frac{1}{2})$ if $\gamma_t \leq 0$. Knowing retailers' responses given $\beta$, the manufacturer solves $\max \pi_m = \mathbb{E}_{\beta} \pi_m = \mathbb{E}_{\beta} \mathbb{E}_{\pi_m} = 0 + \mathbb{E}_{\beta} \mathbb{E}_{\pi_m} = 0$. The solutions are $\beta^*_0 \equiv \arg\max \pi_m = d_1 + d_2, \beta^*_2 = \arg\max \pi_m = d_2$ if $d_2 \leq d_1$ and $\beta^*_2 = d_2$ otherwise. Therefore $\pi_m(\beta^*_0) = \frac{1}{2} - \frac{d_2}{d_1 + d_2}, \pi_m(\beta^*_2) = \frac{1}{2} d_1$ and $\pi_m(\beta^*_2) = \frac{1}{2} d_2(1 - \frac{d_1}{d_2})$ otherwise. It can be easily verified that $\pi_m(\beta^*_0) - \pi_m(\beta^*_2) > 0$ whenever $2d_2 > d_1$ or when $2d_2 \leq d_1 \leq 3d_2$. Therefore the manufacturer prices at $\beta^*_0$ if $d_1 \leq 3d_2$ (which is equivalent to $q_2 < q_1 < 2(q_2 - q_2)$) and prices at $\beta^*_2$ otherwise.
Appendix I: Proof of Proposition 5

When both retailers carry both the store and the national brands, the first order conditions of two retailers can be simplified into \( \phi_1 = \frac{1}{2} \left( 1 + \gamma_1 + \frac{B_1}{\alpha + C_1} \right) \), \( \phi_2 = \frac{1}{2} \left( 1 + \frac{B_1}{\alpha + C_1} \right) \), \( \phi_h = \frac{1}{2} \left( 1 + \gamma_h + \frac{B_h}{\alpha + C_h} \right) \), and \( \tilde{\phi}_h = \frac{1}{2} \left( 1 + \frac{B_h}{\alpha + C_h} \right) \) where \( B_1, B_1, B_h \) and \( B_h \) are quadratic functions of \( \phi \), \( C_1, C_1, C_h \) and \( C_h \) are linear functions of \( \phi \). Observe that none is a function of \( \alpha \). With the above expressions for the first order conditions, we next establish Proposition 5 via a series of lemmas.

Lemma 5. There exists \( t_1 < +\infty \) such that whenever \( t > t_1 \), \( \hat{\sigma}_m(\beta) \) is concave on \([0, d_2]\).

PROOF. First, notice that \( \forall \beta \in [0, d_2] \),

\[
\hat{\sigma}_m(\beta) = [\hat{D}_n1 + \hat{D}_n2] \beta \\
= [(1 - \phi_1(\beta)) + (1 - \phi_2(\beta)) + \frac{1}{\alpha} P(\phi(\beta))] \\
= [(1 - \frac{1 + \beta}{2}) + (1 - \frac{1 + \beta}{2})]^{-1} \left[ \frac{B_1(\phi(\beta))}{2 + C_1(\phi(\beta))} \right]^{-1} \left[ \frac{B_2(\phi(\beta))}{2 + C_2(\phi(\beta))} \right] + \frac{1}{\alpha} P(\phi(\beta))
\]

where \( P(\phi) \) is a quadratic polynomial of \( \phi_1, \phi_2, \tilde{\phi}_1 \) and \( \tilde{\phi}_2 \). Define \( p(\beta) \equiv -\frac{1}{2} \frac{B_1(\phi(\beta))}{2 + C_1(\phi(\beta))} - \frac{1}{2} \frac{B_2(\phi(\beta))}{2 + C_2(\phi(\beta))} + \frac{1}{\alpha} P(\phi(\beta)) \).

Because \( P(\phi(\beta)), C_1(\phi(\beta)), C_2(\phi(\beta)), B_1(\phi(\beta)) \) and \( C_2(\phi(\beta)) \) are bounded and continuously differentiable for \( \forall \beta \in [0, d_2] \), we know that \( \lim_{\beta \to +\infty} p(\beta) = 0 \), \( \forall \beta \in [0, d_2] \). Therefore, for any \( \epsilon > 0 \), there exists \( \alpha_1 = \alpha_1(\epsilon) < +\infty \) such that whenever \( \alpha > \alpha_1 \), \( |p'(\beta)| = |p''(\beta)| < \epsilon \) for \( \forall \beta \in [0, d_2] \). If we take \( \epsilon = 2 \), then whenever \( \alpha > \alpha_1(2) \), \( \hat{\sigma}'_m(\beta) = -\left( \frac{1}{\alpha_1} \right) + \frac{1}{\alpha_2} \) and \( p''(\beta) < 0 \) for \( \forall \beta \in [0, d_2] \). This implies that there exists \( t_1 \equiv (\tilde{\theta}_m) \alpha_1(2) < +\infty \) such that whenever \( t > t_1 \), \( \hat{\sigma}_m(\beta) \) is concave on \([0, d_2]\).

