Information Acquisition and Sharing in a Vertical Relationship*

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June, 2009

* We appreciate the valuable feedback from Eric Bradlow, the Area Editor, and the reviewers. This paper has also benefited from the helpful comments of Tony Cui, Anthony Dukes, Raphael Thomadsen, and seminar participants at CKGSB and at Summer Institute in Competitive Strategy 2008. The usual disclaimer applies. †Assistant Professor, Department of Marketing, Hong Kong University of Science and Technology, Hong Kong, China. ‡Edgar F. Kaiser Professor of Business Administration, Haas School of Business, University of California at Berkeley, Berkeley, CA 94720. Email: mkguo@ust.hk and giyer@haas.berkeley.edu.

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ABSTRACT

Manufacturers can acquire consumer information in a sequential manner and influence downstream retail behavior through sharing the acquired information. This paper examines the interaction between a manufacturer's optimal information acquisition and sharing strategies in a vertical relationship, capturing the impacts of both the flexibility to sequentially control information collection and the flexibility in ex post voluntary sharing. We show that, when information acquisition is sequential, the manufacturer may not acquire perfect information even if it is costless to do so. This self-restriction in information acquisition follows from the manufacturer's motivation to strategically influence retail behavior. When information acquisition is inflexible and constrained to be either zero or perfect information, the manufacturer acquires less (more) information under mandatory (voluntary) sharing. Moreover, voluntary sharing unambiguously leads to more information being generated, because the manufacturer has the option to strategically withhold the acquired information that turns out to be unfavorable. Finally, the conditions under which the manufacturer ex ante prefers a particular sharing format are examined.

Key words: information acquisition; information sharing; channel; disclosure; vertical relationship

1. Introduction

It is ubiquitous that firms are uncertain about consumer preferences and demand, and such uncertainty may arise for a number of reasons. In markets for fashion or seasonal goods (e.g., apparel, cosmetics, motion pictures, sporting goods), consumer tastes are inherently uncertain. The proliferation of new products, in industries ranging from automobile, electronics, to consumer goods, can result in shorter product lifetime and increasingly volatile demand. A challenge for firms selling through a distribution channel is then about how to match supply with uncertain consumer preferences. This has led to a growing need for uncertainty-resolving information. Manufacturers in fashion and seasonal goods industries (e.g., L.L. Bean and Timberland) have invested in sophisticated information gathering and demand forecasting systems (Fisher and Hammond 1994). Another example is Sport Obermayer in the sporting goods industry which instituted information systems based on early demand signals to more accurately forecast demand. Moreover, the acquisition and the sharing of consumer or market information with channel partners to influence their behavior has been recognized in the descriptive and the trade literature as a strategic tool in channel relationship management, i.e., "information power" (Eyuboglu and Atac 1991, Williams and Moore 2007).

Manufacturers of national brands can become better informed of consumer preferences and demand, through market research, product testing, or accessing ongoing syndicated research (Blattberg and Fox 1995). However, downstream firms may lack the necessary expertise and resources and thus have to rely on upstream firms for information (*Agency Sales*, 1992, 2003). For instance, members in the Association of Better Computer Dealers indicated that they frequently obtain information from their focal manufacturer (Mohr and Sohi 1995).[1] In addition, with the proliferation of new products launched, many retailers increasingly realize that, as the scale and scope of the carried items increase, they are less informed about consumer preferences and demand than their upstream manufacturers. Consequently, a growing practice for manufacturers like Kraft, Procter and Gamble, Warner-Lambert is to get involved in the decision making process of downstream retailers through information sharing (Niraj and Narasimhan 2004, Kurtulus and Toktay 2005).

This paper analyzes a manufacturer's incentive to acquire and share consumer preference information in order to manage its retailer's behavior. We address several questions that have not been investigated in the literature. In particular, how much information should be acquired? When should the acquired information be shared with the retailer? How do the information acquisition and sharing decisions interact with each other? Moreover, what type of sharing format should be implemented by the manufacturer? We consider a model in which the channel members have uncertainty about how well the product fits consumer preference, which can be potentially resolved by the manufacturer through acquiring (imperfect) signals. In addition, it is uncertain to the retailer about whether useful information is available to the manufacturer. We highlight the manufacturer's strategic incentive for information acquisition, i.e., manipulating the retailer's belief and thus its behavior through information sharing.

Advances in information collection technologies and Internet-based information gathering have led to increasing sophistication in the manner in which firms track consumer preferences, test new products, and monitor reactions to product modifications. Central to these advances is the increasing flexibility to sequentially control the generation of information. For example, with syndicated databases or online surveys, signals that are useful for decision making (i.e., data) may flow in continuously and sequentially, and a manufacturer can decide, at each point after observing the collected signals, whether to acquire additional signals. We examine the effect of sequentially acquiring information and compare it with the case when information acquisition is "inflexible" whereby the manufacturer can choose to acquire only either none or infinite signals.

Further, we investigate and compare two distinct formats of (truthful) information sharing. In the "ex ante mandatory sharing" case, the manufacturer contractually commits to share all the acquired information with the retailer before the information is actually acquired. For example,

[1] Brown et al. (1983) demonstrate that a retailer's dependence on its supplier is positively related to the extent to which superior upstream information is provided. Eyuboglu and Atac (1991) find that upstream channel members who are perceived to communicate more informative messages exert more control over others' decision.

manufacturers such as Procter and Gamble and Warner-Lambert streamline the sharing process by setting up formal arrangements, such as Collaborative, Planning, Forecasting, and Replenishment (CPFR), that automatically transmit the acquired data to the collaborating retailer (Gal-Or et al. 2008). In contrast, under the "ex post voluntary sharing" format, the manufacturer can decide after information collection whether to share the acquired information. This can represent informal communications between managers or sales personnel of upstream firms and their retail partners. Many firms share insights with their retailers from time to time, but do not contractually commit to share information on a long-term basis. Indeed, according to the extensive surveys conducted by BearingPoint, firms in the US rely primarily on traditional devices (e.g., E-mail, phone, fax, meeting) to communicate with their channel partners (Chain Store Age 2003).

1.1. Summary of Results

The first result of our analysis is that, when information acquisition is sequential, the manufacturer may not acquire an infinite number of signals that perfectly reveal consumer preference, *even though* information acquisition is costless. The manufacturer in equilibrium exercises self-restriction in information acquisition, and continues to generate signals if and only if the posterior belief is between some upper and lower bound. This stems from the manufacturer's motivation to strategically influence retailer behavior. If the information acquisition process reaches a stage where the posterior belief is at a high enough level, the manufacturer may stop the acquisition simply because no better outcome can be achieved in terms of inducing more retail ordering. Conversely, the manufacturer may also terminate information acquisition when a sufficiently low posterior belief is reached, because of the risk that further collection of signals may lead to overly adverse results. In summary, this result identifies "strategic ignorance" in information acquisition as a new mechanism that manufacturers may use to govern the behavior of their retailers.

The equilibrium amount of information generated is influenced by the interaction between the flexibility in information acquisition (sequential versus inflexible) and the flexibility in information sharing (voluntary versus mandatory). First, whether the flexibility to sequentially control information acquisition leads to an increasing incentive to acquire more information depends on the sharing format. When the manufacturer has committed to mandatorily share information, it will choose not to collect any information if information acquisition is inflexible, whereas sequential information acquisition will lead to a positive amount of information being acquired. In contrast, under voluntary sharing, the manufacturer in equilibrium always acquires information if acquisition is inflexible, and as a result the flexibility in information acquisition may actually lead to reduced information generation. Overall, the impact of the flexibility in information acquisition on the equilibrium amount of information acquired is moderated by the flexibility in information sharing.

Interestingly, the flexibility in information sharing can unambiguously induce the manufacturer to generate more information. This obtains from the manufacturer's ability to select whether to disclose the acquired information. Suppose that the manufacturer continues to acquire additional information and when the acquired information turns out to be unfavorable, the manufacturer has the option to withhold the bad news and hence may not necessarily induce unfavorable responses from the retailer. Such information concealment is feasible: The retailer cannot distinguish between the cases when useful information is indeed unavailable and when the manufacturer's acquired information is unfavorable, since no information will be received by the retailer in both cases.

We then examine the manufacturer's preference for the information sharing formats. Counter-intuitively, when the prior belief about consumer preference is sufficiently low, the manufacturer prefers to commit to mandatorily share any information that will be acquired and thereby gives up the ex post flexibility to voluntarily disclose information. This is because more information may induce on average lower retail ordering, which can be alleviated by mandatory sharing commitment that serves as the manufacturer's self-discipline in information generation.

1.2. Related Research

There is a substantial literature starting from Novshek and Sonnenschein (1982) and Vives (1984) on (ex ante mandatory) information sharing between oligopolistic firms. The general theme in this literature is that the equilibrium impact of information sharing depends on the interplay of two effects: First, there is an efficiency effect whereby each firm has better information about supply or demand uncertainty. Second, the sharing of information leads to greater correlation in the strategies of the competing firms. The net impact of these effects differs across the various contexts considered. For example, Gal-Or (1985) and Li (1985) investigate a Cournot oligopoly with uncertainty about a common demand parameter and show that firms will not share information, while Gal-Or (1986) and Shapiro (1986) derive the opposite result with uncertainty about costs. Vives (1984) shows that the incentives to share information change depending on whether the products are substitutes or complements and whether the competition is Cournot or Bertrand.[2] Villas-Boas (1994) studies the effects of information transmission that might result from competitors sharing the same advertising agency. In contrast to the above literature, this paper investigates the acquisition and the transmission of information in a vertical relationship. The economic incentive at play in our paper involves the effect of information acquisition/sharing on retailer behavior.

[2]Raith (1996) presents a general model of information sharing which accommodates and clarifies the effects of information sharing in many of the major models in the literature.

Another stream of research examines the acquisition of information by oligopolistic firms. For example, Li et al. (1987) and Vives (1988) analyze optimal information acquisition about demand uncertainty, and show that the equilibrium level of information acquisition decreases with the cost of information acquisition and the slope of the demand function. Hwang (1993) extends these models to the case of non-identical and increasing marginal costs to show that the firm with smaller marginal costs acquires more information in equilibrium. In these studies, firms make an one-time (static) decision on how much information to acquire, and perfect information acquisition would arise in equilibrium when the acquisition cost is zero. In contrast, we examine sequential information acquisition, which implies that after each signal there is a decision on whether to acquire additional signals. As a result, in our analysis the manufacturer may prefer not to acquire full information, even if it is costless, because of the motivation to control retail behavior. Moreover, we examine how information acquisition interacts with the subsequent sharing decision in a vertical channel.

There are also some papers that focus on information sharing in vertical relationships (e.g., Niraj and Narasimhan 2004, He et al. 2008).[3] Li (2002) is perhaps the first analysis of the incentives of competing retailers to share private information with an upstream manufacturer.[4] It identifies a direct effect due to changes in the actions of the parties involved in the sharing, and an indirect effect due to the changes in the actions of the competing retailer. Gal-Or et al. (2008) examine information sharing between a manufacturer and two competing retailers, which can alleviate demand uncertainty and thus mitigate distortions in wholesale price. Our focus diverges significantly from these papers in that we analyze sequential information acquisition by a manufacturer and its subsequent sharing with the retailer. Moreover, only mandatory sharing is considered in these studies,[5] whereas we examine how the format of sharing (voluntary versus mandatory) affects the amount of information that the manufacturer will acquire.

The rest of the paper is organized as follows. The next section describes the model. Section 3 presents some preliminary results. This is followed by the analysis of mandatory sharing in Section 4. Next, Section 5 addresses the case of voluntary sharing, where the manufacturer's equilibrium ex ante payoffs across the alternative sharing formats are also compared. Some model extensions are analyzed in Section 6. The final section concludes the paper.

[3]There is a substantial literature in supply chain management, originating from Lee et al. (1997), that examines the efficiency-improving role of information sharing in reducing production, logistical, or inventory-related costs (e.g., Cachon and Fisher 2000, Kulp et al. 2004). In this vein, Iyer et. al (2007) investigates the substitutability between information sharing and inventory holding in a distribution channel.

[4]Gal-Or et al. (2007) study whether buyers should share supplier-specific fit information with prospective suppliers in order to extract more surplus in input procurement. Creane (2007) looks at an analogous problem of downstream firms sharing their productivity information with an input supplier.

[5]One exception is Guo (2009) which examines the payoff implications of a downstream retailer sharing privately acquired information with an upstream manufacturer on an ex post voluntary basis.

2. The Model

An upstream manufacturer produces a good and sells to end consumers through a retailer. The manufacturer has a constant and zero marginal cost of production. There is a competitive supply of the product in the upstream market, over which the manufacturer enjoys a margin $d > 0$. The retailer would order the product from the manufacturer only if the per-unit wholesale price ω is not higher than d which represents a measure of competitiveness in the upstream market. The firms are risk neutral and maximize expected payoffs.

Consumers belong to one of two segments whose product valuation are denoted by:

$$V_i = \theta_i Q, \tag{1}$$

where $i \in \{h, l\}$ denotes consumer segments; Q represents the consumers' perception of the fit of the product with their preference; and θ_i captures consumer segment i's valuation conditional on product fit, where $0 < \theta_l < \theta_h$. The relative size of the high and the low consumer segments are α and $1 - \alpha$, respectively, where $\alpha \in (0, 1)$. Consumers know their perceived product fit, which is initially unknown to the firms.[6] Suppose without loss of generality that there are two possible states of nature, $S \in \{G, B\}$, such that $Q = 1$ if the product is "good" (i.e., $S = G$) and $Q = 0$ if it is "bad" instead (i.e., $S = B$). The firms have common prior belief about the true state on consumer preference, i.e., $\Pr(S = G) = \beta$ and $\Pr(S = B) = 1 - \beta$, where $\beta \in (0, 1)$.

This paper deals with information on consumers' perceived product fit, which the manufacturer can acquire (e.g., through market research, focus groups, online surveys, syndicated consumer databases) and transmit to the retailer. This may be particularly relevant for new products, modifications to products, or consumer taste changes (e.g., in fashion markets), where significant firm uncertainty may exist about product fit with consumer preferences. In the base model we assume that the retailer has no independent means to improve its knowledge about the perceived product fit. This represents situations where the manufacturer has superior access to product fit information, and the firms' ability for information collection are asymmetric (*Agency Sales*, 1992, 2003). Nonetheless, we shall extend the model to incorporate downstream information acquisition in Section 6.2, and discuss the implications of retailers exerting influence on upstream suppliers through strategic information acquisition/sharing in Section 7.2.

The timing of the game is shown in Figure 1. In the first stage, the firms sign and commit

[6] Alternatively, one can interpret the consumers' perceived product fit Q as "perceived quality" in that higher quality product fits consumer preference better. To simplify exposition, we will use the terms consumer preference and perceived product fit interchangeably whenever no confusion arises.

Figure 1: Timing of the Base Model

to a contract that includes the specification of a per-unit wholesale price ω for the transfer of the product, and the format of information sharing. Two alternative sharing arrangements therefore arise. The first one is ex ante information sharing in which the manufacturer commits to mandatorily share information with the retailer. For example, manufacturers like Procter and Gamble collaborate with retailers to develop shared databases. In contrast, the manufacturer can choose not to commit to transfer the to-be-acquired information to the retailer. Rather, after acquiring the information in stage 2, the manufacturer will voluntarily decide whether to disclose the acquired information. This case can represent informal communications between sales managers of firms and their downstream partners. The essential difference between these two sharing formats hinges on whether the manufacturer, prior to knowing the content of the acquired information, commits ex ante to share the information. Note that if information sharing cannot be credibly contracted/committed, the manufacturer can always share its private information voluntarily on an ex post basis. We will investigate and compare these two alternative sharing formats, which allows us to capture the impact of the flexibility in information sharing.

Nevertheless, useful information may not always be available. For example, the design of survey instruments may involve bias, the data collection technology may only yield signals that are too complex or irrelevant for decision making, or the capability to process the collected raw data may be absent. Specifically, the availability of useful information is denoted as $I \in \{y, \bar{y}\}$, where y or $\bar{y}$ is realized in the second stage of the game each with a probability of one half and represents the state in which useful information is available or unavailable, respectively.[7] The retailer is uncertain about whether useful information is available to the manufacturer, unless the acquired information is disclosed (either mandatorily or voluntarily). As we will see in the analysis, this uncertainty has implications for the information sharing strategy of the manufacturer. To facilitate notation, we define the manufacturer's and the retailer's expected payoffs in stage 2 as Π' and π' (or Π'' and π''), respectively, conditional on the wholesale price ω and the realized information state being $I = y$ (or $I = \bar{y}$), and taking into account the manufacturer's optimal information acquisition and sharing

[7]Equivalently, the state y ($\bar{y}$) can represent the scenario when the manufacturer's cost of information acquisition is negligible (prohibitively high). We thank an anonymous reviewer for suggesting this alternative interpretation.

strategies. Moreover, the firms' equilibrium sub-game profits in stage 2, prior to the realization of the information state, are defined as $\Pi(w) = (\Pi' + \Pi'')/2$ and $\pi(w) = (\pi' + \pi'')/2$, respectively.

