Problem Set #1

Economics 201B

1. Answer the questions posed by footnotes 3–6 on page 16 of "Lecture Notes for Economics." In addition, please do the exercise suggested at the end of the example on page 16.

2. Suppose a profit-maximizing firm faces demand $X(p) = p^{-2}$. Suppose its cost is $C(x) = x$.

   - What is the elasticity of demand.
   - What is the profit-maximizing price to charge under linear pricing?
   - Suppose $X(p) = p^{-\frac{1}{2}}$. What is the profit-maximizing price now? Something's funny about this problem; what is it?

3. Suppose that it cost you s to initiate communications with a friend and your friend r to receive communications. For instance, s could be the price you pay your long-distance carrier to call your friend's mobile phone and r is the price she pays her cellular-phone provider to talk. The benefit *to you* of communicating with your friend is b_Y, while the benefit *to your friend* is b_F. Assume that the cost to connect the two of you is $c > 0$. Regulations ensure that prices just cover cost; that is, $r + s = c$. Assume that regulators must set prices not knowing your benefit or that of your friend. Assume, however, that regulators know person T's, $T \in \{Y, F\}$, benefit, b_T, is distributed $G_T(\cdot)$ on support $\mathcal{T}$, where $\mathcal{T}$ is a closed interval of $\mathbb{R}_+$. Where appropriate, let $g_T(\cdot)$ be the associated density function. Define $T = \max \mathcal{T}$. Assume $Y + F > c$.

   (a) Suppose that $\mathcal{T} = [0, 1]$ for both T and $c < 2$. Prove that the expected-surplus-maximizing prices for the regulators to set are $r = s = c/2$.

   (b) Suppose that $\mathcal{F} = [f, f]$ (*i.e.,* there is no uncertainty about your friend's benefit, it's f). Assume $y = \min \mathcal{Y} < Y$. Prove that if $y < c < f + Y$, then expected-surplus-maximizing prices are $r = \min\{c, f\}$ and $s = c - r$.

   (c) Relate your analysis here to simple monopoly pricing.

4. A firm can produce as many units, x, of a good as it wishes at a cost of cx^2, where $c > 0$ is a constant. Goods can be shipped overseas, where they command a constant price of p_0. They can also be sold in a domestic market, which has an inverse demand of $P(x)$. Assume $P(0) > p_0$. Derive conditions for the number of units that are sold domestically and

the number sold overseas. Why can we think of p_0 as the marginal cost of selling domestically? (That is, what if there were only the domestic market, but the cost function were $p_0 x$?)

5. A firm faces 100 customers, each of whom has inverse demand $10 - x/2$. Marginal cost is $x/200$. What is the optimal two-part tariff for this firm to offer?