Quiet Bubbles*

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Abstract

Classic speculative bubbles are loud—price is high and so are price volatility and share turnover. The credit bubble of 2003-2007 is quiet — price is high but price volatility and share turnover are low. We develop a model, based on investor disagreement and short-sales constraints, that explains why credit bubbles are quieter than equity ones. First, since debt up-side payoffs are bounded, debt is less sensitive to disagreement about asset value than equity and hence has a smaller resale option and lower price volatility and turnover. Second, while optimism makes both debt and equity bubbles larger, it makes debt mispricings quiet but leaves the loudness of equity mispricings unchanged. Finally, holding fixed average optimism, an increase in disagreement provided leverage is cheap enough can also lead to a large and quiet debt mispricing. Our theory suggests a taxonomy of bubbles.

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1. Introduction

Many commentators point to a bubble in credit markets from 2003 to 2007, particularly in the AAA and AA tranches of the subprime mortgage collateralized default obligations (CDOs), as the culprit behind the Great Financial Crisis of 2008 to 2009.[1] However, stylized facts, which we gather below, suggest that the credit bubble lacked many of the features that characterize classic episodes. These episodes, such as the Internet bubble, are typically loud—characterized by high price, high price volatility, and high trading volume or share turnover as investors purchase in anticipation of capital gains.[2] In contrast, the credit bubble is quiet—characterized by high price but low price volatility and low share turnover. In this paper, we make a first attempt at developing a taxonomy of bubbles based on the nature of the asset being traded. In particular, we show why credit bubbles are quieter and hence fundamentally different than equity ones.

Our theory builds on the investor disagreement and short-sales constraints framework, which has been used to generate loud equity bubbles. In these models, disagreement and binding short-sales constraints lead to over-pricing in a static setting as pessimists sit out of the market (Miller (1977) and Chen et al. (2002)). In a dynamic setting, investors value the potential to re-sell at a higher price to someone with a higher valuation due to binding short-sales constraints (Harrison and Kreps (1978) and Scheinkman and Xiong (2003)).[3] This framework generates a bubble or overpricing in which price trades at above fundamental value. A distinguishing feature of this model is that the resale option is also associated with high price volatility and high share turnover. In contrast to other approaches which rely on investor optimism to generate mispricing, the resale option mechanism is intrinsically tied to loudness associated with share turnover: an asset price is overvalued when short-sales

[1] Indeed, there is compelling evidence that these securities were severely mis-priced relative to their risk-adjusted fundamentals (see Coval et al. (2009)).

[2] See Hong and Stein (2007) for a review of the evidence regarding classic bubbles and Ofek and Richardson (2003) for a focus on the dot-com bubble.

[3] See Hong and Stein (2007) for a more extensive review of the disagreement approach to the modeling of bubbles.

constraints are binding, which also corresponds to high share turnover.[4]

Within this framework, we consider the pricing of a debt security. Investors have disagreement over the underlying asset value. Whereas equity payoffs are linear in the investor beliefs regarding underlying asset value, debt up-side payoffs are capped at some constant and hence are non-linear (concave) in the investor beliefs about fundamental. We make the standard assumption regarding short-sales constraints. There is compelling evidence that such constraints are at least as binding in debt markets as in equity ones.[5] The pricing of debt contracts in this disagreement and short-sales constraints setting is new.

We show that the disagreement or resale framework, which is typically thought of as generating loud equity bubbles, also naturally generates quiet credit bubbles. Since debt up-side payoffs are bounded in contrast to equity, the valuation of debt is less sensitive to disagreement about underlying asset value than equity and hence has a smaller resale option. As a result, a debt bubble is smaller and quieter than an equity bubble. This is due simply to the bounded upside payoff of debt compared equity. The safer is the debt claim, the more bounded is the upside, the less sensitive it is to disagreement and therefore the lower the resale option and the smaller and quieter is the bubble.

In addition to this resale option, the bubble can also have an optimism component. As investor optimism rises holding fixed fundamental value, both debt and equity bubbles naturally grow in size. But debt bubbles become quieter in the process whereas an equity bubble's loudness is invariant to the degree of investor optimism. When investors become more optimistic about the underlying fundamental of the economy, they view debt as being more risk-free with less upside and hence having a smaller resale option. Hence the resale option component becomes a smaller part of the debt bubble and debt mispricing becomes

[4]For other behavioral approaches without this strong turnover implication, see Delong et al. (1990) and Abreu and Brunnermeier (2003). For an agency approach which generates investor optimism, see Allen and Gorton (1993) and Allen and Gale (2000). For the possibility of rational bubbles, see Blanchard and Watson (1983), Tirole (1985), Santos and Woodford (1997), and Allen et al. (1993).

[5]Indeed, short-sales constraints on CDOs were binding until the onset of the financial crisis (see Michael Lewis (2010)). See for more systematic evidence on cost of borrowing in corporate bonds Asquith et al. (2010).

quieter with investor optimism. In contrast, the linearity of the equity payoff implies that an increase in average optimism does not change the sensitivity of equity to disagreements about the underlying asset. As a result, both the price volatility and share turnover of equity bubbles are independent of investors' average optimism.

Alternatively, one can fix investor optimism and consider a comparative static with respect to fundamental value and derive similar predictions. Interestingly, when fundamentals deteriorate, investors perceive the debt claim as riskier – closer to an equity claim – and beliefs start having a stronger influence on the asset valuation. The resale option grows and both price volatility and trading volume increase. This result stands in contrast to the traditional arguments in Myers and Majluf (1984) and Gorton and Pennacchi (1990) that debt claims are less sensitive to private information — our model is the counterpart to this view in a world without private information but with heterogenous beliefs. It turns out that working through debt pricing with heterogeneous beliefs about asset value yields many interesting implications.[6]

Finally, we show that even in a setting with an unbiased average belief, an increase in the dispersion of priors can make bubbles larger and quieter at the same time, provided that leverage is cheap enough. The only ingredients required for this to hold are (1) the concavity of the debt claim and (2) the existence of an interim payoff (an interest payment for debt or dividend for equity) on which investors currently disagree. There are two reasons why mispricing increases with dispersion. The first is the well-known binding short-sales constraints effect in which price reflects the opinions of the optimists about the interim payoff. The second effect is that the resale option value increases since the marginal buyer in the future will be more optimistic. As dispersion of priors increases, optimists today value the interim payoff more highly and buy more shares. Hence, pessimists have fewer shares with which to re-sell to the optimists tomorrow. This decreases turnover. Price volatility

[6]Earlier work on heterogeneous beliefs and bond pricing such as in Xiong and Yan (2010) assume that investors have disagreement about bond prices or interest rates and they do not model the nature of the concavity of debt pay-offs as a function of underlying asset value disagreement.

– which essentially reflects the differences in belief among the optimistic agents across the future states of nature – also decreases with an increase in the dispersion of priors. When the dispersion becomes sufficiently large, the asset resembles a risk-free asset and there is low disagreement among optimists about the value of the asset regardless of the state of nature and hence the low price volatility.

The contribution of our paper is to generalize the disagreement and short-sales constraints models of asset price bubbles by extending it to non-linear payoff function. We show that the concavity of debt-payoffs and their insensitivity to belief differences generate many interesting new insights on asset price bubbles. In this context, we adopt a reduced-form view of leverage: we take the cost of leverage as an exogenous parameter that decreases with the efficiency of the banking sector. In this sense, our taxonomy of bubbles does not depend on the determinants of leverage. An earlier literature has examined how to endogenize leverage when investors trade equity claims—so the credit bubble in their setting is a bubble in lending.[7] In contrast, our paper focuses on bubbles in the trading of actual credits. Another element of our model is the use of heterogeneous priors.[8] We show that heterogeneous priors lead to novel dynamics in debt prices that do not necessarily occur with equity prices.

Our paper proceeds as follows. We present some stylized facts on the recent credit bubble of 2003-2007 in Section 2. The model and main results are discussed in Section 3. We discuss empirical implications of our model in Section 4. We relate our work to studies on the recent financial crisis in Section 5. We conclude in Section 6.

[7] Geanakoplos (2010) shows that leverage effects in this disagreement setting can persist in general equilibrium though disagreement mutes the amount of lending on the part of pessimists to optimists. Simsek (2010) shows that the amount of lending is further mediated by the nature of the disagreement—disagreement about good states leads to more equilibrium leverage while disagreement about bad states leads to less.

[8] Our use of priors is similar to Morris (1996) who showed that small differences in priors can lead long-run divergences in the Harrison and Kreps (1978) setting that we also consider. We interpret our priors assumption as in Morris (1996) who argues that it applies in settings where there is a financial innovation or a new company and hence there is room for disagreement. The subprime mortgage-backed CDOs are new financial instruments and hence the persistence effect that he identifies also holds in our setting. An alternative and more behavioral interpretation of our model is that overconfident investors who overreacted to information as in Odean (1999) and Daniel et al. (1998).

2. Stylized Facts About the Credit Bubble of 2003-2007

We begin by providing some stylized facts regarding the recent credit bubble that motivate our theoretical analysis. By all accounts, subprime mortgage CDOs experience little price volatility between 2003 until the onset of the financial crisis in mid-2007. In Figure 1, we plot the prices of the AAA and AA tranches of the subprime CDOs. The ABX price index only starts trading in January of 2007, very close to the start of the crisis. Nonetheless, one can see that, in the months between January 2007 until mid-2007, the AAA and AA series are marked by high prices and low price volatility. Price volatility only jumps at the beginning of the crisis in mid-2007. This stands in contrast to the behavior of dot-com stock prices—the price volatility of some internet stocks during 1996-2000 (the period *before* the collapse of the internet bubble) exceed 100% per annum, more than three times the typical level of stocks.

Another way to see the quietness is to look at the prices of the credit default swaps for the financial companies that had exposure to these subprime mortgage CDOs. This is shown in Figure 2. The price of insurance for the default of these companies as reflected in the spreads of these credit default swaps is extremely low and not very volatile during the years before the crisis. One million dollars of insurance against default cost a buyer only a few thousand dollars of premium each year. This price jumps at the start of the crisis, at about the same time as when price volatility increases for the AAA and AA tranches of the subprime mortgage CDOs.

The low price volatility coincided with little share turnover or re-selling of the subprime mortgage CDOs before the crisis. Since CDOs are traded over-the-counter, exact numbers on turnover are hard to come by. But anecdotal evidence suggests extremely low trading volume in this market particularly in light of the large amounts of issuance of these securities. Issuance totals around $100 billion dollars per quarter during the few years before the crisis but most of these credits are held by buyers for the interest that they generate. To try to capture this low trading volume associated with the credit bubble, in Figure 3, we plot the average monthly share turnover for financial stocks. Turnover for finance firms is low and only jumps at the onset of the crisis as does turnover of subprime mortgage CDOs according to anecdotal accounts (see, e.g., Michael Lewis (2010)). This stands in contrast to the explosive growth in turnover that coincided with the internet boom in Figure 4—obtained from Hong and Stein (2007). As shown in this figure, the turnover of internet stocks and run-up in valuations dwarfs those of the rest of the market.

