MARKETING SCIENCE
Vol. 29, No. 4, July–August 2010, pp. 650–670
ISSN 0732-2399 | EISSN 1526-548X | 10 | 2904 | 0650

informs

DOI 10.1287/mksc.1090.0546

Limited Memory, Categorization, and Competition

Yuxin Chen
Kellogg School of Management, Northwestern University, Evanston, Illinois 60208,
yuxin-chen@kellogg.northwestern.edu

Ganesh Iyer
Haas School of Business, University of California at Berkeley, Berkeley, California 94720,
giyer@haas.berkeley.edu

Amit Pazgal
Jesse H. Jones Graduate School of Business, Rice University, Houston, Texas 77005, pazgal@rice.edu

This paper investigates the effects of a limited consumer memory on the price competition between firms. It studies a specific aspect of memory—namely, the categorization of available price information that the consumers may need to recall for decision making. This paper analyzes competition between firms in a market with uninformed consumers who do not compare prices, informed consumers who compare prices but with limited memory, and informed consumers who have perfect memory. Consumers, aware of their memory limitations, choose how to encode the prices into categories, whereas firms take the limitations of consumers into account in choosing their pricing strategies. Two distinct types of categorization processes are investigated: (1) a symmetric one in which consumers compare only the labels of price categories from the competing firms and (2) an asymmetric one in which consumers compare the recalled price of one firm with the actual price of the other. We find that the equilibrium partition for the consumers calls for finer categorization toward the bottom of the price distribution. Thus consumers have a motivation to invest in greater memory resources in encoding lower prices to induce firms to charge more favorable prices. The interaction between the categorization strategies of the consumers and the price competition between the firms is such that small initial improvements in recall move the market outcomes quickly toward the case of perfect recall. Even with few memory categories, the expected price consumers pay and their surplus is close to the case of perfect recall. There is thus a suggestion in this model that market competition adjusts to the memory limitations of consumers.

Key words: bounded rationality; memory; behavioral economics; categorization; competitive strategy; pricing strategy
History: Received: November 19, 2008; accepted: September 30, 2009; processed by Miklos Sarvary. Published online in *Articles in Advance* December 30, 2009.

1. Introduction

A common and implicit assumption in the literature on imperfect price competition is that consumers who compare prices across different firms are able to perfectly recall all the prices they encountered and use them in their decision making. However, there is a substantial body of psychological research that examines the effect of memory limitations on consumer choice among available alternatives.[1] Limitations on short-term memory mean that consumers would not be able to perfectly recall relevant price information, and consumers are more likely to face greater short-term memory constraints in environments with higher levels of information. Although imperfect short-term memory on prices is well documented for consumers buying routinely purchased products or products with low involvement (Dickson and Sawyer 1990, Monroe and Lee 1999), such limitations would also be significant in complex product markets where consumer comparisons across firms are based on the recall of not just the posted numerical prices but on the full price that includes other associated monetary aspects such as payment terms, financing, delivery, warranty offers, or other optional features. Faced with memory constraints, consumers make decisions using heuristics that help them to form suitable price impressions.

An important heuristic to deal with the abundance of information is the *grouping* of events, objects, or numbers into categories on the basis of their perceived similarities (see Rosch and Mervis 1975). Accordingly, memory limitations in our model pertain to the categorization scheme that consumers use to encode and recall prices. The purpose of this paper is then to highlight the effect of a specific aspect of limited consumer memory on the pricing strategies

[1] See Alba et al. (1991) for a survey of the information processing literature that examines the effect of memory on consumer choice.

of competing firms—namely, the fact that consumers are unable to recall the exact prices encountered and instead recall only the categories to which the prices belong. Specifically, we develop a model of market competition between firms that face consumers who compare prices but have the ability to recall previously encountered market prices only as categories.

This paper highlights the following aspects of memory limitations: First, we characterize memory limitations as the ability to recall only the category of a previously observed price rather than the actual price realization. Second, we take the analysis a step further by embedding consumers with limited memory in a market setting involving price competition between firms. In equilibrium, the pricing strategies of the firms take into account the memory limitations of consumers. Finally, following Dow (1991), we also consider the problem of consumers who have limited memory but who are aware of this limitation and take it into account to make the best possible decision. In other words, we explicitly consider the "encoding" decision of consumers given the information environment.[2] In doing so, we provide a framework to investigate how memory limitations are affected by and, in turn, influence the market equilibrium.

Consumers in our model can only use some limited number of categories to construct their optimal memory structure of prices charged by the firms. Consider a consumer visiting a store and classifying a product's price as "expensive" or "inexpensive." When the same consumer compares the price to that at a different store, she can only recall that the price in the first store was "high" or "low" but not its exact value. Knowing that she will only remember the category, the consumer will optimally choose the threshold that partitions the prices at the first store into the high or low categories. Firms that compete in the market know that consumers can only remember if a product was expensive or not and take that into account when making their pricing decisions.

The model consists of a duopoly market in which firms compete for consumers who have limited memory. A limited-memory consumer compares the prices of the firms before purchase. The consumer does not recall the actual prices at the firm but instead divides the price distribution into categories and recalls the category that contains the actual price. We consider the effects of two distinct types of categorization processes to represent memory limitations, which are paramorphic representations of the price recall problems that consumers face in actual markets. First, we consider a symmetric categorization process in which the limited-memory consumers compare the category labels of the prices of the firms. This represents a situation in which consumers can only compare imperfect impressions of the total prices offered by both firms. Next, we consider an asymmetric categorization process in which consumers compare the actual price realization at one firm to the recalled category of the price at the other firm. This process is akin to Dow (1991) and highlights the value of remembering price information. A consumer with limited memory contacts both firms in sequence to compare their prices. The consumer observes the actual price charged at the final firm and compares it to the expected price of the recalled category of the first firm.[3] We consider heterogeneity by allowing for several segments of consumers that differ in their memory recall abilities. Besides the limited-memory consumers, we consider a group of uninformed or loyal consumers for whom memory limitations do not matter because they consider shopping only at their favorite firm (as long as the offered prices are below their common reservation price), as well as a group of informed consumers with perfect memory for prices. Both the uninformed and the informed consumers with perfect memory are as in Varian (1980) or Narasimhan (1988).

We find the Nash equilibrium of firms' pricing strategies and the surplus maximizing categorization strategy for limited-memory consumers. Several key results about the effect of limited memory are remarkably consistent across the categorization processes and market conditions. For both the symmetric and the asymmetric categorization processes, we find that the equilibrium categorization structure for the consumers calls for finer categorization toward the bottom of the price distribution. This implies that consumers should devote more memory resources to encoding lower prices to induce firms to charge more favorable prices. The interaction between the categorization strategies of the consumers and the price competition between the firms is such that small initial improvements in recall move the market outcomes quickly toward the case of perfect recall. Thus, even with few memory categories, the expected price consumers pay and their surplus are close to the case of consumers having a perfect recall. There is thus a suggestion in this model that market competition compensates for the imperfect recall of consumers.

We show that the presence of limited-memory consumers along with the perfect-memory consumers can

[2] Thus the model is in the spirit of bounded rationality as defined in Simon (1987), where the decision maker makes a rational choice that takes into account the cognitive limitation of the decision maker.

[3] As in Dow (1991), this process is not to be interpreted as one in which consumers search across the firms. However, in this paper, we also consider the case in which the decision process of the limited-memory consumers involves the initial decision of whether to compare prices at all, after observing the price at the first firm.

soften price competition between the firms. In the asymmetric categorization case, equilibrium profits are higher in a market with both limited-memory and perfect-memory consumers (for a given number of uninformed consumers) than in a market in which all the informed consumers either have perfect memory or have limited memory. In the symmetric categorization case, for a given level of uninformed consumers, the increased presence of limited-memory consumers among the informed consumers leads to greater equilibrium profits. Finally, across the different categorization processes, the expected profits of the firms always increase when there are a greater number of uninformed consumers.

Comparison across the two types of categorization processes reveals some interesting results. Because in the asymmetric categorization process consumers compare a price conditional on a recalled category to an actual price, consumers in this case have better price information than in the symmetric categorization case. Consequently, the asymmetric categorization process may create a more competitive and undifferentiated environment. Consistent with this intuition, we find that the asymmetric categorization process, in general, leads to lower equilibrium profits. We also investigate the robustness of our results to different representations of the category within the asymmetric categorization case. The key results of this paper are robust to different endogenous representations of the category such as the median (rather than the mean), as well as exogenous representations such as the top or the bottom of the category. Indeed, we establish that the model where consumers remember the top of the category in the asymmetric categorization model is equivalent to the symmetric categorization model where consumers compare the price labels from both firms. Interestingly, when consumers use the mean of the category, which is endogenous to the equilibrium firm actions, their equilibrium surplus is actually higher than when consumers use exogenous rules such as the bottom or the top of the category. Thus, if consumers in a market were to learn (for example, through experience over time) to do the best for themselves and were motivated to make the best possible purchase decisions, it would be optimal for them to recall the category mean price rather than to use any exogenous rule. We also extend the asymmetric categorization model to the case in which consumers choose whether to obtain prices from both firms. We show that all the key equilibrium results pertaining to firm pricing and the categorization by consumers are also robust to this extension.

1.1. Related Research

A useful way of modeling bounded rationality in the literature has been to enforce limitations on the information processing of the decision maker. The decision maker cannot perfectly convert the inputs she receives to the optimal outputs she needs to choose. Our paper can be seen as related to a class of such models that involve the specific modeling of partitions or categories. The consumer receives as an input a signal (e.g., prices) that she cannot perfectly recall. The consumer partitions the entire set of potential signals and classifies the one received into a partition. The action taken by the consumer is identical for all signals that fall within a category. In the literature, exogenously given partitions are the most common. Along these lines there is research in game theory that models the use of finite automata mechanisms in repeated games (see Kalai 1991). In contrast to the literature on exogenous partitions, Dow (1991) investigates the question of the optimal choice of the partitions by consumers. However, in Dow, consumers face exogenously fixed price distributions. In our paper, the price distributions are a result of the competitive market equilibrium. In addition, to examine the case that the partitions are exogenously given, we are also interested in the optimal choice of the partition structure by consumers who face price distributions offered by firms in a market equilibrium. Rubinstein (1993) analyzes a model in which a firm price discriminates between consumers who have different memory capacities by choosing a random lottery of prices, which consumers categorize. In Rubinstein's paper, the prices are a result of the choice of a monopoly firm rather than the result of competition between firms.

Our paper is also related to the recent literature on the effect of bounded rationality on market interaction. For example, Basu (2006) characterizes the equilibria of pricing competition between firms that face consumers that do not try to remember the full prices they observe but rather round them to the nearest dollar. He shows that in equilibria (some) firms will always use prices ending in "9," which may be seen as equivalent to pricing at the top of the category in our model. Spiegler (2006) develops a model in which firms are fully rational as in this paper but where consumers are limited in their ability to process information about firm characteristics and form boundedly rational expectations about the firm.[4] Competitive firms in such markets choose actions that obfuscate consumers leading to a loss of consumer welfare. Camerer et al. (2004) present an approach to

[4] Another recent paper by Iyer and Kuksov (2010) examines the role of consumer feelings in quality evaluations and the supply of quality by firms. Consumers are not able to separate the effect of environmental variables from the true quality offered by firms, but they rationally try to infer the true equilibrium quality from their quality perception.

modeling bounded rationality in the form of cognitive limitations of players that leads to lack of rational expectations of the beliefs that agents have about other agents.

In addition, this paper is related to the recent research direction that examines how common biases in consumer decision making affect strategic interaction. For example, Amaldoss et al. (2008) show that the psychological bias called the asymmetric dominance effect may facilitate coordination in games. Another recent example of psychological biases in strategic decision making is Lim and Ho (2007), who study the effect of counterfactual thinking-related payoffs when retailers in a channel are faced with multiblock tariffs.

A different approach to modeling imperfect recall involves modeling the impact of past decisions taken by a consumer on current decision faced by the same individual when the memory recall of the past is limited and consumers cannot recall the exact decision taken (Hirshleifer and Welch 2002, Ofek et al. 2007). Along similar lines, Mullainathan (2002) models some specific psychological aspects of human memory for economic behavior, namely, rehearsal and associativeness, and in doing so provides a structure to understand when individuals will under- or overreact to news.[5] The aim of these papers then is to use memory loss as a basis to explain psychological biases such as inertia in decision making. In contrast, our paper focuses on a specific aspect of memory limitations—namely, the ability of consumers to recall information as categories and investigates its impact on the competitive market equilibrium.

The rest of this paper is organized as follows. Section 2 presents and analyzes the model with symmetric categorization process, and §3 analyzes the model with asymmetric categorization process. In §4, we discuss the robustness of our results across different categorization schemes and consumer decision processes, as well as the external validity of our results. Finally, §5 concludes and provides a summary.

2. The Model

We start by analyzing the simplest possible model of limited memory and categorization to highlight their joint effects on firm competition. Consider two symmetric competing firms indexed by j (where $j = 1, 2$) selling a homogenous product. Let the marginal cost of production of each firm be constant and assume it is equal to zero. The market is comprised of a unit mass of consumers with each consumer requiring at most one unit of the product. Consumers have a common reservation price, which is normalized to one without loss of generality. The consumers would like to purchase the lowest-priced good and thus compare the prices offered by both firms prior to making any purchase decision. However, consumers' recall of market prices are imperfect in that they cannot remember the exact prices they encountered but only the category to which they belong. Specifically, assume that the consumers have limited memory in the sense that they are endowed with $n+1$ memory categories, which they use to encode the price information. Consumers encode all prices as long as they are at or below their reservation price and reject any price that is above and opt out of the market. The entire range of price information is thus divided into $n+1$ mutually exclusive and exhaustive categories or partitions such that any observed market price is classified into one (and only one) category. When making their purchasing decisions, consumers cannot recall the exact prices offered by the firms, but only the category to which the prices belong. Consumers have memory limitations but are aware of that fact and act optimally given their memory constraints.

