Sunk Cost Effect, Self-control, and Contract Design

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Abstract

This paper examines the role of the sunk cost effect as a commitment device in mitigating the self-control problem and analyzes its implications for optimal contract design. Consumers may anticipate the effect *ex-ante*, and strategically use it to mitigate their self-control problems. While the sunk cost effect may lead to a loss of consumption flexibility in the event of high consumption costs, it can serve as a commitment device to enforce self-control. A firm's optimal policy should balance the consumer's demand for flexibility in consumption with the demand for commitment. Under a simple fixed-fee contract sunk costs have a non-monotonic effect on profits for investment goods: i.e., profits first decrease and then increase with the sunk cost effect. The firm can use a two-part tariff or a refundable fixed-fee contract to mitigate the sunk cost effect. The paper also compares the implications of alternative psychological mechanisms underlying the sunk cost effect (regret-based vs. memory-cue-based) for contract design.

Keywords: Sunk cost effect, Present bias, Contract design, Two-part tariff, Refund policy

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Introduction

In many markets, consumers pay a fixed upfront fee to access services and products, for instance, paying the membership fee for a health club, buying a mobile app, paying to take an online course, etc. The fixed fee is typically non-refundable and independent of future usage, therefore is a sunk cost (Dick and Lord, 1998). Since it is irreversible, a sunk cost should be irrelevant to decision-making. Thaler (1980) illustrated with a succinct example of how a consumer violates this principle in decision making:

> *"A man joins a tennis club and pays a \$300 yearly membership fee. After two weeks of playing he develops a tennis elbow. He continues to play (in pain) saying 'I don't want to waste the \$300!'"*

The standard economic argument is that the decision whether to continue playing should only depend on the marginal benefits and marginal costs of playing, and the membership fee (a sunk cost) should not matter. A *sunk cost effect* arises when the consumer has a dis-utility for changing a chosen plan of action because of a sunk cost that has already been incurred and is irreversible, even though a rational consumer should ignore such a cost and only consider the marginal benefits and marginal costs of the action. There is empirical evidence to show that people make decisions conditional on sunk cost, which leads to sub-optimal decisions such as over-consumption (Ho, Wu and Zhang, 2020; Ho, Png and Reza, 2018; Just and Wansink, 2011) and an escalation of commitment in investment (Camerer and Weber, 1999; Staw, 1981). Although it is a textbook example of inconsistent (irrational) behavior in economics (Mankiw, 2011), the philosopher Robert Nozick has argued that consumers can benefit from the sunk cost effect (Nozick, 1994), especially when they have an issue of 'under-consumption' due to self-control problems. He offered the following example:

> *"If I think it would be good for me to see many plays or attend many concerts this year, and I know that when the evening of the performance arrives I frequently will not feel like rousing myself at that moment to go out, then I can buy tickets to many of these events in advance... Since I will not want to waste the money I*

have already spent on the tickets, I will attend more performances than I would if I left the decisions about attendance to each evening."

For investment goods such as health clubs or online education, consumers often face under-consumption problems due to the lack of self-control. For example, while the consumer would prefer to use a health club at a certain frequency in a future period, when that period arrives her actual usage frequency is lower. Although the sunk cost effect can give rise to the over-consumption problem as illustrated in Thaler (1980), it also has the potential to serve as a commitment device to countervail one's self-control problem. Nozick's example above describes precisely such an effect: he anticipates that he will not incur the cost of "rousing himself to go out" on the evening of the concert. However, if he buys the tickets in advance, the sunk cost effect would kick in, counteracting his cost of "rousing himself to go out" as he would not want to waste what he has already spent.

Two questions arise in the presence of the sunk cost effect. First, do consumers anticipate that they will suffer from the sunk cost effect associated with the upfront fixed fee? Second, if this were to be so, what is the optimal contract design and the resulting firm profits when facing consumers who are subject to the sunk cost effect and self-control problem?

There exists anecdotal and empirical evidence to suggest that individuals exploit their own future sunk cost effect to exert more effort in planned tasks. Steele (1996) and Walton (2002) recount stories of individuals who buy expensive exercise machines or gym memberships, reasoning that the high cost will motivate them to exercise more in the future. Using user engagement data on Coursera, a massive open online courses (MOOCs) platform, Goli, Chintagunta and Sriram (2022) show that users might use the sunk cost effect as a commitment device. Coursera provides free access to course content, homework, and a final grade. Customers can pay to acquire a certificate upon course completion. On the platform, because users can opt to defer payment until 24 days after the commencement of the course, consumers are always weakly better off by deferring the payment. Despite this, a significant fraction of paying users make the payment right after the start of the course, which is suggestive of the fact that consumers may be using early payment as a potential commitment device to increase their future engagement. The authors find that solely paying for the courses upfront increases user engagement by 17% to 20% in the weeks following the payment.

To strategically use the sunk cost effect as a commitment device, a consumer should *ex-ante* anticipate the future sunk cost effect when they sign the contract with a fixed fee. Prior studies have demonstrated that people experience the sunk cost effect *ex-post*, and it is an empirical question whether they can anticipate the sunk cost effect *ex-ante*. To examine whether consumers are able to anticipate that the sunk cost will influence their future consumption, we conducted a pilot study involving 186 online participants (see Appendix A for more details). We asked participants to imagine that they are planning to join a health club with a regular membership fee of $2500 a year. Because of a promotion, they only need to pay $2000 ($1500/$500/$100). For each of the four discounted price points, we then asked "*how often do you think you will go to the club?*". We found that as the price was increased from $100 to $2000, participants forecast that they would attend the club around three times more per month.

The model in the present paper consists of a firm selling to a consumer who is subject to both the sunk cost effect and potential self-control problems. Specifically, the consumer has a regret dis-utility when she does not carry out the planned action after paying the upfront fixed fee (e.g., failing to attend the gym after paying the membership fee), and this captures the sunk cost effect. This is consistent with the "aversion to waste" argument by Arkes and Blumer (1985) that the sunk cost effect arises because the consumer does not want a cost incurred in the past to be wasted. The lack of self-control stems from present bias (O'Donoghue and Rabin, 1999; Strotz, 1955), namely, that the consumer cannot resist immediate urges and temptation, behaving myopically in the short run, even though she would prefer otherwise in the long run. That is, the preference is time inconsistent between short-run and long-run self.

We first characterize the optimal fixed-fee contract for investment goods and the implications for the firm and the consumers. After paying the fixed fee, the consumer faces a stochastic consumption cost if she consumes, and a regret dis-utility if she does not. On the one hand, the sunk cost effect of the fixed fee curtails the consumer's consumption flexibility because the consumer may end up consuming even when the realization of the consumption cost is high. As a result, the sunk cost effect reduces the consumer's willingness to pay. On the other hand, the effect can serve as a commitment device to mitigate the under-consumption due to the self-control problem, which increases the willingness to pay. The firm's pricing policy trades off this demand for flexibility with the demand for commitment against the self-control problem.

This trade-off leads to an interesting non-monotonic effect of the sunk cost effect on the optimal fixed fee: as the degree of the sunk cost effect increases, the optimal fixed fee first decreases and then increases. When the degree of the sunk cost effect is small, any increase in the sunk costs increases the regret dis-utility from non-consumption by more than the counter-veiling commitment benefit of mitigating the self-control problem. As a result, the optimal fixed fee decreases with the sunk cost effect. But when the degree of the sunk cost effect is large enough, it can provide sufficient commitment power to mitigate the self-control problem without causing too much inefficient over-consumption. This allows the firm to charge a higher fixed fee that balances the demand for commitment versus that for flexibility. As a result, the optimal fixed fee increases with the sunk cost effect. As a contrast, for leisure goods (e.g., mobile apps for gaming), the firm's optimal fixed fee and profits always decrease with the sunk cost effect. Here the sunk cost effect aggravates the existing concern of over-consumption problems due to the lack of self-control.

Next, we examine the profit implications of the sunk cost effect. We compare the two most commonly observed contractual forms in markets with self-control problems – the fixed fee to a variable pay-per-use contract. The pay-per-use contract which does not induce the sunk cost effect, also provides a benchmark for comparing the fixed-fee contract. When the firm incurs a marginal operating cost each time a consumption takes place, the firm has to balance the upfront lump-sum fee and expected consumption amount associated with the sunk cost effect. The analysis shows that the fixed-fee contract yields higher equilibrium profits than the pay-per-use contract when the firm's marginal cost is low and the consumer's self-control problem is moderate.

We then consider the more general two-part tariff contract. The pay-per-use fee in the two-part tariff can be potentially used to manage the over-consumption caused by the fixed-fee-induced sunk cost effect. While the overall profits will obviously be higher under the more general contract, comparative statics of the firm's profits uncover negative internalities which results in firm profits decreasing with the sunk cost effect even though the pay-per-use fee in the two-part tariff can be used to counteract the over-consumption problem. We also investigate a contract with a refund policy which allows the consumer to cancel the contract and get a refund. Providing a refund can mitigate the negative impact of the sunk cost effect because it insures the consumer in the event of no consumption. However, like in the case of the two part-tariff the presence of the sunk cost effect reduces the firm's profits

We extend the analysis to investigate the implication of an alternative psychological mechanism of the sunk cost effect. Baliga and Ely (2011) and Hong, Huang and Zhao (2019) argue that sunk cost can serve as a device for coping with limited memory and can be a memory/perceptual cue for the consumer regarding the importance of the associated activity. Specifically, the sunk cost only affects the likelihood of consumption but not the utility. When sunk costs act as a memory cue, the optimal fixed fee increases if the extent of the sunk cost effect is sufficiently low, but decreases if the extent of the effect is sufficiently high. This is in contrast to the case of the regret-based sunk cost effect where only a higher level of the sunk cost effect can increase the optimal fixed fee. We find that if the sunk cost effect is memory-cue-based (and not regret-based), offering a two-part tariff contract or a contract with a refund can achieve the first-best profits.

Related Research

The existing research has focused on empirically documenting the sunk cost effect and on explaining the underlying mechanisms of the effect. Several accounts were proposed, such as diminishing sensitivity toward loss (Thaler, 1980), aversion to being wasteful (Arkes and Blumer, 1985), and need to justify a prior action (Staw, 1981). Recently, Baliga and Ely (2011) and Hong et al. (2019) show that the sunk cost effect can endogenously arise as a cue for coping with limited memory.[1] However, there is little work on whether consumers anticipate the effect ex-ante, and the manner in which the anticipated sunk cost effect affects contract design. This paper highlights consumer sophistication and the awareness of the effect, and how it affects consumer choice and the optimal contract design of the firm.

This paper is also related to the large literature on the demand for commitment (DellaVigna and Malmendier, 2004; Jain, 2009; Jain and Li, 2018; Wertenbroch, 1998). Given the lack of self-control, consumers may have the incentive to seek commitment devices (Bryan, Karlan and Nelson, 2010; Carrera, Royer, Stehr, Sydnor and Taubinsky, forthcoming) which helps them to counter their self-control problems. Here we investigate the role of the sunk cost effect as a commitment device. In a related vein, Jain (2009) investigates how consumers should set optimal goals to achieve objectives in the presence of self-control problems. Optimal goals arise because when a consumer does not achieve the goals, she suffers costs from negative emotions. Here the sunk cost effect drives consumer decisions, but the sunk cost is endogenous to firm incentives – i.e., the fixed fee is strategically set by the firm.

This paper is related to the research on price tariff choice biases (e.g., Lambrecht and Skiera (2006) and Miravete (2003)) and usage behavior in telecommunication markets (e.g., Danaher (2002) and Iyengar, Jedidi, Essegaier and Danaher (2011)). Here, we focus on the markets where consumers are concerned about self-control problems. We show that in these markets, the sunk cost effect can be a potential cause of tariff-choice biases. Because the

[1] Hong et al. (2019) model a signaling game between the current self and a future self that suffers from limited memory and self-control problems.

sunk cost effect increases (decreases) the willingness to pay for the fixed fee in the investment (leisure) goods markets, our framework offers an alternative viewpoint regarding two types of tariff-choice biases found in empirical studies: a flat-rate bias in the investment goods markets (e.g., health club attendance), and a pay-per-use bias in the leisure goods markets (e.g., digital content consumption). Iyengar et al. (2011) empirically investigated the role of the access fee in a two-part tariff on usage. Their results supported the combination of the notion of the mental depreciation and the reference price effect (Heath and Fennema, 1996). Consumers tend to mentally depreciate the sunk cost to the future per-use fee, so that they can align the benefits and costs when they evaluate the entire transaction experience. The mental depreciation process shares similar underlying psychological features with the sunk cost effect that consumers try to make the sunk cost worthwhile.

This paper is complementary to Zhang (2015) and Jain and Chen (2022) who also investigate the interaction of the sunk cost effect and time inconsistency and its implication for pricing. Zhang (2015) analyzes the sunk cost effect as a cue that increases the consumption probability and focuses on pricing for investment and leisure goods. Jain and Chen (2022) study firm pricing in a market where heterogeneous consumers have different valuations of a durable product. They look at how pricing and profits are influenced by the self-control problem and the sunk cost effect. The focus here is on examining the optimal contractual design and explaining commonly observed contracts, namely, the fixed-fee contract (which induces the sunk cost effect) and pay-per-use contract (which does not) to understand the trade-offs facing the firms. We also study the role of a two-part tariff and refund policy under the sunk cost effect and show how different psychological foundations of the sunk cost effect have different implications for contract design. In addition, we examine the profits implications of a screening contract menu when there exist heterogeneous consumers in their sensitivity to the sunk cost effect.

The rest of the paper runs as follows: we first set up the model and present the main insights on how the optimal fixed fee is driven by the interaction between sunk cost effect and self-control and discuss the profit implications of the sunk cost effect under the fixed-fee, the pay-per-use, and the two-part tariff contract. In the extensions, we explore the implication of the sunk cost effect for the refund policy and the screening contract as two possible extensions to the main contract design. Before concluding, we investigate the implication of the memory cue as an alternative psychological mechanism underlying the sunk cost effect.

Model Setup

The market consists of a firm selling to a consumer. The firm sells an investment good where the consumption incurs an immediate cost but results in a long-run benefit. Examples of such goods include going to the health club, visiting a dentist, and taking a professional course, among others. The consumer exhibits two behavioral biases: present bias and sunk cost effect.

The present bias refers to the tendency that the consumer behaves overly impatiently for immediate rewards or costs and represents the self-control problem. In the literature, the present bias is modeled using the quasi-hyperbolic $\beta - \delta$ preferences (Laibson, 1997; O'Donoghue and Rabin, 1999; Phelps and Pollak, 1968; Strotz, 1955). Specifically, the $\beta - \delta$ model allows for the standard discounting with δ and captures the present-biased preference with β as follows. At any period t, benefits and costs for $t+1, t+2, ...$ are discounted with $\beta\delta, \beta\delta^2, ...$ with $0 \leq \beta \leq 1$. The discount factor β captures "present biasedness": the discounting between the present period and the next period is $\beta\delta$, while the discounting between any two consecutive periods in the future is δ. Since $\beta\delta \leq \delta$, the consumer is more impatient when considering the trade-offs involving immediate rewards/costs than when considering the trade-offs involving delayed rewards/costs. This discrepancy between the immediate and the future discounting gives rise to a time-inconsistent preference. The smaller is β, the more significant is the time inconsistency in preference. This time inconsistency in preferences results in a potential conflict between today's preferences and the preferences that will be held in the future, and generates the issue of self-control. The quasi-hyperbolic discounting allows for both a standard time-consistent preference ($\beta = 1$) and a present-biased preference ($\beta < 1$).

The sunk cost effect is captured by a dis-utility associated with the sunk cost if the consumer does not consume, which captures the negative feelings analogous to a psychological "regret effect". This modeling choice is consistent with the leading explanation that the sunk cost effect can arise as a result of the aversion to being wasteful (Arkes and Blumer, 1985). In Extensions, we analyze a memory-cue-based model and compare it with the regret-based setup.

In the main model, we consider full consumer sophistication and rational expectations on both the sunk cost effect and time inconsistency. In particular, the consumer anticipates the sunk cost effect and our survey study shows that consumers do. Also, the consumer is sophisticated in anticipating the self-control problem. So, anticipating the self-control problem, the consumer chooses whether or not to take the contract offered by the firm with the hope that when the time to consume comes, the upfront-fee-induced sunk cost effect will overcome her present-bias-induced procrastination facilitating consumption.

Consider an interaction between a consumer and a firm (e.g., a health club) which has a marginal operating cost of a. Following DellaVigna and Malmendier (2004), we set up a two-period model with the following timing (see Figure 1). In period $t = 0$, the firm offers a contract with an upfront fixed fee of L and nothing to pay at the time of consumption. If the consumer rejects the offer, the firm receives zero profit, and the consumer gets a reservation utility $\underline{u}$. If the consumer signs the contract, she pays the fixed fee of L.

In period $t = 1$, the consumer observes the private consumption cost c. The cost is stochastic with a range $0 < c \leq \bar{c}$ follows the distribution $F(\cdot)$, which is smooth, continuous, and differentiable. For example, the cost of attending the gym is related to some random factors such as weather, the consumer's physical condition of the day, etc.. If the consumer decides to proceed with consumption, the consumption cost realization c is incurred and the consumer receives a benefit $0 < b \leq \bar{c}$ subsequently in period $t = 2$. If the consumer decides not to proceed with consumption, a sunk-cost-induced dis-utility $-\gamma L$ is experienced.[2] Note that L is the magnitude of sunk cost and γ is the parameter that captures the magnitude of the sunk cost effect.

