Supplementary Appendix to Bribing Voters: Introducing Expressive Voters and Uncertainty

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1 Introduction

We want to investigate how the presence of expressive voters would affect the predictions of the basic “Bribing Voters” model. In particular, is it still the case that we would conclude that roll calls in legislatures should be public, while ballots in elections should be secret?

The argument can be broken into two: first, would legislators be typically more prone to behave in a way that makes public votes the way to go? Second, even if public votes are good for legislators, would secret ballots be good for general elections?

On the first part, if one makes the neutral assumption that legislators are drawn from the general population, then legislators should display the same expressive preferences as general voters. (In fact, the evidence in political science suggests that legislators are not cynical agents, but rather ideological types for which we would think expressive considerations matter as much as for the general voting population). Thus, if common voters in general elections have an expressive component, so do legislators. However, legislators will always have an extra cost of voting the wrong way under public votes: the cost of being held accountable. In this sense, the basic model entails a normalization: expressive costs are zero for both type of voters (general voters and legislators) and the vote-related costs capture the extra disutility facing legislators voting the wrong way: they may be kicked out of office. Thus, public voting will always make more sense in legislatures than in general elections, because they activate the accountability mechanism, which

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raises the costs of capture.

On the second part, the question is whether we can derive optimality of secret ballots when voters may be expressive, and also considering that they may attach a low probability to being pivotal. There is substantial evidence (both empirical and experimental) that voters are strategic. But not all voters may be that way. So I want to investigate the possibility that at least some are expressive.

2 Extension 1

Assume that voters come in three types: strategic, expressives for “yes” and expressives for “no,” with respective probabilities $1 - p - q$, $p$, and $q$. In other words, let us assume that ‘yes’ voters like the principal’s project (in an amount $\theta$) and would also suffer a disutility $\eta$ when not voting for it. Then ‘no’ voters dislike the principal’s project (so they have outcome-related costs $\theta$) and would suffer a disutility $\eta$ when voting for it; lastly, strategic voters are just like the voters in the baseline model in the paper: they suffer a cost $\theta$ if the principal’s project goes through and they do not care how they vote per se. Assume there is a very large number $N$ of voters, and at least $M < N$ of them must vote “yes” for the principal’s proposal to be approved. Because $N$ is very large I will obviate considerations on integer problems (i.e. I will always assume that $pN$, $qN$ and $(1 - p - q)N$ is a Natural number).

Now I will consider two scenarios. In the first, voter types are observable to the principal, and in the second scenario types are voters’ private information.

**Observable types** When types are observable, the principal knows that a number $pN$ of voters will vote “yes” because they are “yes”-expressives. When $pN < M$ then (given $N$ is very large) the principal is almost sure to lose the vote unless she offers incentives. Suppose that $(1 - q)N > M$ (i.e. the share of expressives for “no” is not too large). Then the principal can offer the pivotal bribes $b = \theta + \varepsilon$ in exchange for a pivotal yes vote to the strategic voters and the game for the latter is analogous to the baseline case in the paper. Capture at no cost follows. To see this, note that expressives for no will vote no. Expressive for yes will vote yes. And strategic voters will know that if more than $M - pN$ of them vote yes the project will go through (inducing a cost $\theta$ to them), and that if less than that share vote yes the project will not go through. This then reduces to a game exactly like that in the baseline model, where the majority requirement is now $M - pN$ (i.e. $pN$
votes are provided by ‘yes’ expressives). Provided that \((1 - q)N > M\) then the principal can simply avoid trying to buy ‘no’ expressives and the baseline model can be seen as dealing with the case of how to get the strategic voters to vote ‘yes’. When there are enough ‘no’ expressives, costless capture does not obtain and in this case secret ballots do not hurt. But there may well be situations when \((1 - q)N > M\) and costless capture will occur. In this instance secret ballots will strictly help, by raising the cost of capture to \(\frac{M - pN}{2}\). From an ex ante perspective, one would want to have secret ballots in elections.