Finally, we provide evidence that short-sales constraints were tightly binding until around 2006 when synthetic mezzanine ABS CDOs allowed hedge funds to short subprime CDOs, thereby leading to the implosion of the credit bubble. In Figure 5, we plot issuance of synthetic mezzanine ABS CDOs which is how hedge funds such as John Paulson's finally were able to short the subprime mortgage CDOs. Notice that there was very little shorting in this market until the end as issuances of this type of CDO are not sizable until 2006. In other words, short-sales constraints were binding tightly until around 2007, consistent with the premise of our model. The collapse coincided with a large supply of these securities in 2007, similar to what happened during the dot-com period. This collapse effect is already modeled in Hong et al. (2006) using disagreement and short-sales constraints. We are interested in seeing whether the properties of credit bubbles (as described in Figures 1-4) arise within the same disagreement and short-sales constraint framework. It is to this analysis that we now turn.

3. Model

3.1. Set-up

Our model has three dates $t = 0$, 1, and 2. There are two assets in the economy. A risk-free asset offers a risk-free rate each period. A risky debt contract with a face value of D has the following payoff at time 2 given by:

$$\tilde{m_2} = \min\left(D, \tilde{G}_2\right), \tag{1}$$

where

$$\tilde{G}_2 = G + \epsilon_2 \tag{2}$$

and G is a known constant and ϵ_2 is a random variable drawn from a standard normal distribution $\Phi(\cdot)$. We think of $\tilde{G}_t$ as the underlying asset value which determines the payoff of the risky debt or the fundamental of this economy. There is an initial supply Q of this risky asset.

There are two groups of agents in the economy: group A and group B with a fraction 1/2 each in the population. In the first part of this paper, both groups share the same belief at date 0 about the value of the fundamental. More specifically, both types of agents believe at $t = 0$ that the underlying asset process is:

$$\tilde{V}_2 = G + b + \epsilon_2, \tag{3}$$

where b is the agents' optimism bias. When $b = 0$, investor expectations are equal to the actual mean of the fundamental G and there is no aggregate bias. The larger is b, the greater the investor optimism.

At $t = 1$, agents' beliefs change stochastically: at t=1, agents in group A believe the asset process is in fact

$$\tilde{V}_2 = G + b + \eta^A + \epsilon_2 \tag{4}$$

while agents in group B believe it is:

$$\tilde{V}_2 = G + b + \eta^B + \epsilon_2, \tag{5}$$

where η^A and η^B are i.i.d and distributed according to a normal law with mean 0 and standard deviation 1. These revisions of beliefs are the main shocks that determine the price of the asset, price volatility and share turnover at $t = 1$.

The expected payoff of an agent with belief $G + b + v$ regarding $\tilde{m}_2$ is given by:

$$\pi(v) = E[\tilde{m_2}|v] = \int_{-\infty}^{D-G-b-v} (G + b + v + \epsilon_2)\phi(\epsilon_2)d\epsilon_2 + D\left(1 - \Phi\left(D - G - b - v\right)\right). \quad (6)$$

This is the expected payoff, conditioned on an agent with a belief $G+b+v$ regarding the mean of $\tilde{G}_2$, of a standard debt claim in the absence of limited liability. If the fundamental shock ϵ is sufficiently low such that the value of the asset underlying the credit is below its face value (i.e. $G+b+v+\epsilon < D$), then the firm defaults on its contract and investors become residual claimant (i.e. they receive $G+b+v+\epsilon$). If the fundamental shock ϵ is sufficiently good such that the value of the fundamental is above the face value of debt (i.e. $G + b + v + \epsilon \geq D$), then investors are entitled a fixed payment D. Our analysis below applies more generally to any (weakly) concave expected payoff function, which would include equity as well standard debt claims. Note also that the unlimited liability assumption is not necessary for most of our analysis, but it allows us to compare our results with the rest of the literature.

Agents are risk-neutral and are endowed with zero liquid wealth but large illiquid wealth W (which becomes liquid and is perfectly pledgeable at date 2). To be able to trade, these agents thus need to access a credit market that is imperfectly competitive. The discount rate is 0 but banks charge a positive interest rate, which we call $\frac{1}{\mu} - 1$, so that μ is the inverse of the gross rate charged by the banks. μ is increasing with the efficiency of the credit market.

Finally, the last ingredient of this model is that our risk-neutral investors face quadratic trading costs given by:

$$c(\Delta n_t) = \frac{(n_t - n_{t-1})^2}{2\gamma}, \quad (7)$$

where n_t is the shares held by an agent at time t. The parameter γ captures the severity of the trading costs – the higher is γ the lower the trading costs. These trading costs allow us to obtain a well-defined equilibrium in this risk-neutral setting.

Note that $n_{-1} = 0$ for all agents, i.e. agents are not endowed with any risky asset. Investors are also short-sales constrained.

3.2. Date-1 equilibrium

Let P_1 be the price of the asset at $t = 1$. At $t = 1$, consider an investor with belief $G + b + \eta$ and date-0 holding n_0. Her optimization problem is given by:

$$J(n_0, \eta, P_1) = \left| \begin{array}{l} \max_{n_1} \left\{ n_1 \pi(\eta) - \frac{1}{\mu} \left((n_1 - n_0) P_1 + \frac{(n_1 - n_0)^2}{2\gamma} \right) \right\} \\ n_1 \geq 0 \end{array} \right. \tag{8}$$

where the constraint is the short-sale constraint.

Call $n_1^\star(\eta)$ the solution to the previous program. If $n_1^\star(\eta) - n_0$ is positive, an agent borrows $(n_1^\star(\eta) - n_0)P_1 + \frac{(n_1^\star(\eta) - n_0)^2}{2\gamma}$ to buy additional shares $n_1^\star(\eta) - n_0$. If $n_1^\star(\eta) - n_0$ is negative, the agent makes some profit on the sales but still has to pay the trading cost on the shares sold $(n_0 - n_1^\star(\eta))$. This is because the trading cost is symmetric (buying and selling costs are similar) and only affects the number of shares one purchases or sells, and not the entire position (i.e. $n_1 - n_0$ vs. n_1). In equation 6, $J(n_0, \eta, P_1)$ is the value function of an agent with belief $G + b + \eta$, initial holding n_0 and facing a price P_1. Clearly, $J(n_0, \eta, P_1)$ is driven in part by the possibility of the re-sale of the asset bought at $t = 0$ at a price P_1.

Our first theorem simply describes the date-1 equilibrium. At date 1, three cases arise, depending on the relative beliefs of agents from group A and B. If agents from group A are much more optimistic than agents from group B ($\pi(\eta^A) - \pi(\eta^B) > \frac{2Q}{\mu\gamma}$), then the short-sales constraints binds for agents from group B. Only agents A are long and the price reflects the asset valuation of agents A ($\pi(\eta^A)$) minus a discount that arises from the effective supply of agents B who are re-selling their date-0 holdings to agents A.

Symmetrically, if agents from group B are much more optimistic than agents from group

A ($\pi(\eta^B) - \pi(\eta^A) > \frac{2Q}{\mu\gamma}$), then the short-sales constraints binds for agents from group A. Only agents B are long and the price reflects the valuation of agents B for the asset ($\pi(\eta^B)$) minus a discount that arises from the effective supply of agents A who are re-selling their date-0 holdings to agents B.

Finally, the last case arise when the beliefs of both groups are close (i.e $|\pi(\eta^A) - \pi(\eta^B)| < \frac{2Q}{\mu\gamma}$). In this case, both agents are long at date 1 and the date-1 equilibrium price is simply an average of both groups' belief ($\frac{\pi(\eta^A)+\pi(\eta^B)}{2}$).

Theorem 1. ***Date-1 equilibrium.***

At date 1, three cases arise.

1. *If* $\pi(\eta^A) - \pi(\eta^B) > \frac{2Q}{\mu\gamma}$*, only agents from group A are long (i.e. the short-sales constraint is binding). The date-1 price is then:*

$$P_1 = \mu\pi(\eta^A) - \frac{Q}{\gamma}.$$

2. *If* $\pi(\eta^B) - \pi(\eta^A) > \frac{2Q}{\mu\gamma}$*, only agents from group B are long (i.e. the short-sales constraint is binding). The date-1 price is then:*

$$P_1 = \mu\pi(\eta^B) - \frac{Q}{\gamma}.$$

3. *If* $|\pi(\eta^A) - \pi(\eta^B)| \leq \frac{2Q}{\mu\gamma}$*, both agents are long. The date-1 price is then:*

$$P_1 = \mu\frac{\pi(\eta^A) + \pi(\eta^A)}{2}.$$

Proof. Let (η^A, η^B) be the agents' beliefs at date 1. Agents from group i are solving the following problem:

$$\left| \begin{array}{l} \max\limits_{n_1} \left\{ n_1\pi(\eta^i) - \frac{1}{\mu}\left((n_1 - n_0)P_1 + \frac{(n_1 - n_0)^2}{2\gamma}\right)\right\} \\ n_1 \geq 0 \end{array} \right.$$

Consider first the case where both agents are long. Then, the date-0 holdings are given by the FOC of the unconstrained problem and yield

$$n_1^A = n_0 + \gamma\left(\mu\pi(\eta^A) - P_1\right) \quad \text{and} \quad n_1^B = n_0 + \gamma\left(\mu\pi(\eta^B) - P_1\right)$$

The date-1 market-clearing condition ($n_1^A + n_1^B = 2Q$) combined with the date-0 market-clearing condition ($n_0^A + n_0^B = 2Q$) gives:

$$P_1 = \frac{\mu\pi(\eta^A) + \mu\pi(\eta^B)}{2},$$

and

$$n_1^A - n_0^A = \mu\gamma\frac{\pi(\eta^A) - \pi(\eta^B)}{2} \quad \text{and} \quad n_1^B - n_0^B = \mu\gamma\frac{\pi(\eta^B) - \pi(\eta^A)}{2}.$$

This can be an equilibrium provided that these date-1 holdings are indeed positive:

$$\frac{2n_0^A}{\mu\gamma} > \pi(\eta^B) - \pi(\eta^A) \quad \text{and} \quad \frac{2n_0^B}{\mu\gamma} > \pi(\eta^A) - \pi(\eta^B).$$

If this last condition is not verified, two cases may happen. Either agents from group B are short-sales constrained ($n_1^B = 0$). In this case, the date-1 market clearing condition imposes that:

$$P_1 = \mu\pi(\eta^A) - \frac{Q}{\gamma}.$$

This can be an equilibrium if and only if group B agents' FOC leads to a strictly negative holding or

$$\pi(\eta^A) - \pi(\eta^B) > \frac{2n_0^B}{\mu\gamma}.$$

Or the agents from group A are short-sales constrained ($n_1^A = 0$). In this case, the date-1 market clearing condition imposes that

$$P_1 = \mu\pi(\eta^B) - \frac{Q}{\gamma}.$$

This can be an equilibrium if and only if group B agents' FOC leads to a strictly negative holding or

$$\pi(\eta^B) - \pi(\eta^A) > \frac{2n_0^A}{\mu\gamma}$$

□

3.3. Date-0 equilibrium

We now turn to the equilibrium structure at date 0. Let P_0 be the price of the asset at $t = 0$. Then at $t = 0$, agents of group $i \in \{A, B\}$ have the following optimization program:

$$\left| \begin{array}{l} \max_{n_0} \left\{ -\frac{1}{\mu} \left(n_0 P_0 + \frac{n_0^2}{2\gamma} \right) + \mathbb{E}_\eta [J(n_0, \eta^i, P_1)] \right\} \\ n_0 \geq 0 \end{array} \right. \tag{9}$$

where the constraint is the short-sale constraint and the expectation is taken over the belief shocks (η^A, η^B).

