A Model of Direct and Intermediated Sales

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We examine a model in which an upstream firm can sell directly online and through heterogeneous intermediaries to heterogeneous consumers engaging in time-consuming search. Direct online sales may be more or less convenient and involve costly returns if the good fits consumers poorly. Direct selling appeals to higher-value consumers and increases the upstream firm’s profits by allowing price discrimination. Competition and segmentation due to direct sales results in lower intermediary prices, making all consumers better off. Thus, entry by an upstream firm increases consumer surplus at the expense of intermediaries with the net result being an increase in social welfare.

Traditionally, services provided by intermediaries, such as managing inventory, breaking bulk shipments, supplying information, marketing, and coordinating transactions, have been sufficiently difficult for producers that a substantial part of the economy has utilized intermediaries.\(^1\) Advances in technology now allow producers to cost-effectively perform many tasks that previously required intermediaries. This trend is evident in 68% of consumer goods manufacturers planning on selling online (Forrester, 2000) and manufacturers’ Web sales taking in 15% of the total retail e-commerce revenue (Boston Consulting Group, 2000).

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\(^1\) See Spulber (1996b) for a discussion of intermediation in the economy and Spulber (1999) for models of intermediation and the theory of the firm in a number of settings.
This paper studies this disintermediation by modeling the sale of goods by an upstream firm directly online to consumers as well as through an intermediated sales channel. Beyond characterizing when an upstream firm sells directly, indirectly, or both, our model shows that the upstream firm attracts the high-value consumers with direct sales, and that the upstream firm always chooses the channel structure that maximizes consumer and social welfare.

We begin by examining a traditional intermediated market using a sequential-consumer-search and intermediary-competition model closely related to Spulber (1996a). A continuum of intermediaries with different transaction costs purchase a good from the upstream firm at a wholesale price. A continuum of consumers with different reservation values search across the intermediaries, discounting their gains as they search, until they find a sufficiently low price and purchase. This discounting implies that higher value consumers search less because they gain the most by avoiding search. The intermediaries compete against each other based on their expectation of the consumers’ search decisions. It is optimal for each intermediary to charge a unique price and the heterogeneity of the intermediaries’ transaction costs results in price dispersion. Consumers’ search decisions are endogenous and based on their expectation of the intermediaries’ prices, and we solve for the unique rational expectations equilibrium.

Spulber (1996a) examines how intermediaries set bid and ask prices to make a market between symmetric buyers and sellers, whereas our model introduces asymmetry between the consumers and the upstream firm by viewing the intermediary as a price taker with respect to the upstream firm and a price maker to the consumers. We extend the model of consumer search and intermediary entry and competition by including the possibility that the upstream firm sells directly online to consumers. This allows us to construct a model related to the search, vertical integration, and channel management literatures to address the effects of direct sales on upstream firm, intermediaries, consumers, and welfare.

2. Katz and Shapiro (1986), Hart and Tirole (1990), O’Brien and Shaffer (1992), McAfee and Schwartz (1994), and others study an upstream firm selling to competing downstream firms with a focus on contracting issues.

3. The marketing literature examines many issues in channel management. The decision to integrate depends on the uniqueness of the manufacturer’s product with highly substitutable products tending to be sold through independent retailers in a decentralized system (McGuire and Staelin, 1983; Trivedi, 1998). Coughlan (1985) shows that middlemen can mitigate direct price competition at the manufacturer level. Choi (1991) considers retailers who carry an assortment of products from multiple manufacturers. Purohit (1997) examines manufacturers selling through dealers and car rental agencies and studies arrangements that can be used to minimize channel conflict. Research focusing on different channel structures and decisions includes Jeuland and Shugan (1983), Moorthy (1987), and others.
After analyzing the exclusively intermediated market, we study the impact of technology allowing an upstream firm to sell directly online to consumers. As a benchmark we also solve the model with only direct sales. Direct online sales also involve search/delay, but with a potentially different discount factor. Next, we allow the upstream firm to utilize both channels and solve for when the upstream firm chooses to use the direct or intermediated channel exclusively or both channels in tandem. Because we are interested in how technology enabling direct sales is changing the ways goods are bought and sold by manufacturers, retailers, and consumers, we focus on online sales by manufacturers making direct sales to systematically differ from intermediated sales. Specifically, we assume that there is a probability the good does not fit consumers and they must physically touch or try the good to be certain. At intermediaries consumers can determine if the good is suitable without purchasing it, but this is not possible for online sales, where, if the good does not fit, it is returned. Returns are costly for the upstream firm and allow the physical intermediaries to provide a different kind of service than is possible for the upstream firm to provide online.

With the option of direct sales, the upstream firm may continue to use the intermediated channel for several reasons: the intermediaries may provide a more efficient channel in terms of transaction and return costs and the correlation between the consumers’ values and search costs due to discounting allows price discrimination through a higher direct price. However, the intermediaries mark up the wholesale price and intermediated sales may involve additional search, which delays those sales. Direct sales attracting higher value consumers makes it optimal for the upstream monopolist to induce lower intermediary prices. Therefore, with direct sales consumers have the option of choosing between the direct price and intermediary prices that are lower than when only the intermediated channel exists. A revealed preference argument guarantees that all consumers are better off when the upstream firm sells directly online. Although the model imposes significant structure, this result appears quite general. Entry by the upstream firm increases its profits, whereas the intermediaries lose sales and profits. The gains by the upstream firm and consumers are larger than the losses by the intermediaries, so overall social welfare increases. Although this result holds everywhere in our model, it is difficult to be sure it extends beyond our setup.

4. We will often omit the online when referring to direct sales, but unless specifically stated otherwise direct sales mean direct online sales. When we refer to intermediated sales we mean intermediated sales through physical stores. The implications our model may have for retailers/intermediaries selling online are discussed in Section 4.
The model makes a number of empirical predictions. First, direct sales will only be used for products where the cost/inconvenience of searching the upstream firm directly is less than the total cost of searching the intermediaries. Second, when direct sales are used the direct price is higher than the average intermediary’s price. This second prediction is consistent with the findings of Carlton and Chevalier (2001)—for both industries for which they have data, fragrances and DVD players—and of Bell, Wang, and Padmanabhan (2003) for a number of other products. Finally, our model makes predictions about which types of goods are well suited for sale online: goods where immediacy is less important, for example, good purchased regularly at predictable times; goods where breaking bulk at intermediaries does not significantly reduce transactions costs as opposed to shipping directly to individual consumers, for example, less bulky goods; goods less widely available at retailers, for example, specialty goods; and goods where touch and feel is less important, for example, commodities, standardized goods, goods with strong brands, or goods where consumer tastes are more homogeneous.

A number of papers examine selling through multiple channels. These studies assume homogeneity of consumers’ values and the costs of buying directly (Liu and Zhang, 2002) or assume a demand structure without modeling individual consumer choice between channels (Purohit, 1997; Bell et al., 2003). For a critique of the limitations of the latter approach see Lee and Staelin (2000). Our search model examines the issue of channel competition with heterogenous consumers’ values and choices, heterogeneity in transaction costs within the intermediated channel, and heterogeneity in transaction and search costs across the intermediated and direct channels.

Our model is also related to a number of papers with search models but where symmetric buyers and sellers can search and trade directly with each other or use an intermediary. Yavas (1994) allows traders to either search for each other or use an intermediary to match them together. Gehrig (1993) allows traders to either search for each other or trade directly with an intermediary who quotes buy (bid) and sell (ask) prices. He finds that traders with the strongest desire to trade—sellers with lowest valuations and buyer with the highest valuations—use the

5. Liu and Zhang (2002) examine the effect of the upstream firm having a direct channel and discuss how it can increase prices and lower consumer welfare. However, their assumption that all consumers always purchase limits the upstream firm’s incentive to lower price with the addition of the direct channel.

6. In a model without search, Fingelton (1997) allows for symmetric buyers and sellers to trade either with intermediaries or directly with each other with an exogenous probability of finding a match.
intermediary and those with weaker desires to trade search for each other or are inactive. Yavas (1996) extends this to endogenize the search intensity of the buyers and sellers. He finds a similar segmentation of which mechanism traders use and that the addition of the price quote intermediary narrows the price dispersion of prices and reduces the search intensity of traders.

In a related and independently developed model, Rust and Hall (2003) extend Spulber’s (1996a) model to study the situation where traders can search middlemen for prices or go directly to a market maker quoting observable prices. Beyond the asymmetry between the seller and buyers, the main difference between Rust and Hall (2003) and this paper is the ability for the seller to trade directly with the buyers. Rust and Hall (2003) also find that traders with a stronger desire to trade use the market makers. The market segmentation in Rust and Hall (2003), Gehrig (1993), and Yavas (1994, 1996) is similar to what we find. The asymmetry between a continuum of consumers and a single upstream firm in our model helps link the literature on search, matching, and market making with the marketing literature focusing on the decisions of an upstream firm with market power.

Section 1 studies the pure intermediated and pure direct market structures where all sales either occur through intermediaries or directly from the upstream firm. In Section 2 we solve for the upstream firm’s optimal choice of market structure, and the equilibrium and its characteristics when both direct and intermediated sales are used. Section 3 analyzes the effect of introducing direct selling on the upstream firm, intermediaries, and consumers. Section 4 discusses the model’s implications for online intermediated sales and channel conflict between the upstream firm and the intermediaries. Section 5 provides a summary and discussion.

1. Individual Market Structures

We first consider a model where a monopolist upstream firm sells its product through a continuum of intermediaries, which differ in their costs for selling to consumers. Intermediaries take the upstream firm’s price as given, but act as price makers to the consumers. Although consumers are price takers from each intermediary, consumers have the option of searching other intermediaries for a better price. The most natural example of this scenario is a manufacturer selling its product to retailers who then sell to consumers.

Many models of intermediation assume symmetry between buyers and sellers (e.g., see Rubinstein and Wolinsky, 1987; Gehrig, 1993; Biglaiser, 1993; Spulber, 1996a; Yavas, 1992, 1994, 1996; Rust and Hall,
We focus on an asymmetric model in which the seller acts as a monopolist and focus on its setting the intermediaries’ wholesale price. These intermediaries compete for consumers who search as in Spulber (1996a). In Sections 1.6 and 2 we extend the model by allowing direct-to-consumer sales.