One may think we have not pushed the expressive voting scenario too much. After all, the principal can avoid dealing with ‘no’ expressives because she can observe who they are. So we now relax this assumption.

**Unobservable types** Our (strong) implementation criterion is that voters must have a dominant strategy (at least weakly) for us to think they will vote yes. The principal can now offer bribes \(b = \theta + \varepsilon\) in exchange for a pivotal yes vote. What happens now? Because ‘no’ expressives would lose \(\eta\) in case they vote yes and are not pivotal, voting yes is never a dominant strategy for them, and they vote against the principal and the latter never has to pay them. In other words, ‘no’ expressives screen themselves out. Expressive ‘yes’ voters and strategic voters all take these offers and vote yes. Again, if \((1 - q)N > M\) there are enough ‘yes’ votes in a very large election and the proposal passes without the principal paying anything. Secret ballots would again help by forcing positive costs of capture. Therefore, at the time of writing a constitution, making ballots secret will be best, as it will make capture costly whenever \((1 - q)N > M\).

Again, one may want to push the assumption of expressive voters even further, and assume that they are the only type of voters that exist.

### 3 Extension 2

Now I want to show that secret ballots will also be safer in situations in which the expected number of strategic voters is very low and the principal must deal exclusively with expressive voters. It is enough then to show that for some parameter configurations, secret ballots would raise the costs of capture, even when voters are expressive and they may attach low probability to the event of being pivotal.

Suppose there are only ‘yes’ and ‘no’ expressive voters. Moreover, we now want to incorporate uncertainty to the setup, so that, with many voters, there is a low perceived probability that each voter will be pivotal. This
incorporates the two relaxations simultaneously and in very strong form. Suppose there are \( n \) voters, all with expressive preferences on top of outcome-related preferences. In other words, if option Y is chosen collectively, a ‘no’ expressive voter incurs a cost \( \theta \) regardless of how he votes, and if he votes for Y, he incurs a cost \( \eta \) regardless of the collective outcome. ‘yes’ expressives may be assumed to care positively about the principal’s project and also suffer a cost \( \eta \) when voting ‘no.’ For simplicity, we take the majority rule to be simple majority: \( M = \frac{n+1}{2} \).

Suppose the principal offers a bribe \( b \) contingent on a pivotal vote for Y. Denoting with \( \#y \) the number of votes cast for Y by the other voters, the expected utility differential between voting yes and no for a ‘no’ expressive is,

\[
U_Y - U_N = P(\#y < \frac{n-1}{2}) (-\eta) + P(\#y = \frac{n-1}{2}) (b - \theta - \eta) + P(\#y > \frac{n-1}{2}) (-\eta).
\]

That is, whenever the voter is not pivotal either Y is selected or not, but the only difference for a voter from voting for Y is paying the expressive vote \( -\eta \). When being pivotal, which requires that \( \frac{n-1}{2} \) other voters voted for Y, doing likewise triggers the approval of the project with cost \( \theta \), it also triggers compensation \( b \), and also triggers the cost \( \eta \). Denoting with \( Piv \) the probability of a voter \( i \) being pivotal \( P(\#y = \frac{n-1}{2}) \), the utility differential is,

\[
Piv. (b - \theta) - \eta.
\]

The presence of uncertainty in this setup complicates things for the principal. Now voters care about their expected utility across a number of contingencies (or voting profiles), each possible with some probability. Therefore, a bribing scheme such that large bribes are paid when a voter is pivotal will not work if the effect of that very scheme is to make sure no one is pivotal in equilibrium. But now the principal can choose the pivotal bribes \( b \) to be slightly lower, so as to make each voter indifferent between voting for Yes or No (no money is offered for non pivotal ‘yes’ votes). Then \( b \) must satisfy \( Piv(b - \theta) - \eta = 0 \), or,

\[
b = \theta + \frac{\eta}{Piv}.
\]

This indicates that if the principal wants to make expressive voters indifferent between voting ‘yes’ and voting ‘no,’ she must compensate them not only for the cost \( \theta \) they would suffer in the event of being pivotal and making the project go through, but also for the cost \( \eta \) that they incur whenever they vote ‘yes.’ Note that if the bribe is only paid when voters are pivotal,
then the transfer that must be paid in that event to compensate for the sure disutility \( \eta \) of voting ‘yes’ increases as the probability of being pivotal goes down.