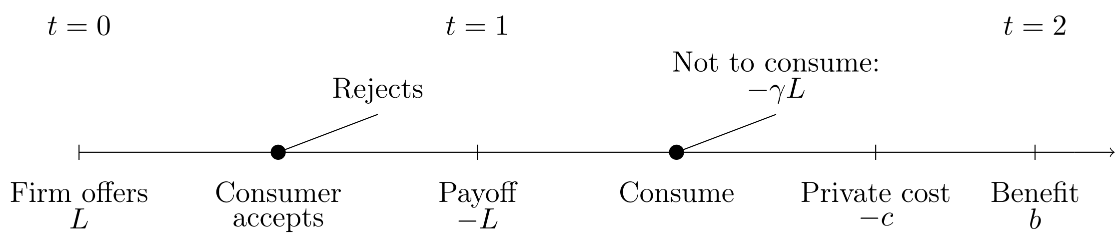

Figure 1: Timing of the Game

We derive the expected utility of the consumer as follows. Suppose the consumer only exhibits time inconsistency. At $t = 0$, the consumer thinks she will consume the product (say, go to the gym) at $t = 1$ if the net discounted benefit is positive, i.e., $\delta\beta(\delta b - c) \geq 0$ which implies $c \leq \delta b$. At $t = 1$, the consumer will *actually* consume if $\beta\delta b - c \geq 0$, or $c \leq \beta\delta b$. If $\beta < 1$, for the consumers with $\beta\delta b < c < \delta b$, consumption is desirable at $t = 0$ but undesirable at $t = 1$, a change in preference due to time inconsistency. Given the distribution of the stochastic cost c, the probability that the consumer will consume is $F(\beta\delta b)$. Note that the time-consistent consumer has a higher likelihood of consuming because $F(\delta b) > F(\beta\delta b)$ if $\beta < 1$. If the consumer exhibits the sunk cost effect, the consumer chooses to consume the good if $\beta\delta b - c \geq -\gamma L$, or $c \leq \beta\delta b + \gamma L$. As a result, the probability of consumption is $F(\beta\delta b + \gamma L)$. Putting all together, the consumer's expected utility from signing the contract is:

$$E(U_{t=0}) = \delta\beta \left(-L + \int_0^{\beta\delta b + \gamma L} (\delta b - c)\, \mathrm{d}F(c) + \int_{\beta\delta b + \gamma L}^{\bar{c}} (-\gamma L)\, \mathrm{d}F(c) \right) \tag{1}$$

As shown in Equation 1, the sunk cost effect has two impacts on the consumer's expected utility. First, the sunk cost effect increases the probability of consumption. Recall that

[2] A linear specification of the regret dis-utility was also used in Filiz-Ozbay and Ozbay (2007) and Jiang, Narasimhan and Turut (2017).

without the sunk cost effect, the probability of consumption is $F(\beta\delta b)$, whereas in the presence of the sunk cost effect, the probability becomes $F(\beta\delta b + \gamma L) > F(\beta\delta b)$. The probability of consumption is affected by both the sunk cost effect γ and time inconsistency governed by β. Second, the sunk cost effect directly affects the consumer's utility. Due to the randomness of the consumption cost, there is some likelihood that the consumer will experience dis-utility $-\gamma L$. Similar to previous research (DellaVigna and Malmendier, 2004; Jain, 2009; O'Donoghue and Rabin, 1999), we assume $\delta = 1$ without loss of generality of the insights.

We begin by demonstrating the impact of the sunk cost effect in the model. Consider the first-order partial derivative of $E(U_{t=0})$ with respect to γ:

$$\begin{aligned}\frac{\partial E(U_{t=0})}{\partial \gamma} = \beta \Bigg[& bLf(\beta b + \gamma L) - (\beta b + \gamma L)Lf(\beta b + \gamma L) \\ & + \gamma L^2 f(\beta b + \gamma L) - \int_{\beta b + \gamma L}^{\bar{c}} L \, \mathrm{d}F(c) \Bigg]\end{aligned} \tag{2}$$

As can be seen, Equation 2 has four terms, which represent a decomposition of the sunk cost effect on consumer expected utility. The first two terms capture the commitment effect — the impact of the sunk cost effect on the expected utility when the consumer consumes. The first term is positive, which is related to the increase in the probability of experiencing the benefit. The second term is negative, which is related to the increased chance of experiencing the cost c. The last two terms capture the impact of the sunk cost effect on the expected regret dis-utility when the consumer fails to consume. The third term is positive, which shows that the increase of the sunk cost effect reduces the likelihood of experiencing regret dis-utility by increasing the likelihood of consuming. The fourth term is negative, which captures the increased dis-utility associated with not consuming. Rearranging Equation 2 gives:

$$\frac{\partial E(U_{t=0})}{\partial \gamma} = \beta \left[\underbrace{(1-\beta)bLf(\beta b + \gamma L)}_{\text{Commitment effect } (+)} - \underbrace{\int_{\beta b+\gamma L}^{\bar{c}} L \, \mathrm{d}F(c)}_{\text{Regret effect } (-)} \right] \tag{3}$$

Equation 3 shows the impact of the sunk cost effect on consumer expected utility is a combination of two effects: first, a non-negative commitment effect. It shows that when the consumer is time inconsistent $\beta < 1$, the commitment effect can have a positive impact on the expected utility because it drives more consumption to mitigate the self-control problem. The second effect is related to regret when the consumer does not consume. Both effects depend on the consumer's degree of time inconsistency.

LEMMA 1. *(i) If the consumer is time consistent (i.e., $\beta = 1$), the sunk cost effect has a negative impact on the consumer's expected utility (i.e., $\frac{\partial E(U_{t=0})}{\partial \gamma} < 0$);*

(ii)If the consumer is time inconsistent ($\beta < 1$) and $F(\cdot)$ is weakly convex, the consumer's expected utility first decreases and then weakly increases with the sunk cost effect γ.

The proof is in Appendix B. If the consumer is time consistent (i.e., $\beta = 1$), a larger sunk cost effect always results in a lower expected utility. This is because a time-consistent consumer does not need a commitment device, but the sunk cost effect generates regret if the consumer does not consume. As the degree of time inconsistency increases, the benefit of the commitment effect looms larger since the sunk cost effect mitigates the self-control problem and motivates the consumer to consume. However, a higher degree of time inconsistency can also have a counter-veiling effect by causing the consumer to fail to consume, resulting in regret dis-utility if the consumer has already paid L. The overall impact of the sunk cost effect on consumer utility depends on which of these two effects dominates. When the sunk cost effect is small, the benefit from the commitment effect is small while the regret effect is large. As a result, the consumer's expected utility decreases with the sunk cost effect. As the sunk cost effect further increases, the commitment effect starts to dominate the regret effect, and the consumer's expected utility increases with the sunk cost effect.[3] When setting the optimal fixed fee, the firm has to consider the trade-off between the commitment effect and regret effect, both of which depend on the degree of the sunk cost effect.

Main Results

The Fixed-fee Contract

Now, let's consider the firm's decision problem of finding an optimal L to maximize profits. If the consumer signs the contract, the firm gains expected profits $E[\Pi_{t=0}]$ from charging an upfront fixed fee L. If the consumer consumes, the firm incurs a marginal cost a. The firm maximizes its profits subject to the consumer's and its own participation constraints. The firm solves the following decision problem:

$$\max_L \left(L + \int_0^{\beta b + \gamma L} (-a) \, \mathrm{d}F(c) \right) \tag{4}$$

subject to the consumer's participation constraint:

$$\beta \left(-L + \int_0^{\beta b + \gamma L} (b - c) \, \mathrm{d}F(c) + \int_{\beta b + \gamma L}^{\bar{c}} (-\gamma L) \, \mathrm{d}F(c) \right) \geq \beta \underline{u}$$

and firm's participation constraint:

$$L + \int_0^{\beta b + \gamma L} (-a) \, \mathrm{d}F(c) \geq 0$$

Without loss of generality, we assume $\underline{u} = 0$. To obtain a closed-form solution for the optimal L, we assume $c \sim U[0, 1]$. Throughout this analysis, we assume that the marginal cost a is sufficiently low (i.e., $a \leq \bar{a}$) and the benefit is sufficiently high (i.e., $\frac{3}{4} \leq b < 1$) so that the firm always gets non-negative expected profits over the full range of parameters

[3] If the regret dis-utility is represented by a more general functional form: $-g(\gamma, L) < 0$, where $\partial g(\gamma, L)/\partial \gamma > 0$ and $\partial g(\gamma, L)/\partial L > 0$, Lemma 1 holds if $\frac{\partial^2 g(\gamma, L)}{\partial \gamma^2} = 0$ (see Appendix B).

and the participation constraint of the firm and the consumer is satisfied.[4] We also restrict $0 \leq \gamma \leq 1$ because the regret of paying γL cannot exceed L.[5] By solving the optimal fixed fee we have:

$$L^* = \begin{cases} \frac{\beta(2-\beta)b^2}{(1+\gamma-\gamma b)+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}} & \text{if } 0 \leq \gamma \leq \bar{\gamma}_1 \\ b - \frac{1}{2} & \text{if } \gamma > \bar{\gamma}_1 \end{cases} \tag{5}$$

PROPOSITION 1. *(i) If the degree of time inconsistency is sufficiently large (i.e.,* $0 \leq \beta \leq \bar{\beta}_1$*), the optimal fixed fee decreases with the sunk cost effect for* $\gamma \leq \bar{\gamma}_1$*.*

(ii) If the degree of time inconsistency is sufficiently small (i.e., $\bar{\beta}_1 < \beta < 1$*), the optimal fixed fee first decreases when* $0 < \gamma < \bar{\gamma}_2$ *and then increases with the sunk cost effect when* $\bar{\gamma}_2 \leq \gamma \leq \bar{\gamma}_1$*.*

(iii) If the degree of the sunk cost effect is larger than the threshold $\bar{\gamma}_1$*, the optimal fixed fee* L^* *is a constant which induces the consumer to consume with probability* 1*.*

Here $\bar{\gamma}_1 = \min\{\frac{2(1-\beta b)}{2b-1}, 1\}$, $\bar{\gamma}_2 = \frac{2(1-b)}{2b-1-b^2(1-\beta)^2}$, and $\bar{\beta}_1 = 1 - \frac{\sqrt{4b-3}}{b}$. The proof can be found in Appendix C.[6]

If the consumer has a severe time-inconsistency problem (i.e., $0 \leq \beta \leq \bar{\beta}_1$), the regret effect would dominate the commitment effect because the consumer has a high chance of failing to consume due to time inconsistency. The higher the degree of the sunk cost effect, the larger the regret dis-utility the consumer will experience, which reduces the willingness to pay.

If the consumer's time inconsistency is not too severe ($\bar{\beta}_1 < \beta < 1$), the optimal fixed fee and the sunk cost effect have a U-shape relationship (see Figure 2). When the sunk cost effect is small, the consumer has a relatively large chance of experiencing the dis-utility

[4] $\bar{a} = min\{b - \frac{1}{2}, \beta b, \frac{(2-\beta)b}{\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}+(1+\gamma+\gamma b-\beta\gamma b)}\}$.

[5] Allowing for $\gamma > 1$ does not qualitatively change the main insights of the paper.

[6] Under the general dis-utility form $-g(\gamma, L)$, the main results on the relationship between the sunk cost effect and the optimal fixed fee in Proposition 1 hold if $g(0, L) = g(\gamma, 0) = g(0, 0) = 0$ and $0 < \frac{\partial g}{\partial L} < 1$ (see the proof in Appendix C).

$-\gamma L$. In addition, although the commitment effect can drive more consumption, the positive commitment effect is not sufficiently large to compensate for increased regret dis-utility. As a result, the willingness to pay and the optimal fixed fee decrease with the sunk cost effect. As the sunk cost effect increases, the consumer will be more likely to consume, resulting in a higher chance of getting the benefit from consuming the investment goods and a lower chance of experiencing regret. Now the commitment effect can dominate the regret effect. As a result, the willingness to pay and the optimal fixed fee are increasing with the sunk cost effect. Note that the presence of the sunk cost effect ($\gamma > 0$) can lead to a higher or lower optimal fixed fee compared to the case where the sunk cost effect is absent ($\gamma = 0$). The presence of the sunk cost effect induces a higher optimal fixed fee when the sunk cost effect is sufficiently high and the consumer's time inconsistency is small.[7]

When the sunk cost effect is sufficiently large (i.e., $\gamma > \bar{\gamma}_1$), the consumer will consume with probability one. In such a case, the firm charges a fixed fee which equals the expected benefits of the consumer who consumes irrespective of the consumption cost c, i.e., $L^* = b - \frac{1}{2}$.

The results have interesting implications. For fully time-consistent consumers and for consumers with severe time-inconsistency problems, the fixed-fee-induced sunk cost effect leads to a lower equilibrium fixed fee. However, for the consumer whose time inconsistency is small, a sufficiently large sunk cost effect associated with the fixed-fee contract can increase the willingness to pay for the contract. Recent empirical studies found estimates of the present-bias parameter $\beta \approx 0.9$ for unpleasant tasks (Augenblick, Niederle and Sprenger, 2015; Imai, Rutter and Camerer, 2021), which suggests that consumer time inconsistency is small in such a market. Thus, the analysis can provide a possible rationalization of the fixed fee bias in DellaVigna and Malmendier (2006). The authors found that the consumers pay more than \$70 monthly membership and on average attend the gym 4.8 times a month, resulting in \$17 per visit. However, the gym also offers a pay-per-use contract that costs only \$10 per visit. In DellaVigna and Malmendier (2006), this is explained by consumer

[7]Here, $\gamma > \frac{4(1-b)}{(2-\beta)\beta b^2}$ and $\frac{1}{b^2}(b^2 + 2 - \sqrt{b^4+4}) < \beta < 1$, the detailed proof is provided in Appendix C.

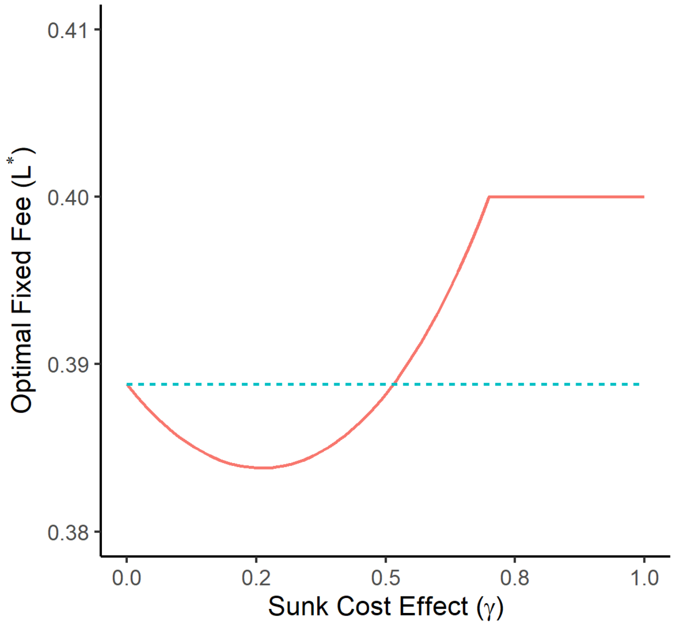

Figure 2: Sunk Cost Effect and Optimal Fixed Fee

Note: In the plot, we demonstrate the case where $b = 0.9$ and $\beta = 0.8$. The solid line is the optimal fixed fee given the degree of sunk cost effect ($\gamma > 0$), and as a contrast, the dashed line is the optimal fixed fee when there is no sunk cost effect ($\gamma = 0$)

overconfidence about their self-control. The alternative explanation is that if the consumer is aware of her self-control problem and sunk cost effect, choosing the fixed-fee contract with an upfront fixed fee induces commitment to greater future usage. Thus the flat-rate bias can arise from sophisticated consumers with rational expectations about their future behavior.

Implications of the Sunk Cost Effect for Firm Profits We now examine the implication of the sunk cost effect for firm profits in investment goods market. If the firm charges the fixed fee L^* in equilibrium, the profit is $\Pi(L^*) = L^* + \int_0^{\beta b+\gamma L^*}(-a)\,\mathrm{d}F(c)$. The sunk cost effect influences the firm's profit in two ways. First, as shown in Equation 5, the sunk cost effect has a non-monotonic impact on the revenue — fixed fee L^*. The sunk cost effect can increase the optimal fixed fee by increasing the willingness to pay when it serves as a commitment device in counteracting the self-control problem. Second, the sunk cost effect increases the consumption probability $F(\beta b + \gamma L^*)$, and hence the probability of incurring the marginal cost a. The first order partial derivative of $\Pi(L^*)$ with respective to γ yields:

$$\frac{\partial \Pi(L^*)}{\partial \gamma} = \frac{\partial L^*}{\partial \gamma} - a \cdot \frac{\partial F(\beta b + \gamma L^*)}{\partial \gamma}$$

As can be seen, the first part is about the impact of the sunk cost effect on revenue, and the second part is about the impact on expected marginal cost. The overall impact of the sunk cost effect on the profits is summarized in the following proposition:

PROPOSITION 2. *(i) Suppose the marginal cost $a = 0$, and the consumer's time inconsistency is sufficiently small ($\bar{\beta}_1 < \beta < 1$), the profits decrease with the sunk cost effect when the sunk cost effect is small, and increase with the sunk cost effect when the effect is large.*

(ii) If the marginal cost is sufficiently high ($a > \bar{a}_1$), the profits decrease with the sunk cost effect.

Here, $\bar{a}_1 = max\{\frac{2(1-\beta b)(2-\beta)\beta b^2-(1-b)(2b-1)+2(1-\beta b)(1-b)^2}{2(1-\beta b)(2-3b)+4b-2}, \frac{(2-\beta)\beta b^2-(1-b)(2-b)}{4-3b}\}$. The proof of Proposition 2 can be found in Appendix D. If the marginal cost $a = 0$, the profits are equal to the fixed fee and $\frac{\partial \Pi(L^*)}{\partial \gamma} = \frac{\partial L^*}{\partial \gamma}$. As shown in Proposition 1, the profits would first decrease and then increase with the sunk cost effect. The firm can benefit from the sunk cost effect only when the sunk cost effect is sufficiently large. If the marginal cost a is sufficiently high, then the profits decrease with the degree of the sunk cost effect. This is because even though the sunk cost effect can potentially increase the revenue L^*, the consumption probability and the firm's expected cost also increases with the sunk cost effect. As the degree of the sunk cost effect increases, the effect on expected cost dominates the effect on the revenue. In general, can the presence of the sunk cost effect ($\gamma > 0$) increase the firm's profits? We can show that when the marginal cost is low enough and the consumer's time-inconsistency problem is not too severe, the sunk cost effect increases firm profits when the effect is sufficiently large.[8]

Leisure Goods Market Consumption of leisure goods such as gaming involves imme-

[8]Here $a \leq min\{\frac{b}{2} - \sqrt{1-b}, \frac{1}{2}[(2-\beta)b-1]\}, min\{\bar{\beta}_1, 1 - \frac{\sqrt{b^2-4(1-a)(1-b+a)}}{b}\} \leq \beta \leq 2 - \frac{1}{b}$, $b \geq 2\sqrt{2} - 2$, and $\gamma \geq \frac{4(1+a-b)}{4a^2+4a(1-b)+(2-\beta)\beta b^2}$, the proof is provided in Appendix D.

diate benefits and delayed costs. The time-inconsistent consumer is concerned about the over-consumption problem associated with the immediate benefits. In the presence of a fixed fee, the sunk cost effect would further exacerbate the over-consumption problem. As a result, a higher degree of the sunk cost effect would lead to a lower expected utility given the fixed fee, and therefore a lower willingness to pay. We summarize the results on the leisure goods market in the following corollary (the proof is in the Appendix E):

COROLLARY 1. *The firm profits weakly decrease with γ irrespective of the marginal cost a.*

Comparison with the pay-per-use contract

Alternatively, the firm can use a variable pay-per-use contract which does not entail sunk costs and does not induce the sunk cost effect. Therefore, it is a contractual benchmark to compare the sunk cost effect created by the fixed-fee contract. Suppose the firm offers a pay-per-use contract with a price $p > 0$, which the consumer only pays when she consumes. As there is no sunk cost effect, the consumption decision is only determined by the benefit and the cost of the consumption. The firm's optimization problem is,

$$max_p \left(\int_0^{\beta b - p} (p - a) \, \mathrm{d}F(c) \right) \tag{6}$$

subject to

$$E[U_{t=0}] = \beta \left(\int_0^{\beta b - p} (b - p - c) \, \mathrm{d}F(c) \right) \geq 0$$

Solving Equation 6 gives:

$$p^* = \frac{1}{2}(a + \beta b) \tag{7}$$

The firm's profits from the pay-per-use contract are $\frac{1}{4}(\beta b - a)^2$, which does not depend on the sunk cost effect. The following proposition compares the profitability comparison between fixed-fee and pay-per-use contracts.