If $I = y$, the manufacturer can acquire (imperfect) signals about the perceived product fit at zero marginal cost per signal.[8] Each signal $s \in \{g, b\}$ is independently generated from the true state S with probability $\gamma \in [1/2, 1]$: $\Pr(g|G) = \Pr(b|B) = \gamma$. Conditional on a signal s, the updated probabilities of the states of the perceived product fit are $\Pr(G|g) = \frac{\beta\gamma}{\beta\gamma+(1-\beta)(1-\gamma)}$ and $\Pr(B|b) = \frac{(1-\beta)\gamma}{\beta(1-\gamma)+(1-\beta)\gamma}$. The parameter γ captures the informativeness of the signals. When $\gamma \to 1/2$, a signal provides no additional information beyond the prior: $\Pr(G|g) = \beta$ and $\Pr(B|b) = 1 - \beta$. When $\gamma \to 1$, the signals reveal the truth with full certainty: $\Pr(G|g) = \Pr(B|b) = 1$.

We consider two alternative scenarios on information acquisition. In the first scenario, information acquisition is inflexible in the sense that the manufacturer can choose to acquire only either none or an infinite number of signals. In contrast, if information acquisition is sequential, at each point when a total of $n \geq 0$ signals have been accumulated and processed, the manufacturer decides whether to generate an additional signal.[9] The comparison between these two scenarios allows us to capture the impact of the flexibility in information acquisition.

In stage 3 the manufacturer can share the acquired information with the retailer who may then update its belief about the perceived product fit. Under mandatory sharing, all acquired signals are truthfully and completely transmitted to the retailer. Therefore the manufacturer's sharing decision is immaterial, and the firms have a common posterior belief. When information sharing is ex post and voluntary, the manufacturer decides whether to transfer the acquired signals to the retailer. The manufacturer truthfully discloses all the acquired signals if it decides to share, with the alternative option to remain silent and disclose none of the signals.[10] The firms' posterior beliefs are then the same if sharing occurs, but may diverge from each other if otherwise.

In the final stage the retailer decides on the ordering units, $x \in [0, 1]$.[11] The retailer has to carry

[8]This may represent the (negligible) cost of converting the data obtained from a vendor or an online survey, if informative (i.e., $I = y$), into useful insights for decision making. For example, many information vendors typically charge a fixed fee for unlimited access to their database. In online surveys, once the fixed cost of setting up the survey is incurred, the cost of eliciting an additional response is negligible. Generally, we can interpret the setup as one in which masses of data can be generated by investing a fixed cost on a data-collection technology, and the subsequent cost of converting each unit of the collected data into managerially-relevant information is minimal.

[9]An equivalent interpretation of the information acquisition problem is that the signals flow in sequentially (e.g., online surveys) and the manufacturer chooses when to stop the information acquisition process.

[10]The truth-telling assumption is standard in the existing literature on information sharing, and implies either that verification costs are negligible or that the firms have long-term reputation concerns.

[11]The implicit assumption is that retail ordering does not occur before information acquisition/sharing. Therefore, this timing represents all the information collected by the manufacturer that is relevant for the retail ordering decision. For example, in the fashion industry or for new product introductions, there is often a significant interval from the time when wholesale transfer contracts are signed to the time when actual retail orders are placed. This interim time is relevant for information acquisition/sharing. We thank the AE for comments on this issue.

the ordered stock to be able to sell to the end consumers. When making the ordering decision, the retailer remains uncertain about the fit of the product with consumer preference unless an infinite number of signals (i.e., almost perfect information) are acquired and disclosed by the manufacturer. Finally, the perceived product fit is revealed and the retail price p is set.[12]

Some elaboration on model assumptions are warranted. We intentionally assume that the determination of the wholesale price precedes the manufacturer's information acquisition, because this enables us to isolate the strategic effect of upstream information acquisition/disclosure on the retailer's behavior from the efficiency effect of improving the manufacturer's own decision making. Nevertheless, we show in Section 6.1 that the main results are robust to making the alternative timing assumption that the wholesale price is determined conditional on the acquired information (i.e., in stage 3). Next, under the assumption of zero marginal cost of information acquisition, the manufacturer can choose to acquire an infinite amount of signals and become (almost) fully informed of consumers' true preference without incurring any additional cost. But the interesting point we want to investigate is the possibility that the manufacturer may choose not to know the truth for sure even when it is completely costless to do so, because of the incentive to strategically manipulate the downstream retailer's belief. It is this role of information sharing in influencing the manufacturer's information acquisition strategy that we highlight in the analysis.

We focus on market conditions under which the relative size of consumer segments satisfies $\alpha\theta_h < \theta_l < \alpha(2-\alpha)\theta_h$. The first inequality rules out the trivial scenario when the low-type consumers are never served, and ensures that a vertically integrated manufacturer would serve the whole market. The second inequality represents the case in which the low-type consumers will not buy in equilibrium in a decentralized channel without further information. This captures the relevant range of market conditions that help us examine the relationship between information acquisition/sharing and the trade-off between greater market coverage and surplus extraction through higher wholesale prices. To solve the game, we use backward induction to insure sub-game perfection.

3. Preliminaries

3.1. Retailer Decisions

Let the retailer's updated belief about consumer preference be $\hat{\beta}$ when making the ordering decision. With probability $\hat{\beta}$, the consumers' product valuations are $V_h = \theta_h$ and $V_l = \theta_l$, respectively; and with probability $1 - \hat{\beta}$, we have $V_h = V_l = 0$. As a result, the retailer would consider only

[12]The results are unchanged when the retail price is determined before the perceived product fit is revealed.

three possible levels of ordering quantity, i.e., $x \in \{0, \alpha, 1\}$, serving none, the high type, or both consumer types, respectively. In particular, the retailer's expected payoff of ordering $x = \alpha$ is $\pi = \max\{\alpha\hat{\beta}\theta_h - \alpha\omega, 0\}$, and ordering $x = 1$ is $\pi = \max\{\hat{\beta}\theta_l - \omega, 0\}$. Define $\beta_l \equiv \frac{\omega}{\theta_h}$, and $\beta_h \equiv \frac{(1-\alpha)\omega}{\theta_l - \alpha\theta_h}$. We can then characterize the retailer's optimal ordering strategy:

$$x = \begin{cases} 0, & \text{if } \hat{\beta} < \beta_l; \\ \alpha, & \text{if } \beta_l \leq \hat{\beta} < \beta_h; \\ 1, & \text{if otherwise.} \end{cases}$$

The optimal ordering quantity increases with the updated belief $\hat{\beta}$, since the retailer makes a trade-off between serving more consumers when the product turns out to be good and saving on over-ordering when the product is bad. If the product is unlikely to be a good ($\hat{\beta} < \beta_l$), the saving incentive dominates and the retailer would order nothing from the manufacturer. When the perceived likelihood that the product is good is sufficiently high ($\hat{\beta} \geq \beta_h$), the retailer would make an order of size one. In addition, an intermediate belief ($\beta_l \leq \hat{\beta} < \beta_h$) would lead the retailer to stock only an intermediate amount such that only the high-type consumers are served.

3.2. Sequential Signal Acquisition and Posterior Beliefs

We now investigate how sequential information acquisition influences the updating of posterior beliefs, starting from an arbitrary initial belief $\dot{\beta}$. Conditional on a series of n_g good signals and n_b bad signals having been acquired, the posterior belief is updated as follows:

$$\Pr(G|n_g, n_b) = \frac{\Pr(G)\Pr(n_g, n_b|G)}{\Pr(G)\Pr(n_g, n_b|G) + \Pr(B)\Pr(n_g, n_b|B)} = \frac{\dot{\beta}}{\dot{\beta} + (1-\dot{\beta})(\frac{1-\gamma}{\gamma})^{(n_g - n_b)}}.$$

Note that the updated posterior belief is dependent only on the difference between the number of good and bad signals. We can then define the posterior belief, updated from an initial belief $\dot{\beta}$, as a function of the number of accumulated "net good" signals $N \equiv n_g - n_b$:

$$\hat{\beta}(N) \equiv \frac{\dot{\beta}}{\dot{\beta} + (1-\dot{\beta})(\frac{1-\gamma}{\gamma})^N}. \tag{2}$$

To facilitate the analysis, suppose that N is a real number, which is without loss of generality when the number of signals becomes sufficiently large. Given $\gamma \in [\frac{1}{2}, 1]$, it is obvious that $\frac{\partial\hat{\beta}(N)}{\partial N} > 0$. Intuitively, this suggests that the larger the number of good signals relative to bad ones, it is believed that the true state is more likely to be the former. Moreover, we have $\lim_{N \to -\infty} \hat{\beta}(N) = 0$

and $\lim_{N\to+\infty} \hat{\beta}(N) = 1$, which implies that the true state can become almost certainly known if an infinite number of signals are acquired.

Suppose that, starting from an initial belief $\dot{\beta}$, information acquisition will not stop until either $N_H > 0$ or $N_L < 0$ signals are generated. Then conditional on the true state being $S \in \{G, B\}$, what is the probability, formally defined as $\Phi^S(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H)$, of reaching the posterior belief $\hat{\beta}_H \equiv \hat{\beta}(N_H) > \dot{\beta}$ before reaching $\hat{\beta}_L \equiv \hat{\beta}(N_L) < \dot{\beta}$? Similarly, what is the unconditional probability $\Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) \equiv \dot{\beta}\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) + (1 - \dot{\beta})\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H)$? To derive these probabilities, let us define $\Psi^S(N) \equiv \Phi^S(\hat{\beta}(N)|\hat{\beta}_L, \hat{\beta}_H)$ as the conditional transition probability that the upper bound N_H is hit before the lower bound N_L, as a function of the current net good signals $N \in [N_L, N_H]$. Suppose that an additional signal is to be generated. Then conditional on $S = G$, the posterior belief would be updated upward to $\hat{\beta}(N + 1)$ or downward to $\hat{\beta}(N - 1)$, with a probability γ or $1 - \gamma$, respectively. This implies that the conditional transition function $\Psi^G(N)$ is also updated to $\Psi^G(N+1)$ or $\Psi^G(N-1)$ with probability γ or $1-\gamma$, respectively. If the true state is $S = B$ instead, then an additional signal would move the updated posterior belief toward $\hat{\beta}(N + 1)$ or $\hat{\beta}(N - 1)$, leading to the updated conditional transition function $\Psi^B(N + 1)$ or $\Psi^B(N - 1)$, with probability $1 - \gamma$ or γ, respectively. The above discussion suggests that we can derive second-order difference equations for $\Psi^G(N)$ and $\Psi^B(N)$, respectively. These difference equations can be solved using the boundary conditions $\Psi^S(N_L) = 0$ and $\Psi^S(N_H) = 1$, where $S \in \{G, B\}$. Noticing that by definition $\Phi^S(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \Psi^S(0)$, we can obtain:

LEMMA 1: $\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{\hat{\beta}_H(\dot{\beta}-\hat{\beta}_L)}{\dot{\beta}(\hat{\beta}_H-\hat{\beta}_L)}$, $\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{(1-\hat{\beta}_H)(\dot{\beta}-\hat{\beta}_L)}{(1-\dot{\beta})(\hat{\beta}_H-\hat{\beta}_L)}$, *and* $\Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{\dot{\beta}-\hat{\beta}_L}{\hat{\beta}_H-\hat{\beta}_L}$.

This lemma captures several interesting features of the probabilities of arriving at a posterior upper bound $\hat{\beta}_H$ before a lower bound $\hat{\beta}_L$, starting from an initial belief $\dot{\beta}$. First, it can be seen that $\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) > \Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) > \Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H)$, for all $\hat{\beta}_L$, $\hat{\beta}_H$, and $\dot{\beta} \in [\hat{\beta}_L, \hat{\beta}_H]$. Thus the accumulated signals are more likely to update the posterior belief toward the upper bound before the lower bound, when the true state is G than when it is B. Intuitively, the g signals move the posterior belief upward while the b ones move it downward. Note also that these probabilities are proportional to the distance between the initial belief and the lower bound (i.e., $\dot{\beta} - \hat{\beta}_L$) relative to the difference between the upper and the lower bound (i.e., $\hat{\beta}_H - \hat{\beta}_L$). All else being equal, the closer the initial belief is to the upper bound (or the farther from the lower bound), the more likely the upper bound is reached before the lower bound. Moreover, one can readily verify that $\Phi^S(\hat{\beta}_L|\hat{\beta}_L, \hat{\beta}_H) = \Phi(\hat{\beta}_L|\hat{\beta}_L, \hat{\beta}_H) = 0$, and $\Phi^S(\hat{\beta}_H|\hat{\beta}_L, \hat{\beta}_H) = \Phi(\hat{\beta}_H|\hat{\beta}_L, \hat{\beta}_H) = 1$, $S \in \{G, B\}$. This suggests that, irrespective of the true state, a posterior bound would be almost surely approached if it is sufficiently close to the initial belief, before reaching the opposite posterior bound.

3.3. Benchmark Models

Before we begin the main analysis, let us consider two benchmark cases. The first one is a vertically integrated channel in which the manufacturer sells directly to end consumers. The optimal information acquisition strategy is to collect an infinite number of signals that (almost) perfectly reveals the product's fit. When the revealed product fit is good (or when no useful information is available) all consumers are served, whereas if the product fit turns out to be bad the manufacturer will not sell the product. Thus the ex-ante expected profit is $\beta\theta_l$. Consider now how contracts in a decentralized channel can be used to achieve this first-best outcome, which involves i) maximizing the total channel profits; and ii) transfering the profits at the retail level back to the manufacturer. Two types of externalities need to be corrected in order to maximize the total system's profits: The contract must induce the manufacturer to collect full information, and as well remove the vertical (double marginalization) externality. In particular, the first-best outcome can obtain in a decentralized channel through two-part tariff contracts with a fixed up front payment (i.e., $\beta\theta_l$) and variable transfer terms that are contingent on verifiable posterior beliefs: $x(\hat{\beta}) = 1$ if $\hat{\beta} > 0$ and $x(\hat{\beta}) = 0$ if $\hat{\beta} = 0$, and $\omega(\hat{\beta}) = \epsilon$ if $\hat{\beta} = 1$ and $\omega(\hat{\beta}) = 0$ if $\hat{\beta} < 1$, where $\epsilon > 0$ is sufficiently small.

The second benchmark case arises when information sharing in a decentralized channel is either infeasible or not credible. For example, if the message sent by the manufacturer is unverifiable (i.e., cheap talk) or if the manufacturer can select only the good signals for disclosure while withholding all the bad ones, then the retailer will completely discard any information disclosed by the manufacturer. As a result, no information would be acquired by the manufacturer, since information acquisition has no role if it is impossible for the manufacturer to influence the retailer's behavior through information sharing. The retailer will hence maintain the prior belief β. We can then readily obtain that in equilibrium the manufacturer's and the retailer's ex ante profits are, respectively, given by:

$$\Pi^{ns} = \begin{cases} \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}; \\ \alpha\min\left\{d, \beta\theta_h\right\}, & \text{if otherwise.} \end{cases}$$

$$\pi^{ns} = \begin{cases} \beta\theta_l - \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}; \\ \alpha\beta\theta_h - \alpha\min\left\{d, \beta\theta_h\right\}, & \text{if otherwise.} \end{cases}$$

The manufacturer's equilibrium ex ante payoff without information sharing, Π^{ns}, is shown in Figure 2. When the competitive margin is sufficiently small, i.e., $d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$, the manufacturer is induced to charge low wholesale prices such that the whole market is served in equilibrium. However, when the manufacturer's competitive margin is sufficiently large, i.e., $d > \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$, the low-type consumers are not served in equilibrium, resulting in a loss of market coverage.

