A simple model of real estate prices and investment

Thomas Chaney* David Sraer† David Thesmar‡

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1 A Model of Investment Under Collateral Constraints

We develop a simple dynamic model of investment in the presence of financing frictions. We derive three main predictions. First, investment is positively affected by the value of a firm's collateral if and only if the firm is financially constrained. In particular, the mere expectation of future binding constraints makes current investment sensitive to real estate prices. Second, we show that, even after controlling for the average Q (i.e., the market value of a firm), the sensitivity of investment to collateral value remains a valid test for the existence of credit constraints. Finally, we show that firms facing stronger financial frictions have greater incentives to lease their real estate.

1.1 Model Setup

The model has three dates: 0, 1, and 2. The discount factor is $r \geq 0$. At time 0, the risk-neutral firm is an ongoing concern with cash flow from current operations c_0 and capital k_0. At dates 0 and 1, the firm has the option to invest in a long-term project that yields gross profit $\pi(k_t, \theta)$ in period $t \in \{1, 2\}$ if the firm has capital k_t and productivity $\theta \in [\underline{\theta}, \bar{\theta}]$. The production function π has the standard properties, that is, it is twice differentiable, increasing with k and θ, weakly concave with k and θ, and such that $\pi_{k\theta} \geq 0$. After date 2, the project stops yielding profit so that it is liquidated (see later for the definition of the liquidation proceeds). That the project has a finite horizon is by no mean necessary to our analysis, but it helps simplify the exposition.

The firm has the opportunity to invest and increase its stock of capital at both dates 0 and 1. Installing capital, however, is costly: To increase the existing stock of capital k to $k + i$, the firm needs to invest $i + G(i, k)$, where $G(,)$ is twice differentiable, increasing in i, decreasing in k, convex, and such that $G_{ik} \leq 0$. The convexity of the adjustment costs is an essential ingredient of our analysis. It is standard in the investment literature since, at least, Lucas (1967). That the adjustment costs depend on the current stock of capital is not necessary but is also standard

*University of Chicago and NBER, tchaney@uchicago.edu
†Princeton University, dsraer@princeton.edu
‡HEC School of Management and CEPR, thesmar@hec.fr

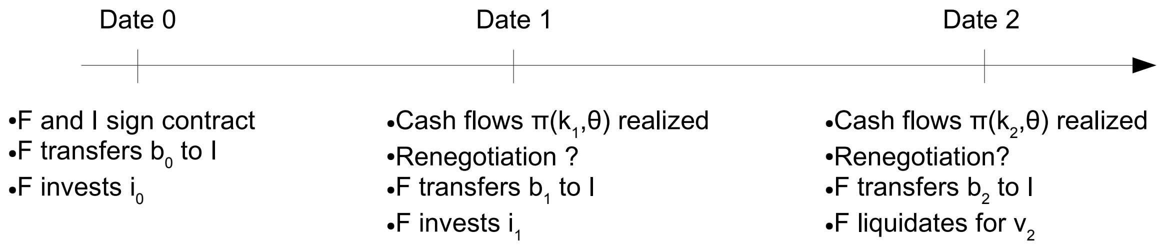

Figure 1: Sequence of Events

in the investment literature and helps us compare our model to neoclassical benchmarks such as Hayashi (1982) and Abel and Eberly (1994). Finally, to operate, the project also needs an additional input (e.g., real estate) in quantity R. We assume in this section that the firm is initially endowed with this asset; the discussion on the decision to acquire or rent it is deferred to Section 1.4.

Investment may require additional external funding if the initial cash flow c_0 and the interim cash flow $\pi(k_1,\theta)$ are insufficient to cover the investment costs. To obtain these funds, the firm contracts with a deep-pocket, risk-neutral outside investor. A financing contract specifies transfers $(b_t)_{t\in\{0,1,2\}} \in \mathbb{R}^3$ from the firm to the investor. We introduced financing frictions along the lines of Hart and Moore (1994). The contract is imperfectly enforced: At each date, the firm can renegotiate a fraction $\frac{1}{\psi} \in]0,1[$ of its outstanding liabilities, where $\frac{1}{\psi}$ is a reduced form parameter that captures the lack of commitment by the firm and, hence, the degree of financial frictions it faces. The firm is assumed to have the entire bargaining power in the renegotiation process. Hence, outside financing is possible only if the firm can transfer control over its assets in case of renegotiation. Importantly, we assume that such an ex post transfer of control rights is possible but that the firm's owner is an essential input in the production function. Thus, the investor's outside option during renegotiation corresponds to the resale value of the assets. To focus the analysis on the non-intensive asset (i.e., real estate), the market value of used capital is set to 0. The unit market value of real estate is $(p_t)_{t\in\{0,1,2\}}$. It is deterministic and persistent: $\frac{\partial p_2}{\partial p_0} = \frac{\partial p_1}{\partial p_0} = 1$.[1] We note $(\nu_t)_{t\in\{0,1,2\}}$, the market value of the firm's collateral, where $\nu_t = p_t \times R$. To prevent renegotiation, the contract must prevent the present value of transfers to the outside investor to be larger than the current value of collateral. We assume that the firm offers the contract to the investor, which has an outside option $B_0 \geq 0$. The sequence of events is summarized in Figure 1. Note that the liquidation value of the firm is ν_2, since capital k has no outside value. We also assume that ex ante and interim liquidations are never efficient. This is the case if, for instance, $(1+r)\pi(k_0,\theta) \geq \nu_t$ for $t = 0, 1$.