Explicitly, let the set of memory categories be $\{C_i\}_{i=1}^{n+1}$ and assume without loss of generality that they are indexed in increasing order of prices ($j > k$ implies that $p > q$ for every $p \in C_j$, $q \in C_k$). Denote the set of cutoff prices as $\{k_i\}_{i=1}^{n}$, where k_i separates category i from category $i+1$. From our previous assumption on the order of the categories, we have that $k_1 < k_2 < \cdots < k_n$. Because the categories are exhaustive and mutually exclusive, k_i belongs to one and only one of them; we assume that each category is a set that is open to the left, which implies $k_i \in C_i$ for $i = 1, \ldots, n$. Therefore $k_{i-1} < p \le k_i \Rightarrow p \in C_i$. Finally, for the sake of completeness, we define k_{n+1} and k_0 to be the highest and lowest possible prices charged by a firm, respectively (clearly, $k_{n+1} = 1$). Therefore the category C_i is defined as the set of prices $(k_{i-1}, k_i]$ for $i = 1, 2, \ldots, n+1$.

Suppose $n = 1$; then one might think of consumers partitioning the observed prices into an "inexpensive" or an "expensive" category, with k_1 being the price above which a product is considered to be expensive. Bounded rationality on the part of the consumers implies that the division into inexpensive and expensive categories is chosen to help the consumers deal with their limitations in recalling the exact price information and aid in making the optimal purchase decision.

[5] The literature on dynamic models of learning also captures the recall aspects through modeling the forgetting of information (see Camerer and Ho 1999). On the empirical side, Mehta et al. (2004) develop a structural econometric model to analyze the role of imperfect recall and the forgetting of consumers for choice decisions of frequently purchased products. Their aim is to characterize the role of the forgetting over time on the brand choice decisions of consumers.

Consumers form the categorization scheme by choosing the set of cutoff prices between adjacent categories in a way that maximizes their expected surplus. Because consumers are aware of their limitations regarding the recall of prices, they will strive to devise the optimal system (categorizations) to deal with it. Thus the model represents agents whose cognitive capacities are *bounded* but who nevertheless are *rational* in recognizing this limitation and accounting for it in making the optimal decision. This is exactly in the spirit of bounded rationality as defined by Simon (1987), where the consumer makes a rational choice that takes into account her cognitive limitation. Our model of consumer categorization also reflects an important aspect of human cognition: although consumers are limited in short-term capacity for remembering market information, they can be sophisticated in identifying patterns and forming optimal decision rules.[6]

The categorization process can also be interpreted as follows: when consumers deal with firms in a market, they would have encountered many prices (and the total *price* faced by consumers may have several aspects). This makes price comparisons imperfect, and the categorization heuristic captures this imperfection. Given this, we can appeal to the finding in cognitive psychology that shows that despite the fact that individuals have limited short-term memory, the mind is sophisticated in constructing heuristics to optimally react to the limitations in information processing. Indeed, in the specific context of the categorization heuristic, Rosch (1978) provides the best support when she argues that individuals aim for "cognitive efficiency" by minimizing the variation with each category. Furthermore, the findings of the more recent experimental literature on the automaticity of categorical thinking is also consistent with our model in that subjects may develop and use complex categorization rules without even being consciously aware of computing it (see Bargh 1994, 1997), while at the same time, their ability to recall information from short-term memory might be limited. This characterization is consistent with our model and price categorization is akin to a long-term heuristic that may be automatically formed by the mind of the consumer because of experience over time with the market prices, which then helps the consumer to make optimal decisions during a specific purchase occasion. Finally, in the context of behavioral pricing, Monroe and Lee (1999) argue that consumers may not be able to perfectly recall the price of a product, but at the same time, they are likely to tell if the product is "too expensive," "a bargain," or "priced reasonably." This also provides support to our model of price categorization for consumers with limited memory.

2.1. Comparing Category Labels: Symmetric Categorization Process

We first consider the case in which the limited-memory consumers encode the prices from both firms in categories and compare only the labels of the categories that the prices from both the firms fall into. In other words, the categorization process is symmetric across the firms. This represents the situations in which consumers have an imperfect impression about the prices from both firms. This case arises in environments that require consumers to process and compare not only the posted price offered by the competing firms but also numerous other informational details that are relevant for the full price faced by consumers. Thus the decision process should not be interpreted as one of search but rather one that highlights price comparisons when there are imperfections in the recall of consumers.

The specific process of the limited-memory consumers is as follows. Consumers encode the prices posted by the firms into categories. To make a purchase decision, they recall and compare the categories associated with the prices of both the firms. Consumers buy from the firm whose recalled price was in a lower category. In the case of a tie, when the recalled prices of the two firms are in the same category, consumers purchase randomly from either firm with equal probability. The parameter n is exogenously given and captures the precision of consumers' price recall or the degree of memory.[7] If $n = 0$, then consumers have *no memory* and all prices fall in a single category. If $n = 1$, consumers divide the firms' prices into two categories: high prices or low prices. As n increases, consumers categorize the firms' prices into finer and finer partitions. Consequently, consumer recall improves, and the recalled price will more closely reflect the actual price charged by the firms.

We analyze the price equilibria for a given category structure and then examine the endogenous case where the consumers optimally choose the categorization to maximize their surplus. The following are the sequence of decisions: given the number of categories $n + 1$, consumers with limited memory optimally choose the categorization cutoffs $\{k_i\}_{i=1}^{n}$ in an optimal way. In the next stage, the two firms decide on

[6] For example, an analyst is unlikely to be able to remember numbers in a data set but can be good at detecting patterns in the data and at forming rules or heuristics and in using those rules.

[7] We can endogenize n if we assume a cost function that is increasing for n. We choose not to do so because adding an extra stage to the model in which n is chosen by consumers does not affect the results. However, at the end of §3.3, we discuss how n, if it is endogenously chosen, can be affected by the competitiveness of the market environment.

their pricing strategies given the consumers' cutoffs. Note that we can also consider the timing in which consumers and firms move simultaneously to, respectively, choose categories and prices without changing the results of the paper.[8] In the last stage, all consumers make their purchase decisions based on price realizations and the decision process described earlier. We solve for the symmetric subgame-perfect Nash equilibrium of the game, which consists of the set of optimal cutoffs chosen by the consumers and the pricing strategies chosen by both firms. The optimal cutoffs chosen by the limited-memory consumers ($\{k_i^*\}_{i=1}^n$) satisfy the requirement that the equilibrium surplus of the limited-memory consumers is maximized. In cases where there are multiple equilibria for firms' pricing strategies given a set of cutoffs $\{k_i\}_{i=1}^n$, we use the selection criterion that firms will play the equilibrium strategies with Pareto-dominant payoffs. We identify the symmetric pure-strategy equilibrium when it exists.

2.2. Analysis and Results

In this section, we present the equilibrium solution of this simple model of symmetric categorization when consumers compare the labels of the categories. Note that consumers in choosing to buy from a firm recall the labels of the categories in which two firms' actual prices occurred. Because consumers can only recall the category labels, the firms will have the incentive to price at the top of each category.

It is useful to begin by considering the simplest possible case of $n = 1$, where consumers can only recall *high* versus *low* prices. If the cutoff between the categories is at k, each firm will optimally use at most two prices k or 1. It is easy to see that both firms pricing at k always constitutes a pure-strategy equilibrium. Both firms pricing at 1 constitutes an equilibrium as long as $\frac{1}{2} \geq k$ and this equilibrium is the Pareto-dominant one for firms. This implies that when consumers can choose k to maximize their surplus, they will, in equilibrium, choose it to be slightly above $\frac{1}{2}$ to strategically induce the firm to charge their equilibrium prices in the *low* category.

In general, given the cutoff points $k_1 \leq \cdots \leq k_n \leq k_{n+1} = 1$ and the corresponding $n+1$ categories, any equilibrium pricing strategy for the firms can assign a positive probability only to prices at the top of each category. The following proposition shows that there is a unique symmetric equilibrium in pure strategies. The proofs for all the propositions and lemmas can be found in the appendix.

Proposition 1. *When consumers optimally choose the cutoffs, there is a unique pure-strategy equilibrium.*[9] *The optimal cutoffs are* $k_i^* = (\frac{1}{2})^{n+1-i} + \varepsilon$ *for every* $i = 1, \ldots, n$, *where* $\varepsilon \ll (1/2)^n$ $\forall n$ *and* $\varepsilon \to 0$. *Both firms charge* $p_j^* = k_1^* = (\frac{1}{2})^n + \varepsilon$. *Each firm makes positive equilibrium profits* $\pi_j^* = (\frac{1}{2})^{n+1} + \varepsilon/2$, *which are decreasing in* n.

Competition in a market with consumers who compare the price category labels results in a symmetric pure-strategy equilibrium in which firms compete by charging prices, which are at the top of lowest category. When consumers choose the categories that maximize their surplus, the categorization scheme consists of a unique set of cutoffs. The first point is that the equilibrium prices are strictly greater than marginal costs and the firms earn positive profits despite the fact that they are undifferentiated and face a single homogenous segment of consumers. Bounded rationality of consumers who can recall prices only as the category labels moves firms away from the Bertrand competition outcome. Because consumers do not distinguish between all prices within a category, firms have the incentive to charge only the highest price within a category. However, the competition between the firms induces them to price only in the lowest category.[10]

Ever since Edgeworth (1925), there has been a literature on possible resolutions to the Bertrand paradox, the idea that undifferentiated firms facing a homogenous consumer market might still be able to price above marginal costs and earn positive equilibrium profits. These resolutions have typically focused on supply-side factors such as capacity constraints (Levitan and Shubik 1972, Kreps and Scheinkman 1983, Iyer and Pazgal 2008) or the nature of cost functions (Baye and Morgan 2002). Proposition 1 adds to this literature by showing the role of bounded rationality on the consumer side as a means to resolve the Bertrand paradox.

An interesting corollary of Proposition 1 is that for $i = 1, \ldots, n+1$, the differences between consecutive cutoffs is given by

$$k_i^* - k_{i-1}^* = \left(\tfrac{1}{2}\right)^{n+2-i}.$$

The difference decreases geometrically as i decreases. This implies that the categorization becomes finer toward the lower end of price range. In fact, when moving from low to high prices, each successive

[8] The equilibria of sequential move game are also equilibria in the simultaneous move game as well. This is because the subgame-perfect Nash equilibrium in the sequential game is also a Nash equilibrium of the simultaneous game with the selection criterion, involving equilibrium strategies with Pareto-dominant payoffs for consumers.

[9] Optimal choice by the consumers involves each individual consumer setting cutoffs that maximizes her own surplus.

[10] Note that in this pure-strategy equilibrium, while the firms' strategy is to price in the lowest category, it is the best response to the consumer's categorization strategy, and, in turn, consumers choose all the n partitions optimally given the firms' pricing strategies.

category is exactly double the size of its lower neighbor ($|C_i| = 2|C_{i-1}|$). This is intuitively appealing because it suggests that consumers pay more attention to and invest more memory resources in encoding lower prices rather than higher prices. The reason behind this is the consumers' strategic goal to induce the firms to lower prices as much as possible. The consumer's equilibrium cutoffs strategy $k_i^* = (\frac{1}{2})^{n+1-i} + \varepsilon$ for every $i = 1, \ldots, n$, is designed such that at each of the higher price levels, the firms are induced to undercut each other. When both firms charge high prices close to the reservation price, a deviation by a firm to a price in a lower category can be profitable even if the undercutting amount required for winning consumers away from the rival firm is of a relatively large amount. However, when both firms charge lower prices, the discount needed to undercut and switch consumers away from the competitor has to be smaller to make such a move profitable. Thus, it is optimal for consumer categorization to have wider partitions at the upper end of the price range. Moreover, as the prices decrease, the partitioning becomes successively finer such that in equilibrium firms are induced to price only in the lowest category with the lowest possible k_1^*.

Furthermore, the equilibrium prices and profits of the firms decrease with the number of categories. Thus as consumers' recall of the market prices improves, the equilibrium prices charged by the firms move toward marginal cost. Indeed, as the degrees of memory increase beyond bound ($n \to \infty$), the model represents the standard model of perfect recall. In this case, the equilibrium prices converge to marginal cost, and thus we are able to recover the standard Bertrand competition outcome as the limiting case of this model with infinite degrees of memory.

Proposition 1 also reveals an important convergence property of the market outcome as the degrees of consumer memory increase toward perfect memory. As n increases, the equilibrium prices and profits converge to the Bertrand outcome of marginal cost pricing at a decreasing rate. Thus, additional categories have a smaller effect in changing the equilibrium price as compared to the initial few categories. Overall, this simple categorization model of limited recall suggests that given strategic market interactions, small amounts of initial improvements in recall can lead to equilibrium choices that are close to the perfect recall outcome.

We can also examine the consumer surplus and the loss of consumer surplus because of limited recall. In the perfect-recall Bertrand outcome, the consumer surplus will be at its maximum of one, and from the Proposition 1, the equilibrium surplus of the limited-recall consumers is $S(n) = 1 - p_j^* = 1 - (\frac{1}{2})^n$. Thus the loss in consumer welfare relative to the case of perfect recall is decreasing in the degrees of memory and the marginal effect is also decreasing. This again implies that the small amounts of initial improvements in memory will lead to substantial gains in consumer welfare.