1.1 Model Overview

Our discussion of the intermediaries’ competition and consumers’ search in this section is relatively brief due to the treatment available in Spulber (1996a) and Rust and Hall (2003). There are three types of actors in the model: an upstream firm, intermediaries, and consumers. A continuum of intermediaries with different transaction costs purchase a good from the upstream firm at a wholesale price. A continuum of consumers with different values search across the intermediaries, discounting their gains as they search, until they find a sufficiently low price and purchase. Individual intermediaries’ prices, $p_I$, are revealed to consumers only through search. All players are risk neutral and optimize their expected gains. The decision making follows three stages:

1. The upstream firm determines a wholesale price $w$ for all intermediaries that is stationary over the time consumers search.  
2. Based on its expectation of the consumers’ search behavior, each intermediary determines whether or not to enter, its price $p_I$, and buys from the upstream firm each period; intermediaries’ prices are also stationary over consumers’ search.
3. Based on the expected distribution of the intermediaries’ prices, each consumer decides whether or not to search and his reservation price $r$. Consumers search until they find an intermediary with a price less than or equal to their reservation price, $p_I \leq r$.

The time spent searching is an implicit transaction cost and all players discount their profits by the same discount factor. Discounting

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7. This approach fits financial intermediaries, particularly specialists, dealers, and market makers well, e.g., Glosten and Milgrom (1985), but does not satisfactorily model intermediaries such as retailers, who set prices to consumers, but take prices from larger firms such as manufacturers or distributors.

8. Our assumption that the monopolist charges a uniform wholesale price can be viewed as stemming from nondiscrimination requirements of the Robinson–Patman Act. We focus on linear pricing by the upstream firm because in search models competition between the intermediaries is typically decreasing in their number (as in Stahl, 1989; Spulber, 1996a). This makes more complicated contracts yield extreme results, e.g., under a two-part tariff the upstream firm will choose a fixed fee to maximize intermediary competition through inducing an arbitrarily small number of intermediaries to enter.
gives the intermediaries market power because continuing to search for a lower price means consumers must temporarily forgo the gains associated with owning the good. Discounting also implies that the consumers’ search costs are increasing in their values.

1.2 Consumers

We begin with consumers’ search decisions conditional on their expectations of the distribution of the intermediaries’ prices. Consumers are price takers from each intermediary, purchase at most one unit of the good, and have heterogeneous valuations for the good, represented by a willingness to pay \( v \) that is uniformly distribution on the unit interval. As in Rust and Hall (2003) each consumer remains in the market for a random (geometrically distributed) length of time before permanently exiting with \( \lambda \in [0, 1] \) being the probability of a consumer exiting the market at the end of each period. Whenever a consumer exits the market, he is replaced by a new consumer who is randomly drawn from the uniform distribution on the unit interval. Consumers entry and exit corresponds to consumers giving up if search takes too long and facilitates focusing on a stationary pricing policy. We assume no consumer exits before having the opportunity to search once.

Consumers have an expectation of the equilibrium distribution of intermediaries’ ask prices \( F(p_i) \), but individual intermediaries’ prices are revealed only through search. Discovering an intermediary’s price takes time, delaying the consumer’s benefit from purchasing the good. The cost of delay is modeled as the time taken for each search—each search can be thought of as a period in the model—and gains from purchasing are discounted by a factor \( \beta \in (0, 1) \).

Beyond searching for price, consumers also face uncertainty about the suitability of the good for their needs. This uncertainty is fully resolved by inspecting/trying the good while searching an intermediary. With probability \( \alpha \) a consumer is unsatisfied with the good, balks at purchasing, and receives zero utility. Consumers that search and find the good unsuitable stop searching. As with other consumers, they exit the market with probability \( \lambda \) at the end of each period.

9. If consumers succeed in making a purchase prior to exiting, the unitary demand assumption implies that they will not make any subsequent transactions afterwards.
10. When \( \beta = 1 \), the intermediaries’ prices are transparent to the consumers and search is costless. Bertrand competition results in only the lowest-cost intermediary surviving in the market and charging the price at its marginal cost. In contrast, \( \beta = 0 \) is the case in which liquidity or immediacy is extremely valued and each consumer buys from the first possible intermediary. These degenerate cases without search are excluded from our analysis throughout.
Goods differ in the ability of consumers to ascertain the quality of products without inspecting it. Commodity products, such as oil, gold, memory modules, and stock shares, have very low values of $\alpha$. Relatively standardized products and goods that can be easily described online, for example, books and CDs, have low values of $\alpha$. Repeat purchases and goods with strong and well-known brands also can have low values of $\alpha$. Goods where taste and perception of quality differ across consumers, for example, furniture, wine, art, and clothes, have higher values of $\alpha$.

A consumer with value $v$ arriving at an intermediary with price $p_I$ maximizes the value function $V(v, p_I)$ which satisfies the Bellman equation

$$V(v, p_I) = \max \left\{ 0, (v - p_I), \beta (1 - \lambda) \int V(v, p'_I) dF(p'_I) \right\}. \quad (1)$$

Because all intermediaries sell the identical good, consumers’s first search determines whether the good is suitable. Therefore, (1) represents a consumer’s choice if the good is suitable and, prior to searching, a consumer has expected utility $(1 - \alpha)\beta \int V(v, p'_I) dF(p'_I)$. Conditional on the good being satisfactory, each consumer with a willingness to pay $v$ has a reservation price $r$ for searching the intermediaries. A consumer will purchase the good from the first intermediary who has price $p_I$ less than or equal to the reservation value $r$. The reservation value is defined by a standard recursive equation (Weitzman, 1979; Spulber, 1996a, and others):

$$v - r = \beta (1 - \lambda) \left[ \int_{p_I}^{\bar{p}_I} (v - p_I) dF(p_I) + \int_{r}^{\bar{p}_I} (v - r) dF(p_I) \right]. \quad (2)$$

where a solution exists. The second term in brackets on the right side of (2) is the consumer’s expected gain from searching subsequent intermediaries if the next intermediary has too high a price. The first term in brackets represents the expected gain if the consumer purchases from the next intermediary. Simplifying the recursive equation by integrating by parts yields

$$v = v(r) \equiv r + \frac{\beta (1 - \lambda)}{1 - \beta (1 - \lambda)} \int_{0}^{r} F(p_I) dp_I. \quad (3)$$

The consumer value corresponding to a reservation price of $r$, $v(r)$, is continuous and strictly increasing on $r \in [0, \bar{r}]$, implying that it has a unique solution and that consumers with higher valuations of the good

11. Nelson (1970) suggested that goods possess either search or experience qualities. Search qualities are those that “the consumer can determine by inspection prior to purchase,” and experience qualities are those that “are not determined prior to purchase” (Nelson, 1974).
will have higher reservation values and engage in less search.\textsuperscript{12} This relationship means that the discounting of gains implies a correlation between consumers’ values and their search costs. This feature is exploited in Section 2 to derive a natural segmentation of consumers between direct and intermediated sales. Consumers who begin searching continue until they find a price beneath their reservation value or exit. Consumers with reservation values below the lowest intermediary price do not search.

1.3 Intermediaries

Similar to consumers’ valuations, intermediaries’ transaction costs, $k_I$, have a uniform distribution on the interval $[k_{\bar{I}}, 1]$ and exhibit constant returns to scale.\textsuperscript{13} The lower bound $k_{\bar{I}} \in [0, 1)$ is the transaction cost of the most efficient intermediary. Differences in transaction costs across firms represent differences in technology, operations, and managerial ability. Intermediaries purchase from the upstream firm in each round of search and do not carry inventory. For consistency and tractability we assume that intermediaries discount their gains in the same manner as consumers.

Based on consumers’ search behavior and other intermediaries’ prices, each intermediary determines whether or not to be active. There are $N^I$ intermediaries in the market, where $k_{\bar{I}} = N^I + k_{\bar{I}}$ is the highest transaction cost an intermediary can have and still attract consumers.\textsuperscript{14} Each intermediary buys from the upstream firm at a price $w^I$ and then sets a price $p^I$ to maximize the present discounted value of profits, trading off profits from current and future transactions. All consumers confront the same positive discount factor, $\beta$, and no consumers are completely informed, resulting in intermediaries following pure strategies.\textsuperscript{15}

\textsuperscript{12} As $v$ approaches the lower bound of the distribution of intermediaries’ prices, the second term of (3) goes to zero, implying that $r$ goes to $v$. A consumer with value equal to the lowest intermediary’s price has $v = r$ and searches until finding the lowest price. This consumer is similar to the informed customers in Varian (1980) and the shoppers in Stahl (1989).

\textsuperscript{13} Transaction costs include all the costs associated with each sale, such as clerical services, managing inventory, breaking bulk shipments, supplying information, marketing, and coordinating transactions. The paper’s results hold under more general costs distributions, e.g., the density of intermediaries with cost $k$ being $k^\gamma$.

\textsuperscript{14} Throughout the paper we use the superscript to denote the appropriate market structure. The endogeneous variables have superscripts $I$, $D$, and $H$ in the intermediated, disintermediated, and hybrid market structures. Subscripts for prices and transactions costs denote either the upstream firm, $U$, or the intermediaries, $I$.

\textsuperscript{15} Models in which some consumers are informed—have zero search cost—whereas others have positive search costs result in an equilibrium in which identical firms follow mixed strategies by using promotions to occasionally attract the informed consumers, while attracting only uninformed consumers when they are not the lowest-price firm (see Salop and Stiglitz, 1977; Varian, 1980; Stahl, 1989).
Hence, price dispersion is due to the heterogeneity in intermediaries’ costs and consumers’ willingness to pay.

Based on the distribution of prices, consumers search each period and buy from the first intermediary with the product charging a price below their reservation value. The implicit assumption is that consumers do not have information about individual intermediaries’ costs. This is less suitable in situations where intermediaries have well known price reputations such as products with significant repeat purchases.