PROPOSITION 3. *(i) When the marginal cost a is sufficiently low (i.e., $a < \bar{a}_2$) and the time inconsistency is moderate (i.e., $\bar{\beta}_2 \le \beta \le \bar{\beta}_3$), the fixed-fee contract is always more profitable than the pay-per-use contract, irrespective of the degree of the sunk cost effect;*

(ii) If the marginal cost a is higher (i.e., $\bar{a}_2 \le a \le min\{\bar{a}_3, \bar{a}\}$) and the consumer is sufficiently time consistent ($\bar{\beta}_4 \le \beta < 1$), the fixed-fee contract is more profitable when the sunk cost effect is low (i.e., $\gamma < \bar{\gamma}_3$); when the sunk cost effect is large (i.e., $\gamma > \bar{\gamma}_3$), the pay-per-use contract is more profitable.[9]

The proof of Proposition 3 can be found in Appendix F. Under the pay-per-use contract, the consumer has to incur both a consumption cost and a per-use fee which reduces the probability of consumption, whereas under the fixed-fee contract, the consumer has a higher probability of consumption even without the sunk cost effect. When the marginal cost is low, the firm can use the fixed-fee contract to extract all the consumer surplus and it results in a higher profit level than the pay-per-use contract.

When the marginal cost is higher, the fixed-fee-induced sunk cost effect leads to two negative impacts. First, the sunk cost effect can increase the probability of consumption and therefore the expected marginal cost for the firm. Second, the consumer who is under a fixed-fee contract faces a potential over-consumption problem if the realized consumption cost is high. These two impacts reduce the profitability of the fixed-fee contract. On the other hand, the pay-per-use contract provides the consumer with more flexibility in consumption. The consumer only needs to pay when the benefit is larger than the realized consumption cost. As a result, the pay-per-use contract can yield higher profits.

The results have some testable implications about which contract the firm would prefer based on their marginal operating costs. For example, in some industries with relatively low marginal costs, such as health clubs, it is beneficial for the firm to offer a fixed-fee

[9] In this proposition, $\bar{a}_2 = \frac{1}{3}[-2 + 2b - \beta b + \sqrt{(-2+2b-\beta b)^2 - 6(1-b)^2 + 3(4-3\beta)\beta b^2}]$, $\bar{a}_3 = \min\{\sqrt{(2-\beta)\beta}b - \frac{\sqrt{2}}{2}\beta b, \bar{a}\}$, $\bar{\beta}_2 = \frac{2}{3} - \frac{\sqrt{4b^2-6(1-b)^2}}{3b}$, $\bar{\beta}_3 = min\{\frac{2}{3} + \frac{\sqrt{4b^2-6(1-b)^2}}{3b}, 1\}$, $\bar{\gamma}_3 = \frac{16a(2-\beta)\beta b^2+4(1-b-a)(\beta b+a)^2-2(\beta b+a)^2\sqrt{4(1-b+a)^2-4(2-\beta)\beta b^2+2(\beta b+a)^2}}{16a^2(2-\beta)\beta b^2+8a(1-b)(\beta b+a)^2+(\beta b+a)^4}$, and $\bar{\beta}_4 = max\{\frac{3}{2b} - 1, \frac{a}{b} + \frac{\sqrt{4b-4a-2}}{b}\}$

contract. In DellaVigna and Malmendier (2004)'s telephone survey of 67 health clubs in the metropolitan Boston area, the researchers asked "which contracts are available at the health club?". Although both the fixed-fee and pay-per-use contracts are feasible, a majority of the clubs initially offered a fixed-fee contract. Only 2 out of 67 clubs initially mentioned the pay-per-use contract. In other industries with relatively high marginal costs, such as personal coaching and dental care, the pay-per-use contract seems more prevalent. For example, tutors in the leading marketplace for private tutors, such as *Apprentus.com* and *Takelessons.com* adopt pay-per-teaching-session contracts.

The Two-part Tariff Contract

The fixed-fee and pay-per-use contracts are special cases of a two-part tariff contract consisting of the lump-sum fixed fee L and the per-use fee p. We now analyze the implication of the sunk cost effect for profits when the firm adopts a two-part tariff contract. The firm solves the following problem:

$$\max_{L,p}\left(L+\int_0^{\beta b-p+\gamma L}(p-a)\,\mathrm{d}F(c)\right) \tag{8}$$

subject to the consumer's participation constraint:

$$\beta\left(-L+\int_0^{\beta b-p+\gamma L}(b-p-c)\,\mathrm{d}F(c)+\int_{\beta b-p+\gamma L}^{1}(-\gamma L)\,\mathrm{d}F(c)\right)\geq 0$$

No sunk cost effect ($\gamma = 0$). In the case when the consumer does not exhibit the sunk cost effect but only time inconsistency, i.e., $\gamma = 0$, the equilibrium profits are $\frac{1}{2}(b-a)^2$. The optimal two-part tariff consists of $L^* = \frac{1}{2}(b-a)\left(b(3-2\beta)-a\right)$ and $p^* = a-(1-\beta)b$.

Under the two-part tariff, the firm can lower p to attenuate the self-control problem. Specifically, to address the under-consumption problem due to the lack of self-control, the firm may even be willing to pay the consumer to participate ($p^* < 0$) when she is sufficiently time inconsistent ($\beta < 1-\frac{a}{b}$), because the fixed fee can be used to extract surplus. A lower per-use fee can induce the time-inconsistent consumer to consume more, and hence the firm reduces p and increases L as the time inconsistency increases. In the equilibrium, the probability of consumption is $F(b-a) = b-a$, and the firm's profits are $\frac{1}{2}(b-a)^2$, and both are independent of the degree of time inconsistency. Note that since the maximum social surplus is $\frac{1}{2}(b-a)^2$, when $\gamma = 0$, the firm can extract all the surplus and achieve the first-best profits (see Appendix G).

Now we analyze the firm profits when the consumer also exhibits the sunk cost effect. The firm solves the profits maximization problem shown in Equation 8. We find that the presence of the sunk cost effect in general has a negative impact on the firm's profits:

PROPOSITION 4. *Under a two-part tariff the firm's profits decrease with the sunk cost effect.*

The proof is in Appendix G. As we just showed in the case where $\gamma = 0$, the two-part tariff contract can fully address the self-control problem and achieve the first-best profit level. In general, under a two-part tariff as γ increases the sunk cost effect gives rise to over-consumption problems due to the commitment effect and the regret dis-utility if the consumer fails to consume. The firm may try to mitigate the over-consumption problem by decreasing L and increasing p, but reducing the consumption probability leads to a higher chance of realizing the regret dis-utility. Conversely, the firm can mitigate the regret dis-utility by increasing the consumption probability (e.g., by increasing L and decreasing p), but that inevitably leads to over-consumption problems. As a result, the two-part tariff cannot fully address the negative impact caused by the sunk cost effect, and the increase in sunk cost effect reduces the firm's profits.

While it should be obvious that the profit level can be higher under the more general two-part tariffs as compared to either the fixed fee or pay-per-use fee, the two-part-tariff still does not generate the first best outcome. The sunk cost effect will generates a net loss in social surplus because when the consumer fails to consume, the total surplus will be reduced by the regret $-\gamma L$ which prevents the firm from achieving the first-best profits. Only when

Contract Type		Severity of Self-control Problem	
		Mild (i.e., high β)	Severe (i.e., low β)
Fixed-Fee Contract (L^*)	Pricing	L^* first decreases and then weakly increases with γ	L^* decreases with γ
	Profits	The sunk cost effect increases profits when γ is high and a is low	The sunk cost effect reduces profits
Pay-per-use contract (p^*)	Pricing	p^* is independent of γ	
	Profits	The sunk cost effect has no effect on profits When γ and a are high, the pay-per-use contract is more profitable than the fixed-fee contract	
Two-part-tariff Contract (L^*, p^*)	Pricing	L^* and p^* have a non-monotonic relationship with γ (see Online Appendix)	
	Profits	The sunk cost effect reduces profits	

Notes: β is the degree of time consistency. γ represents the degree of the sunk cost effect. L^* and p^* represent the optimal fixed fee and per-use fee, respectively.

Table 1: The Impact of the Sunk Cost Effect on Pricing and Profits

the consumer chooses to consume with probability 1, the loss in social surplus $-\gamma L$ will not occur. However, if the consumer chooses to consume with probability 1, there is a probability of $(1 - b + a)$ that the sum of the consumption cost and the marginal cost $c + a$ is higher than the benefit b. Put differently, when the consumer consumes with a probability of 1, the social welfare is $W = b - a - \frac{1}{2}$, is also strictly lower than the first-best outcome $\frac{1}{2}(b-a)^2$ if $b < 1$ and $a > 0$. Therefore, only in the knife edge case when $b = 1$ and $a = 0$, can the firm can possibly achieve the first-best profits $\frac{1}{2}(b-a)^2$. In such a case, the consumer always consumes, and the loss in social surplus $-\gamma L$ never occurs.

Extensions

Refund Policy

In the price contract, the firm can offer a refund f $(0 \leq f \leq L)$ if the consumer decides not to consume. The refund can serve as an instrument to influence the magnitude of the sunk cost effect the consumer commits. Similar to the previous setting, when the consumer chooses to consume at $t = 1$, she gets $\beta b - c$; otherwise, she gets $-\gamma L$. Given the refund policy, the firm refunds f if the consumer decides not to consume and cancel the contract. In this case, her utility is $-\gamma(L-f)+f$.

Assume that the consumer will always get a positive refund if she decides not to consume. Therefore, the consumer chooses to consume if $\beta b - c > -\gamma(L-f)+f$. The probability of consumption is $F(\beta b + \gamma(L-f) - f)$. The refund f reduces the probability of consumption via two channels: first, it reduces the sunk cost to $L-f$, and thus reduces the sunk cost effect; second, since the consumer can get f by canceling the contract, it creates an incentive for the consumer not to consume.

The firm's problem is:

$$\max_{L,f}\left(L+\int_0^{\beta b+\gamma(L-f)-f}(-a)\,\mathrm{d}F(c)+\int_{\beta b+\gamma(L-f)-f}^{1}(-f)\,\mathrm{d}F(c)\right) \tag{9}$$

subject to:

$$\beta\left(-L+\int_0^{\beta b+\gamma(L-f)-f}(b-c)\,\mathrm{d}F(c)+\int_{\beta b+\gamma(L-f)-f}^{1}(-\gamma(L-f)+f)\,\mathrm{d}F(c)\right)\geq 0$$

If we consider the case when the consumer does not exhibit the sunk cost effect but only time inconsistency, i.e., $\gamma=0$, then analogous to the two-part tariff case, by setting the optimal fixed fee and refund, the equilibrium profits are $\frac{1}{2}(b-a)^2$, which is at the first best profit level. The optimal fixed fee and refund are $L^*=\frac{1}{2}(b-a)(b+a)+(a-b+\beta b)(1+a-b)$ and $f^*=a-(1-\beta)b$. As a positive refund incentivizes the consumer not to consume, aggravating the under-consumption problem due to time inconsistency, the optimal refund decreases with the severity of the time-inconsistency problem. Next, we examine the implication of the sunk cost effect for profits.

PROPOSITION 5. *Under the contract consisting of a fixed fee and a refund, the presence of the sunk cost effect reduces the firm profits.*

The proof is in the Online Appendix.

The role of the refund is analogous to the pay-per-use fee in two-part-tariff pricing because both can discourage consumption. The difference is that the per-use fee is to penalize consumption, whereas the refund is to "reward" no consumption. Note that it is never optimal for the firm to offer $L = f$. This is because if a full refund is provided, the consumer will consume if and only if there is a positive surplus, i.e., $\beta b > c$, then the firm has incentives to decrease the refund.

In the presence of the sunk cost effect, if the consumer fails to consume, the total social surplus will be reduced by $-\gamma(L - f)$, which prevents the firm from achieving the first-best outcome. Since it is never optimal for the firm to offer $L = f$, the net loss always exists unless the consumer chooses to consume with probability 1. As the social welfare should be higher than the social cost, i.e., the sum of the consumption cost and the marginal cost $c + a$ is lower than the benefit b (i.e., $c < b - a$); otherwise, the net social surplus is negative. However, if the consumer chooses to consume with probability 1 when $b < 1$ or $a > 0$, there is a probability of $1 - b + a$ that consumption leads to a negative net social surplus. Therefore, the first-best outcome can never be achieved unless $b = 1$ and $a = 0$.

Sunk Cost Effect as a Memory Cue

In this section, we examine an alternate account of the sunk cost effect based on limited memory (Baliga and Ely, 2011; Hong et al., 2019). Consistent with this account, the sunk cost effect can be modeled as a memory/perceptual cue for the consumer regarding the importance of the activity associated with the sunk cost. That is, the sunk cost only affects the likelihood of consumption but does not yield dis-utility. The strength of the cue depends on both how much the consumer paid L and the degree of recall salience. Here we use s ($s \in [0, 1]$) to capture the degree of the recall salience. As a result, the consumer's expected utility from signing the fixed-fee contract is:

$$E[U_{t=0}] = \beta \left(-L + \int_0^{\beta b + sL} (b - c)\, \mathrm{d}F(c) \right) \tag{10}$$

Note that the sunk cost effect is incorporated only in the probability of consumption: $F(\beta b + sL)$, which is larger than the probability without sunk cost $F(\beta b)$. The impact of the sunk cost effect s on expected utility is as follows:

$$\frac{\partial E[U_{t=0}]}{\partial s} = \beta L[(1-\beta)b - sL]f(\beta b + sL) \tag{11}$$

As can be seen from Equation 11, if the consumer is time consistent ($\beta = 1$), then $\frac{\partial E[U_{t=0}]}{\partial s} < 0$, i.e., the consumer's expected utility decreases with a higher degree of sunk cost effect. This result is consistent with that in the main model shown in Lemma 1. However, the mechanism of the negative effect of the sunk cost effect is different. In the main regret-based sunk cost effect model, the negative impact comes from two sources: 1) the dis-utility (regret) when the consumer fails to consume (attend the gym) due to high realized cost; 2) the over-consumption problem associated with the sunk cost effect. Under the memory-cue account, the negative impact stems only from the over-consumption problem. When the consumer is time inconsistent ($\beta < 1$), the consumer's utility is not monotonic in the degree of sunk cost effect (s); specifically, when the consumer is sufficiently time inconsistent, i.e., $\beta < 1 - \frac{sL}{b}$, then the consumer utility increases in s (i.e., $\frac{\partial E[U_{t=0}]}{\partial s} > 0$). If we assume $c \sim U[0,1]$, then we have the following proposition:

PROPOSITION 6. *(i) If the degree of time inconsistency is sufficiently large (i.e., $\beta \leq 1 - \frac{b}{2}$), the optimal fixed fee increases with s.*

(ii) If the degree of the time inconsistency is sufficiently small (i.e.,$1 - \frac{b}{2} < \beta < 1$), the optimal fixed fee first increases when $s < \bar{s}_2$ and then decreases with s when $\bar{s}_2 \leq s \leq \bar{s}_1$;

(iii) If s is sufficiently large (i.e., $s > \bar{s}_1$), the optimal fixed fee is a constant $L^* = b - \frac{1}{2}$ which induces the consumer to consume with probability 1.

Here $\bar{s}_1 = min\{1, \frac{2(1-\beta b)}{2b-1}\}$ and $\bar{s}_2 = \frac{2-2\beta}{b}$. When the consumer's time inconsistency is high, the higher degree of sunk cost effect will result in a higher chance of consumption, canceling out the negative impact of the self-control problem. Thus, the consumer's willingness to pay – the optimal fixed fee increases with the degree of the sunk cost effect. However, if the degree of the time inconsistency is small, although a small degree of the sunk cost effect can increase profits, a higher degree of the effect will drive her to consume under a greater set of undesirable circumstances (i.e., $c > b$), and this results in a lower willingness to pay for the fixed fee.

Two-part Tariff and Refund Policy. Note that when the consumer does not exhibit the sunk cost effect, the two-part tariff and the refund policy generate the same equilibrium profits, i.e., $\frac{1}{2}(b-a)^2$. Interestingly, if the sunk cost effect is memory-cue based and does not generate dis-utility, we can show that the potential negative impact of the effect is eliminated in the equilibrium if the firm uses a two-part tariff or a refund policy. That is, the firm can achieve the first best profit level using the two-part tariff or the refund policy (see the Online Appendix). This result is driven by the fact that by appropriately setting the fixed fee and per-use fee in the two-part tariff (refund in the refund policy), the sunk cost effect and time inconsistency counterbalance each other in equilibrium. Under a two-part tariff, the optimal fixed fee L^* decreases in s. Lowering the fixed fee will give the consumer more flexibility to choose whether to consume in the future, and this mitigates the over-consumption problem due to the sunk cost effect. Meanwhile, the per-use price increases in s since a higher degree of sunk cost effect makes the consumer likely to consume even with a higher per-use fee. Under a refund policy, as the sunk cost effect increases, the firm will increase its refund to mitigate the over-consumption of consumers, as a result, the firm will also set a higher up-front-fixed fee to insure their profits.

