Leaders, Followers, and Risk Dynamics in Industry Equilibrium*

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Abstract

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Abstract

We study own and rival risk in a dynamic duopoly with a homogeneous output good. A competitor’s options to adjust capacity reduce own-firm risk through a simple hedging channel. For example, if a rival possesses a growth option, an increase in industry demand directly enhances current profits but also encourages value-reducing competitor expansion. As a consequence, when a leader and a follower emerge in equilibrium, risk dynamics depart substantially from previously-studied simultaneous move benchmarks. Own-firm and competitor required returns tend to move together through contractions and oppositely during expansions, providing testable new empirical predictions.

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1 Introduction

A corporation’s opportunities to expand, contract, or otherwise alter production can impact its risk and return dynamics, as observed by Berk, Green, and Naik (1999) and subsequent authors.\footnote{See, for example, Aguerrevere (2009), Berk, Green, and Naik (2004), Carlson, Fisher, and Gianmarino (2004, 2006), Cooper (2006), Garlappi (2004), Gomes, Kogan, and Zhang (2003), Hackbarth and Morellec (2008), Johnson (2002), Kogan (2004), Sagi and Seasholes (2007), and Zhang (2005). In the real options area, this literature builds on Brennan and Schwartz (1985) and McDonald and Siegel (1985, 1986).} In an industry setting, a firm’s decisions may additionally affect the required returns of product market rivals, and vice versa. Identifying the distinct impacts of own and rival real options on firm risk can be useful to both finance research and practice. For example, financial analysts often estimate the required return of a project or corporation using not only the historical risk of the firm, but also its industry rivals.\footnote{The widely used Ibbotson Beta Book provides estimates of beta based on a peer group that depends on industry classification, and the use of industry competitors to proxy for own-firm risk is discussed in finance textbooks such as Brealey and Myers (2001), and Ross, Westerfield, and Jaffe (1996).} To evaluate the validity of such practices requires sound theoretical understanding of the drivers of systematic risk, yet existing literature provides little guidance regarding the differential effects of own-firm and rival real options on own-firm and rival required returns.

In this paper, we study own and rival risk in a dynamic duopoly with a homogeneous output good, real options to expand or contract capacity as industry demand changes, and potentially different adjustment costs across firms. For some parameter values firms exercise their options simultaneously, implying that the expected returns of the two firms move together as in the symmetric oligopoly studied by Aguerrevere (2009). By contrast, even arbitrarily small differences in adjustment costs can imply non-simultaneous exercise in which one firm acts as a leader and the second as a follower.\footnote{See Smets (1991), Dixit and Pindyck (1994), and Grenadier (1996).} In such cases, we show that the systematic risk of a firm and its rival may alternately move together or apart over time, depending on industry conditions and the corresponding changing importance of own and rival growth and contraction options. Because of these dynamics, in a leader-follower equilibrium the joint evolution of required returns departs substantially from simultaneous move outcomes.

For both expansions and contractions, our analysis shows that rival real options reduce own-firm risk through a simple hedging channel. For example, when a competitor possesses a growth option any good news about the product market is partially offset by the closer proximity of rival
expansion. Conversely, bad news about industry demand is counterbalanced by a decline in the threat of competitor capacity additions. Hence, all else equal rival growth options reduce own firm risk. Similarly, when the rival possesses a contraction option, industry demand shocks are partially offset by opposite movements in the likelihood of near-term rival asset sales, again reducing own firm risk. The magnitudes of the hedging effects created by rival real options change over time with industry conditions and the distance to the competitor’s option exercise boundaries.

To develop intuition in the simplest case possible, we first consider an industry where one firm is a “strategic dummy” with permanently fixed output, while the second firm has a single option to irreversibly expand or contract its quantity supplied. At each instant, prices are determined by aggregate industry output and both firms receive a flow of profits. The firm possessing an option to adjust capacity has upper and lower bounds for expansion and contraction, and risk dynamics similar to those shown in prior literature focusing on monopolist exercise. Although the strategic dummy has no real options of its own, we show that its valuation equations and risk nonetheless reflect the dynamic output policies of its rival. In particular, the risk of the strategic dummy decreases as its rival moves towards either its expansion or contraction boundary, and immediately jumps up to a constant when the rival exercises its option.4

In the more general case where both firms may expand or contract, the own-firm and rival valuation equations and betas can possess up to four real options components. On an expansion path, the dynamics of leader and follower risk follow a distinctive pattern. As the leader moves closer to exercise, her own risk increases due to growth option leverage, while the follower risk decreases due to the rival hedging effect. Immediately at the instant the leader exercises her growth option, the risks of the two firms jump oppositely by sufficient magnitudes such that the follower risk exceeds leader risk. By contrast, the own-firm and rival-firm effects of contraction options have the same sign, and in an environment of decreasing industry demand the leader and follower risks tend to move together. These theoretical results suggest that the commonly recommended practice of using competitor or industry betas to proxy for own-firm risk should work well in certain environments, but not in others, providing testable new empirical predictions.

Our paper builds on several areas of the literature. Berk, Green, and Naik (1999) and Gomes, Kogan, and Zhang (2003) pioneered investigation of the risk and return implications of real options,

4The discontinuity in risk at the instant of competitor option exercise reflects the generic lack of smooth pasting when other players take discrete actions in a continuous-time game.

Importantly, the framework we adopt overcomes the difficulties with subgame perfection noted by Back and Paulson (2009) in the model of Grenadier (2002) and in other recent papers analyzing equilibrium stopping-time games. Back and Paulson show that while the equilibrium discussed by Grenadier is an “open-loop” Nash equilibrium, it does not satisfy the standard subgame perfection requirement of a Markov perfect “closed-loop” equilibrium. Subgame perfection importantly rules out strategies involving precommitments that are not credible. In the strategies described by Grenadier, firms have an incentive to preempt investment by their rivals but do not do so. Hence while these precommitment strategies form a Nash equilibrium, they do not satisfy subgame perfection. Similar features are present in recent studies building on the Grenadier model, such as Aguerrevere (2003, 2009) and Novy-Marx (2008). Back and Paulson show that in a closed-loop equilibrium of the Grenadier model, all real option values are competed away, which occurs because of the incentive to preempt accompanied by the unlimited ability to expand. By contrast, all of the equilibria we consider satisfy subgame perfection and hence form closed-loop equilibria, but option values remain positive because expansion opportunities are finite.

Other research in the real options literature analyzes equilibrium exercise of expansion or contraction opportunities in a duopoly setting, but does not investigate risk dynamics. Examples that relate most closely to the framework we consider include Smets (1991), Dixit and Pindyck (1994), Grenadier (1996), Huisman and Kort (1999), Boyer, Lasserre, Marriotti, and Moreaux (2004), and Murto (2004). In general, simultaneous exercise of growth options may occur even when firms have

5Other related work includes Novy-Marx (2008), who considers simultaneous-move strategies in an oligopoly related to the Grenadier (2002) setting but where firms have cost differences; Hackbarth and Morellec (2008) who study risk dynamics in a merger setting; Carlson, Fisher, and Giammarino (2009) and Kuehn (2008), who discuss the impact of investment commitment on risk; and Pastor and Veronesi (2009) who discuss the impact of technological innovation on asset price dynamics.
asymmetric adjustment costs, provided assets in place exist (e.g., Pawlina and Kort, 2006). Our framework emphasizes the importance of the product market demand elasticity in determining the boundary between simultaneous-exercise equilibrium and leader-follower equilibria, and hence provides an explicit, empirically measurable link between product market characteristics and risk dynamics. For high demand elasticities, simultaneous exercise can be supported for a large range of asymmetries in adjustment costs. By contrast, when demand elasticities are low, even arbitrarily small adjustment cost asymmetries can lead to leader-follower exercise as the unique equilibrium outcome.\textsuperscript{6} For contraction options, no simultaneous-move equilibria exist.

In all leader-follower equilibria\textsuperscript{7} for both expansions and contractions, the distance between leader and follower triggers remains bounded below even for arbitrarily small adjustment cost asymmetries. Intuitively, the leader’s action, whether expansion or contraction, strategically impacts the incentives of the follower to create a finite separation in their actions. Hence, non-simultaneous exercise can be an important feature of both expansions and contractions, even when firms are ex ante very similar or identical. The risk dynamics that we demonstrate for leader-follower equilibria therefore fill an important gap in the finance literature.

To clarify the contribution of this paper, we emphasize that versions of the equilibria we study have been developed in prior research. However, our paper is the first to 1) analyze risk dynamics for leader follower models, 2) to isolate the effect of rival real options, and 3) to demonstrate dramatically different risk implications for leader-follower models relative to the risk dynamics in simultaneous move outcomes that have been previously considered (e.g., Aguerrevere, 2009). Our general framework conveniently nests many of the real option models, both expansion and contraction, developed in prior work, which illuminates the different implications for comovement of own and rival risk in expansions versus contractions. We also expand the interpretation of

\textsuperscript{6}For many combinations of low demand elasticities and large growth options, even perfectly symmetric firms cannot optimally exercise growth options simultaneously, and a randomly chosen leader arising from mixed strategies is the only possibility. Huisman and Kort (1999) and Boyer, Lassere, Mariotti, and Moreaux (2004) discuss mixed strategies in expansion games, which requires an extension of the strategy space beyond simple state-dependent triggers following Fudenberg and Tirole (1985). See also Smets (1991), Dixit and Pindyck (1994), and Grenadier (1996).

\textsuperscript{7}Our notion of a “leader-follower” equilibrium is synonymous with “non-simultaneous.” Several types of leader-follower equilibria are distinguished below and have been discussed in prior literature. In a “non-preemptive” equilibrium, the leader and follower use the trigger strategies that would arise if the follower were prohibited from acting first and the role of “leader” determined prior to the start of the game. In a “pre-emptive” equilibrium, the threat of action by the follower causes the leader to act earlier than she would if the rules of the game prohibited the follower from acting first. In a mixed strategy equilibrium, which may occur if firms are symmetric, the leader is determined randomly.
prior real options models by showing that demand elasticities are an important determinant of the boundary between simultaneous-move and leader-follower outcomes.

In a related literature on R&D, random technological progress plays a key role in determining the dynamics of risk (e.g., Berk, Green, and Naik, 2004). An important contribution to this literature is Garlappi (2004), who models a multi-stage patent race between two firms. Infinitesimal instantaneous R&D activity, which cannot be committed in advance, provides an opportunity for stochastic advancement, and the risk dynamics of the two firms differ due to random technological progress shocks and corresponding endogenous time-variation in the probabilities of which firm will win the race as well as the value of the invention. By contrast, we focus on a simple intuition that provides a direct link between own and rival corporate expansion and contraction decisions and risk.

