Relationship Trading in OTC Markets*

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September 12, 2017

*We thank Darrell Duffie, Jerôme Dugast, Burton Hollifield, Haoxiang Zhu, and seminar audiences at California Polytechnic University, Carnegie Mellon University, FED Board, Higher School of Economics Moscow, IDC Herzliya, Imperial College, Michigan State University, National Bank of Belgium, NY FED, Rice University, University of Houston, University of Illinois-Chicago, University of Oklahoma, University of Wisconsin at Madison, University of Victoria, WU Vienna, and the 2015 Toulouse Trading and post-trading conference, 2016 AFA, 2016 NBER AP Meeting, 2016 SFI Research Day, 12th Annual Central Bank of France Workshop on the Microstructure of Financial Markets, 2016 LAEF Conference on OTC Markets, 2017 OTC Markets and Their Reform Conference in Rigi for valuable comments and feedback.

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Abstract

We examine the network of trading relations between insurers and dealers in the over-the-counter corporate bond market. Comprehensive regulatory data shows that many insurers use only one dealer while the largest insurers have networks of up to forty dealers. Large insurers receive better prices than small insurers. However, execution costs are a non-monotone function of the network size, increasing once the network size exceeds 20 dealers. To understand these facts we build a model of decentralized trade in which insurers trade off the benefits of repeat business against dealer competition. The model can quantitatively fit the distribution of insurers' network sizes and how prices depend on insurers' size.

JEL Classification: G12, G14, G24

Key words: Over-the-counter market, corporate bond, trading cost, liquidity, decentralization, financial network

We study insurers' choice of trading networks and the corresponding execution prices in the corporate bond market. These bonds are corporations' primary source for raising capital and trade in an over-the-counter (OTC) market with more than 400 active broker-dealers. Insurers are among the largest investors, owning roughly 30% of the $7.8 trillion market capitalization. Insurers' heterogeneous trading needs facilitate the study of how investor heterogeneity impacts both network formation and market prices. Regulatory data provide the identities of the dealers and the more than 4,300 insurers for all transactions from 2001 to 2014.

We first empirically examine insurers' choice of trading networks sizes and how these relate to transaction prices. Insurers form small, but persistent, dealer networks. Figure 1 provides two examples of client-dealer trading relations over time. Panel A shows buys and sells for an insurer trading repeatedly with a single dealer. Panel B shows an insurer trading with multiple dealers over time. Roughly 30% of insurers trade with a single dealer annually. The largest insurers trade with up to 40 dealers in a year. We estimate trading costs as a function of network size N. Costs are non-monotone in N: costs decline with N for small networks and then increase once N exceeds 20 dealers.

In random search models clients repeatedly search for best execution without forming a finite network of dealers (Duffie, Garleanu, and Pedersen, 2005, 2007; Lagos and Rocheteau, 2007, 2009; Gavazza (2016)). The empirical fact that insurers form finite dealer networks suggests that adding dealers must be costly for insurers. Traditional models of strategic search, e.g., Stigler (1961), assume each additional dealer imposes a fixed cost on insurers. Insurers add dealers to improve prices up to the point where the marginal benefit equal the fixed cost. This leads to prices improving monotonically in network size, which is inconsistent with the empirical non-monotonicity of trading costs as a function of network size.

We build a strategic model that produces finite network sizes and non-monotonic trading costs. The model incorporates features from a variety of existing models, including search costs, exclusivity, and loyalty. The model highlights the trade-offs when increasing the size of the dealer network. Additional dealers increase competition, leading to better prices and faster execution. However, the larger network reduces the value of the trading relationship. Dealers compensate for the loss of repeat business by requiring more surplus. This leads to a wider spread between the buy and sell prices. The optimal network size balances these effects.

In our model a single console bond trades on an inter-dealer market which clients can only access through dealers. Dealers have search intensity λ and upon trading with a client then transact at the competitive inter-dealer bid/ask prices. Clients initially start without a bond but stochastically receive trading shocks with intensity η which cause them to simultaneously

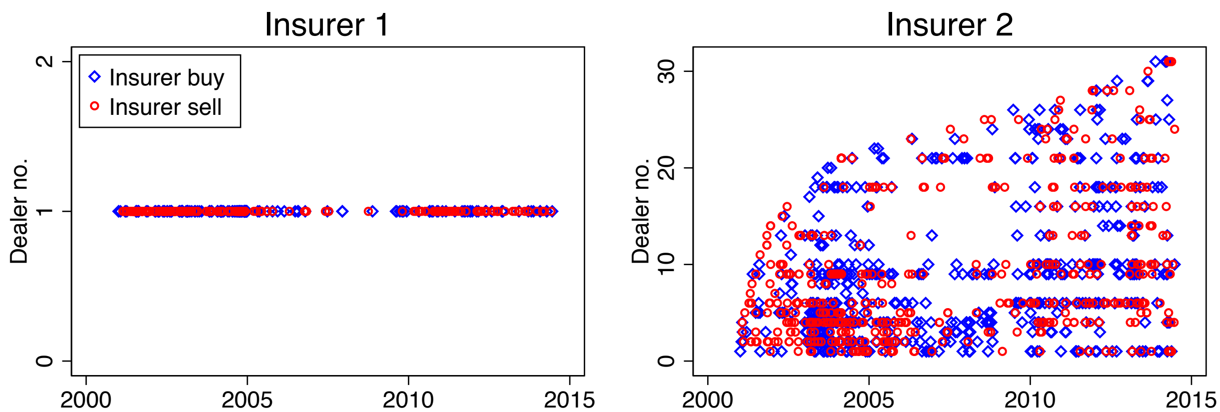


Figure 1: Example of dealer-client trading relations
The figure shows the buy (blue squares) and sell (red circles) trades of two insurance companies with different dealers. We sort the dealers on the vertical axis by the first time they trade with the corresponding insurance company.

contact N dealers to buy; η varies across clients. The client's effective search intensity is $\Lambda = N\lambda$. The first dealer to find the bond captures all benefits from the transaction. Thus, our trading mechanism is identical to repeated winner-takes-all races (Harris and Vickers (1985)). The bond's purchase price is set by Nash bargaining. Once an owner, the client stochastically receives a liquidity shock forcing her to sell the bond. The mechanics of the sell transactions are the same as the buy transaction. Both dealers and clients derive value from repeat transactions which help determine transaction prices, leading to price improvement for more frequent clients in Nash bargaining.

Both the network size and transaction prices are endogenously determined in equilibrium by maximizing buyers' utility. Existing OTC models provide predictions about network size or prices, but not both. Random search models assume investors may contact every other counterparty. Other models allow investors to choose specific networks or markets, but exogenously fix the structure of those networks. For instance, in Vayanos and Wang (2007) investors chose to search for a counterparty between two markets for the same asset: a large market with faster execution but higher transaction costs, and a small market with slower execution but lower transaction costs. Neklyudov and Sambalaibat (2016) use a similar setup as in Vayanos and Wang (2007) but with investors choosing between dealers with either large or small interdealer networks instead of asset markets.

We solve the model in steady state. The model delineates the trade-offs leading to the optimal finite network size. Clients trade off the value of repeat relations with dealers against the benefits of competition among dealers. The benefits of intertemporal dealer competition lead to better prices and faster execution. However, the value of repeat relations declines in the number of dealers. Dealers compensate for losses from repeat business by charging higher spreads. Eventually the costs of having larger network outweigh the benefits and the dealers' spread starts to increase with the network size. This corresponds to the empirical non-monotonicity in trading costs with respect to the network size. The value of repeat relations diminishes more slowly with the addition of dealers for clients with larger trading intensity as dealers compete for larger repeat business. Therefore, these larger clients use more dealers and get better execution as benefits from having larger repeat business outweigh the costs of having larger network.

Finally, we investigate if the model can quantitatively match the insurers' observed network sizes and transaction prices. Doing so requires structural estimation of the model parameters not directly observable in the data, Θ. The clients' trading intensity, η, is the one parameter for which we observe the cross-sectional distribution, $p(\eta)$, in the data. Insurer n's trading shock intensity η_n can be estimated by the average number of bond purchases per year over the sample period. Utilizing multiple years of trade data, we perform the estimation separately for each insurer in the sample, which enables us to construct $p(\eta)$. The model provides the optimal network size, $N^*(\Theta, \eta_n)$, for client n, thus allowing us estimate the unobservable model parameters Θ by matching the model-implied N^* to its empirical counterpart. Using the structurally estimated parameters along with the distribution of trading intensities quantitatively reproduces the distribution of network sizes observed in the data and the dependence of trading costs on network size found in the data. The model estimates reasonable unobserved parameters, e.g, dealers can find the bond within a day or two and insurers' average holding period ranges from two to four weeks. Allowing dealers' bargaining power to decrease with insurers' trading intensity improves the model's fit.

The paper complements the empirical literature on the microstructure of OTC markets and its implications for trading, price formation, and liquidity. Edwards, et al. (2005), Bessembinder et al. (2006), Harris and Piwowar (2007), Green et al. (2007) document the magnitude and determinants of transaction costs for investors in OTC markets. Our paper deepens our understanding of OTC trading costs by using the identities of all insurers along with their trading networks and execution costs. These help explain the substantial heterogeneity in execution costs observed in these studies. O'Hara et al. (2015) and Harris (2015) examine best execution in OTC markets without formally studying investors' optimal network choice.

There exists an empirical literature on the value of relationships in financial markets. Similar to our findings, Bernhardt et al. (2004) show that on the London Stock Exchange broker-dealers offer greater price improvements to more regular customers.[1] Bernhardt et al. (2004) do not examine the client-dealer networks and in the centralized exchange quoted prices are observable. Afonso et al. (2013) study the overnight interbank lending OTC market and find that a majority of banks in the interbank market form long-term, stable and concentrated lending relationships. These have a significant impact on how liquidity shocks are transmitted across the market. Afonso et al. (2013) do not formally model the network and do not observe transaction prices. DiMaggio et al. (2015) study inter-dealer relationships on the OTC market for corporate bonds while our paper focuses on the client-dealer relations.[2]

The role of the interdealer market in price formation and liquidity provision are the focus in Hollifield et al. (2015) and Li and Schürhoff (2015). These studies explore the heterogeneity across dealers in their network centrality and how they provide liquidity and what prices they charge. By contrast, we focus on the heterogeneity across clients and how trading intensity affects their networks and transaction prices.

The search-and-matching literature is vast. Duffie et al. (2005, 2007) provide a prominent treatment of search frictions in OTC financial markets, while Weill (2007), Lagos and Rocheteau (2007, 2009), Feldhütter (2011), Neklyudov (2014), Hugonnier et al. (2015), Üslü (2015) generalize the economic setting. These papers do not focus on repeat relations and do not provide incentives to investors to have a finite size network. Gavazza (2016) structurally estimates a model of trading in decentralized markets with two-sided one-to-one search and bilateral bargaining using aircraft transaction data. At the market-wide level Gavazza (2016) quantifies the effects of market frictions on prices and allocations. We use the structural estimation of our one-to-many search-and-match model to quantify the effects of client-dealer relations on execution quality in the OTC market for corporate bonds.

Directed search models allow for heterogenous dealers and investors, as well as arbitrary trade quantities. These typically rely on a concept of competitive search equilibrium proposed by Moen (1997) for labor market. Examples include Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2014), and Lester et al. (2015). These papers explain assortative matching between clients and dealers and show how heterogeneity affects prices and liquidity. However the matching technology employed by these papers is one-to-one, thus limiting the network size to a single dealer.

[1] For a comprehensive theoretical model of loyalty see Board (2011).

[2] Our paper also relates to a growing literature studying trading in a network, e.g., Gale and Kariv (2007), Gofman (2011), Condorelli and Galeotti (2012), Colliard and Demange (2014), Glode and Opp (2014), Chang and Zhang (2015), Atkeson et al. (2015), Babus (2016), Babus and Hu (2016), and Babus and Kondor (2016). These papers allow persistent one-to-one dealer-client relationships, while the main focus of our model is on client networks.

1 Data

Insurance companies file quarterly reports of trades of long-term bonds and stocks to the National Association of Insurance Commissioners (NAIC). For each trade the NAIC data include the dollar amount of transactions, par value of the transaction, insurer code, date of the transaction, the counterparty dealer name, and the direction of the trade for both parties, e.g., whether the trade was an insurance company buying from a dealer or an insurance company selling to a dealer. The NAIC data preclude intraday analysis as the trades do not include time stamps of the trades. To focus on secondary trading we only include trades more than 60 days after issuance and trades more than 90 days to maturity.

Our final sample covers all corporate bond transactions between insurance companies and dealers reported in NAIC from January 2001 to June 2014. We supplement the NAIC data with a number of additional sources. Bond and issuer characteristics come from the Mergent Fixed Income Security Database (FISD). Insurer holdings and bond ratings come from Lipper eMAXX data. Insurer financial characteristics come from A.M. Best and SNL Financial. The final sample contains 506 thousand insurer buys and 497 thousand insurer sells.

Table 1 reports descriptive statistics for the corporate bond trades (Panel A) and insurers (Panel B) in our 2001-2014 sample. There are 4,324 insurance companies in our sample. Insurance companies fall into three groups based on their product types: (i) Health, 617 companies (14% of the sample); (ii) Life, 1,023 companies (24% of the sample); (iii) P&C 2,684 companies (62% of the sample). Health insurance companies account for 16.3% of trades and 4.4% of yearly trading volume. They trade on average with 6.59 dealers each year. Life insurance companies account for the majority 46.9% of trades and 70.4% of yearly trading volume. They trade on average with 8.06 dealers each year. P&C insurance companies comprise 36.8% of trades and 25.2% of yearly trading volume. They trade on average with 4.81 dealers each year.

The distribution of trading activity is skewed with the top ten insurance companies accounting for 6.3% of trades and 14.3% of trading volume. They use almost 30 dealers which is much higher than the sample average of 5.83 dealers per insurer. The top 100 insurers account for 27.8% of trades as well as for 45.3% of trading volume. The 3,000 smallest insurers use on average 3.76 dealers.