We now construct an equilibrium that is very favorable to the principal. In this equilibrium, the principal chooses a bribe that in equilibrium makes voters indifferent between voting one way or another, and voters randomize between voting one way and the other. Now the principal may have to pay some money in equilibrium (a number \( \frac{n+1}{2} \) voters may be pivotal). Suppose each voter votes Y with probability \( q \). The objective function of the principal then is,

\[
P(Y \text{ wins}) \pi - \frac{n+1}{2} P\left( \#y = \frac{n+1}{2} \right) \cdot b = \\
P(Y \text{ wins}) \pi - \frac{n+1}{2} P\left( \#y = \frac{n+1}{2} \right) \left( \theta + \frac{\eta}{Piv} \right) = \\
P(Y \text{ wins}) \pi - \frac{n+1}{2} P\left( \#y = \frac{n+1}{2} \right) \left( \theta + \frac{\eta}{P\left( \#y = \frac{n+1}{2} \right)} \right).
\]

In the equilibrium we construct, there are two sources of uncertainty for a ‘no’ expressive voter. First, there is an uncertain number of ‘yes’ expressive voters. There may be enough of them to pass the principal’s project even if all ‘no’ voters vote against it; but it is also possible that there are very few of them, so the project would go through only if enough ‘no’ voters switch and vote ‘yes’. The probability of there being \( x \) votes cast for Y depends on the proportion of ‘yes’ expressives and the proportion of ‘no’ expressives. If there is a number \( z \) of ‘yes’ expressives, there will be a baseline number \( z \) of votes for Y, which has probability,

\[
\binom{n}{z} p^z (1 - p)^{n-z}.
\]

Given \( z \) ‘yes’ expressives, who always vote ‘yes’, the probability of \( x \) ‘yes’ votes by ‘no’ expressives (who vote ‘yes’ with probability \( q \)) is,

\[
\binom{n - z}{x} q^x (1 - q)^{n-x}.
\]

Thus, we can compute the probability that a ‘no’ voter perceives of being pivotal as the probability that there are \( \frac{n-1}{2} \) yes votes from the other voters,

\[
\sum_{i=0}^{\frac{n-1}{2}} \binom{n - 1}{i} p^i (1 - p)^{n-1-i} \cdot \binom{n - 1 - i}{\frac{n+1}{2} - i} q^{\frac{n+1}{2} - i} (1 - q)^{n-1 - \frac{n+1}{2}}.
\]
the rationale for this expression is: for any number $i$ of ‘yes’ voters (each with probability $p_i(1 - p)^{n-i}$) that is lower than a strict majority, there exists a number $\frac{n+1}{2} - i$ of ‘no’ voters that, if voting yes, would create a minimum winning majority for ‘yes’, thus making all voters pivotal. The probability of that precise number $\frac{n+1}{2} - i$ of ‘no’ voters voting yes when they randomize with probability $q$ is

$$\left(\frac{n-i-1}{\frac{n+1}{2} - i}\right) \frac{n-1}{2} - i (1 - q)^{n-\frac{n+1}{2} + i}.$$

Then the expression simply sums these probabilities across all possible numbers of ‘yes’ voters that are below a minimum winning majority.