In research subsequent to our own, Bena and Garlappi (2010) study risk dynamics in a patent race framework much closer to the environment we consider.8 Firms with exogenous differences in the probability of success optimally decide when to pay a one-time irreversible fixed fee to enter the race, which then has a random outcome. Due to the presence of lumpy investment, the risk dynamics that emerge in this model are very similar to those shown in our analysis, helping to demonstrate the robustness of our empirical predictions. Bena and Garlappi find empirical evidence from patent filings consistent with the predictions of this model. Bustamante (2010) similarly considers extensions of the framework we consider and discusses empirical evidence.

Section 2 describes the general model. In Section 3, we analyze the simplest case where one firm is a strategic dummy, and show the risk-reducing effects of rival growth options. Section 4 presents the leader-follower equilibrium where firms with asymmetric costs may expand or contract. Section 5 concludes.

2 The Asymmetric Duopoly Model

We present a model in which two strategically interacting firms compete in output levels in a homogeneous goods market, and have options to invest or disinvest in capacity.

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8The theoretical foundations of their model build on Weeds (2002).


2.1 Industry Demand, Production Technologies, and Capital Accumulation

Let \( Q_1^t \) and \( Q_2^t \) denote the output rates of firm one and firm two at instant \( t \), and define the industry output rate \( Q_t = Q_1^t + Q_2^t \). The homogeneous good price is determined by the iso-elastic inverse demand curve

\[
P_t = X_t Q_t^{\gamma-1},
\]

where \( 0 < \gamma < 1 \), and \( X_t \) is an exogenous state variable that represents the level of industry-wide demand. The dynamics of \( X_t \) are specified by

\[
dX_t = g X_t dt + \sigma X_t dW_t,
\]

where \( dW_t \) is the increment of a Wiener process, \( g \) is the constant drift, and \( \sigma^2 \) the constant variance.

Firm \( i \) produces output at time \( t \) using installed capital \( K_i^t \) where \( i \in \{1, 2\} \). Any capital level \( K_i^t \) is associated with a maximum output level \( Q(K_i^t) \geq Q_i^t \). For simplicity, capital levels take one of three discrete values: \( K_i^t \in \{\kappa_0, \kappa_1, \kappa_2\} \), where \( \kappa_0 < \kappa_1 < \kappa_2 \), and for convenience we denote \( q_j \equiv Q(\kappa_j) \) with \( q_0 < q_1 < q_2 \).

Costs of production for firm \( i \) at date \( t \) are given by the increasing function \( F_i^t = f(K_i^t) \). This cost structure emphasizes operating leverage, since total expenditures depend only on the installed capital level \( K_i^t \), as with maintenance costs or other overhead related to plant size. Given the three possible capital levels, there are also three possible levels of fixed operating costs: \( F_i^t \in \{f_0, f_1, f_2\} \), where \( f_0 < f_1 < f_2 \).

To move from one capital state to another, the firm may incur costs or generate cash flows from buying or selling the productive asset, inclusive of any associated adjustment costs. To capture this idea in a general way, we specify for each firm a matrix of discrete transition costs:

\[
\Lambda^i = \begin{bmatrix}
0 & \lambda_{i1}^0 & \lambda_{i1}^2 \\
\lambda_{i0}^1 & 0 & \lambda_{i1}^2 \\
\lambda_{i0}^2 & \lambda_{i2}^1 & 0
\end{bmatrix}.
\]

The instantaneously incurred lump-sum cost for firm \( i \) to move from capital level \( \kappa_m \) to \( \kappa_n \) is given

\[\text{The assumption that the potential output levels } q_j \text{ are the same for firms 1 and 2 is not essential, and is made here for notational convenience. The arguments in the Appendix are valid when the output levels } q_j \text{ differ across firms } i, \text{ hence permitting asymmetric revenue functions.}\]
by $\lambda^t_{mn}$. The only source of heterogeneity between firms in our model is that $\Lambda^1$ and $\Lambda^2$ need not be identical. We assume as an initial condition that at date zero, each firm is endowed with $K^i_0 = \kappa_1$ units of capital.

We finally define indicator variables $D_t^{imn}$ that take the value one at the instant when firm $i$ switches from capital level $\kappa_m$ to $\kappa_n$, and zero elsewhere. We denote by $D_t^i$ the matrix of investment decisions $D_t^{imn}$.

### 2.2 Output, Investment Strategies, and Equilibrium

The economy described above is a dynamic game between firms 1 and 2. At each instant, the managers of the two firms choose output rates $Q^i_t$ and make investment decisions $D^i_t$ knowing the complete history of the game denoted by $\Phi_t = (\{Q^1_s, Q^2_s, K^1_s, K^2_s\}_{s < t}, \{X_s\}_{s \leq t})$, which is common to both managers.

We define the payoff to firm $i$ as the present value of the expected discounted future cash flows. The cash flows at time $t$ derive from revenues in excess of fixed costs $\pi^i_t \equiv P_t Q^i_t - F^i_t$ and from lumpy investment costs related to the decision $D^i_t$. We assume the absence of agency conflicts, so that manager $i$ maximizes the value function

$$V^i_t \equiv E_t \int_t^\infty e^{-r(s-t)} \frac{M_{t+s}}{M_t} \left[ \pi^i_{t+s} ds + 1' (D^i_{t+s} * \Lambda^i) 1 \right], \tag{3}$$

where $1' = [1,1,1]$, $\ast$ represents element-by-element multiplication, and the pricing kernel $M_t$ satisfies $M_0 = 1$ and $dM_t = \frac{\mu - r}{\sigma} M_t dW_t$.

Given the Markov structure of this environment, it is natural to restrict attention to Markov strategies. Manager $i$ can then take actions $Q^i_t$ and $D^i_t$ that depend only on the most recently observed values of the payoff relevant state variables $X_t$ and $K_{t-} \equiv (K^1_{t-}, K^2_{t-})$, where $K^i_{t-} \equiv \lim_{s \uparrow t} K^i_s$. A pure strategy Markov-perfect equilibrium (MPE) of the game is a pair of strategies $(Q^i, D^i), i = 1, 2$, such that the value functions (3) are maximized in every state $(K_{t-}, X_t)$ given the equilibrium strategy of the rival.

It is straightforward to show that any MPE must have quantity choices equal to static Cournot equilibrium output levels. Given our assumption that demand is sufficiently elastic (implied by $\gamma > 0$) and the absence of marginal costs, all firms produce at full capacity. Hence, any MPE
strategy requires \( Q_t^i = Q^i (K_t^i) \). The instantaneous profit functions

\[
\pi_t^i = X_t \left[ Q^1 (K_t^1) + Q^2 (K_t^2) \right]^{\gamma-1} Q^i (K_t^i) - F_t^i
\]

are thus fully determined by the current capital levels \( K_t^1 \) and \( K_t^2 \) and the value of the state variable \( X_t \).

To aid future exposition, it is convenient to define the capital dependent revenue factors

\[
R_{mn}^1 \equiv [Q^1 (\kappa_m) + Q^2 (\kappa_n)]^{\gamma-1} Q^1 (\kappa_m),
\]

\[
R_{mn}^2 \equiv [Q^1 (\kappa_m) + Q^2 (\kappa_n)]^{\gamma-1} Q^2 (\kappa_n),
\]

where \( m, n \in \{0, 1, 2\} \) index the capital levels of firms 1 and 2, respectively. We can then conveniently write the profit function of each individual firm \( i \) as \( \pi^i (K_t^1 = \kappa_m, K_t^2 = \kappa_n, X_t) = X_t R_{mn}^i - F_t^i \).

Given the simplification of the instantaneous output choices \( Q_t^i \), we can henceforth focus attention on the dynamic game of option exercise involving the capital levels \( K_t^i \) and the investment decisions \( D_t^i \). Any Markov strategy can be summarized by a set of exercise boundaries that for each player \( i \) and each capital state \( K_t \) specify regions of the state variable \( X_t \) at which player \( i \) will change his capital level to a new state. We can use standard techniques of backward induction to derive MPE of the dynamic game.

3 Rival Growth Options and Risk

This section considers the simplest case of the general model developed in Section 2. Specifically, we assume that one rival is flexible, and begins with one option to either expand or contract, while the other rival is inflexible and has no ability to change its capital level. This scenario helps us to isolate the two sources of real option risk, own and rival, that can occur in a real options duopoly.

The flexible firm has risk that changes over time only because of its own real option and operating leverage. As in the monopoly case explored in previous literature, the flexible firm has an own-option risk component but no independent source of dynamic industry risk. By contrast, the inflexible firm has no own-option component in its risk loadings, but nonetheless, it is exposed

\footnote{Instantaneous suboptimal actions are ruled out by Markov perfect equilibrium, which requires that all players’ strategies must depend only on payoff relevant state variables.}
to dynamic risk due to the investment decisions of its rival.

To achieve a specification where one firm is flexible and the other inflexible, we set the capital adjustment costs to

\[
\Lambda_1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix},
\]

\[
\Lambda_2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix},
\]

where \( S, I > 0 \). Firm 1, the flexible firm, thus begins at capital level \( \kappa_1 \) and has a single option to change capacity, either by expanding to \( \kappa_2 \) or contracting to \( \kappa_0 \). If it expands, it pays the investment cost \( I \) and if it contracts it receives the salvage value \( S \). Once firm 1 either expands or contracts, it has no further options to change capacity. Firm 2 begins at capital level \( \kappa_1 \) and has no real options.

We now examine the exercise decision and valuation of the flexible firm.

**Proposition 1:** The optimal policy of the flexible firm is to expand at \( X_E > 0 \) and contract at \( X_C < X_E \), where \( X_C \) and \( X_E \) solve the pair of nonlinear equations given in the Appendix. The value of the flexible firm prior to option exercise is:

\[
V_1(K_t, X_t) = V_{A1}(K_t, X_t) + V_{F1}(K_t) + V_{O1}(K_t, X_t),
\]

where \( V_{A1}(K_t, X_t) = \frac{R_{11}X_t}{\delta} \) is the growing perpetuity value of assets in place assuming no future capacity adjustments by either firm, \( V_{F1}(K_t) = -f(K_t^1) / r \) is the perpetuity value of fixed operating costs, \( V_{O1}(K_t, X_t) = B_{11}X_t^{\nu_1} + B_{12}X_t^{\nu_2} \) is the value of growth options, \( B_{11} \) and \( B_{12} \) are positive constants determined by the boundary conditions, and \( \nu_1 > 1 \) and \( \nu_2 < 0 \) are constants given in the Appendix.