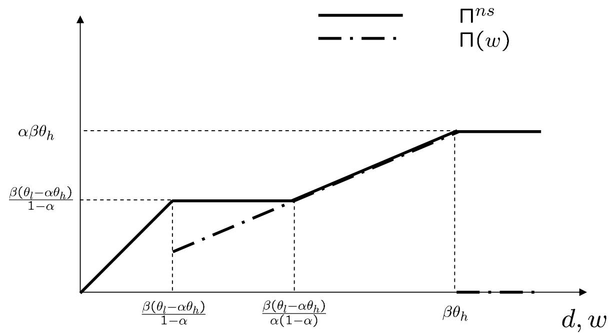

Figure 2: The Manufacturer's Expected Payoffs without Information Sharing

4. Ex Ante Mandatory Information Sharing

In this section we analyze mandatory information sharing in which the manufacturer commits ex ante to share with the retailer all the acquired information. The manufacturer's decision in the third stage of the game hence becomes immaterial, and we can focus on the its second-stage decision about how much information should be acquired conditional on useful information being available (i.e., $I = y$). Note that the firms necessarily share the same posterior belief, which can be denoted as $\hat{\beta}$. In characterizing the manufacturer's optimal information acquisition strategy, we highlight the strategic influence exerted on the retailer's posterior belief and hence on its ordering decision.

4.1. Inflexible Information Acquisition

We start with the scenario when information acquisition is inflexible such that only either none or an infinite number of signals can be generated when $I = y$. The manufacturer's information acquisition decision thus amounts to either maintaining the prior belief β or becoming (almost) fully informed of the true state on consumer preference. Should no information be acquired at all, the retailer's belief will not be updated from the prior. If the manufacturer decides to acquire information, then the retailer's posterior belief is updated to either $\hat{\beta} = 1$ or $\hat{\beta} = 0$, with the ex ante probability β or $1-\beta$, respectively. Given that the belief updating can subsequently influence the retailer's optimal ordering decision, which of these two options is more beneficial for the manufacturer?

Proposition 1: *Under ex ante mandatory information sharing and when information acquisition is inflexible, in equilibrium the manufacturer decides not to acquire information even when the information state is $I = y$. The equilibrium wholesale price and the firms' ex ante payoffs are the same as those in the benchmark without information sharing (i.e., Π^{ns} and π^{ns}).*

Interestingly, when the acquired information is mandatorily shared with the retailer and the acquisition process is inflexible, the manufacturer in equilibrium would not charge wholesale prices under which information acquisition is desirable. Under the equilibrium wholesale price, the manufacturer is strictly better off acquiring no information at all than making the retailer (almost) perfectly informed of consumer preference. Intuitively, providing information to the retailer only leads to more stochastic retail ordering, intensifying the double marginalization and market recession problem when the acquired information indicates bad product fit (i.e., $\hat{\beta} = 0$). Thus, even though the cost of information acquisition is negligible, the manufacturer may not necessarily acquire information if it is constrained to choose between zero and perfect information.

4.2. Sequential (Flexible) Information Acquisition

Let us now investigate the scenario when the manufacturer can decide on the number of signals to acquire in a sequential manner. We start by investigating the manufacturer's optimal stopping rule for information acquisition when $I = y$, and then examine the payoff implications of strategically manipulating the information acquisition process.

4.2.1. Optimal Information Acquisition Strategy

The manufacturer's optimal information acquisition strategy pertains to whether to stop at each updated posterior belief $\hat{\beta} \in (0, 1)$ that is commonly shared by both firms.[13] Recall that the retailer's optimal ordering quantity x increases with $\hat{\beta}$. It is obvious that the manufacturer would not continue information acquisition whenever $\hat{\beta} \geq \beta_h$, or $\hat{\beta} \geq \beta_l$ and $\beta_h > 1$. Conversely, the manufacturer would continue to collect more information whenever $\hat{\beta} < \beta_l$: The manufacturer's payoff would be zero if the retailer's belief remains below β_l, while sampling additional signals may induce the retailer to order $x = \alpha$ if β_l is reached. Similarly, when $\beta_l < \hat{\beta} < \beta_h \leq 1$, information acquisition will not be terminated, because sampling additional signals would not decrease the amount ordered but may induce more ordering if the posterior belief is updated upward to β_h.

What remains to be determined is the information acquisition decision when $\hat{\beta} = \beta_l$ (and $\beta_h \leq 1$). If no additional information is collected, a payoff equal to $\Pi = \alpha\omega$ can be guaranteed. However, collecting additional information would lead to either $\hat{\beta} > \beta_l$ or $\hat{\beta} < \beta_l$, and from the discussion above we know that information acquisition will continue from then on until the posterior belief reaches either β_h or 0, moving the manufacturer's ultimate payoff to either ω or 0, with probability

[13]It is assumed that when $\hat{\beta}$ is arbitrarily close to either 0 or 1, perfect information is (almost) surely obtained and the information acquisition process is hence terminated naturally.

$\Phi(\beta_l|0,\beta_h)$ or $1-\Phi(\beta_l|0,\beta_h)$, respectively. Using Lemma 1, we have $\Phi(\beta_l|0,\beta_h)\omega = \frac{\beta_l}{\beta_h}\omega = \frac{\theta_l-\alpha\theta_h\omega}{(1-\alpha)\theta_h} < \alpha\omega$.[14] Thus, the manufacturer will stop information collection when $\hat{\beta} = \beta_l$.

Proposition 2: *Under ex ante mandatory information sharing and when information acquisition is sequential, the optimal stopping rule of information acquisition when $I = y$ is characterized by two boundary points, $\underline{\beta} \in [0,\beta]$ and $\overline{\beta} \in [\beta,1]$, such that the manufacturer continues to collect information unless the updated posterior belief $\hat{\beta}$ reaches either $\underline{\beta}$ or $\overline{\beta}$. In particular:*

i. If $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, then $\underline{\beta} = \overline{\beta} = \beta$, $\Pi' = \omega$, and $\pi' = \beta\theta_l - \omega$;

ii. If $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, then $\underline{\beta} = \beta_l$, $\overline{\beta} = \beta_h$, $\Pi' = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

iii. If $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, then $\underline{\beta} = \overline{\beta} = \beta$, $\Pi' = \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

iv. If $\omega > \beta\theta_h$, then $\underline{\beta} = 0$, $\overline{\beta} = \beta_l$, $\Pi' = \alpha\beta\theta_h$, and $\pi' = 0$.

This proposition establishes an important result of the paper: The manufacturer will not collect an infinite number of signals to fully resolve the uncertainty on consumer preference, even if the costs of information acquisition (both fixed and marginal) are zero. In other words, the manufacturer in equilibrium exercises self-restriction in information acquisition, which is bounded by two posterior beliefs $\underline{\beta}$ and $\overline{\beta}$, where $|\overline{\beta} - \underline{\beta}| < 1$. The incentive underlying this self-restriction is the strategic influence exerted on the retailer's behavior. Under mandatory sharing, the only way the manufacturer can manipulate the retailer's belief is through controlling the acquisition of information. Nevertheless, collecting more information is a "double-edged sword," moving the posterior belief either upward or downward. As a result, to control the generation of information to its own advantage, the manufacturer would stop the information acquisition process if the updated posterior belief is already sufficiently high or if the expected payoff improvement from obtaining good signals cannot compensate for the potential loss when adverse signals are generated.[15]

Two points are warranted in interpreting the manufacturer's equilibrium self-restriction in information generation. First, under mandatory sharing, the manufacturer does not enjoy any informational advantage over the retailer, i.e., any incompleteness in information is symmetric across the

[14] This is because, under the market condition our analysis concentrates on (i.e., $\theta_l < \alpha(2-\alpha)\theta_h$), a decentralized channel will not find it sufficiently profitable to serve the low-type consumers without any information. If instead $\theta_l \geq \alpha(2-\alpha)\theta_h$, the manufacturer would continue information acquisition when $\hat{\beta} = \beta_l$.

[15] This does not necessarily mean that the signals ultimately generated include on average more good ones than bad ones, or more generally that the empirical sample is biased upward. For example, when β is sufficiently close to β_l from above, it is almost the case that only bad signals would be observed towards the end of information collection. In this case, it is optimal to stop when the updated posterior belief hits β_l.

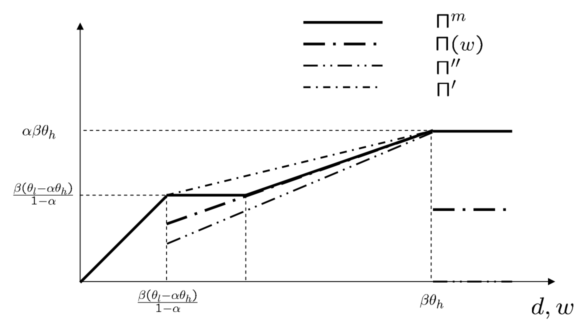

Figure 3: The Manufacturer's Expected Payoffs under Mandatory Sharing ($\beta < \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$)

firms. As a result, the discontinuation of information acquisition and transmission cannot play any signaling role. Second, because information disclosure is truthful, the retailer would always believe in the received information, even though it is known that the acquisition of information has been strategically manipulated by the manufacturer. Therefore, the retailer would not, even if it could, commit ex ante not to accept and act on the ex post shared information.

Nevertheless, in comparison to inflexible information acquisition under which no information is acquired in equilibrium at all, the manufacturer acquires more information when information acquisition can be sequentially controlled. Thus, paradoxically, it is only when the manufacturer can sequentially control the amount of information acquisition that there will be a positive amount of information acquired. In other words, the flexibility in the manufacturer's information acquisition leads to a larger amount of information collection in equilibrium.

One may expect that the manufacturer can derive economic rents from sequentially controlling information acquisition. From Proposition 1(ii), for instance, it is obvious that when $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, the manufacturer's expected payoff under sequential information acquisition when $I = y$ (i.e., Π') is indeed higher than that when $I = \bar{y}$ (i.e., $\Pi'' = \alpha\omega$). Note that when the wholesale price is within this range, the retailer would order only $x = \alpha$ if it maintains its prior belief β. However, with the strategic control of information flow, the manufacturer can induce the retailer to order $x = 1$ when the updated posterior belief reaches the upper bound β_h but never order below $x = \alpha$ even when the lower bound β_l is hit. This is demonstrated in Figure 3.

Interestingly, this strategic effect of sequential information acquisition works to the advantage of the manufacturer without necessarily hurting the retailer. Indeed, conditional on any wholesale price, the retailer's expected payoff remains the same as in the benchmark without information sharing. Again, this is because the information transmitted to the retailer, although strategically manipulated by the manufacturer, is truthful. This suggests that, with the manufacturer's sequential control of information acquisition, there can be a Pareto improvement in the channel members' expected payoffs. Essentially, the demand recession problem of being unable to fully cover the whole market can be mitigated and the retailer's expected ordering quantity can be higher, when the retailer's updated posterior belief is strategically boosted.

4.2.2. Equilibrium Ex Ante Profits

We now characterize the optimal wholesale price and derive the firms' equilibrium ex ante profits.

PROPOSITION 3: *Under ex ante mandatory information sharing and when information acquisition is sequential: (i) If β is sufficiently large (i.e., $\beta > \tilde{\beta}$), in equilibrium the manufacturer does not acquire any information even when $I = y$, where $\tilde{\beta} \in \left(\frac{\theta_l - \alpha\theta_h}{(1-\alpha)\theta_h}, 1\right)$ is given in the Appendix. The manufacturer's equilibrium ex ante payoff is strictly higher than that without information sharing (i.e., Π^{ns}), if and only if β is sufficiently small and d is intermediate; (ii) The retailer's equilibrium ex ante payoff can be strictly higher than that without information sharing (i.e., π^{ns}), if both β and d are intermediate.*

The first point of this proposition is that sequential information acquisition may not strictly improve the manufacturer's ex ante payoff when the prior belief is sufficiently high. This has to do with the uncertainty about the availability of useful information at the time when the wholesale price is determined. Recall that the manufacturer can benefit from sequential information acquisition (i.e., $\Pi' > \Pi''$) when the charged wholesale price is such that the retailer would order $x = \alpha$ in the absence of information sharing (i.e., $\frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega < \beta\theta_h$). When the prior belief is already high, the extent to which the retailer's belief can be manipulated upward is limited, and this constrains the potential benefit of sequential information acquisition. However, with one half probability, no information can be acquired (i.e., $I = \bar{y}$), in which case the manufacturer would have been better off had it charged a lower wholesale price (i.e., $\omega = \frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha}$) under which the whole market is served. As a result, when β is sufficiently large, the equilibrium wholesale price is such that it is optimal for the manufacturer to acquire no information even when $I = y$.

It is only when the prior belief β is sufficiently low that the manufacturer would in equilibrium acquire information and thus benefit from sequentially controlling information acquisition. In this case, the difference in the manufacturer's equilibrium ex ante profits between sequential and inflexible information acquisition, has an inverted-U relationship with the manufacturer's competitive margin d. In other words, the equilibrium ex ante benefit from manipulating the retailer's belief through sequential information acquisition reaches its peak when d is intermediate. On the one hand, the manufacturer can charge a higher wholesale price as d increases, magnifying the benefit of sequential information acquisition in that on expectation there is greater coverage of consumer segments. While on the other hand, both β_l and β_h increase with a higher wholesale price, leading to a lower probability that the retailer's posterior belief reaches β_h before β_l (i.e., a lower likelihood that both consumer segments are covered in equilibrium). Consequently, the effect of d on the difference between the manufacturer's equilibrium ex ante profits is non-monotonic.

Interestingly, the retailer can ex ante benefit from the manufacturer's sequential information acquisition. This occurs when the prior belief β is neither too large such that information acquisition can arise in equilibrium, nor too small such that there exist a set of wholesale prices (i.e., $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$) under which the retailer orders $x = \alpha$ but never $x = 1$. If instead $\omega = \frac{\theta_l-\alpha\theta_h}{1-\alpha}$ is charged, the manufacturer would acquire information to potentially induce the retailer to order more quantity, leading to a discontinuous increase in the manufacturer's expected payoff from that when $\omega = \frac{\theta_l-\alpha\theta_h}{1-\alpha} + \epsilon$ (i.e., a sufficiently small increase in the wholesale price). Therefore, if the competitive margin d is sufficiently close to the cut-off point $\frac{\theta_l-\alpha\theta_h}{(1-\alpha)}$ from above, the equilibrium wholesale price will be $\omega = \frac{\theta_l-\alpha\theta_h}{(1-\alpha)} < d$. In contrast, under inflexible information acquisition, the optimal wholesale price would be $\omega = d$. This means that the prospect of sequentially manipulating information collection may induce the manufacturer to charge a lower equilibrium wholesale price. As a result, both firms can be strictly better off when information flow is strategically controlled by the manufacturer, i.e., a Pareto improving "win-win" situation.

5. Ex Post Voluntary Information Sharing

In this section we look at voluntary information sharing where the manufacturer can decide whether to share information on an ex post basis after the information has been acquired. Thus, in contrast to mandatory sharing, the retailer's updated posterior belief, $\hat{\beta}_r$, may not coincide with that of the manufacturer, $\hat{\beta}_m$, because the manufacturer can choose not to disclose the acquired signals. Formally, define the manufacturer's information disclosure strategy at stage 3 as $m(\hat{\beta}_m) : \hat{\beta}_m \to M \equiv \{\hat{\beta}_m, \otimes\}$, where M is the manufacturer's feasible set of messages conditional on $\hat{\beta}_m$, and $\otimes$ represents "no disclosure." Note that the manufacturer's updated posterior belief $\hat{\beta}_m$ at the time of making the disclosure decision is determined by its information acquisition strategy at stage 2.

Moreover, we need to characterize the updating of the retailer's posterior belief, $\hat{\beta}_r(m)$, in response to the received message $m \in M$. Note first that $\hat{\beta}_r(\hat{\beta}_m) = \hat{\beta}_m$. In addition, the message $m = \otimes$ can be received either when no useful information is available (i.e., $I = \bar{y}$), or when $I = y$ but the manufacturer withholds its acquired information. This implies that the updating of the retailer's posterior belief $\hat{\beta}_r(\otimes)$ and the manufacturer's optimal information acquisition and disclosure strategies may influence each other, which therefore need to be derived together.