The next theorem describes the date-0 equilibrium. In this symmetric setting, it is particularly simple. Both groups of agents are long and hold initial supply Q. The date-0 demand is driven by the anticipation of the date-1 equilibrium. When agents consider a large belief shock, they anticipate they will end up short-sales constrained. In this case, holding n_0 shares at date 0 allows the agents to receive $n_0 P_1$ at date 1 minus the trading cost associated with the reselling of the date-0 holding or $\frac{n_0^2}{2\gamma}$. Or agents consider a small belief shock, and thus anticipate to be long at date 1, i.e. that they will not become too pessimistic relative to the other group. In this case, it is easily shown that their utility from holding n_0 shares at date 0 is proportional to the expected payoff from the asset conditional on the date 1 belief $\pi(\eta^i)$.

Theorem 2. ***Date-0 equilibrium.***

At date-0, each group owns Q shares. The date-0 price is given by:

$$P_0 = \int_{-\infty}^{\infty} \left[\left(\mu\pi(y) - \frac{2Q}{\gamma} \right) \Phi \left(\pi^{-1}[\pi(y) - \frac{2Q}{\mu\gamma}] \right) + \int_{\pi^{-1}[\pi(y) - \frac{2Q}{\mu\gamma}]}^{\infty} \mu\pi(x)\phi(x)dx \right] \phi(y)dy - \frac{Q}{\gamma} \tag{10}$$

Proof. At date 0, group A's program can be written as:

$$\max_{n_0}\left\{\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\mu\gamma}]}\frac{1}{\mu}\left(n_0\left(\mu\pi(y)-\frac{Q}{\gamma}\right)-\frac{n_0^2}{2\gamma}\right)\phi(x)dx\right.\right.$$

$$\left.\left.+\int_{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\mu\gamma}]}^{\infty}\frac{1}{\mu}\left(n_1^\star(x)\pi(x)+(n_0-n_1^\star(x))P_1(x,y)-\frac{(n_1^\star(x)-n_0)^2}{2\gamma}\right)\phi(x)dx\right]\phi(y)dy-\frac{1}{\mu}\left(n_0P_0+\frac{n_0^2}{2\gamma}\right)\right\}$$

Let $G+b+x$ be the date-1 belief of group A agents and $G+b+y$ be the date-1 belief of group B agents. The first integral corresponds to the case where group A agents are short-sale constrained. This happens when $\pi(x)<\pi(y)-\frac{2Q}{\mu\gamma}\Leftrightarrow x<\pi^{-1}\left(\pi(y)-\frac{2Q}{\mu\gamma}\right)$. In this case, group A agents re-sell their date-0 holdings for a price $P_1=\mu\pi(y)-\frac{Q}{\gamma}$ and pay the trading cost $\frac{n_0^2}{2\gamma}$. The second integral corresponds to the case where group A agents are not short-sell constrained and their date-1 holding is given by the interior solution to the FOC, $n_1^\star(x)$. The corresponding payoff is the expected payoff from the date-1 holding with date-1 belief, i.e. $n_1^\star(x)\pi(x)$ plus the potential gains (resp. cost) of selling (resp. buying) some shares $((n_0-n_1^\star(x))P_1(x,y))$ minus the trading costs $(\frac{(n_1^\star(x)-n_0)^2}{2\gamma})$ of adjusting the date-1 holding.

Note that the bounds defining the two integrals depend on the aggregate holding of group A, but group A agents do not have individually any impact on this aggregate holding n_0^A. Thus, they maximize only over n_0 in the previous expression and take n_0^A as given. Similarly, agents consider $P_1(x,y)$ as given (i.e. they do not take into account the dependence of P_1 on the aggregate holdings n_0^A and n_0^B).

To derive the FOC of group A agents' program, use the envelope theorem to derive the second integral w.r.t. n_0. For this integral, the envelope theorem applies as $n_1^\star(x)$ is determined according to the date-1 interior FOC. We thus have:

$$\frac{\partial\left(n_1^\star(x)\pi(x)+(n_0-n_1^\star)P_1(x,y)-\frac{(n_1^\star-n_0)^2}{2\gamma}\right)}{\partial n_0}=P_1(x,y)+\frac{n_1^\star(x)-n_0}{\gamma}=\mu\pi(x).$$

Thus, the overall FOC writes:

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\mu\gamma}]}\left(\pi(y)-\frac{Q}{\mu\gamma}-\frac{n_0}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y)-\frac{2n_0^A}{\mu\gamma}]}^{\infty}\pi(x)\phi(x)dx\right]\phi(y)dy-\frac{1}{\mu}\left(P_0+\frac{n_0}{\gamma}\right)=0$$

The model is symmetric. Hence, it has to be that $n_0^A=n_0^B=Q$. Substituting in the previous FOC gives the date-0 equilibrium price:

$$P_0=\int_{-\infty}^{\infty}\left[\left(\mu\pi(y)-\frac{2Q}{\gamma}\right)\Phi\left(\pi^{-1}[\pi(y)-\frac{2Q}{\mu\gamma}]\right)+\int_{\pi^{-1}[\pi(y)-\frac{2Q}{\mu\gamma}]}^{\infty}\mu\pi(x)\phi(x)dx\right]\phi(y)dy-\frac{Q}{\gamma}$$

□

3.4. Comparative Statics

Now that we have solved for the dynamic equilibrium of this model, we are interested in how mispricing, price volatility and share turnover depend on the following parameters: the structure of the credit claim (D), the bias of the agents' prior (b), the fundamental of the economy (G) and the cost of leverage (μ). We will relate the predictions derived from these comparative statics to the stylized facts gathered in Section 2.

To be more specific, we first define the bubble or mispricing, which we take to be P_0, the equilibrium price, minus $\bar{P}_0$, the price of the asset (1) in the absence of short-sales constraints and (2) with no aggregate bias ($b = 0$). This benchmark or unconstrained price can be written as:[9]

$$\bar{P}_0 = \mu \int_{-\infty}^{\infty} \pi(\eta - b)\phi(\eta)d\eta - \frac{Q}{\gamma}. \qquad (11)$$

Now define $\hat{P}_0$ as the date-0 price when there are no short-sales constraint but the aggregate bias is b. This price is given by

$$\hat{P}_0 = \mu \int_{-\infty}^{\infty} \pi(\eta)\phi(\eta)d\eta - \frac{Q}{\gamma}. \qquad (12)$$

The date-0 price can then be decomposed in the following way:

$$P_0 = \hat{P}_0 + \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y) - \frac{2Q}{\mu\gamma}]} \left(\mu\pi(y) - \mu\pi(x) - \frac{2Q}{\gamma} \right) \phi(x)dx \right) \phi(y)dy. \qquad (13)$$

[9]First, if there is no bias b, then the belief of an agent with belief shock η will be $G + \eta$. Thus, this agent will expect a payoff $\pi(\eta - b)$. Moreover, when there is no short-sales constraint, the formula for the price is similar to equation 10, except that the short-sales constraint region shrinks to 0.

Then we can decompose the bubble into the following two terms:

$$\text{bubble} = \underbrace{\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y) - \frac{2Q}{\mu\gamma}]} \left(\mu\pi(y) - \mu\pi(x) - \frac{2Q}{\gamma} \right) \phi(x) dx \right) \phi(y) dy}_{\text{resale option}} + \underbrace{\hat{P}_0 - \bar{P}_0}_{\text{optimism}} \tag{14}$$

In this simple model, the bubble emerges from two sources: (1) there is a resale option due to binding short-sales constraints in the future and (2) agents are optimistic about the asset payoff and thus drive its price up.

The second quantity we are interested in is price volatility between $t = 0$ and $t = 1$. Price volatility is defined simply by:

$$\sigma_P = \text{Var}_{\{\eta^A, \eta^B\}} \left[P_1(\eta^A, \eta^B) \right] \tag{15}$$

The third object is expected share turnover. It is simply defined as the expectation of the number of shares exchanged at date 1. Formally:

$$\mathbb{T} = \mathbb{E}_{\{\eta^A, \eta^B\}} \left[|n_1^A - n_0^A| \right] \tag{16}$$

The following proposition shows how these three quantities depend on D.

Proposition 1. *An increase in D (the riskiness of debt) leads to an increase in (1) mispricing (2) price volatility, and (3) share turnover.*

Proof. See Appendix. □

Proposition 1 offers a rationale for why debt bubbles are smaller and quieter than equity ones. The main intuition is that because the credit payoff is bounded by D, it is insensitive to beliefs on the distribution of payoffs above D. Thus, when D is low, there is very little scope for disagreement – the credit is almost risk-free and its expected payoff is close to its face value, and is in particular almost independent of the belief about the fundamental value. Thus short-sales constraint are not likely to bind (as short-sales constraints at date 1 arise from large differences in belief about the expected payoff). As a result, the resale option is low (i.e. the asset will most likely trade at its "fair" value at date 1) and thus mispricing is low. This, in turns, leads to low expected turnover as in this model turnover is maximized when the agents' short-sale constraint binds. Similarly, volatility will be low as prices will be less extreme (intuitively, the date-1 price will be representative of the average of the two groups beliefs rather than of the maximum of the two groups' beliefs). As D increases, agents' belief matters more for their valuation of the credit, both because of the recovery value conditional on default and because of the default threshold. In the extreme, when D grows to infinity, the credit becomes like an equity, beliefs become relevant for the entire payoff distribution of the asset and the scope for disagreement is maximum. This leads to more binding short-sales constraint at date 1, and hence more volatility and expected turnover.

A simple remark emerges from this analysis (which relates more fundamentally to the analysis in Harrison and Kreps (1978) and Scheinkman and Xiong (2003)). In this pure resale option model, mispricing and turnover/volatility go hand in hand – an increase in mispricing is associated with more volume and more turnover. There is, however, one mechanism that leads to a decoupling of prices and turnover volatility. When the aggregate bias increases (i.e. b increases), mispricing increases *and* turnover/volatility decrease. Fundamentally, increasing the aggregate bias b decreases the scope for disagreement and thus reduces the resale option, volume and volatility. However, because G is held fixed, the fundamental price of the asset has not changed. Thus, even though the speculative component of the price decreases as b increases, the price overall gets further away from its fundamental value and thus mispricing increases. In this sense, an increase in aggregate optimism leads to a quiet bubble:

Proposition 2. *Assume that $D < \infty$. An increase in aggregate optimism (i.e. b) leads to (1) higher mispricing (2) lower price volatility and (3) lower turnover.*

If the asset is an equity ($D = \infty$), an increase in aggregate optimism leads to higher mis-

pricing but leaves price volatility and turnover unaffected.