The nature of the financial friction, here imperfect enforcement of contract, is not essential to our argument. What matters is that there is an upper bound on the amount the firm can borrow and that this upper bound increases with collateral value. This is a natural feature of most models of financing constraints, such as those of Townsend (1979) and Holmstrom and Tirole (1998).

[1] Introducing uncertainty does not modify our analysis as long as real estate prices are persistent.

1.2 Investment and Collateral

We focus on renegotiation-proof contracts. As we show in Appendix .1, the optimal contract satisfies the following program:

$$\left|\begin{array}{ll} \max\limits_{(i_0,i_1,b_0,b_1)} c_0 - G(i_0,k_0) + \dfrac{\pi(k_1,\theta) - G\left(i_1,k_1\right)}{1+r} + \dfrac{\pi(k_2,\theta)+\nu_2}{(1+r)^2} & \\ (1+r)\left(B_0 - b_0\right) \leq \psi\nu_1 & (\lambda_1) \\ (1+r)^2\left(B_0 - b_0 - \dfrac{b_1}{1+r}\right) \leq \psi\nu_2 & (\lambda_2) \\ G(i_0,k_0) \leq c_0 - b_0 & (\lambda_3) \\ G(i_1,k_1) \leq \pi(k_1,\theta) - b_1 & (\lambda_4) \end{array}\right. \tag{1}$$

The optimal contract thus maximizes the project's net present value under four constraints. Constraints 1 and 2 impose that at dates 1 and 2 a fraction $\mu = \frac{1}{\psi}$ of the discounted value of total payments to the investor have to be lower than the value of collateral. If these conditions are not verified, the firm would successfully renegotiate down future transfers on the fraction of the contract that can be renegotiated. The additional constraints simply state that dividends have to be positive, or, in other words, that investment costs must be covered by cash flows and transfers from the investor.

First Best

The neoclassical benchmark is reached when neither of these constraints are binding. First-best investment is given by the following first-order conditions:

$$\begin{cases} G_1\left(i_1^\star,k_1^\star\right) = \dfrac{\pi_k(k_2^\star,\theta)}{1+r} \\ G_1\left(i_0^\star,k_0\right) = \dfrac{\pi_k(k_1^\star,\theta) - G_2\left(i_1^\star,k_1^\star\right)}{1+r} + \dfrac{\pi_k(k_2,\theta)}{(1+r)^2} \end{cases} \tag{2}$$

These conditions simply ensure that at dates 1 and 2, the marginal cost of investment equates its marginal benefit. First best will be attained, for instance, if (1) the market for outside finance is sufficiently competitive (i.e., B_0 is low enough) (2) the firm has sufficient initial cash c_0, and (3) the firm has a sufficient commitment ability ψ. While first-best investment is unique, the first-best contract is not.

Second Best

A firm is financially constrained if its value is strictly lower than the first-best value. There are two types of constrained firms.

First, some firms are constrained as of the first period, that is, $\lambda_1 > 0$. Their investment in the first period is therefore simply given by the current financing constraint: $G(i_0,k_0) = c_0 + \frac{\psi\nu_1}{1+r} - B_0$. When second-period debt capacity is large enough (because, for instance, p_2 is large enough), their date 1 investment simply equates its marginal cost to its marginal benefit, that is, $G_1(i_1, k_1) = \frac{\pi_k(k_2,\theta)}{1+r}$. When such an investment cannot be financed, the date 1 investment is simply given by the date 1 budget constraint, that is, $G(i_1, k_1) = \pi(k_1, \theta) + \psi\frac{\nu_2-\nu_1}{1+r}$. Independently of date 1 investment, these firms invest more at date 0 when they have initially more cash c_0 or less initial liability B_0. More interestingly, an increase in collateral value at date 0 leads to an increase in their investment.

Second, some firms are constrained only in the second period, that is, their investment is determined through an *intertemporal* budget constraint and a condition equating marginal cost of investment across the two periods:

$$\begin{cases} G(i_0, k_0) + \dfrac{G(i_1, k_1)}{1+r} = c_0 + \dfrac{\pi(k_1, \theta)}{1+r} + \psi\nu_2 - B_0 \\ G_1(i_0, k_0) = \dfrac{\pi_k(k_1, \theta) - G_2(i_1, k_1) + G_1(i_1, k_1)}{1+r} \end{cases} \tag{3}$$

This is, for instance, the case of firms that expect low date 2 but high date 1 collateral value. From an observational point of view, these firms look, at date 0, "as if" they were unconstrained: They do not use their entire debt capacity although their investment is lower than the first-best investment $i_0^\star$. Yet their date 0 investment depends positively on collateral value. This is the object of our first proposition.