Hence, if the sunk cost effect serves only as a memory cue, the negative impact of this bias can be internalized in the two-part tariff contract or the refund policy, while that is not the case when the sunk cost effect stems from regret dis-utility. The contrast between the two psychological accounts helps disentangle the effect on consumption probability from the direct effect on utility, which has different implications on firm profits. The results show that it is important for the firm to understand the psychological mechanism underlying the sunk cost effect.

Discussion and Conclusion

The sunk cost effect is a canonical example of inconsistent behavior and deviations from standard economic theory. It can result in over-consumption, escalation of commitment to investments, and insufficient adaptation to new situations. In Nozick (1994)'s words, the sunk cost effect *"... can be rationally utilized to check and overcome another behavioral bias (the self-control problem)"*. This paper analyzes the role of the sunk cost effect as a self-commitment device and its implication for optimal contract design. For investment goods, a firm balances the demand for flexibility in response to the sunk cost effect and the demand for a commitment to counter the self-control problem when designing an optimal price contract. Under a fixed-fee contract for investment goods, the firm's profits can increase or decrease with the effect in the investment goods market, depending on the degree of the effect. The firm can use a two-part tariff or a refundable fixed-fee contract to mitigate the sunk cost effect. The authors show that under a two-part tariff or a refund policy, the sunk cost effect reduces profits, and the adverse impact on profits arises when the sunk cost effect is regret-based but not memory-cue-based.

Managerial Implications. In our main analysis of the fixed-fee contract in the investment goods markets, we showed that the optimal fixed fee can increase with the degree of the sunk cost effect. Interestingly, in the leisure goods market (e.g., games, smoking, etc) where the consumption involves immediate benefits and delayed cost, the optimal fixed fee decreases with the degree of the sunk cost effect. This is because the sunk cost effect exacerbates the over-consumption problems, and therefore reduces the consumer's willingness to pay. Our analysis can shed light on mobile app pricing. For apps associated with investment goods consumption (e.g., Education apps), the fixed fee can serve as a commitment device for the consumers, and hence the firm can charge a higher fixed fee. For the apps associated with leisure goods consumption (e.g., Game apps), consumers who lack self-control have a concern about over-consumption problems, and the fixed-fee-induced sunk cost effect aggravates the concern. Therefore, the consumer has a lower willingness to incur the fixed fee. The apps' pricing strategy on Apple App Store appears consistent with our model prediction. Among the 119,174 apps under the Education genre, associated with investment goods consumption, about 15% of the apps charge a fixed fee. The average fixed fee charged is \$6.7. On the other hand, among the 193,749 apps under the Games genre, associated with leisure goods consumption only about 8% of the apps charge a fixed fee. The average fixed fee charged is \$3.4. The difference in fixed fees may be due to the consumers' concern for the sunk cost effect and the self-control problem.

We demonstrate that regret-based and memory-cue-based sunk cost effect have different implications on firm profits and contract design. While a two-part tariff can achieve the first best when the sunk cost effect is memory cue based, there is a deadweight loss of surplus and profits will be below the first best level when the effect is regret based. Framing the sunk cost as a memory cue in marketing communications and help firms increase the available surplus. Certain consumer characteristics might be associated with the types of the sunk cost effect they suffer. For example, it has been shown that women are more susceptible to regret emotion than men (Li, Li, Cao and Niu, 2018) and this again may have implications for the nature of marketing communications. Firms can conduct marketing research to identify consumer segments or product markets which are more likely to involve one or both types of sunk cost effects.

In the presence of consumer heterogeneity in their sensitivity to sunk costs, it is common to observe firms offer consumers a menu consisting of both fixed-fee and pay-per-use contracts. Interestingly, it is never optimal for the firm to offer the fixed-fee contract to the sunk-cost-sensitive consumer and the pay-per-use contract to the consumer who is not affected by the sunk cost (see analysis and discussion in the Online Appendix). It is interesting that in a separating equilibrium it is optimal for the firm to design a menu such that the consumer who is prone to the sunk cost effect chooses the pay-per-use contract, whereas the consumer who does not suffer from the sunk cost effect chooses the fixed-fee contract. The screening contract menu is more profitable than a single fixed-fee contract and than a single pay-per-use contract when the difference in the degree of the sunk cost effect is large; otherwise when the difference is small, the two contracts can cannibalize each other, resulting in lower profits.

There are some interesting aspects of the problem that may be pursued in future work. First, it would be useful to consider repeated interactions between the firm and the consumer, and the role of the sunk cost effect in customer retention. In our main model, although the sunk cost effect can mitigate the self-control problem, the consumer might over-consume to avoid regret or might experience regret if she does not consume. In either case, the fixed-fee-induced sunk cost effect reduces consumer welfare and the willingness to continue the contract with the firm. The sunk cost effect might help to explain the empirical results that the users are more likely to churn if the contract includes a fixed fee (e.g., Danaher (2002), Lambrecht and Skiera (2006), and Iyengar et al. (2011). To address this issue, the firm can offer a voucher or allow the consumer to carry over the service in a future period if the consumer decides not to consume in the current period. These tactics may potentially mitigate the regret associated with the sunk cost effect. How the firm should design a contract involving repeated interactions warrants formal analysis.

It may also be interesting to consider the quantity decisions of individual consumers. This in the presence of the sunk cost effect may have implications for pricing structures such as the three-part tariff in telecommunication services (Ascarza, Lambrecht and Vilcassim, 2012) and the bucket pricing discrimination in online rental services (Sun, Li and Sun, 2015). Both pricing schemes include an upfront fixed fee, and a quota for free usage of the service. When the consumption quantity hits the quota, the consumer faces a higher marginal price under a three-part tariff or a cap under the bucket pricing discrimination. Although the upfront fixed fee can result in a high tendency to consume more, the firms may set an optimal quota to mitigate the over-consumption problem due to the sunk cost effect.

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Appendix A Consumer's Anticipated Sunk Cost effect - Pilot Study

We conducted a pilot study to investigate whether people anticipate that their future consumption will be affected by the sunk cost.

Design and Procedure. We recruited 200 US and UK-based participants (76% females, $Mean_{Age} = 29.9.20$; $SD_{Age} = 10.18$) from *Prolific.co* in exchange for monetary compensation. The study was launched and completed on Feb 15th, 2022. The participants were paid approximately US$0.51 for 3 minutes of participation. All the participants read the following scenario.

> *Imagine that you plan to join a health club. The club has fully equipped fitness facilities, a state-of-the-art gymnasium, and relaxation areas for members. Once you become a member of the club, you can enjoy all the facilities and services in the club for free. The regular annual membership fee is $2500, but now the club offers membership at a discounted rate.*

Right after reading the scenario, participants were asked to recall the regular annual membership fee, which serves as an attention check. Then participants were presented with the 4 discounted prices: $2000, $1500, $500, and $100 in random order. They were asked: *If you pay $2000 ($1500/$500/$100) membership fee and join the health club, how often do you think you will go to the club?*. The participants were choosing among 1) never, 2) once a month, 3) twice a month, 4) weekly, 5) twice a week, and 5) more than twice a week. We include an original price for the membership fee to prevent subjects from making inferences about the quality of the services. Since one month is roughly equal to 4 weeks, we interpret the frequency per month as: 0, 1, 2, 4, 8, and 12 per month, corresponding to each choice.

Results. 14 participants failed the initial attention check and were excluded from the study. No data were collected from these participants. As can be seen in Figure 3, A higher membership fee induces a higher anticipated attendance frequency. If the membership fees increase from \$100 to \$2000, participants forecast that the frequency will increase from 6 to 9 times per month. Regression controlling for individual fixed effects yields a significant effect of membership fee on anticipated attendance frequency ($\beta = 0.0015$, clustered S.E. $= 0.0002$, $p < 0.001$).

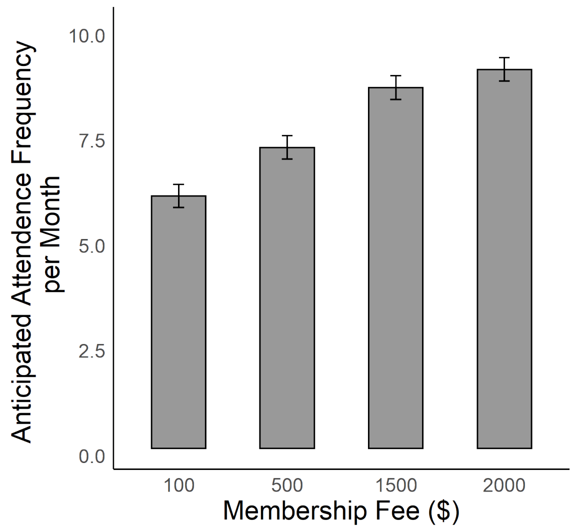

Figure 3: Expected Health Club Attendance

Note: A higher membership fee induces a higher anticipated attendance frequency. The error bars represent the $\pm$ 1 standard error.

The experiment demonstrates that people are aware that if they pay more today, they anticipate that they will go to the health club more frequently in the future. This is critical for the sunk cost to work as the commitment device, since only when the sunk cost effect can be anticipated, consumers can strategically use it to increase (or decrease) future gym attendance.

Appendix B Proof of Lemma 1

In this proof, we first examine the general setting whereby the cost c incurred by the consumer follows a distribution function $F(c)$, and $F(c)$ is smooth, continuous, and differentiable. In addition, the sunk cost effect is captured by a general functional form $g(\gamma, L)$, which satisfies $g(\gamma, L) = 0$ when $\gamma = 0$ or $L = 0$, $\frac{\partial g(\gamma,L)}{\partial \gamma} > 0$, and $\frac{\partial g(\gamma,L)}{\partial L} > 0$. By denoting the consumer's expected utility when signing a fixed-fee contract as $E(U_{t=0})$, we have:

$$E(U_{t=0}) = \beta \left(-L + \int_0^{\beta b + g(\gamma,L)} (b - c) \, \mathrm{d}F(c) + \int_{\beta b + g(\gamma,L)}^{\bar{c}} (-g(\gamma, L)) \, \mathrm{d}F(c) \right) \tag{12}$$

By using Leibniz Rule, we take the first order derivative of $E(U_{t=0})$ with respect to γ:

$$\frac{\partial E(U_{t=0})}{\partial \gamma} = \beta[(1-\beta) b f(\beta b + g(\gamma, L)) \frac{\partial g(\gamma, L)}{\partial \gamma} - \int_{\beta b + g(\gamma,L)}^{\bar{c}} \frac{\partial g(\gamma, L)}{\partial \gamma} \, \mathrm{d}F(c)] \tag{13}$$

When $g(\gamma, L) = \gamma L$, the equation in 13 reduces to $\frac{\partial E(U_{t=0})}{\partial \gamma} = \beta[(1-\beta) b L f(\beta b + \gamma L) - \int_{\beta b + \gamma L}^{\bar{c}} L \, \mathrm{d}F(c)]$.

(i) When $\beta = 1$, $\frac{\partial E(U_{t=0})}{\partial \gamma} = -\int_{\beta b + \gamma L}^{\bar{c}} L \, \mathrm{d}F(c) < 0$, and the consumer utility decreases with γ.

(ii) In this proof, we will show that when the consumer is time inconsistent ($\beta < 1$) and the distribution function $F(\cdot)$ is weakly convex, the consumer's expected utility $E(U_{t=0})$ first decreases and then weakly increases with γ. Note from the analysis earlier that $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma=0} = \beta[(1-\beta) b L f(\beta b) - \int_{\beta b}^{\bar{c}} L \, \mathrm{d}F(c)]$, when $F(\cdot)$ is weakly convex and the density function $f(\cdot)$ is non-decreasing, $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma=0} = \beta[(1-\beta) b L f(\beta b) - \int_{\beta b}^{\bar{c}} L \, \mathrm{d}F(c)] \leq (1-\beta) b L f(\beta b) - (\bar{c} - \beta b) L f(\beta b) = (b - \bar{c}) L f(\beta b) < 0$ when $b < \bar{c}$. Also, $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma L + \beta b = \bar{c}} = \beta L (1-\beta) b f(\bar{c}) > 0$ when $\beta < 1$. Lastly, by taking the second-order derivative of $E(U_{t=0})$ with respect to γ, we have $\frac{\partial^2 E(U_{t=0})}{\partial \gamma^2} = \beta L^2 [(1-\beta) b f'(\beta b + \gamma L) + f(\beta b + \gamma L)] > 0$ since $f(\cdot)$ is non-decreasing. In sum, we have shown that when $\beta < 1$, $b < \bar{c}$ and $F(\cdot)$ is weakly convex, $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma=0} < 0$, $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma L + \beta b = \bar{c}} > 0$ and $\frac{\partial^2 E(U_{t=0})}{\partial \gamma^2} > 0$, indicating that the consumer's expected utility first decreases and then weakly increases with γ.

Next, we will further show that the results above hold when the sunk cost effect follows a general functional form $g(\gamma, L)$ when $\frac{\partial^2 g(\gamma,L)}{\partial \gamma^2} = 0$. When $\gamma = 0$, $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma=0} = \beta \frac{\partial g(\gamma,L)}{\partial \gamma}[(1-$

$\beta)bLf(\beta b) - \int_{\beta b}^{\bar{c}} 1\, \mathrm{d}F(c)]$. Similar to the reasoning earlier, $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma=0} < 0$ when $F(\cdot)$ is weakly convex. When $\gamma L + \beta b = \bar{c}$, $\frac{\partial E(U_{t=0})}{\partial \gamma}|_{\gamma L+\beta b=\bar{c}} = \beta \frac{\partial g(\gamma,L)}{\partial \gamma}(1-\beta)bf(\bar{c}) > 0$ holds when $\beta < 1$. By taking the second-order derivative of $E(U_{t=0})$ with respect to γ, we have $\frac{\partial^2 E(U_{t=0})}{\partial \gamma^2} = \beta \frac{\partial^2 g(\gamma,L)}{\partial \gamma^2}[(1-\beta)bf(\beta b + g(\gamma,L)) - \int_{\beta b+g(\gamma,L))}^{\bar{c}} 1dF(c)] + \beta[\frac{\partial g(\gamma,L)}{\partial \gamma}]^2[(1-\beta)bf'(\beta b + g(\gamma,L)) + f(\beta b + g(\gamma,L))] = \beta[\frac{\partial g(\gamma,L)}{\partial \gamma}]^2[(1-\beta)bf'(\beta b + g(\gamma,L)) + f(\beta b + g(\gamma,L))] > 0$ when $\frac{\partial^2 g(\gamma,L)}{\partial \gamma^2} = 0$. $\square$

Appendix C Proof of Proposition 1

The firm solves the profit maximization problem described in Equation 4. By re-writing the consumer participation constraint in Equation 4 as $\frac{\gamma^2 L^2}{2} - L(1+\gamma(1-b)) + \beta b^2 - \frac{\beta^2 b^2}{2} \geq 0$, we find that this condition is satisfied when $L \geq \frac{(1+\gamma-\gamma b)+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2}$, or $L \leq \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2}$. However, note that when $L \geq \frac{(1+\gamma-\gamma b)+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2}$, $\gamma L + \beta b > \frac{1}{\gamma} + \beta b > 1$, the boundary condition $\beta b + \gamma L \leq 1$ does not hold. In our analysis, we will focus on the solution when $L \leq \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2}$. In equilibrium, the consumer participation constraint must be binding to 0, since otherwise, the firm can increase its fixed fee to extract more surplus from the consumer. By solving $\frac{\gamma^2 L^2}{2} - L(1+\gamma(1-b)) + \beta b^2 - \frac{\beta^2 b^2}{2} = 0$, we have

If $\gamma = 0$, then the optimal fixed fee is: $L^* = \beta b^2 - \frac{\beta^2 b^2}{2}$.

If $\gamma > 0$, the optimal fixed fee is

$$L^* = \frac{(1+\gamma-\gamma b) - \sqrt{(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2}}{\gamma^2} \tag{14}$$

Equivalently, we can re-write $L^* = \frac{\beta(2-\beta)b^2}{(1+\gamma-\gamma b)+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}$.

(i) When $0 < \gamma \leq 1$ and $\beta < 1$, from the implicit function derivative, we have $\frac{\partial L^*}{\partial \gamma} = \frac{L^*(1-b-\gamma L^*)}{-1-\gamma(1-b-\gamma L^*)} \leq 0$ if and only if $\gamma L^* \leq (1-b)$. By plugging in L^* in 14, $\gamma L^* \leq (1-b)$ can be reduced to $(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2 \geq 1$, and the condition can be simplified as $\gamma[(1-b)^2 - (2-\beta)\beta b^2] + 2(1-b) \geq 0$. Note that this condition is linear in γ, and this condition can be satisfied if and only if $\gamma = 0$ and $\gamma = 1$. When $\gamma = 0$, the condition is satisfied automatically since $2(1-b) > 0$; when $\gamma = 1$, the condition is satisfied when $(1-b)^2 - (2-\beta)\beta b^2 + 2(1-b) > 0$, which can be reduced to $\beta \leq \bar{\beta}_1 = 1 - \frac{\sqrt{4b-3}}{b}$. In sum, we have shown that when $\beta \leq \bar{\beta}_1$, L^* is decreasing in γ.

(ii) (iii) Note from the proof in (i), $\frac{\partial L^*}{\partial \gamma} = \frac{L^*(1-b-\gamma L^*)}{-1-\gamma(1-b-\gamma L^*)} \leq 0$ if and only if $\gamma[(1-b)^2 - (2-\beta)\beta b^2] + 2(1-b) \geq 0$. When $\frac{3}{4} < b$ and $\bar{\beta}_1 < \beta$, $\frac{\partial L^*}{\partial \gamma} \leq 0$ holds if $\gamma \leq \bar{\gamma}_2 = \frac{1-b}{[(2-\beta)\beta b^2 - (1-b)^2]}$; otherwise, $\frac{\partial L^*}{\partial \gamma} > 0$ if $\gamma > \bar{\gamma}_2$. Note that the boundary condition $\gamma L^* + \beta b \leq 1$ is equivalent to $\gamma\beta b + \gamma^2 L^* \leq \gamma$, by plugging in L^* in 14, the condition can be reduced to $(1 - \gamma b + \gamma\beta b) \leq \sqrt{(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2}$. By solving this inequality, we get $2(1-\beta b) \geq \gamma(2b-1)$, and the boundary condition is satisfied when $\gamma \leq \frac{2(1-\beta b)}{2b-1}$. As a result, when $\frac{3}{4} < b$ and $\bar{\beta}_1 < \beta < 1$, $\frac{\partial L^*}{\partial \gamma} \leq 0$ when $\gamma \leq \bar{\gamma}_2$ and $\frac{\partial L^*}{\partial \gamma} \geq 0$ when $\bar{\gamma}_2 < \gamma \leq \frac{2(1-\beta b)}{2b-1}$. When $1 \geq \gamma > \frac{2(1-\beta b)}{2b-1}$, $\gamma L^* + \beta b \geq 1$, the consumer will consume with probability one, and the consumer's expected utility is $E(U_{t=0}) = -L + b - \frac{1}{2}$, and $L^* = b - \frac{1}{2}$.