Proof. We first look at mispricing. Note that $\bar{P}_0$ is independent of b. Thus:

$$\frac{\partial \text{mispricing}}{\partial b} = \frac{\partial P_0}{\partial b} = \int_{-\infty}^{\infty} \left(\Phi\left(\underline{x}(y)\right) \Phi(D - G - b - y) + \int_{\underline{x}(y)}^{\infty} \Phi(D - G - b - x)\phi(x)dx \right) \phi(y)dy > 0$$

Formally, the derivative of turnover and price volatility w.r.t. b is equal to the derivative of turnover and price volatility w.r.t. G. Thus, thanks to the proof of proposition 3 below:

$$\frac{\partial \mathbb{T}}{\partial b} < 0 \text{ and } \frac{\partial \mathcal{V}}{\partial b} < 0$$

□

In our model, provided that the payoff function is strictly concave (or equivalently that $D < \infty$), an increase in average optimism makes the bubble bigger and quieter at the same time. This can be contrasted with the case of a straight equity claim, where both volatility and turnover would be left unaffected by variations in the average optimism – even in the case of binding short-sales constraint. This is because differences in opinion about an asset with a linear payoff are invariant to a translation in initial beliefs. Thus while an increase in optimism would obviously inflate the price of an equity, it would not change its price volatility nor its turnover.

Propositions 1-2 explain why the recent credit bubble is quieter than the Internet bubble. These two results rationalize Figures 1-4, which presented stylized facts that the recent credit bubble was quiet in contrast to classic speculative episodes such as the dot-com bubble. The reason is that an increase in optimism makes both credit and equity bubbles big. But it makes credit bubbles quiet while leaving the loudness of equity bubbles unchanged.

In Proposition 2, we held fixed G and considered how a change in b influence properties of the credit bubble. In the next proposition, we hold fix b and consider the comparative static with respect to G.

Proposition 3. *A decrease in G (the riskiness of debt) leads to an increase in (1) mispricing (2) price volatility, and (3) share turnover.*

Proof. See Appendix. □

When fundamentals deteriorate, the credit claim becomes riskier and hence disagreement becomes more important for its valuation. This increase in "disagreement sensitivity" leads to an increase in the resale option (the speculative component of the date-0 price increases as short-sale constraints are more likely to bind at date 1) and hence higher mispricing. This triggers an increase in both price volatility and turnover as argued above and the bubble stops being "quiet". This time-series behavior can be seen in Figure 1 above as price volatility increases in the year preceding the crisis and the collapse of these markets. Notice that the price volatility of even the highest rated AAA tranches begin to exhibit significant price movements in the year preceding the financial crisis.

Our model thus leads to very different predictions than standard model of adverse selections (e.g., see the discussion by Holmstrom (2008)). In these models, a deterioration in the fundamental of the economy destroys the information-insensitiveness of the credit, which reinforces adverse selection and potentially leads to a market breakdown. Thus a worsening of the economy leads to lower trading activity. In our model, however, when the economy worsens, agents realize that disagreement matters for the pricing of the credit in future periods – which drives up the resale option and subsequently increases volatility and trading volume.

Finally, we consider a comparative static with respect to leverage.

Proposition 4. *An increase μ (the cheapness of leverage) leads to an increase in (1) mispricing (2) price volatility, and (3) share turnover.*

Proof. See Appendix. □

Our result for leverage has an even simpler intuition. More leverage (an increase in μ) amplifies the disagreement between the two groups and hence makes short-sales constraints more likely to bind at date 1. As a consequence, the resale option is larger as μ increases.

With a larger resale option comes greater price volatility and turnover. So while leverage will make the credit bubble larger, it will also make it "louder".

The unambiguous comparative static with respect to leverage depends on our assumption of symmetric priors. When priors are dispersed, leverage allows optimistic agents to buy the assets without the participation of pessimistic agents, thereby leading to binding short-sales constraints. This can make the bubble quieter as a result using the same logic associated with optimism leading to quiet bubbles. We illustrate this point in the next section.

3.5. Extension: Interim Payoffs and Dispersed Priors

As we showed in the previous section, an increase in aggregate optimism leads to both larger *and* quieter bubbles while leaving unchanged the loudness of equity bubbles. In this section, we highlight another mechanism that makes credit bubbles both larger and quieter while still holding aggregate optimism fixed. In order to do so, we add two additional ingredients to our initial model. First, we introduce heterogenous priors. Group A agents start at date 0 with prior $G + b + \sigma$ and group B agents start with prior $G + b - \sigma$. Second, we introduce an interim payoff $\pi(G + \epsilon_1)$ that agents receive at date-1 from holding the asset at date 0. As a consequence, agents now hold the asset both for the utility they directly derive from it (consumption) and for the perspective of being able to resell it to more optimistic agents in the future (speculation). More precisely, the $t = 1$ interim cash-flow $\pi(G + \epsilon_1)$ occurs before the two groups of agents draw their date-1 beliefs. We also assume that the proceeds from this interim cash flow, as well as the payment of the date-0 and date-1 transaction costs all occur on the terminal date. This assumption is made purely for tractability reason so we do not have to keep track of the interim wealth of the investors.

Our next proposition shows that, provided that dispersion is large enough, an increase in the initial dispersion of belief, σ, leads to an increase in prices *and* simultaneously to a decrease in share turnover an price volatility. Thus, quiet bubbles emerge when there is sufficient heterogeneity among investors about the fundamental.

Proposition 5. *Provided leverage is cheap enough, there is $\bar{\sigma} > 0$ so that for $\sigma \geq \bar{\sigma}$ only group A agents are long at date 0. For $\sigma \geq \bar{\sigma}$, an increase in σ leads to (1) an increase in mispricing (2) a decrease in trading volume and (3) a decrease in price volatility.*

Proof. See Appendix. □

The intuition for this result is the following. The condition on leverage allow the optimists to have enough buying power to lead to binding short-sales constraints at date 0. In the benchmark setting, high leverage was associated with a louder credit bubble. But it turns out that leverage when there is dispersed priors can leads to large but quiet mispricings.

To see why, first consider the effect of dispersed priors on mispricing. Mispricing (i.e. the spread between the date-0 price and the no-short-sales constraint/ no bias price) increases with dispersion for two reasons. First, group A agents' valuation for the interim payoff increases. This is the familiar Miller (1977) effect in which the part of price regarding the interim payoff reflects the valuations of the optimists as short-sales constraints bind when disagreement increases. Second, as dispersion increases, so does the valuation of the marginal buyers (or the optimists) at date 1 — which leads to an increase in the resale option and hence of the date-0 price.

As dispersion increases, group A agents – who are more optimistic about the interim payoff than group B agents – own more and more shares until they hold all the supply at date 0 (which happens for $\sigma > \bar{\sigma}$).[10] As σ increases, the probability that group B become the optimistic group at date 1 also becomes smaller. As a consequence, an increase in dispersion leads to an increase in the probability of the states of nature where turnover is zero or equivalently where the group A agents hold all the shares at date 0 and 1. So overall expected turnover decreases.

[10]Note that this occurs despite the fact that both types share the same valuation for the date-1 resale option – this is entirely driven by the interim payoff: in a pure resale option setting (i.e. without the interim payoff), all agents would end up long at date 0 as, in the margin, there would be no disagreement about the value of the resale option.

Price volatility also decreases as σ increases. This is because in these states of nature where only group A agents are long, the expected payoff becomes more concave as a function of their belief shock, so that the price volatility conditional on these states decreases. When σ becomes sufficiently large, group A agents are long most of the time and the asset resembles a risk-free asset and price volatility goes down to zero.

A distinct prediction that emanates from our model and indeed all disagreement and short-sales constraints models is that disagreement leads to concentrated positions in the hands of the optimists. This prediction can be seen in accounts of the subprime mortgage CDO bubble during the years of 2003-2008 as being due to the concentrated positions of key institutional players such as AIG-FP which we discuss below (see Michael Lewis (2010)). Big banks' exposures to these structured credits are responsible for their demise. Michael Lewis (2010) provides a detailed account of the history of this market. More specifically, we make the case based on his account and other sources that an example of the extreme optimists with deep pockets in our model is AIG-FG – the financial markets group of AIG. AIG-FG insured 50 billion dollars worth of subprime AAA tranches between 2002 and 2005 at extremely low prices, years during which the issuance in the market was still relatively low, on the order of 100 billion dollars a year. Hence the bubble in CDOs was quiet because AIG-FP ended up being the optimist buyer of a sizable fraction of the supply of these securities. Michael Lewis (2010) provides evidence that AIG-FP, the trading division of AIG responsible for insuring subprime CDOs, and banks like it, may be the optimistic agents from group A in our model and as a result might have influenced prices and issuance in this market.

We begin with a summary of Michael Lewis (2010)'s extremely detailed timeline of the events surrounding the subprime mortgage CDO market's rise between 2003 to 2007 and its implosion in 2008.

The Timeline of subprime Mortgage CDO Bubble

- Before 2005, AIG-FP was de-facto the main optimistic buyer of subprime mortgages because it offered extremely cheap insurance for buyers of AAA tranches of these

CDOs.

- In early years before 2005, AIG-FP ended up with a $50 billion position in the CDO market, which was one-fourth of the total annual issuance then.

- AIG FP stopped insuring new mortgages after 2005, though they maintained insurance on old ones.

- After 2005, other banks including German Banks and UBS that were previously pessimistic started buying.

- After 2006, shorting becomes possible in the market through synthetic shorts on so-called mezzanine tranches (the worst quality subprime names) and the introduction of ABX indices.

- Market begins to collapse in 2007.

Michael Lewis (2010) makes the case, and convincingly from our perspective, that the market for subprime mortgage CDOs might not have taken off between 2003 and 2005 without the extremely cheap insurance offered by AIG-FP, which was then the largest insurance company in the world and one of the few companies with a AAA-rating and perceived invulnerable balance sheet. In light of these facts, it is natural to think about the subprime CDO market from 2003 to 2005/6 as one with high leverage and high dispersion – precisely the conditions required in proposition 5.

For instance, Michael Lewis (2010) writes, "Stage Two, beginning at the end of 2004, was to replace the student loans and the auto loans and the rest with bigger piles consisting of nothing but U.S. subprime mortgage loans...The consumer loan piles that Wall Street firms, led by Goldman Sachs, asked AIG-FP to insure went from being 2 percent subprime mortgages to being 95 percent subprime mortgages. In a matter of months, AIG-FP in effect bought $50 billion in triple B rated subprime mortgage by insuring them against default."

In Figure 6, we plot a figure shown from Stein (2010), who argues that the huge growth in issuance in the non-traditional CDO market was an important sign that a bubble might have taken hold here compared to traditional structured products. Indeed, this plot of issuance activity between 2000 and 2009 shows that activity really jumped in 2004 when AIG begins to insure significant amounts of the the subprime names as described in Michael Lewis (2010).