Proposition 1.1 *The sensitivity of investment to collateral value is strictly positive for constrained firms. Unconstrained firms' investment is unrelated to the value of their collateral.*

Proof See Appendix .2. ∎

It is not surprising that firms constrained at date 0 react to variations in collateral value, as their investment is directly determined by the binding constraint. Less evidently, firms that only face the intertemporal budget constraint will also increase their investment following an increase in collateral value. This is because (1) real estate prices are persistent and (2) there are convex adjustment costs. An increase in collateral value at date 0 also increases collateral value at date 1 and hence relaxes the intertemporal budget constraint. Because there are convex adjustment costs, the firms prefer smoothing out this additional debt capacity across the two periods and hence increase their investment as of date 0. In other words, firms that only expect binding credit constraints *in the future* react to variations in their collateral value. This is important for the interpretation of our empirical results, as this suggests that a broad set of firms might react to variations in real estate prices as soon as they hold real estate assets on their balance sheet.

In our model, the sensitivity of investment to collateral value is not monotonic with the extent of credit constraint. A firm is said to be more credit constrained the further its value is from the first-best value. A simple way to obtain monotonic variations in credit constraint is to increase the commitment ability parameter ψ. Note $V(c_0, k_0, B_0, \nu_0, \theta, \psi)$, the date 0 value of the firm. We know that $\frac{\partial V}{\partial \psi} = \lambda_1\nu_1 + \lambda_2\nu_2$ so that V_ψ is strictly increasing as soon as the firm is constrained. Therefore, as ψ decreases, the firm becomes more credit constrained. Yet the evolution of the sensitivity of investment to collateral value is not necessarily higher as ψ increases. On the one hand, a lower ψ leads to a lower investment i_0 so that the marginal cost of investment is also lower: this implies that the firm will react with a larger increase of investment for a given increase in collateral value. On the other hand, as ψ decreases, the firm can extract a lower debt capacity from its collateral, so that the firm will become less sensitive to variations in collateral value. Which of these effects dominate depends on the convexity of the cost function and on the level of date 0 investment. This result is reminiscent of that in Kaplan and Zingales (1997), who proved that even in a simple static model of investment with costly financing, investment to *cash flow* sensitivities are not monotonic with financing costs.[2]

1.3 Investment Equations

A structural estimation of the parameters of interest of the model (ψ, the concavity of the production function, the convexity of the cost function, etc.) is beyond the scope of this paper. In the reduced-form approach we adopt in the empirical part of this paper, we are more modestly interested in finding an unbiased estimate of the sensitivity of investment to collateral value for the representative firm in our sample. This is already interesting, since(1) a positive estimate constitutes a rejection of the null hypothesis that all firms are unconstrained and (2) it provides a simple way to quantify the average effect of shocks on asset values on aggregate investment, as in Bernanke and Gertler (1989). Of course, this estimate itself depends on the structural parameters of the model but, as we saw in the previous section, the relation between these parameters and the sensitivity of investment to collateral value is not straightforward.

A simple way to think of our reduced-form approach is to consider a linear approximation of the policy function i_0, solution to program 1, around a firm with the median characteristics:

$$i_0 = h(c_0, k_0, B_0, \nu_0, \theta) \approx \gamma + \frac{\partial h}{\partial c_0}(\bar{x})c_0 + \frac{\partial h}{\partial k_0}(\bar{x})k_0 + \frac{\partial h}{\partial B_0}(\bar{x})B_0 + \frac{\partial h}{\partial \nu_0}(\bar{x})\nu_0 + \frac{\partial h}{\partial \theta}(\bar{x})\theta \tag{4}$$

where $\bar{x} = \left(\bar{c_0}, \bar{k_0}, \bar{B}_0, \bar{\nu_0}, \bar{\theta}\right)$ represents the state variables at their median level and γ is a constant.[3]

A potential pitfall in the previous regression is that θ, the firm's future productivity, is not observable. Any correlation between this productivity and the initial value of collateral, ν_0, will thus lead to a biased estimate. Such a correlation can arise either from a correlation (1) between real estate prices and productivity shocks or (2) between productivity shocks and the decision to own real estate. In our empirical work, we will deal with (1) by instrumenting real estate prices. Next, in Section 1.4, we first argue that in the absence of the correlation in (1), firms facing stronger financial frictions have more incentive to lease their properties. Furthermore, we

[2]Note, however, that in some simple cases, there is such a monotonic relation between credit constraints and the sensitivity of investment to collateral value. A firm is also more credit constrained when its investor has a higher outside option B_0, as $V_{B_0} = -\lambda_3$ is strictly negative as soon as the firm is constrained. Consider a firm that is constrained at date 0. An increase in B_0 decreases investment at date 0 and hence the marginal cost of investment at date 0. An increase in collateral value will thus unambiguously lead to a higher increase in investment for higher values of B_0. Formally, $\frac{\partial^2 i_0}{\partial \nu_0 \partial B_0} = -\frac{G_1 1(i_0,k_0)}{G_1(i_0,k_0)}\frac{\partial i_0}{\partial \nu_0}\frac{\partial i_0}{\partial B_0} > 0$. Unfortunately, for firms that are only constrained at date 1, this result no longer holds. While the sensitivity of overall investment $i_0 + i_1$ to collateral value remains higher for higher B_0, the sensitivity of the initial investment, i_0, depends on how higher levels of the investor's outside option B_0 affects the distribution of investment across the two periods and cannot be signed unambiguously.