As in standard real option models, (e.g., McDonald and Siegel, 1985, 1986), the flexible firm value consists of assets in place and its own option value. The real option has two components related to the growth opportunity and contraction option respectively, but their values are not independent since the constants \( B_{11} \) and \( B_{12} \) can only be determined by jointly solving the value matching equations at the exercise boundaries. The positivity of the constants \( B_{11} \) and \( B_{12} \) reflects that ownership of these options is value-enhancing, and the positive and negative signs of the roots \( \nu_1 \) and \( \nu_2 \) reflect that growth options increase with movements in the underlying asset while the
opposite holds for contraction options.

The inflexible firm value consists only of its assets in place, but an externality is imposed by the rival real options.

**Proposition 2.** The value of the inflexible firm is entirely determined by the value of the assets in place net of the present value of fixed costs:

\[
V^2(K_t, X_t) = V^2_A(K_t, X_t) + V^2_F(K_t) + V^2_C(K_t, X_t),
\]

where \( V^2_A(K_t, X_t) = R^2_{11} X_t / \delta \) is the growing perpetuity value of assets in place assuming no future adjustment to capacity by either firm, \( V^2_F(K_t) = -f(K_t^2) / r \), is the perpetuity value of fixed operating costs, \( V^2_C(K_t, X_t) = B^2_1 X_t^{\nu_1} + B^2_2 X_t^{\nu_2} \) is the value externality imposed by competitor growth options, and the constants \( B^2_1 \leq 0, B^2_2 \geq 0 \) are determined by the value matching conditions at the rival exercise boundaries, as described in the Appendix.

The valuation externality imposed by competitor real options has two components related to the rival growth option and contraction option respectively. The negative sign of \( B^2_1 \) reflects that rival expansion options reduce value, while \( B^2_2 \geq 0 \) follows from the value enhancing effect of competitor contractions. We note from Propositions 1 and 2 that contraction options impact own and rival-firm values with the same sign, whereas expansion options have opposite valuation impacts on a firm and its rivals. These valuation effects have implications for risk.

To determine dynamic loadings on the stochastic discount factor, we calculate the elasticity of firm value with respect to \( X_t \) as described in the Appendix.

**Proposition 3.** The dynamic betas for the flexible and inflexible firm are:

\[
\beta^i(K_t, X_t) = 1 + f_1/r \frac{1}{V^i(K_t, X_t)} + \left\{ (\nu_1 - 1) \frac{B^i_1 X_t^{\nu_1}}{V^i(K_t, X_t)} + (\nu_2 - 1) \frac{B^i_2 X_t^{\nu_2}}{V^i(K_t, X_t)} \right\}
\]

prior to option exercise and \( \beta^i(K_t, X_t) = 1 + (f_1/r) / V^i(K_t, X_t) \) afterwards.

Consistent with the valuation equations, the betas for the flexible and inflexible firms consist of three parts. By assumption the revenue beta is equal to 1. The second component is operating leverage, which always increases risk, and the final term for both firms arises from the flexible
firm’s real options. We note that although the structure of beta for both firms is similar, the economic interpretation is very different. The flexible firm’s risk depends only on its own firm-specific decisions, whereas the inflexible firm has no decisions to make and its risk is determined entirely by industry effects.

Examining the flexible firm first, we note that since $\nu_1 > 1$ and $B_1^1 \geq 0$, its own risk rises due to its own option to expand. On the other hand, since $\nu_2 < 0$ and and $B_2^1 \geq 0$, the option to contract reduces risk. The inflexible firm dynamic loadings on the stochastic discount factor are determined by $B_1^2 \leq 0$ and $B_2^2 \geq 0$, implying that its risk decreases due to both the competitor growth option and the competitor expansion option. This simple example illustrates two important points, which we now discuss in more detail.

First, rival real options reduce risk. Intuitively, a competitor’s investment decisions act as a natural hedge against variations in the exogenous state variable. Good news about demand going up will be partially offset by the bad news that the competitor is closer to expanding. Figure 1 gives a graphical presentation of this hedging argument. Before the flexible firm exercises her option, industry demand is indicated by the downward sloping curve $D$ and the industry supplies output at the full-capacity level $Q_1$. Consider now an increase in demand to the level $D'$ that induces the flexible firm to exercise her growth option. The corresponding increase in industry supply causes prices to increase less than to the level $P^*$ corresponding to the old supply curve. Prices rise more moderately to $P_2$ instead of $P^*$, and the dampening in profits caused by the increase in industry supply after a positive demand shock corresponds to the natural hedging effect caused by rival real options.

The second important implication of the simple example developed in this section is that expansion options have an oppositely signed impact on own-firm and rival risk, while contraction options affect both firms’ risk in the same direction. These risk implications follow from the valuation impacts of own and rival real options discussed previously. Contraction options of both one’s own firm and rivals create a hedge against adverse moves in underlying fundamentals. By contrast, own-firm expansion opportunities amplify risk, whereas rival expansion opportunities mitigate the potential for upside gain.

Figure 2 shows the own and rival risk effects discussed above. For simplicity, we assume the inflexible firm has no operating leverage. In the figure, $X_C$ is the critical level of demand at which
the flexible firm shrinks and $X_E$ is critical level at which the flexible firm expands. The diagram illustrates that rival real options reduce risk, and that real options can cause own and competitor risks to move together or in opposite directions. As demand increases and the growth option becomes more important, the flexible firm’s risk increases while the inflexible firm’s risk decreases. By contrast, when demand decreases and the contraction option is more valuable, own and rival firm risk tend to move together. The next section investigates the robustness of these results when both firms possess growth options and exercise is strategic.

4 Dynamic Risk in Asymmetric Industry Equilibrium

We now permit both firms to have real options to expand or contract capacity, and consider the corresponding equilibrium play. We obtain analytical solutions for firm risk and required return in two cases: 1) when both firms have expansion options, and 2) when both firms have contraction options.

4.1 Equilibrium Exercise of Expansion Options

To analyze equilibrium in the case where both firms have only a single growth option, we set the capital adjustment costs to

$$
\Lambda^1 = \begin{bmatrix}
0 & -\infty & -\infty \\
-\infty & 0 & -I \\
-\infty & -\infty & 0
\end{bmatrix} \quad \Lambda^2 = \begin{bmatrix}
0 & -\infty & -\infty \\
-\infty & 0 & -\rho I \\
-\infty & -\infty & 0
\end{bmatrix},
$$

where $\rho \geq 1$ so that the expansion costs of firm 1 are lower than those of firm 2.

We show below that in all Markov-perfect equilibria the low-cost firm invests at least as soon as the high-cost firm. Given this simplification, industry structure can be one of three potential phases: a juvenile industry where neither firm has exercised its growth option, an adolescent industry where the “leader” has exercised and the “follower” has not, and a mature industry where both firms have expanded. Figure 3 depicts the different industry stages.

We divide all possible equilibria into two primary classifications, “simultaneous” and “leader-follower.” By this categorization, a leader-follower equilibrium is defined simply by the absence of simultaneous exercise. All three industry stages occur for a finite period of time with probability
one in a leader-follower equilibrium, whereas in a simultaneous equilibrium the industry structure jumps immediately from juvenile to mature.

Following Pawlina and Kort (2006), determining payoffs under different strategies proceeds by backward induction, and allows determination of the type of equilibrium. For example, assuming firm 1 as the leader and firm 2 as the follower, we first calculate the optimal exercise of firm 2 and then the optimal exercise of firm 1. We similarly calculate value functions and triggers with firm 2 as the leader and firm 1 as the follower. Finally, we calculate value functions and triggers where both firms exercise simultaneously.

To simplify discussion, in the remainder of this section we assume that the initial demand state $X_0$ is strictly less than the leader trigger level of firm 1, which ensures that the juvenile industry state occurs for a finite period of time in equilibrium. We then summarize the types of equilibrium that may occur.

**Proposition 4.** The MPE of the expansion game are characterized by:

1. **Simultaneous Equilibrium:** There exists a value $\rho^{**}(\gamma, q_1, q_2, f_1, f_2, v_1) > 0$ such that for all $1 \leq \rho < \rho^{**}$ the unique MPE involves simultaneous exercise with a trigger that maximizes the low-cost firm’s value. There exist no other simultaneous investment MPE. Hence if $\rho^{**} < 1$, no simultaneous exercise equilibria exist.

2. **Non-Preemptive Leader-Follower Equilibrium:** There exists a value $\rho^{*}(\gamma, q_1, q_2, f_1, f_2, v_1) > 1$ such that for all $\rho > \max[\rho^{*}, \rho^{**}]$, the unique MPE results in the high-cost firm acting as the follower with trigger $X_F^2$ and the low-cost firm acting as the leader with trigger $X_{LN}^1 < X_F^2$, where the triggers given in the Appendix are identical to those obtained if the roles of leader and follower were predetermined prior to the beginning of the game, and the follower could not threaten to preempt the leader’s investment.

3. **Preemptive Leader-Follower Equilibrium:** For $\rho$ satisfying $\rho^* \geq \rho \geq \max[\rho^{**}, 1]$, the unique MPE results in the high-cost firm acting as the follower with trigger $X_F^2$ and the low-cost firm acting as the leader with trigger $X_{LP}^1 < X_F^2$, where the trigger of the leader $X_{LP}^1 \leq X_{LN}^1$ is determined as the indifference point of the high-cost firm between acting as a leader or

---

11See also Smets (1991), Dixit and Pindyck (1994), and Grenadier (1996), who consider leader-follower equilibria in the special case of symmetric firms, where a leader must be chosen by some form of randomization.

12Grenadier (1996) discusses outcomes when the initial state exceeds the leader trigger in the symmetric case.
a follower. Hence the threat of the high-cost firm to preemptively expand causes the low-cost firm to itself preemptively invest just at the instant when the high-cost firm’s preemption threat becomes credible. The leader’s expansion deters growth of the follower in the region between $X_{LP}^1$ and $X_{LP}^2$.