Why did AIG-FP take on such a large position? They did not think home prices could fall—i.e. our large dispersion in priors assumption. Michael Lewis (2010) writes, "Confronted with the new fact that his company was effectively long $50 billion in triple-B rated subprime mortgage bonds, masquerading as triple A-rated diversified pools of consumer loans—Cassano at first sought to rationalize it. He clearly thought that any money he received for selling default insurance on highly rated bonds was free money. For the bonds to default, he now said, U.S. home prices had to fall and Joe Cassano didn't believe house prices could ever fall everywhere in the country at once. After all, Moody's and S&P had rated this stuff triple-A!" Indeed, AIG FP continued to keep their insurance contracts even after 2005. In sum, anecdotal evidence very directly points to the central role of optimistic priors and leverage in influencing the subprime mortgage CDO bubble.

4. Narratives of the Financial Crisis

Our theory complements recent work on the financial crisis. An important narrative is that a wave of money from China needed a safe place to park and in light of the lack of sovereign debt, Wall Street created AAA securities as a new parking place for this money. This search for safe assets let to inflated prices through a variety of channels. Caballero and Krishnamurthy (2009) provides a theory of how search for safe assets let to a shrinking of the risk premium. Gennaioli et al. (2010) focuses on the neglected risks from this demand for safety. Another narrative is that of agency and risk-shifting as in Allen and Gale (2000), which can also deliver asset price bubbles and would seem vindicated by the bailouts of the big banks. Gorton et al. (2010) provide a theory of why these securitized debt products are a good substitute for Treasuries since they are information insensitive except when the economy is hit by a negative shock and they become more sensitive to private information, resulting in a loss of liquidity and trade (Holmstrom (2008)).

Our mechanism shares the most with the last approach in terms of our emphasis on the insensitivity of debt to underlying disagreement about asset values. Focusing on disagreement instead of asymmetric information leads to an opposite prediction compared to theirs, in which there is little trading in good times and more trading in bad times in CDOs. Our prediction is borne out in anecdotal accounts of the rise in speculative trade of CDOs after 2006-2007 but before the financial crisis in 2008. Our approach cannot capture the financial crisis and the ensuing freeze in trade.

Our theory also has a number of implications that are distinct from this earlier work. First, it offers a unified approach to explain both "classic" bubbles such as the dot-com and the recent so-called credit bubble.[11] Second, it offers a story for how a financial crisis of this size could emerge despite our recent experience with bubbles. Anecdotal accounts of the crisis invariably point to how the crisis and the credit bubble caught everyone by surprise. Indeed, this is an especially large conundrum when one considers that sophisticated finance companies such as Goldman Sachs were able to survive and indeed thrive through many prior speculative episodes, including the dot-com boom and bust. So why did companies like Goldman Sachs get caught this time?

Our unified theory offers a rationale for why smart investment banks and regulators failed to see the bubble on time. The low price volatility made these instruments appear safer in contrast to the high price volatility of dot-com stocks which made traders and regulator

[11]Our paper focuses on mispricing of the mortgages as opposed to the homes since evidence from Mian and Sufi (2009) indicate that the run-up in home prices was due to the cheap subprime home loans and that households took out refinancing from the equity of their homes for consumption as opposed to the purchase of additional homes, suggesting that there was limited speculation among households in the physical homes themselves. Khandani et al. (2010) make a similar point that the financial crisis was due to the refinancing of home loans as opposed to the speculative aspects described in the media.

more aware of their dangers. Interestingly, our theory also predicts that riskier tranches of CDOs trade more like equity and hence have more price volatility and turnover, which is also consistent with the evidence. These securities were also less responsible for the failure of the investment banks which were mostly caught with the higher rated AAA tranches of the subprime mortgage CDOs. Other theories for the crisis based on agency or financial innovation cannot explain why banks did not get caught during the dot-com bubble since they had the same agency problems and innovative products to price.

Third, our analysis also has implications for thinking about the Dodd-Frank financial reform package. This legislation establishes, among many things, an Office of Financial Research and a Financial Stability Oversight Committee that are meant to detect the next bubble. The premise of the Office of Financial Research is that by forcing finance firms to disclose their positions, regulators will be in a better position to detect the next speculative episode. Our model suggests that the quietness of the credit bubble would have made it difficult to detect regardless. Hence, regulators should distinguish between loud versus quiet bubbles as the signs are very different.

Fourth, the role of leverage in amplifying shocks has been understood and is clearly critical in melt-down or fire-sale part of the crisis (see Kiyotaki and Moore (1997) and Shleifer and Vishny (1997)). Our analysis shows that leverage can lower price volatility when there is an asset price bubble. Our analysis also echoes Hyman Minsky's warnings of bubbles fueled by credit. All in all, this paper can be thought of as offering an explanation for why bubbles fueled by leverage are more dangerous – because they are quiet.

Finally, our theory is also consistent with the literature analyzing the destabilizing role of institutions in financial markets. This literature has pointed mostly toward constraints as the culprit for demand shocks (see, e.g., Vayanos and Gromb (2010)). Our theory offers a novel approach – namely that that outlier beliefs in institutional settings are more likely to be amplified with access to leverage.

5. Conclusion

With the onset of the financial crisis, the term "bubble" is being used, from our perspective too liberally, to describe any type of potential mispricings in the market ranging from equities to housing and credit. The classic speculative episodes such as the recent dot-com bubble usually come with high price, high price volatility and high turnover. We argue that the credit bubble in subprime mortgage CDOs is fundamentally different from classic speculative episodes as it was quieter —price is high but price volatility and turnover are low. We offer a first attempt at a taxonomy of bubbles that distinguishes between loud equity bubbles and quiet credit bubbles.

This theory builds on the platform of disagreement and short-sales constraints that are key to getting loud bubbles. We show that credit bubbles are quieter than equity ones. The up-side concavity of debt payoffs means debt instruments (especially higher rated ones) are less disagreement-sensitive than lower rated credit or equity. As a consequence, optimism which increases the size of credit and equity bubbles makes credit bubbles quiet but leaves the loudness of equity bubbles unchanged. Future work elaborating on this taxonomy and providing other historical evidence would be very valuable.

A. Appendix

A.1. Proof of Proposition 1

As shown in the text, mispricing can be written as:

$$\text{mispricing} = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y) - \frac{2Q}{\mu\gamma}]} \left(\mu\pi(y) - \mu\pi(x) - \frac{2Q}{\gamma} \right) \phi(x)dx \right) \phi(y)dy + \int_{-\infty}^{\infty} \mu \left(\pi(y) - \pi(y-b)\right) \phi(y)dy$$

Note that $x < \pi^{-1}\left[\pi(y) - \frac{2Q}{\mu\gamma}\right] \Rightarrow x < y$. Moreover, $\frac{\partial(\pi(y)-\pi(x))}{\partial D} = \Phi(D-G-b-x) - \Phi(D-G-b-y)$. Thus, for all $x < \pi^{-1}\left[\pi(y) - \frac{2Q}{\mu\gamma}\right]$, $\frac{\partial(\pi(y)-\pi(x))}{\partial D} > 0$. Similarly, as $b > 0$, $\frac{\partial(\pi(y)-\pi(x))}{\partial D} = \Phi(D-G-y) - \Phi(D-G-b-y) > 0$. Thus, the derivative of mispricing w.r.t. D is strictly positive:

$$\frac{\partial(\text{mispricing})}{\partial D} = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\pi^{-1}[\pi(y) - \frac{2Q}{\mu\gamma}]} \mu \underbrace{\frac{\partial(\pi(y) - \pi(x))}{\partial D}}_{>0} \phi(x)dx \right) \phi(y)dy + \int_{-\infty}^{\infty} \mu \underbrace{\frac{\partial(\pi(y) - \pi(y-b))}{\partial D}}_{>0} \phi(y)dy$$

Thus, as D increases, both the resale option and the mispricing due to aggregate optimism increases, so that overall mispricing increases.

We now turn to expected turnover. To save on notations, call $\bar{x}(y)$ the unique real number such that: $\pi(\bar{x}(y)) = \pi(y) + \frac{2Q}{\mu\gamma}$. Similarly, call $\underline{x}(y)$ the unique real number such that: $\pi(\underline{x}(y)) = \pi(y) - \frac{2Q}{\mu\gamma}$. Obviously, $\underline{x}(y) < y < \bar{x}(y)$. Expected turnover is:

$$\mathbb{T} = \int_{-\infty}^{\infty} \left(\underbrace{\int_{-\infty}^{\underline{x}(y)} Q\phi(x)dx}_{\text{A short-sale constrained}} + \underbrace{\int_{\underline{x}(y)}^{\bar{x}(y)} \mu\gamma \frac{|\pi(y) - \pi(x)|}{2} \phi(x)dx}_{\text{no short-sale constraint}} + \underbrace{\int_{\bar{x}(y)}^{\infty} Q\phi(x)dx}_{\text{B short-sale constrained}} \right) \phi(y)dy$$

We can take the derivative of the previous expression w.r.t. D. Note that the derivative of the bounds in the various integrals cancel out, so that:

$$\frac{\partial \mathbb{T}}{\partial D} = \int_{-\infty}^{\infty} \left(\int_{\underline{x}(y)}^{\bar{x}(y)} \frac{\mu\gamma}{2} \frac{\partial \left|\pi(y) - \pi(x)\right|}{\partial D} \phi(x)dx \right) \phi(y)dy$$

If $y \geq x$, $\frac{\partial|\pi(y)-\pi(x)|}{\partial D} = |\Phi(D-G-b-x) - \Phi(D-G-b-y)| > 0$. Thus turnover is strictly increasing with D.

We now turn to variance. Formally, note: $P_1(\tilde{x}, \tilde{y}, D)$ the date-1 price when one agent has belief shock

$\tilde{x}$, the other $\tilde{y}$ and the face value of debt is D.

$$P_1(\tilde{x},\tilde{y},D) = \begin{cases} \mu\pi(\tilde{x}) - \frac{Q}{\mu\gamma} & \text{if } \pi(\tilde{x}) \geq \pi(\tilde{y}) + \frac{2Q}{\mu\gamma} \\ \mu\frac{\pi(\tilde{x})+\pi(\tilde{y})}{2} & \text{if } |\pi(\tilde{x}) - \pi(\tilde{y})| \leq \frac{2Q}{\mu\gamma} \\ \mu\pi(\tilde{y}) - \frac{Q}{\mu\gamma} & \text{if } \pi(\tilde{y}) \geq \pi(\tilde{x}) + \frac{2Q}{\mu\gamma} \end{cases}$$

We have:

$$\mathbb{E}_{x,y}[\left(\frac{P_1(\tilde{x},\tilde{y}}{\mu}\right)^2] = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\underline{x}(y)}\left(\pi(y) - \frac{Q}{\mu\gamma}\right)^2\phi(x)dx + \int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}\right)^2\phi(x)dx + \int_{\bar{x}(y)}^{\infty}\left(\pi(x) - \frac{Q}{\mu\gamma}\right)^2\phi(x)dx\right)\phi(y)dy \tag{17}$$