2.3. Introducing Consumer Heterogeneity: Adding Uninformed Consumers

We now extend the model to investigate the effect of consumer heterogeneity in firm preference as well as in memory capacity. We first extend the basic model to include a group of uninformed or loyal consumers. Let there be a group of uninformed consumers of size 2γ who randomly purchase from either firm with equal probability as long as the price is below their reservation price.[11] Note that these consumers do not compare prices across the firms, and therefore there is no role for memory in facilitating price comparisons for these consumers. The remaining group of $(1 - 2\gamma)$ consumers are limited-memory consumers with n degrees of memory as in §2.2. The following proposition establishes the equilibrium:

Proposition 2. *When consumers optimally choose the cutoffs and $(1/(2(1-\gamma)))^n > 2\gamma$, there is a unique symmetric pure-strategy equilibrium. The optimal cutoffs are $k_i^* = (1/(2(1-\gamma)))^{n+1-i} + \varepsilon$ for every $i = 1, \ldots, n$, where $\varepsilon \ll (1/(2(1-\gamma)))^n$ $\forall n$ and $\varepsilon \to 0$. Both firms charge $p_j^* = k_1^* = (1/(2(1-\gamma)))^n + \varepsilon$. Each firm makes positive equilibrium profits $\pi_j^* = \frac{1}{2}(1/(2(1-\gamma)))^n + \varepsilon/2$, which are decreasing in n.*

Even in a heterogenous market with uninformed or loyal consumers and consumers with limited memory, there exists a unique symmetric pure-strategy equilibrium in which firms price at the top of the lowest partition as long as γ is not too large. It can be noted that the presence of uninformed consumers in the market who consider buying from only one firm creates market differentiation between the firms. Thus, the higher level of γ represents a more differentiated market with less intense price competition. Therefore, as expected, the equilibrium price and profit of the firms increase with γ. It is also interesting to note that in this equilibrium, the firms' profits are $\pi^* = \frac{1}{2}(1/(2(1-\gamma)))^n$, which are, in fact, greater than γ, the maximum profit that can be attained in a standard model of competition in which consumers are able to compare the actual prices (Varian 1980). All the other key results of §2.2 are preserved with the addition of uniformed consumers. For example, as in the basic model, the categorization becomes finer toward the lower end of price range. Furthermore, in a more

[11] Alternatively, γ of these consumers can also be assumed to consider purchasing only from one of the two firms, whereas the remaining γ consider the other firm.

competitive market with a smaller γ, the partitions at the lower end of the price range become even finer.

In the appendix, we characterize a symmetric mixed-strategy equilibrium in which the categorization by consumers are finer toward the lower end of the price range when the condition $(1/(2(1-\gamma)))^n > 2\gamma$ does not hold. Interestingly, in this mixed-strategy equilibrium, firms charge a price in every category with positive probability. Furthermore, as the number of categories increases beyond bound, this equilibrium also converges to one in which the informed consumers have perfect memory as in the standard model of Varian (1980).

2.4. Adding Heterogeneity in Memory Capacity

Consider now a market that also consists of a group of size 2α fully informed consumers with perfect memory for market prices who can therefore compare the actual prices offered by both firms. As before, there are 2γ uninformed or loyal consumers and, consequently, a group of $2\beta = 1 - 2\gamma - 2\alpha$ consumers with limited memory. We can now investigate the effect of consumer heterogeneity in memory capacity as distinct from heterogeneity in consumer firm loyalty.

Proposition 3. *When consumers optimally choose the categories and* $((\beta+\gamma)/(1-\gamma))^n > \gamma/(\beta+\gamma)$, *there is a unique symmetric mixed-strategy equilibrium. The optimal cutoffs are* $k_i^* = ((\beta+\gamma)/(1-\gamma))^{n+1-i} + \varepsilon$ *for every* $i = 1, \ldots, n$, *where* $\varepsilon \ll ((\beta+\gamma)/(1-\gamma))^n$ $\forall n$ *and* $\varepsilon \to 0$. *Both firms price at the top of the lowest category according to the cumulative distribution function* $F_j(p) = ((\beta+\gamma)/(2\alpha))(k_1^* - p)/p$, *where* $p \in [((\beta+\gamma)/(1-(\beta+\gamma)))k_1^*, k_1^*]$.

With a segment of consumers with perfect memory, there is no longer a pure-strategy equilibrium. There is, however, a unique symmetric mixed-strategy equilibrium in which firms charge prices only in an interval extending from the top of the lowest partition. Once again, the key results of §§2.2 and 2.3 are preserved as the categorization becomes finer toward the lower end of price range in this case. Firms' profits are $\pi_j^* = (\beta+\gamma)((\beta+\gamma)/(1-\gamma))^n$, which are greater than γ. This result is also similar to that of Proposition 2.

We can now compare the results of Proposition 3 with those of Propositions 1 and 2 to better understand the effect consumer heterogeneity in this model. First, notice that when $\alpha \to 0$, the price support of the equilibrium price distribution shrinks to a single point, $k_1^* = (1/(2(1-\gamma)))^n + \varepsilon$, and so we recover the equilibrium price in Proposition 2. On the other hand, if $\gamma \to 0$, then the market consists of only limited-memory and perfect-memory consumers, and we get a market with consumer heterogeneity in memory capacity. In this case, the equilibrium will be one in mixed pricing strategies in the lowest category and in the interval $p \in [(\beta/(1-\beta))k_1^*, k_1^*]$. Firms make positive profits βk_1^*. For a fixed proportion of uninformed or loyal consumers, the increased presence of perfect-memory consumers leads to lower firm profits compared to the one in Proposition 1. Conversely, for a given γ, a greater proportion of limited-memory consumers increases equilibrium firm profits.

3. Asymmetric Categorization Process

In §2, the categorization scheme of the limited-memory consumers was symmetric across the firms. Consumers compared only the labels of the categories that represented the prices charged by both the firms. In this section, we analyze an alternative categorization process along the lines of Dow (1991) and Rubinstein (1993), where consumers compare the actual price at a firm they are at with the recalled category for the price that they encountered at the other firm. As in these earlier papers, this asymmetric categorization process is intended as a framework designed to highlight the value of recalled information.

Specifically, consider a three-stage decision process that is similar to Dow (1991) and in which the limited-memory consumers contact both firms in the following manner:[12] In the first stage, consumers observe the price at a firm and encode this price (half of the consumers observe Firm 1 while the other half observe Firm 2). In the second stage, the consumers observe the exact price at the other firm, compare it with the encoded price recalled from their memory, and decide whether to buy the product at the current firm. If the actual price observed is lower than or equal to the price recalled from memory, the consumer will purchase one unit of the good at the second firm (provided that the price is not higher than the reservation price).[13] Finally, in the third stage, if the consumers did not purchase at the second stage from the second firm, they purchase from the original firm provided that the price there is below the reservation price.[14]

[12] We can extend our model to allow consumers to decide whether to compare prices at all after observing the price at the first firm. The results of this paper are unaffected in this extension. Details are provided in §4.2 and the appendix.

[13] Note that we assume that if the observed price at the second firm is exactly identical to the price that is recalled from memory, the consumer will buy at the second firm.

[14] This decision process can also have other interpretations pertaining to the broader class of problems of communication constraints between agents in organizational settings. For example, it can be seen as reflecting communication constraints when the decision-making team consists of two agents. Agent 1 observes the price in the first firm and sends a message to agent 2, who then has the discretion to decide after observing the price at firm 2. Here, the constraint can be interpreted as a limit on the set of words or messages that can be sent.

Unlike the case where the consumers compare the price labels of both firms (symmetric categorization case), this decision process allows us to represent price comparisons between the two firms when a consumer has better information about one of the two firms. In actual markets, this would represent situations where the consumer has better information about the price offer at the current firm than the price that was previously observed (and encoded) at the competing firm. We have two specific objectives in considering this alternative categorization process. First, we aim to investigate the robustness of the equilibrium consumer and firm strategies to the alternative forms of categorization. Furthermore, we can examine whether the categorization process, considered in this section leads to a relatively more competitive market. Second, as opposed to the symmetric categorization process, in the asymmetric categorization process, the consumer has to compare an actual price to a recalled category. Consequently, the consumer has to assign some *number* in the category to represent its prices. This then allows us to compare the effects of exogenous representations of the category (such as the *top* of the category) versus endogenous statistics (such as the mean or the median of prices charged in the category).

As in §2, consumers with n degrees of memory are endowed with $n+1$ categories $\{C_i\}_{i=1}^{n+1}$ that are indexed in increasing order of prices. They may have to optimally select n cutoff prices $\{k_i\}_{i=1}^{n}$ that separate the categories. Consumers remember the expected price charged in a category as the representative price in that category.[15] Let $\{m_{i,j}\}_{i=1}^{n+1}$ be the set of the mean of the prices that each firm j charges in each category; then $m_{i,j} = E[p_j \mid p_j \in C_i]$. Clearly, $k_{n+1} \geq m_{n+1,j} > k_n \geq m_{n,j} > \cdots > k_1 \geq m_{1,j} > k_0$ for $i = 1, \ldots, n$ and $j = 1, 2$. Figure 1 shows the categorization scheme with the categories, cutoffs, and mean prices.

Except for the categorization process of the limited-memory consumers, we maintain exactly the same features of the market as in §2.4. In other words, the segment of uninformed and perfect-memory consumers are exactly as previously described.

3.1. The General Case

We start by presenting the most general case of $n+1$ categories in a market with all three segments of consumers. It is immediate that the symmetric equilibrium of this model involves mixed strategies. Prior to the explicit derivation of the equilibrium price support, note that the potential price range of each firm is $(b, 1)$, where $b = \gamma/(1-\gamma)$. A firm will never set a price p_j below b because the maximum profit it can obtain is $p_j(\gamma + 2\beta + 2\alpha)$, which is lower than γ, the guaranteed profit it can obtain by setting $p_j = 1$ and selling only to its uninformed consumer group. Hence the $n+1$ categories that consumers use to classify prices are all within $(b, 1)$. Given our notation, we have $k_0 = b = \gamma/(1-\gamma)$.

The following lemma identifies the equilibrium price support.

Lemma 1. *The support of the price distribution used by the firms in a symmetric mixed-strategy equilibrium contains no atoms and is comprised of a union of intervals:* $\bigcup_{i=1}^{n+1}[(b_i, m_i) \cup (v_i, k_i)]$, *where* $k_i > v_i > m_i > b_i > k_{i-1}$ *(i.e., there are two "holes" in the support of the pricing distribution in each category).*

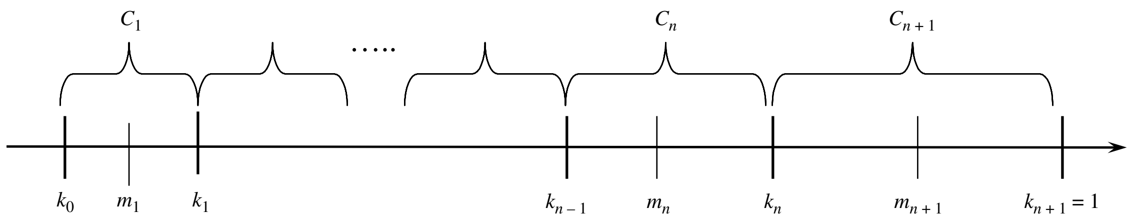
Let $W_j(p)$ be the probability that firm j prices above p, and denote $w_i = W(v_i)$ and $s_i = W(b_i)$ for $i = 1, \ldots, n+1$ (and by definition $s_{n+2} = 0$). Note that s_i is just the probability of pricing in any of the categories between $i, \ldots, n+1$. Note that the equilibrium firm strategies will make the rival indifferent between any of its strategies. For the extreme points of the distribution, we get the following profit expressions for $i = 1, \ldots, n+1$:

$$
\begin{aligned}
p_j = k_i&: \Pi = (\gamma + w_i\beta + s_{i+1}\beta + 2s_{i+1}\alpha)k_i,\\
p_j = v_i&: \Pi = (\gamma + w_i\beta + s_{i+1}\beta + 2w_i\alpha)v_i,\\
p_j = m_i&: \Pi = (\gamma + w_i\beta + s_i\beta + 2w_i\alpha)m_i,\\
p_j = b_i&: \Pi = (\gamma + w_i\beta + s_i\beta + 2s_i\alpha)b_i.
\end{aligned}
\tag{1}
$$

When pricing at k_i, a firm will get four groups of consumers: (1) all of its uninformed consumers; (2) the informed consumers with perfect memory who find a higher price at the other firm; (3) the limited-memory consumers who started with it, recall m_i (rather than the actual price), and encounter a higher price than m_i at the other firm; and (4) finally, the limited-memory consumers who begin with the other firm, observe a price above b_{i+1} and remember a price m_{i+1}. When charging v_i, a firm will get, in addition to the above consumers, all the informed perfect-memory consumers who find a higher price at the other firm. A price of m_i will get a firm the obvious uninformed and informed perfect-memory consumers, as well as all the limited-memory ones that started with the other firm and saw a price higher then b_i (as they remember m_i but will not purchase from the first firm even in a case of a tie) and the limited-memory consumers that started with it and compare to a price above m_i in the other firm. Finally, by pricing at the lower end of the support, a firm will get additional informed consumers with perfect memory as well. To solve for the equilibrium, recall that m_i is the mean of the price distribution within category i and given by $m_i = \int_{b_i}^{k_i}(p(d/dp)(1 - W(p)))\,dp$.

[15] In §4.1, we discuss the effects of other representations of the category such as the median or the top of the category.