4. Random Leader-Follower Equilibrium: If $\rho = 1$ and $\rho^{**} < 1$, no pure strategy MPE is possible. To obtain a mixed strategy MPE requires expanding the strategy space as discussed in Fudenberg and Tirole (1985) and Huisman and Kort (1999). In the mixed strategy equilibrium the leader is randomly chosen at instant $X_{LP}^1 = X_{LP}^2$, and the other firm becomes the follower exercising at $X_F^1 = X_F^2$.

Equilibrium play in the expansion game can thus be categorized by the regions of the parameter space in which each equilibrium holds. To illustrate the proposition, we fix the parameter values $\sigma = 0.2, q_1 = 2, q_2 = 10, f_1 = f_2 = 0, r = 0.05, \delta = 0.03$ and $I = 500$, and diagram in Figure 4 the equilibrium regions in the two-dimensional space of $\gamma$, related to the demand elasticity, and the relative cost difference $\rho$. For high levels of demand elasticity (low $\gamma$), the simultaneous equilibrium can be supported even when expansion cost asymmetries are large. By contrast when the demand elasticity is low (high $\gamma$), arbitrarily small positive cost asymmetries imply that one of the pure strategy leader-follower equilibria must hold.

The link between demand elasticity and the existence of the simultaneous exercise equilibrium relates to the impact of investment on the profits generated by assets in place. When demand elasticity is low, expanding output has a small negative effect on asset-in-place value, and the benefit of waiting for simultaneous investment relative to acting as a leader is not as large. By contrast, when the demand elasticity is very high the value of waiting to invest simultaneously can be everywhere higher than the value of acting as a leader, and simultaneous investment can be supported.

Valuation in the pure-strategy leader-follower equilibria can be conveniently summarized.

**Proposition 5.** In any pure-strategy leader-follower equilibrium, the leader’s value function $V^1(K_t, X_t)$

---

13 See also Thijssen, Huisman, and Kort (2002) and Boyer, Lasserre, Mariotti, and Moreaux (2004).

14 An interesting comparative static that does not appear in Figure 4 is that as $q_2$ increases (corresponding to an increase in the ratio of growth options to assets-in-place), the region corresponding to simultaneous investment shrinks.
is given by

\[
\begin{aligned}
&\left\{
\begin{array}{ll}
\frac{R_1}{\delta}X_t - \frac{f_t}{r} + \left[\left(\frac{R_{21}^1 - R_{11}^1}{\delta}\right)X_t^1 - \frac{(f_2 - f_1 + rI)}{r}\right] \left(\frac{X_t}{X_L^1}\right)^{\nu_1} \\
+ \frac{X_F^2}{\delta} \left[ R_{22}^1 - R_{21}^1 \right] \left(\frac{X_t}{X_F^2}\right)^{\nu_1} & X_t < X_L^1,
\end{array}
\right.
\end{aligned}
\]

\[
\begin{aligned}
&\left\{
\begin{array}{ll}
\frac{R_1}{\delta}X_t - \frac{f_t}{r} + \frac{X_F^2}{\delta} \left[ R_{22}^1 - R_{21}^1 \right] \left(\frac{X_t}{X_F^2}\right)^{\nu_1} & X_L^1 \leq X_t \leq X_F^2,
\end{array}
\right.
\end{aligned}
\]

\[
\begin{aligned}
&\left\{
\begin{array}{ll}
\frac{R_1}{\delta}X_t - \frac{f_t}{r} & X_t > X_F^2,
\end{array}
\right.
\end{aligned}
\]

and the follower’s value function \(V^2(K_t, X_t)\) is equal to

\[
\begin{aligned}
&\left\{
\begin{array}{ll}
\frac{R_1}{\delta}X_t - \frac{f_t}{r} + \frac{f_2 - f_1 + rI}{r(\nu_1 - 1)} \left(\frac{X_t}{X_L^1}\right)^{\nu_1} + \frac{X_F^2}{\delta} \left[ R_{21}^2 - R_{11}^2 \right] \left(\frac{X_t}{X_L^1}\right)^{\nu_1} & X_t \leq X_L^1,
\end{array}
\right.
\end{aligned}
\]

\[
\begin{aligned}
&\left\{
\begin{array}{ll}
\frac{R_1}{\delta}X_t - \frac{f_t}{r} + \frac{f_2 - f_1 + rI}{r(\nu_1 - 1)} \left(\frac{X_t}{X_L^1}\right)^{\nu_1} & X_L^1 \leq X_t \leq X_F^2,
\end{array}
\right.
\end{aligned}
\]

\[
\begin{aligned}
&\left\{
\begin{array}{ll}
\frac{R_1}{\delta}X_t - \frac{f_t}{r} & X_t > X_F^2,
\end{array}
\right.
\end{aligned}
\]

where the optimal leader trigger is \(X_L^1 = X_{LN}^1\) for a non-preemptive equilibrium and \(X_L^1 = X_{LP}^1\) in a preemptive equilibrium.

Both firm values are composed of the growing perpetuity value of the assets in place assuming constant industry structure, the perpetuity value of the fixed costs, the own-firm option value, and the externality imposed by the rival option.

Using the leader’s non-preemptive trigger \(X_{LN}^1 = (f_2 - f_1 + rI)\delta\nu_1 / ([R_{21}^1 - R_{11}^1]r(\nu_1 - 1))\), an alternative decomposition of the leader value function in a juvenile industry can be derived:

\[
V^1(K_t, X_t) = \frac{R_{11}^1}{\delta}X_t - \frac{f_t}{r} + \left[X_L^1 - \left(1 - \frac{1}{\nu_1}\right)X_{LN}^1\right] \frac{[R_{21}^1 - R_{11}^1]}{\delta} \left(\frac{X_t}{X_L^1}\right)^{\nu_1}
\]

\[
+ \frac{X_F^2}{\delta} \left[ R_{22}^1 - R_{21}^1 \right] \left(\frac{X_t}{X_F^2}\right)^{\nu_1},
\]

where as before \(X_L^1 = X_{LN}^1\) applies for a non-preemptive equilibrium and \(X_L^1 = X_{LP}^1\) for a preemptive one. The rival value adjustment is always negative, consistent with the price-reducing effect of competitor expansion discussed in Section 3. The leader’s own growth option value is proportional to a weighted sum of the two triggers \(X_L^1\) and \(X_{LN}^1\). In a non-preemptive equilibrium \(X_L^1 = X_{LN}^1\) it is straightforward to observe that the leader’s own growth option value must be positive since
\( \nu_1 > 1 \). In a preemptive equilibrium, equation (4) interestingly shows that both the preemptive and non-preemptive triggers enter into the valuation equation, and the relative sizes of the two triggers as well as \( \nu_1 \) determine the sign of the own growth option value. For most parameters, the own growth option value is positive, but for example if the risk-free rate and hence \( \nu_1 \) are very large then the leader’s own growth option value can in fact be negative.\(^\text{15}\)

The follower’s value in a juvenile industry similarly can be rewritten:

\[
V^2(K_t, X_t) = \frac{R^2_{11}}{\delta} X_t - \frac{f_1}{r} + \frac{X^2_F}{\delta \nu_1} [R^2_{22} - R^2_{11}] \left( \frac{X_t}{X^2_F} \right)^{\nu_1} + \frac{X^1_L}{\delta} [R^2_{21} - R^2_{11}] \left( \frac{X_t}{X^1_L} \right)^{\nu_1}.
\]

The follower’s own-option effect is always positive and the rival value-adjustment is negative.

The valuation equations in a simultaneous-exercise equilibrium have a different appearance.

**Proposition 6.** In case of simultaneous exercise the value function of each firm \( V^i(K_t, X_t) \) is given by

\[
\begin{cases}
\frac{R^i_{11}}{\delta} X_t - \frac{f_1}{r} + \frac{f_2-f_1+rF^i}{r(\nu_1-1)} \left( \frac{X_t}{X^i_S} \right)^{\nu_1} , & X_t \leq X^i_S, \\
\frac{R^i_{22}}{\delta} X_t - \frac{f_2}{r} , & X_t > X^i_S,
\end{cases}
\]

with the expansion trigger \( X^i_S = \nu_1 \delta (f_2 - f_1 + rF^i)/ [(\nu_1-1)r (R^i_{22} - R^i_{11})] \).

Firm value therefore appears to contain only the assets in place and an option component:

\[
V^i(K_t, X_t) = \frac{R^i_{11}}{\delta} X_t - \frac{f_1}{r} + \frac{X^i_S}{b \nu_1} [R^i_{22} - R^i_{11}] \left( \frac{X_t}{X^i_S} \right)^{\nu_1}.
\] (5)

Hence, the distinguishing feature of the simultaneous exercise equilibrium is that the rival firm value adjustment is not apparent.

Of course, both own-firm and rival effects are implicitly embedded within the growth option component of (5), but there is not a unique decomposition of the change in profits \( R^i_{22} - R^i_{11} \). For example, one possible decomposition for firm 1 is to designate \( R^1_{22} - R^1_{12} \) as the own growth option

\(^\text{15}\)One set of parameters for which the leader own growth option value is negative is \( \gamma = 0.5, \sigma = 0.1, q_1 = 2, q_2 = 10, f_1 = f_2 = 0, r_f = 1.0, \delta = 0.88, I = 100.\)
component and $R^{12}_{1} - R^{11}_{1}$ as the rival effect. On the other hand, it is equally sensible to view $R^{22}_{1} - R^{21}_{1}$ as the competitor effect and $R^{12}_{1} - R^{11}_{1}$ as the own effect. Thus, due to simultaneous exercise the own and rival effects are not separately identified. However, we do note that in both possible decompositions the own effect is positive and the rival effect is negative.

To derive risk implications for all three different types of equilibria, we use similar notation as previously and write for $i = 1, 2$,

$$V^i(K_t, X_t) = V^i_A(K_t, X_t) + V^i_F(K_t) + V^i_O(K_t, X_t) + V^i_C(K_t, X_t),$$

where $V^i_O(K_t, X_t)$ is the own-option component of value, and $V^i_C(K_t, X_t)$ is the rival-option component of value.\(^{16}\) We then show

**Proposition 7.** In all pure strategy equilibria, systematic firm risks for the follower and the leader are given by

$$\beta^i(K_t, X_t) = 1 + \frac{V^i_O(K_t, X_t) + V^i_C(K_t, X_t)}{V^i(K_t, X_t)}(\nu_1 - 1) + \frac{V^i_F(K_t)}{V^i(K_t, X_t)}, \quad (6)$$

for all industry states $K_t$.

