Consumer Feelings and Equilibrium Product Quality *

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ABSTRACT

This paper considers the possibility that a firm can invest not only in the true product quality, but also in activities such as merchandising and store atmospherics that influence consumer perception of the product quality. Consumers make their purchase decisions based on the signal (perception) of quality they experience, where the signal is influenced by both the true product quality valued by the consumer and the affect of the consumer at the time of the signal formation. In this situation, a firm finds it optimal to invest in both product quality and in variables inducing affect, even though rational consumers, in equilibrium, correctly solve back for the true product quality. We uncover an asymmetry in the effects of the cost of producing quality and the cost of inducing affect. As a firm’s cost of quality decreases, the firm will find it optimal to invest more both in the true quality and in the affect inducement, even if it does not have a lower cost of inducing affect. Conversely, if a firm finds it easier to induce affect, then the product quality decreases but affect-inducing activities increase.

Under competition, we find that the firm investing more in quality also invests more in affect creation. An implication of this is that in a competitive environment, consumers can rationally associate an up-lifting store atmosphere, affect inducing merchandising, or mood-creating communication with high quality products even when the firm has no need to signal their private cost of quality information, and when there is no consumption externality of the affect. We also analyze the case in which firms might have different costs and consumers are uncertain about the costs incurred by a given firm. Here again we show that the perceived quality production is positively correlated with both the true quality and the affect inducing activities.
1. Introduction

Many consumption and purchasing situations are influenced by the feelings that consumers experience at the time of decision making. Furthermore, firms take an active role in inducing some of those feelings. Retail and product markets are replete with examples of firms conducting activities that create positive affect. Upscale retailers invest in store “atmospherics” including elements such as lighting, merchandising, pleasant music, attractive salespeople, and even disperse fragrance in the air in order to put consumers in a good mood during the shopping process. Firms use affect inducing ads to get consumers positively disposed towards their product. Real estate agents bake cookies, use brightly colored flowers, paintings, lighting, and fixtures to conduct open house showings. Furthermore, in the real estate context there are also companies such as Showhomes (see http://www.showhomes.com/) which specialize in the business of “staging” or dressing up homes that are up for sale.

Some interesting points may be noted regarding the above practices. First, the affect-inducing activities described above typically do not change the utility that consumers will obtain from the product. Pleasant music or merchandizing in a retail store should not directly impact on the quality of a dress, and the smell of baked cookies in an open house showing should not change the financial valuation of the property by a prospective buyer. Second, these activities involve costs. Therefore, the question arises why retailers would incur these costs rather than offer a lower price to close a sale. While retail merchandising investments are observed across a wide range of retailers, the more significant investments are present at high-end stores such as Nordstrom or Neiman Marcus.

What explains the greater supply of affect inducing activities that are unrelated to product quality by higher quality firms? How do the optimal firm decisions depend on the costs of quality and costs of the affect-inducing activities? Under what conditions would it be rational for consumers to expect affect-inducing activities to be diagnostic of the true quality of the product? We examine these questions in a model that formalizes the role of affect in a consumer’s decision to buy product quality. Specifically, the model captures the information processing problem of a consumer who is unable to fully separate the affect felt from the “true” evaluation of the product quality, but who nevertheless is rational and aware of this non-separability and takes it into account in her decision-making.

Formally, we assume that a consumer receives a signal of product quality (this signal is the quality perception that the consumer experiences), which is influenced by both the true quality as well as by the affective state. The firm may invest not only in product quality, but also in activities that do not

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1For example, staging companies such as Showhomes may charge $2000 to $5000 upfront, plus a monthly rental fee, to dress up homes for sale (see “Secrets of an open house,” May 7, 2004 CNN Money for more details).
change the true quality of the product. Rather these activities influence the consumer perception of
the product quality. We will henceforth use the term “atmospherics” for such activities and the term
“affect” for the change in consumer perceptions caused by atmospherics. The consumer at the time of
decision-making cannot perfectly separate the perception of the true product quality from the affect
induced by the atmospherics which is unrelated to the true product quality. Because the consumer’s
quality perception is influenced by atmospherics, the seller has an incentive to invest in them. The
paper provides an explanation for the role of affect in decision making through an equilibrium theory
of a market in which consumers make rational buying decisions given the information that they have
at the time of the decision, while the firm acts to optimally influence consumer behavior through the
marketing instruments at its disposal.

Jointly considering the interaction between consumers and firms suggests an interesting question
which does not arise when considering only individual level consumer behavior. In the spirit of bounded
rationality, while the consumer may not be able to separate the effect of feelings from her true product
quality evaluation, in a market setting, the consumer knows that a firm engages in atmospherics that
are irrelevant to the true quality. Knowing this, a rational consumer should try to correct for this
perceptual bias in quality evaluations that might be caused by the presence of affect. But if consumers
indeed discount their perceptions, how would that influence firms’ incentives for quality provision and
affect inducement? The main results are as follows.

When consumers cannot separate affect from the true quality evaluation, firms will invest in both
the product quality and atmospherics, even though in equilibrium, rational consumers are able to fully
solve back for the true product quality. In comparative statics, we uncover an interesting asymmetry
in the effects of the cost of quality and the cost of atmospherics. We show that if a firm is better at
producing high quality products (i.e., has a lower marginal cost for quality), it will find it optimal
to invest more both in the true quality and in the atmospherics, even if such a firm does not have a
lower cost of atmospherics activities. On the other hand, if a firm can generate affect at a lower cost,
affect increase but product quality decreases. Our analysis traces this asymmetry between
the effect of the cost of quality and the effect of the cost of atmospherics to the consumer’s ability
to rationally solve back for the true product quality. In other words, because consumers “think”

2In the examples that motivate this paper (real estate sales, retail markets) one might argue that there are situations
where this information processing problem could be solved by other market mechanisms such as consumer returns because
consumers given enough time will learn about the true quality. In these cases the theory applies best to situations when
product return is costly.
3The consideration of rational inference on the part of consumers also means that our model is not about situations
where firms are fooling or deceiving consumers into buying their goods.
about their feelings about quality, the market equilibrium involves a connection between quality and atmospherics provision. We show that the main results of the paper are robust to the case in which the consumers’ quality perceptions include a random error due to uncertain factors in the environment outside of the control of the firm.

To see whether higher product quality should be expected to come along with higher atmospherics, we look at the equilibrium actions of firms competing in a vertically differentiated market. We show that the firm choosing higher quality also chooses to invest more in the atmospherics. An implication of this result is that in a competitive environment, consumers can rationally associate an up-lifting store atmosphere or an emotional advertisement with high quality products even in the absence of signaling motives or any consumption externality of atmospherics.

Another reason for different firms offering different levels of atmospherics and quality is that the firms (not necessarily competing) may have different costs of inducing affect and/or producing quality. In this case, if consumers are uncertain about the firm’s costs, the perceived quality of the product turns out to be also positively correlated with both the atmospherics and the true quality. The extension of the basic model to the case when consumers are uncertain about the firm’s costs of quality or atmospherics also allows us to investigate the possible signaling motives of the firm. We find that when consumers are uncertain about the cost of quality, then similarly to the standard signaling models, a firm with low cost of quality can credibly signal its cost by distorting its perceived quality production over and above its full information level corresponding to the case when consumers are able to observe all costs. But what is more interesting is that when consumers are uncertain about the cost of affect inducement, it is the firm with high cost of affect-inducement that now wants to separate and can credibly signal its difficulty in inducing more affect by distorting its perceived and actual quality to a level which is below the full information level that it would choose if its cost were observed by the consumers.

1.1. Contribution Compared to the Existing Literature

Our approach complements the standard idea that affect-inducing investments may act as signals of product quality to consumers. Costly signaling (Spence 1973) is relevant if a firm wants to communicate its private information about quality or some other characteristic. In that case, by incurring costly expenditures on activities observable by the consumers, such as advertising, a firm might signal high quality. For advertising or other expenditures to work as signals of quality, it is necessary that these signaling activities be more costly for the low quality firm than for the high quality firm (i.e., the single-crossing property must be satisfied). For example, in Milgrom and Roberts (1986) this
property is satisfied through the mechanism of repeat purchases. 4

In contrast, the main point of our paper is that the connection may exist even if there is no difference in the costs of inducing affect for firms with different quality. Therefore, the model of this paper can be seen as explaining the provision of affect-inducing activities in cases where the motivation for the firm to signal private information is either absent or when the single crossing condition cannot be satisfied, thereby making signaling impossible for the firm.5 For instance, in the real estate example, the interaction between a seller and a buyer in the sale of a house is a one-time interaction. Therefore, the seller of a low quality house would be equally inclined to incur expenditures in staging the house as a seller of a high quality house making the signaling of quality through atmospherics impossible. Our model of imperfect consumer information processing contributes by explaining affect inducing investments (that are irrelevant to the true quality) in precisely these cases where either quality signaling motivations are absent, or where signaling is not possible.

This model also differs from the signaling approach in another respect: In our framework the specific amount of affect induced by the firm is unobservable to the consumers, because consumers only observe the perceived quality. Consumers have to rationally infer the amount of affect induced, and it is due to this reason that firms in equilibrium invest in atmospherics. In contrast, it is necessary that the signaling instrument (e.g., advertising or warranties) is observable to consumers in order for firms to successfully use them as quality signaling instruments. Thus, our consumer information processing framework is particularly useful in explaining situations in which affect and other such investments cannot be easily separated out by consumers from what they truly value.