Figure 1 The Categorization Scheme



A firm pricing at $k_{n+1} = 1$ guarantees itself a profits of $\Pi = \gamma + w_{n+1}\beta$, which is the profit in the symmetric mixed-strategy equilibrium. Therefore, firms make profits higher than γ, which is the profit in the extreme case where all the informed consumers have perfect recall ($\beta \to 0$ and $\alpha > 0$). This result is similar to that of Proposition 3 for the case in which the limited-memory consumers recall and use the categories from both firms.

3.2. The Equilibrium as $\alpha \to 0$

The effect of limited memory on competition can be clearly seen from the analysis of the limit market in which all the informed consumers have limited memory of degree n. This also recovers the case that is analogous to that in Proposition 2 of a market with uninformed or loyal and limited-memory consumers. If we take the limit of α approaching zero, then we get $v_i = k_i$, $m_i = b_i$, and $w_i = s_{i+1}$, $i = 1, \ldots, n$ (as well as $w_{n+1} = 0$). In other words, as α approaches zero, the two price support intervals in each category i shrink to two points, $p = k_i$ and $p = m_i$, where the probability of charging prices at the k_is approach zero. Consequently, the set of equations in (1) reduces to

$$p_j = m_i\colon\ \pi = (\gamma + s_{i+1}\beta + s_i\beta)m_i \quad \forall\, i = 1, \ldots, n+1. \tag{2}$$

A few comments about the nature of the equilibrium price support are in order. In the models of competitive price promotions in which the informed consumers have perfect recall, and therefore compare the actual prices of both the firms, the equilibrium price distribution is continuous (for example, see Narasimhan 1988, Raju et al. 1990, or Lal and Villas-Boas 1998). However, limited rationality in the form of the asymmetric categorization process in which the limited-memory consumers compare an actual price to a recalled category leads to firms choosing from only a finite set of possible prices. In empirical studies, it has been observed that the distributions of prices are typically such that most of the probability mass is concentrated around a small number of price points (see, for example, Villas-Boas 1995, Rao et al. 1995). Furthermore, the number of prices charged goes up with the improvement in the degrees of memory. Behavioral studies have pointed out that high involvement environments lead to greater attentional capacity being devoted to encode a relevant piece of information in memory (e.g., Celsi and Olson 1988). Thus one can expect to observe more prices being charged in high-involvement product markets with greater degrees of consumer memory. Proposition 4 states the equilibrium of this limit market case with uninformed and limited-memory consumers.

Proposition 4. *In the limit market (as $\alpha \to 0$) with uninformed consumers and limited-memory consumers who have n degrees of memory, a symmetric mixed-strategy equilibrium will involve each firm charging prices at $p_i = m_i = 2/(1/k_{i-1} + 1/k_i)$ with probabilities $\Pr(m_i) = (\gamma/2\beta)(1/k_{i-1} - 1/k_i)$ for $i = 1, \ldots, n+1$, where the $\{k_i\}_{i=1}^n$ is the set of cutoffs for the limited-memory consumers such that $1 = k_{n+1} > k_n > \cdots > k_1 > k_0 = \gamma/(1-\gamma)$.*

Proposition 4 shows that the prices charged in each category are the harmonic means of the category cutoffs. Note that the profits for the case where all the informed consumers had perfect memory was γ. As seen in the proof of Proposition 4 in the appendix, the equilibrium profits in a market where all the informed consumers compare prices with limited memory is also γ. Furthermore, in this market with only limited-memory consumers, the number of prices charged by firms is equal to the number of categories. Therefore, in the limit market equilibrium, the available memory capacity is aligned with the price information that is required to be recalled, and it is as if a market with perfect recall is mimicked. This results in the firms competing away all but the guaranteed profits that can be made from their uninformed consumers.

Proposition 4 identifies the equilibrium firm strategies as function of the cutoffs of the consumers. We now characterize the equilibrium of this model if the cutoffs satisfy the requirement that the equilibrium surplus of the limited-memory consumers is maximized. This requirement results in a unique Pareto-optimal equilibrium. Note that each firm's equilibrium profits are always γ. Therefore the requirement that at the optimal cutoffs the equilibrium surplus is maximized results in a unique Pareto-optimal equilibrium that is best for the limited-memory consumers. This is also consistent with the idea of boundedly rational consumers doing the best for themselves given the constraints that they face. Proposition 5 identifies these optimal cutoffs k_i^*.

Proposition 5. *As $\alpha \to 0$, the optimal cutoff prices that maximize the equilibrium surplus of the limited-memory consumers are $k_i^* = (\gamma/(1-\gamma))^{(n+1-i)/(n+1)}$. For these cutoffs, each firm's equilibrium prices are $m_i^* = 2/((1+((1-\gamma)/\gamma)^{1/(n+1)})(\gamma/(1-\gamma)))^{(n+1-i)/(n+1)}$ with probability $\Pr(m_i^*) = (\gamma/(1-2\gamma))(((1-\gamma)/\gamma)^{1/(n+1)}-1) \times ((1-\gamma)/\gamma)^{(n+1-i)/(n+1)}$ for $i = 1, \ldots, n+1$.*

We can now summarize the main findings from the analysis of this limit market consisting of only uninformed or loyal and limited-memory consumers and compare these results with those of Proposition 2, where consumers categorize the prices from both firms before comparing them. In Proposition 5, even though consumers have limited memory, the pricing strategies of the firms adjust so that the number of prices charged is aligned with the degrees of consumer memory, and so it is as if consumers can perfectly recall the actual prices that are charged. Thus the market equilibrium adjusts to the memory capacity of consumers, and each firm charges only a single price with positive probability in each category.

Next, we can see from Proposition 5 that for $i = 1, \ldots, n+1$, the differences between consecutive cutoffs are

$$k_i^* - k_{i-1}^* = \left(\frac{\gamma}{1-\gamma}\right)^{(n+1-i)/(n+1)} - \left(\frac{\gamma}{1-\gamma}\right)^{(n+2-i)/(n+1)} = \left(\frac{\gamma}{1-\gamma}\right)^{(n+1-i)/(n+1)} \left(1 - \left(\frac{\gamma}{1-\gamma}\right)^{1/(n+1)}\right). \quad (3)$$

The difference decreases exponentially as i decreases. This implies that the categorization becomes finer toward the lower end of price range. Thus, this result is robust across different types of categorization processes. Furthermore, the probability of charging a particular price is proportional to $((1-\gamma)/\gamma)^{(n+1-i)/(n+1)}$ and thus is also exponentially decreasing in i. As in §2, these results are intuitively appealing, suggesting that the consumers pay more attention in encoding lower prices than higher prices, and this induces firms to respond by charging lower prices with higher probabilities. Also, we get that $\partial k_i^*/\partial\gamma > 0$. The values of the cutoffs increase in less competitive markets with more uninformed consumers, and this result again is robust across the different types of categorization processes. As expected, an increase in the size of the uninformed group of consumers increases each price that the firms will charge and also shrinks the price range $1-b$. From Proposition 2, we can see that firms' profits in the symmetric categorization model where consumers compare the price category labels from both firms are higher than that in the asymmetric categorization model of this section. This is quite intuitive because consumers in the asymmetric categorization model have better price information than in the symmetric categorization model as the actual price from one firm is used in making purchase decisions when categorization is asymmetric. This induces more intense competition between firms and thus reduces their profits.

Finally, another interesting result of this limit market case is the behavior of the expected prices charged by the firms.

Result 1. The expected equilibrium price charged by the firms, as well as the price variance, increases with the degree of consumer memory.

With greater n, consumers with limited memory become more sensitive to price differences between firms. This increased price sensitivity reduces firms' expected profits from the limited-memory consumers because with greater n, those consumers are less likely to make mistakes and more likely to end up buying from the firm that (actually) has the lower price. The strategic responses of the firms are therefore to increase the prices charged, on average, to extract greater surplus from their uninformed consumers. Thus, in the equilibrium, the average market price charged by the firms goes up with improvements in memory and is the highest when the informed consumers have perfect recall. This result may be seen as interesting in that the average market price increases even as the consumer cognition for price comparisons improves.[16] It is important to note that this actually implies that the expected price paid by limited-memory consumers decreases with n because firms profits are invariant with regard to n, and the loyal consumers do pay higher prices, on average. This is because the limited-memory consumers pay the average of the minimum price charged by the firms. In contrast, the uninformed or loyal consumers pay the expected prices charged by the firms, which increases with the degrees of memory.

The above results on equilibrium prices is different from the pure-strategy equilibrium result in Proposition 2, where both uninformed and informed consumers pay the same price and that price decreases with n. Therefore, although in the symmetric categorization case the improvement of memory for the informed consumers provides a positive externality to the uninformed consumers, it leads to a negative externality to the uninformed consumers in the asymmetric categorization case.

3.3. Comparing Limited Memory to Perfect Memory

We now turn to the comparison of this model to the standard model where all the informed consumers

[16] This result is similar in flavor to those previously presented in the literature. For example, in Rosenthal (1980), the average market price increases even as the number of sellers increases; in Iyer and Pazgal (2003), the expected market prices increase with the number of retailers who choose to join a comparison shopping agent.

who compare prices are assumed to have perfect recall. In the standard model (Varian 1980, Narasimhan 1988), $W(p) = \Pr(p_j \geq p) = \gamma/((1-2\gamma)) \times (1/p - 1)$, $b \leq p \leq 1$ and $E(p) = (\gamma/(1-2\gamma)) \times \ln((1-\gamma)/\gamma)$. As the degree of memory increases, the price distribution resembles the standard model, and in the limit as the degree of memory increases beyond bound, it is identical to the distribution in standard model. Formally,

Proposition 6. *Given an arbitrarily small $\delta > 0$, $\exists\, N$ such that for every degree of memory $n > N$ and every price $p \in (\gamma/(1-\gamma), 1)$, we have* (i) *$\exists\, m_j(n)$ such that $|p - m_j(n)| < \delta$ and* (ii) *$|W_n(p) - W(p)| < \delta$, where $W_n(p)$ is the equilibrium probability of each firm pricing above p and $m_j(n)$ is a price charged with positive probability when the limited-memory consumers use n cutoff prices.*

Thus we recover the standard model of perfect recall competition as the limiting case of our model with infinite degrees of memory. Next, consider a convergence measure that is based on the expected price consumers pay. Define the degree of convergence by $\Delta E_n(p) = (E(p) - E_n(p))/(E(p))$, where $E(p)$ is the average equilibrium price in the case of perfect memory:

$$\Delta E_n(p) = 1 - \frac{2(n+1)}{\ln((1-\gamma)/\gamma)} \frac{[((1-\gamma)/\gamma)^{1/(n+1)} - 1]}{[((1-\gamma)/\gamma)^{1/(n+1)} + 1]}. \quad (4)$$

Smaller values of this measure imply greater convergence of the expected price to the case of perfect memory. It is straightforward to verify that $\partial \Delta E_n(p)/(\partial n) < 0$ and $\partial^2 \Delta E_n(p)/(\partial n^2) > 0$. Therefore the convergence is increasing in n, but the marginal gain in convergence is decreasing in n. Thus additional categories have a lower effect in changing the expected price compared with the first few categories. Thus once again, this result is similar to the convergence result that we established with the symmetric categorization process. Also, we have that $\partial \Delta E_n(p)/(\partial \gamma) < 0$ and $\partial^2 \Delta E_n(p)/(\partial \gamma^2) > 0$, which implies that the convergence is faster (with the marginal gain in convergence decreasing) in a less competitive market that has a greater proportion of uninformed consumers. An increase in the proportion of the uninformed consumers means fewer comparison shoppers who have limited memory. This implies that the expected prices in the case of limited-memory approach that for the case of perfect memory. In the extreme case, at $2\gamma = 1$, the difference disappears. It is easy to also verify that $\lim_{n\to\infty}(\Delta E_n(p)) = 0$. Overall, both the symmetric and asymmetric categorization models suggest that given strategic market interactions, small amounts of initial improvements in recall can lead to equilibrium choices that are close to the perfect recall outcome.

We can also examine the expected consumer welfare loss because of limited memory. Since $S_\beta(n) = S - S_\gamma - 2\Pi = 1 - 4\gamma + 2\gamma E_n(p)$, we can define the relative loss in total surplus that all the comparison shopping consumers incur using $n+1$ categorizations versus perfect memory as $\lambda_n = (S_\beta - S_\beta(n))/S_\beta = 2\gamma(E(p) - E_n(p))/(1 - 4\gamma + 2\gamma E(p))$. Again, it is straightforward to show that $\partial\lambda_n/(\partial n) < 0$, $\partial^2\lambda_n/(\partial n^2) > 0$, $\partial\lambda_n/(\partial\gamma) > 0$, and $\partial^2\lambda_n/(\partial\gamma^2) > 0$. Therefore the relative surplus loss for all the limited-memory consumers is decreasing in n, and the rate at which the surplus decreases is decreasing in n. Finally, the relative surplus loss for all the limited-memory consumers increases with the proportion of uninformed consumers in the market. Because $\partial^2\lambda_n/(\partial n^2) > 0$, the relative marginal gain in surplus of the limited-memory consumers from an increase in memory is decreasing with n. In this model, even with just one cutoff ($n = 1$), we get $\lambda_1 < 7\%$ for every feasible value of $\gamma \in (0, 0.5)$. Overall, this suggests that the expected welfare loss to consumers from limited memory of price recall is attenuated when we account for market competition between firms for these consumers, and this result again is similar to what was obtained in the case of the symmetric categorization.

In addition, note that $\partial S_\beta(n)/(\partial\gamma) < 0$ in both the symmetric and asymmetric categorization cases. Thus, for any given degree of memory, the total surplus from all the limited-memory consumers increases as the market becomes more competitive. Hence, when the market is more competitive, consumers have greater value in allocating more memory resources for encoding and storing price information. Interestingly, this is also precisely the situation for which firms will be using more prices in the equilibrium support. This implies that in more competitive product markets, consumers would use more degrees of memory, which should then result in more observed prices being used by firms.