Systematic firm risk in a growing oligopolistic industry is thus driven by a firm’s operating leverage, its own growth options, and the risk reducing effects of rival growth options.

We emphasize several important points regarding Proposition 7. First, the own growth option and rival growth option components of value enter additively into the second term in (6). Hence, the risk effects of own and rival growth options are identical when normalized by dollar values, which provides a remarkable simplification. Second, since $\nu_1 > 1$ and $V^i_C(K_t, X_t) < 0$, rival growth options always reduce risk, independent of whether the equilibrium is simultaneous, pre-emptive, or non-preemptive. Third, when $V^i_O(K_t, X_t) > 0$, which always holds for non-preemptive equilibria, own-firm expansion options increase risk.

Perhaps most importantly, only in the simultaneous equilibrium can we uniquely sign the sum $V^i_O(K_t, X_t) + V^i_C(K_t, X_t)$ at all points in the state space. In particular, in a simultaneous exercise

---

\(^{16}\) We acknowledge that the own and rival components can interact, particularly in the pre-emption equilibrium where the follower real option directly influences the leader trigger. However, conditional on the leader and follower triggers the decomposition into own and rival components is natural.
equilibrium this sum is guaranteed to be positive, and hence the cumulative effect of growth options is always to increase risk, consistent with the results in Aguerrevere (2009). By contrast, in a leader-follower equilibrium the cumulative effect of industry growth options
\[ V_O(K_t, X_t) + V_C(K_t, X_t) \] can generally not be uniquely signed, implying that growth options will alternately increase or decrease risk for a given firm at different points in the state space.

Figure 5 displays the evolution of risk in a growing industry for all three different types of equilibria. In the four panels of the figure, we hold all parameters constant except the expansion cost asymmetry which is set to 2.0 in Panel A, 1.3 in Panel B, 1.1 in Panel C, and 1.0 in Panel D. As a consequence, in Panels A and B the equilibrium type is non-preemptive, in Panel C the equilibrium is preemptive, and in panel D the equilibrium is simultaneous. The equilibrium types are consistent with Figure 2 where the demand elasticity is set to the intermediate value \( \gamma = 0.5 \).

Moving from Panel A to B by decreasing the adjustment cost asymmetry \( \rho \), the follower trigger moves forward closer to the leader trigger, but has no strategic impact on the leader decision of when to exercise, consistent with the nature of the non-preemptive equilibrium. However, in Panel C, the follower trigger moves close enough to the leader trigger that the follower would have an incentive to strategically preempt the leader prior to its non-preemptive trigger. As a consequence, the leader must itself preempt the preemptive investment of the follower, by moving forward its trigger to \( X_{LP}^1 \). Finally, in Panel D the firms are sufficiently symmetric and the option value of waiting relative to acting as a leader sufficiently large that a simultaneous exercise equilibrium can be sustained.

The risk dynamics of the two firms in the leader-follower equilibria in Panels A-C differ markedly from the simultaneous exercise equilibrium in Panel D. For a leader-follower equilibrium, prior to the leader’s exercise the leader’s risk increases more steeply than the follower. Immediately upon the exercise of the leader growth option, the leader risk drops discretely and the follower risk jumps upwards, reversing the risk-ordering of the two firms. The two firms’ risk loadings continue to move apart until the follower growth option is exercised, and no growth options remain. By contrast, under simultaneous exercise in Panel D, the risk of both firms increases equally and always is above one until the exercise trigger and then drops to the level of the cash flow beta.

The dynamics of risk in a leader-follower equilibrium therefore differ dramatically from the simultaneous exercise case. Proposition 4 states that the region of existence of the simultaneous
equilibrium can be small depending on parameter values, and in many cases the simultaneous exercise equilibrium does not exist even for perfectly symmetric firms. These results imply that leader-follower equilibria are important and merit independent study. Our theoretical investigation shows that growth options have opposite effects on own firm and rival risk. Hence, the common practice of proxying for a firm’s risk using industry peer betas may not be appropriate when growth options are an important component of firm values, and this theoretical implication can be tested in future empirical work.

4.2 Equilibrium Exercise of Contraction Options

We now assume that each firm has a single contraction option. Capital adjustment costs are specified by

\[
\begin{align*}
\Lambda^1 &= \begin{bmatrix}
0 & -\infty & -\infty \\
S & 0 & -\infty \\
-\infty & -\infty & 0
\end{bmatrix}, \\
\Lambda^2 &= \begin{bmatrix}
0 & -\infty & -\infty \\
\rho S & 0 & -\infty \\
-\infty & -\infty & 0
\end{bmatrix},
\end{align*}
\]

where \(0 < \rho \leq 1\) so that firm 1 has the high and firm 2 the low salvage value. This implies that firm 1 has an incentive to contract earlier. Our interest again lies in equilibrium play of the two rivals, and we focus on pure strategy equilibria only. In contrast to the case of expansion options, leader-follower exercise is the unique equilibrium, following Murto (2004).

**Proposition 8.** For every \(0 < \rho < 1\) there exists a unique MPE of the contraction game in which the high salvage value firm acts as the leader with trigger \(X^1_C\) and the low salvage value firm acts as the follower with trigger \(X^2_C < X^1_C\). If \(\rho = 1\), there exist two pure strategy MPE, one in which firm 1 acts as the leader and firm 2 as the follower, and in the other firm 2 acts as the leader and firm 1 as the follower. No equilibrium exists with positive probability of simultaneous contraction.

We note that preemption does not play a role in contractions because any rival reduction in output increases rather than reduces firm value. Hence, for symmetric firms the follower value exceeds the leader value.

We again assume both firms initially have capacity \(\kappa_1\). The leader contracts first at the demand trigger \(X^1_C\), and the follower contracts at the trigger \(X^2_C < X^1_C\). We then show:
Proposition 9. The leader’s value function $V^1(K_t, X_t)$ is equal to

$$
\begin{cases}
R_{11}^1 X_t - \frac{f_1}{r} - \frac{f_1 - f_0 + r S}{r(1 - \nu_2)} \left( \frac{X_t}{X_C^1} \right)^{\nu_2} + \frac{X_C^1}{\delta} \left[ R_{00}^1 - R_{11}^1 \right] \left( \frac{X_t}{X_C^1} \right)^{\nu_2} & X_t > X_C^1, \\
R_{01}^1 X_t - \frac{f_0}{r} + \frac{X_C^1}{\delta} \left[ R_{00}^1 - R_{01}^1 \right] \left( \frac{X_t}{X_C^1} \right)^{\nu_2} & X_C^2 \leq X_t \leq X_C^1, \\
R_{00}^1 X_t - \frac{f_0}{r} & X_t < X_C^2,
\end{cases}
$$

and the followers value $V^2(K_t, X_t)$ is

$$
\begin{cases}
R_{11}^2 X_t - \frac{f_1}{r} - \frac{f_1 - f_0 + r S}{r(1 - \nu_2)} \left( \frac{X_t}{X_C^2} \right)^{\nu_2} + \frac{X_C^2}{\delta} \left[ R_{00}^2 - R_{11}^2 \right] \left( \frac{X_t}{X_C^2} \right)^{\nu_2} & X_t > X_C^1, \\
R_{01}^2 X_t - \frac{f_0}{r} + \frac{X_C^2}{\delta} \left[ R_{00}^2 - R_{01}^2 \right] \left( \frac{X_t}{X_C^2} \right)^{\nu_2} & X_C^2 \leq X_t \leq X_C^1, \\
R_{00}^2 X_t - \frac{f_0}{r} & X_t < X_C^2,
\end{cases}
$$

with the contraction triggers $X_C^1 > X_C^2 > 0$ given in the Appendix.

Rewriting the contraction triggers and substituting into the value functions gives

$$
V^1(K_t, X_t) = \underbrace{R_{11}^1 X_t - \frac{f_1}{r}}_{\text{assets in place}} + \underbrace{\frac{X_C^1}{\delta} \left[ R_{00}^1 - R_{11}^1 \right] \left( \frac{X_t}{X_C^1} \right)^{\nu_2}}_{\text{contraction option}} + \underbrace{\frac{X_C^1}{\delta} \left[ R_{00}^1 - R_{01}^1 \right] \left( \frac{X_t}{X_C^1} \right)^{\nu_2}}_{\text{value adjustment}}.
$$

and

$$
V^2(K_t, X_t) = \underbrace{R_{11}^2 X_t - \frac{f_1}{r}}_{\text{assets in place}} + \underbrace{\frac{X_C^2}{\delta} \left[ R_{00}^2 - R_{11}^2 \right] \left( \frac{X_t}{X_C^2} \right)^{\nu_2}}_{\text{contraction option}} + \underbrace{\frac{X_C^2}{\delta} \left[ R_{00}^2 - R_{01}^2 \right] \left( \frac{X_t}{X_C^2} \right)^{\nu_2}}_{\text{value adjustment}}.
$$

The value functions are composed of the growing perpetuity value of assets net of fixed costs assuming a constant industry structure, the perpetuity value of fixed costs, and the own-firm and rival option effects. The own-firm contraction option corresponds to a put and has positive value, consistent with the product of $\nu_2 < 0$ and $[R^1_{11} - R^1_{01}] < 0$. The rival value adjustment also has a positive value, consistent with the increased market price induced by lower industry output.
As in the expansion case, the value functions can be written as:

\[ V^i(K_t, X_t) = V^i_A(K_t, X_t) + V^i_C(K_t, X_t) + V^i_O(K_t, X_t) + V^i_C(K_t, X_t). \]

In contrast to expansion options, the rival effect for downsizing is positive \( V^i_C(K_t, X_t) > 0. \)

The risk dynamics of the two firms follows from the valuation equations.

**Proposition 10.** Systematic firm risks for both firms are

\[
\beta^i(t) = 1 + \frac{V^i_O(K_t, X_t) + V^i_C(K_t, X_t)}{V^i(K_t, X_t)}(\nu_2 - 1) + \frac{f_k/r}{V^i(K_t, X_t)},
\]

for all industry states \( K_t \), where \( \nu_2 < 0 \) and \( V^i_O(K_t, X_t), V^i_C(K_t, X_t) > 0. \)

As in the case of expansion options, the own-firm and rival values of contractions appear additively in the numerator of the second term, again implying that own and competitor contraction options have the same risk implications when normalized by dollar values. In contrast to the case of expansion options the signs of \( V^i_O \) and \( V^i_C \) are always positive, which combined with \( \nu_2 < 0 \) implies that contraction options, whether own or rival, always reduce risk.