This signal extraction feature is one which our model shares with the models of signal jamming that have been developed in other contexts. For example, Fudenberg and Tirole (1986) consider the situation in which the information available to an entrant (who is uncertain about future profitability) is distorted by an incumbent through the use of predatory pricing. The entrant, therefore, has to infer whether adverse profit realizations were due to a low realization state drawn by the nature or because of the predatory action. Stein (1989) considers a signal-jamming model of managerial incentives in a capital market, where managers distort current earnings by borrowing at adverse rates from the

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4In other examples, warranties (Grossman 1981, Gal-Or 1989, Soberman, 2003) are less costly to offer for the high quality firm than for a low quality firm, or increasing the variable fee in the contract and reducing the fixed part of the contract is optimal for a manufacturer with high demand rather than one with low demand (Tirole 1988, p. 177, Desai and Srinivasan 1995). Similarly, in Bagwell and Riordan (1991) holding high initial prices is more efficient for high quality firm and more disadvantageous for a lower cost and low quality firm. Finally, in Padmanabhan, Rajiv and Srinivasan (1997) the firm signals high network externality by initially withholding quality, but by making up for it later through an upgrade, a strategy which is more costly for the low type firm.

5Sections 5.2 and 5.3 extend the analysis by considering cost uncertainty to explore the possibility of signaling.
future. In these models the true variables of interest (the true profitability, or the true earnings) are themselves exogenously stochastic, while distortive actions of the agent (the incumbent or the manager) create jamming because there is exogenous uncertainty to begin with. In our set-up, both the true quality and the noise (atmospherics) are endogenous and are choice variables of the firm. Therefore, a notable difference of our paper from the above papers is that the inference problem for the consumer exists even if there is no exogenous uncertainty. Furthermore, the inference problem is due to the endogenous supply of quality and atmospherics that are linked in equilibrium due to consumer inference. This causes the positive connection between quality and affect when quality is easier to provide, but a negative connection when affect is easier to induce.

The next section presents a model without uncertainty. Section 3 presents comparative statics results and comparison of the model predictions when consumers are rational and naive. Section 4 considers competition and Section 5 discusses the robustness of our findings and also examines the effects of consumer uncertainty. Section 6 discusses the results and concludes.

2. The Model

A monopoly firm produces a single good. There is a unit mass of homogenous consumers who have utility \( U(q, p) = \gamma q - p \) for a unit of the good, where \( q \) is the true quality level, \( p \) is the price, and \( \gamma > 0 \) is a parameter representing the consumer valuation of the true quality.

Consumers do not directly observe \( q \), but have to make an inference about \( q \) from the perceived quality \( \tilde{q} \) which is influenced by both true product quality \( q \) and atmospherics \( a \) according to some perceptual mechanism \( f(q, a) \), which is continuous and twice differentiable. We further assume that \( f(q, a) \) is increasing and non-convex in both arguments, and has non-negative partial cross-derivatives. The consumer feels that the product quality is \( \tilde{q} = f(q, a) \). However, she realizes that she is also potentially being influenced by atmospherics. The true quality and the affect are separately not

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6 Other examples of signal jamming can be found in Holmstrom (1982) analysis of managerial incentives and Riordan’s (1985) article on dynamic conjectural variations. Note that the mixing of noise with a variable of interest is a feature that is also present in the standard principal-agent model (e.g., Ross 1973, Holmstrom 1979) in which the agent has control over the effort (quality) and the principal (the recipient of the output from the agent’s effort) has an imperfect estimate of the agent’s effort level. Unlike the signal jamming models, the noise in the principal-agent set-up is exogenous while the agent’s effort is endogenous.

7 Double moral hazard problems such as in Cooper and Ross (1985) have the property that the final output is dependent on two variables – the quality and the consumer effort – and it is not possible to credibly observe and verify the extent to which each variable has contributed to the output. This is somewhat similar to the unobservability of true quality and affect for the consumer in our model. However, the double moral hazard problem arises from each party in the relationship choosing one variable and the inability to credibly stipulate the individual choices of the two parties in a contract. In our model it is the firm which chooses both the variables.

8 The main results are not affected if consumers are heterogenous in their valuations for true quality. In fact, in section 4 we extend the basic model to one with competition between firms and a market with consumer heterogeneity in quality valuations to show that all the main results remain valid.
observable to the consumer, in the sense that the consumer cannot separately observe how much of
the perceived quality is due to the true quality and how much of it is due to the affect that is unrelated
to true quality. Although we use the general function $f(q, a)$ to derive the equilibrium equations, we
assume the additive form $f(q, a) = q + a$ to illustrate the model solution and implications. However,
in Section 5.2, we also discuss the robustness of the results to other functional forms.

Our model is about the bounded rationality phenomenon that consumers may face an information
processing problem because of which they cannot fully separate out the environment/affect variable
from quality when both are present (but that they rationally try to infer the actions of the firm).
This is what the perceptual mechanism $f(q, a)$ represents. The psychological foundation for this
representation is based on the stream of research in psychology beginning with Schwartz and Clore
(1983) that formalizes the idea termed as the “affect as information hypothesis”. Specifically this
research studies the impact of “incidental” affect which is defined as the affect that is not generated
by the “target of evaluation” but which is nevertheless present in the environment (see Cohen, Pham
and Andrade 2008 for a review of this literature). In the context of our model this incidental affect
is that which is created by the store’s atmospherics (the variable $a$) and not by the target which is
the product’s quality $q$. The robust finding of this literature is that the affect in the environment
biases consumer perception and decision-making unless the consumers are explicitly told about of the
presence of the source that creates the affect, even when there is just a single affect variable used in
the experiment. Indeed, there are also studies specific to consumer choice contexts (Gorn, Goldberg
& Basu 1993) that show that consumer evaluations of the quality of a product are impacted by the
affect that is present in the environment.

Note that by the atmospherics $a$ we mean here things unrelated to the product that impact
consumer perception $\hat{q}$ of the product and do not directly enter the consumer utility function. The
aspects that influence the product’s actual, rather than perceived or expected, value to the consumer
are represented by $q$.9

Note also that the model accommodates the idea that in actual retail environments consumers
may be aware of the existence of the atmospherics creating variables, and yet may not be able to
fully separate the effect of these variables from their quality perception. This is consistent with the
experimental findings in Schwartz and Clore where the subjects could not disentangle the influence of

9Note that if the affect directly enters consumer utility function, but does not interact with the utility of the product,
the results will not change. However, if some part of the affect has a consumption externality value (i.e., if consumers
enjoy the product consumption more if they experience a higher affect while shopping for it), then it can be viewed as a
part of the quality $q$. But the affect we consider is the part which has no consumption externality, but which is induced
by the firm in order to influence the consumer’s perception and evaluation of the true quality.
affect even though they directly perceived the source of the affect variable.¹⁰

A consumer buys the product if her expected utility of the product is non-negative. The unit cost of the good of quality q to the firm is $C(q) = c_q q^2$, and the unit cost of atmospherics a is $C(a) = c_a a^2$. To summarize, the total unit cost for the firm of producing the good of quality q and atmospherics a is:

$C(q, a) = C(q) + C(a) = c_q q^2 + c_a a^2$, where $c_q > 0$ and $c_a > 0$.

The timing of actions in the game is as follows: First, the firm decides on the levels of q and a, followed by choosing the price $p$.¹² Then, consumers observe the price and the perceived quality $\tilde{q}$ and make the purchase decision. We look for the sub-game perfect Nash equilibrium of this game after elimination of strictly dominated strategies.

2.1. Solution

Consumers must make their purchase decisions based on the information they have at the time of their decision-making, i.e., based on $\tilde{q}$ and $p$. Therefore, the profit-maximization problem of the firm can be thought of as consisting of two inter-related components:

1. Decide on the values of all variables, i.e., $\tilde{q}$ and $p$, observed by consumers, and
2. Provide these values in the most cost-efficient way by appropriately choosing the variables under the control of the firm (i.e., q, a, and p).

While the optimal firm’s decision in the first part above depends on consumer behavior, the optimal choice of q and a in inducing $\tilde{q}$ in the second part above does not depend on consumer behavior or beliefs. Since consumers know this and as far as the above discussed cost-efficient way is unique, consumers will be able to infer the true values of q and a from the assumption of profit maximizing behavior of the firm. This results in the consumer-inferred quality function $\hat{q}$, which is a function of the (observed) perceived quality $\tilde{q}$, and which is equal to the true quality level given that the firm is

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¹⁰While in the Schwartz and Clore and other related papers the subjects cannot disentangle the influence of affect even though they are consciously aware of the source of the affect variable, there is also a second stream of research on “automatic processing” which shows that individuals information processing may be affected by stimuli though they may not be consciously aware of it (see the review on unconscious/automatic processing by Bargh 2006). Both these cases are consistent with information processing problem represented by our model that consumers might not be able fully separate out the effect of environment and affect variables from that of quality even if they observe specific physical aspects of the store that create the affect such as lighting or pleasant ambience etc.