[3]γ is given by $i_0(\bar{x}) - \sum_{y \in c_0,k_0,B_0,\nu_0,\theta} \frac{\partial h}{\partial y}\bar{y}$.

show that a larger correlation between real estate prices and investment opportunities does not necessarily provide firms with greater incentive to buy their real estate assets.

Another, traditional solution in the investment literature is to complete the set of observable state variables by controlling for the initial value of the firm V. This amounts to swapping the state variables x for a new set of state variables $y = (c_0, k_0, B_0, \nu_0, V^0)$ and leads us to the estimation of the following linear approximation:

$$i_0 = j(c_0, k_0, B_0, \nu_0, V^0) \approx \rho + \frac{\partial j}{\partial c_0}(\bar{y})c_0 + \frac{\partial j}{\partial k_0}(\bar{y})k_0 + \frac{\partial j}{\partial B_0}(\bar{y})B_0 + \frac{\partial j}{\partial \nu_0}(\bar{y})\nu_0 + \frac{\partial j}{\partial V^0}(\bar{y})V^0, \quad (5)$$

where $\bar{y}$ represents the median vector of state variables and ρ is the new constant.

In our simple model where only one state variable, θ, is unobservable, the estimation of Equation 5 is clearly unbiased. Controlling for V^0, however, is not innocuous, as V is not only a function of θ but also a function of the remaining state variables. Our next proposition discusses the interpretation of this alternative regression.

Proposition 1.2 *Controlling for the value of the firm provides a lower bound on the true sensitivity of investment to collateral value:* $\frac{\partial h}{\partial \nu_0} \geq \frac{\partial j}{\partial \nu_0}$.

The conditional sensitivity of investment to collateral value is (1) strictly negative for unconstrained firms, (2) strictly positive for firms constrained at date 0, and (3) of ambiguous sign for firms constrained at date 1.

Proof See Appendix .3. ∎

The intuition behind the first part of Proposition 1.2 is the following. Consider two firms, firm 1 and firm 2, with the same market value. Assume firm 1 has a higher collateral value than firm 2. This implies that firm 1 has a lower productivity than firm 2. Because date 0 investment is increasing with productivity, the apparent difference in investment between these two firms will be lower than if they had the same productivity.

An unconstrained firm's investment does not react to collateral value. When collateral value rises, however, even the unconstrained firm's value increases, as its final liquidation value is higher. If firm value V^0 is to stay constant, productivity and hence investment must decrease. Therefore, for Neoclassical firms, there is a negative correlation between investment and collateral value once V^0 is controlled for.

Consider now the case of a date 0 constrained firm. Because investment at date 0 is given by the binding budget constraint, it is independent of productivity θ.[4] However, its value is increasing with productivity θ. Thus, to leave unchanged the value V^0, productivity will

[4] One might worry that this particular result is driven by the assumption that date 0 cash flow c_0 is independent of productivity θ. This is not the case. If $c_0 = \pi(k_0, \theta)$ instead, then it is still the case that the conditional sensitivity for the date 0 constrained firm is strictly positive. This is because in order to have a constant value V^0, it takes less than a $\$\frac{1}{k_0}$ decrease in productivity to compensate a \$1 increase in collateral value, while a $\$\frac{1}{k_0}$ decrease in productivity has the same impact on investment as a \$1 increase in collateral value.

decrease but that will not affect date 0 investment. Therefore, for the date 0 constrained firm, the conditional sensitivity is equal to the unconditional sensitivity, which is strictly positive.

Finally, consider the case of a date 1 constrained firm with a $1 increase in collateral value. For its value to remain constant, its productivity needs to strictly decrease. The extent of this decrease depends on the marginal benefit of productivity, π_θ. Furthermore, we know that for this firm date 0 investment is strictly increasing with productivity, as an increase in productivity raises the date 1 marginal product of capital. Hence the downward adjustment in productivity necessary to keep the value constant after an increase in collateral value leads to a strict decrease in date 0 investment. The extent of this decrease in date 0 investment depends on how productivity impacts date 0 investment, which depends on the convexity of the investment cost function and on how productivity affects the marginal benefit of capital. Whether the overall adjustment leads to a strict decrease in date 0 investment is ambiguous and depends on the model's parameters.

Proposition 1.2 is important for the interpretation of our empirical results. When we control for the market to book ratio, that is, for initial firm value V^0, the estimation is a priori unbiased. The estimation of the sensitivity, however, corresponds to a lower bound of the true, unconditional elasticity. Moreover, a positive, significant, conditional sensitivity can be interpreted as a rejection of the hypothesis that all firms are unconstrained in our sample. This is because the sensitivity of unconstrained firms is always strictly negative whereas it is strictly positive for date 0 constrained firms.