Insurers trade in a variety of corporate bonds. The average issue size is quite large at $917 million and is similar across insurer's buys and sells. The average maturity is nine years for insurer buys and eight years for insurer sells. Bonds are on average 2.88 years old with sold bonds being a little older at 3.09 years. Finally, 75% of all bonds trades are in investment

Table 1: Descriptive statistics

The table reports descriptive statistics for trades (Panel A) and insurers (Panel B) in our sample from 2001 to 2014. Panel A reports the average across all trades over the sample period. Panel B reports the yearly average across insurers.

	Panel A: Trades		
	All trades	Insurer buys	Insurer sells
No. of trades (k)	1,003	506	497
Trade par size ($mn)	1.80	1.73	1.87
Bond issue size ($mn)	916.66	921.37	911.87
Bond age (years)	2.88	2.67	3.09
Bond remaining life (years)	8.54	8.94	8.13
Private placement (%/100)	0.08	0.08	0.07
Rating (%/100)			
IG	0.74	0.76	0.72
HY	0.25	0.23	0.28
Unrated	0.01	0.01	0.01
	Panel B: Insurers (N= 4,324)		
	Volume ($mn)	No. of trades	No. of dealers
All insurers	17.32	9.52	5.83
Insurer type			
Health (617, 14%)	10.66	21.74	6.59
Life (1,023, 24%)	103.00	37.71	8.06
P&C (2,684, 62%)	14.08	11.29	4.81
Insurer activity			
Top 10	2,111.88	517.92	29.89
11-100	509.49	233.04	22.07
101-1000	75.66	46.22	11.56
1001+	3.80	4.24	3.76
Insurer characteristics:	Mean (SD)		
Insurer size	4.97 (0.90)		
Insurer RBC ratio	3.36 (0.35)		
Insurer cash-to-assets	3.49 (10.79)		
Life insurer	0.24 (0.42)		
P&C insurer	0.62 (0.48)		
Insurer rated A-B	0.37 (0.38)		
Insurer rated C-F	0.01 (0.07)		
Insurer unrated	0.53 (0.39)		

grade while only 1% are in unrated with the remainder being high yield. Privately placed bond trades form a small minority of our sample at 8%.

The risk-based-capital (RBC) ratio measures an insurer's capital relative to the riskiness of its business. The higher the RBC ratio, the better capitalized the firm. Insurer size is reported assets. The cash-to-asset ratio is cash flow from the insurance business operations divided by assets.

Overall, there exists a large degree of heterogeneity on the client side in our sample. Insurance companies buy and sell large quantities of different corporate bonds and execute these transactions with the number of dealers ranging on average from one to as many as 40.

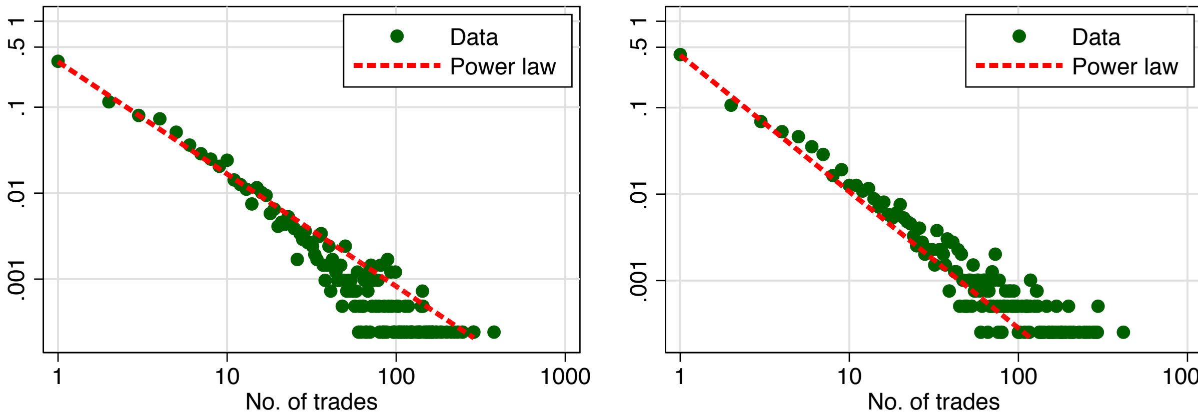


Figure 2: Insurer trading activity
The figure shows the distribution in the number of insurer buys per year (left) and insurer sales (right). We use a log-log scale.

2 Empirical Results on Insurer Trading Networks

This section empirically characterizes insurers' trading intensity, and the size of their trading networks.

2.1 Insurer trading activity

We investigate the determinants of both the extensive margin, i.e., the number of trades, and intensive margin, i.e., the total dollar volume traded, of insurer trading in a given year. Both margins reveal that insurers have heterogenous trading needs.

We start with univariate analysis. The majority of insurers do not trade often at the annual frequency. About 30% of insurers trade just once per year while 1% of the insurers make at least 25 trades per year. This is consistent with the evidence from Table 1 that while the top 100 insurers constitute just 0.23% of the total sample, they account for as much as 32% of all trades in our sample. The mean number of trades per year is 19, with a median of 14, with several insurers making more than 1,000 trades in some years and up to the maximum of 2,200 trades in a year.

Figure 2 shows the distribution in the average number of trades per year across insurers. A large fraction of insurers do not trade in a given month and we therefore report an annual figure. The annual distributions follow a power law with $p(X) \propto .27 \times X^{-1.21}$ for all insurer trades combined. The power law is $.34 \times X^{-1.31}$ for insurer buys (depicted in Panel A) and $.40 \times X^{-1.58}$ insurer sales (Panel B). Visually the two power law distributions for insurer buys and sales in Figure 2 look similar. This suggests insurers buy and sell at similar rates,

Table 2: Insurers' trading activity

The determinants of insurance company trading activity are reported. We measure trading activity by the total dollar volume traded in a given year and, alternatively, by the number of trades over the same time horizon. All dependent variables are log-transformed by 100*log(1+x). All regressors are averaged across all trades of the insurer during the period and lagged by one time period. Estimates are from pooled regressions with time fixed effects. Standard errors are adjusted for heteroskedasticity and clustering at the insurer and time level. Significance levels are indicated by * (10%), ** (5%), *** (1%).

Determinant	(1) Volume ($mn)	(2) No. of trades
Insurer size	21.95***	14.51***
Insurer RBC ratio	-1.53	-4.68***
Insurer cash-to-assets	0.27***	0.26***
Life insurer	4.96***	0.27
P&C insurer	-1.06	-3.89**
Insurer rated A-B	5.13**	5.80***
Insurer rated C-F	1.97	-0.43
Insurer unrated	6.39***	5.79**
Trade par size	-3.62***	-3.22***
Bond issue size	-0.00	0.00
Bond age	-0.79***	-1.25***
Bond remaining life	-0.04	-0.05
Bond high-yield rated	4.62***	4.65***
Bond unrated	-6.67	-12.81*
Bond privately placed	-5.37	-3.56
Variation in trade size	4.50***	1.52***
Variation in issue size	0.00	0.00
Variation in bond age	0.37	0.39
Variation in bond life	0.52***	0.65***
Variation in bond rating	-0.35	0.04
Variation in rated-unrated	39.60*	6.16
Variation in private-public	1.62	-1.60
No varieties traded	9.23***	13.64***
Lagged volume	0.67***	
Lagged no. of trades		0.76***
Year fixed effects	Yes	Yes
R^2	0.789	0.646
N	30,029	30,029

even though these rates vary significantly across insurers.

We next examine what characteristics explain the heterogeneity in trading intensities. Table 2 documents the determinants of the intensive margins (trading volume in $bn, column (1)) and extensive margins (number of trades, column (2)) of the annual trading by insurance companies using pooled regressions with time fixed effects. The specification consists of the trade par size, insurer and bond characteristics, as well as the variation in the trade size and bond characteristics across all trades of the insurer during a given year. Insurer characteristics include its size, cash-to-assets ratio, type, RBC ratio, and rating. Bond and trade characteristics include size, age, maturity, rating, a private placement dummy, and the trade size. Insurer size, RBC ratio, and the dependent variables are log-transformed. All regressors are averaged across all trades of the insurer during the period and lagged by one year.

Logarithms of both measures of trade intensity are persistent; the coefficient on the lagged log-volume is 0.67 and the coefficient on the lagged log-number-of-trades is 0.76. Both coefficients are statistically significant at 1% levels. This evidence is consistent with insurance companies having persistent portfolio rebalancing needs.

Insurer trading strongly correlates with insurer size, type, and quality, with bond types and bond varieties as these variables explain 79% of the variation in annual trading volume and 65% variation in annual number of trades. A ten-fold increase in insurer's size increases trading volume by $2.2 billion. Larger insurance companies and insurers with higher cash-to-assets ratio also trade more often and submit larger orders. Insurers with higher RBC ratios trade less often than insurers with low RBC ratios. Both margins of trading increase with the insurer's rating, i.e., insurers with the lowest rating (C-F) trade less than higher-rated insurers. Life insurers tend to submit larger orders.

Both margins of bond turnover increase as bond ratings decline; lower rated bonds are traded more often and in larger quantities. Insurers tend to trade privately placed bonds less since potentially they just own fewer of them than publicly placed bonds. Both margins of bond turnover decline with par size and bond age indicating that the majority of insurers are long-term investors. Neither measure of trade intensity depends on bond issue size and remaining life as their coefficients are not statistically significant.

Finally, both trading volume and the number of trades decline if an insurer trades more bond varieties. However, a specific variety can have an opposite effect on the trading intensity. For instance, both measures of trading intensity increase with variation in bond rating and bond life. This is consistent with insurers increasing trading intensity when rebalancing their portfolios, i.e. shifting from high-yield to lower yield bonds or from younger to older bonds.

Overall, these analysis highlight large heterogeneity in trading intensities across different insurers. Trading intensity depend on the variety of bonds traded, bond specific characteristics, and the insurer type and quality. We now turn to how these characteristics affect the insurers' choice of dealer network.

2.2 Properties of insurer networks

The previous section's results demonstrate how insurer characteristics explain the intensity of their trading. There is large degree of heterogeneity in the insurers' trading intensity with some insurers trading on average twice per day while others trade just once per year. This suggests that insurers may have similarly heterogeneous demands for their dealer network.

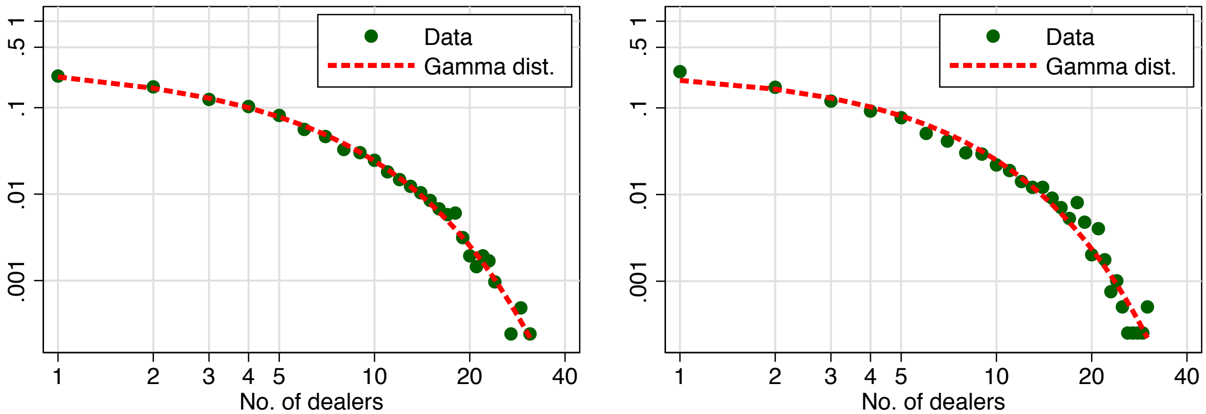


Figure 3: Size of insurer-dealer trading networks
The figure shows the degree distribution for insurer-dealer relations by year for insurer buys (left) and insurer sales (right). We use a log-log scale.

This section studies how many dealers they trade with over time and how persistent are these networks. These results describe the basic network formation mechanism in the OTC markets.

We start with the examples of the insurer-dealer relationship depicted in Figure 1. These show that insurers do not trade with a dealer randomly picked from a large pool of corporate bonds dealers. Instead, insurers buy from the same dealers that they sell bonds to and they engage in long-term repeat, but non-exclusive, relations. We analyze how representative are the examples in Figure 1 and how insurer characteristics determine their network size.

Figure 3 plots the degree distribution across insurers by year, i.e., the fraction of insurers trading on average across all years with the given number of dealers, using a log-log scale. The figure shows insurers trade with up to 31 dealers every year, with some trading with as many as 40 but these represent less than 1/10,000 of the sample. Exclusive relations are dominant with almost 30% of insurers trading with a single dealer in a given year.[3] The degree distributions in Figure 3 follow a power law with exponential tail starting at about 10 dealers. This is consistent with insurers building networks that they search randomly within. Fitting the degree distribution to a Gamma distribution by regressing the log of the probabilities of each N on a constant, the logarithm of N, and N yields the following

[3]Because of the number of insurers that do not trade in a year Figures 2 and 3 are not directly comparable. Figure 2 plots the average number of trades per year, rounded to the next integer and winsorized from below at 1. For about half of the insurer-years there is no trade by an insurer. Figure 3 does not impute a zero for a given year if the insurer does not trade. Hence, 27.7% of insurers use a single dealer in a year when they trade, while 14.8% of insurers trade just once in a year. Thus, many insurers who trade more than once in a year use a single dealer.

coefficients:

$$
\begin{aligned}
\text{For all insurer trades combined:} \quad & p(N) \propto N^{.15} e^{-.20N}, \\
\text{For insurer buys:} \quad & p(N) \propto N^{-.12} e^{-.22N}, \\
\text{For insurer sales:} \quad & p(N) \propto N^{.01} e^{-.24N}.
\end{aligned}
\tag{1}
$$

Table 3 reports the determinants of insurer dealer network sizes using pooled regressions with time fixed effects. We measure the size of the trading network by the number of dealers that an insurance company trades with in a given year. We log-transform all dependent variables by $100 * log(1 + x)$ and average all regressors across all trades by the same insurer during the year and lag them by one time period. We perform the estimation on the whole sample (Column (1)) and, in order to examine how these vary with insurer's size, on subsamples of small and large insurers based on asset size. We classify an insurer as small if it falls in the bottom three size quartiles and, respectively, as large if it falls in the top quartile of the size distribution.