We can take the derivative of the previous expression w.r.t. D. Call $K = D - G - b$:

$$\begin{aligned} \frac{1}{2\mu^2}\frac{\partial\mathbb{E}[(P_1(\tilde{x},\tilde{y})^2]}{\partial D} \\ = \quad & \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\underline{x}(y)}(1-\Phi(K-y))\left(\pi(y) - \frac{Q}{\mu\gamma}\right)\phi(x)dx + \int_{\underline{x}(y)}^{\bar{x}(y)}\left(1 - \frac{\Phi(K-x)+\Phi(K-y)}{2}\right)\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy \\ & + \int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\left(\pi(x) - \frac{Q}{\mu\gamma}\right)\phi(x)dx\right]\phi(y)dy \end{aligned}$$

Note first that:

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\underline{x}(y)}(1-\Phi(K-y))\left(\pi(y) - \frac{Q}{\mu\gamma}\right)\phi(x)dx\right]\phi(y)dy = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y) - \frac{Q}{\mu\gamma}\right)\phi(x)dx\right]\phi(y)dy$$

Now the second term in equation 17 can be decomposed into:

$$\begin{aligned} & \int_{-\infty}^{\infty}\left[\int_{\underline{x}(y)}^{\bar{x}(y)}\left(1 - \frac{\Phi(K-x)+\Phi(K-y)}{2}\right)\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy \\ = \quad & \int_{-\infty}^{\infty}\left[\int_{\underline{x}(y)}^{\bar{x}(y)}(1-\Phi(K-y))\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy \\ & + \underbrace{\int_{-\infty}^{\infty}\left[\int_{\underline{x}(y)}^{\bar{x}(y)}\frac{\Phi(K-y)-\Phi(K-x)}{2}\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy}_{=0} \\ = \quad & \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy \end{aligned}$$

Thus, eventually:

$$\frac{1}{2\mu^2}\frac{\partial \mathbb{E}[(P_1(\tilde{x},\tilde{y})^2]}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\mu\gamma}\right)\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}\right)\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\left(\pi(x)-\frac{Q}{\mu\gamma}\right)\phi(x)dx\right]\phi(y)dy$$

Call $m=\mathbb{E}_{x,y}[\frac{P_1(\tilde{x},\tilde{y},D)}{\mu}]$. The derivative of m^2 w.r.t. to D is simply:

$$\frac{1}{2\mu^2}\frac{\partial\left(\mathbb{E}[P(x,y)]\right)^2}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}m\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}m\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\,m\phi(x)dx\right]\phi(y)dy$$

and $\mathcal{V}=Var(\frac{P_1(\tilde{x},\tilde{y},D)}{\mu})=\mathbb{E}_{x,y}[(P_1(\tilde{x},\tilde{y})^2]-m^2$. We have:

$$\frac{1}{2}\frac{\partial\mathcal{V}}{\partial D} = \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\mu\gamma}-m\right)\phi(x)dx+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}-m\right)\phi(x)dx\right]\phi(y)dy$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(1-\Phi(K-x))\left(\pi(x)-\frac{Q}{\mu\gamma}-m\right)\phi(x)dx\right]\phi(y)dy$$
$$= \int_{-\infty}^{\infty}(1-\Phi(K-y))\left[\int_{-\infty}^{\underline{x}(y)}\left(\pi(y)-\frac{Q}{\mu\gamma}-m\right)d\Phi(x)+\int_{\underline{x}(y)}^{\bar{x}(y)}\left(\frac{\pi(x)+\pi(y)}{2}-m\right)d\Phi(x)+\int_{\bar{x}(y)}^{\infty}\left(\pi(x)-\frac{Q}{\mu\gamma}-m\right)d\Phi(x)\right]d\Phi(y)$$
$$+\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}(\Phi(K-y)-\Phi(K-x))\left(\pi(x)-\frac{Q}{\mu\gamma}-m\right)\phi(x)dx\right]\phi(y)dy$$

First note that $(1-\Phi(K-y))$ is an increasing function of y, as well as $\mathbb{E}[P(x,y)-m|y]$. Thus, these two random variables have a positive covariance and because $\mathbb{E}_y[\mathbb{E}_x[P(x,y)-m|y]]=0$, this implies that $\mathbb{E}_y[(1-\Phi(K-y))\times\mathbb{E}_x[P(x,y)-m|y]]\geq 0$, i.e. the first term in the previous equation is positive.

Now, consider the function: $x\to\pi(x)-\frac{Q}{\mu\gamma}-m$. It is strictly increasing with x over $[\bar{x},\infty($. Call $x^0=\pi^{-1}(\frac{Q}{\mu\gamma}+m)$. Assume first that $\bar{x}(y)>x^0$ (i.e. $y>\pi^{-1}(m-\frac{Q}{\mu\gamma})$).Then for all $x\in[\bar{x},\infty($:

$$(\Phi(K-y)-\Phi(K-x))\left(\pi(x)-\frac{Q}{\mu\gamma}-m\right)>\left(\Phi(K-y)-\Phi(K-x^0)\right)\left(\pi(x)-\frac{Q}{\mu\gamma}-m\right)$$

Now if $\bar{x} < x^0$, then:

$$\begin{aligned}
\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx &= \int_{\bar{x}(y)}^{x^0} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx \\
&\quad + \int_{x^0}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx \\
&\geq \left(\Phi(K-y) - \Phi(K-x^0)\right) \int_{\bar{x}(y)}^{x^0} \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx \\
&\quad + \left(\Phi(K-y) - \Phi(K-x^0)\right) \int_{x^0}^{\infty} \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx \\
&\geq \left(\Phi(K-y) - \Phi(K-x^0)\right) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx
\end{aligned}$$

Thus, for all $y \in \mathbb{R}$,

$$\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx \geq \left(\Phi(K-y) - \Phi(K-x^0)\right) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx$$

This leads to:

$$\begin{aligned}
&\int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} (\Phi(K-y) - \Phi(K-x)) \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx\right] \phi(y)dy \\
\geq\ &\int_{-\infty}^{\infty} \left[\left(\Phi(K-y) - \Phi(K-x^0)\right) \int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx\right] \phi(y)dy
\end{aligned}$$

Now, $\Phi(K-y) - \Phi(K-x^0)$ is a decreasing function of y. $\int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx$ is also a decreasing function of y. Thus, the covariance of these two random variable is positive. But note that:

$$\int_{-\infty}^{\infty} \left[\int_{\bar{x}(y)}^{\infty} \left(\pi(x) - \frac{Q}{\mu\gamma} - m\right) \phi(x)dx\right] \phi(y)dy = \mathbb{P}\left[\pi(x) \geq \pi(y) + \frac{2Q}{\mu\gamma}\right] \times \left(\mathbb{E}\left[P(x,y) | \pi(x) \geq \pi(y) - \frac{2Q}{\mu\gamma}\right] - \mathbb{E}[P(x,y)]\right)$$

Finally, note that the conditional expectation of prices, conditional on binding short-sales constraints has to be greater than the expected price, m. Thus, this last term is positive and finally the variance of date-1 prices is strictly increasing with D:

$$\frac{\partial Var(P_1(\tilde{x}, \tilde{y}, D)}{\partial D} \geq 0$$

We now turn to the comparative static w.r.t. μ. First, note that:

$$\frac{\partial(\text{mispricing})}{\partial\mu}=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\mu\gamma}]}\underbrace{(\pi(y)-\pi(x))}_{>0}\phi(x)dx\right)\phi(y)dy+\int_{-\infty}^{\infty}\underbrace{(\pi(y)-\pi(y-b))}_{>0}\phi(y)dy>0$$

Similarly:

$$\frac{\partial\mathbb{T}}{\partial\mu}=\int_{-\infty}^{\infty}\left(\int_{\underline{x}(y)}^{\bar{x}(y)}\frac{\gamma}{2}\left|\pi(y)-\pi(x)\right|\phi(x)dx\right)\phi(y)dy>0$$

Eventually:

$$\begin{aligned}\frac{\partial Var[P_1(\tilde{x},\tilde{y})]}{\partial\mu} &= \frac{1}{\mu}\mathbb{E}\left[P_1(\tilde{x},\tilde{y})\left(P_1(\tilde{x},\tilde{y})-\mathbb{E}[P_1(\tilde{x},\tilde{y})]\right)\right]+2\mu\frac{Q}{\gamma}\int_{-\infty}^{\infty}\left[\int_{\bar{x}(y)}^{\infty}\left(\pi(x)-\frac{Q}{\mu\gamma}-m\right)\phi(x)dx\right]\phi(y)dy\\ &= \frac{Var[P_1(\tilde{x},\tilde{y})]}{\mu}+\frac{2Q}{\gamma}\left(\mathbb{E}\left[P(x,y)|\pi(x)\geq\pi(y)-\frac{2Q}{\mu\gamma}\right]-\mathbb{E}[P(x,y)]\right)\end{aligned}$$

Both terms in the previous expression are positive (and the first term is strictly positive). Thus:

$$\frac{\partial Var[P_1(\tilde{x},\tilde{y})]}{\partial\mu}>0$$

We now turn to the comparative static w.r.t. G. First, note that $\frac{\partial\pi(y)}{\partial G}=\Phi(D-G-b-y)>0$ and strictly decreasing with y. Now, the derivative of mispricing w.r.t. G is simply

$$\frac{\partial(\text{mispricing})}{\partial G}=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\pi^{-1}[\pi(y)-\frac{2Q}{\mu\gamma}]}\mu\underbrace{\frac{\partial(\pi(y)-\pi(x))}{\partial G}}_{<0}\phi(x)dx\right)\phi(y)dy+\int_{-\infty}^{\infty}\mu\underbrace{\frac{\partial(\pi(y)-\pi(y-b))}{\partial G}}_{<0}\phi(y)dy<0$$

Similarly:

$$\frac{\partial\mathbb{T}}{\partial\mu}=\int_{-\infty}^{\infty}\left(\int_{\underline{x}(y)}^{\bar{x}(y)}\frac{\gamma\mu}{2}\left|\frac{\partial\left(\pi(y)-\pi(x)\right)}{\partial G}\right|\phi(x)dx\right)\phi(y)dy<0$$

Finally, note that:

$$\frac{\partial\mathcal{V}}{\partial G}=Cov(\frac{\partial P_1(x,y)}{\partial G},P_1(x,y))=Cov(\frac{1-\partial P_1(x,y)}{\partial D},P_1(x,y))=-\frac{\partial\mathcal{V}}{\partial G}<0$$

QED.

A.2. Proof of Proposition 3

We now consider the case where group A has prior $G + b + \sigma$ and group B has prior $G + b - \sigma$. Thus, at date 1, beliefs are given by $(G + b + \sigma + \eta^A)$ for group A, with $\eta^A \sim \Phi()$ and $(F - \sigma + \eta^B)$ for group B, with $\eta^B \sim \Phi()$. Agents also receive at date 1 an interim payoff proportional to $\pi()$ from holding the asset at date-0. We first start by solving the date-1 equilibrium. At date 1, three cases arise:

1. Both groups are long. Thus demands are:

$$\begin{cases} n_1^A = n_0^A + \gamma\left(\mu\pi(\sigma + \eta^A) - P_1\right) \\ n_1^B = n_0^B + \gamma\left(\mu\pi(-\sigma + \eta^B) - P_1\right) \end{cases}$$

   The date-1 price in this case is: $P_1 = \frac{1}{2}\mu\left(\pi(\sigma + \epsilon^A) + \pi(-\sigma + \epsilon^B)\right)$. This is an equilibrium if and only if: $\frac{2n_0^A}{\mu\gamma} > \pi(-\sigma + \epsilon^B) - \pi(\sigma + \epsilon^A)$ and $\frac{2n_0^B}{\mu\gamma} > \pi(\sigma + \epsilon^A) - \pi(-\sigma + \epsilon^B)$

2. Only A group is long. The date-1 equilibrium price is then simply: $P_1 = \mu\pi(\sigma + \epsilon^A) - \frac{n_0^B}{\gamma}$

   This is an equilibrium if and only if $\pi(\sigma + \epsilon^A) - \pi(-\sigma + \epsilon^B) > \frac{2n_0^B}{\mu\gamma}$.