5.1. Inflexible Information Acquisition

We start with deriving the manufacturer's optimal information sharing strategy. Suppose that the manufacturer chooses to acquire information when $I = y$ and is (almost) fully informed. When it is revealed to the manufacturer that $S = G$, it is in its best interest to disclose the information and move the retailer's updated posterior belief toward $\hat{\beta}_r = \hat{\beta}_m = 1$. Conversely, when the manufacturer learns that $S = B$, it would choose not to disclose the information. This implies that the manufacturer's optimal information sharing strategy is: $m(1) = 1$, and $m(0) = \otimes$. Therefore, the message $m = \otimes$ would be sent by the manufacturer either when $I = \bar{y}$, or when $I = y$ and the manufacturer conceals its updated posterior belief $\hat{\beta}_m = 0$. The ex ante probabilities for these two scenarios are $1/2$ and $(1-\beta)/2$, respectively. We can use the Bayes theorem to obtain the retailer's updated belief as $\hat{\beta}_r(\otimes) = \frac{\beta}{2-\beta}$.

We can then determine whether the manufacturer should acquire information when useful information is available (i.e., $I = y$), and derive the manufacturer's equilibrium ex ante payoff.

Proposition 4: *Under ex post voluntary information sharing and when information acquisition is inflexible, in equilibrium the manufacturer decides to acquire information when the information state is $I = y$. The manufacturer's equilibrium ex ante payoff is higher than that without information sharing (i.e., Π^{ns}) if and only if $\frac{4\theta_l - 2\alpha(3-\alpha)\theta_h}{2\theta_l + [1-\alpha(4-\alpha)]\theta_h} < \beta < \frac{2\alpha}{1+\alpha}$ and $\frac{2\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]} < d < \min\left\{\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}, \frac{[\alpha(2-\beta)+\beta](\theta_l - \alpha\theta_h)}{2\alpha(1-\alpha)}\right\}$, and (weakly) lower if otherwise.*

Two interesting results pertaining to the effects of the flexibility in information sharing emerge from this proposition. First, in contrast to the mandatory information sharing case, acquiring information is the equilibrium strategy under voluntary sharing when information acquisition is inflexible. This implies that the manufacturer would acquire information when it is not bound to disclose all the acquired information. Intuitively, when information sharing is mandatory, the manufacturer is committed to disclose unfavorable information, which may be undesirable from an ex ante perspective. However, when information sharing is voluntary, the manufacturer can withhold ex post unfavorable information but disclose only favorable information. It is this ex post flexibility that induces the manufacturer to acquire (perfect) information.

Second, the manufacturer may or may not benefit from the flexibility to selectively disclose information. This is driven by the two counteracting effects exerted by voluntary disclosure that, in contrast to mandatory sharing, results in equilibrium (perfect) information collection. On the one hand, acquiring information can lead to a positive payoff effect when the acquired information is favorable such that the consumer segments are more likely to be served than when no information is acquired, i.e., $\hat{\beta}_r(1) = 1 > \beta$. On the other hand, the low-segment consumers may be unserved when the acquired information conveys negative news, while may be otherwise served when the prior belief is maintained by the retailer under zero information acquisition, i.e., $\hat{\beta}_r(\otimes) < \beta$. As a result, there may be a market recession effect of information acquisition. Note that this market recession effect may arise even in the scenario when no useful information is available (i.e., $I = \bar{y}$). This is because the retailer cannot distinguish between the scenarios when useful information is unavailable and when unfavorable information is strategically withheld.

The net effect of the flexibility in information sharing on the manufacturer's ex ante payoff hence hinges on whether on average higher equilibrium market coverage is induced. Intuitively, voluntary disclosure is beneficial if and only if the equilibrium market coverage is increased when the acquired information is favorable but not lowered when the information is unfavorable. This requires that the prior belief is in an intermediate range.[16] On the one hand, β cannot be too small, because both the probability that favorable information would be acquired (i.e., $\beta/2$), and the retailer's posterior belief when no useful information is available or when unfavorable information is acquired (i.e., $\hat{\beta}_r(\otimes)$), increase with β. On the other hand, the prior belief cannot be too large either, because the potential benefit from inducing higher market coverage would become less important as β approaches $\hat{\beta}_r(1) = 1$. Moreover, for voluntary sharing to be beneficial, the competitive margin d should take intermediate values as well. When d is too small, the manufacturer is constrained to set low wholesale prices such that both consumer segments are served in equilibrium even under mandatory sharing. Conversely, when d is sufficiently large, excessively high wholesale prices would be charged in equilibrium, such that no consumer would be served when the retailer's updated posterior belief is $\hat{\beta}_r(\otimes)$ and/or only the high segment would be covered even when $\hat{\beta}_r = 1$.

5.2. Sequential (Flexible) Information Acquisition

5.2.1. Optimal Information Acquisition and Sharing Strategies

When useful information is available, the manufacturer sequentially decides whether to terminate the information acquisition process at each updated posterior belief $\hat{\beta}_m$, and then determines whether to share the acquired information with the retailer. Note that at the time when the shar-

[16] Nevertheless, when the low-segment consumers become sufficiently unimportant (i.e, $4\theta_l - 2\alpha(3-\alpha)\theta_h < 0$) such that market recession is less of a concern, voluntary sharing can be beneficial even when β goes to zero.

ing decision is made, the manufacturer's updated posterior belief is given by $\hat{\beta}_m \in \{\underline{\beta}, \overline{\beta}\}$, where $\underline{\beta} \in [0, \beta]$ and $\overline{\beta} \in [\beta, 1]$ are the boundary points characterizing the optimal stopping rule for information collection. This implies that the manufacturer's optimal disclosure strategy is influenced by its stopping rule for information acquisition, which is in turn affected by whether the manufacturer would choose to disclose the collected information.

As in the mandatory sharing case in Section 4.2, the manufacturer will not acquire more information whenever $\hat{\beta}_m \geq \beta_h$, or $\hat{\beta}_m \geq \beta_l$ and $\beta_h > 1$, and will continue the information acquisition process whenever $\hat{\beta}_m < \beta_l$ or $\beta_l < \hat{\beta}_m < \beta_h \leq 1$, irrespective of the retailer's updated posterior belief $\hat{\beta}_r(\otimes)$. In contrast, the manufacturer would prefer to collect more information when $\hat{\beta}_m$ reaches β_l, if and only if $\hat{\beta}_r(\otimes) \geq \beta_l$ and $\beta_h \leq 1$. This is because now the manufacturer can strategically control the disclosure of information to prevent the reduction in retail ordering even when unfavorable information is generated (i.e., $\hat{\beta}_m = 0$).

We can then determine the manufacturer's equilibrium information acquisition and sharing decisions and the retailer's equilibrium posterior belief $\hat{\beta}_r(\otimes)$. First, it is straightforward that when $\beta \geq \beta_h$ (or $\beta \geq \beta_l$ and $\beta_h > 1$), in equilibrium the manufacturer will not start the information collection process and thus $\hat{\beta}_r(\otimes) = \beta$. Next, consider the equilibrium whereby the manufacturer continues to acquire information until either β_h or 0 is reached. For this to be an equilibrium, the necessary and sufficient condition is $\hat{\beta}_r(\otimes) \geq \beta_l$ and $\beta < \beta_h \leq 1$. Note also that the manufacturer would disclose the acquired information when $\hat{\beta}_m = \beta_h$ is reached and withhold the information when $\hat{\beta}_m = 0$ is reached. Therefore, the message $m = \otimes$ would be sent by the manufacturer when and only when: 1) the information state is $I = \bar{y}$; or 2) the information state is $I = y$ and the manufacturer's updated belief reaches $\hat{\beta}_m = 0$. Noticing that the ex ante probabilities for these two scenarios are 1/2 and $[1 - \Phi(\beta|0, \beta_h)]/2 = (1 - \beta/\beta_h)/2$, respectively, we can obtain:

$$\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + (1 - \beta/\beta_h)/2} = \frac{(1-\alpha)\beta\omega}{2(1-\alpha)\omega - \beta(\theta_l - \alpha\theta_h)}, \tag{3}$$

which sustains $\hat{\beta}_r(\otimes) \geq \beta_l$ if and only if $\beta \geq \frac{2(1-\alpha)\omega}{\theta_l + (1-2\alpha)\theta_h}$.

Consider then the equilibrium whereby information collection is terminated at either $\hat{\beta}_m = \beta_l$ or $\hat{\beta}_m = 0$, which is to be respectively disclosed or concealed. Given the manufacturer's strategies, note that we have $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + (1-\beta/\beta_l)/2} = \frac{\beta\omega}{2\omega - \beta\theta_h}$, which is less than β_l if and only if $\beta < \beta_l$. This implies that this equilibrium would arise if and only if $\beta < \beta_l$. Moreover, unlike the mandatory sharing case, there does not exist an equilibrium whereby the manufacturer continues to acquire information until either β_h or β_l is reached. If otherwise, it must be the case that $\beta_l \leq \beta < \beta_h \leq 1$, which in turn implies $\hat{\beta}_r(\otimes) \geq \beta_l$. But then it is better off for the manufacturer to continue information acquisition when arriving at $\hat{\beta}_m = \beta_l$.

It follows that there does not exist a pure-strategy equilibrium when $\beta_l \leq \beta < \frac{2(1-\alpha)\omega}{\theta_l+(1-2\alpha)\theta_h}$ (and $\beta_h \leq 1$). In the mixed-strategy equilibrium, it must be that: 1) the manufacturer is indifferent and randomizes between continuing and stopping information acquisition when its updated belief reaches β_l; and 2) the retailer is indifferent and randomizes between ordering α and 0 when it receives no information from the manufacturer. Let us define the probability as λ that the manufacturer continues to accumulate signals when $\hat{\beta}_m = \beta_l$, and the probability as ρ (or $1-\rho$) that the retailer orders $x = \alpha$ (or $x = 0$) when $m = \otimes$. The mixed-strategy equilibrium requires:

$$\alpha\omega = \Phi(\beta_l|0, \beta_h)\omega + [1 - \Phi(\beta_l|0, \beta_h)]\rho\alpha\omega, \tag{4}$$

$$\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + \lambda(1-\beta/\beta_h)/2} = \beta_l, \tag{5}$$

which lead to $\rho = \frac{\alpha\beta_h-\beta_l}{\alpha(\beta_h-\beta_l)} = \frac{\alpha(2-\alpha)\theta_h-\theta_l}{\alpha(\theta_h-\theta_l)} \in (0,1)$, and $\lambda = \frac{\beta_h(\beta-\beta_l)}{\beta_l(\beta_h-\beta)} = \frac{(1-\alpha)(\beta\theta_h-\omega)}{(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)} \in [0,1)$, respectively. Summarizing the above discussion, we obtain the following proposition:

Proposition 5: *Under ex post voluntary information sharing and when information acquisition is sequential, the optimal stopping rule of information acquisition when $I = y$ is characterized by two boundary points, $\underline{\beta} \in [0, \beta]$ and $\overline{\beta} \in [\beta, 1]$, such that the manufacturer continues to collect information unless the updated posterior belief $\hat{\beta}_m$ reaches either $\underline{\beta}$ or $\overline{\beta}$; the optimal disclosure strategy is characterized by $m(\underline{\beta}) \in \{\underline{\beta}, \otimes\}$ and $m(\overline{\beta}) = \overline{\beta}$, when the manufacturer stops information acquisition at $\underline{\beta}$ or $\overline{\beta}$, respectively; and the retailer's updated belief is given by $\hat{\beta}_r(m) = m$ if $m \neq \otimes$, and by $\hat{\beta}_r(\otimes)$ if $m = \otimes$. In particular:*

i. If $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, then $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \omega$, and $\pi' = \beta\theta_l - \omega$;

ii. If $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, then $\underline{\beta} = 0$, $\overline{\beta} = \beta_h$, $m(\underline{\beta}) = \otimes$, $\hat{\beta}_r(\otimes) = \frac{(1-\alpha)\beta\omega}{2(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)}$, $\Pi' = \beta(\theta_l - \alpha\theta_h) + \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

iii. If $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, then $\underline{\beta} = 0$ or $\underline{\beta} = \beta_l$ with probability $\lambda = \frac{\beta_h(\beta-\beta_l)}{\beta_l(\beta_h-\beta)}$ or $1-\lambda$ respectively, $\overline{\beta} = \beta_h$, $m(0) = \otimes$, $m(\beta_l) = \hat{\beta}_r(\otimes) = \beta_l$, $\Pi' = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$, and $\pi' = \frac{[\alpha^2\theta_h+(1-2\alpha)\theta_l](\beta\theta_h-\omega)}{\theta_h-\theta_l}$;

iv. If $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, then $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

v. If $\omega > \beta\theta_h$, then $\underline{\beta} = 0$, $\overline{\beta} = \beta_l$, $m(\underline{\beta}) = \otimes$, $\hat{\beta}_r(\otimes) = \frac{\beta\omega}{2\omega-\beta\theta_h}$, $\Pi' = \alpha\beta\theta_h$, and $\pi' = 0$.

In comparison to mandatory sharing (i.e., Proposition 2), voluntary sharing leads the manufacturer to generate a larger amount of information.[17] This implies that, similar to the cases when information acquisition is inflexible (i.e., Sections 4.1 and 5.1), the manufacturer's flexibility to selectively disclose its acquired information can promote more information acquisition. When $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, for example, the lower bound at which the manufacturer stops collecting information is $\underline{\beta} = \beta_l$ under mandatory sharing, but can be extended to $\underline{\beta} = 0$ if information sharing is voluntary. This is because, if information collection continues when the updated posterior belief is at $\hat{\beta}_m = \beta_l$, the acquired unfavorable information (i.e., $\hat{\beta}_m = 0$) can be strategically withheld and therefore does not necessarily induce less retail ordering.

Interestingly, there may exist a mixed-strategy equilibrium (i.e., part *iii* of Proposition 5) where the manufacturer randomizes the lower bound of stopping information acquisition between $\underline{\beta} = \beta_l$ and $\underline{\beta} = 0$. This mixed-strategy equilibrium arises because the retailer's belief when it does not receive any information from the manufacturer, $\hat{\beta}_r(\otimes)$, is positively influenced by the equilibrium lower bound of information acquisition. If the lower bound were placed at β_l for sure, the retailer's updated posterior belief $\hat{\beta}_r(\otimes)$ would be above β_l. But then it is better off for the manufacturer to deviate and (secretly) decrease the lower bound of information acquisition. On the other hand, if the lower bound were placed at zero for sure, the retailer would update its posterior belief $\hat{\beta}_r(\otimes)$ sufficiently downward and below β_l, and thus would order zero amount for sure if no information is received from the manufacturer. However, this discourages the manufacturer from extending the lower bound of information acquisition toward $\underline{\beta} = 0$.

Nevertheless, the flexibility to sequentially determine how much information to acquire does not necessarily promote the amount of information generated in equilibrium. Whether it does so depends on the format of information sharing. Looking first at voluntary sharing, recall from Proposition 4 that when information acquisition is inflexible and the manufacturer is constrained to acquire either none or perfect information, acquiring information is the dominant strategy for the manufacturer. Thus, the equilibrium amount of information generated decreases from perfect information when information acquisition is inflexible to less than perfect information when the manufacturer can sequentially decide how much information to acquire. This result stands in contrast to the case of mandatory sharing, where the equilibrium amount of information generated increases from zero when information acquisition is inflexible (i.e., Proposition 1) to a positive amount when the manufacturer can sequentially decide how much information to acquire (i.e., Proposition 2). Overall, our analysis suggests that the equilibrium amount of information acquired

[17] This can be seen by comparing the amount of information acquisition in parts *ii* and *iii* of Proposition 5 under voluntary sharing to that in part *ii* of Proposition 2 under mandatory sharing. In addition, the respective parameter ranges (on ω) and the amount of information acquisition in the other parts are identical in both propositions.

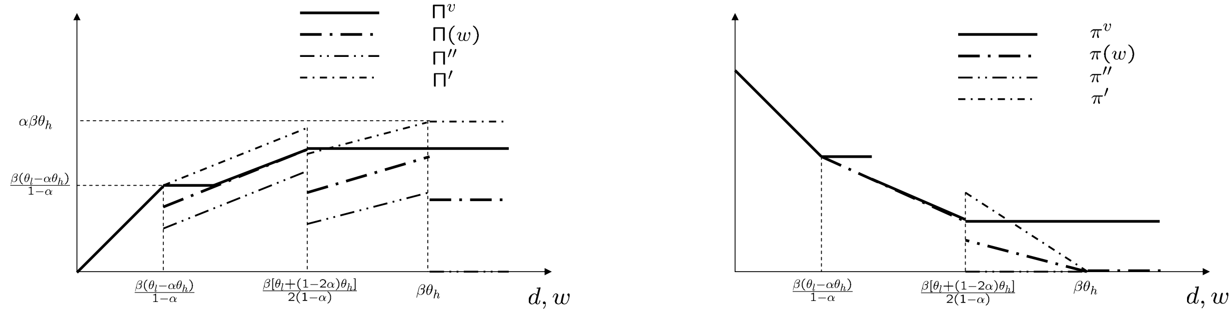

Figure 4: The Firms' Expected Payoffs under Voluntary Sharing ($\beta < \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$)

depends on the interaction between the flexibility in information acquisition (sequential versus inflexible) and the flexibility in information sharing (voluntary versus mandatory).