Because the calculations are very tedious when keeping track of all the possible scenarios depending on the number of ‘yes’ voters, I stack the deck against the principal a bit further and assume there are no ‘yes’ expressives, so that the probabilities in the objective function of the principal can be written as,

$$P(Y \text{ wins}) = \sum_{i=\frac{n+1}{2}}^{n} \binom{n}{i} q^i (1 - q)^{n-i},$$

$$P\left(\#y = \frac{n+1}{2}\right) = \binom{n}{\frac{n+1}{2}} q^{\frac{n+1}{2}} (1 - q)^{n-\frac{n+1}{2}},$$

$$Piv = P\left(\#y = \frac{n-1}{2}\right) = \binom{n-1}{\frac{n-1}{2}} q^{\frac{n-1}{2}} (1 - q)^{\frac{n-1}{2}}.$$

Therefore, the objective function of the principal can be written,

$$\pi \sum_{i=\frac{n+1}{2}}^{n} \binom{n}{i} q^i (1 - q)^{n-i} - \theta \left(\frac{n+1}{2}\right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} \left(\theta + \frac{\eta}{\left(\frac{n-1}{2}\right) q \frac{n+1}{2} (1 - q) \frac{n-1}{2} \right)},$$

or,

$$\pi \sum_{i=\frac{n+1}{2}}^{n} \binom{n}{i} q^i (1 - q)^{n-i} - \theta \frac{n+1}{2} \left(\frac{n+1}{2} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} - \frac{n+1}{2} \left(\frac{n+1}{2} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} = \frac{n+1}{2} \left(\frac{n+1}{2} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} \eta.$$

Note that,

$$\frac{\binom{n+1}{\frac{n+1}{2}}}{\binom{n}{\frac{n-1}{2}}} = \frac{n+1}{2} \frac{\binom{n-1}{\frac{n-1}{2}}}{\binom{n}{\frac{n-1}{2}}} = \frac{n!}{\frac{n+1}{2} \frac{n-1}{2} \frac{n+1}{2}! (n-1)!} = \frac{2n}{n+1}.$$
The objective function of the principal then simplifies to,

\[ \pi \sum_{i=\frac{n+1}{2}}^{n} \binom{n}{i} q^i (1-q)^{n-i} - \theta \frac{n+1}{2} \frac{n}{2} q \frac{n+1}{2} (1-q) \frac{n+1}{2} - nq\eta. \]

Note that the principal’s expected revenues are \( \pi \sum_{i=\frac{n+1}{2}}^{n} \binom{n}{i} q^i (1-q)^{n-i} \), which tend to \( \pi \) when \( n \) goes large provided the optimally chosen \( q \) is larger than \( \frac{n+1}{n} \). The costs to the principal are of two kinds. There is a component that is related to compensating voters for the outcome-related disutility \( \theta \), and this cost goes to zero as \( n \) goes large. \(-\theta \frac{n+1}{2} \frac{n}{2} q \frac{n+1}{2} (1-q) \frac{n+1}{2} \). The other component is related to having to compensate voters for their expressive disutility: \(-n\eta\frac{q}{1-q}\), which grows with \( n \). The principal then must choose the probability \( q \) with which she wants voters to vote ‘yes’. She does not want to induce them to vote ‘yes’ for sure, because in that case the probability of being pivotal goes to zero, and the needed bribes go to infinity. She does not want to induce them to vote yes with probability zero, because then there are never many votes in her favor. She will typically settle for an interior solution. The first order condition of the principal’s problem is long winded. What matters is that for certain parameter configurations there is always an interior solution (otherwise the principal abstains from attempting to exert influence). For example, fix \( n = 51, \theta = 1, \pi = 100, \eta = \frac{1}{2} \). The objective function then clearly has an interior solution for \( q \) higher than \( \frac{1}{2} \) but lower than 1:
That is, voters vote ‘yes’ with positive probability (in fact, more often than not), even when they are only being compensated in the event of being pivotal, which happens with very low probability. The previous analysis implies,

**Proposition 1** Suppose public ballots, a parameter configuration such that the principal wants to exert influence, and that there are only expressive voters for ‘no.’ The principal can then choose an optimal probability $q^*$ with which voters will support her project by setting pivotal bribes $b = \theta + \frac{1}{Piv^*}$, where $Piv^* = \binom{n-1}{\frac{n+1}{2}} q^{n+1}(1-q)^{\frac{n-1}{2}}$. Under this scheme, when $n$ is large the principal avoids compensating voters for the outcome-related costs of the project $\theta$, and only compensates them, in expectation, for their expressive costs.