Lastly, we show that our results here can be extended to a more general functional form of sunk cost effect $g(\gamma, L)$ when $0 < \frac{\partial g}{\partial L} < 1$. When the sunk cost effect is $g(\gamma, L)$, the profit maximization problem in Equation 4 can be written as $max_L \pi = (L - ag(\gamma, L) - a\beta b)$ subject to $\frac{1}{2}g(\gamma, L)^2 - (1-b)g(\gamma, L) - L + \frac{1}{2}\beta(2-\beta)b^2 \geq 0$. Note that the profit π increases in L since $\frac{\partial \pi}{\partial L} = 1 - a\frac{\partial g}{\partial L} \geq 1 - a > 0$, the firm's profit is maximized when the consumer's participation condition is binding to 0 such that $\frac{1}{2}g(\gamma, L)^2 - (1-b)g(\gamma, L) - L + \frac{1}{2}\beta(2-\beta)b^2 = 0$. By using the implicit derivative, we take the first-order partial derivative of the consumer's participation condition with respect to γ, and we have $g(\gamma, L)(\frac{\partial g}{\partial \gamma} + \frac{\partial g}{\partial L}\frac{\partial L}{\partial \gamma}) - (1-b)(\frac{\partial g}{\partial \gamma} + \frac{\partial g}{\partial L}\frac{\partial L}{\partial \gamma}) - \frac{\partial L}{\partial \gamma} = 0$, which leads to $\frac{\partial L}{\partial \gamma} = \frac{-\frac{\partial g}{\partial L}(1-b-g(\gamma,L))}{1+\frac{\partial g}{\partial L}(1-b-g(\gamma,L))}$. Note that when $0 < \frac{\partial g}{\partial L} < 1$, $g(\gamma, L) < L$ and the denominator $1 + \frac{\partial g}{\partial L}(1-b-g(\gamma, L))$ is always positive, and $1 + \frac{\partial g}{\partial L}(1-b-g(\gamma, L)) > 1 - |(1-b-g(\gamma, L)| = min\{b + g(\gamma, L), 2 - b - g(\gamma, L)\} > 0$ since $g(\gamma, L) \leq L < b$. Then $\frac{\partial L}{\partial \gamma} > 0$ if and only if $(1 - b - g(\gamma, L)) < 0$ which is equivalent to $g(\gamma, L) > 1 - b$. By rewriting the consumer's participation condition as $L = [g(\gamma, L) - (1-b)]^2 + (-\frac{1}{2}\beta^2 b^2 + \beta b^2 - \frac{1}{2}(1-b)^2)$, we find that when $\beta < 1 - \frac{\sqrt{2b-1}}{b}$, $(-\frac{1}{2}\beta^2 b^2 + \beta b^2 - \frac{1}{2}(1-b)^2) < 0$, indicating that $[g(\gamma, L) - (1-b)]^2$ is always positive for the full range of parameters since $L > 0$. Note that $g|_{\gamma=0} = 0$, due to the continuity of $g(\gamma, L)$, $g(\gamma, L) - (1-b) < 0$ holds when $\beta < 1 - \frac{\sqrt{2b-1}}{b}$, and as a result $\frac{\partial L}{\partial \gamma} < 0$. On the other hand, when $\beta \geq 1 - \frac{\sqrt{2b-1}}{b}$, $\frac{\partial L}{\partial \gamma} \leq 0$ if $g(\gamma, L) - (1-b) \leq 0$; while $\frac{\partial L}{\partial \gamma} > 0$ if $g(\gamma, L) > 1-b$. In sum, we have shown that when the sunk cost effect follows a more general format $g(\gamma, L)$, which satisfies $g(\gamma, L) = g(0, L) = g(\gamma, 0) = 0$, $0 < \frac{\partial g}{\partial L} < 1$ and $\frac{\partial g}{\partial \gamma} > 0$, the optimal fixed fee L^* decreases with γ when $\beta < 1 - \frac{\sqrt{2b-1}}{b}$; otherwise if $\beta \geq 1 - \frac{\sqrt{2b-1}}{b}$, L^* first decrease with γ when $g(\gamma, L) \leq 1-b$ and then increase with γ when $g(\gamma, L) > 1-b$. $\square$

Appendix D Proof of Proposition 2

(i)Following the analysis in the proof of Proposition 1, we can get this result immediately.

(ii) In this part, we show that when $a > max\{\frac{2(1-\beta b)[(2-\beta)\beta b^2+(1-b)^2]-(1-b)(2b-1)}{2(1-\beta b)(2-3b)+4b-2}, \frac{(1-\beta)\beta b^2-(1-b)(2-b)}{4-3b}\}$, $\Pi(L)$ decreases in γ. By writing the profit function of the fixed-fee contract as $\Pi(L) = (1-a\gamma)L - a\beta b$, we take the first-order derivative of $\Pi(L)$ with respect to γ, and we have $\frac{\partial \Pi(L)}{\partial \gamma} = (1-a\gamma)\frac{\partial L}{\partial \gamma} - aL$. By plugging in $L^* = \frac{\beta(2-\beta)b^2}{1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}} = \frac{A(\gamma)}{B(\gamma)}$, in which $A(\gamma) = \beta(2-\beta)b^2$ and $B(\gamma) = 1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}$, $\frac{\partial \Pi(L)}{\partial \gamma} = (1-a\gamma)\frac{\partial L}{\partial \gamma} - aL$ can be re-written as $\frac{\partial \Pi(L)}{\partial \gamma} = -\frac{A(\gamma)}{B(\gamma)^2}[(1-a\gamma)\frac{\partial B(\gamma)}{\partial \gamma} + aB(\gamma)]$. Note that $A(\gamma) > 0$ and $B(\gamma) > 0$, then we show $\frac{\partial \Pi(L)}{\partial \gamma} < 0$ by showing $f(\gamma) = (1-a\gamma)\frac{\partial B(\gamma)}{\partial \gamma} + aB(\gamma) > 0$ when $\gamma \in (0, \bar{\gamma}_1)$. By plugging in $B(\gamma) = 1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}$ and $\frac{\partial B(\gamma)}{\partial \gamma} = 1-b+\frac{(1-b)(1+\gamma-\gamma b)-\gamma(2-\beta)\beta b^2}{\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}$, we have $f(\gamma) = (1-a\gamma)[1-b+\frac{(1-b)(1+\gamma-\gamma b)-\gamma(2-\beta)\beta b^2}{\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}] + a[1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}] = 1-b+a+a\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2} + (1-a\gamma)\frac{(1-b)(1+\gamma-\gamma b)-\gamma(2-\beta)\beta b^2}{\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}$, and $f(\gamma) > 0$ if $a + a\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2} + (1-a\gamma)\frac{(1-b)(1+\gamma-\gamma b)-\gamma(2-\beta)\beta b^2}{\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}} > 0$, which is equivalent to show $a(\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}) + a(1+\gamma-\gamma b) > \gamma(2-\beta)\beta b^2 - (1-b)(1+\gamma-\gamma b)$. Since $(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2 > (1+\gamma-\gamma b)^2-\gamma^2 b^2 > (1+\gamma-2\gamma b)^2$, we have $\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2} > 1+\gamma-2\gamma b$. By substituting $\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}$ as $1+\gamma-2\gamma b$ into the inequality above, it is sufficient to prove the $f(\gamma) > 0$ by showing $a(2+2\gamma-3\gamma b) > \gamma(2-\beta)\beta b^2-(1-b)(1+\gamma-\gamma b)$. Note that this inequality is linear in γ, then $a(2+2\gamma-3\gamma b) > \gamma(2-\beta)\beta b^2-(1-b)(1+\gamma-\gamma b)$ holds for

$\gamma \in (0, \bar{\gamma}_1)$ if it holds when $\gamma = 0$, $\gamma = 1$ and $\gamma = \frac{2(1-\beta b)}{2b-1}$. When $\gamma = 0$, this inequality reduces to $2a > -(1-b)$, and it holds automatically. When $\gamma = \frac{2(1-\beta b)}{2b-1}$, $a(2+2\gamma-3\gamma b) > \gamma(2-\beta)\beta b^2 - (1-b)(1+\gamma-\gamma b)$ holds when $a > \frac{2(1-\beta b)(2-\beta)\beta b^2-(1-b)(2b-1)+2(1-\beta b)(1-b)^2}{2(1-\beta b)(2-3b)+4b-2}$. If $\gamma = 1$, $a(2+2\gamma - 3\gamma b) > \gamma(2-\beta)\beta b^2 - (1-b)(1+\gamma-\gamma b)$ holds when $a > \frac{(2-\beta)\beta b^2-(1-b)(2-b)}{4-3b}$. In sum, we have shown that when $a > \bar{a}_1 = max\{\frac{2(1-\beta b)(2-\beta)\beta b^2-(1-b)(2b-1)+2(1-\beta b)(1-b)^2}{2(1-\beta b)(2-3b)+4b-2}, \frac{(2-\beta)\beta b^2-(1-b)(2-b)}{4-3b}\}$, $\Pi(L)$ decreases with γ.

Lastly, we identify the conditions under which the fixed-fee contract can be more profitable in the presence of the sunk cost effect when $a > 0$. By denoting the profit of a fixed-fee contract when $\gamma = 0$ as π_0, we have $\pi_0 = \frac{1}{2}\beta(2-\beta)b^2 - a\beta b$. Similarly, by denoting the profit of the fixed-fee contract when $\gamma > 0$ as π_1, we have $\pi_1 = (1-a\gamma)L^* - a\beta b$ if $\gamma \le \frac{2(1-\beta b)}{2b-1}$, in which $L^* = \frac{\beta(2-\beta)b^2}{1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}$; otherwise, $\pi_1 = b - a - \frac{1}{2}$ if $\gamma \ge \frac{2(1-\beta b)}{2b-1}$. By comparing π_0 and π_1 when $\gamma \le \frac{2(1-\beta b)}{2b-1}$, $\pi_1 \ge \pi_0$ if and only if $(1-a\gamma)L^* \ge \frac{1}{2}\beta(2-\beta)b^2$. By plugging in $L^* = \frac{\beta(2-\beta)b^2}{1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}$ into the inequality, $\pi_1 \ge \pi_0$ holds if and only if $(1-\gamma+\gamma b-2a\gamma) \ge \sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}$, simplifying, we have $\pi_1 \ge \pi_0$ holds if and only if $\gamma \ge \frac{4(1+a-b)}{[4a(1-b+a)+(2-\beta)\beta b^2]}$.

To check the boundary condition, we first check $\frac{4(1+a-b)}{[4a(1-b+a)+(2-\beta)\beta b^2]} \le \frac{2(1-\beta b)}{2b-1}$. Note from Proposition 1 that when the profit of the firm is higher in the presence of the sunk cost effect, the optimal fixed fee L^* is increasing in γ. To show $\frac{4(1+a-b)}{[4a(1-b+a)+(2-\beta)\beta b^2]} \le \frac{2(1-\beta b)}{2b-1}$ is equivalent to $\pi(\gamma = \frac{2(1-\beta b)}{2b-1}) = b - a - \frac{1}{2} \ge \pi(\gamma = \frac{4(1+a-b)}{[4a(1-b+a)+(2-\beta)\beta b^2]}) = \pi_0 = \frac{1}{2}\beta(2-\beta)b^2 - a\beta b$. Solving this inequality, we have $(1-\beta b)((2-\beta)b-1) \ge 2(1-\beta b)a$, which is satisfied when $\beta \le 2 - \frac{1}{b}$ and $a \le \frac{1}{2}[(2-\beta)b-1]$. Next, we check $\frac{4(1+a-b)}{[4a(1-b+a)+(2-\beta)\beta b^2]} \le 1$, and it is satisfied when $\beta \ge 1 - \frac{\sqrt{b^2-4(1-a)(1-b+a)}}{b}$. To ensure $b^2 - 4(1-a)(1-b+a) \ge 0$, we have $a \le \frac{b}{2} - \sqrt{1-b}$ and $b \ge 2\sqrt{2} - 2$.

In sum, we have shown that when $a \le min\{\frac{b}{2} - \sqrt{1-b}, \frac{1}{2}[(2-\beta)b-1]\}$, $b \ge 2\sqrt{2}-2$, and $min\{\bar{\beta}_1, 1 - \frac{\sqrt{b^2-4(1-a)(1-b+a)}}{b}\} \le \beta \le 2 - \frac{1}{b}$, the profit of the fixed-fee contract can be higher in the presence of sunk cost effect when $\frac{4(1+a-b)}{[4a(1-b+a)+(2-\beta)\beta b^2]} \le \gamma$. $\square$

Appendix E Proof of Corollary 1

To simplify our exposition, we use the same set of notations as in the analysis for investment goods. If the consumer chooses to consume the goods, she receives a stochastic benefit $b \sim U[0,1]$ at $t=1$, and incurs a deterministic cost $0 \leq c < \frac{1}{2}$ at $t=2$. Otherwise, if the cost is sufficiently high such that $c \geq \frac{1}{2}$, a sophisticated consumer will sign the contract if and only if she is sufficiently time-consistent ($\beta > 2 - \frac{1}{c}$). If the consumer does not consume, she will suffer from the sunk cost effect and experience a disutility $-\gamma L$. From the perspective at $t=0$, the consumer will consume the product, e.g., play the game, at $t=1$ if $b - c > -\gamma L$. As time goes by, at $t=1$, she will actually consume if $b - \beta c > -\gamma L$. Given randomness in benefit b, the probability of consuming is $1 - F(\beta c - \gamma L)$. The firm charges a fixed fee such that $E[U_{t=0}] = \beta \left(-L + \int_0^{\beta c - \gamma L} (-\gamma L) \, \mathrm{d}F(c) + \int_{\beta c - \gamma L}^1 (b - c) \, \mathrm{d}F(c) \right) = \gamma^2 L^2 - 2L(1 + c\gamma) + (1 - \beta c)(1 + \beta c - 2c) = 0$.

If $\gamma = 0$, then $L^* = \frac{1}{2}(1 - \beta c)(1 + \beta c - 2c)$.

When $\gamma > 0$, we have $L^* = \frac{(1+c\gamma) - \sqrt{(1+c\gamma)^2 - \gamma^2(1-\beta c)(1+\beta c - 2c)}}{\gamma^2} = \frac{(1-\beta c)(1+\beta c-2c)}{(1+c\gamma) + \sqrt{(1+c\gamma)^2 - \gamma^2(1-\beta c)(1+\beta c-2c)}}$.

Note from the boundary condition such that $0 \leq \beta c - \gamma L < c - \gamma L$, by using the implicit derivative, we take the first-order derivative of L^* with respect to γ, and we have $\frac{\partial L^*}{\partial \gamma} = \frac{L^*(c - \gamma L^*)}{-1 - \gamma(c - \gamma L^*)} < 0$. The profits of the firm also decrease in the degree of sunk cost effect since $\frac{\partial \Pi}{\partial \gamma} = (1 - a\gamma) \frac{\partial L}{\partial \gamma} - aL < 0$.

Last, we check the boundary condition such that $\beta c - \gamma L > 0$. By plugging in $L^* = \frac{(1+c\gamma) - \sqrt{(1+c\gamma)^2 - \gamma^2(1-\beta c)(1+\beta c-2c)}}{\gamma^2}$, the condition can be reduced to $\gamma < \gamma_L = \frac{2\beta c}{1 - 2c}$. When $\gamma \geq \gamma_L$, the consumer will consume with probability 1, and $L^* = \frac{1}{2} - c$. $\square$

Appendix F Proof of Proposition 3

Following the analysis previously, we use $E(L)$ ($\Pi(L)$) and $E(p)$ ($\Pi(p)$) to denote the expected utility (revenue) for the consumer (firm), when the contract is based on a fixed fee (L) and a pay-per-use fee (p), respectively.

$$E(L) = -L + \int_0^{\beta b+\gamma L} (b-c)\,\mathrm{d}c + \int_{\beta b+\gamma L}^{1} (-\gamma L)\,\mathrm{d}c = \frac{1}{2}\gamma^2 L^2 - L(1+\gamma-\gamma b) + \frac{1}{2}\beta(2-\beta)b^2$$

$$E(p) = \int_0^{\beta b-p} (b-p-c)\,\mathrm{d}c = \frac{1}{2}(2b-p-\beta b)(\beta b-p)$$

$$\Pi(L) = L + \int_0^{\beta b+\gamma L} (-a)\,\mathrm{d}F(c) = (1-a\gamma)L - a\beta b$$

$$\Pi(p) = (\beta b-p)(p-a)$$

First, we check the consumer participation constraint. When the firm offers a fixed-fee contract, following the analysis in the proof of Proposition 1, we have the optimal fixed fee $L^* = \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2}$. When the firm offers a pay-per-use fee contract, it is optimal for a firm to set the price as $p^* = \frac{1}{2}(a+\beta b)$, and the consumer participation constraint is satisfied since $E(p^*) = \frac{1}{2}(2b-\beta b-\frac{\beta b+a}{2})(\beta b-\frac{\beta b+a}{2}) = \frac{1}{8}(4b-3\beta b-a)(\beta b-a) \geq 0$ when $a \leq \bar{a}$. By comparing the revenue of a fixed-fee contract with that of a pay-per-use contract, we find that $\Pi(L) \geq \Pi(p)$ if and only if $(1-a\gamma)\frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2} - a\beta b \geq \frac{1}{4}(\beta b-a)^2$. Simplifying, we find this condition would hold if and only if $A\gamma^2 + B\gamma + C \geq 0$, in which $A = a^2X + 2aYZ + Z^2 > 0$, $B = 2(aZ - aX - YZ) < 0$, $C = (X - 2Z)$, $X = (2-\beta)\beta b^2$, $Y = (1-b)$ and $Z = \frac{1}{4}(\beta b+a)^2$.