5.2.2. Equilibrium Ex Ante Profits

The firms' equilibrium ex ante payoffs under voluntary sharing are shown in Figure 4 for the case when $\beta < \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$. We will focus on the comparison with the mandatory sharing case in Section 4.2.

PROPOSITION 6: *Under ex post voluntary information sharing and when information acquisition is sequential: (i) If both β and d are intermediate, the manufacturer's equilibrium ex ante payoff is higher than that under mandatory information sharing; (ii) If β is sufficiently small, the manufacturer's (retailer's) equilibrium ex ante payoff can be lower (higher) than that under mandatory information sharing.*

This proposition indicates that, when information acquisition is sequential, voluntary sharing can benefit the manufacturer in comparison to mandatory sharing only if the prior belief β is intermediate, but can hurt the manufacturer if the prior belief is sufficiently low. To understand this result, note that the flexibility under voluntary sharing can induce the manufacturer to collect more (unfavorable) information, extending the lower bound of information acquisition to $\underline{\beta} = 0$. This increase in information generation has two offsetting effects on the manufacturer's ex ante payoff. On the one hand, it escalates the likelihood that the upper bound of information acquisition is reached in which case the retailer would order $x = 1$, i.e., $\Phi(\beta|\underline{\beta}, \overline{\beta}) = \frac{\overline{\beta}-\beta}{\overline{\beta}-\underline{\beta}}$ decreases with the lower bound $\underline{\beta}$. This implies an increase in the manufacturer's expected payoff Π' when the information state is $I = y$. On the other hand, collecting more unfavorable information may drive down the retailer's updated posterior belief when no information is disclosed by the manufacturer. That is,

$\hat{\beta}_r(\otimes)$ is lower when the lower bound is extended to $\underline{\beta} = 0$. This in turn has a negative effect on the retailer's ordering quantity and thus on Π'' when the information state is $I = \bar{y}$.

Therefore, whether voluntary sharing leads to a higher equilibrium ex ante payoff for the manufacturer hinges on whether the retailer is not induced to order less when its updated posterior belief is given by $\hat{\beta}_r(\otimes)$. Note that $\hat{\beta}_r(\otimes)$ is positively related to the prior belief β. As a result, when β is not sufficiently small, the increase in the acquired information under voluntary sharing does not lead to lower retail ordering since the mixed-strategy equilibrium will not arise. However, when the prior belief β becomes sufficiently large, the maximum ordering of one unit would be induced in equilibrium even under mandatory sharing. Thus, it is only when β is in an intermediate range that the manufacturer is better off under voluntary sharing.

In contrast, when β is sufficiently small, the mixed-strategy equilibrium will occur in which case the retailer would order less, hurting the manufacturer's equilibrium ex ante profit. As the proof shows, this is especially the case when the low-segment consumers' valuation θ_l is relatively high. Moreover, when this effect becomes sufficiently severe, the manufacturer in equilibrium charges a lower wholesale price than its competitive margin d in order to cope with this demand recession problem. However, in the corresponding scenario under mandatory sharing, the manufacturer would in equilibrium charge $\omega = d$. This explains why there may exist equilibrium conditions under which the manufacturer is worse off, while the retailer is better off, when the former can selectively disclose its acquired information, i.e., a "lose-win" situation.

In summary, the economic forces underlying the effect of the flexibility in information sharing on the manufacturer's equilibrium ex ante profits are similar to those when information acquisition is inflexible (i.e., Section 5.1): Sharing information voluntarily with the retailer leads to an increasing incentive for the manufacturer to acquire information, but generating more information is a "double-edged sword" that can in equilibrium improve the manufacturer's ex ante payoff when and only when on average more retail ordering is induced.

6. Extensions and Robustness of Results

6.1. Flexible Wholesale Price

In the base model the choice of the wholesale price precedes the manufacturer's information acquisition and hence does not respond to the acquired information. Our intention in assuming this timing was to rule out the efficiency effect of information acquisition on the manufacturer's decision on wholesale price, and rather to focus on the strategic effect of information acquisition on retailer behavior. We now consider an alternative model timing whereby the wholesale price is decided in the third stage of the game.[18] Other assumptions in the base model are maintained.

We start with deriving the optimal wholesale price, conditional on d and the retailer's updated posterior belief $\hat{\beta}_r$ following the manufacturer's information acquisition/disclosure decisions. Given the retailer's optimal behavior as laid out in Section 3.1, it is straightforward that the optimal wholesale price is $\omega = \min\left\{d, \frac{\hat{\beta}_r(\theta_l - \alpha\theta_h)}{1-\alpha}\right\}$ if $d \leq \frac{\hat{\beta}_r(\theta_l - \alpha\theta_h)}{\alpha(1-\alpha)}$, and $\omega = \min\left\{d, \hat{\beta}_r\theta_h\right\}$ if otherwise. As a result, the manufacturer's equilibrium sub-game payoff in Stage 3 is given by:

$$\Pi(\hat{\beta}_r) = \begin{cases} \alpha \min\left\{d, \hat{\beta}_r\theta_h\right\}, & \text{if } \hat{\beta}_r \leq \frac{\alpha(1-\alpha)d}{\theta_l - \alpha\theta_h}; \\ \min\left\{d, \frac{\hat{\beta}_r(\theta_l - \alpha\theta_h)}{1-\alpha}\right\}, & \text{if otherwise.} \end{cases} \tag{6}$$

Define $\beta_L \equiv \frac{d}{\theta_h}$ and $\beta_H \equiv \frac{(1-\alpha)d}{\theta_l - \alpha\theta_h}$. We concentrate on the interesting scenario $\beta < \beta_H \leq 1$: The manufacturer will surely stop information acquisition when its (prior or updated) belief is not lower than β_H. We provide the full characterization of the manufacturer's equilibrium information acquisition/sharing strategies in the Supplementary Appendix, and show that the key results are qualitatively similar to those in the base model. In particular, when information acquisition is inflexible, the manufacturer decides to (not to) acquire information if sharing is voluntary (mandatory). In addition, when information acquisition is sequential and sharing is mandatory, the manufacturer continues to collect information until the firms' (symmetric) posterior belief reaches β_H, β_L, or zero. In contrast, when information acquisition is sequential and sharing is voluntary, the manufacturer may continue information collection even when its posterior belief arrives at β_L. This is because the manufacturer can ex post withhold unfavorable information, leading to an increasing ex ante incentive for information acquisition. Indeed, this information withholding effect is stronger here than in the base model, because the concern about acquiring bad signals can be mitigated when the wholesale price is adjusted in response to the acquired information. Therefore, flexible wholesale prices can in equilibrium result in more information acquisition than when the wholesale prices are chosen before the information acquisition.

6.2. Retailer Information Acquisition

In the base model the retailer does not acquire any information but relies on the disclosure by the manufacturer. However, in many real market situations, the retailer may also acquire useful

[18] It does not matter whether information sharing and the wholesale price are determined simultaneously or sequentially in the third stage. Nevertheless, to facilitate exposition, the analysis will proceed as if the wholesale price is set after the information disclosure decision is made.

information about demand uncertainty. As indicated in Gal-Or et al (2008), this may be particularly true in the case of ex ante data pooling arrangements such as Collaborative Planning, Planning and Forecasting. Consider now the case when the retailer also has the ability to acquire information and be informed of the perceived product fit at an acquisition cost c. Let this acquisition cost take two possible values, i.e., $c \in \{c_h, c_l\}$, where c_h is prohibitively high and c_l is negligible. The probability for c_l and c_h are δ and $1-\delta$, respectively, where $\delta \in (0,1)$. The retailer's information acquisition cost (and thus whether the retailer is informed) is unknown to the manufacturer. For example, retailers can analyze sales data they possess or collect local market information to improve their knowledge about consumer preference, where c can represent the retailer's opportunity cost to process and analyze sales data, to conduct market research, or to transmit the derived information to decision makers in the organization (e.g., procurement manager).[19] Under this extension, it is straightforward to show that the manufacturer's equilibrium information acquisition/sharing strategies remain the same as in the base model. Nevertheless, the ex ante probability that the retailer's behavior can be strategically manipulated by the manufacturer is reduced when δ becomes higher. As a result, the manufacturer's economic rents derived from sequentially controlling the information acquisition process are reduced as δ increases, but do not collapse to zero as long as the retailer's information acquisition is imperfect (i.e., $\delta < 1$).

6.3. Continuous Demand

The two-segment assumption in the base model results in a discrete demand function. Consider now an alternative setup in which aggregate consumer demand is continuous and given by:

$$D(p) = a - bp, \tag{7}$$

where $a > 0$ denotes market potential, p is the retailer price, and $b > 0$ captures price sensitivity. Akin to the base model, market potential is uncertain and can take two possible values, $a \in \{a_h, a_l\}$.[20] The firms have common prior belief about the realization of market potential: $\Pr(a = a_h) = \beta$ and $\Pr(a = a_l) = 1 - \beta$, where $\beta \in (0,1)$. The manufacturer can acquire (imperfect) signals to resolve the uncertainty about market potential, in the same manner as in the base model. To facilitate exposition without sacrificing conceptual gist, we focus on the scenario when $0 < a_h - a_l < a_l$ and b is sufficiently low relative to a.

[19] Alternatively, we can assume zero retailer acquisition cost and interpret δ as the degree of informativeness of the retailer's data.

[20] The case when firm uncertainty is about the price sensitivity parameter b produces similar results.

As we show in the Supplementary Appendix, the main insights in the base model also hold for this alternative demand setup, irrespective of whether the wholesale price is set before or after information acquisition. The manufacturer may not acquire perfect information even though doing so is costless, when it can collect information in a sequential manner. Moreover, the flexibility in information acquisition can result in a larger (smaller) amount of information generated when sharing is mandatory (voluntary). In contrast, the flexibility in information sharing can unambiguously induce the manufacturer to acquire more information.

More generally, consider retail demand $D(p, Q)$ that is positively influenced by the uncertain product fit Q. Let $\hat{\beta}$ be the firms' (posterior) belief that the product fits consumer preference. Suppose the retailer's optimal ordering decision in the fourth stage of the game yields $x(\hat{\beta})$, which is an increasing, continuous, and twice-differentiable function of $\hat{\beta}$. The other assumptions are the same as in the base model. We can then readily characterize the manufacturer's optimal information acquisition and sharing strategies, which are qualitatively similar to the base model. For example, suppose that $x(\cdot)$ is S-shaped: $x(\cdot)$ is convex when $\hat{\beta} \in [0, \hat{\beta}']$ and concave when $\hat{\beta} \in [\hat{\beta}', 1]$. Under sequential information acquisition and mandatory sharing, the manufacturer will continue to collect information if and only if the posterior belief reaches either zero or an upper bound $\overline{\beta}$ that is determined by $\frac{\partial x(\overline{\beta})}{\partial \overline{\beta}} = \frac{x(\overline{\beta}) - x(0)}{\overline{\beta}}$, where $\overline{\beta} > \hat{\beta}'$. The case when acquisition is sequential and sharing is voluntary is similar, except that now $\overline{\beta}$ is determined by solving $\frac{\partial x(\overline{\beta})}{\partial \overline{\beta}} = \frac{x(\overline{\beta}) - x(\hat{\beta}_r(\otimes))}{\overline{\beta}}$, where $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + (1-\beta/\overline{\beta})/2}$. Therefore, we conclude that our results are robust to a fairly general set of demand setup.

7. Conclusion

In many markets characterized by fashion or seasonal cycle, short product lifespan, new product proliferation, and the resulting uncertainty about consumer preferences or market demand, firms in vertical relationships can benefit from uncertainty-resolving information. We investigate a manufacturer's optimal information acquisition and sharing decisions in exerting strategic influence on its downstream retailer's behavior. Our paper departs from extant studies that focus mainly on either non-sequential information acquisition or mandatory sharing, each of which is typically analyzed in isolation and usually in the context of horizontal oligopolies. Our analysis is the first to jointly examine both information acquisition and sharing by a manufacturer in a vertical system. This allows us to uncover the strategic interaction between these two decisions, and examine how it is influenced by both the flexibility in information acquisition and the flexibility in sharing format.

7.1. Implications of the Key Findings

The paper obtains several results that provide useful managerial insights into strategic information management in vertical relationships. We demonstrate that a manufacturer can wield information power over downstream retailer not only through selectively withholding unfavorable information, but also through strategically controlling the generation of information. First, when information acquisition is sequential, the manufacturer should exercise self-restriction in information acquisition and refrain from acquiring perfect information, even if it is costless to do so. To influence the retailer's belief, the manufacturer may terminate information collection either when the acquired information is sufficiently favorable or before overly adverse outcome arises. As such, we identify "strategic ignorance" in information acquisition as a new mechanism that can be used by firms to manage channel relationships. The analysis suggests that manufacturers should take this strategic impact into account in their endeavors to resolve uncertainty about consumer preferences, which is particularly relevant as sequential monitoring of information acquisition becomes increasingly prevalent (e.g., online research instruments, syndicated consumer databases).

Interestingly, we show that the manufacturer should collect either more or less information as sequential information acquisition becomes increasingly feasible, depending on the format of information sharing. Under mandatory sharing, the manufacturer can manipulate the retailer's belief only through the control over the information acquisition process, and thus should acquire information only when this strategic control is available. Conversely, voluntary sharing permits the manufacturer to manipulate the retailer's belief through the ex post control over the shared information, which thus motivates the manufacturer to ex ante acquire more information. As a result, the impact of the flexibility in information acquisition on the manufacturer's optimal acquisition strategy is moderated by the flexibility in information sharing. In contrast, irrespective of the level of flexibility in information acquisition, the flexibility to selectively decide whether to share the acquired information should generally lead the manufacturer to collect more information, which follows from the manufacturer's ability under voluntary sharing to disclose favorable information while credibly withholding unfavorable information.

The above insights provide useful prescriptions on how manufacturers should respond to changes in information collection technologies and in information sharing arrangements. When manufacturers move increasingly from standard information collection technologies (e.g., mail surveys, mall intercepts, field testing) to those that allow for sequential monitoring (e.g., online surveys, syndicated databases), how should they adjust their information acquisition policies? The results above suggest that manufacturers may indeed strive to collect more information when they have committed ex ante to mandatorily share the information. In addition, for manufacturers who have set up more formal arrangements to mandatorily share information with downstream firms (e.g., Procter and Gamble, Warner-Lambert), greater scrutiny should be employed regarding the amount of information to generate, than their counterparts who rely primarily on ex post voluntary sharing for channel communication. They should generally be more conservative in information acquisition, especially when they do not have access to information collection technologies that afford sequential control over the data generation process.

Moreover, we investigate the conditions under which the manufacturer should pursue the ex post flexibility in disclosing its acquired information to the retailer. Counter-intuitively, the manufacturer may not necessarily prefer to maintain its flexibility in information sharing. In particular, our results suggest that the manufacturer's preference for the ex ante commitment to mandatorily share information is influenced by the prior belief on consumer preference. When the prior belief is sufficiently low and thus the product is more likely to be perceived as a bad fit, the manufacturer may want to commit to mandatorily share the acquired information and thereby choose to give up the flexibility to voluntarily disclose information (i.e., the ex post strategic influence over the retailer). Committing to mandatorily disclose the collected information can then serve as a self-disciplining device to acquire less information, which may paradoxically induce the retailer to order higher amounts when the likelihood of product failure is high. For example, in many fashion markets only a small fraction of designs may ultimately succeed in any given season. In such environments, ex ante commitments by manufacturers to share the to-be-acquired information can be valuable. Not surprisingly, a number of successful mandatory information sharing arrangements have been documented in markets such as fashion apparels and consumer electronics (Hammond et. al 1991).