¹¹Given the inelastic consumer demand and unit mass of consumers, the results will not change at all whether either of these costs is fixed, marginal (unit), or a mix of the two. The model therefore allows both the packaging (such as container type, color, etc., which result in per-unit costs) and in-store atmospherics (such as music, fragrance, lighting, width of isles, etc., which result in mostly fixed costs) to be possible examples of the affect-inducing activities.

¹²Since no other player makes a move between these three decisions, it is not consequential whether they are made sequentially, as stated above, or simultaneously. However, the sequence defined above is important when we consider, in Section 4, the extension to the competitive case.
profit maximizing.\textsuperscript{13} Given the consumer inference of the true quality $\hat{q}(\bar{q})$, consumers will be willing to pay (at most) $\gamma\hat{q}(\bar{q})$ for the product. Therefore once again, regardless of the actual $q$ and $a$, it is optimal for the firm to charge the price $p = \gamma\hat{q}(\bar{q})$. As a result, we have the following proposition:

**Proposition 1:** In the equilibrium, the firm chooses $q$ and $a$ as to satisfy

$$c_aaf_1(q, a) = c_qfq_2(q, a),$$

where subscripts 1 and 2 denote the derivatives with respect to the first and second variable, and

$$\gamma = 2c_qq + 2c_aa(q)a'(q),$$

where $a(q)$ is the function defined implicitly by Equation (1). Off-equilibrium consumer beliefs on $(q, a)$ are defined by the condition that $q$ and $a$ satisfy $\bar{q} = f(q, a)$ and Equation (1).

**Proof:** The proofs of all Propositions are in the Appendix. $\square$

We now turn to the linear specification of $f(q, a)$ to illustrate the model solution and its implications. Specifically, consider $f(q, a) = q + a$.\textsuperscript{14} In this case, variable $a$ also represents consumer affect, since $a$ is equal to the distortion of consumer perception caused by atmospherics. In this case, equation (1) becomes $c_a a = c_q q$, implying $a(q) = (c_q/c_a)q$ and $\bar{q} = (1 + c_q/c_a)q$. Equation (2) then becomes $\gamma = 2c_q q + 2c_a(c_q/c_a)q(c_q/c_a) = 2c_q q + 2c_a^2 q/c_a$. Therefore, the equilibrium levels of quality, affect, perceived quality, and profits are

$$q^e = \frac{\gamma c_a}{2c_q(c_q + c_a)}, \quad a^e = \frac{\gamma}{2(c_q + c_a)}, \quad \bar{q}^e = \frac{\gamma c_a}{2c_q} \text{ and } \pi^e = \frac{\gamma^2 c_a}{4c_q(c_q + c_a)}.$$  

(3)

Note that this implies that the firm invests in affect inducement even if the consumers are sophisticated enough to be able to infer back the true quality.

3. Implications and Comparative Statics

From the equilibrium derived in Section 2.1, we have the following proposition that presents the comparative statics results about how the equilibrium quality levels and affect are affected by costs of quality and atmospherics production, $c_q$ and $c_a$.

**Proposition 2:** In the equilibrium of the model of Section 2.1 with the specification $f(q, a) = q + a$
1. If quality becomes easier to produce (i.e., if \( c_q \) decreases), the perceived quality, affect, and the true quality all increase. Furthermore, profits increase as well.

2. If affect becomes easier to induce (i.e., if \( c_a \) decreases), affect increases; however, the true quality decreases and the perceived quality remains the same. Furthermore, profits decrease.

The first part of the proposition implies that in comparative statics if firms (across markets) are different in the ability to produce quality, then firms producing higher quality products will also be spending more on inducing affect, even if they do not find it easier to induce affect. In our model, affect stands for the positive consumer feelings (induced by the store environment) which are over and above the feelings induced by higher product quality alone. Therefore, a firm producing higher quality does not necessarily have to induce higher affect. However, the first part of Proposition 2 shows that the connection can be an equilibrium result.

In contrast, according to the second result of Proposition 2, it is also the case that the increased ability to induce affect (lower \( c_a \)) makes the firm worse off when consumers are sophisticated enough to solve back the perceived quality for the true quality.

The asymmetry in comparative statics of the cost of atmospherics versus that of the cost of quality that we see above is due to the ability of the consumers to rationally solve back for the true quality from the perceived quality. To see this more clearly, we can compare the above result with the case of “naive” consumers who do not solve back for the true quality, but rather decide on whether or not to buy based simply on the perceived quality \( \tilde{q} \) (instead of the inferred quality \( \hat{q}(\tilde{q}) \)). When consumers are naive in the above way, the equilibrium level of affect does not depend on the cost of quality, and the equilibrium level of true quality does not depend on the cost of \( a \). However, the perceived level of quality is increasing when either cost decreases (see Appendix for the full analysis). It is interesting to note that in the case of naive consumer behavior, the equilibrium quality level is at the socially optimal level, which is higher than in the case of sophisticated consumers. Therefore, one of the implications is that while naive consumers would over-pay for the products, they would receive better quality products. The comparison of sophisticated and naive consumer behavior cases also implies that the lower level of the true quality when the costs of inducing affect are lower that we have observed in Proposition 2, i.e., the substitution of the investment that the firm makes from quality to affect, is due to the sophisticated consumer behavior, whereby consumers try to infer the true quality from the perceived one.

Furthermore, note that if consumers could directly observe the true product quality, the firm will not invest in atmospherics at all. Note also that if the firm could not change the affect (i.e., if it is
exogenous rather than induced by atmospherics under the firm’s control), the payoff to the firm of increasing the true quality is equal between the cases when consumers observe and do not observe the true product quality. Thus, the difference between consumer directly observing the quality and inferring it is due to the consumer expectation that the optimal firm’s investment in the affect is correlated with optimal firm’s investment in product quality.

4. Competition

In this section, we modify the model to consider a competitive environment with two firms. Given our interest in quality competition, it is natural to consider competition in a differentiated market with consumer heterogeneity in how much they value quality. Accordingly, consider a model of a market with a unit mass of consumers whose valuations for quality $\gamma$ are uniformly distributed on the interval $(0, \gamma_h)$. There are two firms (indexed by $j = 1, 2$) who have the same cost of producing $q$ and $a$ and with cost functions the same as in the previous section. The timing of the game is as in the standard models (Shaked and Sutton 1982): In the first stage firms simultaneously make their product (quality and atmospherics) choices. Then in the next stage, the firms simultaneously make their pricing decisions conditional on the first stage choices. From the arguments as in Shaked and Sutton (1982) and Moorthy (1988) it follows that there is no symmetric sub-game perfect Nash equilibrium in quality choices in this model, and the only possible equilibrium is one in which firms differentiate in quality.

Let firm 1 be the one that produces the higher quality. Let $\tilde{q}_1$ and $\tilde{q}_2$ denote the perceived qualities offered by the two firms. As in the previous section, assume that $f(q, a) = q + a$. Similar to the analysis in the previous section, because the consumer decision depends only on the perceived quality and not on the true quality, each firm $j$ will produce the pair $(q_j, a_j)$ as to satisfy Equation (1), and so $\tilde{q}_j = (\frac{c_q + c_a}{c_a})q_j$. The consumers will therefore infer the true quality as $\hat{q}_j(\tilde{q}_j) = (\frac{c_q}{c_q + c_a})\tilde{q}_j$ (correctly, in the equilibrium).

The Appendix fully derives the equilibrium and shows that in the equilibrium, $q$ and $a$ associated with the firms’ products are as follows:

$$q_1^e = A_1 \frac{\gamma h^2 c_a}{c_q(c_q + c_a)}; \quad a_1^e = A_1 \frac{\gamma h}{(c_q + c_a)};$$

$$q_2^e = A_2 \frac{\gamma h^2 c_a}{c_q(c_q + c_a)}; \quad a_2^e = A_2 \frac{\gamma h}{(c_q + c_a)};$$

Note that we could have considered consumer heterogeneity in quality valuations even for the monopoly case. However, this does not change any of the results of that section.
where $A_1 > A_2 > 0$ are constants defined in the Appendix. This analysis recovers some of the comparative statics results that we presented in the monopoly case from the equilibrium actions of competing firms. The high quality firm offers a higher level of atmospherics and charges a higher price than the low quality firm. Therefore, in a market with competing firms, an outside observer who is able to observe affect inducing activities can rationally associate higher affect provision with high quality. Thus, as opposed to the exogenous cost change rationale provided by the comparative statics results, the analysis above demonstrates the existence of the positive relationship between quality and affect in the presence of market competition. The equilibrium profits of the two firms are

$$\pi_1^e = B_1 \frac{\gamma_h c_a}{c_q(c_q + c_a)}, \quad \text{and} \quad \pi_2^e = B_2 \frac{\gamma_h^4 c_a}{c_q(c_q + c_a)},$$

where $B_1 = A_2^2 \left( \frac{4(A_1 - A_2)}{(4A_1 - A_2)^2} - 1 \right)$ and $B_2 = A_2 \left( \frac{A_1(A_1 - A_2)}{(4A_1 - A_2)^2} - A_2 \right)$ (and we have $B_1 > B_2 > 0$). The equilibrium profits decrease in $c_q$ and increase in $c_a$. This is consistent with the previously obtained insights: When quality is harder to produce both firms produce less quality and the consumers rationally expect lower levels of quality and this reduces profits. However, when affect is harder to induce, consumers expect lower levels of atmospherics and higher levels of quality provision for any given level of perceived quality and this increases the equilibrium firm profits.