Figure 6 summarizes the risk dynamics in equilibrium for contraction options. In Panel A the degree of salvage value asymmetry is large with \( \rho = 0.1 \), and in the remaining three panels \( \rho \) progressively increases until reaching \( \rho = 0.99999 \). As \( \rho \) increases and the follower salvage value increases, its contraction trigger moves closer to the leader’s. However, unlike in the expansion case, the increase in the follower trigger has no strategic impact on the leader’s exercise, which always occurs at the same level of demand. We also note that even in the case where the salvage value is almost one, the difference in the leader and follower triggers is discrete. This is because the exit of the leader raises the incentives of the follower to delay, so that the two triggers cannot occur arbitrarily close together.

The risk dynamics of the two firms in the contraction equilibrium differ, with each firm’s risk dropping faster prior to its own capacity reduction. However, consistent with Proposition 10, both contraction options reduce risk for both firms, and the firms’ risks always move in the same direction.
5 Robustness, Extensions, and Empirical Implications

The model we use in this paper is stylized in order to provide analytical tractability, and to focus on the critical drivers of product-market related risk dynamics. Many potential extensions of the model are computationally feasible using numerical methods, but should not change our predictions about the relationship between product market competition and risk dynamics. For example, incorporating variable costs of production would give rise to a role for operating flexibility. This could potentially dampen the magnitude of the risk dynamics we have shown, but would not qualitatively change our results. Similarly, alternative functional forms, such as using linear rather than isoelastic demand, would have quantitative but not qualitative implications.

One assumption that is critical to our results, however, is the presence of non-convexities in adjustment costs. If we had alternatively chosen a convex specification for adjustment costs, sequential investment and the leader-follower risk dynamics we have shown would disappear. Under convex adjustment costs, firms should always be instantaneously investing or disinvesting in small amounts, unlike the lumpy investment of our model which drives substantial time-series and cross-sectional differences in risk. We believe the specification of non-convex adjustment costs that we have chosen is particularly relevant for firms, given the significant evidence from Cooper and Haltiwanger (2006) of lumpy investment at the plant level.

The importance of non-convexities for our theoretical implications is consistent with the recent literature on R&D. Garlappi (2004) assumes continuous and infinitesimal research costs with the option to mothball at any time, and the risk implications and mechanism driving risk differ from our model. However, in a recent contribution Bena and Garlappi (2010) adopt a different patent race framework consistent with non-convexities, much closer to the assumptions of our model. In their model, as in ours, exogenously heterogeneous firms optimally time lumpy investments in order to gain better access to a product market. The resulting implications for risk in Bena and Garlappi’s model only when instantaneous exploration costs have a fixed component, which produces an effect like operating leverage. By contrast, our risk dynamics are driven by option leverage. Firms exercise real options by making lumpy investments, whereas the firms in Garlappi’s model can only make infinitesimal investments and nature drives the major changes in risk that are determined by random successes in R&D.

The empirical implications also differ. The first random success in Garlappi’s model identifies a leader whose risk is lower than the risk of the follower, which is similar to our model. However, at this point the leader in his model is both more likely to invest and more likely to have a random success. By contrast, in our model, the firm that invests next always has higher risk prior to investment.
model are also broadly similar to ours.\textsuperscript{18} Thus, non-convex costs, whether in standard capacity investments or in R&D outlays, robustly drive similar dynamics in risk for leaders and followers.

The empirical implications of our analysis are broad. In the presence of non-convex adjustment costs and imperfect competition, theory predicts:

- Own firm risk declines with the probability of rival firm expansion or contraction.
- During expansions, own-firm and rival risk tend to move oppositely; industry peer betas are poor proxies for own-firm risk.
- During contractions, own-firm and rival risk tend to move together; industry peer betas are good proxies for own-firm risk.

Importantly, empirical proxies for the drivers of risk in our model are readily available. For example, the predictions above can be tested by examining the dynamics of firm risk around own-firm and rival investment “spikes”, where spikes are defined as in Cooper and Haltiwanger (2006) by abnormally large increases in investment levels. We therefore anticipate future empirical work testing these predictions.

More broadly, this theory provides a novel potential explanation for “asymmetric correlations” in financial markets, wherein the returns of individual assets tend to be more correlated in falling than in rising markets (e.g., Kroner and Ng, 1998; Ang and Chen, 2002). Existing theories propose a variety of financial market imperfections that can explain increased correlation across broad asset classes after negative shocks (e.g., Allen and Gale, 2000; Yuan, 2005; Brunnermeier and Pedersen, 2009). By contrast, our theory predicts asymmetric correlations between individual assets as the natural outcome of product market interactions. Future empirical research may therefore seek to better distinguish the causes of asymmetric correlations across broad asset classes versus those observed in individual stocks within an industry.

6 Conclusion

We study risk dynamics in a duopoly where firms possess real options to expand or contract capacity, with adjustment costs that potentially differ across firms. Prior research derives required returns in

\textsuperscript{18}In particular, in their model as in ours, the risk of the leader falls after the leader’s investment while the risk of the follower rises, and as the follower’s investment threshold approaches, the leader’s risk declines while the follower’s risk increases.
a variety of product market settings: monopoly, where own-firm real options and operating leverage impact returns (Carlson, Fisher and Giammarino, 2004, 2006; Cooper 2006); perfect competition with identical firms, where only industry effects are present (Kogan, 2004); perfect competition with heterogeneous firms, where option values are zero and differences in cost structure drive expected returns (Zhang, 2005); and oligopoly, where both own and rival real options exist but their distinct impacts are not apparent due to simultaneous exercise (Aguerrevere, 2009). In the duopoly setting that we analyze, a variety of leader-follower equilibria exist in which a firm and its rival may exercise growth and contraction opportunities at separate times, allowing identification of the distinct risk impacts of own and competitor growth and contraction options. Non-simultaneous exercise can be the unique equilibrium outcome even when exogenous asymmetries are arbitrarily small or zero, and the risk dynamics that emerge in leader-follower equilibria differ substantially from simultaneous move benchmarks.

We find that a competitor’s options to adjust capacity, whether expansion or contraction, reduce own-firm risk through a simple hedging channel. Intuitively, product market improvements increase the probability of near-term rival expansion, which provides an offsetting decrease in own-firm value. Conversely, negative demand shocks induce competitor contraction, reducing the decline in own-firm value. As a consequence of the risk-reducing effect of competitor real options, own and rival risk tend to move together in contractions, but in opposite directions during expansions.

Financial analysts commonly estimate the required return of a product or corporation using not only the historical risk of the firm, but also its industry rivals, as recommended by standard corporate finance textbooks. Our results suggest that using industry peer betas to proxy for own-firm risk may work well in certain environments, but not in others, in particular where growth options are an important component of firm value. Our study thus highlights the importance of rival real options as an independent source of firm risk dynamics, and provides new theoretical predictions that can be tested in future empirical work.
Appendix

A Proofs

A.1 Proof of Proposition One

Standard arguments (e.g., Carlson, Fisher, and Giamarino, 2004), imply that demand dynamics under the risk neutral measure are

\[ dX_t = (r - \delta)X_t dt + \sigma X_t d\tilde{W}_t, \]  

where \( r > \delta \equiv \mu - g > 0 \). Following Dixit and Pindyck (1994) the continuation value of the flexible firm is obtained from the Bellman equation

\[ \frac{1}{2} \sigma^2 X^2 V_1^{X^2} + (r - \delta)X V_1^X - r V_1^X + X R_{11} - f_1 = 0 \]  

with the boundary conditions

\[
\begin{align*}
V_1(X_E) &= \frac{R_{21}^1 X_E}{\delta} - I - \frac{f_2}{r} \\
V_1(X_C) &= \frac{R_{01}^1 X_C}{\delta} + S - \frac{f_0}{r} \\
V_1^X(X_E) &= \frac{R_{21}^1}{\delta} \\
V_1^X(X_C) &= \frac{R_{01}^1}{\delta}.
\end{align*}
\]

The first two equations ensure value matching at the instants of expansion and contraction respectively. The last two equations guarantee smooth pasting which is a requirement for optimality.

The solution to this system of equations satisfies

\[ V_1(K_t, X_t) = \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + B_1^1 X_t^{\nu_1} + B_2^1 X_t^{\nu_2} \]

where \( B_1^1 \) and \( B_2^1 \) solve

\[
\begin{align*}
(1 - \nu_1) B_1^1 X_E^{\nu_1} + (1 - \nu_2) B_2^1 X_E^{\nu_2} &= -I - \frac{f_2 - f_1}{r}, \\
(1 - \nu_1) B_1^1 X_C^{\nu_1} + (1 - \nu_2) B_2^1 X_C^{\nu_2} &= S - \frac{f_0 - f_1}{r},
\end{align*}
\]

and the constants \( \nu_1 > 1 \) and \( \nu_2 < 0 \) are the positive and negative roots to the characteristic equation \( \sigma^2 \nu (\nu - 1)/2 + (r - \delta) \nu - r = 0 \), satisfying

\[
\nu_{1,2} = \frac{1}{2} \left( \frac{r - \delta}{\sigma^2} \right) \pm \sqrt{\left( \frac{1}{2} \frac{r - \delta}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}.
\]

For given positive values of \( X_E \) and \( X_C \) we observe that \( B_1^1, B_2^1 > 0 \). There is no convenient analytical solution for \( X_E \) and \( X_C \) due to nonlinearity of the system of equations, but the values \( X_E > X_C \) are easily found numerically.
A.2 Proof of Proposition Two

The continuation value of the inflexible firm is derived using $X_E$ and $X_C$ as exogenous barriers (see Dixit, 1993). The valuation equation satisfies

$$\frac{1}{2}\sigma^2 X^2 \frac{d^2 V}{dX^2} + (r - \delta) XV - rV^2 + X R_{11} - f_1 = 0,$$

with two boundary conditions that ensure value matching:

$$V^2(X_E) = \frac{R_{21}^2 X_E}{\delta} - \frac{f_1}{r},$$

$$V^2(X_C) = \frac{R_{01}^2 X_C}{\delta} - \frac{f_1}{r}.$$

Smooth pasting conditions are not needed because the inflexible firm does not choose the boundary levels $X_E$ and $X_C$ and hence these values are not optimizing to the inflexible firm value.