7.2. Discussion and Future Research

One implicit assumption in our model is that consumer preference does not change throughout the game and that retail ordering does not occur before information acquisition (either inflexible or sequential) can be completed.[21] This will be the case if abundant information can be gathered in a relatively short time period (e.g., online surveys, subscription to syndicated databases) and if there is significant lead time between upstream design/production and downstream selling (i.e., new market/product entry). For example, consumer tastes in most markets are relatively stable in the short run and sequential market research studies can be run before the start of retail order taking during a fashion or season cycle (Doeringer and Crean 2006). Note also that a finite number of signals are sufficient to reasonably resolve the firms' uncertainty—the marginal information contained in each additional signal is decreasing. Nevertheless, the results of this paper will continue to hold, even in

[21] We thank the review team for inspiring some of the discussions in this section.

scenarios when consumers change their preferences independently and/or market demand (and thus retail ordering) arises in each period in which one piece of information can be acquired. However, an interesting case for future research is when consumers change their preferences systematically over time because of some endogenous mechanism such as social communication or learning from experiences. In these cases the manufacturer's sequential information acquisition strategy will be more involved since it is conditional not only on the updated posterior belief but also on time.

It is instructive to discuss other (non-strategic) considerations that may influence a manufacturer's relative preference for the different information sharing formats. First, mandatory sharing is normally implemented through data-pooling systems that involve bilateral information exchange by both upstream and downstream firms. As a result, a manufacturer may benefit from such systems if the information provided from downstream can improve the efficiency of the manufacturer's decision making. The current analysis can thus be seen as isolating the commitment from the efficiency effect of such systems. Second, the sharing formats may involve different cost implications. Generally, data-pooling arrangements involve fixed setup costs, while voluntary sharing decisions may incur only marginal costs. Interestingly, however, a lower marginal cost of information sharing does not necessarily lead to a higher ex ante preference for voluntary sharing. This is because a higher marginal cost of information sharing can facilitate strategic information concealment under voluntary sharing, leading to a non-monotonic net effect on ex ante payoffs (Guo 2009). Moreover, mandatory sharing is more likely to be preferred in environments with higher marginal costs of information acquisition, since the manufacturer's expected payoff is hurt more under voluntary sharing that involves more equilibrium information collection.

Another problem which seems to be good candidate for future investigation is the role of explicit and costly mechanisms that will induce truth telling in ex post voluntary sharing arrangements (Ziv 1993). One can also investigate bilateral information exchange in a more comprehensive framework in which both firms in a vertical system can acquire and share information to influence each other's decision making. The analysis will be similar to this paper if each firm has exclusive access to information on different types of uncertainty (e.g., product fit versus demand). Conversely, if the firms can acquire information on the same type of uncertainty, their optimal information acquisition/sharing strategies will interact with each other. Interestingly, however, one may conjecture that a firm may have an incentive to stop the information acquisition process in order to mitigate the chance that the (common) posterior belief is updated by the other firm's acquired signals toward the undesirable direction. Of course, all these issues pertaining to sequential information acquisition and sharing that are currently studied in this paper, are not necessarily unique to vertical relationships and can be examined analogously in the context of horizontal oligopolies.

APPENDIX

Proof of Lemma 1: Let us define $\Psi^S(N) \equiv \Phi^S(\hat{\beta}(N)|\hat{\beta}_L, \hat{\beta}_H)$ as the conditional probability (on $S \in \{G, B\}$) that the net number of good signals reaches N_H before N_L, as a function of the starting point $N \in [N_L, N_H]$. Note that by definition $\Psi^G(N) = \gamma\Psi^G(N+1) + (1-\gamma)\Psi^G(N-1)$, where $N \in \{N_L + 1, \dots, N_H - 1\}$. This is a second-order difference equation, where the boundary conditions are $\Psi^G(N_L) = 0$ and $\Psi^G(N_H) = 1$. We can then obtain the solution:

$$\Psi^G(N) = \frac{1 - (\frac{1-\gamma}{\gamma})^{(N-N_L)}}{1 - (\frac{1-\gamma}{\gamma})^{(N_H-N_L)}}, \text{ where } N \in \{N_L, \dots, N_H\}. \tag{i}$$

Similarly, the second-order difference equation conditional on $S = B$ is $\Psi^B(N) = (1-\gamma)\Psi^B(N+1) + \gamma\Psi^B(N-1)$, where $N \in \{N_L + 1, \dots, N_H - 1\}$, $\Psi^B(N_L) = 0$, and $\Psi^B(N_H) = 1$. The solution is:

$$\Psi^B(N) = \frac{1 - (\frac{\gamma}{1-\gamma})^{(N-N_L)}}{1 - (\frac{\gamma}{1-\gamma})^{(N_H-N_L)}}, \text{ where } N \in \{N_L, \dots, N_H\}. \tag{ii}$$

Note that by definition $\hat{\beta}_L \equiv \hat{\beta}(N_L) = \frac{\dot{\beta}}{\dot{\beta}+(1-\dot{\beta})(\frac{1-\gamma}{\gamma})^{N_L}}$, and $\hat{\beta}_H \equiv \hat{\beta}(N_H) = \frac{\dot{\beta}}{\dot{\beta}+(1-\dot{\beta})(\frac{1-\gamma}{\gamma})^{N_H}}$, which can be inverted to obtain N_L as a function of $\hat{\beta}_L$ and N_H as a function of $\hat{\beta}_H$, respectively. Plugging these inverted solutions into (i) and (ii), respectively, we therefore have $\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \Psi^G(0) = \frac{\hat{\beta}_H(\dot{\beta}-\hat{\beta}_L)}{\dot{\beta}(\hat{\beta}_H-\hat{\beta}_L)}$, and $\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \Psi^B(0) = \frac{(1-\hat{\beta}_H)(\dot{\beta}-\hat{\beta}_L)}{(1-\dot{\beta})(\hat{\beta}_H-\hat{\beta}_L)}$. Finally, we obtain $\Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) \equiv \dot{\beta}\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) + (1 - \dot{\beta})\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{\dot{\beta}-\hat{\beta}_L}{\hat{\beta}_H-\hat{\beta}_L}$.

Q.E.D.

Proof of Proposition 1: Suppose that the manufacturer acquires information when $I = y$. The manufacturer's expected payoff is hence given by $\Pi = \beta\omega$ if $\omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, $\Pi = \alpha\beta\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \theta_h$, and $\Pi = 0$ if otherwise. This expected payoff is higher than that when no information is acquired at all, if and only if $\beta > \alpha$ and $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, or $\beta\theta_h < \omega \leq \theta_h$.

Let us then derive the equilibrium wholesale price. Consider first the case when $d \leq \beta\theta_h$. If furthermore $\beta < \alpha$, the manufacturer's expected payoff with information acquisition is always lower than that when no information is acquired at all. If instead $\beta > \alpha$, information acquisition is desirable if and only if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. However, charging a wholesale price in the range $\omega \in \left(\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\right]$, irrespective of the subsequent information acquisition decision, is ex ante dominated by charging $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$. Similarly, in the case $d > \beta\theta_h$, it is always dominant for the manufacturer to charge $\omega = \beta\theta_h$. As a result, the equilibrium wholesale price is the same as that in the benchmark without information sharing. The proposition follows.

Q.E.D.

Proof of Proposition 2: If $\omega \le \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ (i.e., $\beta \ge \beta_h$), the manufacturer would not start the information acquisition process and the retailer would order $x=1$. It follows that $\underline{\beta}=\overline{\beta}=\beta$, $\Pi'=\omega$, and $\pi'=\beta\theta_l-\omega$.

If $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ (i.e., $\beta_l \le \beta < \beta_h \le 1$), the manufacturer would continue the information acquisition process until $\hat{\beta}$ reaches either $\underline{\beta}=\beta_l$ or $\overline{\beta}=\beta_h$. The manufacturer's expected payoff is then $\Pi'=\Phi(\beta|\beta_l,\beta_h)\omega+[1-\Phi(\beta|\beta_l,\beta_h)]\alpha\omega = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l}+\frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$, and the retailer's is $\pi'=\beta[\Phi^G(\beta|\beta_l,\beta_h)\theta_l+[1-\Phi^G(\beta|\beta_l,\beta_h)]\alpha\theta_h]-[\Phi(\beta|\beta_l,\beta_h)\omega+[1-\Phi(\beta|\beta_l,\beta_h)]\alpha\omega]=\alpha\beta\theta_h-\alpha\omega$.

If $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \le \beta\theta_h$ (i.e., $\beta_l \le \beta < 1 < \beta_h$), the manufacturer would not collect any information and the retailer would order $x=\alpha$, which leads to $\underline{\beta}=\overline{\beta}=\beta$, $\Pi'=\alpha\omega$, and $\pi'=\alpha\beta\theta_h-\alpha\omega$.

Finally, if $\omega > \beta\theta_h$ (i.e., $\beta<\beta_l$), the manufacturer would collect information until either it is learned almost with certainty that $S=B$ or $\hat{\beta}$ reaches β_l, i.e., $\underline{\beta}=0$ and $\overline{\beta}=\beta_l$. The manufacturer's expected payoff is then $\Pi'=\Phi(\beta|0,\beta_l)\alpha\omega=\alpha\beta\theta_h$, and the retailer's is $\pi'=\beta\Phi^G(\beta|0,\beta_l)\alpha\theta_h-\Phi(\beta|0,\beta_l)\alpha\omega=0$.

Q.E.D.

Proof of Proposition 3: When $I=\bar{y}$, no useful information is available and the manufacturer's expected payoff, conditional on $\omega \le d$, is $\Pi''=\omega$ if $\omega \le \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi''=\alpha\omega$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \beta\theta_h$, and $\Pi''=0$ if otherwise. When $I=y$, the manufacturer would acquire information if and only if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ or $\omega > \beta\theta_h$, where its expected payoff is $\Pi'=\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l}+\frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$ or $\Pi'=\alpha\beta\theta_h$, respectively; if otherwise, then $\Pi'=\Pi''$. To derive the equilibrium wholesale price and the firms' ex ante payoffs, we consider two alternative parameter ranges.

Range (i): $\beta \le \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$. In this case, $\beta\theta_h \le \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. Therefore, when $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < d \le \beta\theta_h$, the manufacturer would charge $\omega=d$ if $[\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l}+\frac{[\alpha(2-\alpha)\theta_h-\theta_l]d}{\theta_h-\theta_l}]/2+\alpha d/2 \ge \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, and $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if otherwise. The firms' equilibrium ex ante payoffs are then given by, respectively:

$$\Pi_i^m = \begin{cases} \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \le \frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}; \\ \min\left\{\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)}+\frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]d}{2(\theta_h-\theta_l)}, \alpha\beta\theta_h\right\}, & \text{if otherwise.} \end{cases}$$

$$\pi_i^m = \begin{cases} \beta\theta_l - \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \le \frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}; \\ \alpha\beta\theta_h - \alpha\min\{d, \beta\theta_h\}, & \text{if otherwise.} \end{cases}$$

Note that $\Pi_i^m > \Pi^{ns}$ if and only if $\frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]} < d < \beta\theta_h$, and $\pi_i^m < \pi^{ns}$ if and only if $\frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]} < d < \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$; if otherwise, then $\Pi_i^m=\Pi^{ns}$ and $\pi_i^m=\pi^{ns}$.

Range (ii): $\beta > \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$. In this case, $\beta\theta_h > \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. The manufacturer's expected payoff, conditional on $\omega \le d$, is then $\Pi(\omega)=\omega$ if $\omega \le \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega)=\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)}+\frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]\omega}{2(\theta_h-\theta_l)}$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, $\Pi(\omega)=\alpha\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \le \beta\theta_h$, and $\Pi(\omega)=\alpha\beta\theta_h/2$ if $\omega > \beta\theta_h$. Note that $\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)}+\frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]\omega}{2(\theta_h-\theta_l)}$ is lower than ω when $\omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, and higher than $\alpha\omega$ when $\omega \to \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. Moreover, there exists a $\tilde{\beta} \in \left(\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}, 1\right)$, such that $\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]\omega}{2(\theta_h-\theta_l)}$ is lower than $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\omega \to \frac{\theta_l-\alpha\theta_h}{1-\alpha}$ if and only if $\beta > \tilde{\beta}$.

Therefore, if $\beta \geq \tilde{\beta}$, the manufacturer would charge $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} \leq d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$. As a result, the manufacturer in equilibrium would not acquire any information even when $I = y$, and its equilibrium ex ante payoff Π_{ii}^m is the same as that without information sharing (i.e., Π^{ns}). If $\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h} < \beta < \tilde{\beta}$, the manufacturer would charge $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < d \leq \frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}$, $\omega = \frac{\theta_l-\alpha\theta_h}{1-\alpha}$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < d \leq \frac{(\theta_l-\alpha\theta_h)[(\alpha(3-2\beta)-\alpha^2(1-\beta)+\beta)\theta_h-(1+\alpha)\theta_l]}{2\alpha(1-\alpha)(\theta_h-\theta_l)}$, $\omega = \beta\theta_h$ if $d > \beta\theta_h$, and $\omega = d$ if otherwise. Thus, when $\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h} < \beta < \tilde{\beta}$, the manufacturer in equilibrium would acquire information when $I = y$ if and only if $\frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]} < d < \frac{(\theta_l-\alpha\theta_h)[(\alpha(3-2\beta)-\alpha^2(1-\beta)+\beta)\theta_h-(1+\alpha)\theta_l]}{2\alpha(1-\alpha)(\theta_h-\theta_l)}$, where its equilibrium ex ante payoff Π_{ii}^m is higher than that without information sharing (i.e., Π^{ns}).

When $\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h} < \beta < \tilde{\beta}$ and $\max\left\{\frac{\theta_l-\alpha\theta_h}{1-\alpha}, \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}\right\} < d < \frac{(\theta_l-\alpha\theta_h)[(\alpha(3-2\beta)-\alpha^2(1-\beta)+\beta)\theta_h-(1+\alpha)\theta_l]}{2\alpha(1-\alpha)(\theta_h-\theta_l)}$, note that the optimal wholesale price $\omega = \frac{\theta_l-\alpha\theta_h}{1-\alpha}$ is lower than that when information sharing is infeasible (i.e., $w = d$). This implies that the retailer's equilibrium ex ante payoff under mandatory information sharing and sequential information acquisition can be higher than that without information sharing (i.e., π^{ns}).

Q.E.D.

Proof of Proposition 4: Suppose that the manufacturer acquires information when $I = y$, which leads to $\hat{\beta}_m = 1$ or $\hat{\beta}_m = 0$, with probability β or $1-\beta$, respectively. Because the manufacturer's expected payoff increases with the retailer's updated posterior belief $\hat{\beta}_r$, the manufacturer's optimal disclosure strategy is $m(1) = 1$ and $m(0) = \otimes$. Knowing this, the retailer's updated posterior beliefs are $\hat{\beta}_r(1) = 1$ and $\hat{\beta}_r(\otimes) = \frac{\beta}{2-\beta}$. This is because, the message $m = \otimes$ would be delivered either when no useful information is available or when the manufacturer's updated posterior belief is zero (i.e, $\hat{\beta}_m = 0$). As a result, it is indeed an equilibrium strategy for the manufacturer to acquire information, since $\hat{\beta}_r(1) > \hat{\beta}_r(\otimes)$. To prove the uniqueness of the equilibrium, suppose instead that no information is acquired when $I = y$, which results in $\hat{\beta}_r(\otimes) = \beta$. But then the manufacturer can benefit from deviating and choosing to acquire information: The manufacturer can send $m = \otimes$ and do no worse when the acquired information indicates $S = B$, and can do better when the acquired information indicates $S = G$.

The setting of the wholesale price takes into account these two equilibrium scenarios on the retailer's updated posterior belief: $\hat{\beta}_r \in \{1, \frac{\beta}{2-\beta}\}$. In particular, when $I = y$ and the manufacturer subsequently learns that $\hat{\beta}_m = 1$, it will disclose the information to the retailer whose updated posterior belief is then $\hat{\beta}_r(1) = 1$, the ex ante probability of which is $\beta/2$. In addition, when $I = \bar{y}$, or when $I = y$ and the manufacturer subsequently learns that $\hat{\beta}_m = 0$, the retailer would receive the message $m = \otimes$ and update its belief toward $\hat{\beta}_r(\otimes) = \frac{\beta}{2-\beta}$, the ex ante probability of which is $1 - \beta/2$.