5. Robustness of the Model Implications

We now consider several modifications to the model of Section 2 to understand better how relaxing different assumptions would change the model implications. In Section 5.1, we consider an important extension where the consumer’s perceived quality function also has a random error. In Section 5.2, we consider sensitivity of the results with respect to alternative specifications of the functional form of the perceived quality by considering the possibilities of the atmospherics and true quality entering the perception function with different weights and also the possibility that the perceived quality is multiplicative rather than additive in the underlying factors.

Sections 5.3 and 5.4 relax the full-information assumption. Recall, that in the model of Section 2, consumers are able to figure out the firm’s strategy because they know all the parameters of the production functions of the firm. This full information assumption implies that although consumers do not observe the true quality, they can infer it for sure and without mistakes in equilibrium. In reality, if consumers do not observe quality directly, they may be uncertain about it. This could be because they do not know some of the production function parameters, such as the costs of the firm. We now consider such possibilities. We first consider the case commonly considered in the literature:
namely, uncertainty about the cost of quality (Section 5.3), and then consider the uncertainty about the cost of inducing affect (Section 5.3).

Section 5.5 considers the possibility that consumers have not only a perception of quality, but also perceive the affect, but in a way that they can not deterministically solve these two functions for the true quality. We show that the main implications of the model hold under such an extension and derive some new results.

5.1. Perceived Quality Function with Random Error

In actual market and store environments consumer quality perceptions could be uncertain because they are influenced by a number of random factors outside the control of the firm. For example, the perceived quality could include a random error due to consumers not noticing or accurately processing all the environmental or quality cues, due to some of the environment being random (e.g., whether it is a sunny and/or warm day). Alternatively, it could be due to the error with which consumers perceive the magnitude of the firm’s actions. Let us therefore consider the following modification of the perceived quality function for consumer \( i \):

\[
\tilde{q}_i = q + a + \varepsilon_i, \tag{4}
\]

where the perception error term \( \varepsilon_i \) is uniformly distributed on the interval \([-\varepsilon, \varepsilon]\) for some \( \varepsilon > 0 \) and let the rest of the model to be the same as in Section 2. We then have the following proposition.

**Proposition 3:** Assume that \( f(q, a) = q + a + \varepsilon_i \), where \( \varepsilon_i \sim U(-\varepsilon, \varepsilon) \) with \( \varepsilon < \frac{\gamma}{8c_q} \). Then, in perfect Bayesian equilibrium surviving the Intuitive Criterion, the firm sets the same values of price \( p \), \( q \), and \( a \), and achieves the same profits as when \( f(q, a) = q + a \). This outcome equivalence between the case with and without the random error also holds if \( a \) is observable by the consumers.

Thus, as long as the extent of the uncertainty in the environment is not too large, we have that the equilibrium firm choices are exactly as in the main model, and the insights of the previous section still hold. Furthermore, as in the main model, if either quality or affect were to be separately observable to the consumers, then the firm would still not have any incentive to induce extra affect even when the consumer perceptions of quality are uncertain due to the random error.

Another possibility of the lack of consumer knowledge is that the consumer, while observing the environment, is not sure how this environment impacts her perception. On the other hand, the firm

\[16\] As far as the firm is risk-neutral, the results are the same whether \( \varepsilon_i \) is independent or the same across consumers.
has resources to figure out what influences consumer perceptions. Thus, while consumers may expect that the firm has better information than them. This situation can then be modeled as the case of consumer uncertainty about the firm’s cost of influencing affect that we consider in Section 5.4.

To illustrate a case of lack of consumer knowledge, consider the following example: ambient lighting is needed at any store, but the manner in which lighting influences the quality perception varies. Consumers observe lighting, but do not know whether or by how much it influences their quality perception. Through consumer research, the firm may figure out which lighting influences consumer perceptions in a favorable way. We consider this possibility in Section 5.5: the firm invests in two components of the environment to keep the consumer perception of lighting \( (\tilde{a}) \) at the constant level, but chooses the component that influences \( q \) more. Note that even if consumer observes the exact characteristics of lighting in this example, she will not be able to deduce by how much the lighting changes her quality perception since she does not know which type of lighting influences her perception more. However, she may know that it costs more for the firm to set up lighting in a manner that influences her quality perception in an unusually high amount.

5.2. Functional Forms of the Perceived Quality

Suppose the production of perceived quality follows the more general additively separable form

\[
 f(q, a) = \rho_1 q + \rho_2 a
\]

rather than \( q + a \). One might interpret the \( \rho_1 \) and \( \rho_2 \) as the perceptual weights which the mind assigns to atmospherics and true quality. For example, a relatively high \( \rho_1 \) compared to \( \rho_2 \) would imply that the consumers perception of quality comes primarily from the true quality. In our model this is equivalent to a re-normalization of the cost parameters \( c_q \) and \( c_a \) to \( c_q' = c_q / \rho_1^2 \) and \( c_a' = c_a / \rho_2^2 \), and hence, the implications are the same as in the base model.

It is also possible that instead of the additive functional form, the perceived quality is easier to influence in the upward direction by the supply of atmospherics when the true quality is higher. This could be, for example, due to perception working in terms of ratios rather than in terms of absolute values. This can be modeled by perceived quality function of the form

\[
 f(q, a) = qa
\]

The implications of this case are also similar to those of the base model. The equilibrium quality, atmospherics, and the perceived quality are

\[
 q = \frac{\gamma}{[4c_q]}; a = \frac{\gamma}{[4(c_qc_a)^{1/2}]}, \quad \tilde{q} = \frac{\gamma^2}{[16c_q(c_qc_a)^{1/2}]},
\]

which means that if the cost of quality decreases, then \( q, a \), and the perceived quality all increase (just like in the case of linear \( f(q, a) \)), and if the cost of atmospherics decreases, the quality does not increase, but the supply of atmospherics increases (again, as in the case of linear \( f(q, a) \)).

\[
 17 \text{In this case, Equation (1) reduces to } c_a a^2 = c_q q^2, \text{ i.e., } a = q \sqrt{c_q / c_a}. \text{ Therefore, Equation (2) becomes } \gamma =
\]
5.3. Cost of Quality Uncertainty

We will now relax the assumption of full information about costs. Specifically, in this section, we consider the possibility that consumers are uncertain about the cost of producing quality for the firm. Accordingly, let us modify the model of Section 2 with linear $f(q,a)$ to assume that the cost of quality production $c_q$ instead of being certain and known to consumers, is uncertain and can be either high $c_{q2}$ or low $c_{q1}$ with equal probability.\footnote{It turns out that the exact probabilities of states $h$ or $l$ are inconsequential to the equilibrium (as is common in standard signaling models), so the assumption of equal probability of the states involves no loss of generality.} Furthermore, assume that the firm knows $c_q$, but consumers only know the prior distribution stated above. A low-cost of quality firm will therefore have an incentive to credibly signal its cost. Below, we derive the separating equilibrium of this model.

In the full-information setting, we had that when the cost of quality is high, the firm would produce $q, a$, and, therefore, $\tilde{q}$ at levels below those when the cost of quality is low (see equation (3)). Furthermore, consumers will discount their perception $\tilde{q}$ of quality more when the cost of quality is high. Therefore, one may expect that the high-cost firm may want to pretend to be low-cost firm, and thus, the low-cost firm may have to over-produce quality and/or atmospherics to make sure that it is recognized as the firm with low cost of quality (and therefore, high quality). As the following proposition shows, this in fact leads to an upward distortion of the equilibrium quality and affect production levels for the firm that has a low cost of quality. In the proposition and the discussion that follows we denote the firm type (low or high cost) by 1 and 2 while the quality and affect choices (high or low levels) are denoted by $H$ and $L$.

**Proposition 4:** When $c_q$ is either $c_{q2}$ or $c_{q1}$ and is known to the firm but unknown to the consumers, in the unique perfect Bayesian equilibrium satisfying the intuitive criterion, we have: If $c_q = c_{q2}$, then

\[
q = q_L = \frac{\gamma c_a}{2c_{q2}(c_{q2} + c_a)}, \quad \text{and} \quad a = a_L = \frac{\gamma}{2(c_{q2} + c_a)}.
\]

If $c_q = c_{q1}$, then $q$ and $a$ are

\[
q_H = \frac{\gamma c_a (c_{q2} + c_a + \sqrt{(c_{q2} - c_{q1})(c_{q1} + 2c_a + c_{q2})})}{2(c_{q1} + c_a)^2c_{q2}}, \quad \text{and} \quad a_H = q_H \frac{c_{q2}}{c_a}.
\]

Furthermore, consumers believe that $c_q$ is low if and only if they observe $\tilde{q} \geq f(q_H, a_H)$.