(i) In this proof, we show that $\Pi(L) \geq \Pi(p)$ always holds when $a < \bar{a}_2 = \frac{1}{6}[-2+2b-\beta b+\sqrt{(-2+2b-\beta b)^2 - 3(3\beta^2 b^2 - 4\beta b^2 + 2(1-b)^2)}]$ and $\bar{\beta}_2 \leq \beta \leq \bar{\beta}_3$. By calculating the discriminant of quadratic function $A\gamma^2+B\gamma+C = 0$, we have $\Delta = B^2-4AC \geq 0$ is equivalent to $(Y+a)^2 \geq (X-2Z)$. In other words, $\Pi(L) > \Pi(p)$ always holds when $\Delta = B^2-4AC < 0$ and $(Y+a)^2 < (X-2Z)$. By plugging in $X = (2-\beta)\beta b^2$, $Y = (1-b)$ and $Z = \frac{1}{4}(\beta b+a)^2$, $(Y+a)^2 < (X-2Z)$ can be reduced to $\frac{3}{2}a^2+a(2-2b+\beta b)+\frac{3}{2}\beta^2 b^2-2\beta b^2+(1-b)^2 < 0$. Solving this inequality, we have $a \leq \bar{a}_2 = \frac{1}{3}[-2+2b-\beta b+\sqrt{(-2+2b-\beta b)^2-6(1-b)^2+3(4-3\beta)\beta b^2}]$. To ensure $\bar{a}_2 \geq 0$, we have $\bar{\beta}_2 = \frac{2}{3} - \frac{1}{3b}\sqrt{4b^2-6(1-b)^2} \leq \beta \leq \frac{2}{3} + \frac{1}{3b}\sqrt{4b^2-6(1-b)^2}$.

By denoting $\bar{\beta}_3$ as $min\{\frac{2}{3} + \frac{1}{3b}\sqrt{4b^2 - 6(1-b)^2}, 1\}$, we have shown that when $a < \bar{a}_2$ and $\bar{\beta}_2 \le \beta \le \bar{\beta}_3$, $\Pi(L) \ge \Pi(p)$ always holds.

(ii) In this proof, we show that when $\bar{a}_2 \le a \le min\{\bar{a}, \bar{a}_3\}$ and $\bar{\beta}_4 \le \beta < 1$, $\Pi(L) > \Pi(p)$ if and only if $\gamma < \bar{\gamma}_3 = \frac{16a(2-\beta)\beta b^2 + 4(1-b-a)(\beta b+a)^2 - 2(\beta b+a)^2\sqrt{4(1-b+a)^2 - 4(2-\beta)\beta b^2 + 2(\beta b+a)^2}}{16a^2(2-\beta)\beta b^2 + 8a(1-b)(\beta b+a)^2 + (\beta b+a)^4}$. Using the notation defined earlier, we find that when $a \le \bar{a}_3 = \sqrt{(2-\beta)\beta}b - \frac{\sqrt{2}}{2}\beta b$, $C = (X - 2Z) > 0$ and $\bar{\gamma}_3 = \frac{-B-\sqrt{B^2-4AC}}{2A} > 0$. In this case, when $\bar{a}_3 \ge a \ge \bar{a}_2$, the discriminant $\Delta = B^2 - 4AC$ is positive, and $A\gamma^2 + B\gamma + C > 0$ holds when $\gamma < \bar{\gamma}_3 = \frac{-B-\sqrt{B^2-4AC}}{2A}$ or $\gamma > \frac{-B+\sqrt{B^2-4AC}}{2A}$. However, note that when $\gamma > \frac{-B+\sqrt{B^2-4AC}}{2A}$, the boundary condition $\gamma L + \beta b \le 1$ does not hold, we only keep the solution $\gamma < \bar{\gamma}_3 = \frac{-B-\sqrt{B^2-4AC}}{2A}$. In sum, we have shown that when $\bar{a}_2 \le a \le min\{\bar{a}, \bar{a}_3\}$, $\Pi(L) > \Pi(p)$ if and only if $\gamma < \bar{\gamma}_3 = \frac{-B-\sqrt{B^2-4AC}}{2A}$. Lastly, to ensure the boundary condition $\bar{\gamma}_3 \le 1$, we impose $\bar{\beta}_4 = max\{\frac{3}{2b} - 1, \frac{a}{b} + \frac{\sqrt{4b-4a-2}}{b}\} \le \beta < 1$. In sum, we find that when we have shown that when $\bar{\beta}_4 \le \beta < 1$ and $\bar{a}_2 \le a \le min\{\bar{a}, \bar{a}_3\}$, the fixed-fee contract is more profitable than the pay-per-use fee contract when $\gamma \le \bar{\gamma}_3$; while the pay-per-use fee is more profitable when $\gamma > \bar{\gamma}_3$. □

Appendix G Proof of Proposition 4

In this proof, we show that under a two-part tariff, the firm's profits decrease with the degree of sunk cost effect γ and the presence of the sunk cost effect reduces the firm's profits. Note that in the equilibrium, the consumer participation constraint in Equation 8 is binding to 0 and $L = \int_0^{\beta b - p + \gamma L}(b - p - c)\,\mathrm{d}F(c) + \int_{\beta b - p + \gamma L}^{1}(-\gamma L)\,\mathrm{d}F(c)$, since otherwise, the firm can increase its profit by setting a higher L. Note that consumer participation constraint is equivalent to $\frac{1}{2}[p^2 - 2p(b + \gamma L) + (b + \gamma L)^2] = \frac{1}{2}[1 - (2-\beta)\beta]b^2 + (1+\gamma)L$, we can rewrite p as a function of L such that $p(L) = \gamma L + b - \sqrt{2(1+\gamma)L + (1-\beta)^2 b^2}$. In this case, the profit of the firm in Equation 8 can be written as a function of L and p such that $\pi(p, L) = \int_0^{\beta b - p + \gamma L}(b - c - a)\,\mathrm{d}F(c) + \int_{\beta b - p + \gamma L}^{1}(-\gamma L)\,\mathrm{d}F(c) = \frac{1}{2}(\beta b + \gamma L - p)(2b - 2a - \beta b + \gamma L + p) - \gamma L$. By taking the partial derivative of L with respect to $\pi(p, L)$, we have $\frac{\partial \pi}{\partial L} = \frac{1}{2}(\gamma - \frac{\partial p}{\partial L})(2b - 2a - \beta b + \gamma L + p) + \frac{1}{2}(\gamma + \frac{\partial p}{\partial L})(\beta b + \gamma L - p) - \gamma$. Note that in the equilibrium when L maximizes firm's profits, $\frac{\partial\pi}{\partial L}=0$, and we have $\gamma(1-b+a-\gamma L)=\frac{\partial p}{\partial L}(\beta b-p-b+a)$. Then we show that the profit of firm $\pi(p,L)$ is decreasing in γ. By taking the total derivative of $\pi(p,L)$ with respect to γ, we have $\frac{\mathrm{d}\pi}{\mathrm{d}\gamma}=\frac{\partial\pi}{\partial L}\frac{\partial L}{\partial\gamma}+\frac{\partial\pi}{\partial p}\frac{\partial p}{\partial\gamma}=\frac{1}{2}(L+\gamma\frac{\partial L}{\partial\gamma}-\frac{\partial p}{\partial\gamma}-\frac{\partial p}{\partial L}\frac{\partial L}{\partial\gamma})(2b-2a-\beta b+\gamma L+p)+\frac{1}{2}(L+\gamma\frac{\partial L}{\partial\gamma}+\frac{\partial p}{\partial\gamma}+\frac{\partial p}{\partial L}\frac{\partial L}{\partial\gamma})(\beta b+\gamma L-p)-L-\gamma\frac{\partial L}{\partial\gamma}$. To simplify, we have $\frac{\mathrm{d}\pi}{\mathrm{d}\gamma}=L(b-a+\gamma L-1)+\gamma\frac{\partial L}{\partial\gamma}(b-a+\gamma L-1)+\frac{\partial p}{\partial\gamma}(\beta b-b+a-p)+\frac{\partial p}{\partial L}\frac{\partial L}{\partial\gamma}(\beta b-p-b+a)$. Note that when $\frac{\partial\pi}{\partial L}=0$, $\gamma(1-b+a-\gamma L)=\frac{\partial p}{\partial L}(\beta b-p-b+a)$, then $\frac{\mathrm{d}\pi}{\mathrm{d}\gamma}=L(b-a+\gamma L-1)+\frac{\partial p}{\partial\gamma}(\beta b-b+a-p)$. Next, we show $\frac{\mathrm{d}\pi}{\mathrm{d}\gamma}=L(b-a+\gamma L-1)+\frac{\partial p}{\partial\gamma}(\beta b-b+a-p)<0$.

By denoting $f(L,p)=L+(\beta b+\gamma L-p)(p-a)$ and $h(L,p)=-(1+\gamma)L+\frac{1}{2}(\beta b+\gamma L-p)(2b-p-\beta b+\gamma L)$, the original problem in Equation 8 can be written as $max_{L,p}f(L,p)$ subject to $h(L,p)\geq 0$. By using Lagrangian Multiplier, we have $\frac{\partial f}{\partial L}/\frac{\partial f}{\partial p}=\frac{\partial h}{\partial L}/\frac{\partial h}{\partial p}$. By plugging in $\frac{\partial f}{\partial L}=1+\gamma(p-a)$, $\frac{\partial f}{\partial p}=\beta b+a+\gamma L-2p$, $\frac{\partial h}{\partial L}=-1-\gamma(1-b+p-\gamma L)$ and $\frac{\partial h}{\partial p}=-b-\gamma L+p$, the condition in the Lagrangian Multiplier can deduce $\frac{1+\gamma(p-a)}{\beta b+a+\gamma L-2p}=\frac{1+\gamma(1-b+p-\gamma L)}{b+\gamma L-p}=\frac{\gamma(-1-a+\gamma L+b)}{\beta b-p+a-b}$.

Next, we show $(-1-a+\gamma L+b)<0$ by contradiction. Suppose $(-1-a+\gamma L+b)>0$, since $1+\gamma(1-b+p-\gamma L)>0$ and $b+\gamma L-p>0$, from $\frac{1+\gamma(1-b+p-\gamma L)}{b+\gamma L-p}=\frac{\gamma(-1-a+\gamma L+b)}{\beta b-p+a-b}$ we can deduce $\beta b-p+a-b>0$. Note that $\beta b+\gamma L-p\leq 1$, $\beta b-p+a-b<\beta b+a-b-(\beta b+\gamma L-1)\leq 1+a-\gamma L-b<0$, leading to contradiction.

Knowing that $(-1-a+\gamma L+b)<0$, the total derivative of $\pi(p,L)$ with respect to γ can be written as $\frac{\mathrm{d}\pi}{\mathrm{d}\gamma}=L(b-a+\gamma L-1)+(L-\frac{L}{\gamma L+b-p})\frac{\gamma(b-a-1+\gamma L)(\gamma L+b-p)}{1+\gamma(1+p-\gamma L-b)}$, since $\frac{\partial p}{\partial\gamma}=L-\frac{L}{\sqrt{2(1+\gamma)L+(1-\beta)^2b^2}}=L-\frac{L}{\gamma L+b-p}$ and $[1+\gamma(1+p-\gamma L-b)](\beta b-b+a-p)=\gamma(\gamma L+b-a-1)(b+\gamma L-p)$ (Lagrangian Multiplier). To simplify, we have $\frac{\mathrm{d}\pi}{\mathrm{d}\gamma}=L\frac{b-a+\gamma L-1}{1+\gamma(1+p-\gamma L-b)}<0$.

In sum, we have shown that the profit of the firm under a two-part tariff decreases with the degree of the sunk cost effect γ monotonically such that $\frac{\mathrm{d}\pi}{\mathrm{d}\gamma}<0$, and as a result, the presence of the sunk cost effect reduces the firm's profit. $\square$

Online Appendix: Two-part Tariff Pricing

The figure depicts the relationship between the degree of the sunk cost effect (γ) and the optimal fixed fee (L) and per-use fee (p) under the two-part tariff.

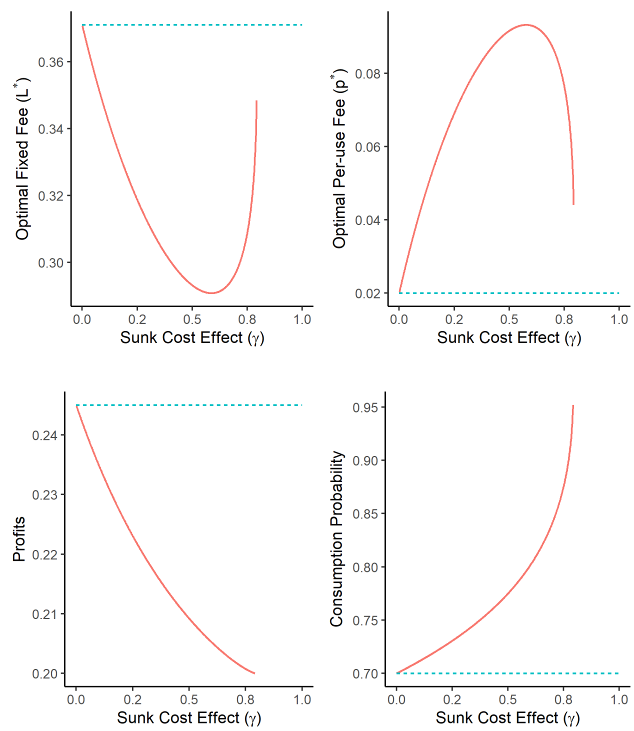

Figure 4: Sunk Cost Effect and Optimal Two-part Tariff

Note: In the plot, we demonstrate the case where $a = 0.2$, $b = 0.9$, and $\beta = 0.8$. The solid line is the optimal fixed fee and per-use fee given the degree of sunk cost effect ($\gamma > 0$), and as a contrast, the dashed line is the optimal fixed fee when there is no sunk cost effect ($\gamma = 0$)

Online Appendix: Refund Policy

Proof of Proposition 5

First, we examine the case when $\gamma = 0$. By rewriting the conditions in Equation 9 as the profit maximization problem such that $\max_{L,f}\Pi(L,f) = L-[\beta b-f+\gamma(L-f)]a-[1-\beta b+f-\gamma(L-f)]f$ subject to $E(U_{t=0}) = -L+\frac{1}{2}[\beta b-f+\gamma(L-f)][2b-\beta b-f+\gamma(L-f)]+(f-\gamma(L-f)) \geq 0$. Note that without sunk cost effect ($\gamma = 0$), the profit maximization problem can be reduced to

$\max_{L,f}\Pi(L,f) = L-(\beta b-f)a-(1-\beta b+f)f$ subject to $E(U_{t=0}) = -L+\frac{1}{2}(\beta b-f)(2b-\beta b-f)+f \geq 0$. Note that in the equilibrium, the consumer's expected payoff $E(U_{t=0})$ is binding to 0, and the equilibrium pricing strategy yields $L^* = \frac{1}{2}(\beta b-f^*)(2b-\beta b+f^*)+(1-\beta b+f^*)f^*$. Plugging this L^* into the profit maximization problem of the firm, the firm chooses the optimal f^* to maximize $\Pi(f)|_{\gamma=0} = \frac{1}{2}(\beta b-f)(2b-\beta b+f-2a)$. By taking the first order derivative of $\Pi(f)$ with respect to f, we have $f^*|_{\gamma=0} = \beta b+a-b$, $L^*|_{\gamma=0} = \frac{1}{2}(b-a)(b+a)+(a-b+\beta b)(1+a-b)$ and $\Pi(L^*,f^*)|_{\gamma=0} = \frac{1}{2}(b-a)^2$. Next, we show that this optimal fixed fee and refund $\{L^*,f^*\}$ achieve the first-best outcome. By denoting the social welfare as the sum of the firm's profits and consumer surplus such that $W = \Pi(L,f)+E(U_{t=0})$, we have $W = \int_0^{\beta b+\gamma(L-f)-f}(b-c-a)\,\mathrm{d}F(c)-\int_{\beta b+\gamma(L-f)-f}^1\gamma(L-f)\,\mathrm{d}F(c) \leq \int_0^{\beta b+\gamma(L-f)-f}(b-c-a)\,\mathrm{d}F(c) = \frac{1}{2}(2b-2a-\beta b-\gamma(L-f)+f)(\beta b+\gamma(L-f)-f) \leq \frac{1}{2}(b-a)^2$, and the equality $W = \frac{1}{2}(b-a)^2$ achieves when $\gamma = 0$ and $b-a = \beta b-f+\gamma(L-f)$ hold. The analysis above shows that the optimal fixed fee and refund $\{L^*,f^*\}$ achieves the first-best outcome when $\gamma = 0$.

Next, we show that the presence of the sunk cost effect reduces the firm's profit by showing $\pi(L,f) < \frac{1}{2}(b-a)^2$ when $\gamma > 0$. Note that in the equilibrium, $E(U_{t=0}) \geq 0$, then $\pi(L,f) \leq W \leq \int_0^{\beta b+\gamma(L-f)-f}(b-c-a)\,\mathrm{d}F(c) \leq \frac{1}{2}(b-a)^2$, and the equality $W = \int_0^{\beta b+\gamma(L-f)-f}(b-c-a)\,\mathrm{d}F(c)$ achieves only when $\int_{\beta b+\gamma(L-f)-f}^1\gamma(L-f)\,\mathrm{d}F(c) = 0$, which is satisfied when $\gamma = 0$, $L = f$ or $\beta b+\gamma(L-f)-f = 1$. However, when $L = f$, the consumer's expected utility and the firm's profit reduce to $E(U_{t=0}) = \frac{1}{2}(\beta b-f)(2b-f-\beta b)$ and $\Pi(L,f) = (\beta b-f)(f-a)$, then the optimal refund is $f^* = \frac{1}{2}(\beta b+a) < \beta b$. That is, $E(U_{t=0}) > 0$ and $\pi(L,f) < W \leq \frac{1}{2}(b-a)^2$ holds with strict inequality. As a result, when $\gamma > 0$, the first best profit can be achieved only if $\beta b+\gamma(L-f)-f = 1$. When $\beta b+\gamma(L-f)-f = 1$, the consumer consumes the product with probability 1, and the social welfare becomes $W = \int_0^1(b-c-a)\,\mathrm{d}F(c) = b-a-\frac{1}{2}$. It is straightforward to see that $W = b-a-\frac{1}{2} \leq \frac{1}{2}(b-a)^2$, and the equality achieves only when $b-a = 1$, which implies $b = 1$ and $a = 0$. In sum, we have shown that the profit of the firm is strictly worse off in the presence of the sunk cost effect ($\gamma > 0$), except for a knife edge value $b = 1$ and $a = 0$. $\square$

Online Appendix: Memory-cue Based Sunk Cost Effect

Proof of Proposition 6

The optimal fixed-fee contract is reached when $\beta\left(-L+\int_0^{\beta b+sL}(b-c)\,\mathrm{d}F(c)\right)=-\frac{1}{2}\beta(s^2L^2+2L(1-bs(1-\beta))-\beta(2-\beta)b^2)=0$.