Let $h(r)$ be the density of consumers’ reservation prices. Given that $N^I$ represent the endogenous number of active intermediaries, the per-intermediary density of consumers equals $h(r)/N^I$. Each active intermediary receives an equal share of consumers searching in each round of search. By the Law of Large Numbers, the number of active consumers with reservation value $r$ that visit the intermediary can be calculated. The number of consumers with reservation value $r$ visiting the intermediary equals the density of consumers per intermediary for the first round of searches, $1 - F(r)$ for the second round of searches, $(1 - F(r))^2$ for the third round of searches, and so on. After adjusting for consumers that exit without purchasing, the intermediary’s demand in the $i$th round of searches is obtained by integrating over the set of consumers with reservation values higher than the intermediary’s price but lower than the upstream firm’s price:

$$D_i(p_I) = (1 - \alpha)(1 - \lambda)^{i-1} \int_{p_I}^{\bar{r}} (1 - F(r))^{i-1} \frac{h(r)}{N^I} dr \quad (i = 1, 2, 3, \ldots).$$

(4)

New consumers enter the market each period and the intermediaries discount sales with each round of search to calculate the discounted demand as a function of their price. Define the discounted demand function $D^I_I(p_I)$:

$$D^I_I(p_I) \equiv \sum_{i=1}^{\infty} \beta^i D_i(p_I) + \lambda \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \beta^i D_i(p_I) = (\bar{r} - p_I) \frac{(1 - \alpha)\beta}{(1 - \beta)N^I}.$$

(5)

The model’s structure is such that each intermediary’s demand is a function of the number of other intermediaries, but not their prices. So for any market price distribution function, the discounted demand is linear and each intermediary’s discounted profits are

16. Note that because individual consumers are infinitesimally small there is no uncertainty about intermediaries’ demand. Therefore, as in Spulber (1996a) and Rust and Hall (2003), the intermediary purchases the exact amount it sells each period.
\[ \Pi_I(k_I) = (p_I - w - k_I)D_I(p_I) = \frac{(1 - \alpha)\beta}{(1 - \beta)N_I} (p_I - w - k_I)(\bar{r} - p_I). \]  

As in Spulber (1996a) and Rust and Hall (2003), this allows for the rational expectations equilibrium to be solved relatively easily. The following Lemma characterizes the consumers’ search behavior and the intermediaries’ competition.

**Lemma 1:** In the intermediated market structure, given the upstream firm’s price \( w^I \), there exists a unique equilibrium in which

(i) the consumers are segmented by cutoff value \( \bar{v}^I = \frac{1}{2}(\bar{r}^I + w^I + k_I) \): \( v < \bar{v}^I \) are inactive consumers; \( v \geq \bar{v}^I \) are consumers who search and purchase from the intermediaries;

(ii) intermediaries are segmented by a cutoff value \( \bar{k}_I = \bar{r}^I - w^I \): \( k_I \leq \bar{k}_I \) are active intermediaries who purchase goods from the upstream firm and then sell them to the consumers; \( k_I > \bar{k}_I \) are inactive intermediaries, \( N_I = \bar{k}_I - k_I \); an intermediary with cost \( k_I \) charges price \( p_I(k_I) = \frac{1}{2}(\bar{r}^I + w^I + k_I) \) and the distribution of the intermediaries’ prices is \( F_I(p_I) = \frac{2}{N_I}(p_I - \bar{v}^I) \), where \( \bar{r}^I, \bar{v}^I, \) and \( \bar{k}_I \) are defined in the Appendix.

Given the upstream firm’s wholesale price, \( w^I \), Lemma 1 provides the specifics of the intermediated market structure shown in Figure 1. Intermediaries’ market power is captured entirely in the lowest cost intermediary’s price and other intermediaries can add only half of their costs to their price. This implies that intermediaries with costs above the difference between the highest reservation value and the wholesale price charge prices above the highest consumer reservation value, making them unable to attract any consumers. Therefore, these intermediaries are inactive. Consumers with values less than the lowest cost intermediary’s price will never find an intermediary with a price less than their reservation value and do not search.

### 1.4 Upstream Firm

Having found the intermediary competition and consumer search equilibrium conditional on the upstream firm’s wholesale price, we now turn to the upstream firm’s problem. The upstream firm has sufficient capacity to meet demand and production costs are normalized to zero. The upstream firm discounts profits in the same manner that intermediaries and consumers discount their gains. The upstream firm’s discounted demand is the integral of the individual intermediaries’ discounted demands given in Lemma 1 and is
The upstream firm’s profit is the product of the wholesale price and discounted demand:

$$\Pi_{UI} = w^I D^I.$$  

(8)

Intermediaries enter until the highest value consumer does not search. This implies that the zero-quantity wholesale price is independent of the discount factor, $\beta$. Although reducing $\beta$ does not
change the intercept of the demand curve, it does steepen the slope. Demand curves of this type result in an optimal price that is independent of the parameter affecting the slope (Klemperer and Meyer, 1986). Therefore, the unique equilibrium in the intermediated market is given in the following proposition where the upstream firm charges the monopoly price, less one-half of the lowest intermediaries’ cost.

**Proposition 1:** In the intermediated market structure there exists a unique equilibrium pricing policy in which the upstream firm charges a wholesale price \( w' = \frac{1-k_I}{2} \).

### 1.5 Characteristics of the Intermediated Market

Before studying direct selling by the upstream firm and its effects, it is useful to understand the role the exogenous parameters play in the intermediated market structure. A lower discount factor decreases cost for consumers searching. A higher rate of exit/entry increases the cost of continued search because the consumer may exit before finding a sufficiently low price. Table I provides a summary of the comparative statics of these parameters on the different market participants.

The higher discount factor increases consumer search, reducing intermediaries’ market power and eliminating higher cost intermediaries’ demand, forcing them out of business. Given that the distribution of the intermediaries’ prices is uniform in Lemma 1, reducing the number of active intermediaries also decreases the variance of the intermediaries’ prices. Changes in the discount factor affects individual intermediaries differently depending on their costs. The increase in consumer search associated with lower search costs results in higher cost intermediaries’
profits decreasing and the lower cost intermediaries’ profits increasing. As search becomes easier for the consumers, aggregate demand increases, but competition between intermediaries increases, reducing their markup. For the higher cost intermediaries, the fall in markup outweighs the rise of demand, decreasing their profits. The reverse is true for the lower cost intermediaries with the demand effect outweighing the reduced markup. The aggregate profits for all intermediaries are
\[(1 - \alpha)\beta NI^2 \frac{1 - \beta}{12(1 - \beta)}\], which increase with the exit rate \(\lambda\) and the discount rate \(\beta\).

Given that fraction \((1 - \alpha)\) of the consumers will find the good unsuitable, consumers survive at least one round of search, and the recursive search equation (2) is written for a consumer at an intermediary, the expected surplus upon arrival for a consumer is \((1 - \alpha)\frac{v - r}{1 - \lambda}\).

As in Rust and Hall (2003), \((1 + \frac{\beta \lambda}{1 - \beta})\) adjusts for the discounted surplus of all current and future consumer who enter the market; aggregate consumer surplus is calculated by

\[
CS^l = (1 - \alpha)\left(1 + \frac{\beta \lambda}{1 - \beta}\right) \int_{v^l}^1 \frac{v - r}{1 - \lambda} dv(r) = \left(1 + \frac{\beta \lambda}{1 - \beta}\right) \frac{(1 - \alpha)\beta(4 - \beta(1 - \lambda))(1 - k_1)^2}{24(4 - 3\beta(1 - \lambda))^2}.
\] (9)

A higher discount factor increases competition between the intermediaries, leading to more active consumers, lower prices, and higher consumer surplus. Thus, consumers and the upstream firm are always better off with a higher discount factor. It can be shown that the sum of the intermediaries’ profits and consumer surplus is one-half the upstream firm’s profits. Hence, social welfare is also increasing in the discount factor \(\beta\):

\[
W^l = \Pi_U^l + \Pi_I^l + CS^l = \left(1 + \frac{\beta \lambda}{1 - \beta}\right) \frac{3(1 - \alpha)\beta(1 - k_1)^2}{8(4 - 3\beta(1 - \lambda))} = \frac{3}{2} \Pi_U^l.
\] (10)

Increasing the entry and exit rate of consumers, \(\lambda\), decreases their desire to search, reducing their individual gains. This impatience is exploited to benefit the intermediaries and the upstream firm. The exit rate of consumers also increases the rate at which new consumers arrive. The impact of the faster arrival rate of new consumers can be large enough to increase total consumer surplus.
1.6 Disintermediated Market Structure

Although most sales have historically been through intermediaries, technology has dramatically decreased the difficulties and associated costs for firms to sell directly to consumers. We abstract away from many of the details of the relatively efficiency of direct sales from intermediated sales by assuming the upstream firm can provide the same services as the intermediaries by incurring a transaction cost $k_U \geq 0$. The cost of direct versus intermediated sales varies across products based on the costs and benefits of the functions intermediaries perform. For example, breaking bulk at the individual consumer rather than at the store level is likely more expensive for large bulky items. Items that are easily described or tested online, for example, CDs or books, may be less costly to provide information to consumers online.

Direct sales enable consumers to view the product pictures and descriptions, and make purchase decisions based upon information provided by the upstream firm on its website. Unlike trying the product at the intermediaries’ physical locations, consumers cannot fully judge the suitability of the product without purchasing it. Accordingly, we assume that consumers purchase the product, find it unsuitable, and return it with probability $\alpha$. In this case the upstream firm incurs the transaction costs $k_U$ and an addition costs, $c$, associated with returns. The costs of returns consist primarily of processing, logistics, restocking, and potential discounting needed to sell no longer “new” units. Thus, the upstream firm keeps the direct sale price, $p_D^U$, with probability $1 - \alpha$, always incurs the transaction costs, $k_U$, and incurs the return cost, $c$, with probability $\alpha$.17

Direct sales are discounted by a factor $\beta_U$ that can be above, below, or the same as the discount factor for the intermediated sales $\beta$. The discount factors correspond to how convenient (time consuming) it is for consumers to use each channel. If prices are equal, the channel with the higher discount factor is more attractive for all consumers with the difference in convenience between channels increasing in consumers’ values. Goods that are information/digital, for example, music, newspapers, electronic airline tickets, and so forth, can be easily found and delivered online. Specialized goods only available at few

17. In reality consumers also incur some costs associated with returns, e.g., time, effort, and, possibly, return shipping. Including these costs greatly complicates the rational expectation equilibrium between consumers’ search decision and intermediaries’ competition/prices, so we assume these costs are zero. One interesting feature this rules out is consumers testing the good in a store and then buying it directly. As we show in Section 2, the direct price is higher than the average intermediaries price, so this testing at an intermediary and buying directly would only occur at the highest price intermediaries. Free riding off the services provided by intermediaries is discussed more generally in Section 4.
stores should also be more convenient to buy online, whereas goods widely held at numerous intermediaries for which consumers value immediate delivery may have a higher discount factor for intermediated sales.