Are there parameter configurations such that secret ballots improve matters? As shown above, the cost of getting a favorable decision when using pivotal bribes under public voting is $\theta \frac{n+1}{2} \left( \frac{n+1}{2} \right) q \frac{n+1}{2} (1-q) \frac{n-1}{2} + nq\eta$, where $q$ is chosen optimally. For $n$ large, the project is approved with probability approaching one. Under secret voting, the only way to induce $\frac{n+1}{2}$ voters who are ‘no’ expressives to vote yes when bribes can only be conditioned on the project going through is to promise full compensation to a minimum winning majority. This yields $\pi$ at a cost $\frac{n+1}{2} (\theta + \eta)$. The principal can always use this method under public votes, too, and she can always be trusted
to use the minimum cost method. So the question is whether there are parameter configurations such that the pivotal bribes will yield a lower cost to the principal. That means that under such configurations, secret ballots would raise the costs of capture, without affecting what the principal would do under other configurations. So now I show that there exist parameter configurations such that the costs of capture under public votes and pivotal bribes is indeed lower than under secret ballots.

We must show that there are parameter configurations such that,

$$\frac{n+1}{2} \theta \frac{n+1}{2} \left( \frac{n}{2} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} + nq \eta < \frac{n+1}{2} \left( \theta + \eta \right).$$

This can be rewritten as,

$$\left[ nq - \frac{n+1}{2} \right] \eta < \theta \frac{n+1}{2} \left( 1 - \left( \frac{n}{n+1} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} \right)$$

$$\left[ \frac{2nq - n + 1}{2} \right] \eta < \theta \frac{n+1}{2} \left( 1 - \left( \frac{n}{n+1} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} \right)$$

$$\left[ \frac{2nq - n - 1}{n+1} \right] \eta < \theta \left( 1 - \left( \frac{n}{n+1} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} \right)$$

$$\left[ \frac{2q - 1}{n + 1} \right] n \eta < \theta \left( 1 - \left( \frac{n}{n+1} \right) q^{\frac{n+1}{2}} (1 - q)^{\frac{n-1}{2}} \right).$$

For $n$ large, these expressions tend to,

$$2q - 1 < \frac{\theta}{\eta},$$

which tells us that whenever outcome related costs are larger than a fraction of the expressive costs, secret ballots will raise the costs of capture. For instance, if the optimal $q$ is 0.6, whenever the outcome-related disutility is more than 20% of the expressive cost secret ballots will improve matters. We then have,

**Proposition 2** For parameter configurations satisfying $2q - 1 < \frac{\theta}{\eta}$ secret ballots raise the costs of capture. For other parameter configurations they do not, and in those cases the imposition of secrecy will be neutral.

Because when the condition $2q - 1 < \frac{\theta}{\eta}$ is not met the principal will use a minimum cost approach (rather than the pivotal approach) imposing secrecy
never hurts, but it helps the committee whenever it is true that $2q - 1 < \frac{q}{q}$. The intuition behind the result is that under public votes the principal can device pivotal bribe schemes that allow her to not pay outcome related costs. In expectation she must cover the expressive costs of a higher number of voters, however, so in the limit these costs should not be too high relative to the outcome-related costs for pivotal bribes to be convenient. When they are, secrecy will get in the principal’s way, forcing her to have to use higher cost methods of influence. What is interesting is that this result is obtained in a setup where voters are numerous, they attach a low probability to the event of being pivotal, and they have expressive voting preferences. A caveat: in the game without ‘yes’ expressives, there are other Nash equilibria where voters vote against the principal. However, adding ‘yes’ expressives with arbitrarily low probability refines away these equilibria.