If $s=0$, then $L^*=\beta b^2-\frac{1}{2}\beta^2b^2$.

If $s>0$, then,

$$L^*=\frac{s(1-\beta)b-1+\sqrt{[1-s(1-\beta)b]^2+s^2(2-\beta)\beta b^2)}}{s^2}$$

By using the implicit function derivative, we have $\frac{\partial L^*}{\partial s}=\frac{(1-\beta)bL-sL^2}{s^2L+1-bs(1-\beta)}$, and $\frac{\partial L^*}{\partial s}>0$ when $sL<(1-\beta)b$. By plugging in $L^*=\frac{s(1-\beta)b-1+\sqrt{[1-s(1-\beta)b]^2+s^2(2-\beta)\beta b^2)}}{s^2}$, the inequality $sL<(1-\beta)b$ can be reduced to $s<\bar{s}_2=\frac{2(1-\beta)}{b}$. In this case, we have shown that $\frac{\partial L^*}{\partial s}>0$ when $s<\bar{s}_2$ while $\frac{\partial L^*}{\partial s}<0$ when $s>\bar{s}_2$. However, to ensure that $\bar{s}_2\leq 1$, we have $2(1-\beta)\leq b$, which can be reduced to $\beta\geq 1-\frac{b}{2}$. Furthermore, in order to guarantee the boundary condition $\beta b+sL^*\leq 1$, by plugging in $L^*=\frac{s(1-\beta)b-1+\sqrt{[1-s(1-\beta)b]^2+s^2(2-\beta)\beta b^2)}}{s^2}$, $sL+\beta b\leq 1$ reduces to $s\leq\frac{2(1-\beta b)}{2b-1}=\bar{s}_1$. When $s>\bar{s}_1$, the consumer consumes with probability one, and the optimal L^* is obtained from $E(U_t=0)=-L+\int_0^1(b-c)\,\mathrm{d}F(c)=0$ and $L^*=b-\frac{1}{2}$.

In sum, when $\beta\leq 1-\frac{b}{2}$, the fixed-fee contract increase with the sunk cost effect. Otherwise, when $1-\frac{b}{2}<\beta<1$, L^* increases with s when $s<\bar{s}_2$, L^* decreases with s when $\bar{s}_2\leq s\leq\bar{s}_1$, while L^* is flat when $\bar{s}_1<s\leq 1$.

Next, we identify the conditions under which the fixed-fee contract is more profitable in the presence of the memory-cue sunk cost effect. Our analysis shows that that $\pi(s>0)>\pi(s=0)$ if and only if $(1-as)L^*-a\beta b>\frac{(2-\beta)\beta b^2}{2}-a\beta b$, which is equivalent to $(1-as)L^*>\frac{(2-\beta)\beta b^2}{2}$. By plugging in $L^*=\frac{s(1-\beta)b-1+\sqrt{[1-s(1-\beta)b]^2+s^2(2-\beta)\beta b^2}}{s^2}=\frac{(2-\beta)\beta b^2}{1-s(1-\beta)b+\sqrt{[1-s(1-\beta)b]^2+s^2(2-\beta)\beta b^2)}}$, this inequality can be reduced to $2(1-as)>1+s\beta b-sb+\sqrt{[1-s(1-\beta)b]^2+s^2(2-\beta)\beta b^2}$, and we have $s<\frac{4(b-a-\beta b)}{(2-\beta)\beta b^2+4a(b-a-\beta b)}$. To ensure $\frac{4(b-a-\beta b)}{(2-\beta)\beta b^2+4a(b-a-\beta b)}>0$, we have $a<b-\beta b$. In

sum, we have shown that when $s < \frac{4(b-a-\beta b)}{(2-\beta)\beta b^2+4a(b-a-\beta b)}$ and $a < b - \beta b$, the fixed-fee contract is more profitable in the presence of the memory-cue sunk cost effect. $\square$

Two-part Tariff and Refund Policy under Memory-Cue-Based Sunk Cost Effect

When the consumer exhibits the memory-cue-based sunk cost effect, using a two-part tariff, the firm is solving the following profit maximization problem:

$$\max_{L,p}\left(L+\int_0^{\beta b-p+sL}(p-a)\,\mathrm{d}F(c)\right) \tag{15}$$

subject to:

$$\beta\left(-L+\int_0^{\beta b-p+sL}(b-p-c)\,\mathrm{d}F(c)\right)\geq 0$$

In our model, we assume the cost c follows a uniform distribution such that $c \sim U[0,1]$, and the profit maximization problem can be written as $\max_{L,p}(L+(p-a)(\beta b-p+sL))$ subject to $\frac{1}{2}\beta\left(-s^2L^2+2\left(s(1-\beta)b-1\right)L+2\beta b^2-2bp+p^2-\beta^2b^2\right)=0$. If we denote the objective function as $f(L,p)$ and the constraint as $g(L,p)=0$. By Lagrange multiplier, $\frac{\partial f(L,p)}{\partial L}/\frac{\partial f(L,p)}{\partial p}=\frac{\partial g(L,p)}{\partial L}/\frac{\partial g(L,p)}{\partial p}$, we obtain the new equality:

$$\frac{1+s(p-a)}{\beta b-2p+sL+a}=\frac{s(1-\beta)b-s^2L-1}{p-b} \tag{16}$$

The condition in 16 is achieved when $(1-\beta)b+p-a=sL$. Combing this condition with that in $\frac{1}{2}\beta\left(-s^2L^2+2\left(s(1-\beta)b-1\right)L+2\beta b^2-2bp+p^2-\beta^2b^2\right)=0$, we have:

$$\begin{cases} L^* = \frac{(b-a)(b(3-2\beta)-a)}{2s(b-a)+2} \\ p^* = \frac{2(a-(1-\beta)b)+s(b^2-a^2)}{2s(b-a)+2} \end{cases} \tag{17}$$

Hence we obtain:

$$\frac{\partial L^*}{\partial s}=\frac{(b-a)^2(a-b(3-2\beta))}{2(s(b-a)+1)^2}<0$$

$$\frac{\partial p^*}{\partial s} = \frac{(b-a)(3b-2\beta b-a)}{2(s(b-a)+1)^2} > 0$$

Lastly, we show that this two-part tariff yields the first-best outcome. Similar to the analysis previously, we define the social welfare as $E[W_{t=0}(L,p)] = E[\Pi_{t=0}(L,p)] + E[U_{t=0}(L,p)]$ and $E[W_{t=0}(L,p)] = \int_0^{\beta b-p+sL}(b-a-c)\,\mathrm{d}F(c) = \frac{1}{2}(2b-2a-\beta b+p-sL)(\beta b-p+sL)$. It is straightforward to see that $E[W_{t=0}(L,p)] = \frac{1}{2}(2b-2a-\beta b+p-sL)(\beta b-p+sL) \leq \frac{1}{2}(b-a)^2$, and the equality achieves when $\beta b-p+sL = b-a$, which is exactly the condition we deduce from 16. As a result, the first-best outcome can be achieved when $L^* = \frac{(b-a)(b(3-2\beta)-a)}{2s(b-a)+2}$ and $p^* = \frac{2(a-(1-\beta)b)+s(b^2-a^2)}{2s(b-a)+2}$.

When the consumer exhibits the memory-cue-based sunk cost effect, when providing a refund to the consumer, the firm solves the profit maximization problem as follows:

$$\max_{L,f}\left(L + \int_0^{\beta b+s(L-f)-f}(-a)\,\mathrm{d}F(c) + \int_{\beta b+s(L-f)-f}^{1}(-f)\,\mathrm{d}F(c)\right) \tag{18}$$

subject to:

$$\beta\left(-L + \int_0^{\beta b+s(L-f)-f}(b-c)\,\mathrm{d}F(c) + + \int_{\beta b+s(L-f)-f}^{1}(f)\,\mathrm{d}F(c)\right) \geq 0$$

Similar to the proof of the two-part tariff under memory-cue-based sunk cost effect, by using Lagrange Multiplier, the profit maximization problem in 18 leads to:

$$\frac{1+s(p-a)}{-1+s(b-\beta b-s(L-f))} = \frac{-1+\beta b+a-2f+s(L+a-2f)}{1-b+f+s(\beta b-b+s(L-f))} \tag{19}$$

Solving the condition in 19, we have $s(L-f) = b-a+f-\beta b$. By plugging this condition into the participation constraint of the consumer, the optimal fee and refund are:

$$\begin{cases} L^* = \frac{(s+1)(b^2-a^2)+2(1-b+a)(a+\beta b-b)}{2s(b-a)+2} \\ f^* = \frac{a-(1-\beta)b+\frac{1}{2}s(b^2-a^2)}{s(b-a)+1} \end{cases} \tag{20}$$

By taking the first-order derivative of s with respect of L^* and f^*, we have

$$\begin{cases} \frac{\partial L^*}{\partial s} = \frac{(1-b+a)(b-a)(3b-a-2\beta b)}{2[s(b-a)+1]^2} > 0 \\ \frac{\partial f^*}{\partial s} = \frac{(b-a)(3b-a-2\beta b)}{2[s(b-a)+1]^2} > 0 \end{cases} \tag{21}$$

Note from our earlier analysis that the optimal social welfare $W = \frac{1}{2}(b-a)^2$ is achieved when $\beta b - f + s(L - f) = b - a$. As a result, the optimal refund as shown in 20 generates the first-best profit for the firm since the consumer's expected payoff is 0 in the equilibrium. □

Online Appendix: Screening Contracts

Suppose that consumers are heterogeneous in their sensitivity to the sunk cost, and their preferences are not observable by the firm. This sets up the possible screening incentives for the firm. According to the telephone survey conducted by (DellaVigna and Malmendier, 2004), the majority of health clubs offer a menu of contracts specifically consisting of a fixed-fee and a pay-per-use contract. *Coursera.com* offers "Coursera Plus" — a fixed-fee contract under which the user can earn unlimited certificates in the contract period. The platform also offers the "Single learning program" — a pay-per-use type contract under which the user has to pay for every single certificate. The menu of contracts can potentially serve as a screening device to let consumers self-select based on their types. Below we characterize the optimal contract menu when there is unobserved consumer heterogeneity in the degree of the sunk cost effect. We analyze the profit implications when the firm offers a menu of contracts.

Assume there are two types of consumers: Type 1 would ignore the sunk cost i.e., $\gamma = 0$, whereas Type 2 is prone to the sunk cost effect and would experience regret dis-utility if she does not consume i.e., $\gamma > 0$. Assume that the two types of consumers are evenly distributed and that the firm will serve both types in equilibrium. The firm's profits from the fixed-fee and pay-per-use contract are denoted as $\Pi(L)$ and $\Pi(p)$, respectively.

$$\Pi(L) = L + \int_0^{\beta b + \gamma L} (-a) \, \mathrm{d}F(c)$$

$$\Pi(p) = \int_0^{\beta b - p} (p - a)\,\mathrm{d}F(c)$$

On the other hand, a consumer chooses between the fixed-fee (L) and pay-per-use (p) contract by comparing its expected payoff $E[U(p)]$ and $E[U(L)]$. When $E[U(L)] \geq E[U(p)]$, a consumer prefers a fixed-fee contract; otherwise, the consumer chooses a pay-per-use fee contract.

$$E[U(p)] = \int_0^{\beta b - p} (b - p - c)\,\mathrm{d}F(c)$$

$$E[U(L)] = -L + \int_0^{\beta b + \gamma L} (b - c)\,\mathrm{d}F(c) + \int_{\beta b + \gamma L}^{1} (-\gamma L)\,\mathrm{d}F(c)$$

For both types of consumers, the fixed-fee contract induces higher consumption compared to a pay-per-use contract because it offers a zero marginal price for every consumption. However, as Type 2 consumer is prone to the sunk cost effect, the fixed-fee contract induces a higher consumption probability for Type 2 than Type 1 consumer. As a result, although the fixed-fee induced sunk cost effect might help Type 2 consumer counteract the self-control problem, the fixed-fee contract generates a higher expected marginal cost for the firm from Type 2 than from Type 1 consumer, which gives less incentive for the firm to let Type 2 consumer choose the fixed-fee contract.

There are two possible separating equilibria: 1) Type 1 consumer chooses the fixed-fee contract and Type 2 consumer chooses the pay-per-use contract; 2) Type 1 consumer chooses the pay-per-use contract and Type 2 consumer chooses the fixed-fee contract. Also, there are two possible pooling equilibria: 1) both Type 1 and 2 choose the fixed-fee contract; 2) both Type 1 and 2 choose the pay-per-use contract. The solution strategy is we first set up the individual rationality (IR) and incentive compatibility (IC) constraints for the consumers and the firm in each possible equilibrium. Next, we solve the optimal L and p in each possible equilibrium, and check whether the optimal L and p can guarantee the IR and IC conditions in that equilibrium. The following lemma summarizes the results:

LEMMA 2. *(i) When both the marginal cost of the firm and Type 2 consumer's degree of the sunk cost effect are moderate (i.e., $\bar{a}_4 < a < \bar{a}_5$ and $\bar{\gamma}_4 < \gamma < \bar{\gamma}_5$), there exists a separating equilibrium in which Type 1 consumer will choose the fixed-fee contract while Type 2 consumer will choose the pay-per-use contract.*[10]

(ii) The equilibrium price menu consists of

$$\begin{cases} L^* = \frac{1}{18}(a+b+\beta b)(5b-a-\beta b) \\ p^* = \frac{1}{3}(a+b+\beta b) \end{cases} \tag{22}$$

The proofs are presented at the end of the section.

In the separating equilibrium: Type 1 consumer ($\gamma = 0$) will choose the fixed-fee contract while Type 2 consumer ($\gamma > 0$) will choose the pay-per-use contract. This seems counter-intuitive because one might expect the Type 2 consumer to benefit more from a fixed-fee contract since the sunk cost effect can work as a commitment device inducing her to consume more, especially when the degree of time inconsistency is high. However, from Type 2 consumer's perspective, she has a low willingness to pay when the sunk cost effect is in the medium range (as shown in Figure 2). From the firm's perspective, if the Type 2 consumer chooses the fixed-fee contract, her sunk cost effect results in a higher consumption probability and higher expected marginal cost, which can lower the firm's expected profits. As a result, the firm prefers Type 1 consumers to choose the fixed-fee contract.

Interestingly, in the presence of heterogeneity in the degree of the sunk cost effect, it is not always optimal for the firm to offer a menu of contracts. The firm can improve its profits by offering a single fixed-fee contract or a single pay-per-use contract. This is because in the contract menu, the pay-per-use contract yields a positive consumer surplus which reduces the attractiveness of the fixed-fee contract and the firm's ability to charge a high fixed fee. Therefore, under the menu of contracts, the firm leaves a positive surplus for both

[10] Here $\bar{a}_4 = \frac{19b-5\beta b-18-\sqrt{(19b-5\beta b-18)^2+5(1+\beta)(7-5\beta)}}{5}$, $\bar{a}_5 = min\{\frac{1}{5}(7-5\beta)b, (2\beta-1)b\}$, $\bar{\gamma}_4 = \frac{1}{a}[1 - \frac{2(2a+2\beta b-b)}{5b-a-\beta b}]$, and $\bar{\gamma}_5 = \frac{36(1-b)}{(a+b+\beta b)(5b-a-\beta b)}$

types of consumers, resulting in lower profits; whereas under a fixed-fee contract, the firm either extracts all the surplus from Type 1 consumer or from Type 2 consumer. Therefore, when the contract menu consists of both a fixed-fee contract and a pay-per-use contract, the pay-per-use contract can cannibalize the fixed-fee contract in the equilibrium, which may lower the overall profits. The following proposition identifies the condition when the screening contract menu can improve the firm's profits:

PROPOSITION 7. *The screening contract menu separating two types of consumers can improve firms' profits when the degree of sunk cost effect is high (i.e., $\frac{2(1-\beta b)}{2b-1} \leq \gamma \leq 1$), the consumer is sufficiently time consistent (i.e., $\beta \geq \frac{3}{2b} - 1$) and the operational cost is moderate (i.e., $\bar{a}_6 < a < \bar{a}_7$).*[11]

The proofs are presented at the end of the section. When Type 2 consumer's sunk cost effect is sufficiently large, offering a fixed-fee contract to Type 2 consumers will increase the probability of consumption. If the firm's marginal cost is moderate, the sunk cost effect increases the firm's expected costs, thus negatively affecting the firm's profits. Hence, in such a case, it is beneficial for the firm to offer Type 2 consumer the pay-per-use contract. Meanwhile, as Type 1 consumer's behavior is not driven by the sunk cost effect, the fixed fee does not induce a higher expected cost for the firm due to the sunk cost effect. As a result, the firm would like to offer the fixed-fee contract to Type 1 consumer.

It is noteworthy that there does not exist a separating equilibrium where Type 1 consumer chooses the pay-per-use contract and the Type 2 consumer chooses the fixed-fee contract. When the marginal cost of the firm and the degree of Type 2 consumer's sunk cost effect is low, both Type 1 and Type 2 consumers will choose a fixed-fee contract. That is, a pooling equilibrium would emerge. This is because compared to the pay-per-use contract, the consumer does not incur any monetary cost when consuming the product under the fixed-fee contract. For Type 2 consumer, the sunk cost effect further induces a higher chance

[11] $\bar{a}_6 = \max\{\frac{19b-5\beta b-18-\sqrt{(19b-5\beta b-18)^2+5(1+\beta)(7-5\beta)}}{5}, 2\beta b-b-3+\sqrt{3(1-\beta b)(3+2b-\beta b)+12b-6}\}$ and $\bar{a}_7 = \min\{\frac{\sqrt{3}+1}{2}b-\beta b, (2\beta-1)b, \frac{1}{5}(7-5\beta)b\}$.

of consumption. Therefore, both Type 1 and Type 2 consumers would have a higher chance of consumption, which increases the willingness to pay for a fixed-fee contract. In such an equilibrium, the firm would set the per-use fee so high (i.e., $p > \beta b$) that no consumers would find it attractive. On the other hand, when the marginal cost of the firm is high, both types choose the pay-per-use contract, resulting in another pooling equilibrium. This is because the pay-per-use contract leads to a lower consumption level, reducing the expected marginal cost for the firm.