Before exploring how the upstream firm may utilize direct sales in conjunction with the intermediated channel, we examine the case where all sales are disintermediated. The upstream firm’s direct price is denoted \( p^D \). As in the intermediated market structure there exists a cutoff value for which consumers with values below that cutoff are not active in the market. With only direct sales there is only one firm for consumers to search, so in equilibrium the consumers know the upstream firm’s price and search is irrelevant. Hence, consumers with willingness to pay \( v \geq p^D \) purchase the product, making the upstream firm profit function

\[
\Pi^D = \left(1 - \alpha \right) p^D - k_U - \alpha c \right) \beta U \left(1 - \frac{\beta U}{1 - \beta}\right).
\]

**Proposition 2:** In the disintermediated market structure, there exists a unique equilibrium pricing policy in which (i) the upstream firm charges price \( p^D = \frac{1}{2} + \frac{k_U + ac}{2(1 - \alpha)} \); (ii) consumers are segmented by \( \frac{1}{2} + \frac{k_U + ac}{2(1 - \alpha)} \).

The above proposition shows that the upstream firm charges the monopoly price plus compensation for its transaction and return costs. Demand is decreasing with both these costs, as are the upstream firm’s profit and consumer welfare. The probability of a return increases prices and decreases profits.

### 2. Hybrid Market Structure

Having analyzed intermediated and direct sales in isolation, this section studies when and how innovations in technology that allow firms to sell directly online to their consumers will be utilized by an upstream firm to replace the intermediaries partially or entirely. As with the individual channels, the upstream firm chooses wholesale price \( w^H \) to the intermediaries and direct price \( p^H \) to the consumers. We assume that intermediaries can purchase at either the wholesale or direct price. Hence, there is no benefit to the upstream firm setting the direct price below the wholesale price and \( w^H \leq p^H \). The timing of players’ decisions in the hybrid market is as in the intermediated market with the addition of the upstream firm’s deciding the direct price. As in Section 1, intermediaries charge prices \( p^H(k_I) \) as a function of their transaction costs.
Conditional on the wholesale price, the intermediaries’ competition is as in the intermediated market structure in Section 1 with additional competition from the upstream firm’s direct sales.

Consumers choose to search and purchase from the intermediaries or directly from the upstream firm based on their comparison of expected gains through the two channels. Consumers can combine the channels by either searching the intermediaries and then purchasing directly or by purchasing directly, returning the product, and then purchasing via an intermediary. However, as we shall see in Proposition 3, the equilibrium prices ensure that these strategies where a consumer utilizes both channels are never followed.

As in the intermediated market structure, the highest value consumer purchasing from the intermediaries will not search. Consequently the expected gain from searching the intermediaries for the highest value consumer that searches is that consumer’s value minus the expected intermediary price discounted by the delay and probability of balking prior to purchasing, \((1 - \alpha)\beta(v - E[p^H_I])\). Consumers’ reservation values are calculated as in the intermediated market structure, resulting in the marginal consumer’s gain being \((1 - \alpha)\beta(v - E[p^H_I]) = (1 - \alpha)\frac{v - E[p^H_I]}{1 - \lambda}\). The expected gain from purchasing directly is \((1 - \alpha)\beta_U(v - p^H_U)\) and the equilibrium segmentation is defined by the consumer with value \(\tilde{v}^H\) who is indifferent between the two channels: \((1 - \alpha)\frac{\tilde{v}^H - p^H_U}{1 - \lambda} = (1 - \alpha)\beta_U(\tilde{v}^H - p^H_U)\), where \(\tilde{v}^H\) is the reservation value for a consumer with value \(\tilde{v}^H\). Consumers with values less than \(\tilde{v}^H\) search the intermediaries and purchase from the first one with the good with a price below their reservation price. Consumers with values above \(\tilde{v}^H\) purchase directly from the upstream firm. There is also a value below \(\tilde{v}^H\) purchase directly from the upstream firm. There is also a value below which consumers do not search the intermediaries, \(v^H\). This cutoff value corresponds to the lowest price any intermediary charges.

The fact that the highest value consumer always purchases directly is a condition for coexistence of both channels. Consumers search to determine the intermediaries’ prices and suitability of the good, and the cost of doing so increases in consumers’ values. Therefore, if the highest value consumer prefers the intermediaries, then all consumers go to the intermediaries and there is no direct channel. Another way to think about this is that time-consuming search makes the relative convenience of buying directly positively correlated with consumers value. Whenever this is true, we expect the highest value consumers to buy directly. Finally, if, in reality, intermediaries attract the highest

18. An alternative way of thinking of this is that if any intermediary charges a price above the highest-value customer’s reservation price, that intermediary receives zero sales. Hence, there is no benefit to charging above that price and the highest-value customer purchases from the first intermediary.
value consumers, that can be viewed as evidence that the intermediaries offer services that appeals to high-value consumers which are not in our model. The result that consumers valuing trade most choose direct sales is analogous to the segmentation in the search models with symmetric buyers and sellers in Gehrig (1993), Yavas (1996), and Rust and Hall (2003), where these consumers use the market maker.

Using the above equilibrium equations for the marginal consumer, the following lemma provides a necessary condition for both channels to coexist: search costs must be lower for direct sales than for intermediary sales. If direct sales are less convenient it is impossible for the upstream firm to segment consumers between channels: all consumers prefer one channel or the other.

**Lemma 2:** Equilibria in which both channels are used can exist only when $\beta_U > \beta$.

The weighted demand function $D^H_I(p_I)$ is defined similarly as in the intermediated market (5). As with the intermediated structure, the structure of intermediary competition and consumer search results in the demand function being independent of the price distribution function. The intermediary’s discounted profits are

\[
\Pi^H_I(p_I(k_I)) = (p_I - w^H - k_I) D^H_I(p_I) = \frac{(1 - \alpha) \beta}{(1 - \beta) N^H_I} (p_I - w^H - k_I)(\bar{r} - p_I).
\]

The following lemma assumes the necessary relationship between the wholesale price and direct price so that both channels are used and describes the equilibrium competition between intermediaries conditional on those prices.

**Lemma 3:** In the hybrid market structure, given the upstream firm’s wholesale price $w^H$ and direct price $p^H_U$, when $w^H + k_I \leq p_U \leq 1 - \frac{\beta}{\rho_U(4 - 4\beta(1 - \lambda))}(1 - w^H - k_I)$ and $\beta_U > \beta$, the equilibrium of the consumer search and intermediary competition is:

(i) consumers are segmented by cutoff values $\bar{v}^H$ and $\bar{v}^H$: consumers with $v < \bar{v}^H$ are inactive; consumers with $\bar{v}^H \leq v \leq \bar{v}^H$ purchase from the intermediaries; consumers with $v > \bar{v}^H$ purchase from the upstream firm;

(ii) intermediaries are segmented by a cutoff value $\bar{k}^H_I$: intermediaries with $k_I < k_1 \leq \bar{k}^H_I$ are active purchase goods from the upstream firm and sell to the consumers; intermediaries with $k_I > \bar{k}^H_I$ are inactive, where $\bar{k}^H_I = \bar{r}^H - w^H$; an intermediary with cost $k_I$ charges price $p^H_I(k_I) = \frac{1}{2}(\bar{r}^H + w^H + k_I) = \bar{v}^H + \frac{1}{2}(k_I - k_1)$ and the distribution of the intermediaries’
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The structure of the hybrid market is depicted in Figure 2. As in the intermediated market, intermediaries with costs above threshold \( \bar{k}_I^H \) are not active. Intermediaries’ prices are a linear function of the wholesale price, direct price, and transaction costs. The form of the intermediaries’ prices is as in the disintermediated market in Lemma 1, but the price of the lowest cost intermediary is affected by the direct price. Direct sales compete for the higher value consumers, reducing the intermediaries’ market power. Each intermediary’s discounted demand is increasing in the direct price and decreasing in its own price and the number of intermediaries.

The upstream monopolist’s maximization problem as a function of the direct price \( p_I^H \) and the wholesale price \( w_I^H \) is

\[
\max_{p_I^H, w_I^H} \Pi_I^H = w_I^H \cdot D_I^H + \left( (1 - \alpha) p_I^H - k_U - \alpha c \right) \cdot D_I^H.
\]

(12)
The upstream firm’s constrained optimization is formulated using the objective function in (12) and the constraints on prices in Lemma 3 to determine which channels are utilized and the optimal prices. The following proposition provides the unique market equilibrium that summarizes the upstream firm’s optimal choice for utilizing the different possible channel configurations.

**Proposition 3:** When the upstream firm is able to sell directly to the consumers, there are cutoff values \( \bar{k}_U = \frac{1 - \alpha - \frac{(1 - \alpha)\beta}{\beta_U(4 - 3\beta(1 - \lambda))}(1 - \bar{k}_I) - \alpha c}{1 - \bar{k}_U} \) and \( k_U = (1 - \alpha)\bar{k}_I - \alpha c \) that determines the upstream firm’s optimal market structure:

(i) when \( k_U > \bar{k}_U \) or \( \beta_U < \beta \), the upstream firm only uses the intermediated channel and the equilibrium is given in Proposition 1;

(ii) when \( k_U < \bar{k}_U \) and \( \beta_U \geq \beta \), the upstream firm only sells directly and the equilibrium is given in Proposition 2;

(iii) when \( \bar{k}_U \leq k_U \leq \bar{k}_I \) and \( \beta_U \geq \beta \), the upstream firm satisfies part of the demand through direct selling. With a hybrid market structure there exists a unique equilibrium pricing policy in which the upstream firm charges direct price \( p_U^H = \frac{1}{2} + \frac{k_U + \alpha c}{2(1 - \alpha)} \) and wholesale price \( w^H = \frac{1 - k_I}{2} \).

