Derivations for Hidden-Information Model of Section I.D

This is a standard mechanism-design problem (see, e.g., Bolton and Dewatripont, 2005, Chapter 2). In light of the revelation principle (Myerson, 1979), there is no loss of generality in restricting attention to direct revelation mechanisms. Specifically, a bonus plan is a mechanism \(\langle x(\hat{\theta}), b(\hat{\theta})\rangle\), where \(\hat{\theta} \in \{B, G\}\) is the CEO’s announced value of the attribute \(\theta\), \(x: \{B, G\} \rightarrow \mathbb{R}_+\), and \(b: \{B, G\} \rightarrow \mathbb{R}_+\). Given the restriction that payments be non-negative, this problem differs from the standard hidden-information model insofar as the CEO cannot be punished if he announces \(\hat{\theta} / \in \{B, G\}\). In essence, the CEO has the option of sending a null message that corresponds to producing an \(x = 0\) and receiving \(b = 0\). Given the assumptions on the cost function, a null message yields the CEO zero utility.

Any mechanism must satisfy participation constraints (i.e., the CEO cannot do better to send the null message) and truth-telling constraints (i.e., the CEO cannot do better to pretend the attribute is other than it truly is). These constraints are:

\[
\begin{align*}
    b(G) - C(x(G), G) &\geq 0, & (\text{ir}_G) \\
    b(B) - C(x(B), B) &\geq 0, & (\text{ir}_B) \\
    b(G) - C(x(G), G) &\geq b(B) - C(x(B), G), & (\text{ic}_G) \\
    b(B) - C(x(B), B) &\geq b(G) - C(x(G), B), & (\text{ic}_B)
\end{align*}
\]

where \((\text{ir}_\theta)\) is the participation (individual rationality) constraint given the attribute is truly \(\theta\) and \((\text{ic}_\theta)\) is the truth-telling (incentive compatibility) constraint given the attribute is truly \(\theta\).

Define \(V(\theta) = b(\theta) - C(x(\theta), \theta)\); that is, \(V(\theta)\) is the equilibrium utility of the CEO when the attribute is truly \(\theta\). Observe

\[
b(\theta') - C(x(\theta'), \theta) = \left(b(\theta') - C(x(\theta'), \theta')\right) + \left(C(x(\theta'), \theta') - C(x(\theta'), \theta)\right) \\
= \begin{cases} 
    V(\theta') + I(x(\theta')), & \text{if } \theta' = B, \theta = G \\
    V(\theta') - I(x(\theta')), & \text{if } \theta' = G, \theta = B
\end{cases}.
\]
Using (1) the constraints can be rewritten as
\begin{align*}
V(G) &\geq 0, \\
V(B) &\geq 0, \\
V(G) &\geq V(B) + I(x(B)), \text{ and} \\
V(B) &\geq V(G) - I(x(G)).
\end{align*}
(\text{IR}_{G}^{'}) (\text{IR}_{B}^{'}) (\text{IC}_{G}^{'}) (\text{IC}_{B}^{'})

The assumptions made in the text, including 
\( C(0, \theta) = 0 \) for both \( \theta \), imply
\[ \int_{0}^{x} \left( \frac{\partial C(z, B)}{\partial x} - \frac{\partial C(z, G)}{\partial x} \right) dz = C(x, B) - C(x, G) = I(x) \]
if \( x > 0 \). Hence, \( I(x(B)) \geq 0 \), which means that (\text{IC}_{G}^{'}) implies (\text{IR}_{G}^{'}); that is, the latter constraint is slack. Constraints (\text{IC}_{G}^{'}) and (\text{IC}_{B}^{'}) can be combined to yield
\[ I(x(G)) \geq V(G) - V(B) \geq I(x(B)). \]
(2)

Expression (2) implies that \( x(G) \geq x(B) \). Given an \( x(G) \), the owners do best to make \( b(G) \) as small as possible. Hence, the first inequality in (2) will be slack, which means that (\text{IC}_{B}^{'}) is slack. To summarize, the only relevant constraints are (\text{IR}_{B}^{'}) and (\text{IC}_{G}^{'}).

The owners seek to design the mechanism to maximize their expected profit,
\[ \psi \left( R(x(B)) - b(B) \right) + (1 - \psi) \left( R(x(G)) - b(G) \right) \]
subject to the relevant constraints. Substituting these constraints into (3) and using the definition of \( V(\cdot) \), yields the unconstrained program:
\[ \max_{x(B), x(G)} \psi \left( R(x(B)) - C(x(B), B) \right) + (1 - \psi) \left( R(x(G)) - C(x(G), G) - I(x(B)) \right). \]

Rearranging terms, the results in the text follow.

2 Derivations for Hidden-Action Model of Section I.D

The CEO’s expected utility is
\[ (1 - \delta)b_1 + \delta b_0 - \chi(0) \]
if he chooses action \( x = 0 \). It is \( b_1 - \chi(1) \) if he chooses action \( x = 1 \).

He will clearly prefer \( x = 0 \) to \( x = 1 \) if \( b_0 = b_1 \). Among all contracts with that property, the cheapest for the owners is \( b_0 = b_1 = 0 \) (recall negative compensation is not permitted). Hence, the owners will offer \( (0, 0) \) if they wish to induce \( x = 0 \).

If the owners wish to induce \( x = 1 \), then the CEO’s choosing \( x \) is incentive compatible only if
\[ b_1 - \chi(1) \geq (1 - \delta)b_1 + \delta b_0 - \chi(0). \]
Recalling that $\chi(0) = 0$, this can be rewritten as

\[(b_1 - b_0) \geq \frac{\chi(1)}{\delta}. \quad (4)\]

The owners are certain to pay $b_1$ if they induce $x = 1$; hence, the owners want to make $b_1$ as small as possible given (4). Because $b_0 \geq 0$, it is readily seen they should set $b_0 = 0$ and, thus, $b_1 = \chi(1)/\delta$. This is what was given in the text.

**REFERENCES**