By comparing the equilibrium values of quality and affect to those in the full information case, one can see that the levels of quality and affect in the high-cost case are at the optimal (full-information) levels, whereas the production levels in the low-cost case are distorted upwards in both the affect and quality dimensions. Thus, as in the previous comparative statics results, we observe that affect is
higher when the quality is produced at a higher level due to lower cost of quality production. Finally, it is interesting to note (see Equation (11) in the Appendix) that the extent to which the low cost firm will distort its production of perceived quality decreases with $c_a$. Thus in markets where it is harder to induce affect, it is easier for the low cost of quality firm to separate and credibly signal its type.

5.4. Cost of Affect Inducement Uncertainty

Suppose now that it is the cost of atmospherics $c_a$ that is uncertain to consumers (while known to the firm), but the cost $c_q$ of quality is certain and known to both consumers and the firm. Let the cost of atmospherics be either high $c_{a2}$ or low $c_{a1}$ with equal probability. In this set-up, the comparative statics results in Section 3 might suggest that the low-cost firm should produce the same perceived quality but lower true quality than the high-cost firm. This may suggest a pooling equilibrium which might then mean that consumers would discount the perceived quality in the same amount for the low- and high-cost firm (as would happen in a pooling equilibrium). But this would imply that the low-cost firm would want to produce a higher perceived quality than the high-cost firm. However, if it does so, consumers would identify it as a low-cost-of-atmospherics firm. As the following proposition shows, the equilibrium satisfying the intuitive criterion is one in which the low-cost firm is producing at the optimal level given that consumers recognize that its cost of affect production is indeed low, and the high-cost firm produces both affect and quality at a lower level.

**Proposition 5:** When $c_a$ is either $c_{a1}$ or $c_{a2}$ and is known to the firm but unknown to the consumers, in the unique perfect Bayesian equilibrium satisfying the intuitive criterion, we have: If $c_a = c_{a1}$ then

$$q = q_H = \frac{\gamma c_{a1}}{2c_q(c_q + c_{a1})}, \quad \text{and} \quad a = a_H = \frac{\gamma}{2(c_q + c_{a1})},$$

while if $c_q = c_{a2}$ then $q$ and $a$ are

$$q_L = \frac{\gamma c_{a2}(c_q + c_{a1}) - \sqrt{(c_{a2} - c_{a1})c_q(c_q c_{a2} + 2c_{a2}c_{a1} + c_q c_{a1})}}{2c_q c_{a1}(c_q + c_{a2})^2}, \quad \text{and} \quad a_L = q_L \frac{c_q}{c_{a2}}.$$

Furthermore, consumers believe that the firm has the low $c_a$ if and only if they observe $\tilde{q} \geq f(q_H, a_H)$.

The above presents an interesting counter-point to the results from signaling models involving quality. The usual result in the literature (as in the previous section) is that a low cost of quality firm can separate and credibly signal its quality by distorting its quality level over and above its full information level (see for example Balachander and Srinivasan 1994). But unlike in the case of uncertain cost of quality, if the cost of inducing affect is uncertain, then the distortion in affect and quality production is downwards and is done by the high cost firm. Thus, it is the high cost firm here that wants to separate and credibly signal its inability to induce too much affect. This is because under full
information affect has no consumption value for the consumer and therefore the high cost of affect firm has the incentive to separate itself from the low cost of affect firm which can more easily invest in environmental variables which have no consumption value. It does so by providing both the level of affect and the level of quality lower than what would have provided if its costs were observed by the consumers. At the same time, just as in the cost-of-quality uncertainty and the competition cases, the result is that the affect induction turns out to be positively correlated with the quality production.

5.5. Consumers Having Perceptions of Both Quality and Affect.

The main model in Section 2 assumes that consumers have a perception of quality while not directly observing the level of affect. In this section, we present an extension which captures two important features of the problem in the observed markets. First, consumers are likely to have not only a perception of quality, but also a perception of affect. Second, firms in reality might choose a wide range of actions which influence the feelings of consumers. This section provides a formal framework that allows one to think about both quality and affect perceptions (and the multiplicity of actions that firms might use to influence these perceptions) in a manner that still maintains the essential feature that the true quality can not be explicitly derived from the multiple perceptions. This implies that the firm’s actions in the non-quality variables still influence the consumer quality inference.

A theoretical issue that arises when introducing an affect perception defined by a function \( \tilde{a} = g(q, a) \), is that once consumers observe two values, they may be able to fully solve for the two variables that the firm uses. However, in reality product quality and especially affect may be generated through a range of different inputs that are in the firm’s control. Product quality usually has different dimensions and the consumer valuation of quality might be a composite measure based on all the dimensions. Similarly, consumer feelings can also be impacted by different actions of the firm such as packaging, lighting, color theme, aroma, merchandising, music etc. Some of the decision variables of the firm may enter and enhance the true consumer utility (which we call the true quality), and some of them may affect the perceived quality and affect felt. Even if all of them enter the true utility (quality) in at least some amount, they may enter the quality perceptions and affect felt with weights different from the weights they enter the true utility (quality) with.

To extend the model to the case of perceptions of quality and affect, but still allow consumers to have imperfect information, we consider the case when consumers have perceptions of both quality and affect, but the number of firm’s decisions is greater than the number of perceptions. Specifically, let us assume that along with the quality choice, the firm chooses two instruments of affect creation, \( a_1 \) and \( a_2 \), and these two instruments generate affect in a different way than the way they enter...
quality perception. Formally, the quality perception and affect felt are, correspondingly, \( \tilde{q} = f(q, a) \),
and \( \tilde{a} = g(q, a) \), where \( a = (a_1, a_2) \). Conceptually, consumers have a perception of quality, but it is influenced by affect (or, in other words, by the supply of atmospherics that induce affect), and consumers have a perception of affect, that may or may not be influenced by the true quality. Furthermore, the perceptions are such that consumers are not able to explicitly derive the true quality. For analytical tractability, assume that the functions \( f \) and \( g \) are linear. Without loss of generality, we can then normalize the parameters in the function \( f \) to the identity (by rescaling the cost parameters), to obtain \( \tilde{q} = q + a_1 + a_2 \), while keeping the specification of \( g \) in a general linear form: \( \tilde{a} = \delta q + \alpha a_1 + \beta a_2 \).

We further extend the cost specification to be, as in the main model, quadratic in costs of each component: \( c(q, a_1, a_2) = c_q q^2 + c_{a_1} a_1^2 + c_{a_2} a_2^2 \). Note that the interesting case for analysis is \( \alpha \neq \beta \) (the atmospherics components enter affect perception in a different manner than the manner in which they enter the quality perception). Under this condition consumers will not be able to explicitly deduce their true utility \( \gamma q \). The following proposition summarizes the equilibrium outcome:

**Proposition 6:** Denote \( D = 2c_q(\alpha - \beta)^2 + 2c_{a_1}(\alpha - \delta)^2 + 2c_{a_2}(\beta - \delta)^2 \). Then the firm chooses:

\[
\begin{align*}
q &= ((\beta - \delta)^2 c_{a_1} + (\alpha - \delta)^2 c_{a_2}) \gamma / D, \\
a_1 &= (\alpha - \beta)(\alpha - \delta) \gamma / D \quad \text{and} \quad a_2 = (\beta - \alpha)(\beta - \delta) \gamma / D,
\end{align*}
\]

so that \( \tilde{q} = \gamma / (2c_q) \) and \( \tilde{a} = \gamma \delta / (2c_q) \). Furthermore, quality and the expenditure on quality increase when either the cost of quality decreases, or when the cost of \( a_1 \) or that of \( a_2 \) increases, while the expenditure on affect production (the sum of expenditures on \( a_1 \) and \( a_2 \)) and the effect of affect on quality perception increase when the cost of quality decreases or the cost of affect (either component) decreases.

Note that the manner in which the firm changes the allocation of resources on quality and atmospherics depending on the costs of quality and atmospherics, is exactly the same as in the main model (see Proposition 2 for the corresponding comparative statics). As the above proposition shows, the comparative statics on the level of quality remain the same. The comparison of comparative statics on the supply of atmospherics is a little bit more subtle, because there are now multiple atmospherics variables. To establish that these comparative statics are similar as well, we note the following: In the main model, consumers did not have a perception of affect, but their quality perception was influenced by the affect production (or atmospherics). In other words, the affect was defined as the amount of perceptual distortion on the true quality that the firm induces. The equivalent measure in the current setup is \( \tilde{q} - q = a_1 + a_2 \), which in equilibrium turns out to be equal to \( (\alpha - \beta)^2 \gamma / D \) (see appendix). In comparative statics, this measure behaves exactly as \( a \) does in the main model.
It is also the case that the consumer perception of affect in this model remains unaffected when the cost of atmospherics (either component) changes. This can be interpreted as the firm attempting to not reveal that it is engaging in affect-inducing activities to consumers. Another interesting result is that if the true quality does not influence the affect felt (i.e., $\delta = 0$), then in equilibrium, the firm chooses atmospherics in such a way that consumers have no perception of them ($\tilde{a} = 0$). However, it must be noted that even as $\tilde{a} = 0$, the supply of atmospherics $a_1$ and $a_2$ are such that they together influence the quality perception upward. In other words, the firm sets the values of different affect-generating components so that consumers can not distinguish between this opportunistic firm behavior and the possibility that the firm does not use affect-inducing activities at all (i.e., sets $a_1 = a_2 = 0$).