The equilibrium segmentation of intermediaries and consumers is given in Lemma 3.

The upstream firm uses direct sales because these sales do not suffer from double marginalization, increase competition between intermediaries, and enable price discrimination. Part (i) of Proposition 3 shows that if buying directly is less convenient or the upstream firm’s transaction or expected return costs are too high, then the upstream firm will not sell directly. Part (ii) gives the other extreme: if buying directly is more convenient and the upstream firm’s transaction and expected return costs are low enough, then the upstream firm will only sell directly. Part (iii) provides the intermediate case where the upstream firm’s transaction and expected return costs are neither too high nor too low and both the direct and intermediated channels are utilized. Note that the return costs can result in upstream firm using the intermediaries even when the upstream firm’s transaction costs are lower than that of the lowest cost intermediary: \( k_U = (1 - \alpha)\bar{k}_I - \alpha c < \bar{k}_I \).

Figure 3 shows the regions of the parameter space for each market structure. The upstream firm’s desire to sell online is determined by three exogenous factors: the relative convenience for the consumers (the direct versus intermediated discount factors), the difficulty of inspecting and trying goods prior purchasing online and the associated costs of returns, and the relative efficiency of transacting directly online versus through traditional intermediaries. Proposition 3 and Figure 3 show how these
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FIGURE 3. CUTOFF VALUE FOR UPSTREAM FIRM’S TRANSACTION COSTS BETWEEN THE INTERMEDIATED AND HYBRID MARKET STRUCTURES ($k_U$) AS A FUNCTION OF THE INTERMEDIATED DISCOUNT FACTOR ($\beta$) AND THE UPSTREAM FIRM’S TRANSACTION COSTS ($k_U$) GIVEN TWO VALUES OF THE DIRECT DISCOUNT FACTOR: $\beta'_U > \beta'_0$ (DASHED LINE) AND $\beta_U > \beta_0$ (SOLID LINE), WHERE $\beta'_U < \beta_U$. THE TWO INTERSECTIONS OF $k_U$ WITH THE HORIZONTAL $k_U$ LINE, $\beta_0$ AND $\beta'_0$, ARE $\frac{4\beta_U}{1+3\beta_U(1-\lambda)}$ AND $\frac{4\beta'_U}{1+3\beta'_U(1-\lambda)}$, RESPECTIVELY

factors interact. When introducing the parameters determining these factors, the types and examples of goods that have relatively high or low values along these dimensions were discussed.

If the upstream firm’s transaction and expected return costs are low enough relative to the lowest cost intermediary, $k_U + \frac{\alpha c}{1-\alpha} < k_I$, the upstream firm’s decision is independent of the relative discount factors and it only sells directly. Goods with a low probability of being returned are those that are standardized, repeatedly purchased, or well known, for example, have strong brand names. When a good has zero probability of a return, direct selling alone is utilized if and only if the upstream firm has lower transaction costs than the lowest cost intermediary. Together these suggest why a firm like Dell might sell directly to consumers: computers are fairly standardized goods, Dell has a strong brand name, and the transaction costs (particularly in terms of inventory) for it to provide somewhat customized products is lower than any retailer could match. Digital or information goods, for example, music, newspapers,
electronic airline tickets, and so forth, that can be easily tested and delivered online are ideal for direct selling.

When the upstream firms’ transaction and return costs are high enough that the intermediaries should be used, the discount factors for direct and intermediated sales play a role in the upstream firm’s choice of market structure. When intermediated search costs fall—the discount factor $\beta$ is higher—the cutoff value of the upstream firm’s choice between an intermediated market and a hybrid market structure drops and the intermediaries become relatively more attractive than direct sales and the upstream firm utilizes both channels for a narrower range of transaction costs. When direct search costs fall—the discount factor $\beta_U$ is higher—the intermediaries become less attractive and the upstream firm utilizes both channels for a wider range of transaction costs. The interaction of $\beta$ and $\beta_U$ is shown by the $\overline{k}_U$ curves’ outward shift as $\beta_U$ increases. The derivatives of the cutoff value for upstream firm’s transaction costs between the hybrid and intermediated market structures $\overline{k}_U$, $\frac{\partial \overline{k}_U}{\partial \beta} < 0$, $\frac{\partial \overline{k}_U}{\partial \beta_U} > 0$, and $\frac{\partial^2 \overline{k}_U}{\partial \beta \partial \beta_U} > 0$, are formalized in the following corollary.

**Corollary 1:** The cutoff value of the upstream firm’s equilibrium choice between an intermediated market and hybrid market structures is increasing in the direct discount factor $\beta_U$ and decreasing in the intermediated discount factor $\beta$. The effect of increasing the direct discount factor $\beta_U$ is increasing in the intermediated discount factor $\beta$.

The discount factors can be thought of as the relative convenience of the two channels with greater convenience corresponding to a higher discount factor. Goods carried by relatively few retailers are less convenient to purchase through intermediaries and would have a lower discount factor $\beta$ relative to the direct discount factor $\beta_U$. Goods that are needed more immediately are less convenient to purchase online due to the possibility of delays due to shipping. However, for goods that are repeatedly purchased at predictable times, the shipping delays may be inconsequential and, therefore, direct online purchasing may be very convenient. Cosmetics fit this later category, which helps explain Carlton and Chevalier’s (2001) findings that fragrances are widely available online from manufacturers. In addition, online sales by Clinique and Lancome are often cited as examples of direct selling by manufacturers.

Figure 3 shows that as the online transactions and return costs increase, the impact of differences in the discount factors become less important. In the extreme, if $k_U > (1 - \alpha) + \alpha c$, then direct selling is never used. This may explain the relatively lack of online direct selling
for bulky goods with heterogeneity of consumer tastes such as furniture. For example, Ethan Allen takes some orders over its web site, but fills them at the nearest retail store. In addition, Carlton and Chevalier (2001) attempted to examine refrigerators sales, but were unable to find any manufacturers selling directly through their web sites.

Our model focuses on only the upstream selling directly online. It is also possible that intermediaries may also want to sell online and in practice this is common for some goods, for example, airline tickets, books, and CDs. Incorporating additional types of intermediaries would greatly complicate the rational expectations equilibrium resulting from the consumers’ search and intermediaries’ competition, but the results from our model do suggest when online intermediated sales may be more likely. We defer this discussion to Section 4, along with a discussion of reasons outside the model that an upstream firm may not want to sell directly, until after we more fully develop our model’s structure and implications.

Using Lemma 3 and Proposition 3 to compare the direct price and the average intermediate price yields the following corollary.

**Corollary 2:** The direct price is higher than the average intermediate price and is equal if \( k_U = k_{\bar{U}} \). If search the upstream firm is costless—\( \beta_U = 1 \)—then the direct price is equal to the highest intermediaries’ price.

Because buying directly also involves discounting/inconvenience, once at an intermediary higher value consumers are willing to pay more than the direct price. However, if the direct market is costless to search, then consumers will never pay more from an intermediary. If the direct price were lower than the average intermediary price, then the highest value consumers going to the intermediaries would switch to buying directly. This would cause the highest price/cost intermediaries to exit, lowering the average intermediary price. This would continue until equilibrium was reached with the direct price greater than the average intermediated price.\(^{19}\) This is consistent with the findings of Carlton and Chevalier (2001)—for both industries for which they have data, fragrances and DVD players—and of Bell et al. (2003) for a number of products.

In Rust and Hall (2003), trade with a market maker occurs without search costs. This makes the market makers price an exact bound on intermediaries’ prices, which in our model would be equivalent to \( \beta_{U} = 1 \). The direct price in our model also limits how high intermediaries’

\(^{19}\) Although the assumption of uniformly distributed consumers valuation is necessary to determine the critical values and price distributions, it is not necessary for the pricing result in Corollary 2 (similar to Gehrig, 1993). We thank an anonymous referee for pointing this out.
prices can be, but the relationship is more complicated and depends on
the discount factors, return probability, and distribution of intermedi-
aries’ prices. Similar to our model, Yavas (1992, 1994, 1996) allows the
agent on either side of the market to buy from a market marker after
searching with their gain discounted. This also provides a bound on
the prices that can result from direct negotiation between buyers and
sellers.

The upstream firm uses the intermediated channel for several
reasons. First, the correlation between the consumers’ values and
search costs due to discounting allows price discrimination through
a higher direct price. Second, the intermediaries may provide a more
efficient channel in terms of transaction and return costs. There are two
drawbacks of using intermediated sales: the intermediaries mark up
the wholesale price and intermediated sales involve additional search,
which delays those sales. The equilibria given in Proposition 3, represent
the upstream firm’s optimal trade-off of these factors.

Comparing the prices in part (iii) of Proposition 3 to those in
Propositions 1 and 2 demonstrates that there is no wholesale or direct
price distortion in the hybrid market structure compared with the
exclusively intermediated and disintermediated market structures. This
is due to the uniform distribution assumption for consumers’ valuation.
The intuition behind this is as follows. The introduction of direct
sales attracts the high-value consumers, changing the distribution of
consumers buying through the intermediaries and providing the up-
stream firm with an incentive to lower the wholesale price to maximize
profits from intermediated sales. However, lowering the wholesale price
causes the lowest of the higher value consumers buying directly to
switch to searching the intermediaries, reducing the direct demand and
revenue. In addition, only half of the decrease in the wholesale price
is passed along to consumers, and a decrease in the wholesale price
causes more intermediaries to enter, increasing search and reducing
intermediated demand. The net effect of these depends on the relative
number of high-value and low-value consumers and the rate at which
intermediaries enter. The uniform distribution of consumers’ values
causes these effects to cancel each other out.20 Given that consumers’
values are unlikely to be uniformly distributed in practice, it is important
to understand whether the paper’s results are sensitive to this. We delay
the discussion of this until after analyzing the impact of direct sales in
Section 3.