1.4 Ownership Decision

Controlling for the firm's value in the investment Equation 4 might not be always the ultimate answer to endogeneity issues. While our simple model entails a unique source of unobserved heterogeneity, productivity, a more complex model with multiple unobservable variables would make this strategy less efficient. In this section, we try to understand the predictions of our model on the nature of the endogeneity that might plague the estimation of Equation 4. Remember that there are two potential sources of correlation between productivity shocks and collateral value: (1) Real estate prices can be correlated with productivity shocks and (2) ownership decision might be driven by productivity. We deal with the first source of correlation empirically, by instrumenting local real estate prices. We now ask whether there are theoretical reasons to worry about the second source of correlation.

Assume the firm can decide, just before date 0, either to buy the real estate at a price ν_0 or to rent it at a fee (f_0, f_1, f_2). We assume that there is perfect competition among renters, so that $F = \sum_{t=0}^{2} f_t = \nu_0 - \frac{\nu_2}{1+r}$. The decision to buy also entails an upfront decision to take on additional debt or to spend some existing cash to finance the acquisition of the asset. We first remark that firms without sufficient initial financial capacity cannot acquire the asset: If $c_0 < B_0$, that is, if the firm's initial cash position is low or if the investor has a strong outside option, the firm will not be able to find the liquidity to pay the upfront price ν_0. Buying the asset would require an initial transfer from the investor at least equal to $\nu_0 - c_0 > \nu_0 - B_0$, which would raise the firm's liability to the investor above B_0 and would trigger immediate renegotiation of the contract.

Consider now a firm with $c_0 > B_0$. This firm is indifferent between using cash or debt to acquire the asset, since $V^0_{B_0} + 1 = V^0_{c_0}$. To simplify the analysis, but without loss of generality, assume that $c_0 > \nu_0$ and that the firm uses only cash to finance the acquisition. To make the analysis starker, we assume that (1) without collateral ν_0 the first-best investments cannot be financed and (2) with collateral ν_0 first-best investments can be sustained if and only if $\psi \geq \psi^\star > (1+r)^2$.[5] We now deliver our final proposition, which relates the ownership decision to the value of the commitment parameter ψ.

Proposition 1.3 *There exists a threshold $\bar{\psi} \in [1+r, (1+r)^2]$ such that the firm strictly prefers buying the real estate asset if and only if $\psi > \bar{\psi}$.*

Proof See Appendix A.1. ∎

The key element in the analysis of Proposition 1.3 is that the net present value of buying is similar to that of renting, *except* for future liquidity motives. Because we have assumed that without collateral, firms are credit constrained, this liquidity rationale is relevant in the decision to buy versus lease the asset. Firms that decide to purchase the asset trade off a lower cash position in exchange for a stronger collateral value. Consider first the case where $\psi < 1+r$. A dollar of date 0 cash brings $(1+r)$ and $(1+r)^2$ dollars of liquidity at dates 1 and 2, respectively. A dollar of date 0 additional collateral yields only $\psi < 1+r < (1+r)^2$ dollars of additional liquidity at dates 1 and 2. Thus, such a firm should always be better off not buying the asset at date 0. Similarly, consider a firm with $\psi > (1+r)^2$. Now, the benefit of an additional dollar of collateral in terms of future liquidity is always larger than that of an additional dollar of debt, as the firm can extract an important amount of liquidity from its real estate. Finally, we can prove that if a firm prefers not buying the asset, then a similar firm with a lower commitment ability ψ cannot strictly prefer buying the asset: This is because both firms have the same value without collateral and, when they do own real estate, the firm with the higher commitment ability should always have a larger value.

Although Proposition 1.3 is important for understanding the selection process at work in our data, it unfortunately does not provide us with an unambiguous interpretation of the potential selection bias. This proposition asserts that firms with a lower commitment ability, and therefore more credit constrained firms (as $V_\psi \geq 0$), are more likely to own than rent their properties. We already noticed, however, in Section 1.2 that the sensitivity of investment to collateral value is not necessarily monotonic with the commitment ability ψ. On the one hand, firms with larger ψ invest more and hence their investment react less to a given appreciation of collateral value ; on the other hand, firms with larger ψ are able to extract more liquidity from an increase in real estate prices. We interpret this result as a sign that, in our simple framework, there is no reason to expect a priori a selection bias of a given sign.

The discussion in this section assumes that we are working in our standard framework with no correlation between real estate prices and investment opportunities. At first glance, one might worry that firms with a high correlation between real estate prices and investment opportunities might have more incentive to buy their real estate assets, since these become better hedges for future financing constraint risks. This would be the case if $V_{\theta\psi}$ would unambiguously be

[5] The alternative cases deliver similar intuitions and are therefore left for Appendix A.1.

positive, that is, if higher productivity would make the marginal value of collateral greater or, in other words, if firms would be more constrained with an increase in productivity. This is, however, not always the case. Here again, two opposing effects are at play: On the one hand, a higher productivity raises the marginal value of capital and hence first-best investment; on the other hand, a higher productivity increases interim cash flows and hence might decrease the intertemporal budget constraint, in particular when the initial capital k_0 is large enough. This implies that $V_{\nu\theta}$ cannot be signed in general. Hence, it is not generally true that real estate is necessarily a good hedge for firms whose productivity shocks are correlated with real estate prices. This implies that there is no way to predict a systematic selection bias in the decision to own versus lease real estate assets, even when real estate prices are correlated with investment opportunities.