6. Discussion and Summary

Feelings and affect seem to be present in many consumer decision making situations. But little attention has been paid to how affect might feature in decision-making by firms. Using a consumer information acquisition assumption about the non-separability of the true quality and affect, this paper provides insights into the observed strategies of firms in choosing product quality and affect inducing activities. It also points to an interesting asymmetry in a firm’s motivation to supply quality and affect. If it is easier for a firm to produce high quality products (i.e., it has a lower marginal cost for quality), the firm will also find it optimal to invest more in the true quality as well as in affect inducement, even if such a firm does not have lower cost of inducing affect. In contrast, if a firm can generate affect at a lower cost, the product quality supplied decreases and affect increases.

The above insight seems to have empirical validity as evidenced by the frequent incidence of high quality along with affect creating activities. Store atmosphere, emotional advertising, classy merchandising, and sophisticated salespeople are often associated with high quality products even in the absence of signaling motives. Our analysis traces this association to two specific aspects of markets: i) markets where quality rather than affect is easy to produce and ii) markets where consumers are sophisticated enough to attempt to solve back for the true quality.

Given the above, our model adds to the understanding of the supply of affect in markets. One alternative explanation for why sellers invest in affect is that consumers value affect directly. In our model if consumers have direct utility for some aspect, then that aspect would act exactly like the quality variable in the model. Our point is that even if there are aspects of affect that have no direct utility for the consumer, they can still be offered by the firm in equilibrium. Second, the point of our paper is also that the positive link between affect and quality will be observed even in the absence of
correlation in the valuations for quality and affect. This distinction has practical importance, because it helps firms to decide on the attributes of store ambience consumers may not directly care about (such as specific variations of the packaging, music, lighting), but which still end up influencing their purchase through the rational consumer inference.¹⁹

¹⁹For example, in the “Why You Buy” feature run by ABC News “20/20” on 3/29/1996 one can see that real life examples of how retailers supply music, lighting, packaging colors and other ambience elements which while not necessarily entering the consumer’s utility directly, may induce perceptions of product attributes which in turn influences purchase.
Appendix

Proof of Proposition 1

We first solve for the optimal inter-relationship between $q$ and $a$ that the firm should use to create the consumer perceived quality $\hat{q}$. This is the problem of cost minimization subject to generating a given value of $\hat{q}$ and its solution is independent of consumer behavior (and/or beliefs). Since the cost function is quadratic with zero derivatives at 0, while derivatives of $f(q, a)$ are strictly positive, any solution must have $a > 0$ and $q > 0$, i.e., corner solutions are not possible. Since the quadratic cost is arbitrarily large when either argument is large enough, the (finite) solution exists. Therefore, the cost minimization problem reduces to looking for the critical corner solutions.

Furthermore, by implicit differentiation, we obtain $a'(q) = (c_a f_2 + c_q q f_21 - c_q a f_11)/(c_q f_1 + c_a a f_12 - c_q q f_22) > 0$. Therefore, $a(q)$ is strictly increasing in $q$, which means that $f(q, a(q))$ is strictly increasing in $q$ as well, and therefore, is an invertible function of $q$. Let $\bar{q} = \hat{q}(\bar{q})$ be the solution of $\bar{q} = f(q, a(q))$ for $q$ given $\hat{q}$. Then, consumers observing $\hat{q}$ and any price expect the true quality to be $\hat{q}^*(\hat{q})$, they value the product at $\gamma \hat{q}(\hat{q})$. Therefore, consumers buy if and only if the price is at or below $\gamma \hat{q}(\hat{q})$. Therefore, in equilibrium, the firm charges the price $p = \gamma \hat{q}(\hat{q})$, and consumers buy. Therefore, the firm’s profit is $\pi = \gamma \hat{q}(\hat{q}) - C(q, a(q))$. The firm’s maximizes the above profit as a function of quality choice $q$, where the affect $a$ has to be set by the firm according to the function $a(q)$ defined by the implicit Equation (1). The first-order condition for the optimality of the firm’s behavior is $\gamma = C'(q) + C'(a(q))a'(q) \equiv 2c_q q + 2c_a a(q)a'(q)$, where the left hand side is the marginal benefit of increasing the quality for the firm (increase in revenue), and the right hand side is the derivative of the cost function $C(q, a(q))$ with respect to $q$. The maximum profit is achieved at an interior point, since the derivative of the cost functions at zero is zero. Furthermore, the maximum is achieved (at a finite value of $q$ and $a$) due to costs of $q$ and $a$ being quadratic. Therefore, at the point of maximum profits, the first-order condition must be satisfied. This proves the proposition. Furthermore, if Equation (2) has a unique solution, then together with Equation (1), it defines the equilibrium choices of the firm.
Proof of Proposition 2.

Differentiating the equilibrium the perceived quality, affect, the true quality levels and the equilibrium profits from Equation (3) with respect to the costs of quality and affect production, we obtain that
\[
\begin{align*}
\frac{\partial q}{\partial c_q} < 0, \quad \frac{\partial \pi}{\partial c_q} > 0, \quad \frac{\partial a}{\partial c_a} < 0, \quad \frac{\partial \pi}{\partial c_a} < 0, \quad \frac{\partial p}{\partial c_q} < 0, \quad \frac{\partial \pi}{\partial c_q} = 0, \quad \frac{\partial a}{\partial c_a} = 0, \quad \frac{\partial \pi}{\partial c_a} > 0,
\end{align*}
\]
from which the claims of the proposition follow.

Solution of the Model with Competition (Defined in Section 4).

The expected utility of a type \( \gamma \) consumer from the product of firm \( j \) will be given by
\[
EU_{ij} = \gamma \hat{q}_j - p_j = \left( \frac{\gamma c_u}{c_q + c_a} \right) \hat{q}_j - p_j.
\]

Since some consumers have very low valuation, not all types of consumers are served in equilibrium. The consumer type who is indifferent between buying and not buying from the low quality firm is \( \gamma_c = \frac{\hat{p}_1 - \hat{p}_2}{\hat{q}_1 - \hat{q}_2} \)
and the marginal consumer who is indifferent between buying and not buying from the low quality firm is \( \gamma_h = \frac{\hat{p}_2}{\hat{q}_2} \). This implies that the demand for the high quality firm is \( d_1 = (\gamma_h - \gamma_c) \) and that for the low quality firm is \( d_2 = (\gamma_c - \gamma_h) \). The corresponding profit functions of the firms are,
\[
\pi_1 = d_1 p_1 - C(\hat{q}_1), \quad \text{and} \quad \pi_2 = d_2 p_2 - C(\hat{q}_2).
\]

Since \( a = \frac{\gamma_c}{\gamma_c} q \), the cost functions of the firms can be written as \( C(\hat{q}_j) = c_q \hat{q}_j^2 + \frac{\gamma_c^2}{\gamma_c} \hat{q}_j^2 \).

Since we have a two-stage game we use backwards induction and start with the final stage of the game involving pricing choices of the firms as a function of the quality choices. Then we move back to solve for the first-stage quality choices given the prices. Simultaneously solving the first order conditions for the prices yields \( p_1(q_1, q_2) = 2 \gamma_h q_1(q_1 - \hat{q}_2)/(4q_1 - q_2) \) and \( p_2(q_1, q_2) = \gamma_h (q_1 - q_2) q_2/(4q_1 - q_2) \). The second order conditions on profit maximum with respect to the price choice are always satisfied. Substituting the above prices in the profit functions and then solving first order conditions for the optimal choice of \( q_1 \) and \( q_2 \), we obtain
\[
q_1^* = A_1 \frac{\gamma_c^2 c_u}{c_q(c_q + c_a)}, \quad q_2^* = A_2 \left( \frac{\gamma_h^2 c_u}{c_q(c_q + c_a)} \right),
\]
where \( A_2 \approx 0.024119 \) is the (unique) solution of \( 58956A_2^2 - 13057A_2 + 944A_2 - 16 = 0 \), and \( A_1 = (24080 - 949072A_2^2 - 2162935A_2^2)/[235824(5 - 256A_2 + 2015A_2^2)] \approx 0.12783 \). Substituting \( q_1^* \) and \( q_2^* \) in the profit functions lead to
\[
\pi_1^* = A_1^2 \left( \frac{4(A_1 - A_2)}{(4A_1 - A_2)^2} - 1 \right) \frac{\gamma_h^4 c_u}{c_q(c_q + c_a)}, \quad \text{and} \quad \pi_2^* = A_2 \left( \frac{A_1(A_1 - A_2)}{(4A_1 - A_2)^2} - A_2 \right) \frac{\gamma_h^4 c_u}{c_q(c_q + c_a)},
\]
where the constants in front of the last fraction of each profit function are approximately equal to \( 1.2218 \times 10^{-2} \) and \( 7.6582 \times 10^{-4} \), correspondingly. To check the second order conditions, we derive \( \frac{\partial^2 \pi}{\partial q^2} = -2.054 \frac{c_u(c_q + c_a)}{\gamma_c^2} < 0 \).
\[ q \leq a = \begin{cases} q_1 & \text{if } \delta > q/\gamma \\ q_2 & \text{if } \delta < q/\gamma \end{cases} \]

Thus \( q_1 \) and \( q_2 \) defined above satisfy the second order conditions, and therefore, they lead to the profit maximum for both firms given that \( q_1 > q_2 \). One can also easily check that deviations by either firm to make \( q_1 \leq q_2 \) result in a lower profit to the deviating firm. This proves that \( q_1 \) and \( q_2 \) are the equilibrium quality choices. It remains to be noted that the equilibrium affect choices are \( a_j^* = \frac{c}{c_a} q_j^* \) for \( j = 1, 2 \).