Range (i): $\beta < \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$. In this case, $\frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha} < \hat{\beta}_r(\otimes)\theta_h < \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha} < \hat{\beta}_r(1)\theta_h$. The manufacturer's expected payoff is $\Pi(\omega) = \omega$ when $\omega \leq \frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha}$. Similarly, the manufacturer's expected payoff is $\Pi(\omega) = \beta\omega/2 + \alpha(1-\beta/2)\omega$ when $\frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \hat{\beta}_r(\otimes)\theta_h$, $\Pi(\omega) = \beta\omega/2$ when $\hat{\beta}_r(\otimes)\theta_h <$

$\omega \leq \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha}$, and $\Pi(\omega) = \alpha\beta\omega/2$ when $\frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \hat{\beta}_r(1)\theta_h$. Note that the manufacturer's expected payoff increases with ω within each of the above four ranges, and discontinuously drops at the boundary points. Note also that $\Pi(\omega) = \omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)}$ when $\omega \to \frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = \beta\omega/2 + \alpha(1-\beta/2)\omega \to \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)}$ when $\omega \to \hat{\beta}_r(\otimes)\theta_h$, $\Pi(\omega) = \beta\omega/2 \to \frac{\beta(\theta_l-\alpha\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha}$, and $\Pi(\omega) = \alpha\beta\omega/2 \to \alpha\beta\theta_h/2$ when $\omega \to \hat{\beta}_r(1)\theta_h$. Moreover, $\frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)}$, $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} > \frac{\beta(\theta_l-\alpha\theta_h)}{2(1-\alpha)}$, and $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} > \alpha\beta\theta_h/2$. Noticing that $\beta\omega/2 + \alpha(1-\beta/2)\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)}$ when $\omega = \frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)[\alpha(2-\beta)+\beta]}$, we can then obtain the manufacturer's equilibrium ex ante payoff, when $\beta < \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$:

$$\Pi_i^{iv} = \begin{cases} \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)}\right\}, & \text{if } d \leq \frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)[\alpha(2-\beta)+\beta]}; \\ \min\left\{[\alpha(2-\beta)+\beta]d/2, \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)}\right\}, & \text{if otherwise.} \end{cases}$$

Comparing Π_i^{iv} with Π^{ns}, we obtain $\frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)[\alpha(2-\beta)+\beta]} < \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$, $\frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $[\alpha(2-\beta)+\beta]d/2 > \alpha d$, $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} < \alpha\beta\theta_h$, and $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if and only if $\beta < \frac{4\theta_l-2\alpha(3-\alpha)\theta_h}{2\theta_l+[1-\alpha(4-\alpha)]\theta_h}$. Moreover, $[\alpha(2-\beta)+\beta]d/2 > \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if and only if $d > \frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]}$, and $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} > \alpha d$ if and only if $d < \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}$. Therefore, when $\beta < \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, we obtain that $\Pi_i^{iv} > \Pi^{ns}$ if and only if $\beta > \frac{4\theta_l-2\alpha(3-\alpha)\theta_h}{2\theta_l+[1-\alpha(4-\alpha)]\theta_h}$ and $\frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]} < d < \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}$.

Range (ii): $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$. In this case, $\frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha} < \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha} < \hat{\beta}_r(\otimes)\theta_h < \hat{\beta}_r(1)\theta_h$. Similarly, the manufacturer's expected payoff is $\Pi(\omega) = \omega$ when $\omega \leq \frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = \beta\omega/2 + \alpha(1-\beta/2)\omega$ when $\frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = \alpha\omega$ when $\frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \hat{\beta}_r(\otimes)\theta_h$, and $\Pi(\omega) = \alpha\beta\omega/2$ when $\hat{\beta}_r(\otimes)\theta_h < \omega \leq \hat{\beta}_r(1)\theta_h$. Note that $\Pi(\omega) = \omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)}$ when $\omega \to \frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = \beta\omega/2 + \alpha(1-\beta/2)\omega \to \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = \alpha\omega \to \frac{\alpha\beta\theta_h}{2-\beta}$ when $\omega \to \hat{\beta}_r(\otimes)\theta_h$, and $\Pi(\omega) = \alpha\beta\omega/2 \to \alpha\beta\theta_h/2$ when $\omega \to \hat{\beta}_r(1)\theta_h$. Moreover, $\frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\alpha\beta\theta_h}{2-\beta}$, and $\frac{\alpha\beta\theta_h}{2-\beta} > \alpha\beta\theta_h/2$. Therefore, we can obtain the manufacturer's equilibrium ex ante payoff $\Pi_{ii}^{iv} = \max\left\{\min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)}\right\}, \min\left\{[\alpha(2-\beta)+\beta]d/2, \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}\right\}, \min\left\{\alpha d, \frac{\alpha\beta\theta_h}{2-\beta}\right\}\right\}$.

Comparing Π_{ii}^{iv} with Π^{ns}, we obtain $\frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $[\alpha(2-\beta)+\beta]d/2 > \alpha d$, $\frac{\alpha\beta\theta_h}{2-\beta} < \alpha\beta\theta_h$, and $\frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} < \alpha\beta\theta_h$. Therefore, when $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, $\Pi_{ii}^{iv} > \Pi^{ns}$ if and only if $\frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} > \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $[\alpha(2-\beta)+\beta]d/2 > \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, and $\frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} > \alpha d$, which give rise to $\beta < \frac{2\alpha}{1+\alpha}$, $d > \frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]}$, and $d < \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}$, respectively.

Noticing that $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)} > \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}$ if and only if $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, we can combine the ranges (i) and (ii) to obtain that $\Pi^{iv} > \Pi^{ns}$ if and only if $\frac{4\theta_l-2\alpha(3-\alpha)\theta_h}{2\theta_l+[1-\alpha(4-\alpha)]\theta_h} < \beta < \frac{2\alpha}{1+\alpha}$ and $\frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]} < d < \min\left\{\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}, \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}\right\}$.

Q.E.D.

Proof of Proposition 5: If $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ (i.e., $\beta \geq \beta_h$), the manufacturer would not start the information acquisition process and the retailer would order $x = 1$. It follows that $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = m(\overline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \omega$, and $\pi' = \beta\theta_l - \omega$.

If $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ (i.e., $\frac{2(1-\alpha)\omega}{\theta_l+(1-2\alpha)\theta_h} \leq \beta < \beta_h \leq 1$), the manufacturer would continue to collect information until its updated posterior belief reaches either $\hat{\beta}_m = \beta_h$ in which case the acquired information would be disclosed and the retailer would order $x = 1$, or $\hat{\beta}_m = 0$ in which case the acquired information would be withheld and the retailer would order $x = \alpha$. It follows that $\underline{\beta} = 0$, $\overline{\beta} = \beta_h$, $m(\underline{\beta}) = \otimes$, $m(\overline{\beta}) = \overline{\beta}$, and $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+(1-\beta/\beta_h)/2} = \frac{(1-\alpha)\beta\omega}{2(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)} \geq \beta_l$. The manufacturer's expected payoff is $\Pi' = \Phi(\beta|0,\beta_h)\omega + [1-\Phi(\beta|0,\beta_h)]\alpha\omega = \beta(\theta_l-\alpha\theta_h)+\alpha\omega$, and the retailer's is $\pi' = \beta[\Phi^G(\beta|0,\beta_h)\theta_l + [1-\Phi^G(\beta|0,\beta_h)]\alpha\theta_h] - [\Phi(\beta|0,\beta_h)\omega + [1-\Phi(\beta|0,\beta_h)]\alpha\omega] = \alpha\beta\theta_h - \alpha\omega$.

If $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ (i.e., $\beta_l \leq \beta < \frac{2(1-\alpha)\omega}{\theta_l+(1-2\alpha)\theta_h}$ and $\beta_h \leq 1$), then there exists no pure-strategy equilibrium. In the (partially) mixed-strategy equilibrium, the upper bound where the manufacturer stops information acquisition is $\overline{\beta} = \beta_h$, and the manufacturer discloses the upper bound to the retailer (i.e., $m(\overline{\beta}) = \overline{\beta}$). However, the manufacturer randomizes the lower bound of information acquisition between $\underline{\beta} = 0$ and $\underline{\beta} = \beta_l$. The probability λ that the manufacturer continues to collect information until $\hat{\beta}_m = 0$ is such that the retailer is indifferent between ordering $x = \alpha$ and $x = 0$ when the message $m = \otimes$ is received, i.e., $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+\lambda(1-\beta/\beta_h)/2} = \beta_l$. This leads to $\lambda = \frac{\beta_h(\beta-\beta_l)}{\beta_l(\beta_h-\beta)} = \frac{(1-\alpha)(\beta\theta_h-\omega)}{(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)}$. In addition, the manufacturer would withhold (disclose) the acquired information when the lower bound $\underline{\beta} = 0$ ($\underline{\beta} = \beta_l$) is reached, i.e., $m(0) = \otimes$ and $m(\beta_l) = \beta_l$. Moreover, when the retailer receives the $m = \otimes$ message, it would randomize between ordering $x = \alpha$ and $x = 0$, with probability ρ and $1-\rho$, respectively. To make the manufacturer indifferent between $\underline{\beta} = 0$ and $\underline{\beta} = \beta_l$, it is required that $\alpha\omega = \Phi(\beta_l|0,\beta_h)\omega + [1-\Phi(\beta_l|0,\beta_h)]\rho\alpha\omega$, which leads to $\rho = \frac{\alpha\beta_h-\beta_l}{\alpha(\beta_h-\beta_l)} = \frac{\alpha(2-\alpha)\theta_h-\theta_l}{\alpha(\theta_h-\theta_l)}$. It follows that the manufacturer's expected profit is $\Pi' = \Phi(\beta|\beta_l,\beta_h)\omega + [1-\Phi(\beta|\beta_l,\beta_h)]\alpha\omega = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$, and the retailer's is $\pi' = \beta\{\lambda[\Phi^G(\beta|0,\beta_h)\theta_l + [1-\Phi^G(\beta|0,\beta_h)]\alpha\theta_h] + (1-\lambda)[\Phi^G(\beta|\beta_l,\beta_h)\theta_l + [1-\Phi^G(\beta|\beta_l,\beta_h)]\alpha\theta_h]\} - [\Phi(\beta|\beta_l,\beta_h)\omega + [1-\Phi(\beta|\beta_l,\beta_h)]\alpha\omega] = \frac{[\alpha^2\theta_h+(1-2\alpha)\theta_l](\beta\theta_h-\omega)}{\theta_h-\theta_l}$.

If $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$ (i.e., $\beta_l \leq \beta < 1 < \beta_h$), the manufacturer would not collect any information and the retailer's order is $x = \alpha$, which leads to $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = m(\overline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$.

Finally, if $\omega > \beta\theta_h$ (i.e., $\beta < \beta_l$), the manufacturer would continue information collection until either it is learned almost with certainty that $S = B$ or its updated posterior belief reaches β_l, i.e., $\underline{\beta} = 0$ and $\overline{\beta} = \beta_l$. In addition, the manufacturer would withhold (disclose) the acquired information when the lower (upper) bound is reached, i.e., $m(\underline{\beta}) = \otimes$ and $m(\overline{\beta}) = \overline{\beta}$. This implies that $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+(1-\beta/\beta_l)/2} = \frac{\beta\omega}{2\omega-\beta\theta_h} < \beta_l$. The manufacturer's expected payoff is then $\Pi' = \Phi(\beta|0,\beta_l)\alpha\omega = \alpha\beta\theta_h$, and the retailer's is $\pi' = \beta\Phi^G(\beta|0,\beta_l)\alpha\theta_h - \Phi(\beta|0,\beta_l)\alpha\omega = 0$.

Q.E.D.

Proof of Proposition 6: When $I = \bar{y}$, the manufacturer's expected payoff, conditional on $\omega \leq d$, is $\Pi'' = \omega$ if $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi'' = \alpha\omega$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ or $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, $\Pi'' = \rho\alpha\omega$ if $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, where $\rho = \frac{\alpha(2-\alpha)\theta_h-\theta_l}{\alpha(\theta_h-\theta_l)}$, and $\Pi'' = 0$ if otherwise.

When $I = y$, the manufacturer's expected payoff is given in Proposition 5. That is, $\Pi' = \omega$ if $\omega \leq \frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha}$, $\Pi' = \beta(\theta_l - \alpha\theta_h) + \alpha\omega$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, $\Pi' = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$ if $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, $\Pi' = \alpha\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, and $\Pi' = \alpha\beta\theta_h$ if $\omega > \beta\theta_h$.

Range (i): $\beta < \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$. In this case, $\beta\theta_h < \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. The manufacturer's expected payoff, conditional on $\omega \leq d$, is then $\Pi(\omega) = \omega$ if $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l-\alpha\theta_h)+2\alpha\omega]/2$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}$, $\Pi(\omega) = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$ if $\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)} < \omega \leq \beta\theta_h$, and $\Pi(\omega) = \alpha\beta\theta_h/2$ if $\omega > \beta\theta_h$. Note that $\Pi(\omega) = \omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l - \alpha\theta_h) + 2\alpha\omega]/2 \to \frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}$, and $\Pi(\omega) = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l} \to \frac{\beta\theta_h[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}{2(\theta_h-\theta_l)}$ when $\omega \to \beta\theta_h$. Moreover, $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)} < \alpha\beta\theta_h$, $\alpha\beta\theta_h/2 < \frac{\beta\theta_h[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}{2(\theta_h-\theta_l)} < \alpha\beta\theta_h$, and $\frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)} > \frac{\beta\theta_h[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}{2(\theta_h-\theta_l)}$ if $[1-\sqrt{(1-\alpha)^3}]\theta_h < \theta_l < \alpha(2-\alpha)\theta_h$. Therefore, when $[1-\sqrt{(1-\alpha)^3}]\theta_h < \theta_l < \alpha(2-\alpha)\theta_h$, the firms' equilibrium ex ante profits are given by, respectively:[22]

$$\Pi_i^v = \begin{cases} \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{(1+\alpha)\beta(\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}; \\ \min\left\{[\beta(\theta_l-\alpha\theta_h)+2\alpha d]/2, \frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)}\right\}, & \text{if otherwise.} \end{cases}$$

$$\pi_i^v = \begin{cases} \beta\theta_l - \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{(1+\alpha)\beta(\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}; \\ \alpha\beta\theta_h - \alpha\min\left\{d, \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}\right\}, & \text{if otherwise.} \end{cases}$$

We can then readily obtain that $\Pi_i^v < \Pi_i^m$ if d is sufficiently large. Similarly, we have $\pi_i^v > \pi_i^m$ if $d > \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}$. This proves part (ii) of the proposition.

Range (ii): $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$. In this case, $\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)} > \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. The manufacturer's expected payoff, conditional on $\omega \leq d$, is then $\Pi(\omega) = \omega$ if $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l - \alpha\theta_h) + 2\alpha\omega]/2$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, $\Pi(\omega) = \alpha\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, and $\Pi(\omega) = \alpha\beta\theta_h/2$ if $\omega > \beta\theta_h$. Note that $\Pi(\omega) = \omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l-\alpha\theta_h)+2\alpha\omega]/2 \to \frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, and $\Pi(\omega) = \alpha\omega \to \alpha\beta\theta_h$ when $\omega \to \beta\theta_h$. Moreover, $\frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} < \alpha\beta\theta_h$ given $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, and $\frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if and only if $\beta > \frac{2\alpha}{1+\alpha}$. Therefore, the manufacturer' equilibrium ex ante payoff is $\Pi_{ii}^v = \max\left\{\min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, \min\left\{[\beta(\theta_l-\alpha\theta_h)+2\alpha d]/2, \frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}\right\}, \min\{\alpha d, \alpha\beta\theta_h\}\right\}$. It is then obvious that given $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, we can obtain that $\Pi_{ii}^v > \Pi_{ii}^m$ if $\beta < \frac{2\alpha}{1+\alpha}$ and $\frac{(1+\alpha)\beta(\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)} < d < \frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}$, and $\Pi_{ii}^v = \Pi_{ii}^m$ if otherwise. This proves part (i) of the proposition.

Q.E.D.

[22] The case when $\alpha\theta_h < \theta_l < [1-\sqrt{(1-\alpha)^3}]\theta_h$ is similar, but the computation is more involved.