Column (1) indicates that insurer size and type, bond characteristics, and bond varieties matter for the size of the dealer network. Large insurers, which Table 2 shows have larger trading intensity, trade with more dealers. Insurers with demand for larger bond variety have larger networks even controlling for their size (column (2) and (3)). Higher quality insurers, i.e., insurers with higher cash-to-assets ratio and higher ratings, have larger networks, but this matters only for smaller insurers as column (2) indicates. This is potentially due to it being cheaper for a dealer to set up a credit account for higher quality insurers. These factors matter more for small insurers because they face larger adverse selection problems in forming permanent links with dealers. Insurers with greater variety in the bond they trade have larger networks. Overall these findings suggest insurers' network choice is endogenous and dependent on multiple factors. Competition and specialization jointly determine investors' trade choices.

Table 3 shows persistence in the size of the network with the coefficient on the lagged network size being 0.75 (column (1)). This result is mostly due to large insurers, because this coefficient equals only 0.62 for small insurers. Table 4 examines this in more detail by reporting statistics for the frequency with which insurers adjust their network size. We compute the likelihood that an insurer uses a certain number of dealers in a given year and compare it to the corresponding number in the following year. The transition probabilities are reported in Table 4. Trading relations are persistent from year to year. This is especially true for exclusive relations as the probability of staying with a single dealer each year is equal to 0.61. Insurers with more than one dealer are very unlikely to switch to a single dealer

Table 3: Size of insurers' trading network

The table reports the determinants of the size of insurers' trading network. We measure the size of the trading network by the number of different dealers that an insurance company trades with in a given year. See caption of Table 2 for additional details. Standard errors are adjusted for heteroskedasticity and clustering at the insurer and time level.

Determinant	(1) All insurers	(2) Small insurers	(3) Large insurers
Insurer size	9.95***	7.93***	5.04***
Insurer RBC ratio	-3.58***	-3.64***	0.11
Insurer cash-to-assets	0.15***	0.15***	0.11**
Life insurer	-0.13	0.75	-2.77**
P&C insurer	-2.60***	-1.20	-3.59***
Insurer rated A-B	4.12***	6.00***	0.13
Insurer rated C-F	-1.54	-0.18	-5.42***
Insurer unrated	3.05*	3.98*	-2.86**
Trade par size	-1.92***	-1.23***	-1.24***
Bond issue size	-0.00	-0.00	0.00
Bond age	-0.91***	-0.77***	-1.10***
Bond remaining life	-0.08	-0.11*	0.10
Bond high-yield rated	1.23	-2.93*	2.56
Bond unrated	-13.74***	-12.50***	-7.73
Bond privately placed	-3.85	1.10	-10.22
Variation in trade size	0.51	-0.10	0.81*
Variation in issue size	0.00	0.00	0.00
Variation in bond age	0.50**	0.56**	0.16
Variation in bond life	0.50***	0.41***	0.11
Variation in bond rating	0.05	0.19	0.36*
Variation in rated-unrated	3.60	-11.38	-24.50
Variation in private-public	1.20	-4.33	3.17
No varieties traded	5.88***	2.71***	12.50**
Lagged no. of dealers	0.75***	0.62***	0.75***
Year fixed effects	Yes	Yes	Yes
R^2	0.614	0.350	0.593
N	30,029	18,033	11,996

Table 4: Persistence in insurers' trading network

Switching probabilities p(No. of dealers in $t+1$|No. of dealers in t) for using a network size conditional on the insurer's past behavior for each year.

	No. of dealers next year			
No. of dealers this year	1	2-5	6-10	>10
1	0.61	0.30	0.06	0.03
2-5	0.20	0.54	0.20	0.06
6-10	0.06	0.31	0.40	0.24
>10	0.01	0.07	0.17	0.75

as the annual switching probabilities are equal to 0.20 for insurers with 2 to 5 dealers and 0.06 for insurers with 6 to 10 dealers. Insurers with the largest networks (> 10 dealers) tend to maintain large networks over time, with the probability of staying with a large network being 0.75. The distribution of insurers shown in Figure 3 together with the stable network sizes are difficult to reconcile with a "pure" random search model à la Duffie et al. (2005, 2007).

In the next section we study the relations between client-dealer networks and execution costs.

3 Insurer trading costs and networks

Tables 2 and 3 suggest that bond characteristics impact insurers' trading intensity. To control for bond, time, and bond-time variation, we compare transaction prices to daily bond-specific Bank of America-Merrill Lynch (BAML) bid (sell) quotes. BAML is the largest corporate bond dealer, transacting with more than half of all insurers for almost 10% of both the trades and volume. The BAML (bid) quotes can be viewed as representative quotes for insurer sales and enable us to measure prices relative to a transparent benchmark price. The BAML quotes essentially provide bond-time fixed effects, which would be too numerous to estimate in our sample. Our relative execution cost measure in basis points is defined as

$$\text{Execution cost (bp)} = \frac{\text{BAML Quote} - \text{Trade Price}}{\text{BAML Quote}} * (1 - 2 * \mathbf{1}_{Buy}) * 10^4, \tag{2}$$

where $\mathbf{1}_{Buy}$ is an indicator for whether the insurer is buying or selling. Because some quotes may be stale or trades misreported, leading to extreme costs estimates, we winsorize the distribution at 1% and 99%.

Execution costs depend on the bond being traded, time, whether the insurers buys or sells, the insurer's characteristics, dealer identity and characteristics, and the insurer network size. To examine the relationship-specific effects on execution costs we control for bond and time fixed effects. In principle if the BAML perfectly controls for bond-time effects, the additional bond and time fixed effects are unnecessary.

The relationship component of transaction costs depends on the properties of the insurers' networks. Figure 3 and equation (1) indicate that the degree distribution for insurer-dealer relations follows a Gamma distribution. Therefore, we include both the size of the network, N, and its natural logarithm, $ln(N)$, as explanatory variables. We control for seasonality using time fixed effects, α_t, and for unobserved heterogeneity using either bond characteristics or bond fixed effects, α_i. The other explanatory variables consist of either insurers' or dealers' characteristics, or both. We estimate the following panel regression for execution costs in bond i at time t:

$$\text{Execution cost}_{it} = \alpha_i + \alpha_t + \beta N + \gamma lnN + \theta X_{it} + \epsilon_{it}. \tag{3}$$

The set of explanatory variables X includes characteristics of the bond, as well as features of the insurer and dealer.

Table 5 provides trading cost estimates from panel regressions. We adjust standard errors for heteroskedasticity and cluster them at the insurer, dealer, bond, and day level. The coefficient on insurer buy captures the average bid-ask spread of roughly 40 basis points. Column (1) of Table 5 shows that execution costs decline with insurer network size. An insurer with an additional dealer has trading cost 0.22 basis point lower. Large insurers pay on average lower execution costs. An insurer with 10 times as many assets has trading cost 3.72 basis points lower. Better capitalized insurers (higher RBC ratio) get better prices. Column (2) adds the logarithm of N to the specification reported in Column (1). The coefficient on N switches from -0.22 reported in Column (1) to 0.32, while the coefficient on the logarithm of N is -6.29. Both coefficients are statistically significant at 1%. This result indicates that the execution costs are non-monotone in the network size. Improvements in execution quality from having a larger dealer network are exhausted at $N = \frac{6.29}{0.22} \approx 20$. Clients with networks of 40 dealers and 10 dealers pay, on average, the same bid-ask spread of 40 basis points. This finding goes against the traditional wisdom that inter-dealer competition improves prices. It is also inconsistent with classic static strategic network formation models (e.g., Jackson and Wolinsky (1996)). In these models a client trades off fixed costs of adding an extra dealer against better execution due to increased dealer competition thus making price a monotonically decreasing function of the network size. In the next section we use this and other network-related empirical evidence to motivate an alternative strategic model of finite network formation in which clients and dealer share the benefits of repeated interactions.

Columns (3) and (4) replace bond and dealer fixed effects with bond and dealer characteristics. NYC-located dealers offer better prices to all insurers and more diversified dealers charge, on average, higher prices. Bond characteristics matter for execution costs as insurers receive worse prices for special bonds and better prices for bonds with larger issue size. Insurers get better prices on unrated bonds.

The next section uses our evidence on networks and execution costs to motivate our model of the OTC markets.

Table 5: Execution costs and investor-dealer relations

The table reports the determinants of execution costs. Execution costs are expressed in basis points relative to the Merrill Lynch quote at the time of the trade. Standard errors are adjusted for heteroskedasticity and clustered at the insurer, dealer, bond, and day level. See caption of Table 2 for additional details.

Determinant	(1)	(2)	(3)	(4)
Insurer no. of dealers	-0.22***	0.32***	0.32***	0.32***
ln(Insurer no. of dealers)		-6.29***	-6.51***	-6.55***
Insurer size	-3.72***	-3.59***	-3.52***	-3.95***
Insurer RBC ratio	-3.51***	-4.19***	-4.68***	-5.40***
Insurer cash-to-assets	-0.04**	-0.04**	-0.04**	-0.03*
Life insurer	4.43***	4.47***	5.73***	7.21***
P&C insurer	1.72**	1.73**	1.99**	2.82***
Insurer rated A-B	-0.47	0.01	-0.18	-0.50
Insurer rated C-F	11.29*	11.13*	10.90*	12.18*
Insurer unrated	0.74	0.62	0.55	0.14
Insurer buy	39.56***	39.27***	39.71***	40.17***
Trade size × Buy	-0.26***	-0.25***	-0.20**	-0.18*
Trade size × Sell	0.53***	0.50***	0.48***	0.52***
Bond issue size			-0.00***	-0.00***
Bond age			0.58***	0.63***
Bond remaining life			0.81***	0.82***
Bond HY rated			4.54***	4.16***
Bond unrated			-5.96***	-6.53***
Bond privately placed			3.38***	3.26***
Dealer size				-5.37***
NYC dealer				-6.66***
Primary dealer				2.43
Dealer leverage				-5.68**
Dealer diversity				0.41***
Dealer dispersion				0.07
Local dealer				0.23
Dealer distance				-0.20
Dealer leverage missing				-6.41**
Dealer dispersion missing				6.09
Bond fixed effects (16,823)	Yes	Yes	No	No
Dealer fixed effects (401)	Yes	Yes	Yes	No
Day fixed effects (3,375)	Yes	Yes	Yes	Yes
R^2	0.154	0.155	0.103	0.098
N	918,279	918,279	918,987	891,875

4 Model

The model is stylized but still rich enough to allow for the structural estimation of its primitives from the NAIC data.

4.1 Setup and Solution

The economy has a single risk-free perpetual bond paying a coupon flow C. The risk-free discount rate is constant and equal to r, so that the present value of the bond is $\frac{C}{r}$. To model client-dealer interactions, we keep several attractive features of Duffie et al. (2005) type models such as liquidity supply/demand shocks on the client side and random search with constant intensity. Following Lester et al. (2015), the bond trades on a competitive market accessible only to dealers.[4] Unlike the frictionless inter-dealer market in Lester et. al. (2015), in our model dealers face search frictions as in Duffie et al. (2005). Dealers buy bonds at an exogenously given price M^{ask} from other dealers and sell it to other dealers at an exogenously given price M^{bid}. A bid-ask spread $M^{ask} - M^{bid} \geq 0$ reflects trading costs or cost of carry.

Each client chooses a network of dealers, N, without knowledge of other clients' decisions. When a client wants to buy/sell a bond, she simultaneously contacts all N dealers in her network. Upon being contacted each dealer starts searching the competitive dealer market for a seller/buyer with a search intensity λ.[5] All dealers in the client's network search independently of each other. Therefore, the effective rate at which a client with N dealers in her network finds a counterparty equals λN. When the client receives a subsequent trading shock all dealers in the network are contacted to reverse the initial transaction.

Each client pays a cost K per transaction.[6] The cost K is any cost of a client contacting dealers, e.g., the time required to make each phone call and any fixed costs required to hire more in-house traders. Clients trading more frequently incur K more often. While K does not depend directly on networks size, clients who choose a larger network trade faster, thus, incurring K more often. Clients' search mechanism can be viewed as a winner-takes-all race with the dealer first to find the bond winning the race. The prize is the spread $P^b - M^{ask}$

[4] The interdealer market obviates the need to track where the entire stock of the asset is held at every moment in time.

[5] Bessembinder et al. (2016) show that corporate bond dealers increasingly hold less inventory and facilitate trade via effectively acting as brokers by simultaneously buying and selling the same quantity of the same bond.

[6] The costs of additional dealers could alternatively be modeled as per dealer or per dealer per transaction. Per dealer costs consist of costs of forming a credit relationship and any other costs of maintaining the relationship independent of the number of trades. Such per dealer costs will immediately lead to clients with larger trading intensity using more dealers.

when the client buys and $M^{bid} - P^s$ when the client sells, where P^b (P^s) is the price at which the client buys (sells) the bond from (to) the dealer.

Clients transition through ownership and non-ownership based on liquidity shocks. At these transitions clients act as buyers and sellers. The discounted transition probabilities and transaction prices link their valuations across the owner, non-owner, buyer, and seller states.