Lemma 6. There exists \( t_2 < +\infty \) such that whenever \( t > t_2 \), \( \hat{\sigma}_m(\beta) \) is concave on \([d_2, d_1]\).

PROOF. The proof is similar to that of Lemma 5 and is omitted here.

Define \( \beta^*_1 = \max_{\beta \in [0, d_2]} \hat{\sigma}_m(\beta) \) and \( \beta^*_2 = \max_{\beta \in [d_2, d_1]} \hat{\sigma}_m(\beta) \) we then have the following:

Lemma 7. There exists \( t_3 < +\infty \) such that whenever \( t > t_3 \), \( \beta^*_1 \) is an interior point in \([0, d_2]\).

PROOF. Notice that \( \hat{\sigma}'_m(\beta) = 1 - \beta\left( \frac{1}{\alpha_1} \right) + \frac{1}{\alpha_2} \) and \( p''(\beta) < 0 \) for \( \beta < (0, d_2) \). Then for any \( d_1, d_2 > 0 \), when \( \beta \to 0^+ \), \( \hat{\sigma}'_m(\beta) = 1 + p''(\beta) < 0 \) whenever \( \alpha > \alpha_1(1) \). When \( \beta \to d_2^+ \), \( \hat{\sigma}'_m(\beta) = -\frac{\alpha_2}{\alpha_1} + p''(\beta) \). Then whenever \( \alpha > \alpha_1(2) \), \( \hat{\sigma}'_m(\beta) < 0 \). Moreover, whenever \( \alpha = \alpha_1(d_1) \) we also have \( \alpha > \alpha_1(2) \), and hence from the proof of Lemma 5, \( \hat{\sigma}_m(\beta) \) is concave on \([0, d_2]\). Therefore \( \exists t_3 \equiv (\tilde{\theta}_m) \alpha_1(d_2) < +\infty \) such that \( \forall t > t_3 \), \( \beta^*_1 \) is an interior point in \([0, d_2]\).

Lemma 8. For any \( \epsilon > 0 \), there exists \( t_4(\epsilon) < +\infty \) such that whenever \( t > t_4 \), \( \beta^*_2 \) is an interior point in \([d_2, d_1]\) if \( d_2 < (\frac{1}{\alpha} + \epsilon)d_1 \) and equals \( d_2 \) if \( d_2 > (\frac{1}{\alpha} + \epsilon)d_1 \).

PROOF. First, when \( \beta \in [d_2, d_1] \), \( \hat{\sigma}_m(\beta) = [1 - \frac{1 + \beta}{2}] + p_1(\beta) \) in which \( p_1(\beta) \equiv -\frac{1}{2} \frac{B_1(\phi(\beta))}{2 + C_1(\phi(\beta))} + \frac{1}{2} \frac{(1 - \phi_1(\beta))A_1 + \frac{1}{2}d_2(1 - \phi_1(\beta))^2 - \frac{1}{2}d_2(1 - \phi_2(\beta))^2}{2 + C_1(\phi(\beta))} \). Similar to \( p(\beta) \) in Lemma 5, for any \( \epsilon > 0 \), there exists \( \alpha_2 = \alpha_2(\epsilon) < +\infty \) such that whenever \( \alpha > \alpha_2 \), \( |p_1(\beta)| < \epsilon \) for \( \forall \beta \in [d_2, d_1] \). If we take \( \epsilon = \frac{\alpha_1}{\alpha_2} \), then \( \hat{\sigma}'_m(\beta) = \left( \frac{\beta \alpha_2}{\alpha_1} \right) \beta = -\frac{\alpha_2}{\alpha_1} + p_1(\beta) < 0 \) whenever \( \alpha > \alpha_2 \).