20. The distribution of intermediary costs turns out to be less crucial and it can be
shown that the prices in Proposition 3 hold for costs densities of the form $k^\gamma$.
We now characterize the hybrid market equilibrium given in Lemma 3 and part (iii) of Proposition 3 by examining comparative statics on how changes in the discount rates, transaction costs, return costs, and consumers’ entry and exit rate and return probability affect the upstream firm, intermediaries, and consumers. Table II provides a summary of the comparative statics. We will focus our attention on those comparative statics which differ or are not present for the intermediated market structure in Table I.

Increasing the intermediary discount factor reduces the delay on intermediated sales and induces customers to switch from direct to intermediated sales. The net effect of these two is an increase in upstream firm sales. The upstream firm’s profits are also increasing in the intermediary discount factors.

The intermediaries’ prices, discounted demands, and individual and aggregate profits are increasing in $\beta$. These contrast with the intermediated market structure only where only the low-cost intermediaries’ profits increase and aggregate intermediary profits only increase when the intermediated discount factor is sufficiently high. These differences arise from the fact that direct sales replace the highest-cost intermediaries in the hybrid market structure. Not surprisingly, increasing
the direct discount rate causes consumers to switch to direct sales, reducing intermediary demand and increasing competition between intermediaries. This leads to fewer active intermediaries with lower discounted demand and profits.

Consumer surplus is calculated by appropriately combining the consumer surplus from direct sales (as in Proposition 2) and intermediated sales (as in equation 9) using the consumer segmentation and optimal prices in Lemma 3 and Proposition 3:

\[
CS^H = (1 - \alpha) \left(1 + \frac{\beta \lambda}{1 - \beta}\right) \left(\beta_U \int_{\bar{v}_H}^1 (v - p_{ij}^H) dv + \int_{\bar{v}_H}^{\bar{v}_I} \frac{v - r}{1 - \lambda} dv(r)\right).
\]

(13)

Because the intermediaries’ prices are increasing in the intermediated discount factor, the value of the marginal consumer, who is indifferent between searching the intermediaries and not searching at all, \(v^H\), is also increasing in \(\beta\). This implies that the fraction of active consumers is decreasing in the intermediated discount factor. When searching the intermediaries becomes less costly, some lower value direct consumers switch from the direct to the intermediated channel. This can be seen by the equilibrium consumer cutoff value for direct and intermediated sales, \(\bar{v}^H\) in Lemma 3, decreasing in the intermediated discount factor.

The effect of higher transaction costs on the upstream firm is similar to the disintermediated market: the upstream firm’s demand and profits fall whereas the direct price rises. This reduces competition with the intermediaries, allowing them to raise their prices, which increases their profits. Because both the direct and intermediated prices rise, fewer consumers purchase from both the upstream firm and the intermediaries and each consumer’s surplus falls.

Increasing the return probability, the cost of returns, and the upstream firm’s transaction costs all raise the direct price and make the online purchasing less attractive. This results in some marginal consumers switching to searching the intermediaries. These relatively high-value consumers allow entry of additional higher cost intermediaries. This increases intermediaries’ prices, which, in turn, causes some lower value consumers to no longer search and purchase.

Total social welfare is increasing in both discount factors and decreasing in the consumers’ return probability and the upstream firm’s transaction and return costs:

\[
W^H = \Pi_U^H + \Pi_I^H + CS^H = \frac{3}{2} \Pi_U^H.
\]

(14)
Having characterized the equilibrium in the hybrid market structure by examining the comparative statics, we next study how the addition of direct sales impacts the different market participants.

3. Impact of Direct Sales

This section focuses on how an upstream firm selling directly to their consumers impacts the different market participants. We compare the upstream firm, intermediaries, and consumers under the mixed-channel (hybrid) market structure in Section 2 and the two exclusive-channel (intermediated and disintermediated) market structures in Section 1. We begin with the upstream firm, which controls the choice of market structure and utilizes the second channel only when it is to his advantage.

**Proposition 4:** With the addition of direct sales the upstream firm’s discounted demand and profits increase.

When the upstream firm can sell directly, the intermediaries are partially replaced when the upstream firm’s costs are sufficiently low. Given that the upstream firm is replacing the intermediaries, it is not surprising that direct sales make intermediaries strictly worse off. As shown in the following proposition, with direct sales there are fewer intermediaries which have lower prices, lower markups, lower discounted demand, and lower profits individually and in aggregate.

**Proposition 5:** With the addition of direct sales: (i) there are fewer intermediaries; (ii) each intermediary charges a lower price and has a lower markup; (iii) each intermediary has lower discounted demand and profits, and aggregate intermediary profits are lower.

Direct sales attract the highest value consumers from the intermediaries. Consequently, each intermediary’s demand decreases and the highest cost intermediaries exit the market. Thus, the direct channel reduces the intermediaries’ market power, resulting in lower intermediary prices, markups, and profits. Given that intermediaries’ prices are lower with direct sales, consumers choose between the lower priced intermediated channel and direct sales to maximize their expected gains, so by revealed preference they are better off. In addition, compared with the intermediated market, intermediaries’ lower prices induce more lower value consumers to purchase.

**Proposition 6:** With the addition of direct sales: (i) more consumers purchase; (ii) each individual consumer has greater surplus; (iii) total consumer surplus is higher.
We now return to the discussion of the results’ sensitivity to the assumption of uniform consumers values. What is needed for consumer surplus to increase with the addition of direct sales is for intermediary prices to decline. When direct sales attract the higher value consumers, which is the only time they are used in our model, this removes the top part of the intermediaries’ demand curve and results in it being optimal for the upstream firm to induce lower intermediary prices. The revealed preference argument then guarantees that all consumers are better off with both channels. Although the model imposes significant structure to get closed form solutions, this result appears quite general.

**Proposition 7:** With the addition of direct sales, (i) the total welfare is higher; (ii) the sum of the upstream and intermediaries profits are higher if and only if the transaction cost of the upstream firm is sufficiently low.

The upstream firm and consumers are better off with direct sales whereas the intermediaries are worse off, making the effect on overall welfare unclear. The transaction and return costs for direct sales may be higher than the intermediaries’ transactions costs. On the other hand, the lower intermediary markups reduce deadweight loss. Proposition 7 shows that the potential loss from higher upstream firm transaction and return costs is outweighed by the lower prices due to reduction in double marginalization through more intense intermediary competition. This ensures that more consumers purchase and total welfare increases with direct sales. Unfortunately, unlike the increase in consumer surplus, it is difficult to argue that this result always holds outside of the model.21

Propositions 4 and 5 demonstrate the conflict between the intermediated and direct channels because using direct sales benefits the upstream firm and hurts the intermediaries. One example of where this issue arises is in car dealerships. In the United States many states have laws against car manufacturers selling directly to consumers. Proposition 6 demonstrates that this ban is harmful to consumers and part (i) of Proposition 7 indicates that it reduces overall welfare. Part (ii) of Proposition 7 shows if the upstream firm transaction’s costs are not too high relative to the dealers, the increase in the upstream firm’s profits is greater than the decrease in the intermediaries’ profits. In this case the channel conflict could be mitigated and a Pareto improvement realized with direct sales and side payments from the upstream firm to the

21. In a model where total demand is independent of price, Liu and Zhang (2002) discuss how the upstream firm selling directly may lead to higher prices and lower social welfare.
intermediaries.\textsuperscript{22} State laws that restrict the shipping of wine directly to consumers have similar impact.

Given that this conflict between the upstream firm and intermediaries exists, we next explore the upstream firm’s optimal strategy when direct selling is possible, but the intermediaries refuse to participate in the hybrid model.

### 3.1 Direct Sales versus Intermediation

Here we assume that the upstream firm must choose between exclusively direct or exclusively intermediated sales. The following Proposition compares the equilibrium and participants in the intermediated and disintermediated market structures.

**Proposition 8:** When the upstream firm must choose between direct or intermediated sales:

(i) it always chooses the option that maximizes social welfare;

(ii) if $k_U$ is low, then the upstream firm chooses direct sales, which is the option that maximizes consumer surplus;

(iii) if $k_U$ is high, then the upstream firm chooses intermediated sales, which is the option that maximizes consumer surplus if $k_U$ is high enough, but not if $k_U$ is intermediate.

Parts (i) and (ii) of Proposition 8 shows that when the upstream firm’s costs are low enough, direct selling is preferred to intermediation in terms of consumer and total surplus. Part (iii) provides the complement: when the upstream firm’s costs are high enough, intermediation is preferred. Together these demonstrate that the upstream firm always chooses the socially optimal channel. However, the upstream firm’s channel choice is not optimal for consumers if its costs are high enough that it would choose the intermediated sales, but not too high.

### 4. Discussion

While discussing the hybrid market structure we presented some examples of manufacturers selling directly on the Internet. Because widespread adoption of the technology making online direct sales significantly less costly for both manufacturers and consumers is fairly recent, there is little empirical research available on this. Carlton and Chevalier’s (2001) and Bell et al. (2003) being exceptions. Our model

\textsuperscript{22} Including payments to intermediaries that would be driven out of business. This is similar to the result in Rust and Hall (2003) where a monopolist market maker that drives all middlemen out of business can improve welfare.
provides some predictions for which goods direct online sales are most likely to occur: goods where immediacy is less important, for example, good purchased regularly at predictable times; goods where breaking bulk at intermediaries does not significantly reduce transactions costs as opposed to shipping directly to individual consumers, for example, less bulky goods; goods less widely available at retailers, for example, specialty goods; and goods where touch and feel is less important, for example, commodities, standardized goods, goods with strong brands, or where consumer tastes are more homogeneous. Many of these predictions are about products well suited for online sales not just by manufacturers, but by online intermediaries as well.