**Proof of Proposition 3.**

Consider first the observable case when in addition to \( \tilde{q} \), consumers also observe \( a \). In this case, when \( \tilde{q} = q + \epsilon \), consumers know the quality exactly before making their purchase decision, and thus, the consumer expectation \( \tilde{q} \) is always equal to the true \( q \). Thus, consumer willingness to pay is \( \gamma q \). Therefore, the optimal \( q, a, \) and \( p \) for the firm are \( q = \frac{\gamma}{2c}, a = 0 \), and \( p = \gamma q \) with all consumers buying.

When \( \tilde{q} = q + \epsilon \), consumers observing \( q \) and \( \epsilon \) still do not have perfect information about \( \tilde{q} \). This situation is equivalent to consumers observing \( q + \epsilon \) and \( a \). Conceivably, the firm may use \( a \) to signal the choice of \( q \) to consumer. However, as we show below, \( a = 0 \) with \( q \) and \( p \) as in the case \( \epsilon = 0 \) is still a Perfect Bayesian Equilibrium (PBE) satisfying the Intuitive Criterion. In this equilibrium, the consumer strategy is to buy if \( \gamma(\tilde{q} + \epsilon) - p \geq 0 \). In other words, consumers buy, unless they know for sure that the firm deviated by setting a lower \( q \). To see that this is an equilibrium, note that if the firm would deviate by setting a lower \( q \), say \( q = p/\gamma - \delta \), consumer demand would reduce by at least \( \frac{\delta}{2c} \) due to the consumers with \( \epsilon \in [-\epsilon, -\epsilon + \delta) \) who detect the deviation. If the firm does not deviate, all consumers buy. Therefore, such a deviation is strictly suboptimal if \( \epsilon < \frac{\gamma}{c} \), which is the condition assumed.

To see that this equilibrium is a PBE satisfying the Intuitive Criterion, note that in any combination of \( p, q, \) and \( a \), the PBE requires that consumers with \( \gamma(\tilde{q} + \epsilon) - p < 0 \) do not buy. Therefore, by the argument above, it can not be optimal to set \( p > q/\gamma \). However, with \( p = q/\gamma \) and all consumers buying, the firm’s profit are maximal if \( p, q, \) and \( a \) are set as suggested above. Therefore, any deviation results in strictly lower profits no matter what consumer off-equilibrium beliefs are, and thus, the Intuitive Criterion is satisfied.

Now consider the case when \( a \) is not observed by the consumers. In this case, by the same argument as in the main model, elimination of strictly dominated strategies requires the firm to always set \( a \) and \( q \) to satisfy \( a = (c_q/c_a)q \). Therefore, consumers must also believe that \( a = (c_q/c_a)q \) is satisfied. Therefore, in equilibrium, \( \tilde{q} = q + a + \epsilon_i = \frac{c_q}{c_a}(q + \epsilon_i') \), where \( \epsilon_i' = c_a \epsilon_i / (c_a + c_q) \). Thus, the model is equivalent to consumers observing \( q' = q + \epsilon_i' \) and \( p \) before the purchase, and the firm not able to produce \( a \), but producing quality \( q \) at cost \( cq^2 \), where \( c = c_q(c_a + c_q)/c_a \). Applying the proof of the case of observed \( a \), we can obtain that the equilibrium levels of \( p, q, \) and \( a \) when \( \epsilon_i = 0 \) as in section 3 still hold in a PBE satisfying Intuitive Criterion when \( \epsilon' < \frac{\gamma}{2c} \), i.e., when \( \epsilon < \frac{\gamma}{2c} \), which was assumed.

Note that a similar proof would work if \( \tilde{q} = (q + a)(1 + \epsilon_i) \), i.e., if the magnitude of the perception error is larger when the underlying variable is larger.

**Proof of Proposition 4.**

If consumers believe that a firm has some cost \( c_q \) then they will discount the perceived quality by \( 1 - \frac{c_a}{c_q + c_a} \). Thus if consumers believe that the firm has the lower cost of quality \( (c_{q1}) \), then they will discount the perceived quality.
by a lower amount and as shown in (3) the profits are also higher. Therefore, the firm’s profits are higher if it is able to convince consumers that it has the lower cost of quality production. Given this and because increasing the perceived quality is less costly for the firm with lower cost of quality, this firm can credibly signal its low cost by increasing the perceived quality.

Therefore, in a separating PBE the following properties must hold:

i. Each type of firm produces an equilibrium level of perceived-quality production, the lower one of which we denote by \( \tilde{q}_L \) and the higher by \( \tilde{q}_H \) and the incentive compatibility constraints, i.e., that neither firm wants to deviate to make consumers believe it is of the other type, are satisfied. In particular:

ii. In equilibrium, the high-cost firm produces the optimal level \( \tilde{q}_L \) associated with its cost \( c_{q2} \), given that consumers will correctly recognize the firm’s type (since consumers already have the worst possible belief about this level of production). The low-cost firm produces at a level \( \tilde{q}_H \) such that the high-cost firm does not prefer to deviate to this level and be perceived as a low-cost type rather than produce at the other level and be perceived as the high-cost type.

iii. Consumer beliefs are that a firm is low (high) cost upon observing a perceived quality production of \( \tilde{q}_H \) (\( \tilde{q}_L \)) and they make their purchase decisions consistent with these beliefs.

Furthermore, according to the Intuitive Criterion, if any firm produces at an off-equilibrium level and if only the low-cost firm could possibly benefit for any off-equilibrium beliefs, then the off-equilibrium beliefs for this action should be that the firm is the low-cost type. In other words, any perceived-quality choice at a level higher than or equal to \( q_{Hic} \) which the high-cost firm has no incentive to mimic (instead of \( \tilde{q}_L \)) even if consumers were to believe it to be the low-cost firm under such production has to be perceived as having been made by the low-cost firm.

In other words, for a separating equilibrium, the perceived quality choice \( \tilde{q}_H \) of the low-cost firm is at least as high as the maximum of (a) \( q_{Ho} \) which is the full information level that would be optimal for the firm given that its cost is \( c_{q1} \) and (b) \( q_{Hic} \) which is what would be necessary for incentive compatibility such that the high-cost firm does not want to pretend to be low-cost. The Intuitive Criterion implies that the perceived quality choice \( \tilde{q}_H \) of the low-cost firm is exactly equal to the maximum of the above.\(^{20}\)

Formally, the Intuitive Criterion for deviations from the equilibrium of the low cost firm is as follows: Suppose the low cost firm deviates to some \( \tilde{q}_{Hdev} > \tilde{q}_H \), then the firm should get lower profits in this deviation than in the equilibrium no matter what the beliefs are that consumers assign for the information set pertaining to the deviation. The choice of the low-cost firm is \( \tilde{q}_H = \max(\tilde{q}_{Ho}, \tilde{q}_{Hic}) \). Now if \( \tilde{q}_{Ho} > \tilde{q}_{Hic} \), then the equilibrium choice for the low cost firm must be \( \tilde{q}_{Ho} \) and for all deviations \( \tilde{q}_{Hdev} \) consumers will continue to believe that firm is the low cost one. Therefore the equilibrium choice of \( \tilde{q}_H = \tilde{q}_{Ho} \) will satisfy the intuitive criterion because any other \( \tilde{q}_{Hdev} \) will imply higher costs for the low cost firm irrespective of consumer beliefs. Therefore, all

\(^{20}\)See Cho and Kreps (1987), for this characterization of the Intuitive Criterion for games with only two sender types and also note that for two-type sender games the Intuitive Criterion is the same as the refinement based on equilibrium dominance.
such deviations are ruled out. Now suppose $\tilde{q}_{Hic} > \tilde{q}_{Ho}$, then any $\tilde{q}_{Hdev} > \tilde{q}_{Hic}$ is ruled out because it means an even higher perceived quality choice than what is dictated by cost minimization for the firm. To rule out any deviation $\tilde{q}_{Ho} < \tilde{q}_{H} < \tilde{q}_{Hic}$, we specify the off-equilibrium belief that consumers believe the firm to be a high cost firm if they see a perceived quality in this range.