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A Proofs

A.1 Derivation of the Optimal Contract

We can write the optimal contract as

$$\left|\begin{array}{l}\max\limits_{((b_t),(x_t),(i_t))} c_0 - G(i_0,k_0) - b_0 - x_0 + \dfrac{\pi(k_1,\theta) - G\left(i_1,k_1\right) - b_1 - x_1}{1+r} + \dfrac{\pi(k_2,\theta) + \nu_2 - b_2 - x_2}{(1+r)^2} \\ b_0 + \dfrac{b_1}{1+r} + \dfrac{b_2}{(1+r)^2} \geq B_0 \\ b_1 + \dfrac{b_2}{(1+r)} \leq \psi\nu_1 \\ b_2 \leq \psi\nu_2 \\ x_0 \geq 0 \\ x_1 + x_0(1+r) \geq 0 \\ x_2(1+r)^2 + x_1 + x_0(1+r) \geq 0 \\ G(i_0,k_0) \leq c_0 - b_0 \\ G(i_1,k_1) \leq \pi(k_1,\theta) - b_1\end{array}\right. \tag{6}$$

It maximizes the expected net payment to the firm under the investor's participation constraint (constraint 1), two no-renegotiation constraints (constraints 2 and 3), three positive saving balance constraints (constraints 4 through 6), and two positive dividend constraints (constraints 7 and 8). The private savings are redundant with the transfers to the bank. Abusing notations, let us call $b_t = b_t + x_t$. Then, program 6 becomes

$$\left|\begin{array}{l}\max\limits_{((b_t),(i_t))} c_0 - G(i_0,k_0) - b_0 + \dfrac{\pi(k_1,\theta) - G\left(i_1,k_1\right) - b_1}{1+r} + \dfrac{\pi(k_2,\theta) + \nu_2 - b_2}{(1+r)^2} \\ b_0 + \dfrac{b_1}{1+r} + \dfrac{b_2}{(1+r)^2} \geq B_0 \\ b_1 + \dfrac{b_2}{(1+r)} \leq \psi\nu_1 \\ b_2 \leq \psi\nu_2 \\ G(i_0,k_0) \leq c_0 - b_0 \\ G(i_1,k_1) \leq \pi(k_1,\theta) - b_1\end{array}\right. \tag{7}$$

The participation constraint is necessarily binding, so that program 7 can be transformed into

$$\left|\begin{array}{l}\max\limits_{((b_t),(i_t))} c_0 - G(i_0,k_0) + \dfrac{\pi(k_1,\theta) - G\left(i_1,k_1\right)}{1+r} + \dfrac{\pi(k_2,\theta) + \nu_2}{(1+r)^2} - B_0 \\ (1+r)\left(B_0 - b_0\right) \leq \psi\nu_1 \\ (1+r)^2\left(B_0 - b_0 - \dfrac{b_1}{1+r}\right) \leq \psi\nu_2 \\ G(i_0,k_0) \leq c_0 - b_0 \\ G(i_1,k_1) \leq \pi(k_1,\theta) - b_1\end{array}\right. \tag{8}$$

Finally, adding the constant B_0 to the previous program yields program 1.

A.2 Proof of Proposition 1.1

Consider first a firm such that none of the constraints in program 1 are binding. Its investments satisfy the first-order condition when the four Lagrange multipliers in program 1 are zero and therefore verify system 2. Clearly, these investments $i_0^\star$ and $i_1^\star$ are independent of both ν_1 and ν_2 and hence of ν_0.

Consider now a firm such that $\lambda_1 > 0$, that is, the date 0 budget constraint is binding. Then $G(i_0, k_0) = c_0 - B_0 + \frac{\psi\nu_1}{1+r}$. Differentiating with respect to ν_0 and reminding the reader that $\frac{\partial \nu_1}{\partial \nu_0} = 1$, we have $\frac{\partial i_0}{\partial \nu_0} = \frac{\psi}{1+r}\frac{1}{G_1(i_0,k_0)} > 0$.