4. Robustness of Findings

4.1. Comparisons of the Categorization Schemes

In this section, we compare and contrast the results across the symmetric and asymmetric categorization processes to highlight the similarities and explain the differences. Although the actual pricing strategies used by firms can obviously be driven by the specific features of the categorization process or by the nature of consumer heterogeneity, we uncover a remarkable degree of robustness in the general economic effects of categorization in a market settings. The first robust effect is about the manner in which consumers categorize price. Across the different categorization processes, the different consumer heterogeneity conditions, and irrespective of whether the equilibrium is one in pure or mixed strategies, we find that the categorization is finer toward the lower end of the price range. Intuitively, this suggests the general point that consumers who are bounded in their recall have a strategic motivation to invest memory resources in encoding lower prices to induce the firms to charge favorable prices with higher probability.

Next, we recover a convergence property of the market across the different categorization processes. We find that the interaction of categorization and market competition is such that small initial improvements in recall move the market outcomes quickly toward the perfect recall outcome. Thus the initial few increases in categories lead to equilibrium pricing choices and consumer surplus, which are close to the case where the number of categories are infinite. As expected, we also find that consumer heterogeneity and the presence of uninformed or loyal consumers in the market increase the equilibrium profits of the firms. Finally, we also find that the effect of increasing n is, in general, to weakly reduce the equilibrium profits of the firms.

There are also interesting differences between the effects of the two types of categorization processes, which are intuitively appealing. The asymmetric categorization process in which consumers compare a category to actual prices can be seen as creating a more competitive and undifferentiated environment for firms than the symmetric categorization process. Consistent with this intuition, our analysis shows that the asymmetric categorization process can lead to more competitive markets and lower equilibrium firm profits. This is evident from the comparison of the basic model of a market with only limited-memory consumers (or the model with both limited-memory and uninformed or loyal consumers) across the two cases. For example, in the symmetric categorization case when all consumers compare labels of categories, as can be seen from Proposition 1, the firms make positive equilibrium profits. However, in the analogous case, where consumers compare a recalled category to actual prices, we get the Bertrand outcome, and firms' equilibrium profits are zero. As we mentioned earlier, the intuition behind this result is that consumers in the asymmetric categorization case have better price information than in the symmetric categorization case because the actual price from one firm is used in purchase decisions in the former case.

We also investigated whether the findings of our model with asymmetric categorization are robust to different representations of the category. We have assumed that consumers represent the recalled category by the mean of the category and compare the mean to the actual price at the current firm. However, we have also analyzed the model in which consumers represent the category by the median price in the category. We find that all the qualitative results of §3 are preserved even if consumers use the median. Similarly, the main results of §3 continue to hold even if consumers use exogenous markers as representation of the category such as the top, middle, or bottom of the category. Indeed, we find that the model where consumers remember the top of the category in the asymmetric categorization model is mathematically equivalent to the symmetric categorization model of §2. Finally, it is interesting to note that when consumers use the mean of the category, their equilibrium surplus is actually higher than when consumers use any exogenous rule to represent the category. This is because the mean of the category is endogenous to the equilibrium actions of the firm. Thus, if consumers in a market were to learn through experience over time to do the best for themselves and were motivated to make the optimal long-term purchase decisions, it is likely that they would learn to recall the category mean price rather than using an exogenous rule.

4.2. The Decision Whether to Compare Prices Under Asymmetric Categorization

The asymmetric decision process of the limited-memory consumers, which we have considered in §3, implies as in Dow (1991) that the consumers necessarily have contact with both firms but that the price information from one of the firms is imperfect. Here, we provide an extension that allows consumers to decide whether to compare prices at all after observing the price at the first firm. This extension can be viewed as being consistent with the interpretation of the decision process as search with optimal stopping. After observing the price at the first firm, consumers decide whether to search or to stop (with zero incremental search cost) and obtain the price at the second firm. We show that all the results of §3 are robust to this extension.

If the decision process involves the choice of whether to go to the second firm, then the consumer upon observing the price at the first firm will have to decide whether to stop and buy at that firm or to compare prices by obtaining the price at the second firm. The consumer might optimally decide not to compare prices if the benefit of obtaining the second price is sufficiently small. This can occur in this model if the limited-memory consumers encounter a sufficiently low price at the first firm so that (even with zero search costs) they would be worse off going to the second firm because they might not recall the low price at the first firm precisely.

In the limit market case of $\alpha \to 0$, it follows immediately that the consumer is never worse off by deciding to obtain the price from the second firm after having observed a price at the first firm. Note from Proposition 4 that firms charge only the prices m_i with positive probability in each category. Thus, if the limited-memory consumer encounters a price in any category $i > 1$, she will be strictly better off going to the second firm to obtain its price given zero incremental cost of search. In addition, for the lowermost category $i = 1$, the consumer will be no worse off. Therefore the consumer will always have the incentive to search and obtain the price at the second firm in the decision process after having observed the price at the first firm, and consequently, all the results discussed above for the limit market case are not affected even if the consumer explicitly chooses whether to compare prices.

Consider now the market with all three consumer segments. For the case of no categories ($n = 0$), which is analyzed in the appendix, there is a threshold price u below which the limited-memory consumers will not compare prices with the second firm after observing the price at the first firm. If they indeed do decide to obtain the price at the second firm, they will encode the first price as a higher price $\hat{m}$, which is the mean price conditional on $p > u$. If the consumer is at the second firm, then she will recall the first firm's price as the conditional mean $\hat{m}$. The equilibrium support will be $(b, u) \cup (d, \hat{m}) \cup (v, 1)$ (where $b < u < d < \hat{m} < v < 1$). From the profit expressions at the extreme points of the distribution and from the definition of the conditional mean, we can derive the equilibrium of this model. Finally, for the general case of n categories and $\alpha > 0$, in the lowermost category $i = 1$, the equilibrium price support is similar to that described above with a threshold price u above which the consumer will obtain the price at the second firm. Then, as in the case of $n = 0$, the consumer will compare the price at the second firm with $\hat{m}$ the recalled price at the first firm. Interestingly, for this general case, we can show that equilibrium consumers will use the threshold price u for only the lowest category $i = 1$, and therefore the main results of §§3.1 and 3.2 will hold. For example, as before the equilibrium profits with consumer heterogeneity in memory capacity are higher than when all the informed consumers are homogenous in their memory capacity (i.e., only perfect-memory or only limited-memory consumers).

4.3. External Validity and Implications of the Results

Our analysis suggests that firms would use a finite number of prices in markets where consumers have limited memory and that consumers should devote greater memory resources to encoding lower prices resulting in finer categorization toward the bottom of the equilibrium price distribution. Accordingly, the interval between two adjacent prices charged by a firm with positive probability in equilibrium shrinks toward the lower end of the price distribution.

There are empirical findings that are consistent with the above implications of our model. For example, Villas-Boas (1995) and Rao et al. (1995) found that the distributions of prices are such that most of the probability mass is concentrated around a small number of price points, and a bimodal distribution is typical. Their findings are also consistent with the implication of our results, which suggest that using a very small number of categories (e.g., $n = 1$) would often be sufficient for consumers as this leads to small surplus loss in a competitive marketplace. As a reaction to a small n, firms would also only adopt a small number of price points.

Furthermore, Krishna and Johar (1996) show through a series of experiments that lower-priced deals are more easily recalled by consumers than deals involving higher prices and that firms have incentive to offer deals that have smaller price differences. In their study of consumer perceptions of promotional activity, Krishna et al. (1991, p. 8) collected price information in a New York supermarket for a period of 12 weeks on nine brand-size combinations with "considerable variance in terms of product class purchase frequency, market share, and frequency of promotion combinations." Among the nine brand-size, seven used only two price points and the remaining two used only three price points. Interestingly, for both the two brand size combinations that used three price points, the gap between the highest price and middle price was larger than the gap between the middle price and the lowest price (Krishna et al. 1991, Table 3). These findings offer further evidence that supports the results of our model.

The implications of our model are applicable across different product classes. On the one hand, for frequently purchased products and products purchased with low involvement and time pressure, consumers are likely to rely on nonconscious, automatic processing in making purchase decisions and thus allocate limited-memory resources in recalling prices (Monroe and Lee 1999). Not surprisingly, researchers have found exact price recall to be low for small-ticket supermarket items (Dickson and Sawyer 1990). On the other hand, in complex purchasing situations (e.g., for automobiles or appliances) the full price facing the consumer consists of not only the quoted posted price but also various types of discounts, trade-in payment, financing offers, warranty, delivery schedule, service and shipping fees, assembly charges, etc. When consumers compare across firms, it is the *impression* of the complex price comprised of several facets in addition to the posted product price that is relevant, making price comparisons imperfect. In these situations, the simple strategy of noting down the prices on a piece of paper rather than (imperfectly) recalling them will not resolve the consumers' problem because what is relevant is not only the numerical posted price but the full price impression, which includes all the other informational details and specifications that are pertinent for comparison across the firms.[17]

5. Summary and Discussion

In this paper, we take an initial step toward understanding the effects of limited memory and categorization by consumers on price competition between firms. We focus on a specific aspect of memory limitations—namely, the inability of consumers to recall exact price information. Limited-memory consumers can only recall the category to which a particular price belongs to. This paper investigates price comparisons with limited memory in a competitive market and analyzes the interaction between consumer price categorization and the equilibrium pricing strategies used by firms.

The analysis establishes several effects of limited consumer recall that are remarkably consistent across the different categorization processes and market conditions. When consumers compare either category labels (symmetric categorization) or a label to an observed price (asymmetric categorization), we find that the optimal strategy for the consumers calls for finer categorization toward the bottom of the equilibrium price distribution. This implies that in equilibrium, consumers should devote greater memory resources to encoding lower prices to induce firms to put more emphasis and charge more favorable prices. We establish a robust convergence result that emerges from the interaction between the categorization strategies of the consumers and the price competition between the firms: i.e., for both categorization methods, small initial improvements in memory capacity shift the equilibrium market outcomes quickly toward the perfect recall outcome. So even with a few memory categories, the expected price consumers pay and their surplus are quite close to case of perfect recall. There is thus a suggestion in our model that the existence of market competition mitigates the negative consequences of imperfect recall for consumers.

There are several interesting questions that are related to our investigation of limited memory. The problem of allocation of limited-memory resources to different tasks, such as recalling several product attributes or the prices of different products that the consumer buys, seems to be an interesting one to pursue. In this paper, we model memory limitations as the inability of consumers to recall exact price information; instead, they only recall the category to which the price belongs. Alternatively, imperfect memory recall can be modeled as consumers recalling the price with an added random noise, consumers recalling only the nearest round amount (as in Basu 2006), or consumers recalling a price distribution instead of the exact realization. The last approach will be analogous to that used in the search and consideration set formation literature (Mehta et al. 2003). It might also be useful to explore other memory mechanisms and their effects on a firm's decision making. Memory can also be thought of as a device to carry information over time. The information-theoretic characterization of memory is especially relevant in markets for frequently purchased goods across different shopping occasions. It would also be interesting to consider the analysis of our paper as it would unfold in a multiperiod setup in which firms repeatedly set prices and consumer categorization strategies evolve as a result. Finally, on the experimental side, it would be interesting to understand how the distributional characteristics of market variables such as price or product quality affect their encoding into the consumer memory. Overall, the analysis of limited recall in market settings can be a fruitful area for future investigation.

Acknowledgments

The authors thank the editor, the area editor, two anonymous reviewers, and Dmitri Kuksov, Glenn MacDonald, Jesse Shapiro, Jose Silva, Marko Tervio, and J. Miguel Villas-Boas for helpful comments. The authors also thank seminar participants at the Berkeley Micro Theory Seminar, Carnegie Mellon University, Harvard Business School, University of Michigan, University of Minnesota, New York University, University of Southern California, Wharton School, and Yale University.

Appendix

Proof of Proposition 1. Each firm can potentially use exactly $n+1$ prices conditional on the choice of the partitions by the consumers. Firm 1 will have demand only if its price is the same or lower than firm 2's price. Clearly, both firms charging at the top of the lowest partition ($p_1 = p_2 = k_1$) is an equilibrium. A firm that raises prices will lose all consumers, whereas lowering the price will only bring lower revenues from half of the market. The payoff matrix for firm 1 is given by (the payoff for firm 2 can be specified in an analogous way)

$$\Pi_1(p_1 = k_r, p_2 = k_t) = \begin{cases} 0 & \text{if } r > t, \\ \dfrac{k_r}{2} & \text{if } r = t, \\ k_r & \text{if } r < t, \end{cases} \qquad \textit{for } r, t = 1, \ldots, n+1. \tag{5}$$

[17] Indeed, the possibility of consumers being able to costlessly note down the price provides a partial justification for the consideration of the perfect-memory consumer segment. The perfect-memory consumers can be seen as those consumers who can costlessly and perfectly note down and codify all the relevant *full price* information that is necessary for across firm comparisons. On the other hand, the limited-memory consumers find it costly (or are unable) to note down all the relevant information and therefore rely on their limited recall of the information.