A client starting as a non-owner with valuation $\widehat{V}^{no}$ is hit by stochastic trading shocks to buy with intensity η. The client contacts her network of N dealers leading to her transiting to a buyer state with valuation $\widehat{V}^b$. In steady state valuations in these two states are related by

$$\widehat{V}^{no} = \underbrace{\widehat{V}^b \frac{\eta}{r+\eta}}_{\text{Value from Trading}} + \underbrace{V^r \frac{r}{r+\eta}}_{\text{Relation-Specific Value}} . \tag{4}$$

Here $V^r(\eta, N)$ captures exogenously given relation-specific flows to the client from her dealers. These flows are separate from any value generated from trading in the bond. In practice they include transacting in other securities, the ability to purchase newly issued securities, as well as other non-monetary transfers such as investment research. We are largely agnostic regarding $V^r(\eta, N)$ as a function of the trading intensity and network size, η and N, and let the functional form be determined by the data-driven estimation. We, however, assume that $V^r(\eta, N)$ is a monotonic function of N and satisfies the following Inada condition $\lim_{N\to\infty} \frac{V^r(\eta,N)}{N} = 0$ which guarantees that the optimal network size is always finite. Below we will show that $V^r(\eta, N)$ does not affect transaction prices, but does impact clients choice of network size.

The buyer purchases the bond from her network at the expected price $\mathrm{E}[P^b]$ and transitions into being an owner with valuation $\widehat{V}^o$. In steady state $\widehat{V}^b$ satisfies the Bellman equation linking it to $\widehat{V}^o$

$$\widehat{V}^b = \frac{1}{1+rdt}[\lambda N dt(\widehat{V}^o - \mathrm{E}[P^b] - K) + (1 - \lambda N dt)\widehat{V}^b + rV^r dt], \tag{5}$$

yielding

$$\widehat{V}^b = (\widehat{V}^o - \mathrm{E}[P^b] - K)\frac{\lambda N}{r+\lambda N} + V^r \frac{r}{r+\lambda N}. \tag{6}$$

While clients are owners they receive a coupon flow C and have valuation $\widehat{V}^o$. Non-owners do not receive the coupon flow. With intensity κ an owner receives a liquidity shock forcing her to become a seller with valuation $\widehat{V}^s$. In steady state valuations in these two states are linked according to the following Bellman equation

$$\widehat{V}^o = \frac{1}{1+rdt}[dtC + \kappa dtV^s + (1-\kappa dt)\widehat{V}^o + rV^r dt], \tag{7}$$

yielding the expression for $\widehat{V}^o$

$$\widehat{V}^o = \frac{C}{r+\kappa} + \underbrace{\widehat{V}^s \frac{\kappa}{r+\kappa}}_{\text{Value From Future Sale}} + V^r \frac{r}{r+\kappa}, \tag{8}$$

where the second term captures the value from future sales. The liquidity shock received by the owner reduces the value of the coupon to $C(1-L)$ until she sells the bond. After receiving the liquidity shock she contacts her dealer network expecting to sell the bond for $\mathrm{E}[P^s]$. Upon selling she becomes a non-owner, completing the valuation cycle. Valuations $\widehat{V}^s$ and $\widehat{V}^{no}$ are related by

$$\widehat{V}^s = \frac{1}{1+rdt}[dtC(1-L) + \lambda N dt(\mathrm{E}[P^s] + \widehat{V}^{no} - K) + (1-\lambda N dt)\widehat{V}^s + rV^r dt], \tag{9}$$

which can be solved for $\widehat{V}^s$ as

$$\widehat{V}^s = \frac{C(1-L)}{r+\lambda N} + (\mathrm{E}[P^s] + \widehat{V}^{no} - K)\frac{\lambda N}{r+\lambda N} + V^r \frac{r}{r+\lambda N}. \tag{10}$$

This sequence of events continues in perpetuity and, therefore, we focus on the steady state of the model.

The above valuation equations depend upon the expected transaction prices. The realized transaction prices are determined by bilateral Nash bargaining. The client's reservation values are determined by the differences in values between being an owner and non-owner and a buyer and seller. Similarly, the dealers' reservation values arise from their transaction cycle. Each dealer acts competitively, i.e., without taking into account the effect of her actions on the actions of other dealers. In addition, we assume that each dealer internalizes only trade-specific value of her relation with each client. When a client contacts her dealer network each dealer simultaneously starts looking for the bond at rate λ and expects to pay the inter-dealer ask price M^{ask} for the bond. The value to the dealer searching for the bond satisfies

$$U^b = \frac{1}{1+rdt}[\lambda dt(P^b - M^{ask}) + \lambda N dt U^o + (1-\lambda N dt)U^b], \tag{11}$$

thus yielding

$$U^b = \underbrace{(P^b - M^{ask})\frac{\lambda}{r+\lambda N}}_{\text{Transaction Profit/Loss}} + \underbrace{U^o\frac{\lambda N}{r+\lambda N}}_{\text{Value of Future Business}} \quad . \tag{12}$$

The last term in the expression for U^b captures the expected value of the future business with the same client which happens with frequency λN. This client, who is now the owner of the bond, becomes a seller with intensity κ and contacts dealers in her network to sell the bond. This generates a value $U^o = U^s\frac{\kappa}{r+\kappa}$ per dealer, where U^s represents the valuation of the dealer searching to sell the bond. The dealer expects to resell the bond at rate λ for the inter-dealer bid price M^{bid} and earn a markup of $M^{bid} - P^s$. She also anticipates that with intensity λN the same client will approach her in the future to buy back the bond. Future business from the same client generates $U^{no} = U^b\frac{\eta}{r+\eta}$ in value to the dealer, thus leading to the following Bellman equation for U^s:

$$U^s = \frac{1}{1+rdt}[\lambda dt(M^{bid} - P^s) + \lambda N dt U^{no} + (1 - \lambda N dt)U^s], \tag{13}$$

which can be solved to obtain

$$U^s = \underbrace{(M^{bid} - P^s)\frac{\lambda}{r+\lambda N}}_{\text{Transaction Profit/Loss}} + \underbrace{U^{no}\frac{\lambda N}{r+\lambda N}}_{\text{Value of Future Business}} \quad . \tag{14}$$

Valuations U^{no} and U^o lead to price improvement for repeat business.

As in most OTC models, prices are set by Nash bargaining resulting in:

$$P^b = (\widehat{V}^o - \widehat{V}^b)w + (M^{ask} - U^o)(1-w), \tag{15}$$
$$P^s = (\widehat{V}^s - \widehat{V}^{no})w + (M^{bid} + U^{no})(1-w). \tag{16}$$

Prices are the bargaining-power (w) weighted average of the reservation values of the client and dealer. The above equations assume that the dealer loses all future business from the client if the bilateral negotiations fail. Upon dropping a dealer the client maintains her optimal network size by forming a new link with another randomly picked identical dealer. Thus, by agreeing rather than not, the dealer receives U^o. As a consequence, dealers face intertemporal competition for future clients. This is a novel assumption missing from the existing models of OTC markets.

Each client's valuation, $\widehat{V}^k$, $k \in \{b, o, s, no\}$, can be written as a sum of its trade-specific,

V^k, and the relation-specific, V^r, values

$$\widehat{V}^k = V^k + V^r. \tag{17}$$

Substituting (17) into relations (4), (6), (18), and (10) yields the following relations for trade-specific client valuations

$$\begin{aligned} V^{no} &= V^b \frac{\eta}{r+\eta}, \\ V^b &= (V^o - \mathrm{E}[P^b] - K)\frac{\lambda N}{r+\lambda N}, \\ V^o &= \frac{C}{r+\kappa} + V^s \frac{\kappa}{r+\kappa}, \\ V^s &= \frac{C(1-L)}{r+\lambda N} + (\mathrm{E}[P^s] + V^{no} - K)\frac{\lambda N}{r+\lambda N}. \end{aligned} \tag{18}$$

Correspondingly, transaction prices depend only on trade-specific client's valuations

$$P^b = (V^o - V^b)w + (M^{ask} - U^o)(1-w), \tag{19}$$

$$P^s = (V^s - V^{no})w + (M^{bid} + U^{no})(1-w). \tag{20}$$

Bargaining power could differ for buys and sells. For ease of exposition, we equate them here. In the subsequent structural estimation we allow for different bargaining powers when the insurer is looking to buy a bond, w^b, and when the insurer is selling a bond, w^s. The valuations and prices provide ten equations and ten unknowns. Proposition 1 in the Appendix provides the closed form solutions (35) and (36) for transaction buy and sell prices respectively.

Expressions (35) and (36) are nonlinear functions of the model primitives and N, which makes the analytical analysis difficult. However, we can verify that prices are well-behaved functions of the network size, N, in the large network limit, $N \to \infty$. $N \to \infty$ implies $\frac{\lambda N}{r+\lambda N} \to 1$ and clients' search friction in terms of time is zero. Dealers' valuations, in this case denoted with subscript $N \to \infty$, satisfy the system of equations $U^b_{N\to\infty} = U^s_{N\to\infty}\frac{\kappa}{r+\kappa}$ and $U^s_{N\to\infty} = U^b_{N\to\infty}\frac{\eta}{r+\eta}$. These only have a trivial solution $U^s_{N\to\infty} = U^b_{N\to\infty} = 0$, implying that dealers compete away all rents from future relations with clients. Clients have valuations $V^o_{N\to\infty} - V^b_{N\to\infty} = P^b_{N\to\infty} + K$ and $V^s_{N\to\infty} - V^{no}_{N\to\infty} = P^s_{N\to\infty} - K$ yielding the following expressions for transaction prices:

$$P^b_{N\to\infty} = M^{ask} + \frac{w}{1-w}K, \tag{21}$$

$$P^s_{N\to\infty} = M^{bid} - \frac{w}{1-w}K. \tag{22}$$

Dealers receive no relationship-based rents, but they charge clients a spread $\frac{w}{1-w}2K$ per roundtrip transaction over the inter-dealer spread. The coefficient $\frac{w}{1-w}$ indicates that the non-zero bargaining power enables dealers to extract some value from a client. This is because every dealer charges the same price thus making clients' threat of ending the relationship and forming a new link not credible (Diamond's (1971) paradox). Equations (21) and (22) illustrate the importance of making $K > 0$ in the model. If K is equal to zero a client can choose an infinitely large network. This increases dealer competition such that the client trades at the inter-dealer prices, effectively becoming a dealer herself. Therefore, the finite network is never optimal if K is equal to zero.

Overall, dealers' surplus comes from both the immediate value of trade (the spread) and the value of future transactions. As the probability of transacting with the same client in the future declines with the size of the client's network, dealers may charge higher spreads, $P^b - M^{ask}$ and $M^{bid} - P^s$, when the network size becomes large. This result is similar to findings by Vayanos and Wang (2007) where an asset with more buyers and sellers has lower search times and trades at worse prices relative to its identical-payoff counterpart with fewer buyers and sellers. However, Vayanos and Wang (2007) do not model client-dealer repeated interactions.

We verify that the model can yield a finite network size by considering a limiting case of a very large search intensity, $\lambda \to \infty$. In this case a single dealer can instantaneously find the bond and the optimal size of the network is one. Taking this limit in equations (12) and (14) yields

$$U^{no}_{\lambda\to\infty} = \frac{1}{N}\frac{\frac{\eta}{r+\eta}}{1-\frac{\kappa}{r+\kappa}\frac{\eta}{r+\eta}}\left[P^b_{\lambda\to\infty} - M^{ask} + \frac{\eta}{r+\eta}(M^{bid} - P^s_{\lambda\to\infty})\right], \tag{23}$$

$$U^{o}_{\lambda\to\infty} = \frac{1}{N}\frac{\frac{\kappa}{r+\kappa}}{1-\frac{\kappa}{r+\kappa}\frac{\eta}{r+\eta}}\left[M^{bid} - P^s_{\lambda\to\infty} + \frac{\kappa}{r+\kappa}(P^b_{\lambda\to\infty} - M^{ask})\right].$$

Solving for transaction prices from (15-16) and using that $(V^o - V^b)_{\lambda\to\infty} = P^b_{\lambda\to\infty} + K$,

$(V^s - V^{no})_{\lambda\to\infty} = P^s_{\lambda\to\infty} - K$ we obtain

$$P^b_{\lambda\to\infty} = M^{ask} + \frac{w}{1-w}K - U^o_{\lambda\to\infty}, \quad (24)$$
$$P^s_{\lambda\to\infty} = M^{bid} - \frac{w}{1-w}K + U^{no}_{\lambda\to\infty}.$$

Expressions (24) also show that $U^o_{\lambda\to\infty}$ and $U^{no}_{\lambda\to\infty}$ represent the repeat relation buy discount and sell premium, respectively. Equations (23) show that both $U^o_{\lambda\to\infty}$ and $U^{no}_{\lambda\to\infty}$ are strictly decreasing with the size of the network, N. Therefore, the client optimally chooses a single dealer.

In order to find the optimal network size N^* in the general case, we maximize the total valuation of the "first-time" owner, i.e., the client buying the asset, $\widehat{V}^b$. This is because the client has to take possession of the asset in the first place. Maximizing $\widehat{V}^b$ accounts for all trade and relation benefits the client receives from a larger network. Proposition 2 given in the Appendix demonstrates that, for a given client type described by the model primitives $\{L, K, w, \kappa, \lambda, \eta\}$, there may exist an optimal network size $N^* = N(L, K, w, \kappa, \lambda, \eta)$.

Next we provide intuition for why the model yields finite size networks greater than one. Without loss of generality we assume that V^r is a monotonically increasing function of the network size N.[7] Because equation (40) is nonlinear in N the existence of the solution is not guaranteed for an arbitrary parameter set $\{L, K, w, \kappa, \lambda, \eta\}$. Given the complexity of the problem it is convenient to consider the optimal network resulting solely from the repeated trading, i.e., N^{**} maximizing V^b from $\frac{dV^b}{dN} = 0$. $\frac{dV^r}{dN} > 0$ implies that $N^{**} < N^*$ thus making N^{**} the lower bound of N^*. As a result a sufficient condition for existence of a finite N^{**} also applies to N^*. In addition, our analysis helps to separate the trade-specific and the relation-specific network formation mechanisms.