Finally, consider a firm such that $\lambda_1 = 0$ but $\lambda_2 > 0$. Its investments are given by the intertemporal budget constraint and the condition equating marginal costs of investment across periods as in system 3. Differentiating this system with respect to ν_0 yields the following system:

$$\begin{cases} G_1(i_1,k_1)\left(\frac{\partial i_0}{\partial \nu_0} + \frac{\partial i_1}{\partial \nu_0}\right) = \frac{\psi}{(1+r)^2} \\ \frac{\partial i_0}{\partial \nu_0}\underbrace{\left(G_{11}\left(i_0,k_0\right) - \frac{\pi_{kk}(k_1,\theta) - G_{22}\left(i_1,k_1\right) + G_{12}(i_1,k_1)}{1+r}\right)}_{>0} = \frac{\partial i_1}{\partial \nu_0}\underbrace{\frac{G_{11}(i_1,k_1) - G_{21}\left(i_1,k_1\right)}{1+r}}_{>0} \end{cases} \tag{9}$$

so that

$$\frac{\partial i_0}{\partial \nu_0} = \frac{\psi}{(1+r)^2}\frac{1}{G_1\left(i_1,k_1\right)}\frac{1}{1+\frac{((1+r)G_{11}(i_0,k_0)-\pi_{kk}(k_1,\theta)+G_{22}(i_1,k_1)-G_{12}(i_1,k_1))}{G_{11}(i_1,k_1)-G_{21}(i_1,k_1)}} > 0$$

A.3 Proof of Proposition 1.2

Call $\theta\left(c_0, k_0, B_0, \nu_0, V, \psi\right)$ the function such that for all V and ν_0

$$V^0\left(c_0, k_0, B_0, \nu_0, \theta\left(c_0, k_0, B_0, \nu_0, V, \psi\right)\right) = V$$

Call x the initial vector of state variables, $x = (c_0, k_0, B_0, \nu_0, \theta, \psi)$, and y the modified vector of state variables, $y = (c_0, k_0, B_0, \nu_0, V, \psi)$. The policy function i_0 associated with program 1 is defined as a function of the initial state variable:

$$i_0 = h\left(x\right) = h\left(c_0, k_0, B_0, \nu_0, \theta(y), \psi\right) = j(y)$$

Therefore, $\frac{\partial j}{\partial \nu_0} = \frac{\partial h}{\partial \nu_0} + \frac{\partial \theta}{\partial \nu_0}\frac{\partial h}{\partial \theta}$. In other words, when two firms have the same value V^0 but different collateral values, productivity must adjust by $\frac{\partial \theta}{\partial \nu_0}$. As investments depend on productivity, this adjustment has an impact on investment decisions. Therefore, the conditional sensitivity is the "true" sensitivity, $\frac{\partial h}{\partial \nu_0}$ plus the impact on investment implied by the adjustment, $\frac{\partial \theta}{\partial \nu_0}\frac{\partial h}{\partial \theta}$.

Consider the case of an unconstrained firm. Its value depends on ν only through the final resale value, so that, using the envelope theorem, $\frac{\partial \theta}{\partial \nu} = -\frac{V_{\nu_0}}{V_\theta} = -\frac{1}{(1+r)\pi_\theta\left(k_1^\star,\theta\right)+\pi_\theta\left(k_2^\star,\theta\right)}$.

So the conditional sensitivity is therefore given by

$$\frac{\partial j}{\partial \nu_0} = \underbrace{\frac{\partial h}{\partial \nu_0}}_{=0} - \frac{1}{(1+r)\,\pi_\theta\left(k_1^\star,\theta\right) + \pi_\theta\left(k_2^\star,\theta\right)}\frac{\partial i_0^\star}{\partial \theta}$$

The derivative of $i_0^\star$ with respect to θ is given by differentiating system 2:

$$\begin{aligned} &\frac{\partial i_0^\star}{\partial \theta} \times \left\{(1+r)\,G_{11}\left(i_0^\star,k_0\right) + \frac{\pi_{kk}\left(k_2^\star,\theta\right)}{1+r}\left(\pi_{kk}\left(k_1^\star,\theta\right) - \left(G_{22} - 2G_{12} + G_{11}\right)\left(i_1^\star,k_1^\star\right) + \left(G_{11}G_{22} - \left(G_{12}\right)^2\right)\left(i_1^\star,k_1^\star\right)\right)\right\} \\ = &\left(G_{11}\left(i_1^\star,k_1^\star\right) - \frac{\pi_{kk}\left(k_2^\star,\theta\right)}{1+r}\right)\left(\pi_{k\theta}\left(i_1^\star,k_1^\star\right) + \frac{\pi_{k\theta}\left(i_2^\star,k_2^\star\right)}{1+r}\frac{\left(G_{11}-G_{21}\right)\left(i_1^\star,k_1^\star\right)}{(1+r)G_{11}\left(i_1^\star,k_1^\star\right) - \frac{\pi_{kk}\left(i_2^\star,k_2^\star\right)}{1+r}}\right) \end{aligned}$$

Using our assumptions on G (i.e., that $G_{11} > 0,\ G_{22} > 0,\ G_{12} < 0$, and $G_{11}G_{22} - \left(G_{12}\right)^2 \geq 0$), as well as the concavity of π in k and $\pi_{k\theta} \geq 0$, we can conclude that $\frac{\partial i_0^\star}{\partial \theta} > 0$. This in turns implies that $\frac{\partial j}{\partial \nu_0} < 0$. This is the first part of Proposition 1.2.

Consider now the date 0 constrained firm. Its investment is given by the binding budget constraint, $G(i_0, k_0) = c_0 - B_0 + \frac{\psi\nu}{1+r}$, and is independent of θ. Hence

$$\frac{\partial j}{\partial \nu_0} = \underbrace{\frac{\partial h}{\partial \nu_0}}_{>0} - \frac{\partial \theta}{\partial \nu} \times \underbrace{\frac{\partial i_0}{\partial \theta}}_{=0} > 0$$

This is the second part of Proposition 1.2.