Consumers will optimally choose their cutoffs so that the unique equilibrium will be at lowest partition and that the lowest cutoff will be at the lowest possible value. We start by considering the highest possible prices. For $p_1 = k_{n+1} = 1$, $p_2 = k_{n+1} = 1$ not to be an equilibrium, the cutoffs must satisfy $k_{n+1}/2 < k_n$; similarly, for $p_1 = k_n$, $p_2 = k_n$ not to be an equilibrium, we need $k_n/2 < k_{n-1}$. Similarly, we need $k_{r+1}/2 < k_r$ for $r = 1, \ldots, n$. Iterated substitution leads to the following condition on k_r ($r = 1, \ldots, n$):

$$k_r > \left(\tfrac{1}{2}\right)^{n+1-r}.$$

Clearly, the best choices for consumers are $k_r^* = (\frac{1}{2})^{n+1-r} + \varepsilon$ for any infinitesimal $\varepsilon > 0$. Both firms price at the top of the lowest partition and split the market generating profits of $\pi_j^* = \frac{1}{2}(\frac{1}{2})^n + \varepsilon/2$. Furthermore, given the choice of all consumers, no single consumer can benefit from unilaterally changing her cutoff points. Q.E.D.

Proof of Proposition 2. If both firms price at the lowest cutoff $p_1 = p_2 = k_1$, each makes a profit of $k_1/2$. For any pair of prices, the payoff matrix for firm 1 is given by (and analogously for Firm 2)

$$\Pi_1(p_1 = k_r,\ p_2 = k_t) = \begin{cases} \gamma k_r & \text{if } r > t, \\ \dfrac{k_r}{2} & \text{if } r = t, \\ (1-\gamma)k_r & \text{if } r < t, \end{cases} \quad \textit{for } r, t = 1, \ldots, n+1. \tag{6}$$

The firm with the lower price gets all the consumers except those loyal to its rival. When both firms have equal prices, they split the market. For $r = 1, \ldots, n$, the pure strategies ($p_1 = k_r$, $p_2 = k_r$) constitute a strict equilibrium if no firm wants to deviate. The most profitable deviation for, say, firm 1 is to charge the highest possible price of $p_1 = 1$, making a profit of $\pi_1 = \gamma$ (and the same holds for firm 2). Therefore, no firm will have the incentive to deviate and charge the reservation price if

$$\Pi_j(p_1 = k_r,\ p_2 = k_r) = \frac{k_r}{2} > \gamma. \tag{7}$$

Moreover, no firm has an incentive to lower the price to the category immediately below if

$$\Pi_j(p_1 = k_r,\ p_2 = k_r) = \frac{k_r}{2} > (1-\gamma)k_{r-1}. \tag{8}$$

For the topmost category, the equilibrium condition is only

$$\Pi_j(p_1 = k_{n+1},\ p_2 = k_{n+1}) = \tfrac{1}{2} > (1-\gamma)k_n. \tag{9}$$

For the bottom category, it is

$$\Pi_j(p_1 = k_1,\ p_2 = k_1) = \frac{k_1}{2} > \gamma.$$

If $k_1/2 \geq \gamma$ and for every $r = 2, \ldots, n$ conditions (8) are not satisfied and condition (9) is not satisfied as well, we get that the unique symmetric equilibrium in pure strategies is for both firms to price at k_1. If conditions (8) and (9) are not satisfied, we have for $r = 2, \ldots, n+1$,

$$\frac{k_r}{2} \leq (1-\gamma)k_{r-1}.$$

This condition implies $\frac{1}{2} = k_{n+1}/2 \leq (1-\gamma)k_n$ or $k_n \geq 1/(2(1-\gamma))$. Applying the above condition iteratively, we get $k_r \geq (1/(2(1-\gamma)))^{n+r-1}$, and finally $k_1 \geq (1/(2(1-\gamma)))^n$. Adding the condition $k_1/2 \geq \gamma$ guarantees that pricing at k_1 is an equilibrium for both firms. Thus, we get the condition

$$k_1 \geq \left(\frac{1}{2(1-\gamma)}\right)^n \geq 2\gamma.$$

Because $\gamma < \frac{1}{2}$, we have $2\gamma \geq \gamma/(1-\gamma)$; so we are guaranteed that $k_1 > k_0 = \gamma/(1-\gamma)$.

We still need to check that no pure-strategy nonsymmetric equilibrium exists. Consider the payoff matrix (6) if $r > t+1$; then the payoff to firm 2 is $(1-\gamma)k_t$, but a deviation to pricing at k_{t+1} will yield a profit of $(1-\gamma)k_{t+1}$, which is higher, so in any potential pure-strategy equilibrium, firms must price at most one category apart. Now, assume that firm 1 prices at k_t and firm 2 prices at k_{t+1} then firm 2 makes a profit of γk_{t+1}. If it lowers its price to k_t, it would make $k_t/2$. If $k_t/2 > \gamma k_{t+1}$ then pricing at k_t and k_{t+1} cannot be an equilibrium. However, we know even more; we know $k_t/2 > \gamma$ ($k_t \geq k_1 > 2\gamma$), and thus we are done.

Consumers will try to choose their strategies to induce as low a k_1 as possible, so we get $k_r^* = (1/(2(1-\gamma)))^{n+r-1} + \varepsilon$ for a very small $\varepsilon > 0$. Firms price at $p_1^* = p_2^* = (1/(2(1-\gamma)))^n + \varepsilon$ and make a profit of $\frac{1}{2}(1/(2(1-\gamma)))^n + \varepsilon/2$. Q.E.D.

Proof of Proposition 3. Suppose that firms price only in the lowest partition; namely, in the interval (l_1, k_1). The equilibrium condition requires firms to get the same profit from each potential price (note that the β consumers see no difference between prices in the same partition):

$$l_1(2\alpha + \beta + \gamma) = \Pi,$$
$$k_1(\beta + \gamma) = \Pi.$$

A firm that decides to price in a higher partition will see its demand shrink to γ; hence the best potential deviation is to price at $k_{n+1} = 1$. To guarantee no deviation, then we must have that $\Pi \geq \gamma$, which is equivalent to $k_1(\beta + \gamma) \geq \gamma$ or, as presented in the proposition, $k_1 \geq \gamma/(\beta + \gamma)$.

For completeness, we can solve for the entire equilibrium and get $l_1(2\alpha + \beta + \gamma) = k_1(\beta + \gamma)$ or $l_1 = k_1((\beta + \gamma)/(1 - \beta - \gamma))$, and for every $p \in (l_1, k_1)$, the probability $W(p)$ of each firm charging below p is given by

$$p(2\alpha W(p) + \beta + \gamma) = k_1(\beta + \gamma),$$
$$W(p) = \frac{(\beta + \gamma)}{2\alpha}\left(\frac{k_1}{p} - 1\right). \tag{10}$$

When consumers choose their partitions optimally, they aim to minimize k_1 while making sure that pricing in any other partition is not an equilibrium. Assume that both firms pricing in the same partition, say, (l_i, k_i), constitutes an equilibrium. Then we must have $i = 2, \ldots, n+1$:

$$l_i(2\alpha + \beta + \gamma) = k_i(\beta + \gamma) = \Pi_i.$$

Firms will not increase their price if $\Pi \geq \gamma$, but this is true since $k_1 \geq \gamma/(\beta + \gamma)$ implies $k_i \geq \gamma/(\beta + \gamma)$ or $\Pi_i = k_i(\beta + \gamma) \geq \gamma$. Hence the only way to prevent both firms from pricing in the same partition is to ensure that firms will have the incentive to deviate to a lower partition

$$\Pi_i < k_{i-1}(1-\gamma),$$
$$k_i(\beta+\gamma) < k_{i-1}(1-\gamma).$$

Thus the conditions $k_i((\beta+\gamma)/(1-\gamma)) < k_{i-1}$, $i = 2,\ldots,n+1$, guarantee that both firms pricing in the same partition (other than the lowest one) will not constitute an equilibrium. In other words, $k_n > (\beta+\gamma)/(1-\gamma)$, $k_{n-1} > ((\beta+\gamma)/(1-\gamma))^2$ and generally $k_i > ((\beta+\gamma)/(1-\gamma))^{n+1-i}$ guarantee that pricing only at the lowest partition constitute the unique symmetric equilibrium. Because the consumers have the incentive to induce a k_1 as low as possible, they would choose $k_i^* = ((\beta+\gamma)/(1-\gamma))^{n+1-i} + \varepsilon$, and the firms would price according to $W(p) = ((\beta+\gamma)/2\alpha)(k_1^*/p - 1)$ for $p \in (((\beta+\gamma)/(1-\beta-\gamma))k_1^*, k_1^*)$. To complete the proof, we need to show that the firms will not want to price in two different partitions. Following the logic of the proof of Proposition 2, if firm 1 prices in the interval (l_r, k_r) while firm 2 prices in (l_t, k_t) such that $r > t+1$. Then the payoff to firm 2 is at most $(1-\gamma)k_t$, but a deviation to pricing at k_{t+1} will yield a profit of $(1-\gamma)k_{t+1}$, which is higher so firms must price at most one category apart. Now, assume that firm 1 prices at (l_{t+1}, k_{t+1}) and firm 2 prices at (l_t, k_t). Then firm 1 makes a profit of at most γk_{t+1} if it lowers its price to k_t, it would make at least $k_t(\gamma+\beta)$. Because we assumed $k_1 \geq \gamma/(\beta+\gamma)$, we have $k_t > k_1 \geq \gamma/(\beta+\gamma) > (\gamma/(\beta+\gamma))k_{t+1}$, and thus $k_t(\gamma+\beta) > \gamma k_{t+1}$ and the deviation by firm 2 is profitable. Hence, in equilibrium, both firms will charge prices in the same category. Q.E.D.

Proof of Lemma 1. Let $W_j(p) = \Pr(p_j \geq p)$ be the probability that firm $j = 1, 2$ charges a price above p. In a symmetric equilibrium, $W_j(p) = W(p)$. As in Lemma 1, the equilibrium price support will have no mass points. The demand for a firm whose price approaches m_i from below in a symmetric equilibrium will be $\gamma + \beta W(k_{i-1}) + (2\alpha+\beta)W(m_i)$. Next, for the price $m_i + \epsilon$, (for $\epsilon \to 0$), the firm's demand changes discontinuously to $\gamma + \beta W(k_i) + (2\alpha+\beta)W(m_i)$. Therefore, any such price will be dominated by m_i, implying that the equilibrium distribution will have a hole from m_i up to some $v_i > m_i$. Define the minimum price charged in the category i to be b_i. Therefore, by the definition of the mixed-strategy equilibrium, we should have

$$\begin{aligned}\pi(b_i) &= b_i(\gamma+\beta W(v_i)+\beta W(b_i)+2\alpha W(b_i))\\ &= \pi(k_{i-1}) = k_{i-1}(\gamma+\beta W(v_{i-i})+\beta W(b_i)+2\alpha W(b_i)).\end{aligned}$$

Now, because $W(v_{i-i}) > W(v_i)$, it follows that $b_i > k_{i-1}$. Therefore, there is a hole in the distribution between b_i and k_{i-1}. The remaining prices in (b_i, m_i) and (v_i, k_i) are part of the equilibrium price support because of standard arguments as in Varian (1980) and Narasimhan (1988). Q.E.D.

Proof of Proposition 4. Note that for every positive α, the support of the equilibrium price distribution is comprised of two intervals in each category and each price in the support leads to the same profit. Thus, at the limit as $\alpha \to 0$, the profit from charging m_i must equal the profit from charging k_i $\forall i = 1,\ldots,n+1$. Charging a price of $p = k_{n+1} = 1$ gives a profit of γ, which is the equilibrium profit. When charging $p = k_i$, the expected profit for a firm is

$$\pi_i = (\gamma + 2s_{i+1}\beta)k_i = \gamma \quad i = 1, 2, \ldots, n. \tag{11}$$

From this, we can show that

$$s_i = \frac{\gamma(1-k_{i-1})}{2\beta k_{i-1}} \quad i = 2, 3, \ldots, n+1, \tag{12}$$

and $s_1 = \gamma(1-k_0)/(2\beta k_0) = 1$ by definition. By noting that $\Pr(m_i) = s_i - s_{i+1}$, we get

$$\Pr(m_i) = \frac{\gamma(1-k_{i-1})}{2\beta k_{i-1}} - \frac{\gamma(1-k_i)}{2\beta k_i} = \frac{\gamma}{2\beta}\left(\frac{1}{k_{i-1}} - \frac{1}{k_i}\right). \tag{13}$$

To calculate the values of the prices charged in each category, we use (2):

$$\begin{aligned} m_i &= \frac{\gamma}{\gamma+s_i\beta+s_{i+1}\beta} = \frac{\gamma}{\gamma+\gamma(1-k_{i-1})/(2k_{i-1})+\gamma(1-k_i)/2k_i}\\ &= \frac{2}{1/k_{i-1}+1/k_i}. \end{aligned} \tag{14}$$

Q.E.D.