Thus, according to Equation (3), the quality and affect production levels of the high-cost firm are as in part 1 of the Proposition, and the production levels of the low-cost firm are at least as high as to make the high-cost firm indifferent between its equilibrium production levels with the associated consumer belief that the firm is high-cost, and the deviation to the low-cost production levels with the associated consumer beliefs that it is a low-cost firm. This indifference is given by the following equation:

$$\gamma^2 c_a = \frac{\gamma \tilde{q}_{Hc} c_a}{c_1 + c_a} - C_h(\tilde{q}_H),$$

(9)

where the left hand side is the profits of the high-cost firm given the equilibrium level of its quality and affect (see Equation (3)), $\tilde{q}_H$ is the perceived quality under which consumers start assuming that this firm is one with low-cost and start discounting the perceived quality by $c_a/(c_1 + c_a)$ instead of by $c_a/(c_2 + c_a)$, and $C_h(\tilde{q}_H)$ is the cost of inducing the quality perception $\tilde{q}_H$ by the high-cost firm. Since for a production of any perceived quality $\tilde{q}$, the optimal choice of affect production for the high-cost firm is $qc_2/c_a$ where $q$ is the choice of quality, the cost for high-cost firm to produce the perceived quality $\tilde{q}_H$ is equal to $c_2c_a\tilde{q}_H^2/(c_2 + c_a)$. Substituting this in the above equation and solving for $\tilde{q}_H$, we obtain:

$$\tilde{q}_H = \frac{\gamma(c_2 + c_a \pm \sqrt{(c_2 - c_1)(c_2 + 2c_a + c_2))}}{2(c_2 + c_a)c_2}.$$  

(10)

The higher value (positive sign in front of the square root) is the one pertaining to the incentive compatibility constraint, since it is the one corresponding to the deviation above rather than below the equilibrium value for the high-cost firm production.

We have to check whether or not the incentive compatibility constraint is binding for the low-cost firm as follows: Note that the optimal production level of perceived quality for the low cost firm if consumers correctly recognize it as a low-cost firm would be $\tilde{q}_{Ho} = \gamma/(2c_1)$, which is indeed smaller than the $\tilde{q}_H = \tilde{q}_{Hic}$ determined by the incentive compatibility constraint. Because the incentive compatibility constraint is binding, the intuitive criterion described above is satisfied given the cost minimization incentive of the firm and the off-equilibrium consumer belief that the firm is a low type if a deviation to any perceived quality below $\tilde{q}_{Hic}$ is observed. The difference $\tilde{q}_{Hic} - \tilde{q}_{Ho}$ is the extent of distortion in the perceived quality choice of the low cost firm over its full information level.

$$\text{Distortion} = \frac{\gamma(c_2 - c_1 + \sqrt{(c_2 - c_1)(c_2 + 2c_a + c_2))}}{2(c_1 + c_a)c_2} > 0.$$  

(11)

The optimal quality and affect production levels for the low-quality firm now follow immediately from the cost-minimization conditions given the perceived quality (see Section 2.1).

Proof of Proposition 5.

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The method of proof is similar to the one of the previous proposition. Recall that consumers discount the perceived quality by \( \frac{c_q - c_a}{c_q + c_a} \). So the difference now is that, since consumers will discount the perceived quality less if they think the firm is the high-cost of affect one, the high-cost firm would like to convince consumers that it is indeed high-cost. It will be able to do this by reducing the perceived quality level, because such a reduction, will have the same negative effect on consumer willingness to pay, but will save more in costs for the high-cost firm than for the low-cost firm.

The incentive compatibility constraint now determines the value of the perceived quality produced by the high-quality firm (or the low-cost of affect firm) and is the solution \( \tilde{q}_L \) of the following equation representing the low-cost firm indifference.

\[
\frac{\gamma^2 c_{q1}}{4c_q(c_q + c_{a1})} = \frac{\gamma \tilde{q}_L c_{a2}}{c_q + c_{a2}} - \frac{c_q c_{a1} \tilde{q}_L^2}{c_q + c_{a1}}.
\]

Solving the above equation for \( \tilde{q}_L \), we obtain

\[
\tilde{q}_L = \frac{\gamma \left( c_{a2} (c_q + c_{a1}) - \sqrt{(c_{a2} - c_{a1})c_q(c_{a2}c_q + 2c_{a2}c_{a1} + c_q c_{a1})} \right)}{2c_q c_{a1}(c_q + c_{a2})}.
\]

As in the last proposition, this incentive compatibility constraint is binding and the reduction in perceived quality of the high-cost of affect firm, relative to what it would choose if its costs were observed is

\[
\text{Distortion} = \frac{\gamma \left( \sqrt{(c_{a2} - c_{a1})c_q(c_{a2}c_q + 2c_{a2}c_{a1} + c_q c_{a1})} - c_q(c_{a2} - c_{a1}) \right)}{c_{a1}c_q(c_q + c_{a2})} > 0.
\]

In this case the distortion represents the amount of the downward distortion in perceived quality by high-cost of affect firm. The optimal levels of \( q \) and \( a \) production for the high-cost of affect firm now follow from the perceived quality value \( \tilde{q}_L \) defined above.

**Proof of Proposition 6.**

Similarly to the proof of Proposition 1, since consumer behavior, and therefore, revenue, only depends on \( \tilde{a} \) and \( \tilde{q} \), if the firm decides on certain values of these variables, it should optimally choose \( q, a_1 \), and \( a_2 \) as to minimize it’s cost. Hence, the optimal behavior by the firm must satisfy the following conditional cost minimization property: \( q, a_1 \), and \( a_2 \) are such that \( C(q, a_1, a_2) \) is minimized subject to the constraint that \( \tilde{q} \) and \( \tilde{a} \) are at the given level. Using Lagrangian multipliers to solve this condition, we write \( L(q, a_1, a_2, \lambda, \mu, \tilde{a}, \tilde{q}) = C(q, a_1, a_2) + \lambda(f(q, a_1, a_2) - \tilde{q}) + \mu(g(q, a_1, a_2) - \tilde{a}) \). Differentiating this function with respect to \( q, a_1 \), and \( a_2 \) (and using the functional forms of \( f(\cdot) \), \( g(\cdot) \) and \( C(\cdot) \)), we obtain the following system of equations

\[
\begin{align*}
2c_q q + \lambda + \mu \delta &= 0, \\
2c_{a1} a_1 + \lambda + \mu \alpha &= 0, \\
2c_{a2} a_2 + \lambda + \mu \beta &= 0,
\end{align*}
\]

Solving this system for \( \lambda, \mu \), and \( a_2 \), we obtain that a necessary condition for the firm’s profit maximization is

\[
a_2 = \frac{c_q \alpha - \delta c_{a1} a_1 + \beta c_{a1} a_1 - \beta c_q q}{(\alpha - \delta)c_{a2}}.
\]

Since the above equation must be satisfied regardless of consumer behavior (and beliefs, if present), the firm
always follows it, and hence, rational consumers know that it holds. Furthermore, as far as the above equation
is satisfied, firm’s decisions on $a_1, a_2,$ and $q$ are uniquely determined by $\tilde{q}$ and $\bar{a}$. Therefore, consumers can infer
the true value of $q$ from $\tilde{q}$ and $\bar{a}$ without error as far as the firm follows the above equation. This means that
the firm’s profit under any $q, a_1,$ and $a_2$ satisfying the above equation, is $\pi = \gamma q - C(q, a_1, a_2)$. Differentiating
this profit function with respect to $a_1$ and $q$, after substituting $a_2 = a_2(q, a_1)$ determined by Equation (16),
results in the system of two linear equations on $a_1$ and $q$ with two unknowns. The solution is unique, thereby
giving the point of profit maximum. The solution values are reported in Equation (5). The comparative statics
reported in the statement of the proposition immediately follow from these values. The proposition is proven.

**The Case of Naive Consumers.**

This case means that consumers buy the product if and only if $\gamma \tilde{q} - p \geq 0$. It is still optimal for the firm
to generate perceived quality through affect production and true quality that are related as in equation (1).
Suppose the firm is considering generating a marginal increase in perceived quality. Then the marginal cost of
increasing perceived quality so that the true quality would increase by $dq$ is, as before, $(2c_q q + 2c_a a(q)a'(q)) dq$,
where $a(q)$, as before, is defined by solving equation (1) for $a$ as a function of $q$. However, the marginal benefit
of such an increase in the perceived quality is the full change in the perceived quality, i.e., it is

$$
\gamma f'(q, a(q)) dq = \gamma (f_1(q, a(q)) + f_2(q, a(q))a'(q)) dq
$$

rather than $\gamma dq$. Therefore, the equilibrium conditions are as in Proposition 1 with the difference that Equation (2) is replaced by

$$
\gamma (f_1(q, a(q)) + f_2(q, a(q))a'(q)) = 2c_q q + 2c_a a(q)a'(q).
$$

If $f(q, a) = q+a$, Equation (18) becomes $\gamma (1+c_q/c_a) = 2c_q q + 2c_a (c_q/c_a) q(c_q/c_a)$, leading to the equilibrium
levels of quality, affect, and perceived quality of

$$
q = \frac{\gamma}{2c_q}, \quad a = \frac{\gamma}{2c_a}, \quad \bar{q} = \frac{\gamma(c_q + c_a)}{2c_q c_a}.
$$

In this case, the costs the firm incurs on generating quality, affect, and the total costs are, respectively,

$$
C(q) = \frac{\gamma^2}{4c_q}, \quad C(a) = \frac{\gamma^2}{4c_a}, \quad C(\bar{q}) = \frac{\gamma^2(c_q + c_a)}{4c_q c_a},
$$

whereas the profit is

$$
\pi = \frac{\gamma^2(c_q + c_a)}{4c_q c_a}.
$$
REFERENCES