Although our focus is on the manufacturer selling directly to consumers, a similarly structured model could allow for online retailers to purchase at the wholesale price and then sell to consumers. As previously mentioned this greatly complicates the rational expectations equilibrium resulting from competition between the intermediaries and search by the consumers. The simplest first step would be to not have the upstream firm sell directly, but allow a single online intermediary, with features similar to upstream firm’s online sales in our model, to compete with the physical intermediaries. In such a model, as in this paper, if both channels coexist, the higher value consumers would buy online. The predictions for which goods are suitable for online sales should also be similar. In our model, the upstream firm can internalize the discrimination in prices across channels. If the upstream firm can only choose one wholesale price for both online and physical intermediaries, then it is possible that wholesale price would be higher than without the online intermediary. This could potentially reduce consumer surplus and social welfare.

We expect the distinction between online intermediaries and online direct sales is important in some instances. Goods that are suitable for online sales, but where producers have weak brands are unlikely to be sold directly online. However, these goods are likely to be sold online by intermediaries/retailers. Music and books, where the artists, not the labels and publishers, have strong bands, fall into this category and are actively sold online by intermediaries. In contrast we expect to see goods well suited for online sales with upstream firms having strong brands sold directly online with or without online intermediated sales as well. Electronic airline tickets and jewelry, both inexpensive to deliver directly to consumers relative to their prices, are examples of such goods. Online travel web sites such as Expedia and Orbitz were some of the most successful initial web sites and the airlines themselves quickly followed with their own direct ticket sales online. In 2004, Amazon.com made
significant push into jewelry sales whereas firms with strong brand names, for example, Tiffany’s, sell directly online.

Intermediaries also provide a number of important services that are not in our model of sales of a single identical good. An intermediary selling goods from multiple competing manufacturers can facilitate comparison of goods and reduce search costs. An intermediary selling goods from complementary manufacturers can reduce search and transaction costs by allowing aggregation of shopping across products into a single trip. Online as well as physical intermediaries can provide these services. Online intermediaries typically facilitate product comparison across manufacturers explicitly or implicitly by posting users reviews and rating.

Our model contains an important difference between physical stores, where goods can be touched and felt, and online sites, where the goods can only be pictured or described. As discussed above, this difference is important in determining which goods are most suitable for online sales. In our model the upstream firm’s direct online price is higher than the average intermediaries price. If the online price were lower than the intermediaries price, then consumers might choose to test a product at a physical intermediary, but then purchase it online. Store space and inventory are necessary for providing the ability to try on and inspect goods. The costs associated with these could lead to online sales “free riding” on efforts by physical intermediaries. Other marketing efforts at risk for free riding include advertising and brand building and providing information via knowledgeable sales people. Although this sort of free riding and channel conflict is absent in our model, it may be important in practice when investment by intermediaries is important and is the focus of Carlton and Chevalier (2001) and Bell et al. (2003). These paper’s demand structures implicitly assume an exogenous explanation for why any consumers continue to buy at physical intermediaries. By using a consumer choice and search model, rather than a given demand structure, this paper provides one endogenous explanation for how direct and intermediated sales can coexist.

As technology has made it easier to sell directly to consumers, issues regarding how firms will employ it and how it will affect the organization of firms and markets are of increasing interest. This paper furthers our understanding along the particular dimension of the incidence and impact of upstream firms selling online directly to consumers. Further understanding of the other issues raised in this section, such as channel conflict, free riding, and intermediated online sales, are promising research directions.
5. Conclusion

We examine how the sale of goods by firms to consumers can be altered by new technologies that enable upstream firms to sell online directly to consumers. Direct sales provide less of an opportunity to inspect and try on goods, making returns more likely. Direct sales can be more or less convenient, in terms of time consuming search, than intermediated sales. We find the unique equilibrium, and study when the upstream firm utilizes both direct and intermediated sales in parallel or either one exclusively. Direct sales benefit the upstream firm by allowing price discrimination due to the correlation between the consumers’ values and search costs and a reduction in double marginalization. The upstream firm may continue to use the intermediated channel because the intermediaries may provide a more efficient channel in terms of transaction and return costs as well as allow price discrimination. When the intermediated discount factor (relative to the direct discount factor) and the upstream firm’s transaction and return costs (relative to the intermediaries’ transaction costs) are both high enough, the intermediated channel is used exclusively. When the upstream firm’s transactions and return costs are not too high and the intermediated discount factor is sufficiently low, both channels are used simultaneously.

Direct sales result in price discrimination, lower equilibrium consumer search costs, and increased intermediary competition. The net effect is that the addition of the second channel makes all consumers better off. However, intermediaries lose sales and compete more intensely, making them worse off. The gains by the upstream firm and consumers are greater than the losses by the intermediaries. Thus, social welfare increases with the addition of direct sales.

Because the intermediaries are made worse off by the upstream firm’s direct sales, the intermediaries may not participate in the hybrid strategy. We show that if the upstream firm’s transaction costs are low enough, the upstream firm’s increase in profits is greater than the decrease in the intermediaries’ profits, implying that the channel conflict can be eliminated through side payments. If this channel conflict can not be eliminated and the upstream firm is forced to choose between using either direct or intermediated sales exclusively, we show that the upstream firms always chooses the channel that maximizes social welfare, but not always consumer surplus.

The decision whether to use direct sales is also affected by the fixed cost of setting up the channel. As advances in technology lower these costs, we expect to see more and more firms using direct sales. As our results show, these trends benefit upstream firms with market power and consumers, but intermediaries will suffer from increased competition from direct sales and each other.
A Model of Direct and Intermediated Sales

APPENDIX

Because it is clear from the position of the lemmas and propositions in the text the superscripts on the endogenous variables referring to the market structure are often omitted in the Appendix.

Proof of Lemma 1. Each intermediary chooses price \( p_I \) to maximize their profits, implying \( p_I^* = \frac{1}{2}(\bar{r} + w + k_I) \). Hence, \( p_I \in \left[ \frac{1}{2}(\bar{r} + w + k_I), \bar{r} \right] \) and \( k_I \in [k_I, \bar{r} - w] \). The equilibrium set of active intermediaries is the convex set \( [k_I, \bar{k}_I] \), and \( N = \bar{r} - w - k_I \) and \( \bar{k}_I = \bar{r} - w \). For \( p_I \in \left[ \frac{1}{2}(\bar{r} + w + k_I), \bar{r} \right] \), the equilibrium intermediaries’ price distribution is

\[
F(p_I) = \Pr \{ p_I(k_I) \leq p_I \} = \frac{2p_I - \bar{r} - w - k_I}{N}.
\]

(A1)

Substituting the above equilibrium distribution \( F(p_I) \) into (3) gives

\[
v = v(r) \equiv r + \frac{\beta(1 - \lambda)}{1 - \beta(1 - \lambda)} \int_r^{\bar{r}} \frac{2p_I - \bar{r} - w - k_I}{N} dp_I
\]

\[
= r + \frac{\beta(1 - \lambda)(r - p_I^*)^2}{(1 - \beta(1 - \lambda))N}.
\]

(A2)

Let \( \bar{v} = 1 \) and \( r = \bar{p}_I = \frac{1}{2} (\bar{r} + w + k_I) \), solving for \( \bar{r} \) and \( \bar{v} \) yields \( \bar{r} = \frac{4 - \beta(1 - \lambda)(4 - w - k_I)}{4 - \beta(1 - \lambda)(4 - w - k_I)} \) and \( \bar{v} = \frac{1}{2} (\bar{r} + w + k_I) \), so the intermediary with transaction cost \( \bar{k}_I \) gets zero demand and zero profit from the consumers. The marginal consumer has value \( v \) and is indifferent between searching the intermediaries for the lowest price and not participating. \( \square \)

Proof of Proposition 1. Given the upstream firm’s profit function (8) in Section 1.4, it is straightforward to verify that the unique equilibrium in the intermediated market is the upstream firm choosing an optimal wholesale price of \( w = \frac{1 - k_I}{2} \). \( \square \)

Proof of Lemma 2. The equilibrium conditions can be written as \((1 - \alpha)\beta_U(\bar{v}^H - p_{U,1}^H) = (1 - \alpha)\beta(\bar{v}^H - E(p_{U,1}^H)) \). For both channels to be used \( 0 < \bar{v}^H < 1 \). The equilibrium equation together with \( \bar{v}^H < 1 \) implies that \( \beta - \beta_U \) must have the same sign as \( \beta E(p_{U,1}^H) - \beta_U p_{U,1}^H \). The equilibrium equation together with \( \bar{v}^H > 0 \) implies that \( \beta_U p_{U,1}^H > \beta E(p_{U,1}^H) \). Clearly these both cannot hold when \( \beta < \beta_U \), so in this case no equilibria exit in which both channels are used. \( \square \)

Proof of Lemma 3. Each intermediary chooses price \( p_I \) to maximize their profit function, implying \( p_I^* = \frac{1}{2}(\bar{r} + w + k_I) \). Because \( r \leq p_I \leq \bar{r} \), we have \( p_I \in \left[ \frac{1}{2}(\bar{r} + w + k_I), \bar{r} \right] \), and \( k_I \in [k_I, \bar{r} - w] \). The equilibrium set of active intermediaries is the convex set \( [k_I, \bar{k}_I] \), and \( N = \bar{r} - w - k_I \).
For $p_I \in \left(\frac{1}{2}(\bar{r} + w + k_I), \bar{r}\right)$, the equilibrium intermediaries’ price distribution is

$$F(p_I) = \Pr \{ p_I^*(k_I) \leq p_I \} = \frac{2p_I - \bar{r} - w - k_I}{N}.$$  \hfill (A3)

Substituting the above equilibrium distribution $F(p_I)$ into (3) gives

$$v = v(r) \equiv r + \frac{\beta (1 - \lambda)}{1 - \beta (1 - \lambda)} \int_{p_I}^{\bar{r}} \frac{2(p_I - \bar{r} - w)}{\bar{r} - w} dp_I$$

$$= r + \frac{\beta (1 - \lambda) (r - p_I)^2}{(1 - \beta (1 - \lambda)) N}$$ \hfill (A4)

$\bar{v}$ and $\bar{r}$ are obtained from the indifferent consumer’s utility

$$\frac{1 - \alpha}{1 - \lambda} (\bar{v} - \bar{r}) = (1 - \alpha) \beta U (\bar{v}^H - p_U^H) = (1 - \alpha) \beta U (\bar{v}^H - E(p_I)).$$ \hfill (A5)