The derivative $\frac{dV^b}{dN} = \frac{\lambda N}{r+\lambda N}\left(\frac{rV^b}{\lambda N^2} + \frac{dV^o}{dN} - \frac{dP^b}{dN}\right)$ must be positive on $[1, N^{**})$ and equal to zero at $N = N^{**}$. $\frac{dV^b}{dN}$ has three terms. The first one is due to the direct effect of the larger network on the speed of execution as faster execution improves buyer's valuation. The second term reflects the marginal effect of the network size on the owner's value, while the third term represents the effect of the network size on the transaction buy price. We will focus on the case of a buy transaction price, P^b, improving with the network size, $\frac{dP^b}{dN} < 0$. As both the speed of execution and P^b improve with N, the value of being the owner and, consequently, the seller of the bond must decline with N, $\frac{dV^o}{dN} = \frac{\kappa}{r+\kappa}\frac{dV^s}{dN} < 0$, but not faster

[7] It is straightforward to derive the corresponding results when V^r monotonically decreases with N.

than the combined value improvement from increasing N

$$\left|\frac{dV^o}{dN}\right| \leq \frac{rV^b}{\lambda N^2} - \frac{dP^b}{dN},\ N \in [1, N^{**}]. \tag{25}$$

Using relation (10) the sufficient condition for $\frac{dV^s}{dN} < 0$ can be written as

$$V^s + \frac{\lambda N^2}{r}\left(\frac{dP^s}{dN} + \frac{\eta}{r+\eta}\frac{dV^b}{dN}\right) \leq \frac{C(1-L)}{r}. \tag{26}$$

Inequality (26) implies that in order for the seller's value to be decreasing with N, V^s plus the present value of all future marginal increases in V^b with N is no greater than the value of holding the discounted asset net of the present value of all marginal sale price declines from increasing the size of the network. Finally, by differentiating equation (15) with respect to N we obtain the necessary condition for P^b to be improving with N

$$\left|\frac{dU^o}{dN}\right| < \frac{w}{1-w}\left(\frac{dV^b}{dN} - \frac{dV^o}{dN}\right), \tag{27}$$

where we have used that $\frac{dU^o}{dN} < 0$. We conjecture that there exists an interval of network sizes belonging to $[1, N^*]$, on which $\frac{dV^o}{dN} < 0$ and the inequality (27) is satisfied for a range of values of model's primitives. While this conjecture cannot be proven analytically, it is clear that when dealers have high bargaining power, w is close but not equal to 1, the right hand side of (27) can be made larger than its left hand side. Therefore, the optimal network size $N^{**} > 1$, and consequently $N^* \geq N^{**}$, exists and P^b improves with $N \in [1, N^{**}]$ if inequalities (25-27) are simultaneously satisfied.

It then follows from the expression (16) that the sell price must decline with the network size, $\frac{dP^s}{dN} = (\frac{dV^s}{dN} - \frac{\eta}{r+\eta}\frac{dV^b}{dN})w + \frac{dU^{no}}{dN}(1-w) < 0$, as U^{no} is monotonically decreasing with N and V^b is monotonically increasing with N on $[1, N^{**})$. As a result the buyer's value is maximized by trading off more frequent buys at a discounted buy price against more frequent sells at dealer marked up sell price. The effect of N on buy and sell transaction prices is not, however, symmetric due to the time lag between each buy and sell as well as the different effect of relations on P^b and P^s. Therefore, instead of focusing on the individual transaction prices, we investigate the sign of the marginal effect of increasing the network size on the bid-ask spread $SP \equiv P^b - P^s$

$$\frac{dSP}{dN} = -w\left(\frac{r}{r+\kappa}\frac{dV^s}{dN} + \frac{r}{r+\eta}\frac{dV^b}{dN}\right) - (1-w)\left(\frac{dU^o}{dN} + \frac{dU^{no}}{dN}\right). \tag{28}$$

For the network sizes below the trade-specific optimal network size N^{**} we must have that $\frac{dV^b}{dN} > 0$ while the sign of $\frac{dV^s}{dN}$ can be either positive (when $\frac{dP^s}{dN} > 0$) or negative (when $\frac{dP^s}{dN} < 0$). Both values dealers derive from long-term relations with clients monotonically decline with the size of the network, $\frac{dU^{o,no}}{dN} < 0$. Therefore the bid-ask spread improves with the size of the network as long as the following inequality is satisfied

$$\frac{r}{r+\eta}\frac{dV^b}{dN} > -\frac{r}{r+\kappa}\frac{dV^s}{dN} - \frac{1-w}{w}\left(\frac{dU^o}{dN} + \frac{dU^{no}}{dN}\right). \tag{29}$$

At $N = N^*$ we have that $\frac{dV^b}{dN}|_{N=N^*} < 0$ thus implying that $\frac{dSP}{dN}|_{N=N^*} > 0$ as long as $\frac{dV^s}{dN}|_{N=N^*} \leq 0$. This implies that $\frac{dSP}{dN}$ switches its sign from negative to positive, i.e., the inequality (29) is violated, at some $\widetilde{N} < N^*$. Likewise, at $N = N^{**}$ we have that $\frac{dV^b}{dN}|_{N=N^{**}} = 0$ thus leading to $\frac{dSP}{dN}|_{N=N^{**}} > 0$ as long as $\frac{dV^s}{dN}|_{N=N^{**}} \leq 0$, thus implying that the sign of $\frac{dSP}{dN}$ switches from negative to positive at at some $\widetilde{N} < N^{**} < N^*$.

A finite optimal size of the trade-specific network follows from several important trade-offs. When a client adds another dealer to her network the speed of execution, measured by $\frac{\lambda N}{r+\lambda N}$, improves. The additional dealer also improves the bid-ask spread but the improvement declines with the size of the network. As the size of the network becomes large, the value of the relationship, shared by more dealers, declines by so much that dealers have to compensate by charging higher markups and the bid-ask spread starts to widen. Therefore, the spread incorporates the indirect effects additional dealers have on both client and dealer reservation values. At this point the client starts trading off the benefit due to increased execution speed against the cost due to wider bid-ask spread when deciding to add another dealer to the network. The optimal size of the network is set when the benefits from transacting with the larger number of dealers equal the costs. When the relation-specific non-traded value, $V^r(\eta, N)$, is added the client is willing to accept even wider bid-ask spreads in exchange for greater non-trade-relation-specific benefits. However, as marginal non-traded value improvement from a larger network declines with the size of the network, the overall optimal network size N^* is reached when relation specific benefits together with benefits from transacting faster are balanced by the wider bid-ask spread.

4.2 Empirical predictions

While there is no client heterogeneity in the model, each client creates her own access to the interdealer market and the interdealer market can link clients together. Therefore, the model's empirical predictions can be considered via comparative statics with respect to the client's trading intensity, η. Based on our empirical findings, we examine how larger trading intensity impacts clients' choice of network size and the transaction prices they receive.

We first discuss the effect of η on transaction prices while keeping the size of the network fixed. When $\eta = 0$ a client currently owning the bond will never be an owner again after she sells. Therefore, all of the dealers' surplus comes from the price at which the sale occurs and none from future trades. In this case, a dealer's buy reservation value is lower, resulting in a wider bid-ask spread. When η is large, dealers derive significant value from repeated trades, leading to smaller bid-ask spreads. This implies that, conditional on network size, clients with greater trading intensity (higher η) get better execution than clients with smaller trading intensity (lower η). This is consistent with the empirical findings in Table 2 that insurers' trading activity increases in insurer size and that trading costs decline in insurer size in Table 5 .

Increasing the network size increases buyers' valuation, $\widehat{V}^b$, through improved speed of execution,[8] greater inter-temporal dealer competition,[9] and increased relation-specific (non-trading) value. The optimal network size, N^*, as a function of η balances these gains against lower buyer's valuation due to the intra-temporal dealer competition leading to the loss of repeat trading business.

There exist a set of model parameters for which the gains from a larger network increase faster in η than the losses: $\frac{\partial^2 \widehat{V}^b}{\partial N \partial \eta}|_{N=N^*} < 0$. In these cases, the optimal network size is increasing in η. Intuitively, dealers' profit from repeated trades improves with the increasing trading intensity and dealers offer better execution. Clients with greater trading intensity benefit from a larger network which improves the speed of execution and may generate larger non-trading relation-specific value. Therefore, consistent with Table 3, the model can generate larger insurers with more frequent trading needs having larger dealer networks. Ultimately, the model must be structurally estimated to test its ability to fit the distribution of network sizes from Figure 3 as well as execution costs as a function of the network size from Table 5.

There exist several explanations for why large active insurers receive better prices than small inactive insurers in Table 5. A structural estimation helps to uncover which mechanism is more important in the data. One explanation for the price improvement is that active insurers have more repeat trade business with the dealers who internalize the benefits from future business and grant price improvements. An alternative, complementary, explanation is that large insurers have higher bargaining power than small insurers vis-a-vis the dealers. A third explanation is that insurers' trading activity in corporate bonds is correlated with the value of other businesses they conduct with the dealers. A structural estimation can

[8]Speed is proportional to $\widehat{V}^b$ which is increasing with η.

[9]The buy transaction price may or may not be improving with N, $\frac{dP^b}{dN} \gtrless 0$. However, the cross derivative $\frac{\partial^2 P^b}{\partial N \partial \eta}$ is positive.

quantify the importance of each channel as well as the non-trading benefits of relations. We structurally estimate the model in the next section.

4.3 Model estimation

Settings such as labor markets and marriages markets involve one-to-one matching. These literatures structurally estimate models to examine their quantitative fit to the data (for example, Poste-Vinay and Robin (2002) and Eckstein and Van den Berg (2007) for labor search and Hitsch et al. (2010) and Choo (2015) for marriage search). Gavazza (2016) estimates a one-to-one search-and-bargaining model of a decentralized market using aircraft transaction data. We follow this approach for our one-to-many network formation model.[10] An interesting empirical finding in Table 5 is that trading costs are non-monotone in network size. While the model can produce such non-monotonicity, it is unclear if it can quantitatively match the empirical relationship between network size and trading costs.

Figures 2 and 3 present insurers' heterogeneous trading intensities and network sizes. To estimate the model's distribution of network sizes we infer the distribution of trading intensities, η_n, $n = 1, ..., \mathcal{N}$, across insurance companies n. Section 2.1 characterizes the compound distribution of trading activity. If trading shocks occur at Poisson times, the intensity η_n of the shocks can be estimated by the expected number of buy trades per year. This yields the maximum likelihood estimator $\widehat{\eta_n} = \frac{1}{T}\sum_{t=1}^{T} X_{nt}$, where X_{nt} is the number of bond purchases by insurer n in year t. To utilize the multiple years of trade data, we perform the estimation separately for each insurer. This yields a cross-sectional distribution of trading intensities, which we index by $p(\eta)$. Section 2.1 and Figure 2 show the distribution of the insurer trades per year follows approximately a power law. For insurer buys, this distribution is best described by $p(\eta) = 0.34 \times \eta^{-1.31}$.

The other model parameters $\Theta = (L, K, \kappa, \lambda, w^s, w^b)$ are not directly observable in the data, so they are estimated structurally. Section 2.2 and Figure 3 show the degree distribution of client-dealer relations follows a mixed power-exponential law. The distance between the empirical distribution and the model-generated network sizes N^* depends on the parameters Θ. We fit this by minimizing the probability-weighted distance between the data and model:

$$\sum_{N=1}^{max(N)} p(N)[N - N^*(\Theta, \eta(p(N)))]^2, \tag{30}$$

[10] The identification of the model shares several similarities with Gavazza (2016). As in Gavazza (2016) the identification of unobserved parameters relies on key moments in the data. Unlike the aggregate moments in Gavazza (2016), we utilize heterogeneity in insurers' trading intensities and networks to facilitate our estimation.

where $p(N)$ is the empirical network-size distribution and $N^*(\Theta, \eta)$ is the model-implied network size for the set of parameters Θ given η. Inverting the power law distribution for trading intensities from Panel A of Figure 2 yields $\eta(p) = (\frac{p}{.34})^{-\frac{1}{1.31}}$. It then follows from equation (1) that for insurer buys $p(N) = .28e^{-.22N}N^{-0.12}$. Therefore, by substituting $p(N)$ into the expression for $\eta(p)$ we obtain the mapping of trading intensities into the network sizes, $\eta(p(N)) = (\frac{.28}{.34})^{-\frac{1}{1.31}} e^{\frac{.22}{1.31}N} N^{\frac{0.12}{1.31}}$. We use this relation in (30).

The model also predicts how percentage trading costs, $(P^b - P^s)/.5(P^b + P^s)$, depend on the parameters Θ and η. Empirically, the estimated relation between trading costs and network size is in column (4) of Table 5 as $c(N) = 51 + 0.32N - 6.29\, lnN$. The probability-weighted distance between the empirical and the model-generated trading costs c^* depends on the parameters Θ and is:[11]

$$\sum_{N=1}^{max(N)} p(N)[c(N) - c^*(\Theta, \eta(p(N)))]^2. \tag{31}$$

A minimum-distance estimator chooses the model parameters to minimizes the sum of the distances (30) and (31). To fit the data the model's optimal N^* is constrained to be an integer. This makes the the objective functions (30) and (31) non-smooth. To accommodate this we optimize using simulated annealing with fast temperature decay and fast reannealing.

To ensure that the estimated parameters remain in their natural domains, i.e., positive or between zero and one, in the estimation we transform model parameters as follows:

$$\begin{aligned} L = \Phi(\theta_L) \in (0,1), \quad K = e^{\theta_K} \geq 0, \quad \kappa = e^{\theta_\kappa} \geq 0, \quad \lambda = e^{\theta_\lambda} \geq 0, \\ w^s = \Phi(\theta^0_{w^s} + \theta^1_{w^s} ln\, \eta) \in (0,1), \quad w^b = \Phi(\theta^0_{w^b} + \theta^1_{w^b} ln\, \eta) \in (0,1), \end{aligned} \tag{32}$$

where $\Phi \in (0,1)$ is the normal cdf. Our results are robust to alternative transformations in place of Φ and e.