Proof of Lemma 2

In this proof, we investigate the equilibrium outcome when the degree of sunk cost effect (γ) is differentiated while the degree of time consistency level (β) is the same. The solution strategy in this proof is that we first set up the individual rationality (IR) and incentive compatibility (IC) constraints for the consumers and the firm in each possible equilibrium, and then check whether the optimal pricing set L and p of the firm guarantee the IR and IC conditions. We specify the consumer's expected payoff and the firm's profits as follows:

$$E(L) = \left(-L + \int_0^{\beta b+\gamma L} (b-c)\,\mathrm{d}F(c) + \int_{\beta b+\gamma L}^{1} (-\gamma L)\,\mathrm{d}F(c)\right) = \frac{1}{2}\gamma^2 L^2 - L(1+\gamma-\gamma b) + \frac{1}{2}\beta(2-\beta)b^2$$

$$E(p) = \int_0^{\beta b-p} (b-p-c)\,\mathrm{d}F(c) = \frac{1}{2}(2b-p-\beta b)(\beta b-p)$$

$$\Pi(L) = L + \int_0^{\beta b+\gamma L} (-a)\,\mathrm{d}F(c) = (1-a\gamma)L - a\beta b$$

$$\Pi(p) = \int_0^{\beta b-p} (p-a)\,\mathrm{d}F(c) = (\beta b-p)(p-a)$$

Following the notations in our paper, we assume the Type 1 consumer ignores the sunk cost effect such that $\gamma = 0$, whereas the Type 2 consumer would experience regret dis-utility if she does not consume, i.e., $\gamma > 0$. In this proof, we use IC_1 and IC_2 to denote the incentive compatibility constraints for Type 1 and Type 2 consumers, respectively; while the individual rationality constraints for Type 1 and Type 2 consumers are denoted as IR_1 and IR_2. The incentive compatibility constraints for the firm to provide contracts to Type 1 and Type 2 consumers are denoted as IC_{F1} and IC_{F2}. In addition, we use $E_i(U(m))$ to denote the expected utility of Type i ($i = 1, 2$) consumer when choosing a contract m ($m = p, L$), and $\Pi_i(m)$ is the expected revenue of the firm by offering a contract m to a Type i consumer. There are four different possible equilibrium outcomes specified as follows.

Case 1: Type 1 consumer chooses a pay-per-use fee contract, Type 2 consumer chooses a fixed-fee contract

When Type 1 consumer chooses a pay-per-use contract (p) while Type 2 consumer chooses a fixed-fee contract (L), the following IC and IR constraints must be satisfied:

$$IC_1 : (E_1(U(p)) > E_1(U(L))) : \frac{1}{2}(2b - \beta b - p)(\beta b - p) > -L + \frac{1}{2}\beta(2 - \beta)b^2$$

$$IC_2 : (E_2(U(p)) < E_2(U(L))) : \frac{1}{2}(2b - \beta b - p)(\beta b - p) < \frac{1}{2}\gamma^2 L^2 - L(1 + \gamma - \gamma b) + \frac{1}{2}\beta(2 - \beta)b^2$$

$$IR_1 : (E_1(U(p)) > 0) : (2b - p - \beta b)(\beta b - p) > 0$$

$$IR_2 : (E_2(U(L)) > 0) : \frac{1}{2}\gamma^2 L^2 - L(1 + \gamma - \gamma b) + \frac{1}{2}\beta(2 - \beta)b^2 > 0$$

$$IC_{F1} : (\Pi_1(L) < \Pi_1(p)) : L - \beta ab < (\beta b - p)(p - a)$$

$$IC_{F2} : (\Pi_2(L) > \Pi_2(p)) : (1 - a\gamma)L - \beta ab > (\beta b - p)(p - a)$$

However, we find that IC_{F1} and IC_{F2} contradict each other, given the same level of time consistency (β). As a result, the outcome in Case 1 cannot be an equilibrium.

Case 2: Type 1 consumer chooses a fixed-fee contract, Type 2 consumers choose a pay-per-use contract

In Case 2, Type 1 consumer chooses a fixed-fee contract while Type 2 consumer chooses a pay-per-use contract. The IC and IR conditions are specified as follows:

$$IC_1(E_1(U(p)) \leq E_1(U(L))): \frac{1}{2}(2b-\beta b-p)(\beta b-p) \leq -L+\frac{1}{2}\beta(2-\beta)b^2$$

$$IC_2(E_2(U(p)) \geq E_2(U(L))): \frac{1}{2}(2b-\beta b-p)(\beta b-p) \geq \frac{1}{2}\gamma^2L^2-L(1+\gamma-\gamma b)+\frac{1}{2}\beta(2-\beta)b^2$$

$$IR_1(E_1(U(L))>0): -L+\frac{1}{2}\beta(2-\beta)b^2>0$$

$$IR_2(E_2(U(p)>0): \frac{1}{2}(2b-\beta b-p)(\beta b-p)>0$$

$$IC_{F1}(\Pi_1(L)>\Pi_1(p)): L-\beta ab>(\beta b-p)(p-a)$$

$$IC_{F2}(\Pi_2(L)\leq \Pi_2(p)): (1-a\gamma)L-\beta ab \leq (\beta b-p)(p-a)$$

The firm maximizes its total profits $\Pi(L,p)=L-a\beta b+(\beta b-p)(p-a)$ by setting the optimal pricing $\{L^*,p^*\}$. To maximize $\Pi(L,p)$, IC_1 must be binding such that $2bp-p^2=2L$ since otherwise the firm can increase its profits by setting a higher value of L. By plugging in $L^*=bp-\frac{1}{2}p^2$ into $\Pi(L,p)$, $\Pi=-\frac{3}{2}p^2+p(a+b+\beta b)-2a\beta b$, and the maximized profit is achieved when $p^*=\frac{1}{3}(a+b+\beta b)$ and $L^*=\frac{1}{18}(a+b+\beta b)(5b-a-\beta b)$, in which case $\pi^*=\frac{1}{6}(a+b+\beta b)^2-2a\beta b$. Next, we check whether other IC and IR conditions above are satisfied under the optimal contract $\{L^*,p^*\}$.

First, note that IR_1 satisfies automatically when IC_1 and IR_2 are satisfied. Then we only need to check IC_2, IR_2, IC_{F1} and IC_{F2} when $p^*=\frac{1}{3}(a+b+\beta b)$ and $L^*=\frac{1}{18}(a+b+\beta b)(5b-a-\beta b)$. When IC_1 is binding to 0, IC_2 is satisfied if and only if $\frac{1}{2}\gamma^2L^2-\gamma L(1-b)\leq 0$, which can be reduced to $\gamma \leq \frac{2(1-b)}{L^*}=\frac{36(1-b)}{(a+b+\beta b)(5b-a-\beta b)}$. To ensure IR_2 is satisfied, we have $\beta b-p>0$ which is equivalent to $a<(2\beta-1)b$. In addition, to ensure IC_{F1}, we have $L-a\beta b>(\beta b-p)(p-a)$, which can be reduced to $a<\frac{1}{5}(7-5\beta)b$. Lastly, we check the IC_{F2} by checking $(1-a\gamma)L<(\beta b-p)(p-a)$, by plugging in $p^*=\frac{1}{3}(a+b+\beta b)$ and $L^*=\frac{1}{18}(a+b+\beta b)(5b-a-\beta b)$, the condition in IC_{F2} can be reduced to $\gamma>\frac{1}{a}[1-\frac{2(2a+2\beta b-b)}{5b-a-\beta b}]$. To ensure that $\frac{1}{a}[1-\frac{2(2a+2\beta b-b)}{5b-a-\beta b}]<\gamma<\frac{36(1-b)}{(a+b+\beta b)(5b-a-\beta b)}$, we further restrict the value of a

and we have $a > \frac{19b-5\beta b-18-\sqrt{(19b-5\beta b-18)^2+5(1+\beta)(7-5\beta)}}{5}$.

In sum, we have shown that when $\bar{a}_4 = \frac{19b-5\beta b-18-\sqrt{(19b-5\beta b-18)^2+5(1+\beta)(7-5\beta)}}{5} < a < min\{\frac{1}{5}(7-5\beta)b, (2\beta-1)b\} = \bar{a}_5$ and $\bar{\gamma}_4 = \frac{1}{a}[1-\frac{2(2a+2\beta b-b)}{5b-a-\beta b}] < \gamma < \frac{36(1-b)}{(a+b+\beta b)(5b-a-\beta b)} = \bar{\gamma}_5$, Type 1 consumer will choose a fixed-fee contract $L^* = \frac{1}{18}(a+b+\beta b)(5b-a-\beta b)$ while Type 2 consumer will choose a pay-per-visit contract $p^* = \frac{1}{3}(a+b+\beta b)$.

Case 3: Both Types of consumers choose a fixed-fee contract

When both Type 1 and Type 2 consumers choose a fixed-fee contract, the following conditions must be satisfied.

$$IC_1(E_1(U(p)) \leq E_1(U(L))) : \frac{1}{2}(2b-\beta b-p)(\beta b-p) \leq -L + \frac{1}{2}\beta(2-\beta)b^2$$

$$IC_2(E_2(U(p)) \leq E_2(U(L))) : \frac{1}{2}(2b-\beta b-p)(\beta b-p) \leq \frac{1}{2}\gamma^2 L^2 - L(1+\gamma-\gamma b) + \frac{1}{2}\beta(2-\beta)b^2$$

$$IR_1(E_1(U(L)) \geq 0) : -L + \frac{1}{2}\beta(2-\beta)b^2 \geq 0$$

$$IR_2(E_2(U(L)) \geq 0) : \frac{1}{2}\gamma^2 L^2 - L(1+\gamma-\gamma b) + \frac{1}{2}\beta(2-\beta)b^2 \geq 0$$

$$IC_{F1}(\Pi_1(L) \geq \Pi_1(p)) : L - \beta ab \geq (\beta b - p)(p-a)$$

$$IC_{F2}(\Pi_2(L) \geq \Pi_2(p)) : (1-a\gamma)L - \beta ab \geq (\beta b - p)(p-a)$$

When both types of consumers choose a fixed-fee contract, the firm maximizes its profit $\Pi = (2-a\gamma)L - 2a\beta b$, and it is straightforward to see that the profit increases with L. Note from IR_1 and IR_2, we have $L^* \leq min\{\frac{1}{2}\beta(2-\beta)b^2, \frac{1+\gamma-\gamma b-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2\beta(2-\beta)b^2}}{\gamma^2}\}$, and the firm realizes its maximized profit when the condition is binding. To achieve this, the firm sets a high value for the pay-per-visit contract to avoid self-cannibalization such that $p^* = \beta b$. As a result, in equilibrium, $L^* = min\{\frac{1}{2}\beta(2-\beta)b^2, \frac{1+\gamma-\gamma b-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2\beta(2-\beta)b^2}}{\gamma^2}\}$, and $\Pi = (2-a\gamma)L^* - 2a\beta b$.

Next, we check the two IC conditions of the firm. Since IC_{F2} is a sufficient condition of IC_{F1}, we only need to ensure the condition in IC_{F2}. Note that when $p = \beta b$, the IC conditions satisfy automatically, we only need to check whether the profit of the firm is positive such that $(1-a\gamma)L^* \geq a\beta b$. When $\gamma \leq \frac{4(1-b)}{\beta(2-\beta)b^2}$, $\frac{1}{2}\beta(2-\beta)b^2 \geq \frac{1+\gamma-\gamma b-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2\beta(2-\beta)b^2}}{\gamma^2}$ and $L^* = \frac{1+\gamma-\gamma b-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2\beta(2-\beta)b^2}}{\gamma^2}$, and the condition in IC_{F2} can be reduced to $(1-a\gamma)\frac{\beta(2-\beta)b^2}{1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2\beta(2-\beta)b^2}} \geq a\beta b$. Simplifying, we find that when $a \leq \frac{1}{2}(2-\beta)b$ and $\gamma \leq \frac{3b-1-\beta b+a-\sqrt{(3b-1-\beta b+a)^2-2(3b-1-\beta b)(2b-\beta b-2a)}}{2a(3b-1-\beta b)}$, the condition in IC_{F2} holds.

In sum, we have shown that there is an equilibrium whereby both types of consumers choose a fixed-fee contract when $\gamma \leq min\{\frac{3b-1-\beta b+a-\sqrt{(3b-1-\beta b+a)^2-2(3b-1-\beta b)(2b-\beta b-2a)}}{2a(3b-1-\beta b)}, \frac{4(1-b)}{\beta(2-\beta)b^2}\}$ and $a \leq \frac{1}{2}(2-\beta)b$ and $L^* = \frac{1+\gamma-\gamma b-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2\beta(2-\beta)b^2}}{\gamma^2}$.

Case 4: Both types of consumers choose a pay-per-use fee contract

In Case 4, both Type 1 and Type 2 consumers choose a pay-per-use contract, and the following IC and IR conditions must be satisfied.

$$IC_1(E_1(U(p)) \geq E_1(U(L))) : \frac{1}{2}(2b-\beta b-p)(\beta b-p) \geq -L + \frac{1}{2}\beta(2-\beta)b^2$$

$$IC_2(E_2(U(p)) \geq E_2(U(L))) : \frac{1}{2}(2b-\beta b-p)(\beta b-p) \geq \frac{1}{2}\gamma^2L^2 - L(1+\gamma-\gamma b) + \frac{1}{2}\beta(2-\beta)b^2$$

$$IR_1(E_1(U(p)) \geq 0) : \frac{1}{2}(2b-\beta b-p)(\beta b-p) \geq 0$$

$$IR_2(E_2(U(p)) \geq 0) : \frac{1}{2}(2b-\beta b-p)(\beta b-p) \geq 0$$

$$IC_{F1}(\Pi_1(L) \leq \Pi_1(p)) : L - \beta ab \leq (\beta b-p)(p-a)$$

$$IC_{F2}(\Pi_2(L) \leq \Pi_2(p)) : (1-a\gamma)L - \beta ab \leq (\beta b-p)(p-a)$$

In this case, the profit of the firm is $\Pi(p) = 2(\beta b-p)(p-a)$, and the optimal pricing is $p^* = \frac{1}{2}(\beta b+a)$. Note that the condition in IC_{F1} is a sufficient condition for IC_{F2}, and IR_1 and IR_2 satisfy when $a \leq \beta b$. We only need to check the conditions in IC_1, IC_2 and IC_{F1}. Combining the conditions in IC_1, IC_2 and IC_{F1} together, we have $\frac{1}{4}(\beta b+a)^2 \geq \frac{1}{8}(\beta b+a)(4b-\beta b-a) \geq \frac{1}{32}\gamma^2(\beta b+a)^4 - \frac{1}{4}(\beta b+a)^2(1+\gamma-\gamma b)$. Solving the inequality, we have $a \geq \frac{4}{3}b-\beta b$ and $\gamma \geq \frac{4(1-b)+\sqrt{16(1-b)^2-4(4b-3\beta b-3a)(\beta b+a)}}{(\beta b+a)^2}$. In sum, we have shown that when $a \geq \frac{4}{3}b - \beta b$ and $\gamma \geq \frac{4(1-b)+\sqrt{16(1-b)^2-4(4b-3\beta b-3a)(\beta b+a)}}{(\beta b+a)^2}$, both types of consumers will choose a pay-per-use contract and $p^* = \frac{1}{2}(\beta b + a)$. □

Proof of Proposition 7

In this section, we identify the condition under which the menu of contracts outperforms the fixed-fee contract and pay-per-use contract. From the analysis earlier, the profit from the menu of contracts is $\pi_m = \frac{1}{6}(a + \beta b + b)^2 - 2a\beta b$, the profit from a pay-per-use contract is $\pi_p = \frac{1}{2}(a + \beta b)^2 - 2a\beta b$, and the profit from a fixed-fee contract is $\pi_L = (2 - a\gamma)L^* - 2a\beta b$ if $\gamma \leq \frac{2(1-\beta b)}{2b-1}$, in which $L^* = min\{\frac{1}{2}\beta(2-\beta)b^2, \frac{\beta(2-\beta)b^2}{1+\gamma-\gamma b+\sqrt{(1+\gamma-\gamma b^2-\gamma^2\beta(2-\beta)b^2}}\}$; while $\pi_L = 2(b - \frac{1}{2}) - a - a\beta b$ when $\frac{2(1-\beta b)}{2b-1} \leq \gamma \leq 1$. By comparing the profits of π_m and π_p. it is straightforward to see $\pi_m \geq \pi_p$ is satisfied when $a \leq \frac{\sqrt{3}+1}{2}b - \beta b$.

Next, we examine the condition under which the menu of contracts outperforms the fixed-fee contract and $\pi_m \geq \pi_L$. Note that when $\frac{2(1-\beta b)}{2b-1} \leq \gamma \leq 1$, $\pi_m \geq \pi_L$ is satisfied if and only if $\frac{1}{6}(a + \beta b + b)^2 - 2a\beta b \geq 2b - a - a\beta b - 1$, which can be reduced to $a \geq 2\beta b - b - 3 + \sqrt{3(1-\beta b)(3 + 2b - \beta b) + 12b - 6}$. In addition, to ensure $\frac{2(1-\beta b)}{2b-1} \leq \gamma \leq 1$, we need $\beta \geq \frac{3}{2b} - 1$ and $b > \frac{3}{4}$. Combing the conditions of the separating equilibrium as shown in the proof of Lemma 2, we find that the menu of contracts can be more profitable than both the fixed-fee contract and the pay-per-use contract when $\bar{a}_6 = max\{\frac{19b-5\beta b-18-\sqrt{(19b-5\beta b-18)^2+5(1+\beta)(7-5\beta)}}{5}, 2\beta b - b - 3 + \sqrt{3(1-\beta b)(3 + 2b - \beta b) + 12b - 6}\} \leq a \leq min\{\frac{\sqrt{3}+1}{2}b - \beta b, \frac{1}{5}(7 - 5\beta)b, (2\beta - 1)b\} = \bar{a}_7$ and $\bar{\gamma}_6 = max\{\frac{1}{a}[1 - \frac{2(2a+2\beta b-b)}{5b-a-\beta b}], \frac{2(1-\beta b)}{2b-1}\} \leq \gamma \leq min\{\frac{36(1-b)}{(a+b+\beta b)(5b-a-\beta b)}, 1\} = \bar{\gamma}_7$. In a numerical example, when $\beta = 0.95$, $b = 0.8$ and $a = 0.25$, the menu of contracts can be more profitable than both the fixed-fee contract and the pay-per-use contract when $0.8 \leq \gamma \leq 1$. □