3. Only B group is long. The date-1 equilibrium price is then simply: $P_1 = \mu\pi(-\sigma + \epsilon^B) - \frac{n_0^A}{\gamma}$. This is an equilibrium if and only if $\pi(-\sigma + \epsilon^B) - \pi(\sigma + \epsilon^A) > \frac{2n_0^A}{\mu\gamma}$

At date 0, group A program can be written as [12] :

$$\max_{n_0}\left\{n_0\pi(\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\mu\gamma}]-\sigma}\frac{1}{\mu}\left(n_0\left(\mu\pi(y-\sigma) - \frac{n_0^A}{\gamma}\right) - \frac{n_0^2}{2\gamma}\right)\phi(x)dx + \int_{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\mu\gamma}]-\sigma}^{\infty} n_0\pi(\sigma + x)\phi(x)dx\right]\phi(y)dy - \frac{1}{\mu}\left(n_0P_0 + \frac{n_0^2}{2\gamma}\right)\right\}$$

The FOC of group A's agents program is given by (substituting n_0^A for n_0 in the FOC):

$$0 = \pi(\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\mu\gamma}]-\sigma}\frac{1}{\mu}\left(\mu\pi(y-\sigma) - \frac{2n_0^A}{\gamma}\right)\phi(x)dx + \int_{\pi^{-1}[\pi(y-\sigma)-\frac{2n_0^A}{\mu\gamma}]-\sigma}^{\infty}\pi(\sigma + x)\phi(x)dx\right]\phi(y)dy - \frac{1}{\mu}\left(P_0 + \frac{n_0^A}{\gamma}\right)$$

Similarly, at date 0, group B agents' program can be written as:

$$\max_{n_0}\left\{\pi(-\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\mu\gamma}]+\sigma}\frac{1}{\mu}\left(n_0\left(\mu\pi(y+\sigma) - \frac{n_0^B}{\gamma}\right) - \frac{n_0^2}{2\gamma}\right)\phi(x)dx + \int_{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\mu\gamma}]+\sigma}^{\infty} n_0\pi(-\sigma + x)\phi(x)dx\right]\phi(y)dy - \frac{1}{\mu}\left(n_0P_0 + \frac{n_0^2}{2\gamma}\right)\right\}$$

[12] In group A agents' program, we note n_0^A group A agents aggregate holding – each agent from group A takes n_0^A as given.

Group B agents' FOC:

$$0 = \pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\mu\gamma}]+\sigma}\frac{1}{\mu}\left(\mu\pi(y+\sigma)-\frac{2n_0^B}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y+\sigma)-\frac{2n_0^B}{\mu\gamma}]+\sigma}^{\infty}\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy-\frac{1}{\mu}\left(P_0+\frac{n_0^B}{\gamma}\right)$$

Consider now an equilibrium where only group A is long, i.e. $n_0^A = 2Q$ and n_0^B=0. In this case, the date-0 price is given by:

$$P_0 = \mu\pi(\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\mu\gamma}]-\sigma}\left(\mu\pi(y-\sigma)-\frac{4Q}{\gamma}\right)\phi(x)dx+\int_{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\mu\gamma}]-\sigma}^{\infty}\mu\pi(\sigma+x)\phi(x)dx\right]\phi(y)dy-\frac{2Q}{\gamma}$$

This is an equilibrium if and only if:

$$P_0 > \mu\pi(-\sigma)+\int_{-\infty}^{\infty}\left[\int_{-\infty}^{y+2\sigma}\mu\pi(y+\sigma)\phi(x)dx+\int_{y+2\sigma}^{\infty}\mu\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy$$

We now show that P_0 is increasing with σ (noting $K = D - G - b$):

$$\begin{aligned}
\frac{1}{\mu}\frac{\partial P_0}{\partial \sigma} &= \pi'(\sigma)+\int_{-\infty}^{\infty}\left[\int_{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\mu\gamma}]-\sigma}^{\infty}\Phi(K-\sigma-x)\phi(x)dx-\int_{-\infty}^{\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\mu\gamma}]-\sigma}\Phi(K-y+\sigma)\phi(x)dx\right]\phi(y)dy\\
&= \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(\pi^{-1}[\pi(y+\sigma)+\frac{4Q}{\mu\gamma}]+\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(\pi^{-1}[\pi(y-\sigma)-\frac{4Q}{\mu\gamma}]-\sigma\right)\Phi(K-y+\sigma)\phi(y)dy\\
&\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(y+2\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(y-2\sigma\right)\Phi(K-y+\sigma)\phi(y)dy\\
&\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\phi\left(y+2\sigma\right)\Phi(K-\sigma-y)\phi(y)dy-\int_{-\infty}^{\infty}\phi\left(y\right)\Phi(K-y-\sigma)\phi(y+2\sigma)dy\\
&\geq \pi'(\sigma)+\int_{-\infty}^{\infty}\Phi(K-\sigma-y)\left[\phi\left(y+2\sigma\right)\phi(y)-\phi\left(y\right)\phi(y+2\sigma)\right]dy
\end{aligned}$$

Call $\psi(\sigma) = \phi\left(y+2\sigma\right)\phi(y)-\phi\left(y\right)\phi(y+2\sigma)$. $\psi'(\sigma) = 2\phi(y+2\sigma)\left(\phi(y)+(y+2\sigma)\phi(y)\right)$ Thus, ψ is increasing if and only if: $2\sigma > -y-\frac{\phi(y)}{\phi(y)}$. Now consider the function $\kappa : y \in \mathbb{R} \rightarrow y\phi(y)+\phi(y)$. $\kappa'(y) = \phi(y) > 0$. Thus, κ is increasing strictly with y. But $\lim_{y\to-\infty}\kappa'(y) = 0$. Thus: $\forall\ y$, $\kappa(y) > 0$. Thus, for all $\sigma > 0$, $-y-\frac{\phi(y)}{\phi(y)} = -\frac{\kappa(y)}{\phi(y)} < 0 < 2\sigma$ so that ψ is strictly increasing with σ, for all $\sigma > 0$ and $y \in \mathbb{R}$. Now $\psi(0) = 0$. Thus, $\psi(y) > 0$ for all $\sigma > 0$. As a consequence:

$$\frac{1}{\mu}\frac{\partial P_0}{\partial \sigma} \quad \geq \quad \pi'(\sigma)+\int_{-\infty}^{\infty}\Phi(K-\sigma-y)\psi(y)dy > 0$$

Thus, when the equilibrium features only group A long at date 0, the price is strictly increasing with dispersion σ.

We now simply show that in this equilibrium, turnover is strictly increasing. Turnover is $2Q$ when group B only is long at date 1 (i.e. $\pi(-\sigma+\eta^B) > \pi(\sigma+\eta^A) + \frac{4Q}{\mu\gamma}$), it is given by: $\mu\gamma\frac{\pi(y-\sigma)-\pi(x+\sigma)}{2}$ when group B and group A are long at date 1 ((i.e. $\pi(-\sigma+\eta^B) < \pi(\sigma+\eta^A) + \frac{4Q}{\mu\gamma}$ and $\pi(-\sigma+\eta^B) > \pi(\sigma+\eta^A)$ and it is 0 if only group A is long at date 1 (i.e. $\pi(-\sigma+\eta^B) < \pi(\sigma+\eta^A)$). Thus, conditionning over η^B, expected turnover can be written as:

$$\mathbb{T} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\mu\gamma})-\sigma} 2Q\phi(x)dx + \int_{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\mu\gamma})-\sigma}^{y-2\sigma} \mu\gamma\frac{\pi(y-\sigma)-\pi(x+\sigma)}{2}\phi(x)dx\right]\phi(y)dy$$

Again, the derivative of the bounds in the integrals cancel out and the derivative is simply:

$$\frac{\partial\mathbb{T}}{\partial\sigma} = \int_{-\infty}^{\infty}\int_{\pi^{-1}(\pi(-\sigma+y)-\frac{4Q}{\mu\gamma})-\sigma}^{y-2\sigma} -\mu\gamma\frac{\Phi(K-y+\sigma)+\Phi(K-x-\sigma)}{2}\phi(x)dx\phi(y)dy < 0$$

Now consider the equation defining the equilibrium where only group A is long at date 0. This condition is:

$$\delta(\sigma) = P_0(\sigma) - \left(\mu\pi(-\sigma) + \int_{-\infty}^{\infty}\left[\int_{-\infty}^{y+2\sigma}\mu\pi(y+\sigma)\phi(x)dx + \int_{y+2\sigma}^{\infty}\mu\pi(-\sigma+x)\phi(x)dx\right]\phi(y)dy\right) > 0$$

Notice that the derivative of the second term in the parenthesis can be written as:

$$\begin{aligned} & \int_{-\infty}^{\infty}\left[\int_{-\infty}^{y+2\sigma}\mu\pi'(y+\sigma)\phi(x)dx - \int_{y+2\sigma}^{\infty}\mu\pi'(-\sigma+x)\phi(x)dx\right]\phi(y)dy \\ = & \int_{-\infty}^{\infty}\phi(y+2\sigma)\pi'(y+\sigma)\phi(y)dy - \int_{-\infty}^{\infty}\phi(y-2\sigma)\pi'(y-\sigma)\phi(y)dy \\ = & \int_{-\infty}^{\infty}\Phi(K-y-\sigma)\left(\phi(y+2\sigma)\phi(y)-\phi(y)\phi(y+2\sigma)\right)dy \end{aligned}$$

We thus have:

$$\frac{1}{\mu}\frac{\partial\delta}{\partial\sigma} \geq \left(\pi'(\sigma)+\pi'(-\sigma)\right) > 0$$

Thus, there is $\bar{\sigma} > 0$ such that for $\sigma \geq \bar{\sigma}$, the equilibrium has only group A long at date 0, the price increases with dispersion and turnover decreases with dispersion. QED.

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Figure 1: ABX Prices

The figure plots the ABX 7-1 Prices for various credit tranches including AAA, AA, A, BBB, and BBB-.