As discussed above, insurers' trading shocks are heterogeneous and directly observable in the data. In the model κ measures clients' selling intensity after having bought. Because the empirical number of buys and sells are equal, and buys and sells follow a similar power law, there is not evidence of heterogeneity in κ. Therefore, κ does not vary across insurers. Because we fit the model to the two observable outcomes of network size and trading costs we limit the number of model parameters that vary across insurers by assuming L and K are constant. The competitive interdealer spread is set to match the average execution costs

[11] The model is solved numerically. Figure 3 shows there are many fewer observed number of network sizes than trading intensities. Therefore, fitting the model using the distribution of trading intensities is substantially slower than using the distribution of the number of dealers. The downside of using the number of dealers is that each dealer network size corresponds to a range of trading intensities.

Table 6: Estimated model parameters

The table reports the estimated model parameters $\theta = (\theta_L, \theta_K, \theta^j_{w^i}, \theta_\kappa, \theta_\lambda, \theta_V)$ in Panel A. Estimates are from the minimum-distance estimation. Standard errors are reported in parenthesis and scaled by 100. Panel B reports the implied values for the model parameters $K = e^{10*\theta_K}$, $L = \Phi(\theta_L)$, $\kappa = e^{10*\theta_\kappa}$, $\lambda = e^{10*\theta_\lambda}$, $w^i = \Phi(\theta^0_{w^i} + \theta^1_{w^i} ln\,\eta)$ for $i = s, b$, where $\Phi \in (0,1)$ is the normal cdf, and non-trade value $V^r = e^{\theta^0_V}\eta^{\theta^1_V}N^{\theta^2_V}$.

	(1) w^b, w^s constant	(2) $w^b(\eta), w^s(\eta)$
Panel A: Parameter estimates		
θ_K	-1.23 (0.50)	-0.65 (0.04)
θ_L	5.02 (43.48)	4.18 (0.98)
θ_κ	0.26 (0.01)	0.30 (0.15)
θ_λ	0.60 (0.38)	0.51 (0.10)
$\theta^0_{w^b}$	2.99 (0.68)	2.55 (0.18)
$\theta^1_{w^b}$		-1.09 (0.24)
$\theta^0_{w^s}$	1.93 (0.94)	1.32 (0.23)
$\theta^1_{w^s}$		0.02 (0.11)
θ^0_V	-3.61 (0.80)	-1.75 (0.19)
θ^1_V	0.09 (0.76)	0.30 (0.20)
θ^2_V	-0.51 (0.93)	0.15 (0.19)
Minimum distance	1,729.80	27.03
S.D. residuals	4.68	0.58
S.D. residuals network	1.35	0.40
S.D. residuals prices	4.48	0.42
Panel B: Implied model parameters		
K*1,000	0.01	1.56
L	1.00	1.00
κ	13.51	19.95
λ	384.99	165.88
w^b (S.D.)	0.98 (0.00)	0.85 (0.05)
w^s (S.D.)	0.96 (0.00)	0.89 (0.01)
Non-trade value V^r (S.D.)	0.02 (0.00)	0.30 (0.02)

of insurers by choosing $C = 1$, $r = .1$, $M^{ask} = (1 + 0.0018)\frac{C}{2r}$, and $M^{bid} = (1 - 0.0018)\frac{C}{2r}$. Because insurers can potentially access all dealers λ is constant across insurers.

The evidence in Table 5 shows trading costs depend on insurer size even after controlling for network size. The most straightforward way to accommodate this is to allow dealer bargaining power to vary across insurers, e.g., large active insurers have higher effective bargaining power than small inactive insurers. We report the model fit under both constant (specification 1) and variable (specification 2) bargaining power by holding w^b and w^s constant and allowing w^b and w^s to be functions of η, respectively.

Finally, we set the functional form of the non-trade value to be Cobb-Douglas

$$V^r = e^{\theta_V^0} \eta^{\theta_V^1} N^{\theta_V^2}, \ \theta_V^1 + \theta_V^2 < 1, \tag{33}$$

with a constant "technology" $e^{\theta_V^0}$ and elasticities to trading intensity and network size given by θ_V^1 and θ_V^2 respectively. The Cobb-Douglas functional form captures the complementarity between trading intensity, measured by η, and network size, while precluding infinitely large non-trade value.

The model can be solved assuming the client maximizes her value as a buyer or as a seller. To simplify exposition we use her value as a buyer. Panel A of Table 6 reports the estimates for the transformed model parameters from (32). Panel B provides the original model parameters $\Theta = (L, K, \kappa, \lambda, \ w^s, w^b)$. All parameters are significantly different from zero and appear well identified as the standard deviations of the residuals are small. Specification 2 allows bargaining power to vary across insurers and generates a substantially lower minimum distance between the model and empirical distributions than does the uniform bargaining power in specification 1.

The estimated liquidity shock parameter L in both specifications is 100% of the flow income from the bond, suggesting a high willingness to pay for immediacy. The cost K of obtaining quotes for each transaction is small, 0.01 (specification 1) and 1.56 (specification 2) basis points per trade, respectively. The estimated selling shock intensity κ is between 13.51 (specification 1) and 19.95 (specification 2), a holding period from two and a half to four weeks. Longer holding periods, smaller κ, reduce the value of repeat relations and increase network size. Without heterogeneity in κ the model needs relatively short holding periods to reproduce the large fraction of insurers using a single dealer.

The dealers' search efficiency λ is estimated to be 385 in specification 1, corresponding to dealers taking two-thirds of a day $(250/\lambda)$ to locate a bond. The dealers' search efficiency λ is estimated to be 166 (1.5 days) in specification 2, which allows bargaining power to depend on η. Given overall corporate bond trading frequencies, these estimates seem reasonable.

Dealers' bargaining power on the buy and the sell sides in specification 1 is large, 98% and 96%, suggesting that dealers capture most of the trade surplus in this specification. In specification 2, when bargaining power depends on insurers' type, dealers' average bargaining power remains large and fairly symmetric across buys, 85%, and sells, 89%. Dealers bargaining power on sales is relatively insensitive to insurer trading frequency. In contrast, $\theta_{w^b}^1 = -1.09$ indicates that dealers' bargaining power when buying declines significantly with insurers trading intensity η.[12] Dealers' bargaining power with insurers with trading intensity

[12]The asymmetry in buy-sell bargaining power's sensitivity to η arises from our choice to maximum the

$\eta = 1$ is close to one. As insurer trading intensity increase to five, dealers' bargaining power falls to about one half. Dealers have almost zero bargaining power when the largest insurers are buying. This could arise from insurers being buy-and-hold investors whereby a dealer can more easily locate a seller than a buyer. The heterogeneity in bargaining across insurers and across buy and sell transactions suggests richer modeling of the price setting process may enable deeper understanding of OTC trading.

The estimated non-trade value, V^r, is quite small in specification 1. The estimated non-trade value, V^r, is fifteen times larger in specification 2 and it is monotonically increasing with the size of the network and trading intensity. The decreasing returns to scale parameter restriction is satisfied. The trade intensity elasticity is twice the network size elasticity.

Figures 4 and 5 visually examine the quality of the model's fit. They each provide four plots illustrating the fits for their respective specification. The top left graph plots the number of dealers as a function of their trading intensity both in the data (circles) and in the model (dashed line). Similar to Figure 3, the top right graph plots the degree distribution for insurer-dealer relations in the data (circles) compared to the model-implied distribution under the estimated parameters (dashed line). In both cases the distance between the two lines captures the model fit. To further visualize the goodness of model's fit, the bottom left graph plots the degree distribution for insurer-dealer relations in the data against the model-implied distribution under the estimated parameters. Each circle is labeled with the corresponding network size. The deviation from the 45-degree line measures model fit.

The specification with uniform dealer bargaining power fits the network distribution up to 16 dealers. In the bottom left graph in Figure 4 the blue circles are on the 45-degree line for all values of N less than 16. However, the goodness of its fit deteriorates for larger networks and the model does not generate networks larger than 22 dealers.

The bottom right corner graph in Figure 4 plots the empirical relation between trading costs and network size from column (4) of Table 5 (circles), $c(N^*) = 51 + 0.32N^* - 6.29\, lnN^*$, against its model-implied counterpart (dashed line).[13] The distance between the two lines is a measure for model fit, showing the parameter estimates from specification 1 do not well describe the relationship between network sizes and trading costs in the data. The model's relation is too weak as the line is too flat. In addition, the non-monotonicity in the relation is barely visible. This suggests that uniform bargaining power limits the variation in the benefits and costs of having a larger network.

In the model network size impacts trading costs, insurers with different η choose different

clients' valuation as a buyer.

[13]Because Table 5 it estimated on individual transactions and contains many control variables, it is simpler to plot the functional relationship rather than some other transformation of the underlying data.

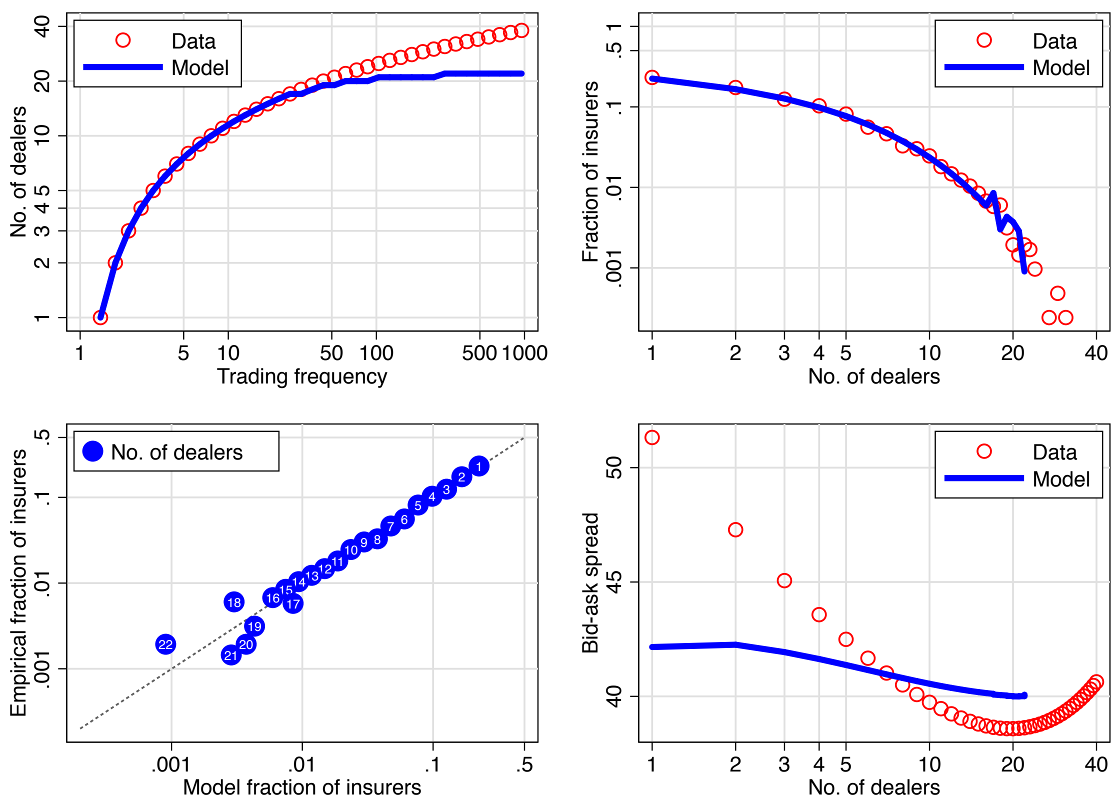


Figure 4: Model fit with constant bargaining power w^b, w^s

The figure shows the model fit for the base specification of constant dealers' bargaining power w. The top left plot shows the network size of insurer-dealer relations for insurer buys as a function of the insurers' trading frequency in the data (circles) compared to the model-implied distribution under the estimated parameters (solid line). The top right plot shows the degree distribution for network size in the data (circles) compared to the model-implied distribution under the estimated parameters (solid line). The bottom left plot compares the empirical probability of a given network size to the model-implied probability. The bottom right plot shows the bid-ask spread in the data as a function of the insurers' network size in the data (circles) compared to the model-implied bid-ask spread under the estimated parameters (solid line).

$N^*(\eta)$, and η impacts trading costs independent of the network size. Understanding how these interact in the model provides insight into why the model with uniform bargaining power weakly fits the relationship between network sizes and trading costs. The spread can be written as $SP(\eta(N^*), N^*)$. Its full derivative with respect of N^* is

$$\frac{dSP}{dN^*} = \frac{\partial SP}{\partial N^*} + \frac{\partial SP}{\partial \eta}\frac{d\eta}{dN^*}. \tag{34}$$

The top left graph in Figure 4 is the inverse of $\frac{d\eta}{dN^*}$. The top right graph in Figure 4 weighs this by the observed empirical distribution of trading intensities. Therefore, fitting the model requires fitting $\frac{d\eta}{dN^*}$ and $\frac{dSP}{dN^*}$, and $\frac{dSP}{dN^*}$ depends on $\frac{d\eta}{dN^*}$. The model determines $\frac{\partial SP}{\partial N^*}$ and $\frac{\partial SP}{\partial \eta}$.

The first term in (34) corresponds to the derivative of spread with respect to optimal network size in the model. As discussed earlier, at the optimum this derivative is positive. The second term in (34) corresponds to the effect of insurers with different η choosing different N^* and η impacting trading costs independent of the network size. The value of future business causes the bid-ask spread to improve with η, $\frac{\partial SP}{\partial \eta} < 0$, and the optimal network size is a non-decreasing function of η, $\frac{d\eta}{dN^*} \geq 0$, the second term in (34) is always negative. The opposing signs of the two terms explain how the model can produce the non-monotonic relationship between network size and trading costs in the data. To fit the trading cost-network size relationship in Table 5 the full derivative in (34) must be initially negative and then become positive at 22 dealers. The slope of the line for the model with uniform bargaining power in neither initially negative enough nor does it increase noticeably enough at 22.