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Supplementary Appendix for "Information Acquisition and Sharing in a Vertical Relationship"

Flexible Wholesale Price:

We now examine the manufacturer's optimal information acquisition/sharing strategies when the wholesale price is set in the third stage. We start with the cases when information acquisition is inflexible. If furthermore information sharing is mandatory, the manufacturer's expected payoff when it acquires information would be $\Pi = \beta\Pi(1) + (1-\beta)\Pi(0) = \beta\Pi(1)$, where $\Pi(\cdot)$ is given by (6). It can be readily verified that $\Pi(\beta) \geq \beta\Pi(1)$ for all $\beta \in (0,1)$. It then follows that, similar to Proposition 1, in equilibrium the manufacturer does not acquire information even when the information state is $I = y$. Consider then the case when information acquisition is inflexible and sharing is voluntary. Noticing that $\Pi(\cdot)$ is an (weakly) increasing function in the retailer's updated posterior belief $\hat{\beta}_r$, we can then readily follow the proof for Proposition 4 to obtain that acquiring information is the unique equilibrium strategy for the manufacturer when $I = y$.

1. Sequential Information Acquisition and Mandatory Sharing:

We first examine whether the manufacturer should continue the information acquisition process when the firms' (symmetric) posterior belief reaches $\hat{\beta} \in (0,1)$. Suppose that the manufacturer continues to collect more information. This will generate either a good or a bad signal, updating the firms' (symmetric) posterior belief upward to $\hat{\beta}^+$ or downward to $\hat{\beta}^-$, respectively.

Lemma 2: *When information acquisition is sequential and sharing is mandatory, the sufficient condition for the manufacturer to continue to collect information when $\hat{\beta}$ is reached is given by:*

$$\frac{\Pi(\hat{\beta}^+) - \Pi(\hat{\beta})}{\hat{\beta}^+ - \hat{\beta}} \geq \frac{\Pi(\hat{\beta}) - \Pi(\hat{\beta}^-)}{\hat{\beta} - \hat{\beta}^-}.$$

Proof: Note that $\hat{\beta}^- < \hat{\beta} < \hat{\beta}^+$. This implies $\Pi(\hat{\beta}^-) \leq \Pi(\hat{\beta}) \leq \Pi(\hat{\beta}^+)$. Therefore, rearranging the condition in the lemma leads to $\Pi(\hat{\beta}) \leq \frac{\hat{\beta}-\hat{\beta}^-}{\hat{\beta}^+-\hat{\beta}^-}\Pi(\hat{\beta}^+) + \frac{\hat{\beta}^+-\hat{\beta}}{\hat{\beta}^+-\hat{\beta}^-}\Pi(\hat{\beta}^-) = \Phi(\hat{\beta}|\hat{\beta}^-,\hat{\beta}^+)\Pi(\hat{\beta}^+) + [1-\Phi(\hat{\beta}|\hat{\beta}^-,\hat{\beta}^+)]\Pi(\hat{\beta}^-)$, where the left-hand side of the inequality represents the manufacturer's expected payoff if information acquisition is stopped at $\hat{\beta}$, and the right-hand side represents the manufacturer's expected payoff if an additional signal is generated. The lemma follows immediately.

Q.E.D.

Note that this lemma holds no matter whether $\Pi(\hat{\beta})$ is continuous at $\hat{\beta}$. When it is continuous, this sufficient condition is equivalent to the condition that the right-sided derivative of $\Pi(\hat{\beta})$ is not lower than the left-sided derivative at $\hat{\beta}$. As a corollary, the manufacturer will continue to collect more information when $\Pi(\hat{\beta})$ is either linear or convex at $\hat{\beta}$.

We are then ready to characterize the manufacturer's optimal information acquisition strategy. The manufacturer will continue the information acquisition process when $\hat{\beta} < \beta_L$ or $\beta_L < \hat{\beta} < \beta_H$. However, the manufacturer will not collect more information when β_L is reached, because $\Pi(\beta_L) = \alpha d > \Phi(\beta_L|0, \beta_H)\Pi(\beta_H) + [1 - \Phi(\beta_L|0, \beta_H)]\Pi(0) = \frac{(\theta_l - \alpha\theta_h)d}{(1-\alpha)\theta_h}$. It is obvious that the manufacturer's optimal information acquisition strategy is similar to that in the base model.

2. Sequential Information Acquisition and Voluntary Sharing:

Now the retailer's updated posterior belief $\hat{\beta}_r$ may diverge from that of the manufacturer (i.e., $\hat{\beta}_m$), if the latter is not disclosed. We first characterize whether the manufacturer should continue the information acquisition process when its posterior belief reaches $\hat{\beta}_m \in (0, 1)$, conditional on the retailer's updated belief $\hat{\beta}_r(\otimes)$ when no information is received from the manufacturer.[23] Define the manufacturer's updated posterior belief as $\hat{\beta}_m^+$ or $\hat{\beta}_m^-$ when a good or a bad signal is generated, respectively. Similar to Lemma 2, we have:

LEMMA 3: *When information acquisition is sequential and sharing is voluntary, the sufficient condition for the manufacturer to continue to collect information when $\hat{\beta}_m$ is reached is given by:*

$$\hat{\beta}_m \leq \hat{\beta}_r(\otimes) \ \text{or} \ \frac{\Pi(\hat{\beta}_m^+) - \Pi(\hat{\beta}_m)}{\hat{\beta}_m^+ - \hat{\beta}_m} \geq \frac{\Pi(\hat{\beta}_m) - \Pi(\hat{\beta}_m^-)}{\hat{\beta}_m - \hat{\beta}_m^-}.$$

Proof: Consider first the case when $\hat{\beta}_m \leq \hat{\beta}_r(\otimes)$. If the manufacturer stops collecting information, its expected payoff is $\Pi(\hat{\beta}_r(\otimes))$ since it can choose to withhold $\hat{\beta}_m$. Similarly, the manufacturer's expected payoff is at least (equal to) $\Pi(\hat{\beta}_r(\otimes))$ if a good (bad) signal is generated. This implies that the manufacturer should continue the information acquisition process.

Next, consider the case when $\hat{\beta}_m > \hat{\beta}_r(\otimes)$. The manufacturer will choose to disclose $\hat{\beta}_m$ and its expected payoff would be $\Pi(\hat{\beta}_m)$, if it stops information acquisition at $\hat{\beta}_m$. Moreover, by the continuity of information acquisition, we have $\hat{\beta}_r(\otimes) < \hat{\beta}_m^- < \hat{\beta}_m < \hat{\beta}_m^+$. As a result, the manufacturer's

[23]Note that $\hat{\beta}_r(\otimes)$ is not directly conditional on either the manufacturer's updated posterior belief $\hat{\beta}_m$ or the wholesale price ω. This is because the manufacturer's type (i.e., $\hat{\beta}_m$) does not directly influence its payoff, and hence cannot be signaled by the wholesale price. In other words, when the manufacturer does not disclose, the only equilibrium on wholesale pricing is a pooling equilibrium. Nevertheless, as in the base model, the equilibrium characterization of $\hat{\beta}_r(\otimes)$ is dependent on the manufacturer's optimal information acquisition/disclosure strategies.

expected payoff is at least $\Pi(\hat{\beta}_m^+)$ or at least $\Pi(\hat{\beta}_m^-)$, when its updated posterior belief following the collection of an additional signal turns out to be $\hat{\beta}_m^+$ or $\hat{\beta}_m^-$, respectively. The rest of the proof is then similar to that for the mandatory sharing case (i.e., Lemma 2) and obtains readily.

Q.E.D.

It follows that, similar to the mandatory sharing case, the manufacturer will continue the information acquisition process when $\hat{\beta}_m < \beta_L$ or $\beta_L < \hat{\beta}_m < \beta_H$. Moreover, the manufacturer (weakly) prefers to continue information acquisition at $\hat{\beta}_m = \beta_L$, if and only if $\Pi(\beta_L) = \alpha d \leq \Phi(\beta_L|0, \beta_H)\Pi(\beta_H) + [1 - \Phi(\beta_L|0, \beta_H)]\Pi(\hat{\beta}_r(\otimes))$ since the manufacturer will not disclose when $\hat{\beta}_m$ (sufficiently) reaches zero. This condition can be reduced to $\hat{\beta}_r(\otimes) \geq \beta^\# \equiv \frac{[\alpha(2-\alpha)\theta_h - \theta_l]d}{\alpha\theta_h(\theta_h - \theta_l)} \in (0, \beta_L)$.

We can then derive the manufacturer's equilibrium information acquisition/sharing decisions. First, consider the equilibrium whereby the manufacturer continues to acquire information until either β_H or 0 is reached. For this to be an equilibrium, the necessary and sufficient condition is $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + (1-\beta/\beta_H)/2} \geq \beta^\#$, which implies $\beta \geq \beta' \equiv \frac{2(1-\alpha)[\alpha(2-\alpha)\theta_h - \theta_l]d}{\alpha[1-\alpha(3-\alpha)]\theta_h^2 + 2\alpha\theta_h\theta_l - \theta_l^2}$. Next, consider the equilibrium whereby the manufacturer continues to acquire information until either β_L or 0 is reached. For this to be an equilibrium, the necessary and sufficient condition is $\beta < \beta_L$ and $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + (1-\beta/\beta_L)/2} < \beta^\#$, which implies $\beta < \beta'' \equiv \frac{2[\alpha(2-\alpha)\theta_h - \theta_l]d}{\alpha(3-\alpha)\theta_h^2 - (1+\alpha)\theta_h\theta_l} \in (0, \beta_L)$. Moreover, in contrast to the mandatory sharing case, there does not exist an equilibrium whereby the manufacturer continues to acquire information until either β_H or β_L is reached. If otherwise, it must hold that $\beta_L \leq \beta < \beta_H$, which in turn implies $\hat{\beta}_r(\otimes) \geq \beta_L$. But then from Lemma 3 it is better off for the manufacturer to continue information acquisition when $\hat{\beta}_m = \beta_L$ is hit.

Finally, from the above discussion, when $\beta'' \leq \beta < \beta'$, there does not exist pure-strategy equilibrium. In the mixed-strategy equilibrium, it must be that: 1) the manufacturer is indifferent and randomizes between continuing and stopping information acquisition when its updated belief reaches β_L; and 2) the retailer's updated posterior belief when it receives no information from the manufacturer satisfies $\hat{\beta}_r(\otimes) = \beta^\#$. Let us define the probability as λ that the manufacturer continues to accumulate signals when $\hat{\beta}_m = \beta_L$. Note that the manufacturer will disclose if information collection is stopped at $\hat{\beta}_m = \beta_L$ in the mixed-strategy equilibrium, since $\beta_L > \hat{\beta}_r(\otimes)$. The mixed-strategy equilibrium then requires $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + \lambda(1-\beta/\beta_h)/2} = \beta^\#$, which determines $\lambda = \frac{\beta_H(\beta - \beta^\#)}{\beta^\#(\beta_H - \beta)} \in (0, 1)$.

It is clear that, in comparison to mandatory sharing, there is more information generated in equilibrium since the probability is not always zero that the manufacturer continues information acquisition at $\hat{\beta}_m = \beta_L$. Moreover, in comparison to the base model, flexible wholesale pricing may lead to more equilibrium information acquisition, e.g., $\beta'' < \beta_L$.

Continuous Demand:

We begin by characterizing the retailer's optimal decisions. The retailer's profit, at the time of setting the retail price, is $\pi = p \min\{x, a - bp\}$. This leads to the optimal retail price $p(x) = \max\{\frac{a-x}{b}, \frac{a}{2b}\}$. The retailer's expected payoff, when the ordering decision is made, is then:

$$\pi(x) = \begin{cases} [\hat{\beta}_r a_h + (1-\hat{\beta}_r)a_l - x]x/b - x\omega, & \text{if } x \le a_l/2; \\ \hat{\beta}_r(a_h - x)x/b + (1-\hat{\beta}_r)a_l^2/(4b) - x\omega, & \text{if } a_l/2 < x \le a_h/2; \\ \hat{\beta}_r a_h^2/(4b) + (1-\hat{\beta}_r)a_l^2/(4b) - x\omega, & \text{if otherwise.} \end{cases}$$

This yields the optimal retail ordering:

$$x(\hat{\beta}_r) = \begin{cases} [\hat{\beta}_r a_h + (1-\hat{\beta}_r)a_l - b\omega]/2, & \text{if } \hat{\beta}_r \le b\omega/(a_h - a_l); \\ (\hat{\beta}_r a_h - b\omega)/(2\hat{\beta}_r), & \text{if otherwise.} \end{cases}$$

1. Inflexible Wholesale Price:

Suppose that the wholesale price is set prior to information acquisition. The manufacturer's expected payoff at the time of making the information acquisition decision is given by $\Pi = x(\hat{\beta}_r)\omega$. When information acquisition is inflexible and sharing is mandatory, the manufacturer's expected payoff of acquiring information would be $\beta(a_h - b\omega)\omega/2 + (1-\beta)(a_l - b\omega)\omega/2$, which is (weakly) lower than the expected payoff without information acquisition (i.e., $x(\beta)\omega$) for all $\beta \in (0, 1)$. It follows that in equilibrium the manufacturer does not acquire information even when the information state is $I = y$. Consider then the case when information acquisition is inflexible and sharing is voluntary. Noticing that $\Pi = x(\hat{\beta}_r)\omega$ is an increasing function in the retailer's updated posterior belief $\hat{\beta}_r$, we can then readily obtain that acquiring information is the unique equilibrium strategy when $I = y$.

Next, consider the case when information acquisition is sequential and sharing is mandatory. Note that the manufacturer's expected payoff is continuous, linear in the firms' (symmetric) posterior belief $\hat{\beta}$ when $\hat{\beta} < b\omega/(a_h - a_l)$, and concave when $\hat{\beta} > b\omega/(a_h - a_l)$. Following Lemma 2, we know that the manufacturer will not stop the information collection process for all $\hat{\beta} < b\omega/(a_h - a_l)$. Therefore, at any $\hat{\beta} \ge b\omega/(a_h - a_l)$, the manufacturer will continue to collect information, if and only if it does not stop at any posterior belief that is lower than $\hat{\beta}$ and furthermore it is satisfied that $x(\hat{\beta}) < \frac{\hat{\beta}}{\hat{\beta}^+}x(\hat{\beta}^+) + \frac{\hat{\beta}^+ - \hat{\beta}}{\hat{\beta}^+}x(0)$. This latter condition is equivalent to $\frac{x(\hat{\beta}^+) - x(\hat{\beta})}{\hat{\beta}^+ - \hat{\beta}} > \frac{x(\hat{\beta}) - x(0)}{\hat{\beta}}$, where the left-hand side of the inequality represents the (right-sided) derivative of $x(\cdot)$ at $\hat{\beta} \ge b\omega/(a_h - a_l)$. In summary, the manufacturer will continue the information acquisition process if and only if the posterior belief reaches either zero or an upper bound $\overline{\beta}$ that satisfies $\frac{\partial x(\overline{\beta})}{\partial \overline{\beta}} = \frac{x(\overline{\beta})-x(0)}{\overline{\beta}}$, where $\overline{\beta} > b\omega/(a_h - a_l)$. Simplifying, we obtain $\overline{\beta} = 2b\omega/(a_h - a_l + b\omega)$.

Finally, the case when information acquisition is sequential and sharing is voluntary is similar to the mandatory sharing case, except that now $\overline{\beta}$ is determined by solving $\frac{\partial x(\overline{\beta})}{\partial \overline{\beta}} = \frac{x(\overline{\beta})-x(\hat{\beta}_r(\otimes))}{\overline{\beta}}$, where $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+(1-\beta/\overline{\beta})/2}$. This is because the manufacturer will not disclose to the retailer the lower bound of information acquisition (i.e., zero) if the lower bound is reached. Noticing that $\hat{\beta}_r(\otimes) > 0$ and $x(\cdot)$ is concave at $\overline{\beta}$, it is straightforward that the upper bound of information acquisition is higher under voluntary sharing than under mandatory sharing.

2. Flexible Wholesale Price:

Suppose then that the wholesale price is set after information acquisition. We can readily obtain that the optimal wholesale price is $\omega^* = d$ and the manufacturer's equilibrium sub-game payoff in Stage 3 is:

$$\Pi(\hat{\beta}_r) = \begin{cases} [\hat{\beta}_r a_h + (1-\hat{\beta}_r)a_l - bd]d/2, & \text{if } \hat{\beta}_r \leq bd/(a_h - a_l); \\ (\hat{\beta}_r a_h - bd)d/(2\hat{\beta}_r), & \text{if otherwise.} \end{cases}$$

It follows that the impact of the retailer's updated posterior belief $\hat{\beta}_r$ on the manufacturer's payoff, and thus the manufacturer's equilibrium information acquisition/sharing strategies, are qualitatively the same as those when the wholesale price is set prior to information acquisition.