Proof of Proposition 5. We select the optimal cutoffs k_i^* that satisfy the following conditions: (i) the equilibrium consumers' surplus are at the maximum, and (ii) no individual consumer will have the incentive to deviate from k_i^* given that the other consumers are also using these cutoffs. The first condition implies identifying the cutoffs that maximize the surplus of the limited-memory consumers S_β.[18] Clearly, the total surplus is $S = 1$ (a unit mass of consumers with a common reservation price of 1) and the total producer surplus of the two firms is $2\pi = 2\gamma$. The consumer surplus of the uninformed consumers is given by $S_\gamma = 2\gamma[1 - \sum_{i=1}^{n+1}\Pr(m_i)m_i]$. Thus the limited-memory consumers' surplus will be $S_\beta = S - S_\gamma - 2\pi = 1 - 4\gamma + 2\gamma\sum_{i=1}^{n+1}\Pr(m_i)m_i$. From (13) and (14), we can see that the expected price paid by the uninformed consumers is just $E_n(p) = \sum_{i=1}^{n+1}\Pr(m_i)m_i = (\gamma/\beta)\sum_{i=1}^{n+1}((k_i - k_{i-1})/(k_i + k_{i-1}))$. Hence the necessary condition for maximizing S_β is that for $i = 1,\ldots,n+1$, we have

$$\frac{\partial S_\beta}{\partial k_i} = \frac{2\gamma}{\beta}\frac{(k_{i-1}-k_{i+1})(-k_{i-1}k_{i+1}+k_i^2)}{(k_i+k_{i-1})^2(k_{i+1}+k_i)^2} = 0.$$[19]

This implies that the condition for a maximum is $-k_{i-1}^*k_{i+1}^* + k_i^{*2} = 0$. Thus the cutoff prices $\{k_i^*\}_{i=1}^n$ form a geometric sequence with the boundary conditions $k_{n+1}^* = 1$, $k_0^* = \gamma/(1-\gamma)$. Hence, for $i = 1,\ldots,n$, we have the optimal cutoffs to be

$$k_i^* = \left(\frac{\gamma}{1-\gamma}\right)^{(n+1-i)/(n+1)}. \tag{15}$$

Substituting Equation (15) into (14) and (13) yields the desired expression in the proposition for m_i^* and $\Pr(m_i^*)$. Next, we have to show the second condition that no consumer has the incentive to deviate from the optimal $\{k_i^*\}_{i=1}^n$

[18] Note that this is equivalent to the maximization of each limited-memory consumer's surplus.

[19] It is tedious but straightforward to calculate the Hessian, $\partial S_\beta^2/(\partial k_j \partial k_i)$, and show that the necessary conditions are indeed sufficient.

given that other consumers choosing $\{k_i^*\}_{i=1}^n$ as given above. The reasons are as follows: if a consumer deviates from the $\{k_i^*\}_{i=1}^n$ given above, firms' pricing strategy will not change because of the assumption that there is a large number of consumers in the market. Then, if such a deviation results in C_i containing either one or two prices from $\{m_i^*\}_{i=1}^{n+1}$ being charged at equilibrium, the consumer will have the same surplus. However, if such deviation leads C_i to contain three prices from $\{m_i^*\}_{i=1}^{n+1}$ (denoting them as $m_h > m_m > m_l$), the consumer can never be better off. Denote the new mean price of C_i to be $\bar{m}$. If $\bar{m} = m_m$, the consumer is indifferent. If $\bar{m} > m_m$, the consumer who observes m_l at the first firm will recall it as $\bar{m}$ and will buy from the second firm if the price there is m_m. If $\bar{m} < m_m$, the consumer who observes m_h at the first firm will recall it as $\bar{m}$ and will not buy from the second firm if the price there is m_m. In both cases, the consumer is worse off. Similarly, the consumer can never be better off if a deviation leads to C_i to contain more than three prices from $\{m_i^*\}_{i=1}^{n+1}$. Hence, an individual consumer has no incentive to deviate given other consumers' strategy. Therefore the $\{k_i^*\}_{i=1}^n$ given above are indeed the optimal cutoff prices in the symmetric equilibrium. Q.E.D.

Proof of Result. We can explicitly calculate the expected equilibrium price of each firm, which is

$$E_n(p) = \sum_{i=1}^{n+1} \Pr(m_i^*) m_i^* = \frac{2(n+1)\gamma}{(1-2\gamma)} \frac{[((1-\gamma)/\gamma)^{1/(n+1)}-1]}{[((1-\gamma)/\gamma)^{1/(n+1)}+1]}. \quad (16)$$

Direct calculation shows that $\partial E_n(p)/(\partial n) > 0$. A similar calculation shows that $\partial \sigma_n^2(p)/(\partial n) > 0$, where $\sigma_n^2(p)$ is the variance of the equilibrium price distribution. Q.E.D.

Proof of Proposition 6. When consumers' degree of memory is n, the maximum size of a category is $|1-k_n(n)| = 1-(\gamma/(1-\gamma))^{1/(n+1)}$ (recall that the categories are finer toward the lower end of the price range). Thus, for $n > N_1 = \ln(\gamma/(1-\gamma))/\ln(1-\delta) - 1$, we have that the size of the largest category is smaller than δ. Let $p \in C_j(n)$ be the mean of the prices in this partition; $m_j(n) \in C_j(n)$ is charged with positive probability and is δ close to p. Recall from Equation (12) that the probability of a firm pricing in category $C_j(n)$ or higher is just $s_j(n) = (\gamma/(1-2\gamma)) \times (1/(k_{j-1}(n)) - 1)$, where $k_{j-1}(n)$ is the lower cutoff bound of $C_j(n)$. Hence, if $p < m_j(n)$, we have $W_n(p) = s_j(n)$; otherwise, $W_n(p) = s_{j+1}(n)$. Recall that for the standard model of Varian (1980) or Narasimhan (1988), which is equivalent to the case of perfect recall, the probability of charging a price above p is $W(p) = (\gamma/(1-2\gamma))(1/p-1)$; thus we have (for $p < m_j(n)$)

$$\begin{aligned}|W_n(p) - W(p)| &= |s_j(n) - W(p)| = \frac{\gamma}{1-2\gamma}\left|\frac{1}{k_{j-1}(n)} - \frac{1}{p}\right| \\ &= \frac{\gamma}{1-2\gamma}\frac{|p-k_{j-1}(n)|}{k_{j-1}(n)p} < \frac{\gamma}{1-2\gamma}\frac{\delta}{p(p-\delta)} \\ &< \frac{\gamma}{1-2\gamma}\frac{\delta}{(\gamma/(1-\gamma))(\gamma/(1-\gamma)-\delta)}.\end{aligned}$$

If $p > m_j(n)$, then

$$\begin{aligned}|W_n(p) - W(p)| &= |s_{j+1}(n) - W(p)| = \frac{\gamma}{1-2\gamma}\left|\frac{1}{(k_j(n)} - \frac{1}{p}\right| \\ &< \frac{\gamma}{1-2\gamma}\frac{\delta}{p \times p} < \frac{\gamma}{1-2\gamma}\frac{\delta}{p(p-\delta)}.\end{aligned}$$

Define η such that

$$\frac{\gamma}{1-2\gamma}\frac{\eta}{\frac{\gamma}{1-\gamma}(\frac{\gamma}{1-\gamma}-\eta)} < \delta,$$

so for any $n > N_2 = \ln(\gamma/(1-\gamma))/(\ln(1-\eta)) - 1$, we get that $|W_n(p) - W(p)| < \delta$. Finally, $N = \max(N_1, N_2)$. Q.E.D.

Mixed-Strategy Equilibrium for Symmetric Categorization

Consider the case of a market with limited-memory consumers and uninformed consumers and investigate the case where $(1/(2(1-\gamma)))^n < 2\gamma$. For this case, we characterize a mixed-strategy equilibrium, which is as follows.

Given $\gamma/(1-\gamma) \le k_1 \le \cdots \le k_n \le k_{n+1} = 1$, we can characterize the completely mixed symmetric equilibrium to be given by the unique solution to the set of equations that will be derived below. Assume that each firm prices at k_j with probability q_j $j = 1, \ldots, n+1$. Then, the profit for firm j when pricing at k_j is given by $j = 1, \ldots, n$:

$$\Pi_i(k_j) = k_j\left(\gamma + \beta q_j + 2\beta \sum_{m=j+1}^{n+1} q_m\right).$$

Pricing $k_{n+1} = 1$ yields

$$\Pi_i(k_{n+1}) = (\gamma + \beta q_{n+1}).$$

Pricing at the k_1 yields

$$\Pi_i(k_1) = k_1[\gamma + \beta(2-q_1)].$$

For a totally mixed-strategy equilibrium, we need $\Pi_i(k_j) = \Pi = \mathit{constant}$ for $j = 1, \ldots, n+1$ as well as $\sum_{m=1}^{n+1} q_m = 1$. Thus we have $n+2$ equations with $n+2$ unknowns that possess a unique solution. Subtracting the equation for k_{j+1} from the one for k_j yields the following set of equations for $j = 1, \ldots, n$:

$$q_j + q_{j+1} = \frac{\Pi}{\beta}\left(\frac{1}{k_j} - \frac{1}{k_{j+1}}\right) > 0.$$

The specific solution to this set of equations depends on the parity of n. Thus suppose, for instance, that n is an odd number, then a possible solution can consist of consumers choosing $q_{j+1} = 0$, $j = n, n-2, n-4, \ldots$. The proof is as follows: if n is odd and $q_{j+1} = 0$, $j = n, n-2, n-4, \ldots$, then we have $\Pi = \gamma + \beta q_{n+1} = \gamma$, and

$$\begin{aligned} q_{n+1} &= 0, \\ q_n &= \frac{\Pi}{\beta}\left(\frac{1}{k_n} - \frac{1}{k_{n+1}}\right) = \frac{\gamma}{\beta}\left(\frac{1}{k_n} - 1\right), \\ q_{n-1} &= 0, \\ q_{n-2} &= \frac{\gamma}{\beta}\left(\frac{1}{k_{n-2}} - \frac{1}{k_{n-1}}\right), \\ &\ldots . \end{aligned}$$

Then

$$\begin{aligned} q_{n-1} + q_n = \frac{\gamma}{\beta}\left(\frac{1}{k_{n-1}} - \frac{1}{k_n}\right) &= \frac{\gamma}{\beta}\left(\frac{1}{k_n} - 1\right) \\ \to \frac{1}{2k_n} &= \left(1 + \frac{1}{k_{n-1}}\right). \end{aligned}$$

Similarly, we have

$$q_{n-3}+q_{n-2}=\frac{\gamma}{\beta}\left(\frac{1}{k_{n-3}}-\frac{1}{k_{n-2}}\right)=\frac{\gamma}{\beta}\left(\frac{1}{k_{n-2}}-\frac{1}{k_{n-1}}\right)$$
$$\to \frac{1}{2k_{n-2}}=\left(\frac{1}{k_{n-1}}+\frac{1}{k_{n-3}}\right),$$
$$\ldots,$$

where we can define $k_0=\gamma/(1-\gamma)$ (the lower bound).

If we solve the whole system of equations with consumers' surplus maximization, we will get

$$k_i^*=\left(\frac{\gamma}{1-\gamma}\right)^{(n'+1-i')/(n'+1)} \quad \text{for } i=n+1, n-1, n-3, \ldots,$$
$$\frac{1}{2k_{i-2}^*}=\left(\frac{1}{k_{i-1}^*}+\frac{1}{k_{i-3}^*}\right) \quad \text{for } i=n+2, n, n-2, \ldots,$$

where $n'=(n+1)/2$ and $i'=(i+1)/2$. We can then notice that k_i^* $(i=n+1,\ n-1,\ n-3,\ldots)$ are like the k_i^* in the asymmetric categorization model with limited memory and informed consumers and k_i^* $(i=n,\ n-2,\ n-4,\ldots)$ are like the m_i^* (means) in the asymmetric categorization model. It can also be easily noted that this solution gives consumers greater surplus than the pure-strategy equilibrium. As we have shown in the asymmetric case, the above equilibrium converges to the perfect recall solution as in Varian (1980) and Narasimhan (1988) as the number of categories increases beyond bound.

The Decision to Compare Prices Under Asymmetric Categorization

The asymmetric categorization model in this paper assumes that the limited-memory consumers have contact with both the firms by assumption. We now present the analysis of the case in which the decision process of the limited-memory consumers allows them to decide whether to compare prices at all after the price at the first firm is observed. Given the price that a consumer encounters at the first firm, the consumer has to decide whether to continue and obtain the price from the second firm. If the consumer encounters a low enough price at the first firm, then she might decide not to compare prices at the second firm.

The $n=0$ Case. We start with the case of $n=0$. Denote by u the threshold price below in which the limited-memory consumers will not obtain the price at the other firm. Define $\hat{m}$ to be the mean price conditional on $p>u$. Then the equilibrium support will be $(b,u)\cup(d,\hat{m})\cup(v,1)$, where $b<u<d<\hat{m}<v<1$. Define $W(p)$ as $\Pr(p\geq p)$ and $f(p)$ as the probability density function of price. We have that

$$\pi=\gamma+\beta W(\hat{m}) \quad (p=1),$$
$$\pi=[\gamma+\alpha W(v)+\beta W(\hat{m})]v \quad (p=v),$$
$$\pi=[\gamma+(\alpha+\beta)W(\hat{m})+\beta W(u)]\hat{m} \quad (p=\hat{m}),$$
$$\pi=[\gamma+\alpha W(d)+\beta W(\hat{m})+\beta W(u)]w \quad (p=d),$$
$$\pi=[\gamma+\alpha W(u)+\beta+\beta W(u)]u \quad (p=u),$$
$$\pi=[\gamma+\alpha+\beta+\beta W(u)]b \quad (p=b),$$
$$\pi=[\gamma+\alpha W(p)+\beta W(\hat{m})]p \quad (v<p<1),$$
$$\pi=[\gamma+\alpha W(p)+\beta W(\hat{m})+\beta W(u)]p \quad (d<p<\hat{m}),$$
$$\pi=[\gamma+\alpha W(p)+\beta+\beta W(u)]p \quad (b<p<u).$$

Using the above expressions and the fact that $W(\hat{m})=W(v)$, $W(d)=W(u)$, $W(1)=0$, and $W(b)=1$, and defining $W(\hat{m})=h$ and $W(u)=g$, we can compute that in the mixed-strategy equilibrium the following hold:

$$W(p)=\frac{\gamma+\beta h}{ap}-\frac{\gamma+\beta h}{a} \quad (v\leq p\leq 1),$$
$$W(p)=\frac{\gamma+\beta h}{ap}-\frac{\gamma+\beta h+\beta g}{a} \quad (d\leq p\leq \hat{m}),$$
$$W(p)=\frac{\gamma+\beta h}{ap}-\frac{\gamma+\beta+\beta g}{a} \quad (b\leq p\leq u).$$