The log-log plot in the top left of Figure 4 shows that fitting $\frac{d\eta}{dN^*}$ requires that $N^* = 1$ for a large range of η. If dealers place a high value on repeat business then spreads are very sensitive to increasing the number of dealers. In the model high dealer bargaining power, w, increases the value of repeat business. Therefore, the fit in specification 1 matches the large fraction of insurers with less than five dealers by setting the dealers' bargaining power close to one. Unfortunately, this same mechanism makes it costly to have larger dealer networks, so the model produces too few insurers with dealer networks larger than 16 and no networks larger than 22. Finally, high dealer bargaining power makes the relationship weak between spreads and network size. If dealer bargaining power declines with insurer trading intensity then larger networks may be optimal and the relationship between network size and trading costs may be larger. Specification 2 allows for bargaining power to vary with η.

The model fit improves drastically when dealers' bargaining power depends on η. All four plots in Figure 5 show a close correspondence between the model and data. The graphs

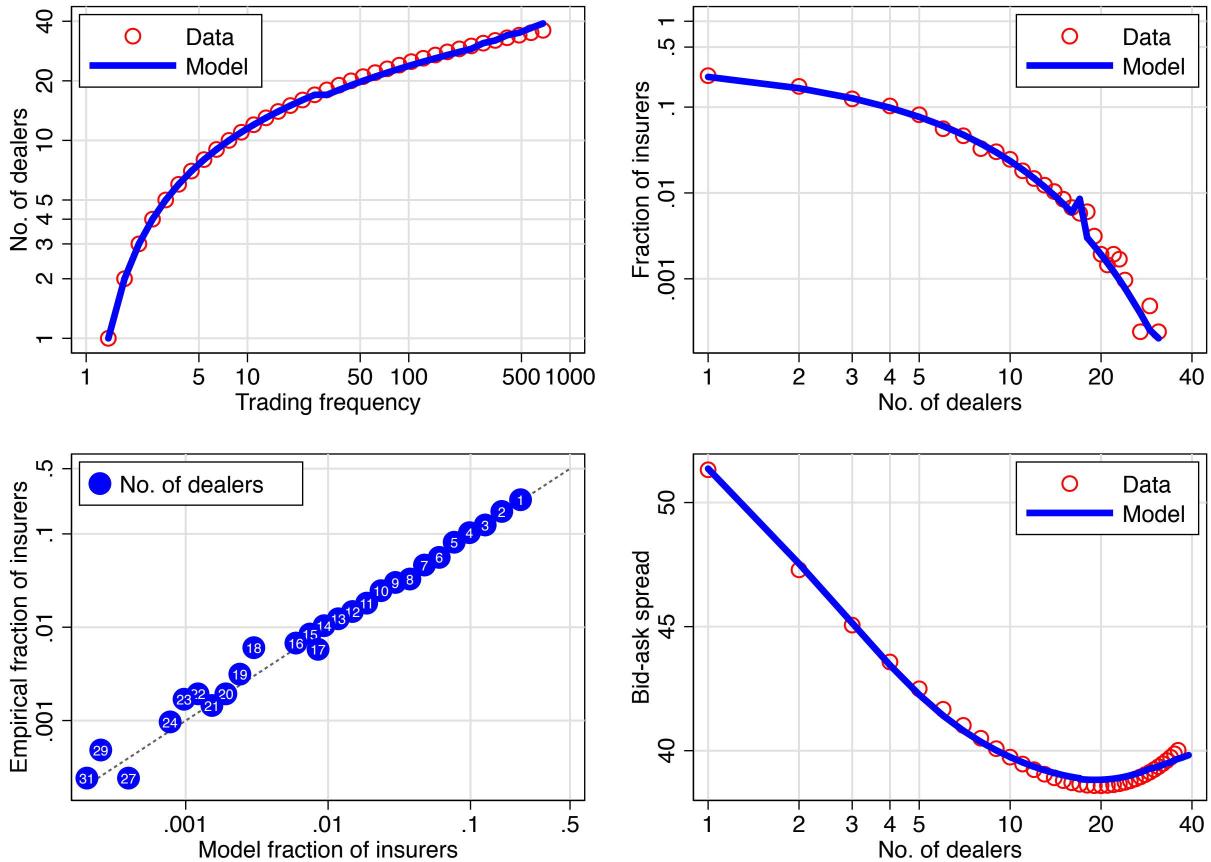
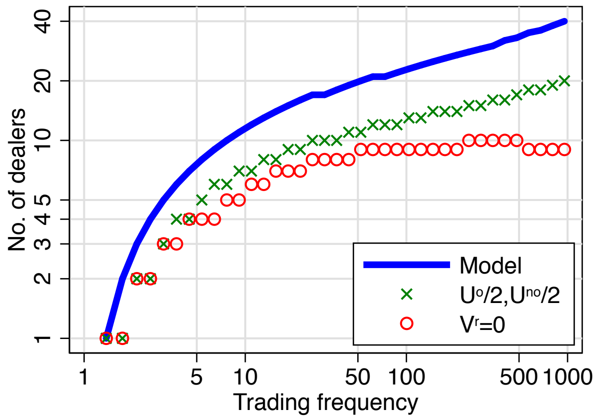


Figure 5: Model fit with client-specific bargaining power $w^b(\eta), w^s(\eta)$
The figure shows the model fit when dealers' bargaining power is a function of trading intensity, $w^b(\eta)$ and $w^s(\eta)$. Please see caption of Figure 4 for further details.

show network sizes greater than 22 and the U-shape in the bid-ask spread starting at 20 dealers. In this specification dealers' bargaining power is greater than 0.8 for insurers with trading intensity less than five.[14] Dealer bargaining power for insurer buys is about 0.5 when trading intensity approaches 10. When η exceeds 30 dealer bargaining power for insurer buys is small. This enables the model to produce large network sizes for insurers with large η as the value dealers place on repeat business increases with η. In addition, bargaining power varying with η has a direct effect on trading costs. Larger insurers have larger bargaining power, lowering their trading costs. This strengthens the relationship between trading costs and network sizes.

To summarize, the theoretical model with the dealers' bargaining power declining with a trading intensity η fits the data well. In particular, it is important to have the dealer's

[14]Figure 2 shows that the majority of insurers have trading intensity less than five.



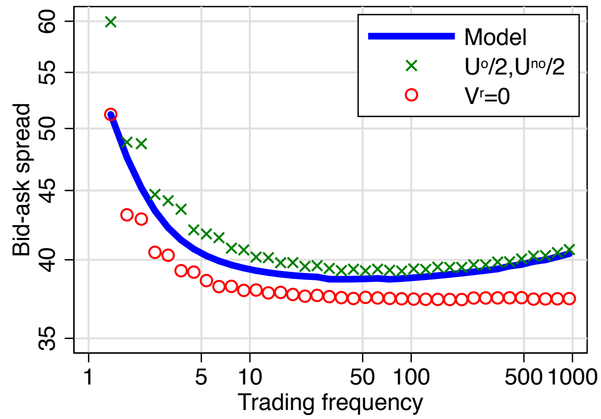


Figure 6: Counterfactual analysis
The left plot shows the insurer-dealer network size as a function of the insurers' trading frequency, η. The right plot shows the bid-ask spread as a function of η. Each plot shows the corresponding variable in under the estimated parameters from specification 2 in Table 6 (solid line) compared to the counterfactuals that would arise if: (i) dealers have suffered a permanent loss of 50% of the repeat trade business (crosses); (ii) the non-trade relationship value is zero (circles). We use log-log scale.

bargaining power on the ask side to depend on η. That is, small inactive insurers have very weak outside options both when they buy and sell. By contrast, large active insurers have strong outside options when they buy and weak bargaining power when they sell.

The parameter estimates can help quantify the value of both the repeat client-dealer trading and not-trade value V^r. We use the model and estimated parameters to construct counterfactuals to capture the impact of repeat trade business on client-dealer networks and prices. We compute counterfactual networks and bid-ask prices under the assumption that the dealers give only partial price improvement for the future repeat trading from the same insurer. In this case, the search and bargaining proceeds in the same way, but the dealers in the network are implicitly re-chosen with probability $\frac{1}{2}$ after every trade. This is counterfactual scenario 1. In this counterfactual scenario, the bid and ask price equations in the model are adjusted by dividing both U^o and U^{no} in (15) and (16) by two. To capture the impact of V^r on networks and transaction prices we compute counterfactual networks and bid-ask prices under the assumption that clients fully benefit from the repeat trading but receive no relation-specific value from dealers, $V^r = 0$. This is counterfactual scenario 2.

The left plot in Figure 6 corresponds to the top left plot in Figure 5 and shows the optimal network sizes, N^*. Because the network sizes change under the counterfactuals, the right plot in Figure 6 shows the bid-ask spread as a function of trading intensity, η. Both graphs in Figure 6 use the estimated parameters from specification 2 in Table 6. Each graph shows the results from specification 2 (solid line), counterfactual 1 (crosses), and counterfactual 2 (circles).

Figure 6 highlights the fundamental trade-offs the insurer faces in our model. When dealers' repeat trade benefits are reduced (counterfactual 1) dealers charge wider bid-ask spreads. Dealers in smaller networks lose more repeat trade surplus than dealers in large networks because each dealer's per-trade loss is scaled by the network size. Smaller insurers with lower trading intensities have smaller networks. Consequently, the widening in bid-ask spread is slightly larger for insurers with lower trading intensities than for insurers who trade very frequently. Both large (high η) and small (low η) insurers respond to wider bid-ask spread by reducing the size of their networks. The magnitude of this effect is much smaller for insurers with low η as they already have small networks.

When the non-trade value V^r is set to zero in counterfactual 2 insurers reduce their network sizes. Because V^r is larger for insurers with larger trading intensities, they reduce the size of their networks more than insurers with low η. Dealers respond to the network size reduction by charging better transaction prices thus leading to narrower bid-ask spreads for insurers of all η-types. The magnitude of the bid-ask spread improvement is larger(smaller) for insurers with high(low) η since they reduce the size their network by more(less).

The U-shape in the bid-ask spread is not present without the non-trade relationship value V^r. This highlights the role V^r increasing in N^* plays in the non-monotonicity of relationship between network size and the bid-ask spread. As V^r increases with N^*, insurers have an incentive to choose larger networks than in the absence of the non-trade value because larger transaction costs are offset by the non-trade value. Because V^r increases with η as well, this incentive is especially strong for insurers with very large trading intensities and large networks, thus leading to the U-shape in the bid-ask spread for large networks.

5 Conclusion

Over-the-counter markets are pervasive across asset classes and often considered poorly functioning due to lack of transparency and fragmented trading imposing search frictions. Regulators have attempted to address these concerns through increases in transparency and the implementation of Dodd-Frank legislation. Competitors have entered with more centralized markets. Changes in regulations and market structure will impact heterogenous investors differently. However, there is limited theory closely linked to empirical work to guide these decisions.

We use comprehensive regulatory corporate bond trading data for all U.S. insurance companies to study how investors choose their size of trading networks and how this impacts prices they receive. We document that 30% of insurers only trade with a single dealer each year. However, there exist few clients with very large networks. The overall degree distribution of network sizes is a hybrid of a power law with an exponential tail and it can be closely approximated by the Gamma distribution. Trading costs decline with network size but for networks greater than 20 dealers trading costs begin to increase with the network size. To understand these and several other empirical outcomes we develop a parsimonious model for OTC trading between clients and dealers. Clients trade off the value of repeat relations with dealers against the benefits of competition among dealers. The value of repeat relations declines in the number of dealers as the increased competition erodes the chance to transact. Dealers compensate for losses from repeat business by charging higher spreads. The value of repeat relations diminishes more slowly with the addition of dealers for clients with larger trading intensity as dealers compete for larger repeat business. Therefore, these larger clients use more dealers and get better execution as benefits from having larger repeat business swamp the costs of having larger network. Dealers provide better prices to these larger clients because their repeat business is more valuable. Eventually the costs of having larger network outweigh the benefits and the spread starts to increase with the network size. Using the structurally estimated model parameters we find that small insurers benefit most from repeat relations. This suggests that care is needed in regulations affecting heterogeneous investors.

The paper provides a first step in understanding how investor heterogeneity impacts observed patterns of trading and prices in OTC markets. A number of other important dimensions require further study. First, what is the impact of dealer heterogeneity on network formation and prices? Second, how do long-lasting client-dealer relations form and severe? Third, is Nash bargaining a realistic price setting mechanism in OTC markets?

Appendix

Data Filters

Parts 3 and 4 of Schedule D filed with the NAIC contains purchases and sales made during the quarter, except for the last quarter. In the last quarter of each year, insurers file an annual report, in which all transactions during the year are reported. Part 3 of Schedule D reports all long-term bonds and stocks acquired during the year, but not disposed of, while Part 4 of Schedule D reports all long-term bonds and stocks disposed of. In addition, all long-term bonds and stocks acquired during the year and fully disposed of during the current year

Table 7: Data filters

Filter	Full sample	Corp. bonds
1. All trades from original filings (includes all markets, all trades since 2001)	19.1	4.5
2. Remove all trades that do not involve a dealer (e.g., paydown, redemption, mature, correction)	6.6	3.1
3. Remove duplicates, aggregate all trades of the same insurance company in the same CUSIP on the same day with the same dealer	6.5	3.1
4. Map dealer names to SEC CRD number, drop trades without a name match, drop trades with a dealer that trades less than 10 times in total over the sample period	6.1	2.9
5. Drop if not fixed coupon (based on eMaxx data), drop if outstanding amount information is in neither eMaxx nor FISD	5.3	2.5
6. Drop if trade is on a holiday or weekend	5.2	2.5
7. Drop if counterparty is “various”	4.1	2.1
8. Drop trades less than 90 days to maturity or less than 60 days since issuance (i.e., primary market trade)	2.8	1.5
9. Merge with FISD data, keep only securities that are not exchangeable, preferred, convertible, issued by domestic issuer, taxable muni, missing the offering date, offering amount, or maturity, and offering amount is not less than 100K	1.0	1.0

are reported in the special Part 5 of Schedule D. NAIC’s counterparty field reports names in text, which can sometimes be mistakenly typed. The bank with the most variation in spelling is DEUTSCHE BANK. We manually clean the field to account for different spellings of broker-dealer names.