Define $\phi \equiv \frac{\beta \beta_U}{\beta_U - \beta (1 + 3\beta_U (1 - \lambda))}$ and note that $\phi$ is increasing in $\beta_U$ and decreasing in $\beta_U$ and $\lambda$. Let $\bar{r}^H = p_U^H + \phi(\frac{1}{\beta_U} - (1 - \lambda))(p_U^H - w^H)$ and $\bar{r}^H = p_I^H \left(\bar{r} + w + k_I\right)$, solving for $\bar{v}^H$ and $v^H$ yields $\bar{v}^H = p_U^H + \frac{\phi}{\beta_U} (p_U^H - w^H)$ and $v^H = \frac{\bar{r}^H + w^H}{2}$. The consumer with value $\bar{v}$ is indifferent between buying from the upstream firm or searching the intermediaries. Whereas the consumer with value $v$ is indifferent between searching the intermediaries for the lowest price and not participating. \hfill \Box

**Proof of Proposition 3.** The upstream firm’s demand from direct sales excluding returns is $D_U^H = (1 + \frac{\beta_U}{1 - \beta}) \beta U (1 - \bar{v}^H) = (1 + \frac{\beta_U}{1 - \beta}) \times [\beta U (1 - p_U^H) - \phi(p_U^H - w^H)]$. The aggregate discounted demand from sales through the intermediaries is $D_I^H = \int_{0}^{\bar{r}^H} D_I^H(p_I(k_I)) dk_I = \frac{\beta_U}{4(1 - \beta)} (\bar{r}^H - w^H - k_I) = (1 - \alpha)(1 + \frac{\beta}{1 - \beta}) \phi(p_U^H - w^H)$. Thus, the upstream firm maximizes its total profit conditional on positive demands from both channels:

$$\max_{p_U^H, w^H} \Pi_U = w^H \cdot D_U^H + [(1 - \alpha) p_U^H - k_U - \alpha c] \cdot D_U^H$$

$$\quad D_U^H \geq 0$$

subject to

$$D_I^H \geq 0$$

$$p_U^H \geq w^H \geq 0.$$  

The constraints result in the feasible range of $w^H + k_I \leq p_U^H \leq 1 - \frac{\beta}{\beta_U (4 - 3\beta_U (1 - \lambda))} (1 - w^H - k_I)$. Solving the first-order condition yields
\[ p_{H}^{H*} = \frac{1}{2} + \frac{k_{U} + \alpha c}{2(1 - \omega)} \] and \[ w^{H*} = \frac{1 - k_{I}}{2} \]. To prove that the solution is the unique global optimum, it is straightforward to verify that the Hessian, \[ \frac{1}{(1 - \omega)} \frac{(1 - \beta)(1 - \lambda)}{\beta} \phi \left[-\beta U(4 - 3\beta(1 - \lambda)) - \beta \right], \] is negative definite.

Comparing the upstream firms’ profit in the hybrid structure to intermediated and disintermediated structures:

\[ \Pi_{U}^{H} - \Pi_{U}^{I} = \frac{(1 - \beta(1 - \lambda))(4 - 3\beta(1 - \lambda))\beta U_{\phi}(k_{U} - k_{I})^2}{4(1 - \alpha)(1 - \beta)}; \]

\[ \Pi_{U}^{H} - \Pi_{U}^{D} = \frac{(1 - \beta(1 - \lambda))\phi}{4(1 - \alpha)(1 - \beta)}(k_{U} - k_{I})^2, \]

can also be used to derive the optimal marketing channel choice for the upstream firm. The comparison makes it clear that when \( 0 < k_{U} < k_{U} < \bar{k}_{U} \), the hybrid strategy is optimal; when \( k_{U} > \bar{k}_{U} \), only intermediated sales are used; and when \( k_{U} < \bar{k}_{U} \), only direct sales are utilized.

Proof of Proposition 4. The upstream firm only chooses direct sales when it increases its profits. When \( k_{U} < k_{U} < \bar{k}_{U} \), the upstream firms demand increase with direct sales:

\[ (D_{I}^{H} + D_{U}^{H}) - D_{I}^{I} = \frac{(1 - \beta(1 - \lambda))(\beta U + \alpha \phi)}{2(1 - \alpha)(1 - \beta)}(k_{U} - k_{I}) > 0. \]

Because \( v^{H} < v^{I} \), there are more active consumers in the hybrid market. Each consumer is searching less due to the lower prices charged by the intermediaries and a portion of the high value consumers switch to the direct channel. So the total discounted demand from the upstream firm increases with the increase of direct sales when \( k_{U} = \bar{k}_{U} \) intermediated sales are zero. Decreasing \( k_{U} \) further increase demand from direct sales.

Proof of Proposition 5. \[ \Pi_{U}^{H} = \frac{(1 - \beta(1 - \lambda))\phi^2(\alpha + (1 - \alpha)k_{U})^2}{3(1 - \alpha)(1 - \beta)} \]. It is straightforward to show that \( \frac{\partial \Pi_{U}^{H}}{\partial k_{U}} > 0 \) and \( \Pi_{U}^{H}(k_{U}) = \Pi_{U}^{I} \). Therefore, \( k_{U} < \bar{k}_{U} \) implies that \( \Pi_{U}^{I}(k_{U}) < \Pi_{U}^{I} \) and the intermediaries’ total profits are lower in the hybrid market (with the entry of direct sales) and equal at the boundary. When only the direct channel is used intermediated sales and profits are zero.

Proof of Proposition 6. \[ CS^{H} = \frac{(1 - \beta(1 - \lambda))\phi}{2(1 - \alpha)(1 - \beta)} [3(4\beta U(1 - \alpha - \alpha c - k_{U})^2 - (1 - \alpha - \alpha c - k_{U})\beta((1 - \alpha)(1 + 3\beta U(1 - k_{U})(1 - \lambda) - k_{I}) + 3\alpha U(1 - \lambda) \times (k_{U} + c)) - (1 - \alpha)\beta(1 - c)(k_{U} + \alpha c - (1 - \alpha)k_{I}) - 8(1 - \beta(1 - \lambda))\phi(k_{U} + \alpha c - (1 - \alpha)k_{I})^2] = \frac{\Pi_{U}^{I}}{2} - \Pi_{U}^{H} \]. Because \( \frac{\partial \Pi_{U}^{H}}{\partial k_{U}} < 0 \) and \( \frac{\partial \Pi_{U}^{I}}{\partial k_{U}} > 0 \), it is
straightforward to show that \( \frac{\partial C^H}{\partial k_U} < 0 \) and \( C^H(\bar{k}_U) = C^I \). Therefore, \( k_U < \bar{k}_U \) implies that \( C^H(k_U) > C^I \) and consumer surplus is higher with direct sales (the hybrid market structure) and equal at the boundary. When \( k_U = k_U \) intermeditated sales are zero. Decreasing \( k_U \) further increases consumer surplus from direct sales.

**Proof of Proposition 7.** If \( k_U > \bar{k}_U \) then the upstream firm will choose only intermediated sales and channel profits are unaffected by the possibility of direct sales. Otherwise the difference in channel profits \( (\Pi^H_U + \Pi^H_I) - (\Pi^I_U + \Pi^I_I) \) is

\[
\frac{\phi^2}{(1-\alpha)} \left(1 + \frac{\beta \lambda}{1-\beta}\right) \times \left( \frac{8\beta_U}{\beta^2} - \frac{4(1 + 9\beta_U(1-\lambda))}{3\beta} + \frac{(5 + 27\beta_U(1-\lambda))(1-\lambda)}{6} \right) \times (k_U - k_U)(k_U^{(1)} - k_U),
\]

(A6)

where \( k_U < k_U^{(1)} = \bar{k}_U - \frac{8(1-\alpha)\beta^2(1-\beta(1-\lambda))(1-\lambda)}{\phi(4-3\beta(1-\lambda))(48\beta_U - 8\beta_U(1-\lambda))) + \beta^2(5+2\beta_U(1-\lambda)(1-\lambda)} \) < \( \bar{k}_U \). The terms before \( (k_U^{(1)} - k_U) \) in (A6) are positive, implying that the sum of the profits of the upstream firm and the intermediaries is higher in the hybrid market than in the intermediated market, \( \Pi^H_U + \Pi^H_I > \Pi^I_U + \Pi^I_I \), when \( k_U \in (k_U, k_U^{(1)}(1)] \) and lower, \( \Pi^H_U + \Pi^H_I < \Pi^I_U + \Pi^I_I \), when \( k_U \in (k_U^{(1)}, \bar{k}_U) \).

Given \( k_U \leq \bar{k}_U \), \( W^H \) is decreasing with \( k_U \) and \( W^H(\bar{k}_U) = W^I \), implying that social welfare is larger in the hybrid market and equal at the boundary. When \( k_U = \bar{k}_U \) intermediated sales and profits are zero. Decreasing \( \bar{k}_U \) increases upstream firm profits and consumer surplus, improving welfare.

**Proof of Proposition 8.** With \( \Pi^I_U = \frac{(1-\alpha)\beta(1-\beta(1-\lambda))}{4(1-\beta)}(1 - k_U) \) and \( \Pi^I_D = \frac{(1-\alpha)\beta(1-\beta(1-\lambda))}{4(1-\beta)}(1 - k_U + \frac{ac}{\alpha}) \) it is straightforward to show that the upstream firm uses direct sales if \( 1 - (1 - k_U)\sqrt{\frac{\beta}{4(1-\beta)(1-\lambda)}} > k_U + \frac{ac}{1-\alpha} \) and uses only intermeditated sales otherwise. Comparing \( C^I = \frac{2\Pi^I_U}{6(4-3\beta(1-\lambda))} \) and \( C^D = \frac{1}{2}\Pi^D_U \) shows that consumers are better off in the direct market when \( k_U + \frac{ac}{1-\alpha} < 1 - \frac{1-k_U}{4-3\beta(1-\lambda)}\sqrt{\frac{\beta(4-\beta(1-\lambda))}{3\beta_U}} \), and worse off otherwise.

\( \Box \)
References