Therefore, from the profit expressions at the extreme points in the distribution, we have that $v=(\gamma+\beta h)/(\gamma+\alpha h+\beta h)$, $\hat{m}=(\gamma+\beta h)/(\gamma+\alpha h+\beta h+\beta g)$, $d=(\gamma+\beta h)/(\gamma+\alpha g+\beta h+\beta g)$, $u=(\gamma+\beta h)/(\gamma+\alpha g+\beta+\beta g)$, and $b=(\gamma+\beta h)/(\gamma+\alpha+\beta+\beta g)$. From the definition of $\hat{m}$ and the expressions for $W(\cdot)$ derived above, we have

$$\hat{m}=\frac{\gamma+\beta h}{a}\left[\int_v^1\frac{1}{p}\,dp+\int_d^{\hat{m}}\frac{1}{p}\,dp\right]$$
$$=\frac{\gamma+\beta h}{a}\ln\left(\frac{1}{v}\frac{\hat{m}}{d}\right).$$

Now, from the definition of u and the expressions for $W(\cdot)$ derived, we can derive the optimal stopping rule that determines the threshold price below which the consumer will not compare prices:

$$0=\int_d^{\hat{m}}(u-p)f(p)\,dp+\int_b^u(u-p)f(p)\,dp$$
$$\Rightarrow u(1-h)=\frac{\gamma+\beta h}{a}\ln\frac{\hat{m}}{d}\frac{u}{b}.$$

Then, using the expressions of v, $\hat{m}$, d, u, and b derived earlier, we have

$$\frac{\gamma+\beta h}{\gamma+\alpha h+\beta h+\beta g}=\frac{\gamma+\beta h}{a}\ln\left(\frac{\gamma+\alpha h+\beta h}{\gamma+\beta h}\,\frac{\gamma+\alpha g+\beta h+\beta g}{\gamma+\alpha h+\beta h+\beta g}\right),$$
$$\frac{\gamma+\beta h}{\gamma+\alpha g+\beta+\beta g}(1-h)=\frac{\gamma+\beta h}{a}\ln\left(\frac{\gamma+\alpha g+\beta h+\beta g}{\gamma+\alpha h+\beta h+\beta g}\,\frac{\gamma+\alpha+\beta+\beta g}{\gamma+\alpha g+\beta+\beta g}\right).$$

The equilibrium (h,g) can be solved from the above equations given that by definition $g\geq h$. As $\alpha\to 0$, we have

$$\pi=\gamma+\beta h, \quad \pi=[\gamma+\beta h]v,$$
$$\pi=[\gamma+\beta h+\beta g]m, \quad \pi=[\gamma+\beta h+\beta g]d,$$
$$\pi=[\gamma+\beta+\beta g]b, \quad \pi=[\gamma+\beta+\beta g]u.$$

Hence $b=u$, $\hat{m}=d$, $v=1$. Also, as $\alpha\to 0$, $W(p)=(\gamma+\beta h)/ap-(\gamma+\beta h)/a$ $(v\leq p\leq 1)$ leads to $h=W(v)=(\gamma+\beta h)/ap-(\gamma+\beta h)/a=0$. Therefore, based on

$$\frac{\gamma+\beta h}{\gamma+\alpha g+\beta+\beta g}(1-h)=\frac{\gamma+\beta h}{a}\ln\left(\frac{\gamma+\alpha g+\beta h+\beta g}{\gamma+\alpha h+\beta h+\beta g}\,\frac{\gamma+\alpha+\beta+\beta g}{\gamma+\alpha g+\beta+\beta g}\right),$$

we can show that $g = 0$ and that this would result in the same solution for the case of the $n = 0$ case in the model of §3. Thus, $\hat{m} = d = v = 1$ and all the probability mass is on u. Hence, when $\alpha \to 0$, the profit equation at u is the same as the profit function at m in this paper. Consequently, all the results of this paper at $\alpha \to 0$ will be preserved.

The General Case of n Categories. In the general n-category case, for any $\alpha > 0$, we can prove that the stopping rule only applies to the lowest category in the equilibrium; i.e., $i = 1$. The proof is as follows. Suppose there exists some category $i > 1$ in which there is a threshold price u_i below which the limited-memory consumers do not compare prices with the other firm. Because $i > 1$ if the limited-memory consumers do not search upon encountering a price below u_i, then the firms have the incentive to undercut each other for only the perfect-memory consumers in that category. Therefore, (v_{i-1}, u_i) will be on the equilibrium support. This implies that $k_{i-1} \in (v_{i-1}, u_i)$ should be on the equilibrium support. However, consumers have incentive to search and compare prices at k_{i-1} but not at $k_{i-1} + \varepsilon$. Thus, there must be a mass point at k_{i-1} to make the profit at k_{i-1} equal to the profit at $k_{i-1} + \varepsilon$ as $\varepsilon \to 0$. However, a mass point at k_{i-1} cannot be part of an equilibrium because the other firm will have incentive to undercut it with a mass point at $k_{i-1} - \varepsilon$. Hence there is a contradiction, and so there is not an equilibrium in which the limited-memory consumers do not compare prices for $i > 1$. Therefore, in equilibrium, the threshold only applies to category 1 regardless of the size of α.

For category $i = 1$, the equilibrium price support is $(b_1, u) \cup (d, \hat{m}) \cup (v_1, k_1)$, where $b_1 < u < d < \hat{m} < v_1 < k_1$. $\hat{m}$ is the mean price and u is the threshold price below which the limited-memory consumers will not obtain a price at the second firm. Define $g = W(d) = W(u)$; the profit equations for category 1 corresponding to (1) in this paper become

$$\begin{aligned}
p_j = k_1&: \pi = (\gamma + w_1\beta + s_2\beta + 2s_2\alpha)k_1,\\
p_j = v_1&: \pi = (\gamma + w_1\beta + s_2\beta + 2w_1\alpha)v_1,\\
p_j = \hat{m}&: \pi = (\gamma + w_1\beta + g\beta + 2w_1\alpha)\hat{m},\\
p_j = d&: \pi = [\gamma + w_1\beta + g\beta + 2g\alpha]d,\\
p_j = u&: \pi = [\gamma + g\beta + \beta + 2g\alpha]u,\\
p_j = b_1&: \pi = [\gamma + g\beta + \beta + 2\alpha]b_1.
\end{aligned} \tag{17}$$

As in the $n = 0$ case, when $\alpha \to 0$, we have $v_1 \to k_1$, $\hat{m} \to v_1$, $d \to \hat{m}$, $b_1 \to u$, and only (b_1, u) is charged with positive probability. Therefore we can see that u is also the unconditional mean of category 1; i.e., $u = m_1$. Thus, when $\alpha \to 0$, Equations (17) here become

$$p_j = k_1: \pi = (\gamma + 2s_2\beta)k_1,$$

$$p_j = m_1: \pi = [\gamma + s_2\beta + \beta]m_1.$$

The above equations are the same as those for $p_j = k_1$ and $p_j = m_1$ at $\alpha \to 0$ given in this paper (without the optimal stopping rule). Hence, all results in this paper are preserved.

References

Alba, J., J. W. Hutchinson, J. Lynch. 1991. Memory and decision making. T. S. Robertson, H. Kassarjian, eds. *Consumer Behavior.* Prentice Hall, Englewood Cliffs, NJ, 1–49.

Amaldoss, W., J. R. Bettman, J. W. Payne. 2008. Biased but efficient: An investigation of coordination facilitated by asymmetric dominance. *Marketing Sci.* **27**(5) 903–921.

Bargh, J. A. 1994. The four horsemen of automaticity: Awareness, intention, efficiency, and control in social cognition. T. K. Srull, R. S. Wyer Jr., eds. *Social Cognition,* Vol. 3. Lawrence Erlbaum Associates, Hillsdale, NJ, 1–40.

Bargh, J. A. 1997. The automaticity of everyday life. R. S. Wyer Jr., ed. *The Automaticity of Everyday Life: Advances in Social Cognition,* Vol. 10. Lawrence Erlbaum Associates, Mahwah, NJ, 1–61.

Basu, K. 2006. Consumer cognition and pricing in the nines in oligopolistic markets. *J. Econom. Management Strategy* **15**(1) 125–141.

Baye, M. J., J. Morgan. 2002. Winner-take-all price competition. *Econom. Theory* **19**(2) 271–282.

Camerer, C., T.-H. Ho. 1999. Experience-weighted attraction learning in normal form games. *Econometrica* **67**(4) 827–874.

Camerer, C., T.-H. Ho, J.-K. Chong. 2004. A cognitive hierarchy model of games. *Quart. J. Econom.* **119**(3) 861–898.

Celsi, R. L., J. C. Olson. 1988. The role of involvement in attention and comprehension processes. *J. Consumer Res.* **15**(2) 210–224.

Dickson, P. R., A. G. Sawyer. 1990. The price knowledge and search of supermarket shoppers. *J. Marketing* **54**(3) 42–53.

Dow, J. 1991. Search decisions with limited memory. *Rev. Econom. Stud.* **58**(1) 1–14.

Edgeworth, F. 1925. The pure theory of monopoly. F. Edgeworth, ed. *Papers Relating to Political Economy,* Vol. 1. Burt Franklin, New York, 111–142.

Hirshleifer, D., I. Welch. 2002. An economic approach to the psychology of change: Amnesia, inertia, and impulsiveness. *J. Econom. Management Strategy* **11**(3) 379–421.

Iyer, G., D. Kuksov. 2010. Consumer feelings and equilibrium product quality. *J. Econom. Management Strategy* **19**(1) 137–168.

Iyer, G., A. Pazgal. 2003. Internet shopping agents: Virtual colocation and competition. *Marketing Sci.* **22**(1) 85–106.

Iyer, G., A. Pazgal. 2008. Procurement bidding with restrictions. *Quant. Marketing Econom.* **6**(2) 177–204.

Kalai, E. 1991. Artificial decisions and strategic complexity in repeated games. *Essays in Game Theory.* Academic Press, New York, 131–157.

Kreps, D. M., J. A. Scheinkman. 1983. Quantity precommitment and Bertrand competition yields Cournot outcomes. *Bell J. Econom.* **14**(2) 326–337.

Krishna, A., G. V. Johar. 1996. Consumer perceptions of deals: Biasing effects of varying deal prices. *J. Experiment. Psych. Appl.* **2**(3) 187–206.

Krishna, A., I. S. Currim, R. W. Shoemaker. 1991. Consumer perceptions of promotional activity. *J. Marketing* **55**(2) 4–16.

Lal, R., J. M. Villas-Boas. 1998. Price promotions and trade deals with multi-product retailers. *Management Sci.* **44**(7) 935–949.

Levitan, R., M. Shubik. 1972. Price duopoly and capacity constraints. *Internat. Econom. Rev.* **13**(1) 111–122.

Lim, N., T.-H. Ho. 2007. Designing price contracts for boundedly rational customers: Does the number of blocks matter? *Marketing Sci.* **26**(3) 312–326.

Mehta, N., S. Rajiv, K. Srinivasan. 2003. Price uncertainty and consumer search: A structural model of consideration set formation. *Marketing Sci.* **22**(1) 58–84.

Mehta, N., S. Rajiv, K. Srinivasan. 2004. Role of forgetting in memory-based choice decisions: A structural model. *Quant. Marketing Econom.* **2**(2) 107–140.

Monroe, K. B., A. Y. Lee. 1999. Remembering versus knowing: Issues in buyers processing of price information. *J. Acad. Marketing Sci.* **27**(2) 207–225.

Mullainathan, S. 2002. A memory-based model of bounded rationality. *Quart. J. Econom.* **117**(3) 735–774.

Narasimhan, C. 1988. Competitive promotional strategies. *J. Bus.* **61**(4) 427–449.

Ofek, E., M. Yildiz, E. Haruvy. 2007. The impact of prior decisions on subsequent valuations in a costly contemplation model. *Management Sci.* **53**(8) 1217–1233.

Raju, J. S., V. Srinivasan, R. Lal. 1990. The effects of brand loyalty on competitive price promotional strategies. *Management Sci.* **36**(3) 276–303.

Rao, R. C., R. V. Arjunji, B. P. S. Murthi. 1995. Game theory and empirical generalizations concerning competitive promotions. *Marketing Sci.* **14**(3) G89–G100.

Rosch, E. 1978. Principles of categorization. E. Rosch, B. B. Lloyd, eds. *Cognition and Categorization*. Lawrence Erlbaum Associates, Hillsdale, NJ, 27–48.

Rosch, E., C. B. Mervis. 1975. Family resemblances: Studies in the internal structure of categories. *Cognitive Psych.* **7**(4) 573–605.

Rosenthal, R. W. 1980. A model in which an increase in the number of sellers leads to a higher price. *Econometrica* **48**(6) 1575–1579.

Rubinstein, A. 1993. On price recognition and computational complexity in a monopolistic model. *J. Political Econom.* **101**(3) 473–484.

Simon, H. A. 1987. Bounded rationality. J. Eatwell, M. Milgate, P. Newman, eds. *The New Palgrave: A Dictionary of Economics*. W. W. Norton & Company, New York, 15–18.

Spiegler, R. 2006. Competition over agents with boundedly rational expectations. *Theoret. Econom.* **1**(2) 207–231.

Varian, H. R. 1980. A model of sales. *Amer. Econom. Rev.* **70**(4) 651–659.

Villas-Boas, J. M. 1995. Models of competitive price promotions: Some empirical evidence from the coffee and saltine crackers markets. *J. Econom. Management Strategy* **4**(1) 85–107.