The value function satisfies

$$V^2(K_t, X_t) = \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + B_1^2 X_{t}^{\nu_1} + B_2^2 X_{t}^{\nu_2},$$

where $B_1^2$ and $B_2^2$ are the solutions to the equations

$$B_1^2 X_{E}^{\nu_1} + B_2^2 X_{E}^{\nu_2} = \frac{R_{21}^2 - R_{11}^2}{\delta} X_E,$$

$$B_1^2 X_{C}^{\nu_1} + B_2^2 X_{C}^{\nu_2} = \frac{R_{01}^2 - R_{11}^2}{\delta} X_C.$$

It is straightforward to observe that $B_1^2 < 0$ and $B_2^2 > 0$.

A.3 Proof of Proposition Three

Following the arguments in Carlson, Fisher, and Giammarino (2004) betas are given by

$$\beta^i(K_t, X_t) = \frac{\partial V^i(K, X)}{\partial X} \frac{X}{V(K, X)}.$$

Taking partial derivatives and substituting firm values from Propositions 1 and 2 into this expression gives the result.

A.4 Proof of Proposition Four

The arguments build on Pawlina and Kort (2006). We provide detailed logic to keep the proof self contained, and extend their proof along the following dimensions: (i) we permit operating leverage; (ii) we accommodate the case where $\rho = 1$ so that firms 1 and 2 are identical ex ante; and (iii) we provide formulas for the value functions for all $X \in (0, \infty)$ and all industry stages juvenile, adolescent, and mature.

The structure of the proof is to provide in Part 1 the value functions of firm 1 and 2 under different strategies:

\[\text{[See also the paper by Mason and Weeds (2008) in which non-preemptive, preemptive and simultaneous-move equilibria in a simple real option game are discussed.]}\]
i) Nonpreemptive leader-follower: firm 1 expands before firm 2 at a demand level that satisfies firm 1’s smooth pasting condition.

ii) Preemptive leader-follower: firm 1 expands before firm 2 but at a demand level forced by firm 2’s preemption threat. Firm 1’s exercise will not satisfy smooth pasting because the exercise is not at an unconstrained optimal level.

iii) Simultaneous move: both firms expand simultaneously.

iv) Off-equilibrium: firm 2 leads.

Following Part 1, in Parts 2-4, we provide conditions under which equilibrium holds.

**Part 1: Value function calculations**

i) Non-preemptive leader-follower: The value of firm 1 satisfies

\[
\frac{1}{2} \sigma^2 X^2 V_\lambda^1 + (r - \delta) X V_\lambda^1 - r V^1 - X R_{11}^1 - f_1 = 0,
\]

with the boundary conditions

\[
V^1(X_L^1) = \frac{R_{11}^1 X_L^1}{\delta} - \frac{f_2}{r} - I + B (X_L^1)^{\nu_1}
\]

\[
V^1(X_F^2) = \frac{R_{22} X_F^2}{\delta} - \frac{f_2}{r}
\]

\[
V^1(X_L^1) = \frac{R_{11}^1}{\delta} + B \nu_1 (X_L^1)^{\nu_1-1}
\]

where \(X_L^1\) is the exercise trigger optimally set by firm 1, \(X_F^2 > X_L^1\) is the trigger level when the follower exercises the option, and the constant \(B\) is calculated through backward induction using the follower’s trigger. The solution is

\[
V^1_L(X) = \frac{R_{11}^1 X_L}{\delta} - \frac{f_1}{r} + \frac{(R_{21}^1 - R_{11}^1) X_L}{\delta \nu_1} \left( \frac{X}{X_L} \right)^{\nu_1}
\]

\[
+ \frac{(R_{22}^1 - R_{21}^1) X_F^2}{\delta} \left( \frac{X}{X_F^2} \right)^{\nu_1},
\]

where

\[
X_L^1 = X_{LN}^1 = \frac{\nu_1}{\nu_1 - 1} \frac{\delta (f_2 - f_1 + r I)}{r (R_{11}^1 - R_{21}^1)}.
\]

The value of firm 2 satisfies:

\[
\frac{1}{2} \sigma^2 X^2 V_\lambda^2 + (r - \delta) X V_\lambda^2 - r V^2 - X R_{11}^2 - f_1 = 0,
\]

with the boundary conditions

\[
V^2(X_L^1) = \frac{R_{21}^2 X_L^1}{\delta} - \frac{f_1}{r} + C (X_L^1)^{\nu_1}
\]

\[
V^2(X_F^2) = \frac{R_{22} X_F^2}{\delta} - \frac{f_2}{r} - \rho I
\]

\[
V^2_X(X_F^2) = \frac{R_{22} X_F^2}{\delta},
\]
where $C$ is a constant. We obtain

$$V_f^2(X) = \frac{R_{11}^2 X_f}{\delta} - \frac{f_1}{r} + \frac{(R_{22}^2 - R_{11}^2) X_f^2}{\delta} \left( \frac{X}{X_f} \right)^{\nu_1}$$

where

$$X_f^2 = \frac{\nu_1}{\nu_1 - 1} \frac{\delta (f_2 - f_1 + r p I)}{r R_{22}^2 - R_{21}^2}.$$  

$$\text{(21)}$$

\[ ii) \text{ Preemptive leader follower:} \] In a preemptive equilibrium, firm 1 chooses a trigger $X_f^1$ that does not satisfy the smooth pasting condition (14). The firm value is

$$V_f^1(X) = \frac{R_{11}^1 X}{\delta} - \frac{f_1}{r} + \frac{(R_{22}^1 - R_{11}^1) X_f^2}{\delta} \left( \frac{X}{X_f^1} \right)^{\nu_1}$$

$$+ \left[ \frac{(R_{22}^1 - R_{11}^1) X_f^1}{\delta} - \frac{f_2 - f_1 + r I}{r} \right] \left( \frac{X}{X_f^1} \right)^{\nu_1}.$$  

$$\text{(22)}$$

\[ iii) \text{ Simultaneous move:} \] If both firms exercise simultaneously at the trigger level $X_S^i$ the value functions satisfy

$$V_S^i(X) = \frac{R_{11}^i X}{\delta} - \frac{f_1}{r} + \frac{(f_2 - f_1 + r I^i)}{r (\nu_1 - 1)} \left( \frac{X}{X_S^i} \right)^{\nu_1}$$

with the trigger levels

$$X_S^i = \frac{\nu_1}{\nu_1 - 1} \frac{\delta (f_2 - f_1 + r I^i)}{r R_{22}^i - R_{11}^i}.$$  

$$\text{(23)}$$

\[ iv) \text{ Off-equilibrium, firm 2 leads:} \] If firm 2 does not act as the follower and instead exercises as the leader its value function is

$$V_L^2(X) = \frac{R_{11}^2 X}{\delta} - \frac{f_1}{r} + \frac{(R_{22}^2 - R_{11}^2) X_f^2}{\delta} \left( \frac{X}{X_L^2} \right)^{\nu_1}$$

$$+ \left( \frac{R_{22}^2 - R_{12}^2}{\delta} X_f^2 \right) \left( \frac{X}{X_L^2} \right)^{\nu_1}.$$  

$$\text{(24)}$$

$$\text{(25)}$$

with $X_L^2$ as the leader’s trigger for firm 2 and $X_f^1$ the follower’s trigger for firm 1.

\[ \text{Part 2: Conditions for preemptive and non-preemptive equilibria:} \] In a preemptive equilibrium, firm 1 expands at the instant when firm 2 is indifferent between acting as the leader or the follower. The difference between firm 2’s value with immediate exercise and its value as a follower is given by:

$$G(X, \rho) \equiv \frac{(R_{12}^2 - R_{22}^2) X}{\delta} - \frac{(f_2 - f_1 + r p I)}{r} + \frac{(R_{22}^2 - R_{12}^2) X_f^1}{\delta} \left( \frac{X}{X_f^1} \right)^{\nu_1}$$

$$\text{(26)}$$

$$\text{(27)}$$

This function is strictly concave in $X$, since $\nu_1 > 1$, $(R_{22}^2 - R_{12}^2) < 0$, and $(R_{22}^1 - R_{21}^1) > 0$. It therefore has a unique maximum which is characterized by $\partial G(X, \rho)/\partial X = 0$. Denoting this value $\bar{X}$, the function $G$ has zero, one, or two real roots, depending on whether $G(\bar{X}, \rho) < 0$, $G(\bar{X}, \rho) = 0$, or $G(\bar{X}, \rho) > 0$.  

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or $G(\hat{X}, \rho) > 0$, respectively. In addition, $G$ can be seen to be strictly decreasing in $\rho$ by substituting $X_F$ from equation (22) into (27).

Let the couple $(X^*, \rho^*)$ solve the system of equations

$$\begin{align*}
G(X^*, \rho^*) &= 0, \quad (28) \\
\frac{\partial G(X^*, \rho^*)}{\partial X} &= 0. \quad (29)
\end{align*}$$

For any $\rho \geq \rho^*$ the follower does not have an incentive to become the leader, since there are no real solutions to $G(X, \rho) = 0$, and in equilibrium firm 1 acts as the leader and firm 2 acts as the follower. For $\rho < \rho^*$ firm 2 has an incentive to become the leader. This incentive exists for all values of $X$ in the interval $[X_{1LP}^1, X_F^2]$, where $X_{1LP}^1$ is the smallest solution to $G(X, \rho) = 0$. If the leader’s investment trigger satisfies $X_{1}^1 < X_{1LP}^1$ the follower value of firm 2 exceeds its leader value and firm 2 does not have an incentive to change its follower role. If, however, $X_{1}^1 > X_{1LP}^1$ the follower has an incentive to preempt the leader, which in turn causes the leader to choose $X_{1LP}^1$ as the investment trigger.

**Part 3: Conditions for simultaneous equilibrium:** Since $I^1 = I < \rho I = I^2$ the only candidate for a simultaneous equilibrium is trigger level $X_{1LP}^1 < X_{2LP}^2$. In a simultaneous equilibrium: (i) the value of firm 1 being the leader has to be smaller than moving simultaneously with firm 2, and (ii) firm 2 has to find it profitable to move simultaneously with firm 1 and not to wait and act as the follower. The difference between the firm 1 leader value assuming immediate exercise and the value from waiting for simultaneous exercise is given by

$$\Delta(X, \rho) = \frac{(R_{21} - R_{11})}{\delta}X + \frac{f_2 - f_1 + rI}{r} + \frac{(R_{22} - R_{21})}{\delta}X_F^2 \left( \frac{X}{X_F^2} \right)^{\nu_1} + \frac{(R_{12} - R_{11})}{\delta\nu_1} \frac{X_{1LP}^1}{X_S} \left( \frac{X}{X_S} \right)^{\nu_1}. \quad (30)$$

This function is strictly concave in $X$, since $\nu_1 > 1$, $(R_{22} - R_{21}) < 0$, and $(R_{12} - R_{11}) > 0$. It therefore has a unique maximum which is characterized by $\partial \Delta(X, \rho)/\partial X = 0$. In addition, $\Delta$ can be seen to be strictly decreasing in $\rho$ by substituting $X_F^2$ from equation (22) into (30).