Now, because \( \hat{\sigma}'_m(d_2) = \frac{1}{\alpha_2} + p_1(\beta) \). For \( \forall \beta > 0 \), using the fact that \( |p_1(\beta)| < \epsilon \) whenever \( \alpha > \alpha_2 \), we have \( \frac{1}{\alpha_2} + |p_1(\beta)| < \left( \frac{1}{\alpha_2} - \epsilon, \frac{1}{\alpha_2} + \epsilon \right) \) whenever \( \alpha > \alpha_2 \). Therefore, if \( \frac{\alpha_2}{\alpha_1} > \frac{1}{\alpha^2} + \frac{1}{\alpha^2} + p_1(\beta) \) which gives \( \hat{\sigma}'_m(d_2) < 0 \), and if \( \frac{\alpha_2}{\alpha_1} < \frac{1}{\alpha_2} - \epsilon \), we must have \( \frac{\alpha_2}{\alpha_1} < \frac{1}{\alpha_2} + p_1(\beta) \) which gives \( \hat{\sigma}'_m(d_2) > 0 \). If \( \hat{\sigma}'_m(d_2) < 0 \), \( \beta^*_2 \) is set at \( d_2 \). If \( \hat{\sigma}'_m(d_2) > 0 \), \( \beta^*_2 \) is an interior point in \([d_2, d_1]\).

Finally, \( \forall \epsilon > 0 \), take \( t_4(\epsilon) \equiv (\tilde{\theta}_m) \max\{\alpha_2(\frac{1}{\alpha^2}), \alpha_2(\epsilon)\} \), we have Lemma 8.
Lemma 9. For any $\epsilon > 0$, there exists $t_0 = t_0(\epsilon) < +\infty$ such that whenever $t > t_0$, the manufacturer sets $\beta_1^*$ whenever $3d_2 > d_1 + \epsilon$ and sets $\beta_2^*$ whenever $3d_2 < d_1 - \epsilon$.

PROOF. \forall \epsilon > 0, define $c_0 = \mu(\epsilon) < +\infty$ from the lemmas above, we know that $\beta_1^* = \frac{d_1d_2}{d_1 + d_2} + O(\frac{1}{t})$, $\beta_2^* = \frac{d_2}{2} + O(\frac{1}{t})$ if $\frac{d_2}{d_1} < \frac{1}{2} - c_0$ and $\beta_2^* = d_2$ if $\frac{d_2}{d_1} > \frac{1}{2} + c_0$. We can thus conclude that $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) = \frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 + O(\frac{1}{t})$ if $\frac{d_2}{d_1} < \frac{1}{2} - c_0$. If $\frac{d_2}{d_1} < \frac{1}{2} - c_0$ fails to hold, we know that $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq \frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 + O(\frac{1}{t})$ because $\beta_2^*$ sometimes cannot assume the value yielding the interior local optimum. When this happens, from mathematical manipulation we get $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq \frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 + O(\frac{1}{t}) = \frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 + O(\frac{1}{t}) > 0$ for $\epsilon_0$ values that are small. Therefore the manufacturer always sets the wholesale price at $\beta_1^*$ when $\frac{d_2}{d_1} < \frac{1}{2} - c_0$ fails to hold.

If $\frac{d_2}{d_1} < \frac{1}{2} - c_0$, there exists $t_5 = t_5(c_0) < +\infty$ such that whenever $t > t_5 \equiv t_0(c_0)$, $\frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 - c_0 \leq \hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq \frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 + c_0$. With this, if $3d_2 \geq d_1 + 8c_0 \frac{d_1 + d_2}{d_1}$, then $\frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 - c_0 \geq 0$, which leads to $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq 0$. If $3d_2 \leq d_1 + 8c_0 \frac{d_1 + d_2}{d_1}$, then $\frac{1}{2} \frac{d_1d_2}{d_1 + d_2} - \frac{1}{8} d_1 + c_0 \leq 0$, which leads to $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq 0$. Define $\epsilon_1 = 8c_0 \sup_{(d_1,d_2)} \frac{d_1 + d_2}{d_1} = 16c_0$ and $\epsilon_2 = 8c_0 \inf_{(d_1,d_2)} \frac{d_1 + d_2}{d_1} = 8c_0$; then $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq 0$ if $3d_2 \geq d_1 + \epsilon_1$ and $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq 0$ if $3d_2 \leq d_1 - \epsilon_2$.

Recall $\epsilon = 16c_0$, then $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq 0$ if $3d_2 \geq d_1 + \epsilon$ and $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq 0$ if $3d_2 \leq d_1 - \epsilon$.

Recall that $d_i = \frac{q_i - q_{i+1}}{q_i}$ for $i = 1, 2$. With this, we can immediately obtain Proposition 5 from Lemma 6.

Appendix J: Proof of Proposition 6

If $q_{i1} < q_{i2} = q_i$, the store brand and the national brand are treated as exactly the same product by R2. She will carry only the one with the lower variable cost. Therefore the national brand manufacturer needs to set a wholesale price less than $c_n$ in order to distribute the national brand through R2, but then he will get a nonpositive profit. Hence the national brand manufacturer distributes the national brand through R1 only.