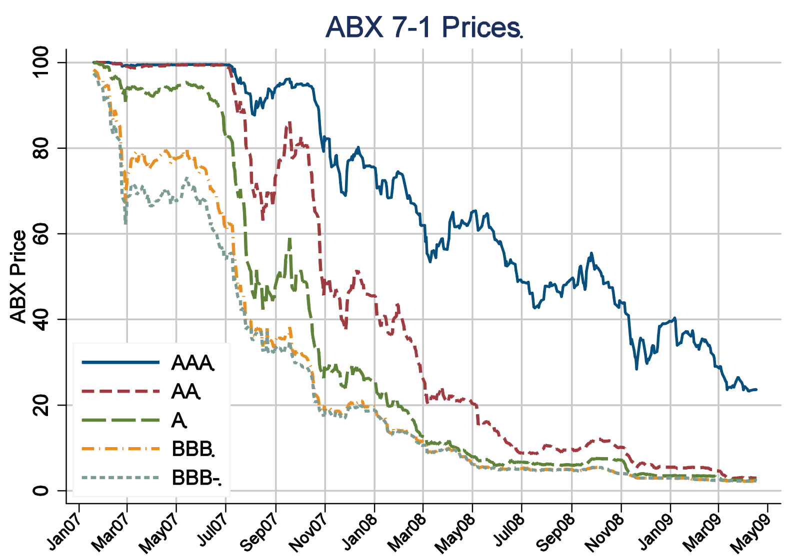

Figure 2: CDS Prices of Basket of Finance Companies

The figure plots the average CDS prices for a basket of large finance companies between December 2002 and December 2008.

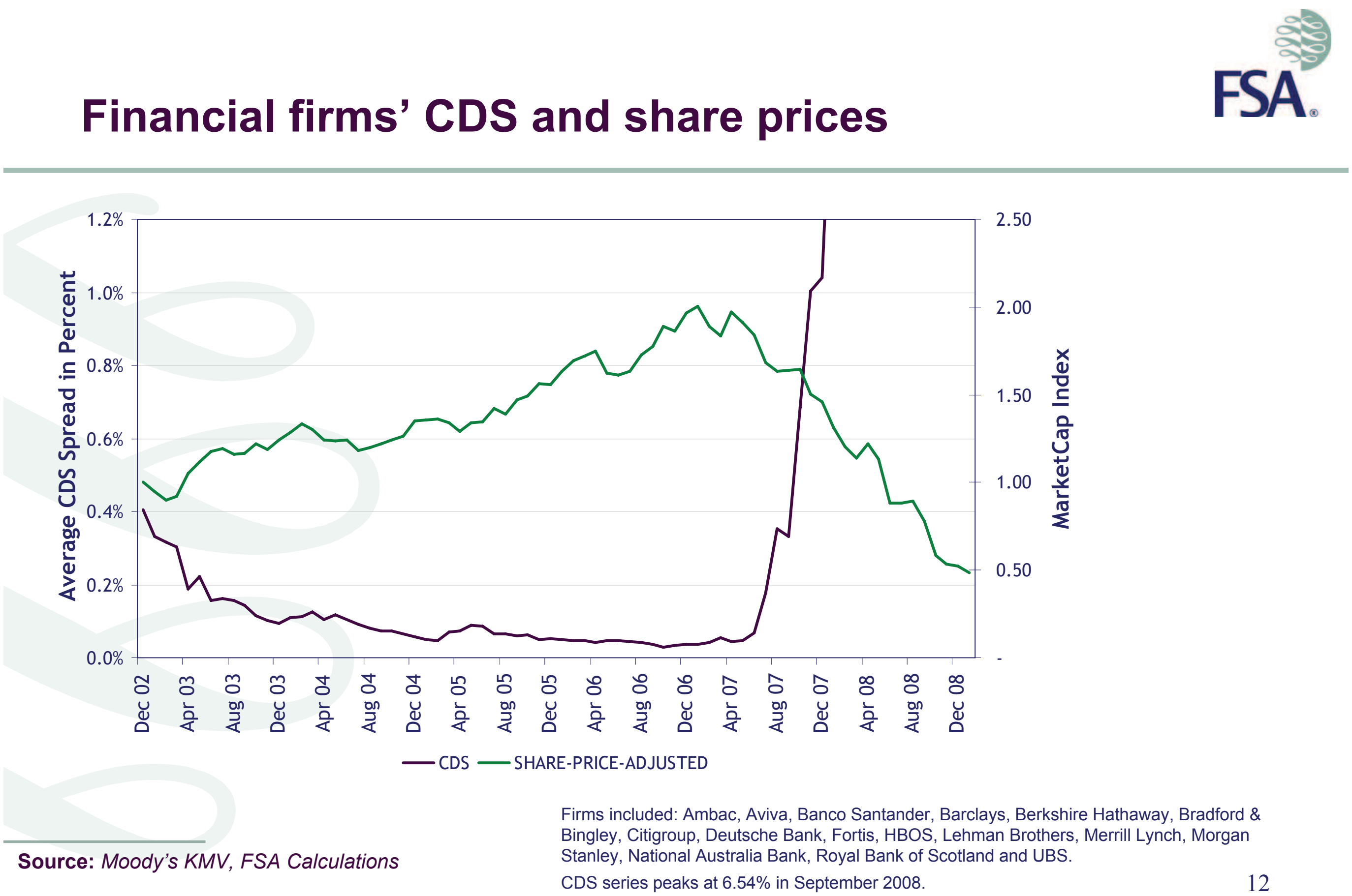

Figure 3: Monthly Share Turnover of Financial Stocks

The figure plots the average monthly share turnover of financial stocks.

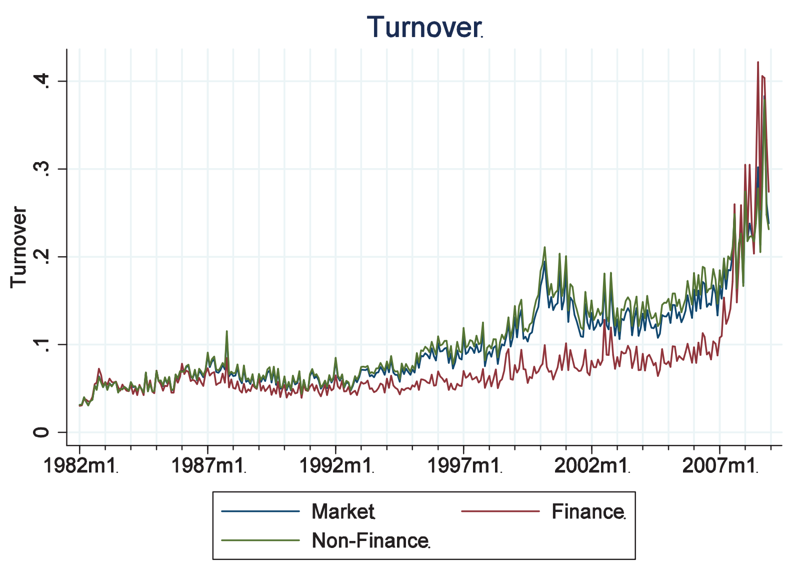

Figure 4: Monthly Share Turnover of Internet Stocks

The figure plots the average monthly share turnover of internet stocks compared to the rest of the market.

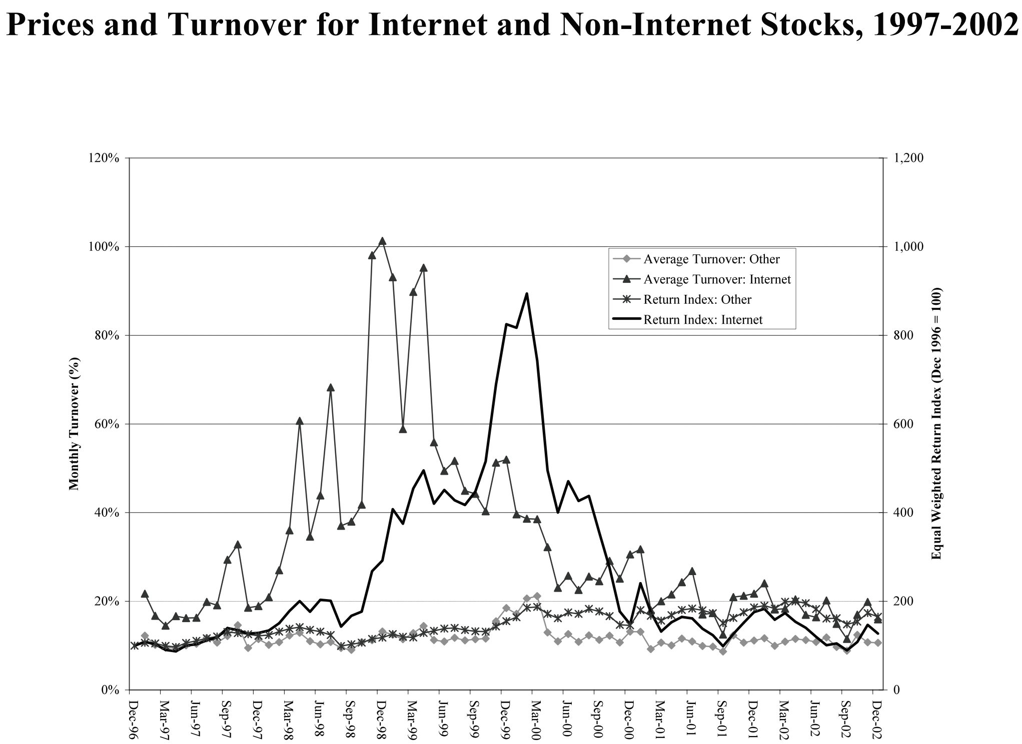

Figure 5: Synthetic Mezzanine ABS CDO Issuance

The figure plots the issuance of synthetic mezzanine ABS CDOs by year.

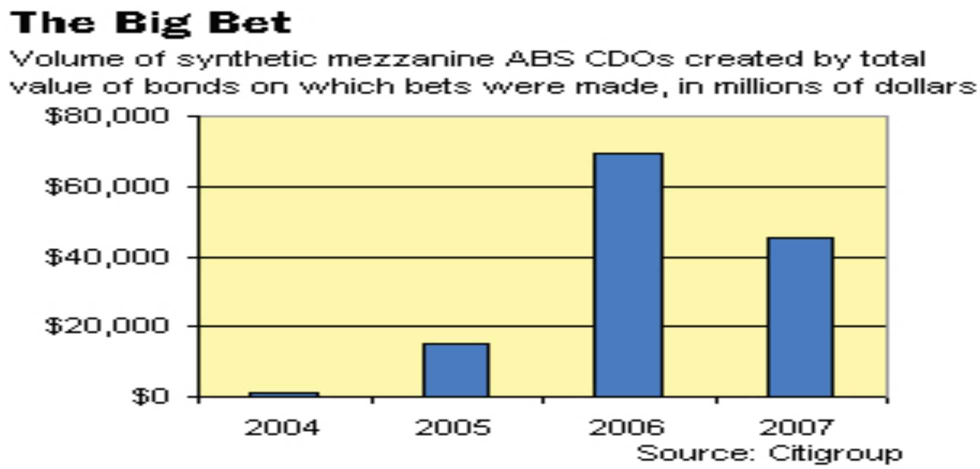

Figure 6: Traditional and Non-Traditional Issuance of Asset-Backed Securities (Quarterly)

The figure plots the issuance of traditional and non-traditional asset-backed securities by quarter.