Following the logic that was used in Part 2 above, it can be shown that there exists $X^{**}$ and $\rho^{**}$ such that $\Delta(X^{**}, \rho^{**}) = 0$ and $\Delta_X(X^{**}, \rho^{**}) = 0$, implying that for all $\rho < \rho^{**}$ firm 1 prefers the simultaneous exercise equilibrium. (If $\rho^{**} < 1$, a simultaneous equilibrium is not possible.) Comparing equations (22) and (25), $R_{21} < R_{11}$ implies that $X_F^2 < X_{2LP}^2$, thus if the conditions for optimal simultaneous exercise are satisfied for firm 1, they are also satisfied for firm 2.

**Part 4: Random leader follower equilibrium:** If $\rho = 1$ and $\rho^{**} < 1$, none of the pure strategy equilibria above are possible. To define a mixed strategy equilibrium requires an expansion of the strategy space following Fudenberg and Tirole (1985) and Huisman and Kort (1999). In such a mixed strategy equilibrium the leader is chosen randomly at the trigger level $X_{1LP}^1$ while the other firm acts as the follower.

**A.5 Proof of Proposition Five**

The leader’s value function in case of a non-preemptive equilibrium is given by (15) while in case of a preemptive equilibrium when the expansion trigger does not satisfy the smooth pasting condition it is given by (23). In both cases the follower’s value function is given by (21).
A.6 Proof of Proposition Six
In the case where both firms simultaneously invest their value functions are given by (24).

A.7 Proof of Proposition Seven
Follows immediately from the firms’ value functions and the definition of beta.

A.8 Proof of Proposition Eight
The value function of firm 2 acting as a follower is
\[ V^2_F(K_t, X_t) = \begin{cases} 
\frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{\rho} + \frac{f_1 - f_0 + \rho S}{\delta(1 - \nu_2)} \left( \frac{X_t}{X_C^2} \right)^{\nu_2} & X_t \geq X_C^2 \\
\frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{\rho} + \rho S & X_t \leq X_C^2,
\end{cases} \]
where
\[ X_C^2 = \frac{\nu_2 \delta (f_1 - f_0 + \rho S)}{(1 - \nu_2) r [R_{00}^2 - R_{01}^2]} \]
is the contraction trigger when firm 2 acts as the follower. Now assume, instead, that firm 2 having the smaller salvage value acts as the leader. The value function of firm 2 at the time when it contracts as the leader is
\[ V^2_L(K_t, X_t) = \begin{cases} 
\frac{R_{10}^1 X_t}{\delta} - \frac{f_0}{\rho} + \frac{X_C^2 \rho |R_{00}^1 - R_{10}^1|}{\delta} \left( \frac{X_t}{X_C^1} \right)^{\nu_2} & X_t \geq X_C^1 \\
\frac{R_{10}^1 X_t}{\delta} - \frac{f_0}{\rho} + \rho S & X_t \leq X_C^1,
\end{cases} \]
where
\[ X_C^1 = \frac{\nu_2 \delta (f_1 - f_0 + r S)}{(1 - \nu_2) r [R_{00}^1 - R_{10}^1]} \]
is the trigger when firm 1 contracts as the follower and firm 2 acts as the leader. Given our assumptions on the revenue functions it follows that trigger (34) is strictly greater than trigger (32) for \(0 < \rho < 1\). From this property and the value functions (31) and (33) it can be shown that
\[ G(X_t, \rho) \equiv V^2_L(X_t, \rho) - V^2_F(X_t, \rho) \leq 0 \]
holds for all \(X_t\). Hence, firm 2 never has an incentive to become the leader. Therefore sequential exercise of contraction options is the unique pure strategy MPE \(\rho < 1\). Sequential exercise of firms from the industry has been shown in the paper by Murto (2004) for asymmetric firms, as well. His model differs from ours, however, since firms have to pay a cost when exiting the market that is smaller then the current fixed costs instead of receiving a positive salvage value. In case of symmetric firms, \(\rho = 1\), Proposition 2 in Murto (2004) can be applied which establishes the existence of two pure strategy MPE.

A.9 Proof of Proposition Nine
The derivation of the leader and the follower value functions follows the same arguments used in Proposition 4 with the difference that instead of a call the contraction option corresponds to a put option. The leader’s value function satisfies
\[ \frac{1}{2} \sigma^2 X^2 V^1_L + (r - \delta) X V^1_L - r V^1 + X R_{11}^1 - f_1 = 0, \]
with the boundary conditions

\[
V^1(X^1_C) = \frac{R_{01}X^1_C}{\delta} - \frac{f_0}{r} + S + B(X^1_C)^{\nu_2},
\]
\[
V^2(X^2_C) = \frac{R_{01}X^2_C}{\delta} + B\nu_2 (X^1_C)^{\nu_2-1},
\]
\[
V^1(X^1_C) = \frac{R_{00}X^2_C}{\delta} - \frac{f_0}{r},
\]
resulting in the value function stated in the proposition. The leader’s contraction trigger is given by

\[
X^1_C = \frac{\nu_2 \delta(f_1 - f_0 + rS)}{(1 - \nu_2)r [R_{01} - R_{11}]}.
\]

The follower’s value function satisfies

\[
\frac{1}{2} \sigma^2 X^2 V^2_{XX} + (r - \delta)XV^2_X - rV^2 + XR^2_{11} - f_1 = 0,
\]

with the boundary conditions

\[
V^2(X^2_C) = \frac{R_{00}X^2_C}{\delta} - \frac{f_0}{r} + \rho S,
\]
\[
V^2(X^2_C) = \frac{R_{00}X^2_C}{\delta},
\]
\[
V^2(X^1_C) = \frac{R_{01}X^1_C}{\delta} - \frac{f_1}{r} + D(X^1_C)^{\nu_2},
\]
that results in the value function stated in the proposition. The follower’s trigger is given by

\[
X^2_C = \frac{\nu_2 \delta(f_1 - f_0 + r\rho S)}{(1 - \nu_2)r [R_{00}^2 - R_{01}^2]}.
\]

A.10 Proof of Proposition Ten

Follows immediately by using the value functions stated in Proposition 9 and the definition of \(\beta\).
References


Figure 1: The hedging effect of a rival real option. The figure illustrates price responses to demand shocks with and without an expansion response by a rival. With current industry output fixed at $Q_1$ an upward shift in demand from $D$ to $D'$ will result in an increase in the product price from $P_1$ to $P^*$. If demand crosses the threshold for investment by the rival, an increase in industry output to $Q_2$ dampens the price response, so that prices increase only to $P_2$. 
Figure 2: Risk dynamics of the flexible and inflexible firm. This figure represents the relationship between the level of the demand state $X_t$ and the systematic risk $\beta$ of firms within an industry. The solid curve illustrates risk dynamics for the flexible firm. Flexible firm risk derives from assets in place, a contraction option, and an expansion option when $X_c < X_t < X_e$ and from assets in place alone otherwise. The risk-reducing effect of the contraction option is largest (smallest) and the risk-increasing effect of the expansion option is smallest (largest) when $X_t = X_c$ ($X_t = X_e$). Exercising either option causes a discontinuous change in risk to $\beta = 1$. The dashed curve illustrates risk dynamics for the inflexible firm. Rival options unambiguously decrease inflexible firm risk, so that the inflexible firm’s beta is relatively low at both of the critical points $X_c$ and $X_e$. The parameter values are $\gamma = 0.5, \sigma = 0.2, q_0 = 1, q_1 = 2, q_2 = 10, f_0 = f_1 = f_2 = 0, r_f = 0.05, \delta = 0.03, S = 50$, and $I = 500$. 
Figure 3: Timeline of the duopoly investment game. This figure illustrates the endogenous timing of industry expansions in a leader-follower equilibrium.
Figure 4: Equilibrium regions in the expansion game. This figure shows how inverse demand elasticity $\gamma$ and investment cost differentials $\rho$ relate to the equilibrium predicted by the expansion game. The simultaneous-move equilibrium occurs in the region $1 \leq \rho \leq \rho^{**}$, the preemptive leader-follower equilibrium exists in the region $\max\{\rho^{**},1\} < \rho \leq \rho^*$, and the non-preemptive leader-follower equilibrium exists in the region $\rho \geq \max\{\rho^*,\rho^{**}\}$. The random-leader equilibrium is the only possibility when $\rho = 1$ and $\rho^{**} < 1$, indicated by asterisks. The parameter values used for the figure are $\sigma = 0.2$, $q_1 = 2$, $q_2 = 10$, $f_1 = f_2 = 0$, $r_f = 0.05$, $\delta = 0.03$, and $I = 500$. 

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$\gamma$

$\rho$

simultaneous-move

preemptive leader-follower

non-preemptive leader-follower

random-leader
Figure 5: Risk dynamics in expansions. This figure illustrates leader (dashed line) and follower (solid line) risk dynamics in the expansion game. In Panels A and B the equilibrium is non-preemptive leader-follower, in Panel C it is preemptive leader-follower, and in Panel D it is simultaneous. In Panels A-C, leader risk is above (below) follower risk prior to (following) leader expansion. Leader and follower firm risk dynamics are identical only in the simultaneous equilibrium of Panel D. The parameter values are $\gamma = 0.5$, $\sigma = 0.2$, $q_1 = 2$, $q_2 = 10$, $f_1 = f_2 = 0$, $r_f = 0.05$, $\delta = 0.03$, and $I = 500$. 
Figure 6: Risk dynamics in contractions. This figure illustrates leader (dashed line) and follower (solid line) risk dynamics in the contraction game for various values of $\rho$. Leader risk is below (above) follower risk prior to (following) leader contraction. The parameter values are $\gamma = 0.5$, $\sigma = 0.2$, $q_0 = 1$, $q_1 = 2$, $f_0 = f_1 = 0$, $r_f = 0.05$, $\delta = 0.03$, and $S = 50$. 