We compile the information in Parts 3, 4, and 5 of Schedule D to obtain a comprehensive set of corporate bond transactions by all insurance companies regulated by NAIC.

We apply various FISD-based data filters based on Ellul et al. (2011) to eliminate outliers and establish a corporate bond universe with complete data. The data filters are in Table 7 which summarizes the number of observations that is affected by each step. We exclude a bond if it is exchangeable, preferred, convertible, MTN, foreign currency denominated, puttable or has a sinking fund. We also exclude CDEB (US Corporate Debentures) bonds, CZ (Corporate Zero) bonds, and all government bond (including municipal bonds) based on the reported industry group. Finally, we also drop a bond if any of the following fields is missing: offering date, offering amount, and maturity. We restrict our sample to bonds with the offering amount greater than \$10 million, as issues smaller than this amount are very illiquid and hence are rarely traded. Ellul et al. (2011) have used \$50,000 which we find restrictive for our purpose. We windsorize the cash-to-asset ratio at 80 percent to remove extreme values.

Proofs

PROPOSITION 1: *Bid and ask prices are*

$$P^b = \left[\frac{C(1-L)}{\lambda N}\Psi_1 + \frac{C}{r+\kappa}\Psi_2 + K\Pi_1(\kappa,\eta)\right] w \tag{35}$$
$$+ \left[M^{ask}\Pi_2 + M^{bid}\Pi_3\frac{\kappa}{r+\kappa}\right](1-w),$$
$$P^s = \left[\frac{C(1-L)}{\lambda N}\Psi_3 - \frac{C}{r+\kappa}\Psi_4 - K\Pi_1(\eta,\kappa)\right] w \tag{36}$$
$$+ \left[M^{bid}\Pi_2 + M^{ask}\Pi_3\frac{\eta}{r+\eta}\right](1-w).$$

where the coefficients Ψ_1*,* Ψ_2*,* Ψ_3*,* Ψ_4*,* $\Pi_1(x,y)$*,* Π_2*, and* Π_3 *are given below in the proof.*

Proof: Define $\Lambda = \lambda N$, $\widetilde{\lambda} = \frac{\lambda}{r+\Lambda}$, $\widetilde{\Lambda} = \frac{\lambda N}{r+\lambda N}$, $\widetilde{\eta} = \frac{\eta}{r+\eta}$, and $\widetilde{\kappa} = \frac{\kappa}{r+\kappa}$. We can rewrite both the client's and the dealer's valuations as

$$\begin{aligned}
V^b &= \frac{C}{r+\kappa}\widetilde{\Lambda} - (P^b+k)\widetilde{\Lambda} + V^s\widetilde{\kappa}\widetilde{\Lambda} = \frac{\widetilde{\Lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[\frac{C(1-l)}{r+\Lambda}\widetilde{\kappa} + \frac{C}{r+\kappa} - (P^b+K) + (P^s-k)\widetilde{\kappa}\widetilde{\Lambda}\right], \\
V^s &= \frac{C(1-l)}{r+\Lambda} + (P^s-k)\widetilde{\Lambda} + V^b\widetilde{\eta}\widetilde{\Lambda} = \frac{\widetilde{\Lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[\frac{C(1-l)}{\Lambda} + \frac{C}{r+\kappa}\widetilde{\eta}\widetilde{\Lambda} + (P^s-K) - (P^b+K)\widetilde{\eta}\widetilde{\Lambda}\right],
\end{aligned}$$

and

$$\begin{aligned}
U^s &= (M^{bid}-P^s)\widetilde{\lambda} + U^b\widetilde{\eta}\widetilde{\Lambda} = \frac{\widetilde{\lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[(M^{bid}-P^s) + (P^b-M^{ask})\widetilde{\eta}\widetilde{\Lambda}\right], \\
U^b &= (P^b-M^{ask})\widetilde{\lambda} + U^s\widetilde{\kappa}\widetilde{\Lambda} = \frac{\widetilde{\lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[(P^b-M^{ask}) + (M^{bid}-P^s)\widetilde{\kappa}\widetilde{\Lambda}\right].
\end{aligned}$$

After some algebra one obtains

$$\begin{aligned}
V^o - V^b &= \frac{C}{r+\kappa} + V^s\widetilde{\kappa} - V^b \\
&= \frac{C}{r+\kappa} + \frac{\widetilde{\Lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[\frac{C(1-l)}{\Lambda}\widetilde{\kappa} - \frac{C(1-l)}{r+\Lambda}\widetilde{\kappa} - \frac{C}{r+\kappa}(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda}) + \right. \\
&\quad \left. +(P^b+K)(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda}) + (P^s-K)\widetilde{\kappa}(1-\widetilde{\Lambda})\right], \\
V^s - V^{no} &= V^s - V^b\widetilde{\eta} \\
&= \frac{\widetilde{\Lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[\frac{C(1-l)}{\Lambda}(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda}) - \frac{C}{r+\kappa}\widetilde{\eta}(1-\widetilde{\Lambda}) + \right. \\
&\quad \left. +(P^b+K)\widetilde{\eta}(1-\widetilde{\Lambda}) + (P^s-K)(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda})\right].
\end{aligned}$$

Substituting these expressions into (15) and (16) yields

$$
\begin{aligned}
P^b &= \frac{C}{r+\kappa}w \\
&+\frac{\widetilde{\Lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[\frac{C(1-l)}{\Lambda}\widetilde{\kappa}-\frac{C(1-l)}{r+\Lambda}\widetilde{\kappa}-\frac{C}{r+\kappa}(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda})+(P^b+K)(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda})+(P^s-K)\widetilde{\kappa}(1-\widetilde{\Lambda})\right]w \\
&+M^{ask}(1-w)-\frac{\widetilde{\lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[(M^{bid}-P^s)+(P^b-M^{ask})\widetilde{\eta}\widetilde{\Lambda}\right]\widetilde{\kappa}(1-w), \\
P^s &= \frac{\widetilde{\Lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[\frac{C(1-l)}{\Lambda}(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda})-\frac{C}{r+\kappa}\widetilde{\eta}(1-\widetilde{\Lambda})+(P^b+K)\widetilde{\eta}(1-\widetilde{\Lambda})+(P^s-K)(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda})\right]w \\
&+M^{bid}(1-w)+\frac{\widetilde{\lambda}}{1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2}\left[(P^b-M^{ask})+(M^{bid}-P^s)\widetilde{\kappa}\widetilde{\Lambda}\right]\widetilde{\eta}(1-w).
\end{aligned}
$$

These expressions can be rewritten as

$$
\begin{aligned}
P^bA_1(w) &= \left[\frac{C(1-l)}{\Lambda}\widetilde{\kappa}\widetilde{\Lambda}+\frac{C}{r+\kappa}\right](1-\widetilde{\Lambda})w+KA_2(\widetilde{\kappa},w) \\
&\quad +M^{ask}A_3(w)-M^{bid}\widetilde{\kappa}\widetilde{\lambda}(1-w)+P^s\widetilde{\kappa}A_4(w) \\
P^sA_1(w) &= \left[\frac{C(1-l)}{\Lambda}(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda})-\frac{C}{r+\kappa}\widetilde{\eta}(1-\widetilde{\Lambda})\right]\widetilde{\Lambda}w-KA_2(\widetilde{\eta},w) \\
&\quad -M^{ask}\widetilde{\eta}\widetilde{\lambda}(1-w)+M^{bid}A_3(w)+P^b\widetilde{\eta}A_4(w),
\end{aligned}
\tag{37}
$$

where we have defined

$$
\begin{aligned}
A_1(w) &\equiv 1-\widetilde{\Lambda}w-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2(1-w)+\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda}\widetilde{\lambda}(1-w), \\
A_2(x,w) &\equiv [1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda}-x(1-\widetilde{\Lambda})]\widetilde{\Lambda}w, \\
A_3(w) &\equiv [1-\widetilde{\kappa}\widetilde{\eta}(\widetilde{\Lambda})^2+\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda}\widetilde{\lambda}](1-w), \\
A_4(w) &\equiv (1-\widetilde{\Lambda})\widetilde{\Lambda}w+\widetilde{\lambda}(1-w), \\
A_5(w) &\equiv A_1(w)-\widetilde{\kappa}\widetilde{\eta}\frac{A_4(w)^2}{A_1(w)}.
\end{aligned}
\tag{38}
$$

The system of equations (37) can be solved to yield expressions (35) and (36), where we have defined

$$
\begin{aligned}
\Psi_1 &\equiv \frac{\widetilde{\kappa}\widetilde{\Lambda}}{A_5(w)}\left[1-\widetilde{\Lambda}+(1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda})\frac{A_4(w)}{A_1(w)}\right], \\
\Psi_2 &\equiv \frac{(1-\widetilde{\Lambda})}{A_5(w)}\left[1-\widetilde{\kappa}\widetilde{\eta}\frac{A_4(w)}{A_1(w)}\widetilde{\Lambda}\right], \\
\Psi_3 &\equiv \frac{\widetilde{\Lambda}}{A_5(w)}\left[1-\widetilde{\kappa}\widetilde{\eta}\widetilde{\Lambda}+\widetilde{\kappa}\widetilde{\eta}(1-\widetilde{\Lambda})\frac{A_4(w)}{A_1(w)}\right], \\
\Psi_4 &\equiv \frac{\widetilde{\eta}(1-\widetilde{\Lambda})}{A_5(w)}\left[\widetilde{\Lambda}-\frac{A_4(w)}{A_1(w)}\right], \\
\Pi_1(x,y) &\equiv \frac{1}{A_5(w)}\left[A_2(x)-xA_2(y)\frac{A_4(w)}{A_1(w)}\right], \\
\Pi_2 &\equiv \frac{1}{A_5(w)}\left[A_3-\widetilde{\kappa}\widetilde{\eta}\frac{A_4(w)}{A_1(w)}\widetilde{\lambda}\right], \\
\Pi_3 &\equiv \frac{1}{A_5(w)}\left[A_3\frac{A_4(w)}{A_1(w)}-\widetilde{\lambda}\right].
\end{aligned}
\tag{39}
$$

PROPOSITION 2: *The optimal size of the insurer's dealer network N^* is given by the following condition*

$$\frac{rV^b}{(\lambda N^*)^2}\left(1+\frac{\kappa}{r+\kappa}\frac{\eta}{r+\eta}\left(\frac{\lambda N^*}{r+\lambda N^*}\right)^2\right)=-\left(\frac{r+\lambda N^*}{\lambda N^*}-\frac{\eta}{r+\eta}\frac{\kappa}{r+\kappa}\frac{\lambda N^*}{r+\lambda N^*}\right)\frac{dV^r}{d\lambda N^*}+ \tag{40}$$
$$+\frac{\kappa}{r+\kappa}\frac{r}{(r+\lambda N^*)^2}\left(\frac{C(1-L)}{r}-P^s+K\right)+\frac{dP^s}{d\lambda N^*}\frac{\kappa}{r+\kappa}\frac{\lambda N^*}{r+\lambda N^*}-\frac{dP^b}{d\lambda N^*}.$$

Proof: Since $\frac{d\widehat{V}^b}{dN}=\frac{dV^b}{dN}+\frac{dV^r}{dN}$ we need to calculate the derivative of V^b with respect to N. We start by rewriting the expression (6) as

$$V^b=\left(\frac{C}{r+\kappa}+\frac{C(1-L)}{r+\lambda N}\frac{\kappa}{r+\kappa}+\left(E[P^s]+V^b\frac{\eta}{r+\eta}-K\right)\frac{\kappa}{r+\kappa}\frac{\lambda N}{r+\lambda N}-P^b-K\right)\frac{\lambda N}{r+\lambda N}, \tag{41}$$

and solve it for V^b to obtain

$$V^b=\frac{\frac{\lambda N}{r+\lambda N}}{1-\frac{\kappa}{r+\kappa}\frac{\eta}{r+\eta}(\frac{\lambda N}{r+\lambda N})^2}\left[\frac{C}{r+\kappa}+\frac{C(1-L)}{r+\lambda N}\frac{\kappa}{r+\kappa}+(P^s-K)\frac{\kappa}{r+\kappa}\frac{\lambda N}{r+\lambda N}-(P^b+K)\right]. \tag{42}$$

Taking into account that

$$\frac{d\left(\frac{\lambda N}{r+\lambda N}\right)}{d\lambda N}=\frac{r}{(r+\lambda N)^2}, \tag{43}$$

we obtain the following expression for the derivative in question

$$\frac{dV^b}{d\lambda N}=\left(\frac{r+\lambda N}{\lambda N}-\frac{\eta}{r+\eta}\frac{\kappa}{r+\kappa}\frac{\lambda N}{r+\lambda N}\right)^{-1}\left[\frac{rV^b}{(\lambda N)^2}\left(1+\frac{\kappa}{r+\kappa}\frac{\eta}{r+\eta}\left(\frac{\lambda N}{r+\lambda N}\right)^2\right)-\right. \tag{44}$$
$$\left.-\frac{\kappa}{r+\kappa}\frac{r}{(r+\lambda N)^2}\left(\frac{C(1-L)}{r}-P^s+K\right)+\frac{dP^s}{d\lambda N}\frac{\kappa}{r+\kappa}\frac{\lambda N}{r+\lambda N}-\frac{dP^b}{d\lambda N}\right], \tag{45}$$

which after setting $\frac{d\widehat{V}^b}{dN}=0$ leads to 40 after some algebra.

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