Finally, consider a date 1 constrained firm. Its investment is given by system 3. In particular, we can obtain the derivative of initial investment i_0 with respect to θ:

$$\frac{\partial i_0}{\partial \theta} = (1+r)^2 \frac{\partial i_0}{\partial \nu_0} \left(\frac{\pi_\theta (k_1, \theta)}{1+r} + \frac{G_1 (i_1, k_1)}{G_{11} (i_1, k_1) - G_{12} (i_1, k_1)} \frac{\pi_{k\theta} (k_1, \theta)}{1+r} \right) > 0$$

This allows one to compute the conditional sensitivity:

$$\begin{aligned} \frac{\partial j}{\partial \nu_0} &= \frac{\partial h}{\partial \nu_0} - \frac{\partial \theta}{\partial \nu_0} \times \frac{\partial h}{\partial \theta} \\ &= \frac{\partial i_0}{\partial \nu_0} \frac{\left((\psi - 1) \lambda_3 \frac{\pi_\theta (k_1, \theta)}{1+r} + \frac{\pi_\theta (k_2)}{1+r} - (1 + \psi \lambda_3) \frac{G_1 (i_1, k_1)}{G_{11} (i_1, k_1) - G_{12} (i_1, k_1)} \frac{\pi_{k\theta} (k_1 \theta)}{1+r} \right)}{(1 + \lambda_3) \frac{\pi_\theta (k_1, \theta)}{1+r} + \frac{\pi_\theta (k_2, \theta)}{1+r}} \end{aligned}$$

As remarked in Proposition 1.2, the sign of this last expression is ambiguous. On the one hand, the larger the impact of productivity on profit, and hence on value, the smaller the adjustment in θ needs to be, and hence the closer the conditional sensitivity to the unconditional sensitivity. On the other hand, the larger the impact of productivity on the marginal product of capital, the larger the effect of productivity on investment and, hence, the smaller the conditional sensitivity relative to the unconditional sensitivity.

A.4 Proof of Proposition 1.3

Consider, as is the case in Proposition 1.3, the case where with no collateral the firm cannot finance first best. Assume that firm best can be financed only with collateral and commitment ability $\psi \geq \psi^\star > (1+r)^2$. Call $z = (k_0, B_0, \theta, \psi)$ the vector of fixed state variables. The firm prefers buying the real estate asset as long as

$$V(z, c_0 - \nu_0, \nu_0) > V(z, c_0, 0) - \left(\nu_0 - \frac{\nu_2}{1+r} \right)$$

Consider the function defined over $[0, \nu_0]$: $\rho(x) = V(z, c_0 - x, x) + x \frac{\nu_2}{\nu_0 (1+r)} - x$. We know that

$$\rho'(x) = (\lambda_1 + \lambda_2) \psi - \lambda_3 = (\psi - (1+r)) \lambda_1 + \left(\psi - \left(1 + r^2\right) \right) \lambda_2$$

If $\psi < \psi^\star$, then it must that the firm is constrained for all $x \in [0, \nu_0]$. In particular, consider first the case where $\psi < 1 + r$. Therefore, $\rho'(x)$ must be strictly negative for all x, so that ρ is strictly decreasing over $[0, \nu_0]$ and the firm strictly prefers renting the asset.

Assume now that $\psi \in]\,(1+r)^2, \psi^\star[$. Then $\rho'(x)$ is strictly negative for all x so that ρ is strictly increasing over $[0, \nu_0]$ and the firm strictly prefers buying the asset.

Consider the case where $\psi > \psi^\star$. By assumption, first best is reached with collateral but not without, so that the firm ends up buying the asset.

To conclude the proof, suppose $\psi^\star > \psi^1 > \psi^2$ such that the firm strictly prefers renting the asset when $\psi = \psi^2$ but (weakly) prefers renting the asset when $\psi = \psi^1$. This is not possible. Without the asset, both firms make the same profit $V(z, c_0, 0)$, as the value of the firm is independent of ψ if the firm does not have any collateral. Moreover, we know that the value of the firm is increasing with ψ, so our assumption amounts to

$$\begin{aligned} V^2(z, c_0 - \nu_0, \nu_0) &< V^2(z, c_0, 0) - \left(\nu_0 - \frac{\nu_2}{1+r} \right) \\ &< V^1(z, c_0, 0) - \left(\nu_0 - \frac{\nu_2}{1+r} \right) \\ &< V^2(z, c_0 - \nu_0, \nu_0), \end{aligned}$$

which is not possible if $\psi^1 > \psi_2$. Hence, the existence of the threshold $\bar{\psi}$.

The alternative cases can be derived using similar arguments. First, if the firm with no collateral can reach first best, then the firm weakly prefers renting the asset. Second, if the firm with no collateral is constrained but first best is reached with collateral for $\psi \geq \psi^\star$ with $\psi^\star < \bar{\psi}$, then the previous argument applies.